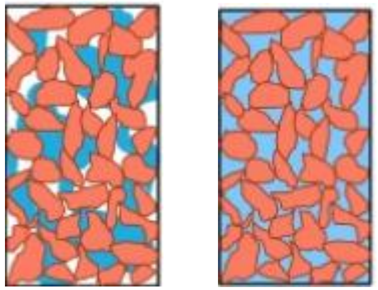
Dr. Khalil Qatu

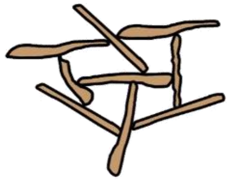
ENCE 331: Permeability

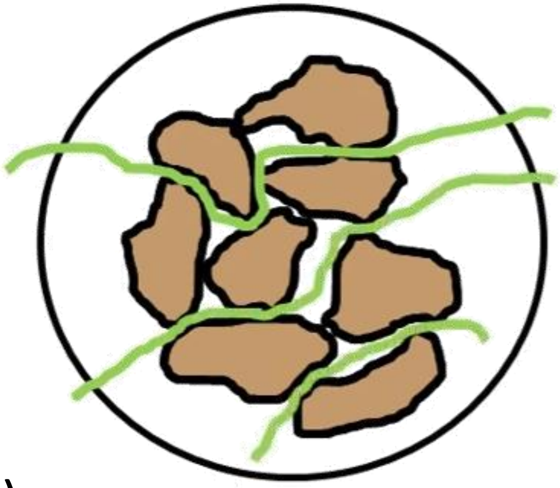
What is permeability ?

- Permeability is the capacity of soil to allow water pass through it.

- Factors affecting permeability:
    - Grain size (D_{10})
    - Void ratio (e)
    - Particle shape (angular vs. spherical)
    - Soil structure
    - Degree of saturation (partially saturated vs. fully saturated)
    - Adsorbed water
    - Stratification of soil
    - Fluid viscosity
    - Temperature







Irregular and narrower

Regular and open

Water flow through soil

$$h = \frac{u}{\gamma_w} + \frac{v^2}{2g} + Z$$

~zero

↑ Pressure head ↑ Velocity head ↑ Elevation head

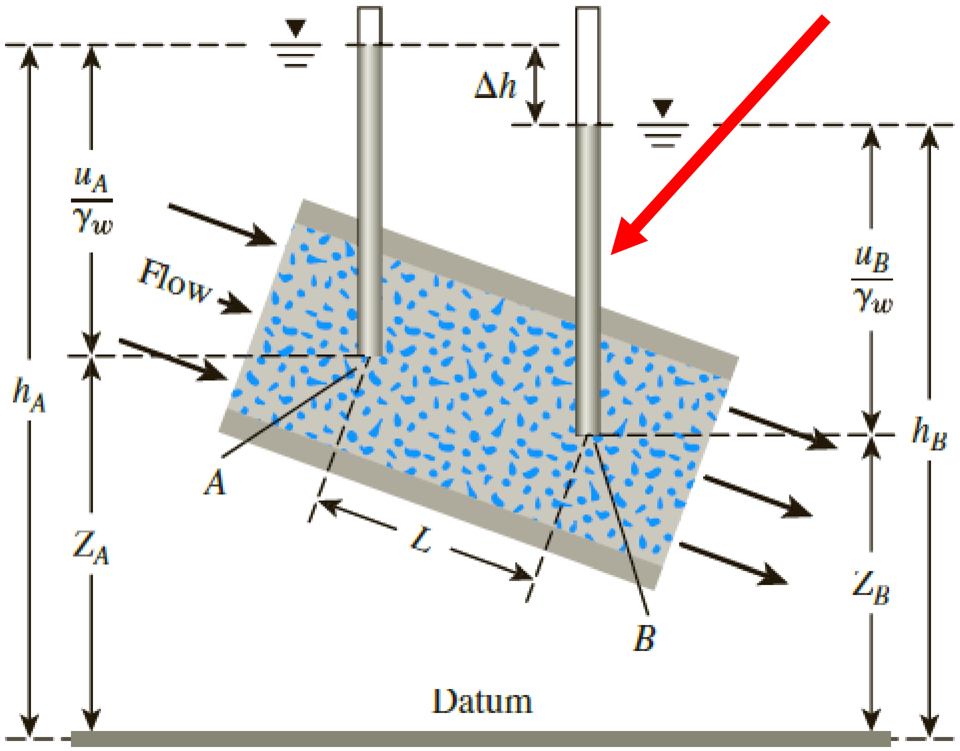
units

$$\Delta h = h_A - h_B = \left(\frac{u_A}{\gamma_w} + Z_A\right) - \left(\frac{u_B}{\gamma_w} + Z_B\right)$$

$$i = \frac{\Delta h}{L}$$

Hydraulic gradient

Piezometer



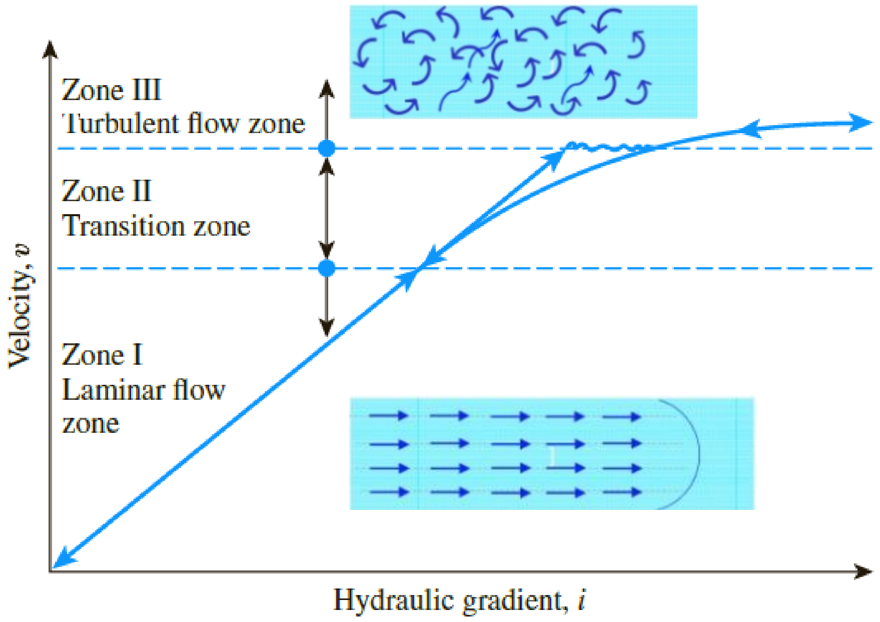
Darcy's law

- Discharge velocity vs. hydraulic gradient

$$v \propto i \longrightarrow v = ki$$

v = *discharge velocity*, which is the quantity of water flowing in unit time through a unit gross cross-sectional area of soil at right angles to the direction of flow.

k = hydraulic conductivity (otherwise known as the coefficient of permeability)



Discharge: Volume of water flowing in unit of time (q)

$$q = vA \Rightarrow q = kiA$$

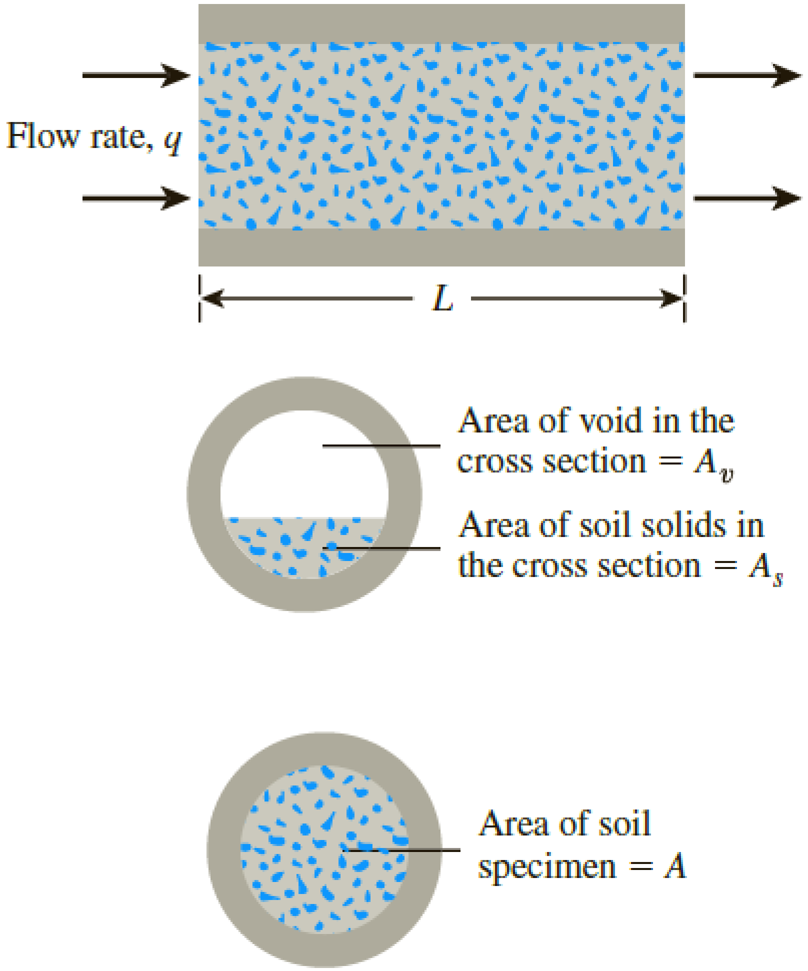
Steady-state conditions

Seepage velocity (v_s)

$$q = vA = A_v v_s$$

$$v_s = \frac{v\,(A_v + A_s)}{A_v} = \frac{v\,(A_v + A_s)\,L}{A_v L} = \frac{v\,(V_v + V_s)}{V_v}$$

$$v_s = \frac{v}{n}$$

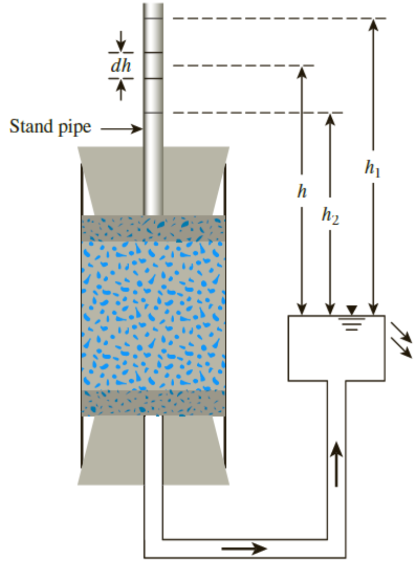


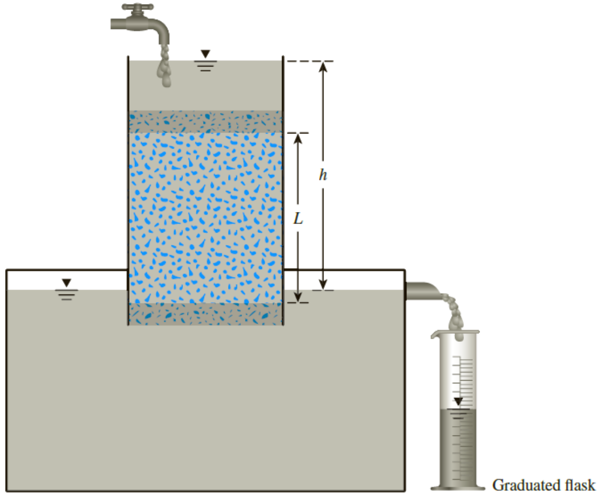
Hydraulic conductivity (k)

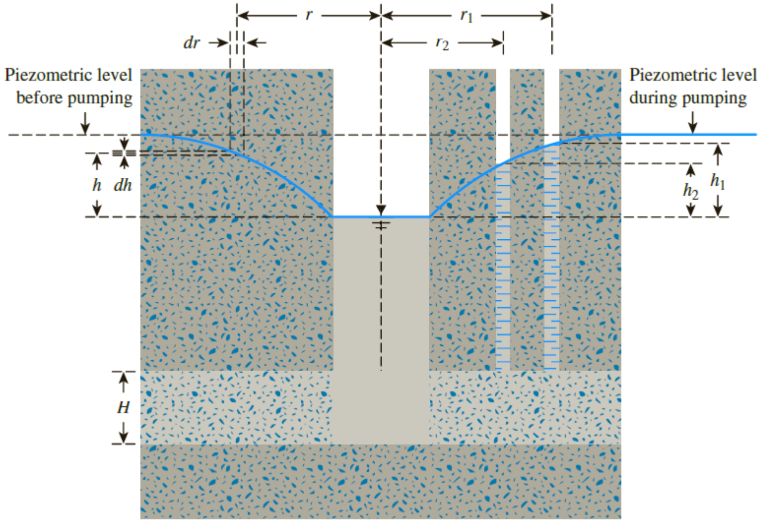
Soil type	*k* cm/sec	*k* ft/min
Clean gravel	100–1.0	200–2.0
Coarse sand	1.0–0.01	2.0–0.02
Fine sand	0.01–0.001	0.02–0.002
Silty clay	0.001–0.00001	0.002–0.00002
Clay	<0.000001	<0.000002

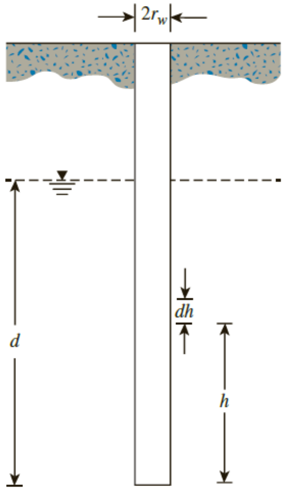
How to find (k)

- Lab tests
  - Constant head test
  - Falling head test
- In-situ tests
  - Pumping from wells
  - Pumping into auger holes









Laboratory tests

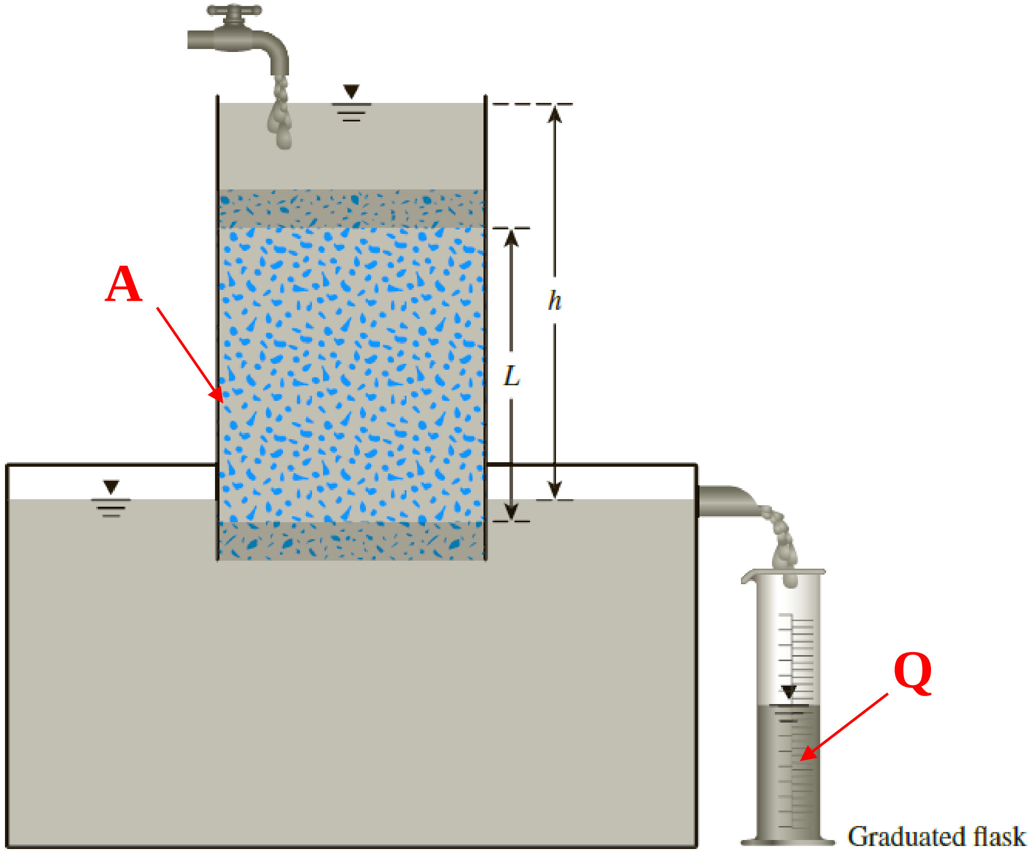
- Constant head test

$$Q = Avt = A(ki)t$$

$$i = \frac{h}{L}$$

$$Q = A\left(k\frac{h}{L}\right)t$$

$$k = \frac{QL}{Aht}$$



Laboratory tests

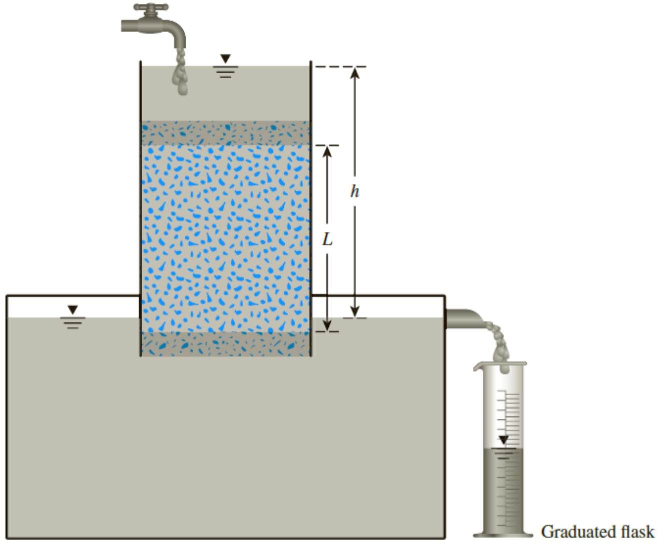
- Constant head test
  - Example:

    The results of a constant-head permeability test for a fine sand sample having a diameter of 150 mm and a length of 300 mm are as follows:

    Constant head difference = 500 mm

    Time of collection of water = 5 min

    Volume of water collected = 350 cm^3



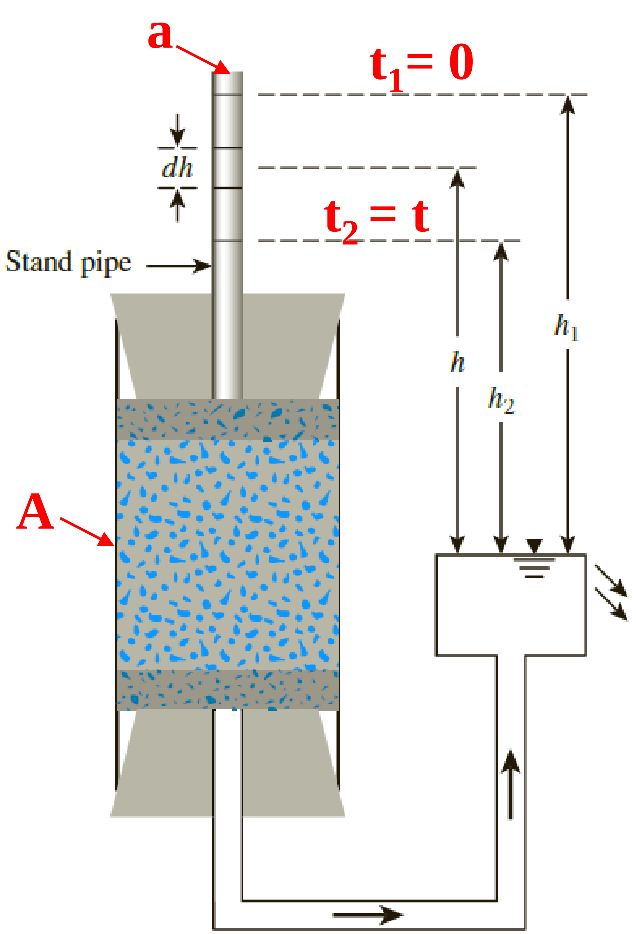
Laboratory tests

- Falling head test

$$q = k\frac{h}{L}A = -a\frac{dh}{dt} \Rightarrow dt = \frac{aL}{Ak}\left(-\frac{dh}{h}\right)$$

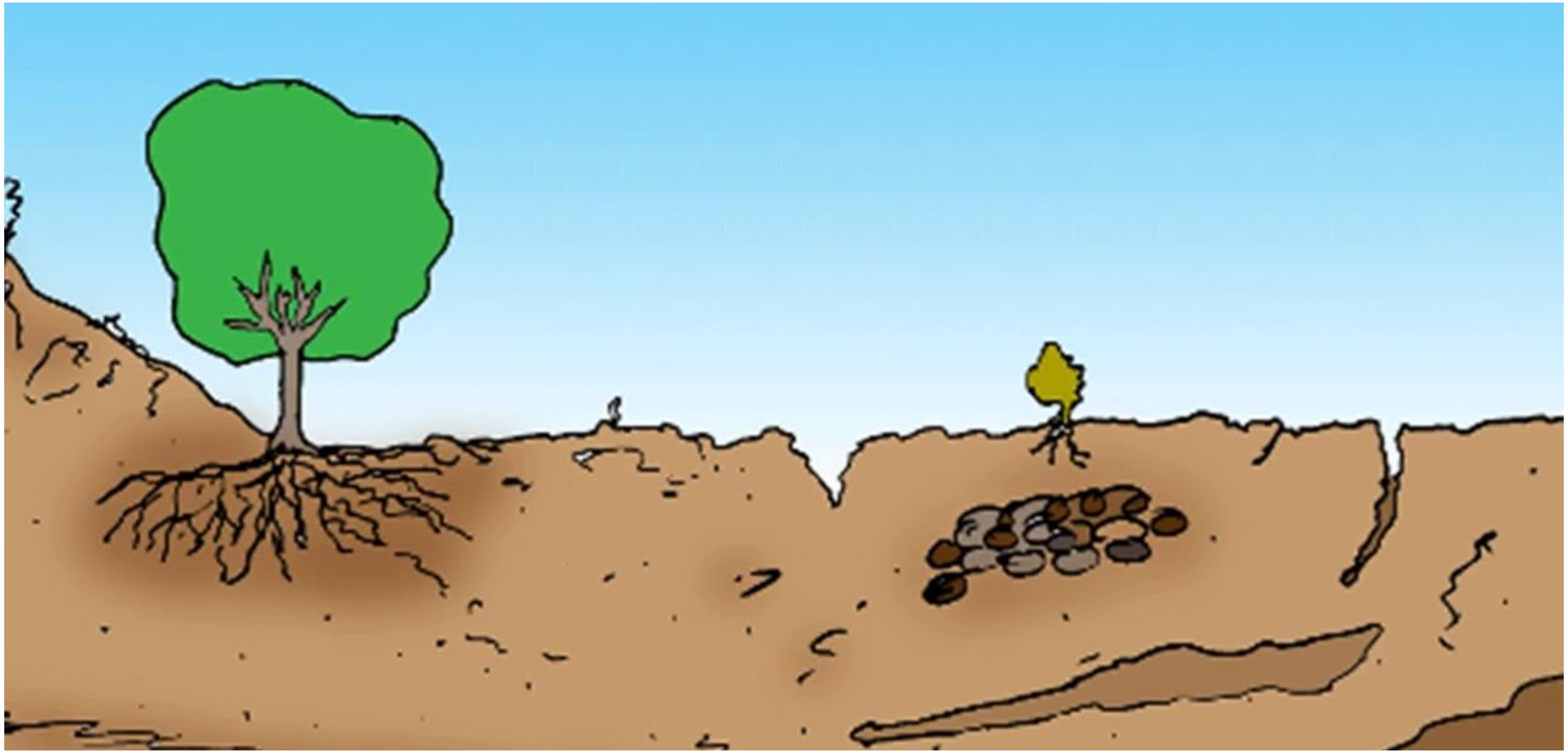
$$t = \frac{a\,L}{A\,k}\ln\frac{h_1}{h_2}$$

$$k = \frac{a\,L}{A\,t}\ln\frac{h_1}{h_2} \Rightarrow k = 2.303\frac{a\,L}{A\,t}\log_{10}\frac{h_1}{h_2}$$



In-situ Tests

- Why??



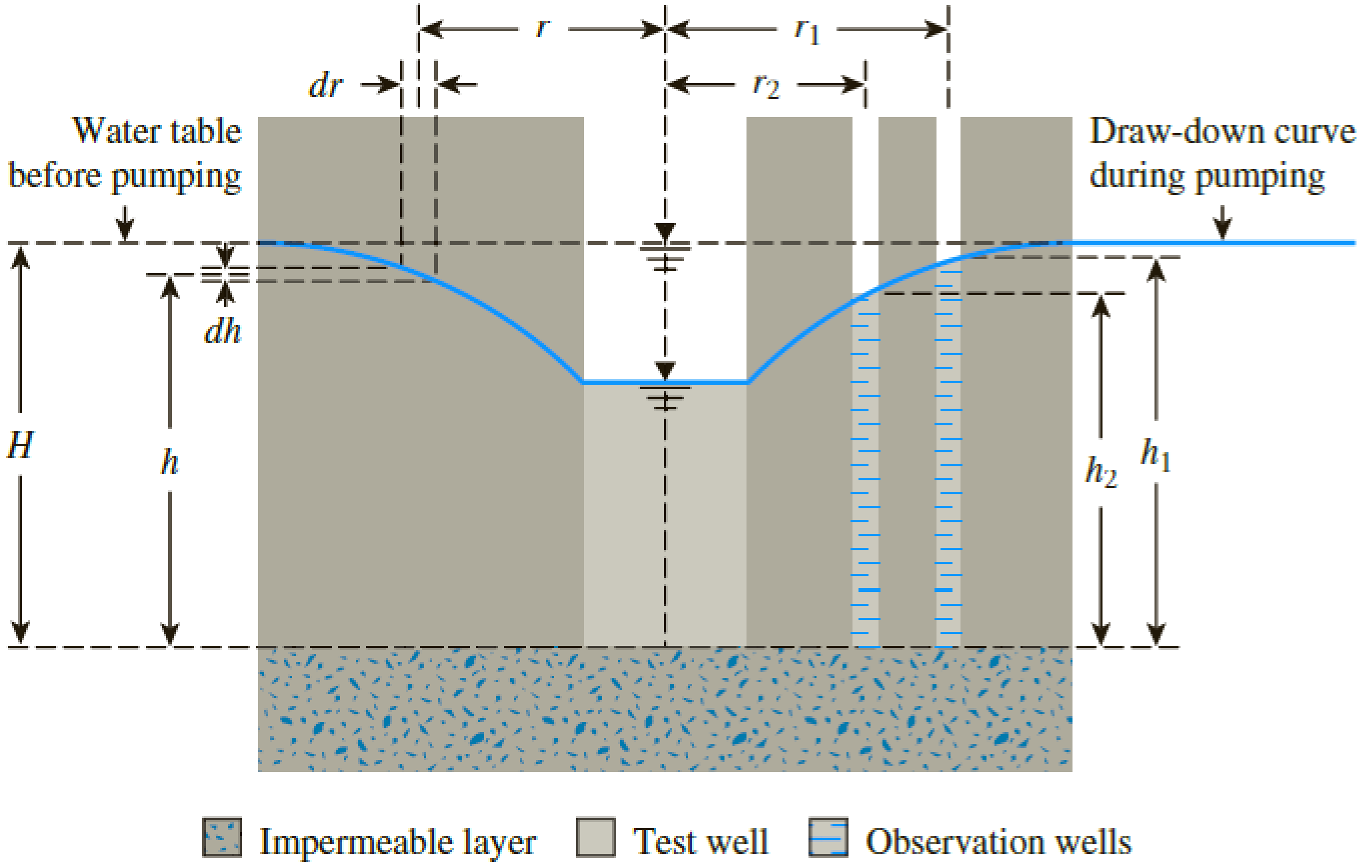
In-situ tests

- Pumping from wells (unconfined aquifer)

$$q = kiA$$

$$q = k\left(\frac{dh}{dr}\right)2\pi rh$$

$$k = \frac{2.303q\ \log_{10}\left(\frac{r_1}{r_2}\right)}{\pi(h_1^2 - h_2^2)}$$



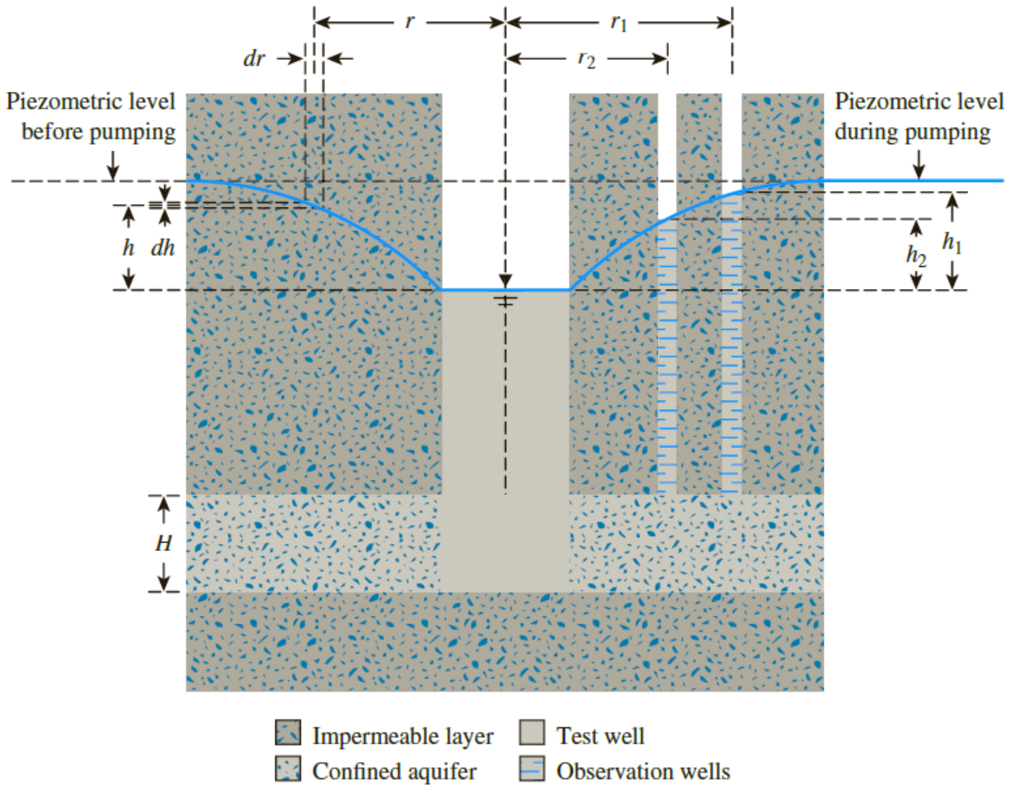
In-situ tests

- Pumping from wells (confined aquifer)

$$q = kiA$$

$$q = k\left(\frac{dh}{dr}\right)2\pi rH$$

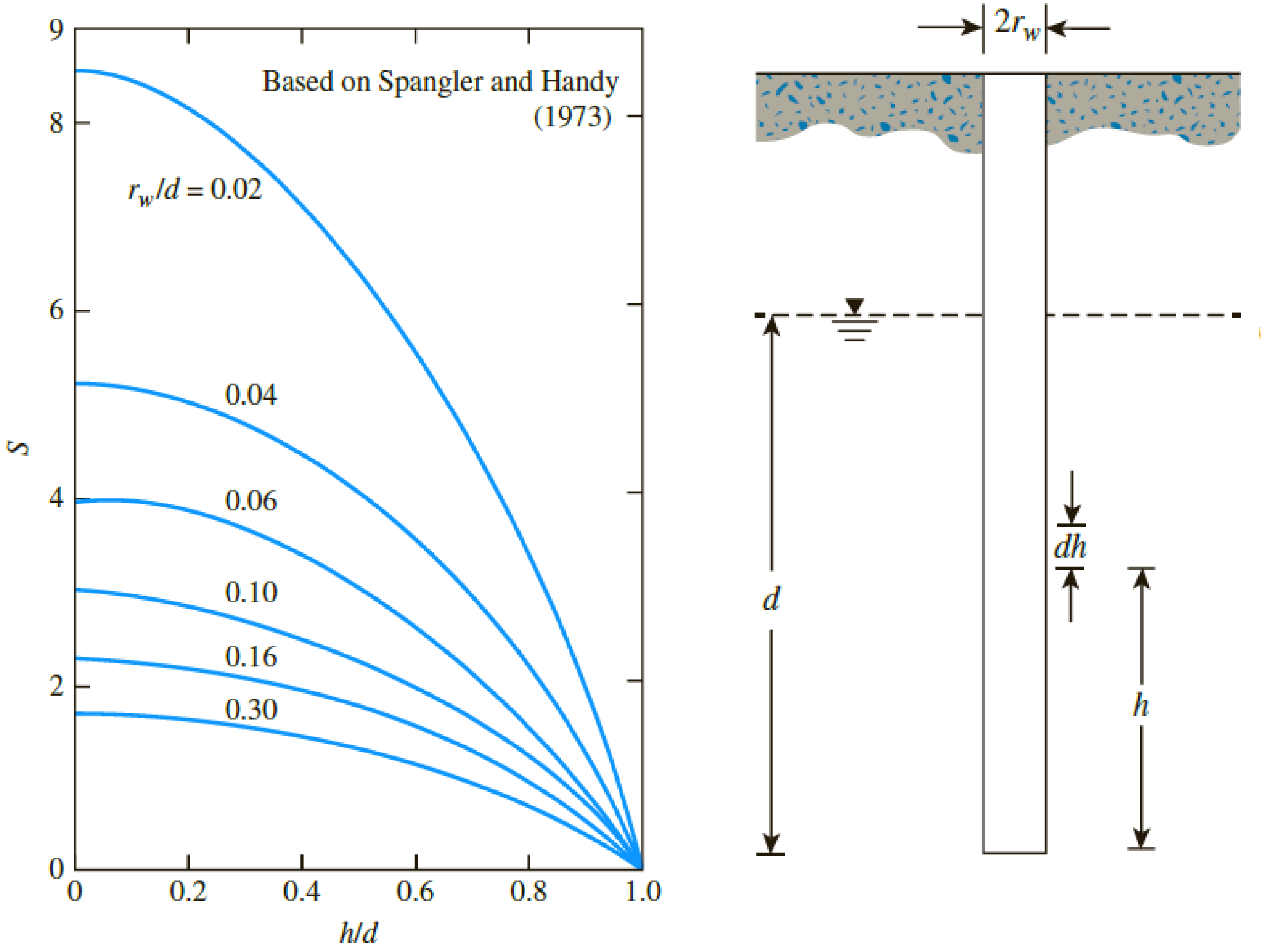
$$k = \frac{q\ \log_{10}\left(\frac{r_1}{r_2}\right)}{2.727H(h_1 - h_2)}$$



In-situ tests

- Pumping from auger holes

$$k = 0.617\frac{r_w}{Sd}\frac{dh}{dt}$$



Empirical Formulas

Uniform Sand

$$k = c\, D_{10}$$

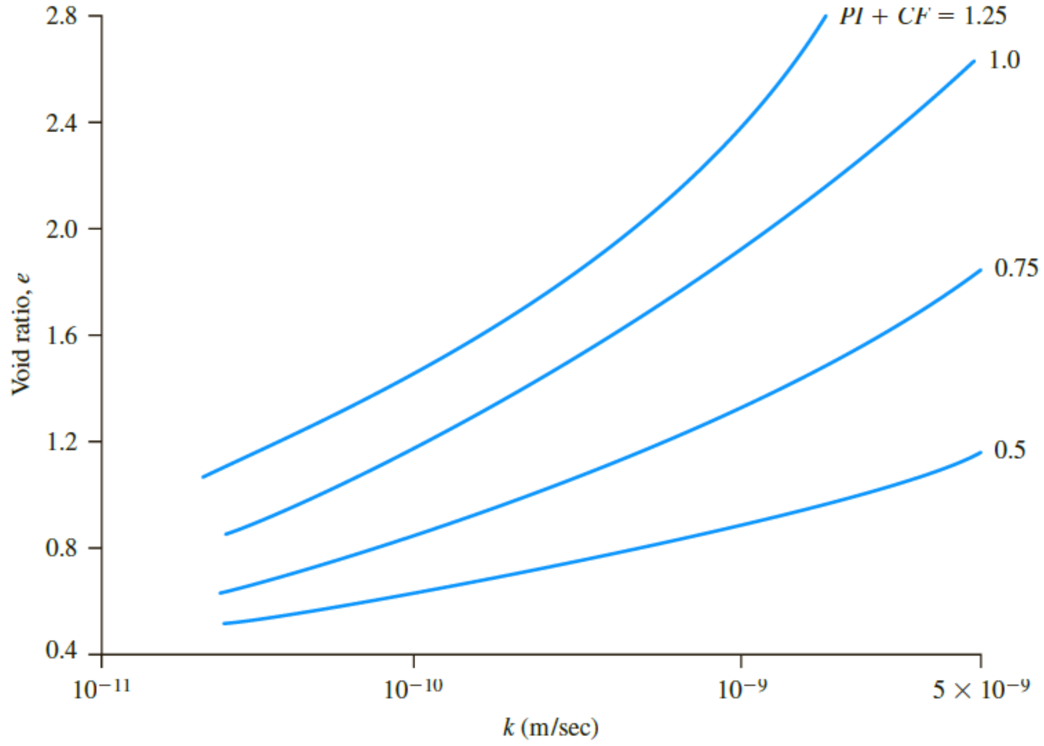
k: permeability (cm/s)
c: const. 1-1.5
D_{10} = effective diameter (mm)

Sand/gravels
(May include some Silts without plasticity)

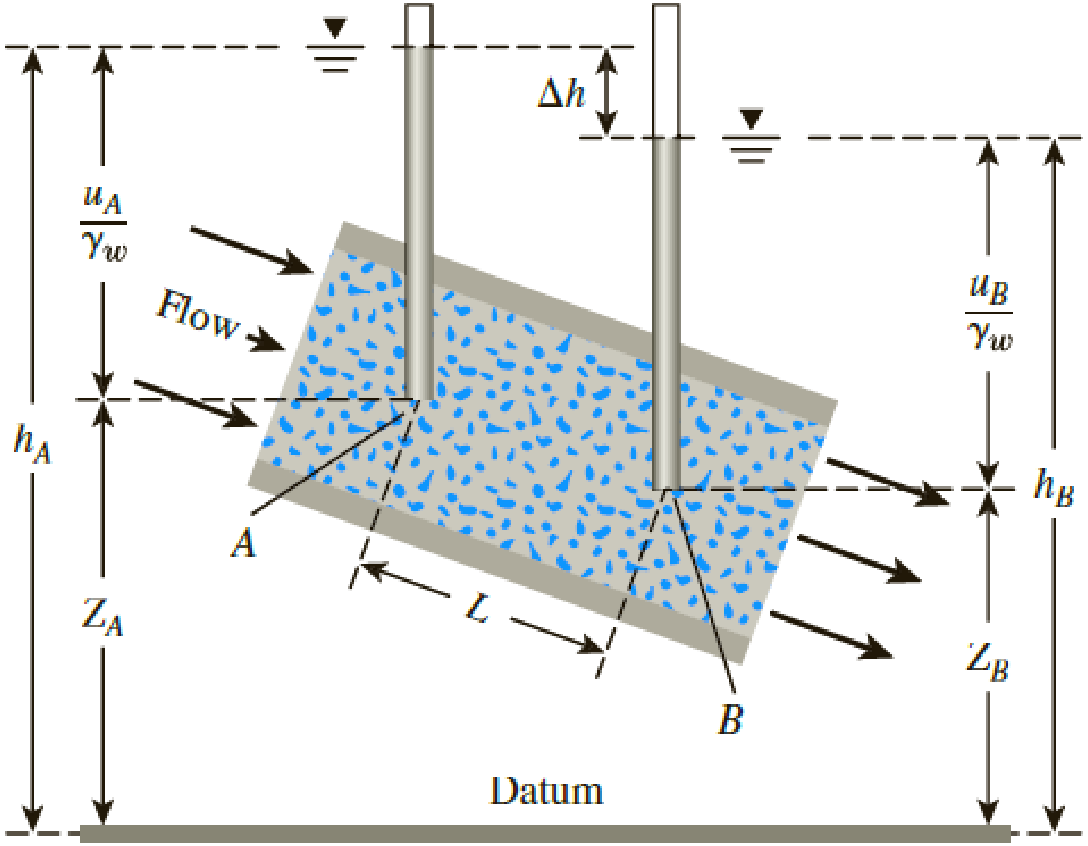
$$k(\text{cm/s}) = 2.4622\left[D_{10}^2 \frac{e^3}{(1+e)}\right]^{0.7825}$$

$$k(\text{cm/sec}) = 35\left(\frac{e^3}{1+e}\right)C_u^{0.6}(D_{10})^{2.32}$$

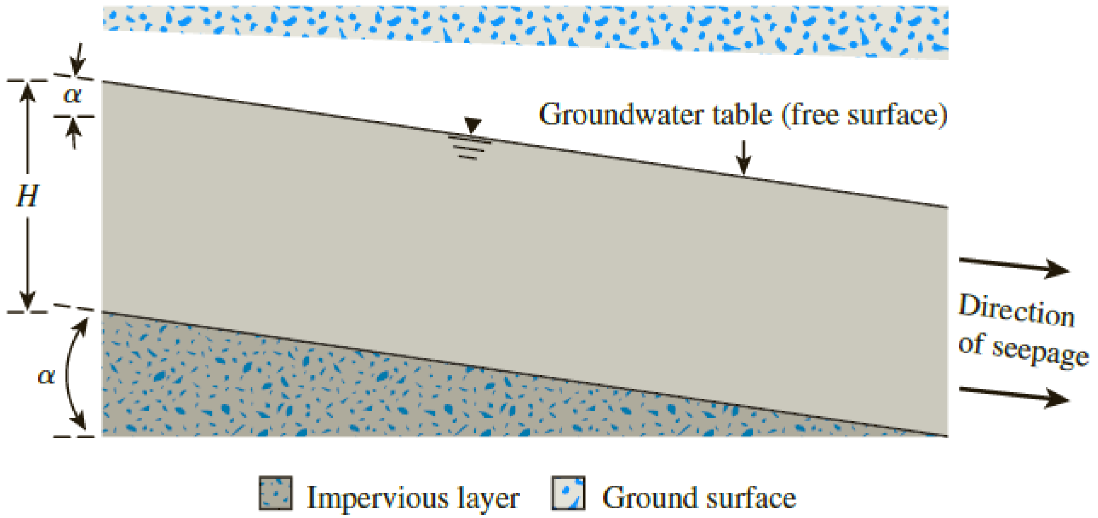
Cohesive soils

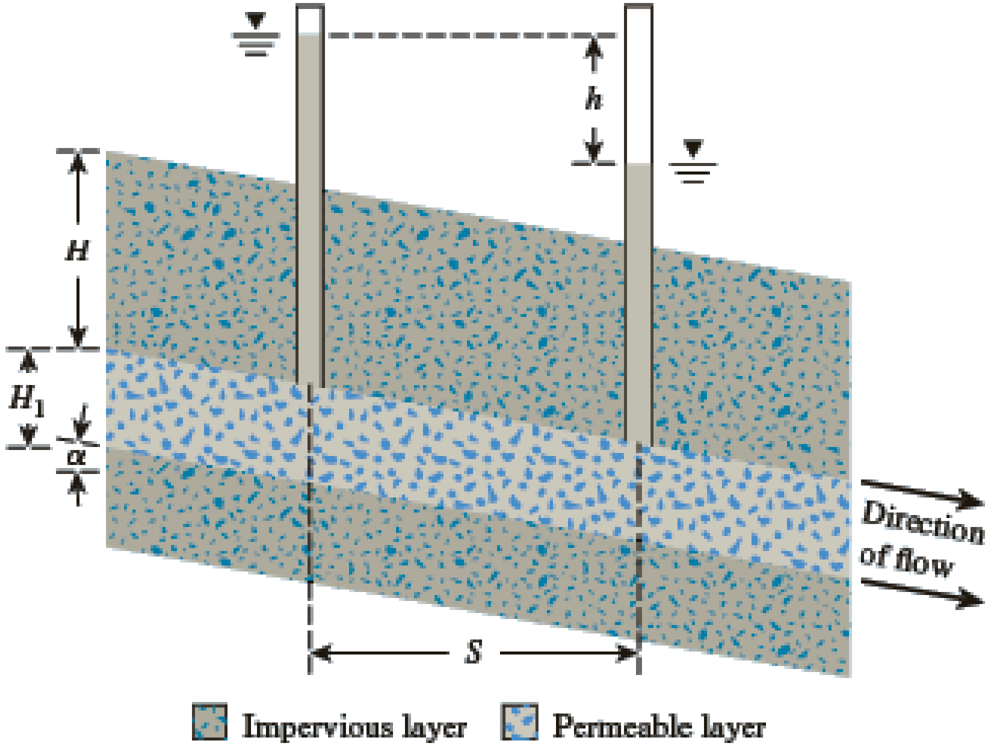


Directional permeability



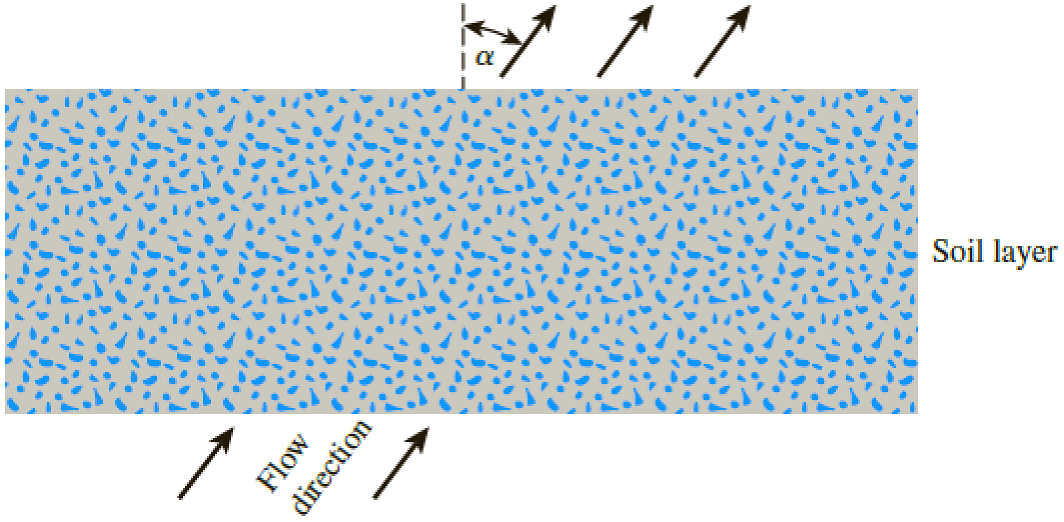
Directional permeability





Directional permeability

- Most soils are not isotropic with respect to permeability
- The magnitude of k changes with respect to the direction of flow
- The magnitudes of k_V and k_H in a given soil depend on several factors, including the method of deposition in the field.



Soil type	k_H/k_V	Reference
Organic silt with peat	1.2 to 1.7	Tsien (1955)
Plastic marine clay	1.2	Lumb and Holt (1968)
Soft clay	1.5	Basett and Brodie (1961)
Varved clay	1.5 to 1.7	Chan and Kenney (1973)
Varved clay	1.5	Kenney and Chan (1973)
Varved clay	3 to 15	Wu et al. (1978)
Varved clay	4 to 40	Casagrande and Poulos (1969)

Directional permeability

- Flow is parallel to stratification

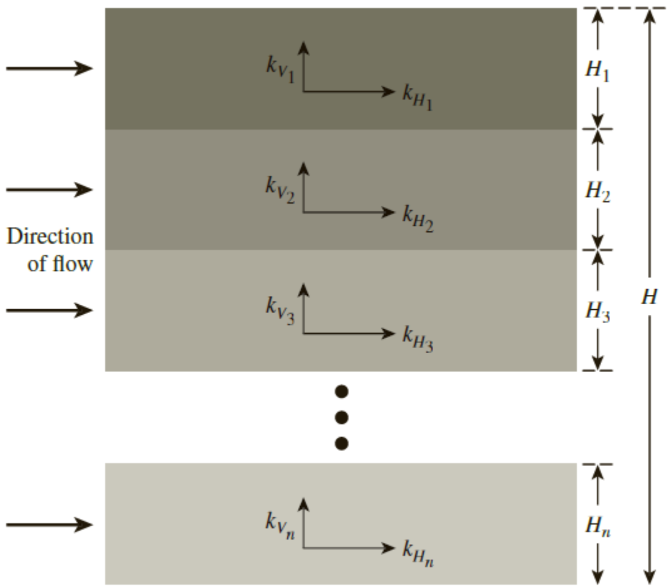
$$q = kiA$$

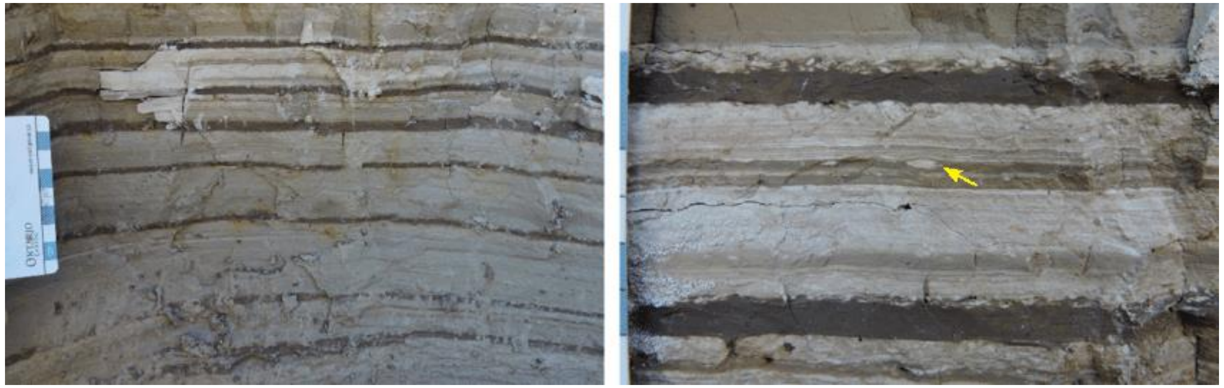
$$q = q_1 + q_2 + \ \dots + q_n$$

$$k_{eq(H)} * i \ * 1 \ * H = k_{H1} i_1 * 1 * H_1 + k_{H2} i_2 * 1 * H_2 + \ \dots + k_{H_n} i_n * 1 * H_n$$

$$i = i_1 = i_2 = \ \dots = i_n$$

$$k_{H(eq)} = \frac{1}{H}(k_{H_1}H_1 + k_{H_2}H_2 + k_{H_3}H_3 + \cdots + k_{H_n}H_n)$$





Directional permeability

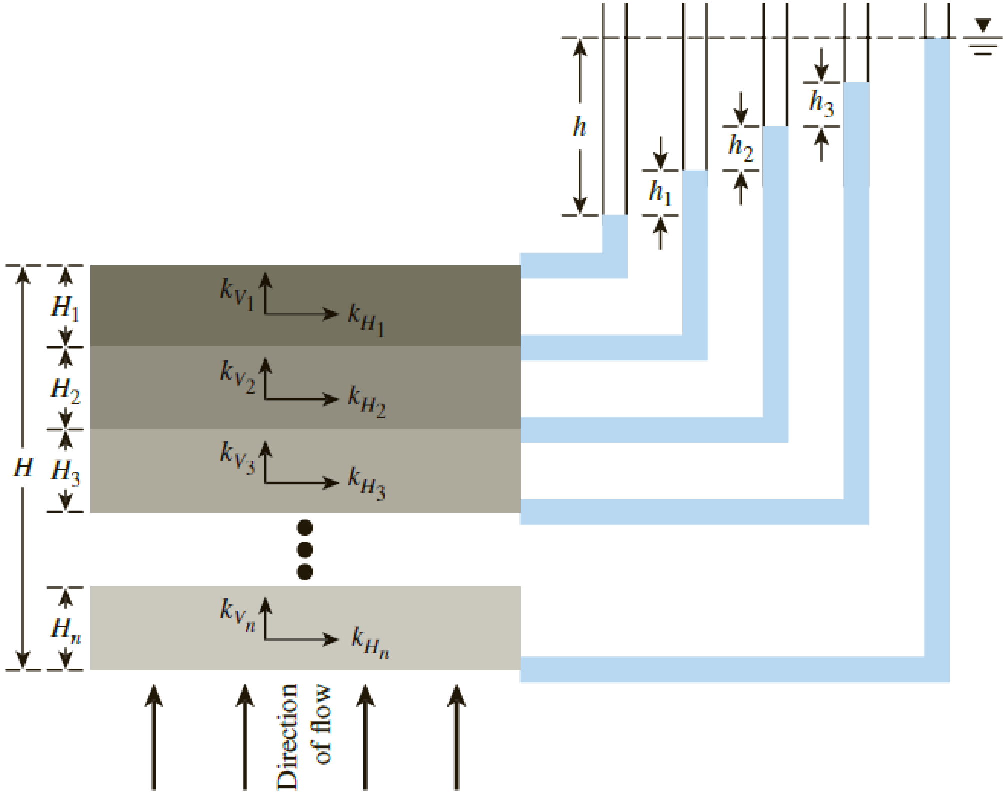
- Flow is Normal to stratification

$$q = kiA$$

$$q = q_1 = q_2 = \ \dots = q_n$$

$$i = i_1 + i_2 + \ \dots + i_n$$

$$k_{v(\mathrm{eq})} = \frac{H}{\left(\frac{H_1}{k_{V_1}}\right) + \left(\frac{H_2}{k_{V_2}}\right) + \left(\frac{H_3}{k_{V_3}}\right) + \cdots + \left(\frac{H_n}{k_{V_n}}\right)}$$

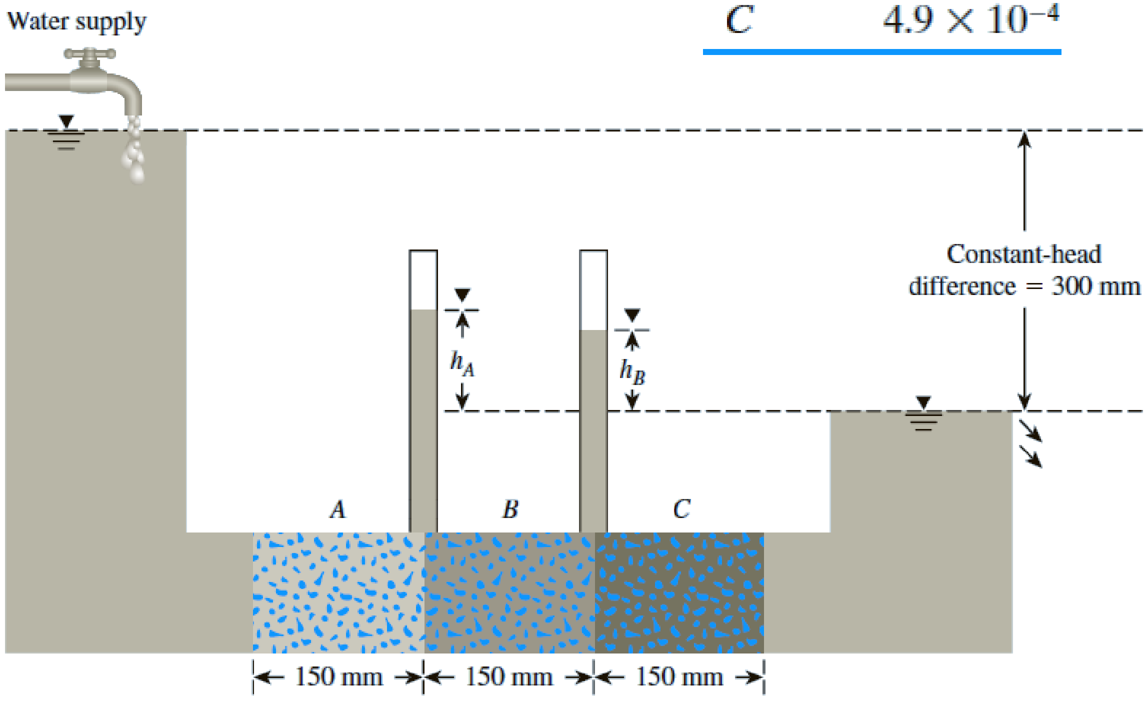


Example

The Figure shows three layers of soil in a tube that is 100 mm 3 100 mm in cross section. Water is supplied to maintain a constant-head difference of 300 mm across the sample. The hydraulic conductivities of the soils in the direction of flow through them are as shown, Determine:

- Equivalent permeability
- Discharge (q) (rate of water supply)
- h_A and h_B

Soil	k (cm/sec)
A	10^{-2}
B	3×10^{-3}
C	4.9×10^{-4}



Example

. Water is supplied to maintain a constant-head difference across the sample. The hydraulic conductivities of the soils in the direction of flow through them are as shown, Determine:

- Equivalent permeability
- Discharge (q) (rate of water supply)
- Total head at points A,B,C, and D.