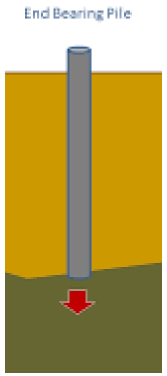
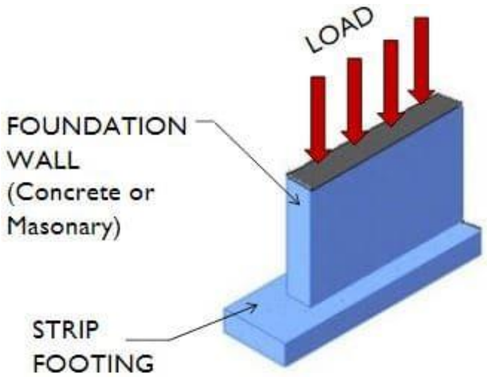
Dr. Khalil Qatu

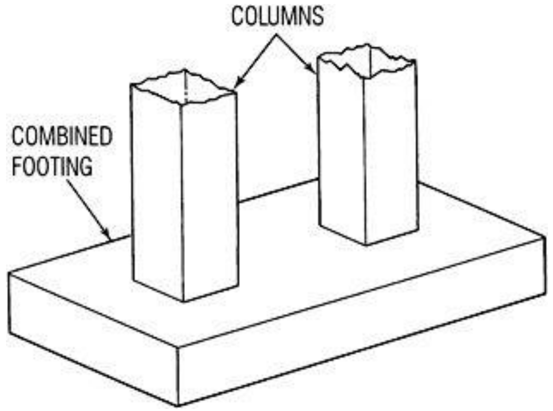
ENCE 331: Stresses in Soil

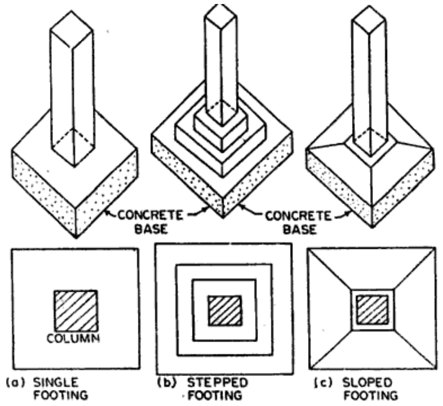
Stress in soil & foundation construction

- Construction of a foundation causes changes in the stress, usually a net increase.
- The net stress increase in the soil depends on
  - The load per unit area to which the foundation is subjected.
  - The depth below the foundation at which the stress estimation is desired.
  - and other factors.
- It is necessary to estimate the net increase of vertical stress in soil that occurs as a result of the construction of a foundation
- This stress increase can be used to calculate settlement under the foundation



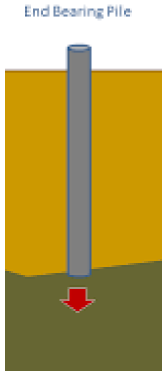


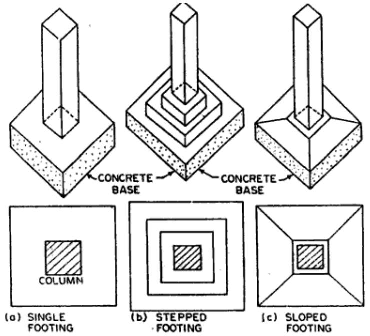


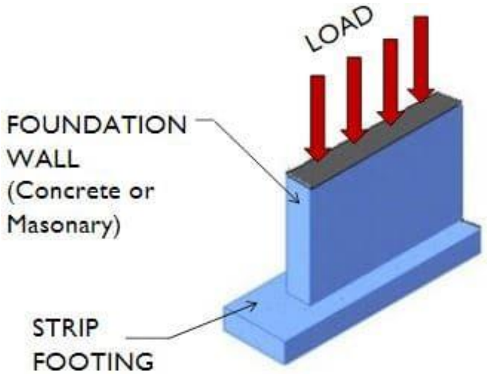


Stress in soil & foundation construction

- Different type of foundation causes different type of loading on the soil
  - Determination of vertical stress increase at a certain depth due to the application of load on the surface. The loading type includes:
    - Point load
    - Line load
    - Uniformly distributed vertical strip load
    - Linearly increasing vertical loading on a strip
    - Embankment type of loading
    - Uniformly loaded circular area
    - Uniformly loaded rectangular area







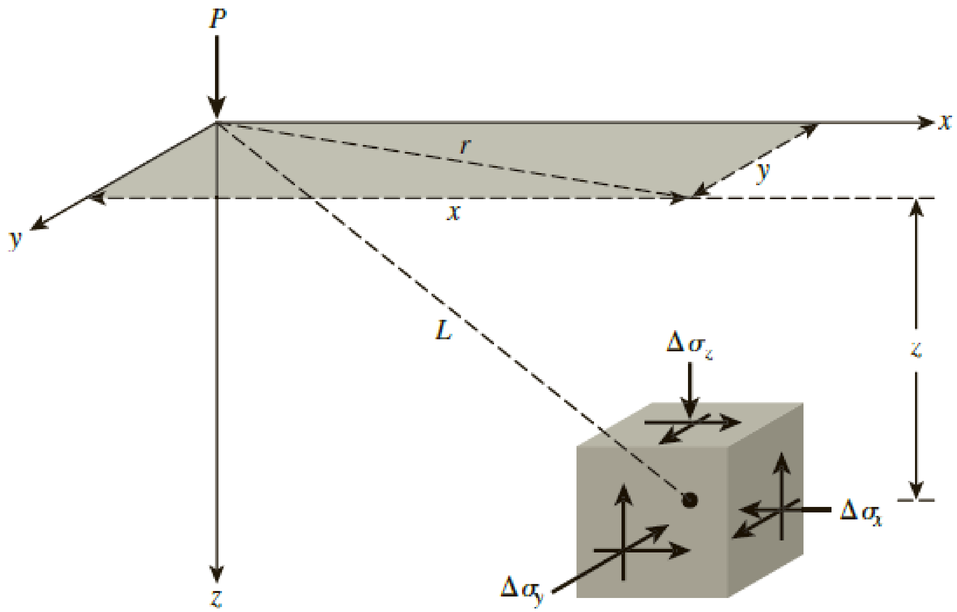
- In most cases, soil is considered as homogeneous, elastic, and isotropic medium

Stresses Caused by a Point Load

- Boussinesq (1883) solved the problem of stresses produced at any point as the result of a point load applied on the surface of an infinitely large half-space
  - The increase in the normal stresses are:
    - Horizontal normal stresses, depend on the Poisson's ratio of the medium

$$\Delta\sigma_x = \frac{P}{2\pi}\left\{\frac{3x^2z}{L^5} - (1-2\mu)\left[\frac{x^2-y^2}{Lr^2(L+z)} + \frac{y^2z}{L^3r^2}\right]\right\}$$

$$\Delta\sigma_y = \frac{P}{2\pi}\left\{\frac{3y^2z}{L^5} - (1-2\mu)\left[\frac{y^2-x^2}{Lr^2(L+z)} + \frac{x^2z}{L^3r^2}\right]\right\}$$



    - Vertical normal stress is independent of Poisson's ratio.

$$\Delta\sigma_z = \frac{3P}{2\pi}\frac{z^3}{L^5} = \frac{3P}{2\pi}\frac{z^3}{(r^2+z^2)^{5/2}}$$

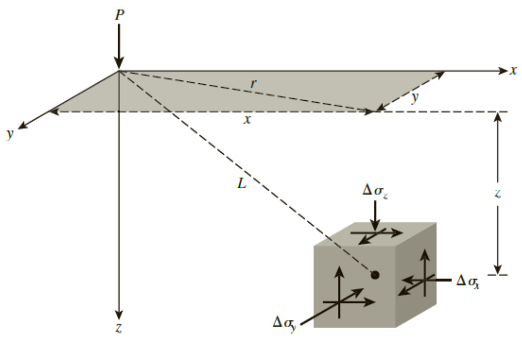
where $r = \sqrt{x^2 + y^2}$

$L = \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + z^2}$

μ = Poisson's ratio

Stresses Caused by a Point Load

- Boussinesq (1883) solved the problem of stresses produced at any point as the result of a point load applied on the surface of an infinitely large half-space
  - The Vertical normal stress can be rewritten as:



$$\Delta\sigma_z = \frac{P}{z^2}\left\{\frac{3}{2\pi}\frac{1}{[(r/z)^2+1]^{5/2}}\right\} = \frac{P}{z^2}I_1$$

  - The Influence factor I_1 depends only on the location of the point we want to calculate the stress at.
  - The term r/z is a dimensionless term, which makes it easy to generate a table containing the value of I_1 with the variation of r/z

Stresses Caused by a Point Load

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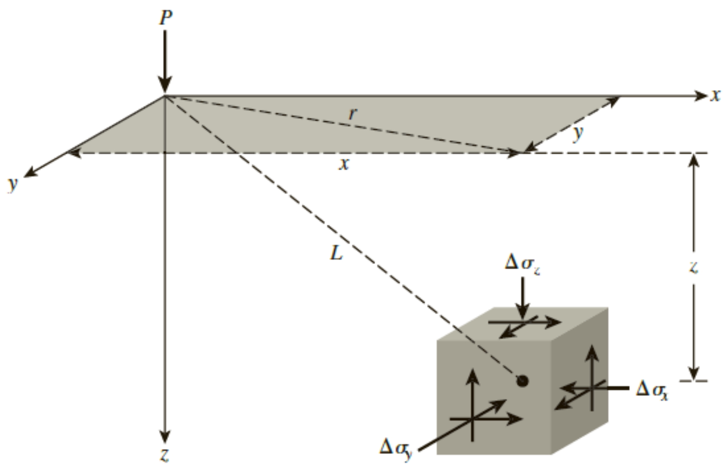
Table 10.1 Variation of I_1 for Various Values of r/z [Eq. (10.14)]

r/z	I_1	r/z	I_1	r/z	I_1
0	0.4775	0.36	0.3521	1.80	0.0129
0.02	0.4770	0.38	0.3408	2.00	0.0085
0.04	0.4765	0.40	0.3294	2.20	0.0058
0.06	0.4723	0.45	0.3011	2.40	0.0040
0.08	0.4699	0.50	0.2733	2.60	0.0029
0.10	0.4657	0.55	0.2466	2.80	0.0021
0.12	0.4607	0.60	0.2214	3.00	0.0015
0.14	0.4548	0.65	0.1978	3.20	0.0011
0.16	0.4482	0.70	0.1762	3.40	0.00085
0.18	0.4409	0.75	0.1565	3.60	0.00066
0.20	0.4329	0.80	0.1386	3.80	0.00051
0.22	0.4242	0.85	0.1226	4.00	0.00040
0.24	0.4151	0.90	0.1083	4.20	0.00032
0.26	0.4050	0.95	0.0956	4.40	0.00026
0.28	0.3954	1.00	0.0844	4.60	0.00021
0.30	0.3849	1.20	0.0513	4.80	0.00017
0.32	0.3742	1.40	0.0317	5.00	0.00014
0.34	0.3632	1.60	0.0200		

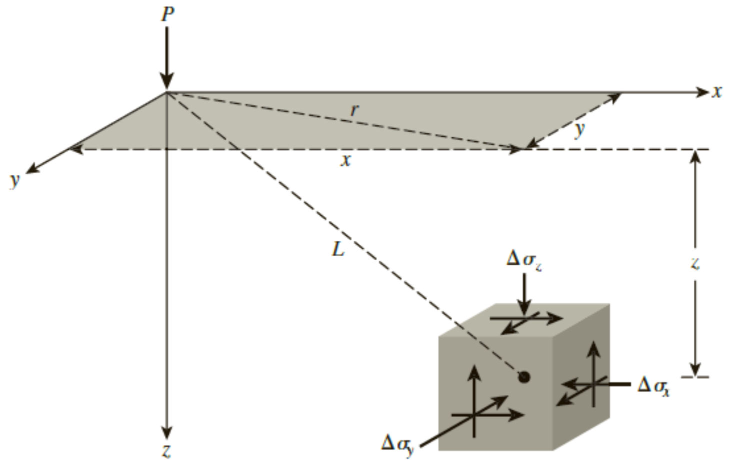


Stresses Caused by a Point Load

- Example: Consider a point load P = 5 kN Calculate the vertical stress increase $\Delta\sigma_z$ at:
  - z = 0, 2 m, 4 m, 6 m, 10 m, and 20 m.
  - x = 0, and 3
  - y = 4 m.

$$\Delta\sigma_z = \frac{P}{z^2}\left\{\frac{3}{2\pi}\frac{1}{[(r/z)^2+1]^{5/2}}\right\} = \frac{P}{z^2}I_1$$

#	z	X	Y	R	r/z	I_1	$\Delta\sigma_z$ (kPa)
1	0.01	0	4	4	400	0	0.0000
2	2	0	4	4	2	0.0085	0.0106
3	4	0	4	4	1	0.0844	0.0264
4	6	0	4	4	0.6667	0.187	0.0260
5	10	0	4	4	0.4	0.3294	0.0165
6	20	0	4	4	0.2	0.4329	0.0054
7	0.01	3	4	5	500	0	0.0000
8	2	3	4	5	2.5	0.00345	0.0043
9	4	3	4	5	1.25	0.0415	0.0130
10	6	3	4	5	0.8333	0.1306	0.0181
11	10	3	4	5	0.5	0.2733	0.0137
12	20	3	4	5	0.25	0.41005	0.0051



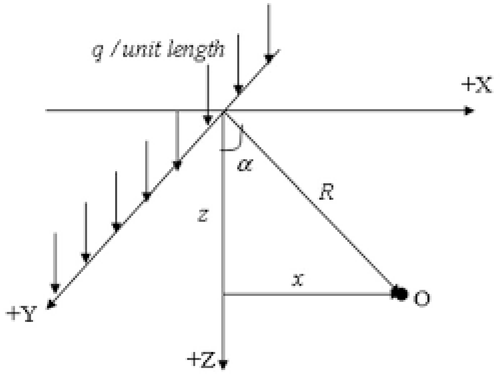
Vertical Stress Caused by a Vertical Line Load

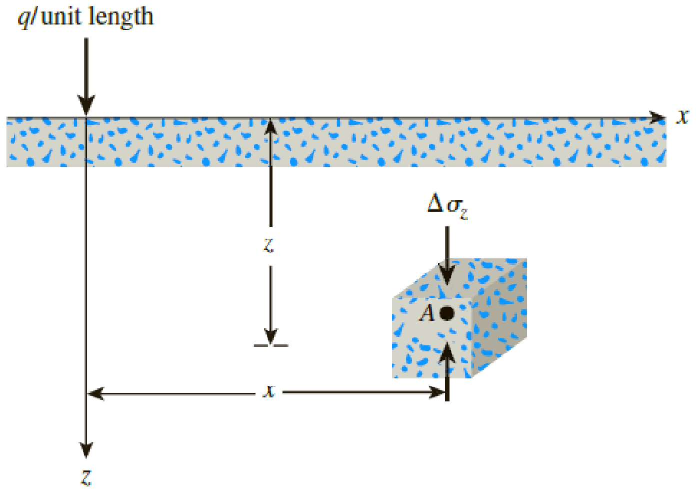
- In this case, the load extends infinitely in Y axis.
  - This is called plane strain conditions (no normal strain in the Y axis)
  - The Vertical stress increase at any point :

$$\Delta\sigma_z = \frac{2qz^3}{\pi(x^2 + z^2)^2}$$

  - This equation can be rewritten

$$\frac{\Delta\sigma_z}{(q/z)} = \frac{2}{\pi[(x/z)^2 + 1]^2}$$





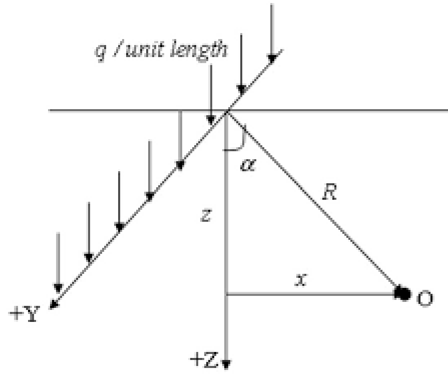
Vertical Stress Caused by a Vertical Line Load

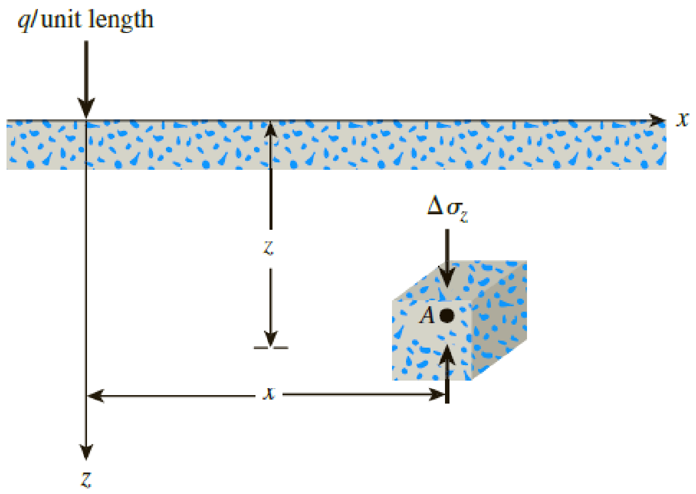
- To makes it easy to generate a table containing the variation of value of left side of the equation with the variation of the ratio x/z

• To makes it easy to generate a table containing the variation of value of left side of the equation with the variation of the ratio x/z

Table 10.2 Variation of $\Delta\sigma_z/(q/z)$ with x/z [Eq. (10.16)]

x/z	$\Delta\sigma_z/(q/z)$	x/z	$\Delta\sigma_z/(q/z)$
0	0.637	1.3	0.088
0.1	0.624	1.4	0.073
0.2	0.589	1.5	0.060
0.3	0.536	1.6	0.050
0.4	0.473	1.7	0.042
0.5	0.407	1.8	0.035
0.6	0.344	1.9	0.030
0.7	0.287	2.0	0.025
0.8	0.237	2.2	0.019
0.9	0.194	2.4	0.014
1.0	0.159	2.6	0.011
1.1	0.130	2.8	0.008
1.2	0.107	3.0	0.006



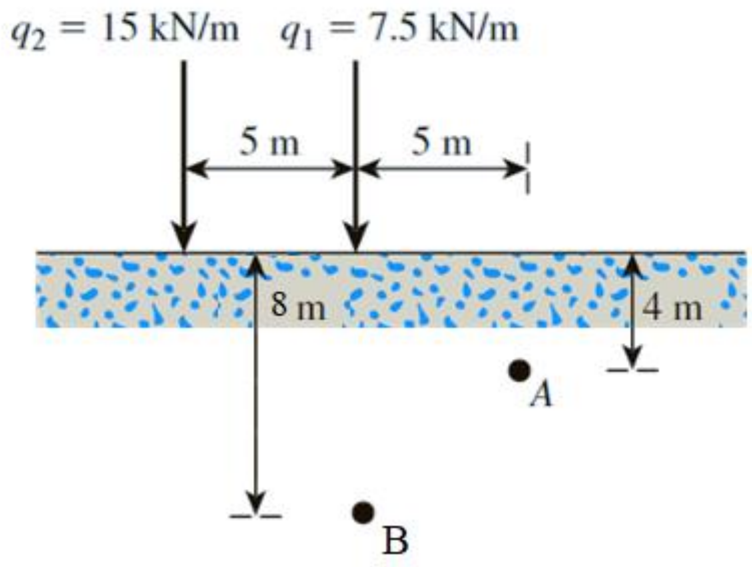


Vertical Stress Caused by a Vertical Line Load

- Example:

  A Soil medium is subjected to the two line-loads at the ground surface.

  - Determine the increase of the vertical stress at point A and B.
  - What is the total stress at point A after loading



Vertical Stress Caused by a Vertical Strip Load

- The load is Finite Width and Infinite Length
  - The increase in the vertical stress:
    - $x \geq \frac{B}{2}$: the Point not under the foundation directly

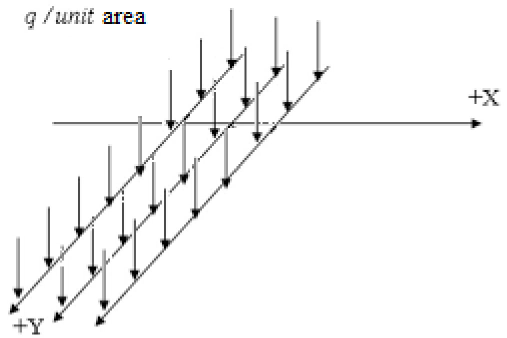
$$\Delta\sigma_z = \frac{q}{\pi}\left\{\tan^{-1}\left[\frac{z}{x-(B/2)}\right] - \tan^{-1}\left[\frac{z}{x+(B/2)}\right] - \frac{Bz[x^2 - z^2 - (B^2/4)]}{[x^2 + z^2 - (B^2/4)]^2 + B^2z^2}\right\}$$

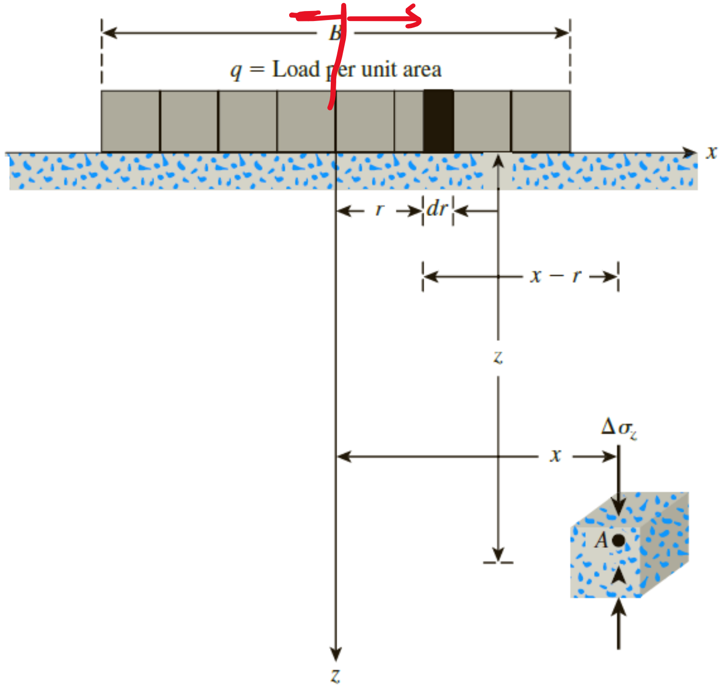
    - $x < \frac{B}{2}$: the Point the foundation directly

Unit??

$$\Delta\sigma_z = \frac{q}{\pi}\left\{\pi + \tan^{-1}\left[\frac{z}{x-(B/2)}\right] - \tan^{-1}\left[\frac{z}{x+(B/2)}\right] - \frac{Bz[x^2 - z^2 - (B^2/4)]}{[x^2 + z^2 - (B^2/4)]^2 + B^2z^2}\right\}$$

    - These equations also can be re-arranged and generate tables for the variation of $(\Delta\sigma_z/q)$





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Vertical Stress Caused by a Vertical Strip Load

Table 10.4 Variation of $\Delta\sigma_z/q$ with $2z/B$ and $2x/B$ [Eq. (10.19)]

2z/B	2x/B = 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.000
0.10	1.000	1.000	0.999	0.999	0.999	0.998	0.997	0.993	0.980	0.909	0.500
0.20	0.997	0.997	0.996	0.995	0.992	0.988	0.979	0.959	0.909	0.775	0.500
0.30	0.990	0.989	0.987	0.984	0.978	0.967	0.947	0.908	0.833	0.697	0.499
0.40	0.977	0.976	0.973	0.966	0.955	0.937	0.906	0.855	0.773	0.651	0.498
0.50	0.959	0.958	0.953	0.943	0.927	0.902	0.864	0.808	0.727	0.620	0.497
0.60	0.937	0.935	0.928	0.915	0.896	0.866	0.825	0.767	0.691	0.598	0.495
0.70	0.910	0.908	0.899	0.885	0.863	0.831	0.788	0.732	0.662	0.581	0.492
0.80	0.881	0.878	0.869	0.853	0.829	0.797	0.755	0.701	0.638	0.566	0.489
0.90	0.850	0.847	0.837	0.821	0.797	0.765	0.724	0.675	0.617	0.552	0.485
1.00	0.818	0.815	0.805	0.789	0.766	0.735	0.696	0.650	0.598	0.540	0.480
1.10	0.787	0.783	0.774	0.758	0.735	0.706	0.670	0.628	0.580	0.529	0.474
1.20	0.755	0.752	0.743	0.728	0.707	0.679	0.646	0.607	0.564	0.517	0.468
1.30	0.725	0.722	0.714	0.699	0.679	0.654	0.623	0.588	0.548	0.506	0.462
1.40	0.696	0.693	0.685	0.672	0.653	0.630	0.602	0.569	0.534	0.495	0.455
1.50	0.668	0.666	0.658	0.646	0.629	0.607	0.581	0.552	0.519	0.484	0.448
1.60	0.642	0.639	0.633	0.621	0.605	0.586	0.562	0.535	0.506	0.474	0.440
1.70	0.617	0.615	0.608	0.598	0.583	0.565	0.544	0.519	0.492	0.463	0.433
1.80	0.593	0.591	0.585	0.576	0.563	0.546	0.526	0.504	0.479	0.453	0.425
1.90	0.571	0.569	0.564	0.555	0.543	0.528	0.510	0.489	0.467	0.443	0.417
2.00	0.550	0.548	0.543	0.535	0.524	0.510	0.494	0.475	0.455	0.433	0.409
2.10	0.530	0.529	0.524	0.517	0.507	0.494	0.479	0.462	0.443	0.423	0.401
2.20	0.511	0.510	0.506	0.499	0.490	0.479	0.465	0.449	0.432	0.413	0.393
2.30	0.494	0.493	0.489	0.483	0.474	0.464	0.451	0.437	0.421	0.404	0.385
2.40	0.477	0.476	0.473	0.467	0.460	0.450	0.438	0.425	0.410	0.395	0.378
2.50	0.462	0.461	0.458	0.452	0.445	0.436	0.426	0.414	0.400	0.386	0.370
2.60	0.447	0.446	0.443	0.439	0.432	0.424	0.414	0.403	0.390	0.377	0.363
2.70	0.433	0.432	0.430	0.425	0.419	0.412	0.403	0.393	0.381	0.369	0.355
2.80	0.420	0.419	0.417	0.413	0.407	0.400	0.392	0.383	0.372	0.360	0.348
2.90	0.408	0.407	0.405	0.401	0.396	0.389	0.382	0.373	0.363	0.352	0.341
3.00	0.396	0.395	0.393	0.390	0.385	0.379	0.372	0.364	0.355	0.345	0.334
3.10	0.385	0.384	0.382	0.379	0.375	0.369	0.363	0.355	0.347	0.337	0.327
3.20	0.374	0.373	0.372	0.369	0.365	0.360	0.354	0.347	0.339	0.330	0.321
3.30	0.364	0.363	0.362	0.359	0.355	0.351	0.345	0.339	0.331	0.323	0.315
3.40	0.354	0.354	0.352	0.350	0.346	0.342	0.337	0.331	0.324	0.316	0.308
3.50	0.345	0.345	0.343	0.341	0.338	0.334	0.329	0.323	0.317	0.310	0.302
3.60	0.337	0.336	0.335	0.333	0.330	0.326	0.321	0.316	0.310	0.304	0.297
3.70	0.328	0.328	0.327	0.325	0.322	0.318	0.314	0.309	0.304	0.298	0.291
3.80	0.320	0.320	0.319	0.317	0.315	0.311	0.307	0.303	0.297	0.292	0.285
3.90	0.313	0.313	0.312	0.310	0.307	0.304	0.301	0.296	0.291	0.286	0.280
4.00	0.306	0.305	0.304	0.303	0.301	0.298	0.294	0.290	0.285	0.280	0.275
4.10	0.299	0.299	0.298	0.296	0.294	0.291	0.288	0.284	0.280	0.275	0.270
4.20	0.292	0.292	0.291	0.290	0.288	0.285	0.282	0.278	0.274	0.270	0.265
4.30	0.286	0.286	0.285	0.283	0.282	0.279	0.276	0.273	0.269	0.265	0.260
4.40	0.280	0.280	0.279	0.278	0.276	0.274	0.271	0.268	0.264	0.260	0.256
4.50	0.274	0.274	0.273	0.272	0.270	0.268	0.266	0.263	0.259	0.255	0.251
4.60	0.268	0.268	0.268	0.266	0.265	0.263	0.260	0.258	0.254	0.251	0.247
4.70	0.263	0.263	0.262	0.261	0.260	0.258	0.255	0.253	0.250	0.246	0.243
4.80	0.258	0.258	0.257	0.256	0.255	0.253	0.251	0.248	0.245	0.242	0.239
4.90	0.253	0.253	0.252	0.251	0.250	0.248	0.246	0.244	0.241	0.238	0.235
5.00	0.248	0.248	0.247	0.246	0.245	0.244	0.242	0.239	0.237	0.234	0.231

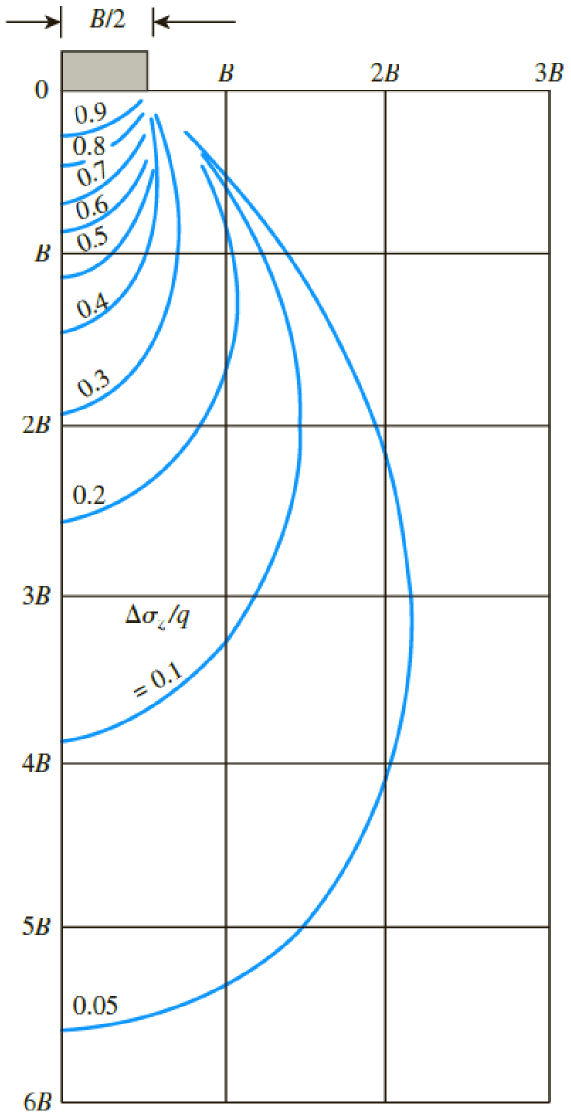
Table 10.4 *(continued)*

2z/B	2x/B = 1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.10	0.091	0.020	0.007	0.003	0.002	0.001	0.001	0.000	0.000	0.000
0.20	0.225	0.091	0.040	0.020	0.011	0.007	0.004	0.003	0.002	0.002
0.30	0.301	0.165	0.090	0.052	0.031	0.020	0.013	0.009	0.007	0.005
0.40	0.346	0.224	0.141	0.090	0.059	0.040	0.027	0.020	0.014	0.011
0.50	0.373	0.267	0.185	0.128	0.089	0.063	0.046	0.034	0.025	0.019
0.60	0.391	0.298	0.222	0.163	0.120	0.088	0.066	0.050	0.038	0.030
0.70	0.403	0.321	0.250	0.193	0.148	0.113	0.087	0.068	0.053	0.042
0.80	0.411	0.338	0.273	0.218	0.173	0.137	0.108	0.086	0.069	0.056
0.90	0.416	0.351	0.291	0.239	0.195	0.158	0.128	0.104	0.085	0.070
1.00	0.419	0.360	0.305	0.256	0.214	0.177	0.147	0.122	0.101	0.084
1.10	0.420	0.366	0.316	0.271	0.230	0.194	0.164	0.138	0.116	0.098
1.20	0.419	0.371	0.325	0.282	0.243	0.209	0.178	0.152	0.130	0.111
1.30	0.417	0.373	0.331	0.291	0.254	0.221	0.191	0.166	0.143	0.123
1.40	0.414	0.374	0.335	0.298	0.263	0.232	0.203	0.177	0.155	0.135
1.50	0.411	0.374	0.338	0.303	0.271	0.240	0.213	0.188	0.165	0.146
1.60	0.407	0.373	0.339	0.307	0.276	0.248	0.221	0.197	0.175	0.155
1.70	0.402	0.370	0.339	0.309	0.281	0.254	0.228	0.205	0.183	0.164
1.80	0.396	0.368	0.339	0.311	0.284	0.258	0.234	0.212	0.191	0.172
1.90	0.391	0.364	0.338	0.312	0.286	0.262	0.239	0.217	0.197	0.179
2.00	0.385	0.360	0.336	0.311	0.288	0.265	0.243	0.222	0.203	0.185
2.10	0.379	0.356	0.333	0.311	0.288	0.267	0.246	0.226	0.208	0.190
2.20	0.373	0.352	0.330	0.309	0.288	0.268	0.248	0.229	0.212	0.195
2.30	0.366	0.347	0.327	0.307	0.288	0.268	0.250	0.232	0.215	0.199
2.40	0.360	0.342	0.323	0.305	0.287	0.268	0.251	0.234	0.217	0.202
2.50	0.354	0.337	0.320	0.302	0.285	0.268	0.251	0.235	0.220	0.205
2.60	0.347	0.332	0.316	0.299	0.283	0.267	0.251	0.236	0.221	0.207
2.70	0.341	0.327	0.312	0.296	0.281	0.266	0.251	0.236	0.222	0.208
2.80	0.335	0.321	0.307	0.293	0.279	0.265	0.250	0.236	0.223	0.210
2.90	0.329	0.316	0.303	0.290	0.276	0.263	0.249	0.236	0.223	0.211
3.00	0.323	0.311	0.299	0.286	0.274	0.261	0.248	0.236	0.223	0.211
3.10	0.317	0.306	0.294	0.283	0.271	0.259	0.247	0.235	0.223	0.212
3.20	0.311	0.301	0.290	0.279	0.268	0.256	0.245	0.234	0.223	0.212
3.30	0.305	0.296	0.286	0.275	0.265	0.254	0.243	0.232	0.222	0.211
3.40	0.300	0.291	0.281	0.271	0.261	0.251	0.241	0.231	0.221	0.211
3.50	0.294	0.286	0.277	0.268	0.258	0.249	0.239	0.229	0.220	0.210
3.60	0.289	0.281	0.273	0.264	0.255	0.246	0.237	0.228	0.218	0.209
3.70	0.284	0.276	0.268	0.260	0.252	0.243	0.235	0.226	0.217	0.208
3.80	0.279	0.272	0.264	0.256	0.249	0.240	0.232	0.224	0.216	0.207
3.90	0.274	0.267	0.260	0.253	0.245	0.238	0.230	0.222	0.214	0.206
4.00	0.269	0.263	0.256	0.249	0.242	0.235	0.227	0.220	0.212	0.205
4.10	0.264	0.258	0.252	0.246	0.239	0.232	0.225	0.218	0.211	0.203
4.20	0.260	0.254	0.248	0.242	0.236	0.229	0.222	0.216	0.209	0.202
4.30	0.255	0.250	0.244	0.239	0.233	0.226	0.220	0.213	0.207	0.200
4.40	0.251	0.246	0.241	0.235	0.229	0.224	0.217	0.211	0.205	0.199
4.50	0.247	0.242	0.237	0.232	0.226	0.221	0.215	0.209	0.203	0.197
4.60	0.243	0.238	0.234	0.229	0.223	0.218	0.212	0.207	0.201	0.195
4.70	0.239	0.235	0.230	0.225	0.220	0.215	0.210	0.205	0.199	0.194
4.80	0.235	0.231	0.227	0.222	0.217	0.213	0.208	0.202	0.197	0.192
4.90	0.231	0.227	0.223	0.219	0.215	0.210	0.205	0.200	0.195	0.190
5.00	0.227	0.224	0.220	0.216	0.212	0.207	0.203	0.198	0.193	0.188

https://www.ajdesigner.com/phpinterpolation/bilinear_interpolation_equation.php

Vertical Stress Caused by a Vertical Strip Load

- The figure shows a contour plot of the variation in ratio $(\Delta\sigma_z/q)$ at any point with the distance from the foundation
  - What can you notice from the plot ??

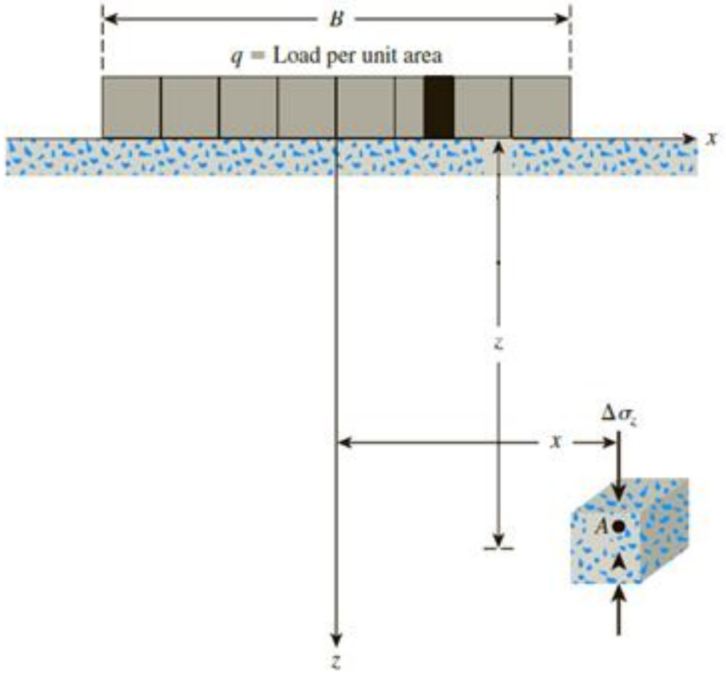


Vertical Stress Caused by a Vertical Strip Load

- Example:

  Given: B = 4 m and q = 100 kN/m^2. For point A, z = 1 m and x = 1 m.

  Determine the vertical stress at A.



Linearly Increasing Vertical Loading on an Infinite Strip

- The vertical stress increase $\Delta\sigma_z$ at a point A(x,z) can be given as

$$\Delta\sigma_z = \frac{q}{2\pi}\left(\frac{2x}{B}\alpha - \sin 2\delta\right)$$

- Re-arranging this equation and generate tables for the variation of $(\Delta\sigma_z/q)$

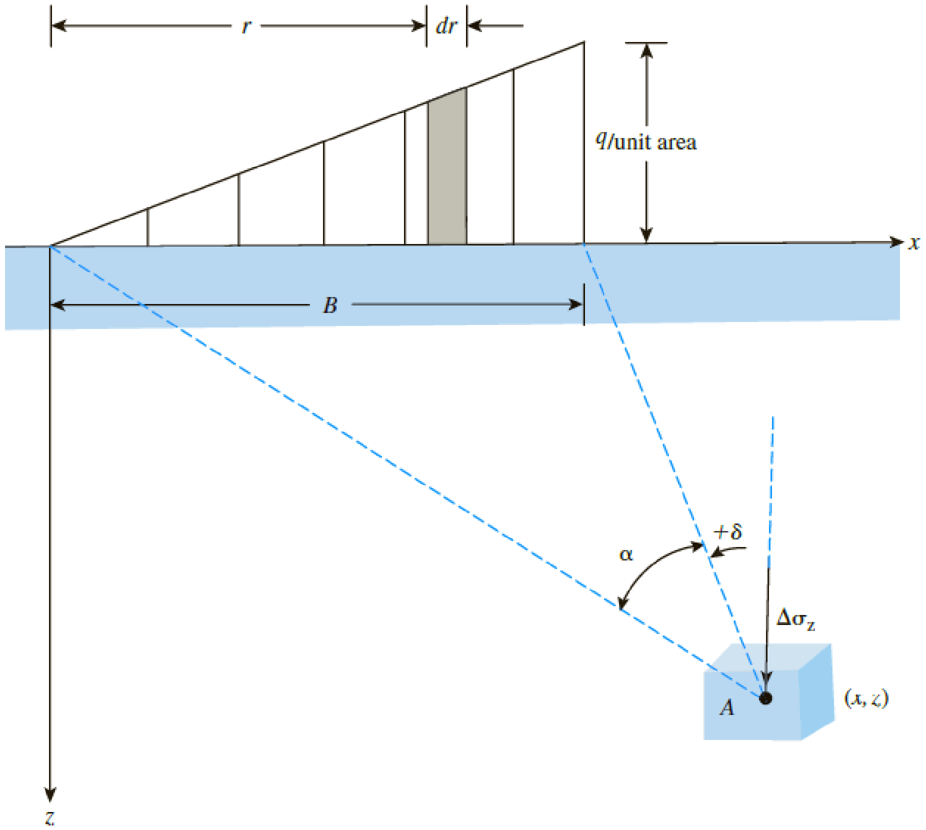


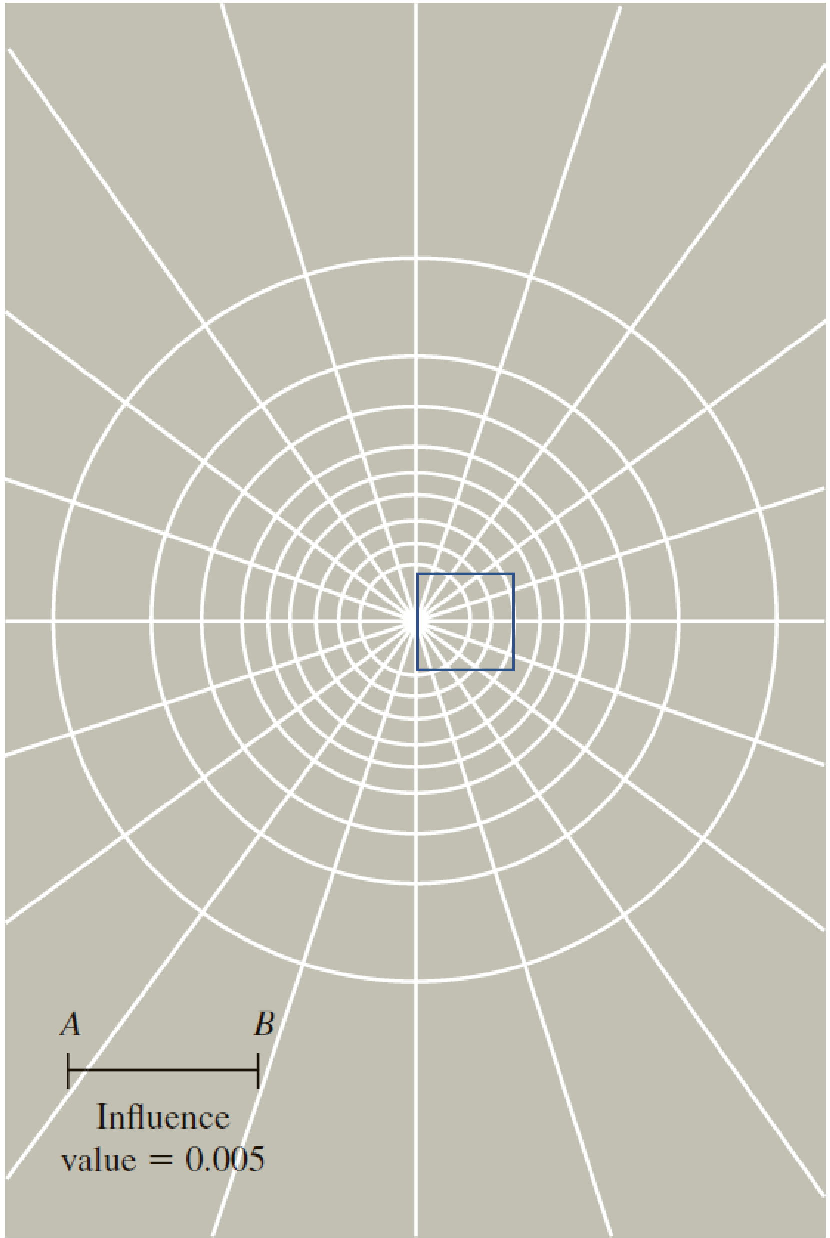
Table 10.6 Variation of $\Delta\sigma_z/q$ with $2x/B$ and $2z/B$ [Eq. (10.21)]

$\frac{2x}{B}$	2z/B: 0	0.5	1.0	1.5	2.0	2.5	3.0	4.0	5.0
−3	0	0.0003	0.0018	0.00054	0.0107	0.0170	0.0235	0.0347	0.0422
−2	0	0.0008	0.0053	0.0140	0.0249	0.0356	0.0448	0.0567	0.0616
−1	0	0.0041	0.0217	0.0447	0.0643	0.0777	0.0854	0.0894	0.0858
0	0	0.0748	0.1273	0.1528	0.1592	0.1553	0.1469	0.1273	0.1098
1	0.5	0.4797	0.4092	0.3341	0.2749	0.2309	0.1979	0.1735	0.1241
2	0.5	0.4220	0.3524	0.2952	0.2500	0.2148	0.1872	0.1476	0.1211
3	0	0.0152	0.0622	0.1010	0.1206	0.1268	0.1258	0.1154	0.1026
4	0	0.0019	0.0119	0.0285	0.0457	0.0596	0.0691	0.0775	0.0776
5	0	0.0005	0.0035	0.0097	0.0182	0.0274	0.0358	0.0482	0.0546

Influence charts vertical stress Caused by any uniformly loaded shape

- Instead of dealing with different equations this influence chart was created to find the increase in vertical stress due and uniformly loaded area.
- The Chart is divided into 9 concentric circles and 20 radial lines forming influence elements
- Each square area has an influence value of 0.005
  - Step1: Determine the depth z below the uniformly loaded area at which the stress increase is required.
  - Step 2: Plot the plan of the loaded area with a scale of z equal to the unit length of the chart (AB).
  - Step 3: Place the plan (plotted in step 2) on the influence chart in such a way that the point below which the stress is to be determined is located at the center of the chart
  - Step 4: Count the number of elements (M) of the chart enclosed by the plan of the loaded area.
  - Step 5: The increase in the pressure at the point under consideration is given by

$$\Delta\sigma_z = (IV)qM$$

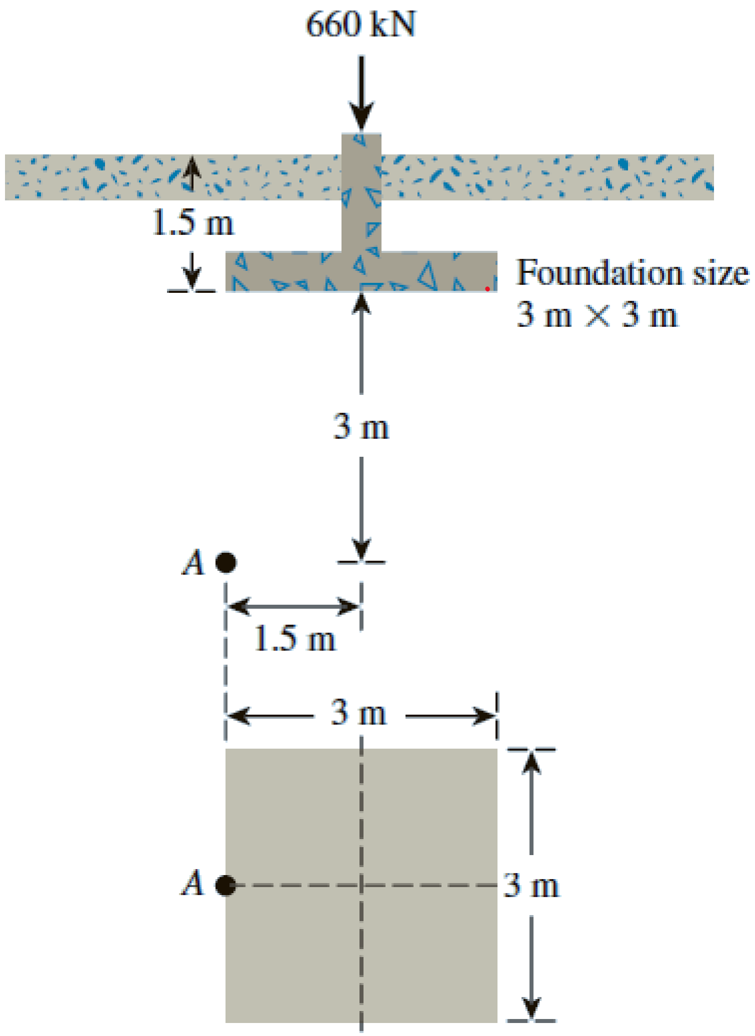


Influence charts vertical stress Caused by any shape

- Example

The cross section and plan of a column foundation are shown in the Figure.

Find the increase in vertical stress produced by the column footing at point A.



$$\Delta\sigma_z = (IV)qM$$

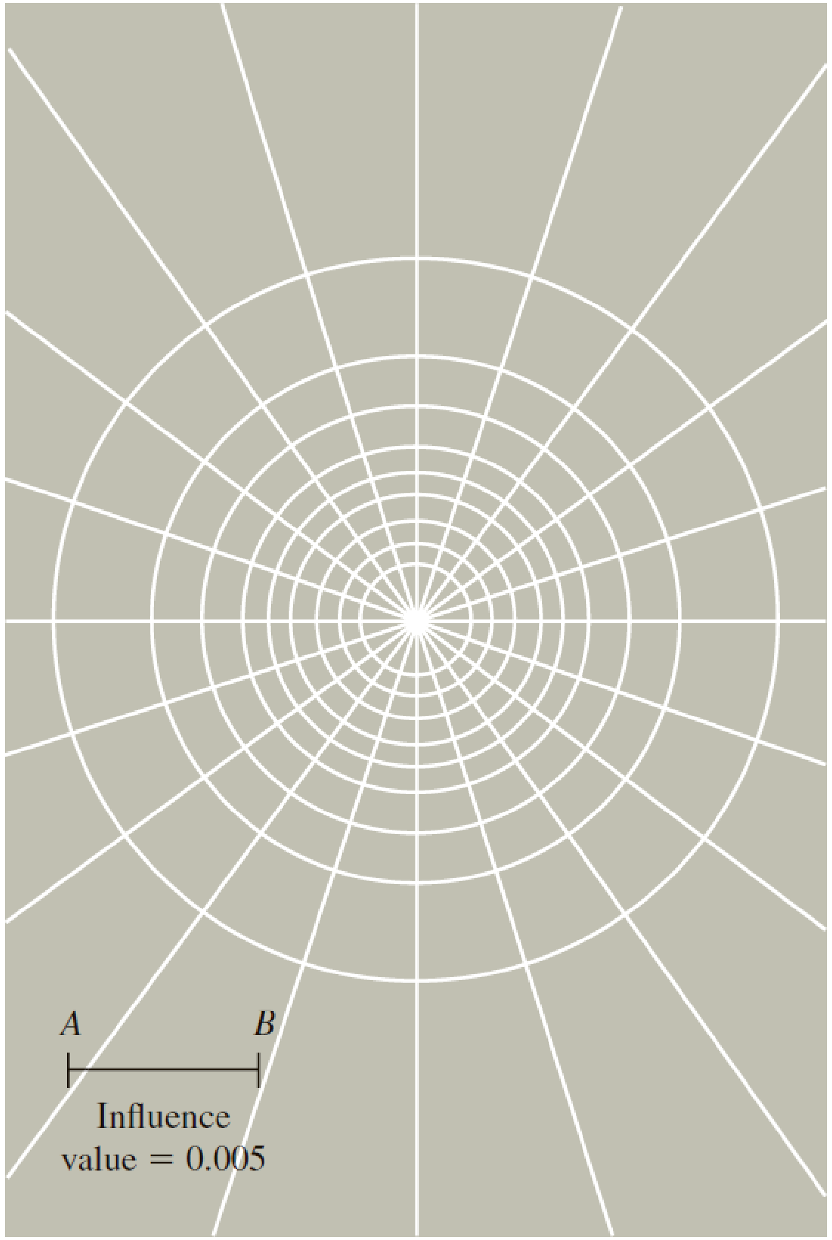
Influence charts vertical stress Caused by any shape

- Example

The cross section and plan of a column foundation are shown in the Figure.

Find the increase in vertical stress produced by the column footing at point A.

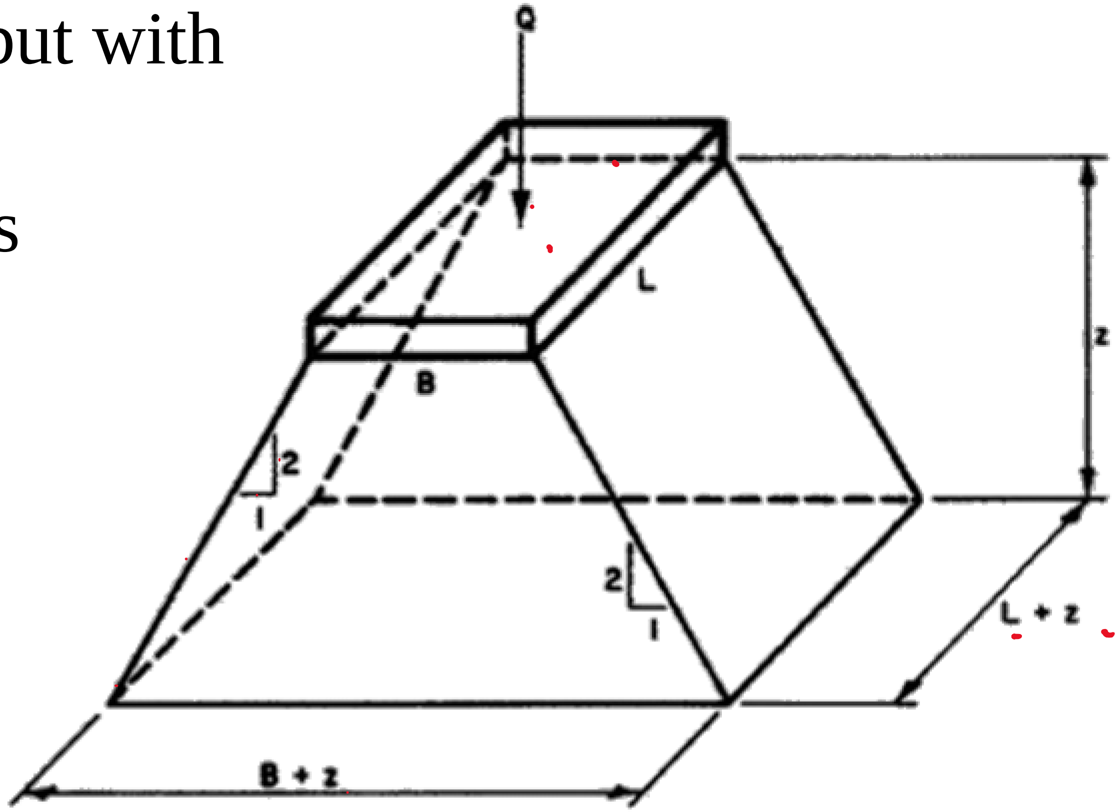
$$\Delta\sigma_z = (IV)qM$$



Approximate method vertical stress Caused by any shape

- This method Assumes that the load transfers in 2:1 slope with depth
- This results in an area with the same shape but with increased dimensions
- The total load on the foundation (Q in kN) is uniformly distributed over the new area
- The stress increase at depth z:

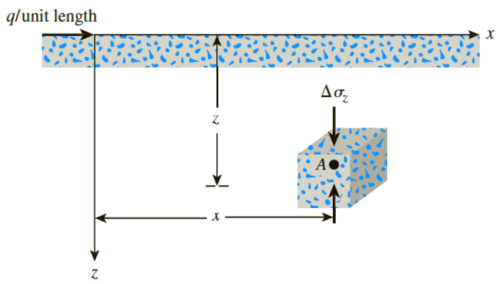
$$\Delta\sigma_z = \frac{Q}{(B+z)(L+z)}$$



Vertical Stress Caused by a Horizontal Line Load

- Horizontal flexible line load on the surface of a semi-infinite soil mass.

$$\Delta\sigma_z = \frac{2q}{\pi}\frac{xz^2}{(x^2+z^2)^2}$$



- To makes it easy, we can re-write the equation and generate a table containing the variation of value of left side of the equation with the variation of the ratio x/z

$$\frac{\Delta\sigma_z}{(q/z)} = \frac{2(x/z)}{\pi[(x/z)^2+1]^2}$$

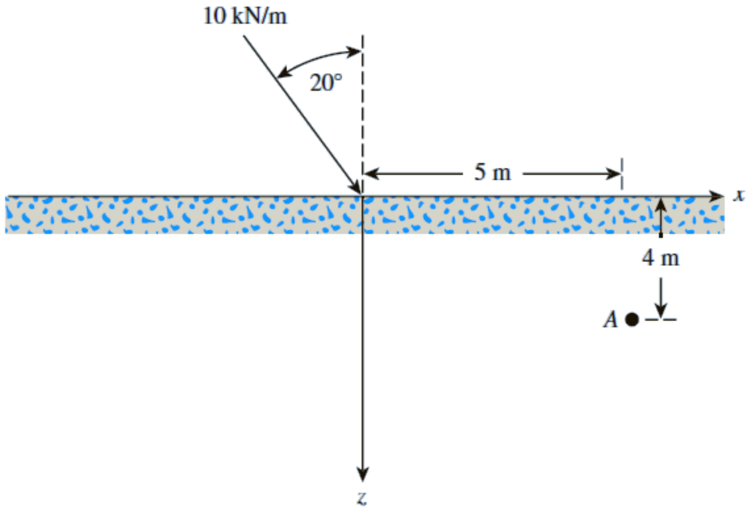
Table 10.3 Variation of $\Delta\sigma_z/(q/z)$ with x/z

x/z	$\Delta\sigma_z/(q/z)$	x/z	$\Delta\sigma_z/(q/z)$
0	0	0.7	0.201
0.1	0.062	0.8	0.189
0.2	0.118	0.9	0.175
0.3	0.161	1.0	0.159
0.4	0.189	1.5	0.090
0.5	0.204	2.0	0.051
0.6	0.207	3.0	0.019

Vertical Stress Caused by a Horizontal Line Load

- Example:

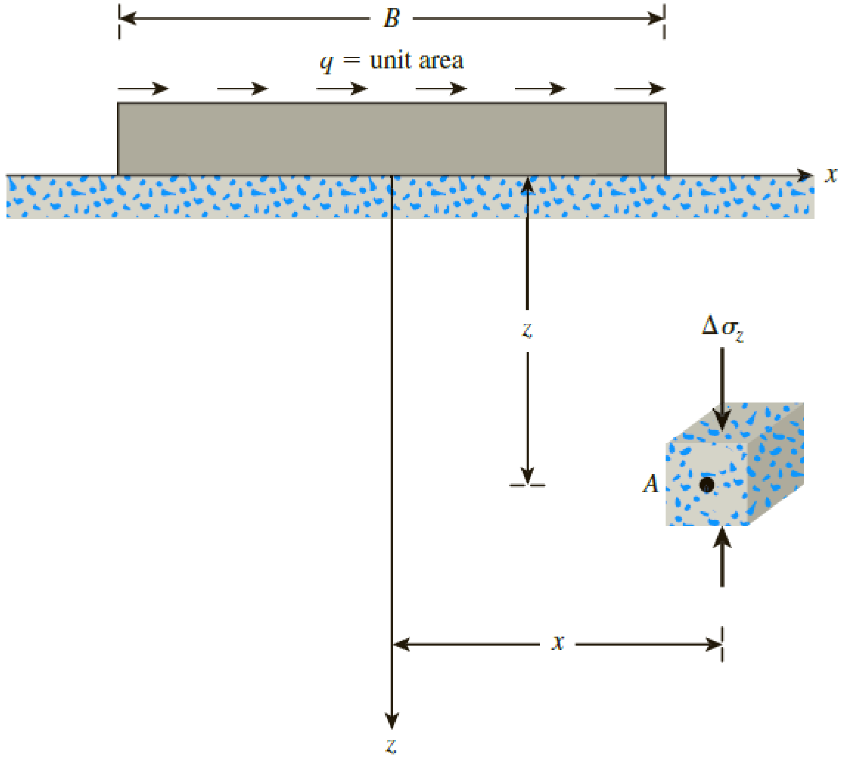
  An inclined line load with a magnitude of 10 kN/m is shown. Determine the increase of vertical stress at point A due to the line load.



Vertical Stress Caused by a Horizontal Strip Load

- The vertical stress increase $\Delta\sigma_z$ at a point A(x,z) can be given as

$$\Delta\sigma_z = \frac{4bqxz^2}{\pi[(x^2 + z^2 - b^2)^2 + 4b^2z^2]}$$



- Re-arranging this equation and generate tables for the variation of $(\Delta\sigma_z/q)$

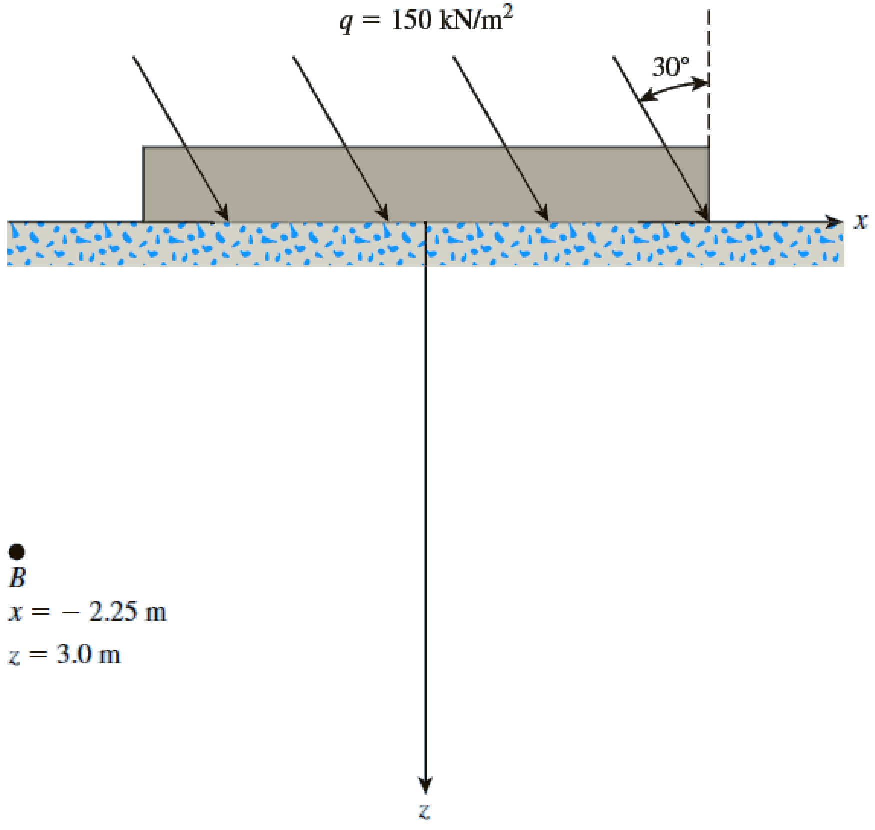
Table 10.5 Variation of $\Delta\sigma_z/q$ with z/b and x/b [Eq. (10.20)]

	x/b					
z/b	0	0.5	1.0	1.5	2.0	2.5
0	—	—	—	—	—	—
0.25	—	0.052	0.313	0.061	0.616	
0.5	—	0.127	0.300	0.147	0.055	0.025
1.0	—	0.159	0.255	0.210	0.131	0.074
1.5	—	0.128	0.204	0.202	0.157	0.110
2.0	—	0.096	0.159	0.175	0.157	0.126
2.5	—	0.072	0.124	0.147	0.144	0.127

Vertical Stress Caused by a Horizontal Strip Load

- Example:

  Consider the inclined strip load shown in the figure. Determine the vertical stress $\Delta\sigma_z$ at B (x = –2.25 m, z = 3 m). Given: width of the strip 5 = m.



Vertical Stress at Any Point below a Uniformly Loaded Circular Area

- For this two-dimensional loading condition, the vertical stress increase may be expressed as

$$\Delta\sigma_z = q(A' + B')$$

where A' and B' are functions of z/R and r/R.

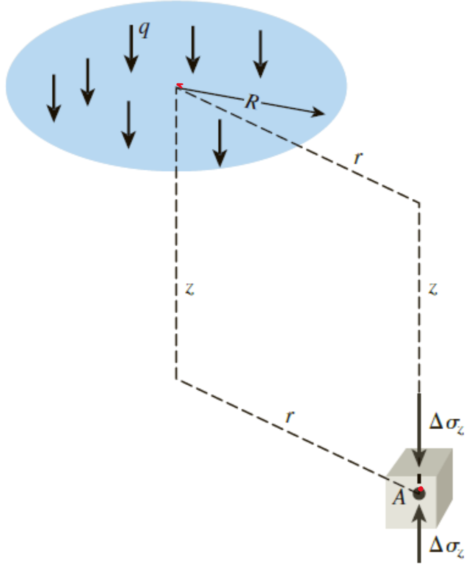


Table 10.8 Variation of A' with z/R and r/R*

z/R	r/R = 0	0.2	0.4	0.6	0.8	1	1.2	1.5	2
0	1.0	1.0	1.0	1.0	1.0	0.5	0	0	0
0.1	0.90050	0.89748	0.88679	0.86126	0.78797	0.43015	0.09645	0.02787	0.00856
0.2	0.80388	0.79824	0.77884	0.73483	0.63014	0.38269	0.15433	0.05251	0.01680
0.3	0.71265	0.70518	0.68316	0.62690	0.52081	0.34375	0.17964	0.07199	0.02440
0.4	0.62861	0.62015	0.59241	0.53767	0.44329	0.31048	0.18709	0.08593	0.03118
0.5	0.55279	0.54403	0.51622	0.46448	0.38390	0.28156	0.18556	0.09499	0.03701
0.6	0.48550	0.47691	0.45078	0.40427	0.33676	0.25588	0.17952	0.10010	
0.7	0.42654	0.41874	0.39491	0.35428	0.29833	0.21727	0.17124	0.10228	0.04558
0.8	0.37531	0.36832	0.34729	0.31243	0.26581	0.21297	0.16206	0.10236	
0.9	0.33104	0.32492	0.30669	0.27707	0.23832	0.19488	0.15253	0.10094	
1	0.29289	0.28763	0.27005	0.24697	0.21468	0.17868	0.14329	0.09849	0.05185
1.2	0.23178	0.22795	0.21662	0.19890	0.17626	0.15101	0.12570	0.09192	0.05260
1.5	0.16795	0.16552	0.15877	0.14804	0.13436	0.11892	0.10296	0.08048	0.05116
2	0.10557	0.10453	0.10140	0.09647	0.09011	0.08269	0.07471	0.06275	0.04496
2.5	0.07152	0.07098	0.06947	0.06698	0.06373	0.05974	0.05555	0.04880	0.03787
3	0.05132	0.05101	0.05022	0.04886	0.04707	0.04487	0.04241	0.03839	0.03150
4	0.02986	0.02976	0.02907	0.02802	0.02832	0.02749	0.02651	0.02490	0.02193
5	0.01942	0.01938				0.01835			0.01573
6	0.01361					0.01307			0.01168
7	0.01005					0.00976			0.00894
8	0.00772					0.00755			0.00703
9	0.00612					0.00600			0.00566
10								0.00477	0.00465

Table 10.8 (*continued*)

z/R	r/R = 3	4	5	6	7	8	10	12	14
0	0	0	0	0	0	0	0	0	0
0.1	0.00211	0.00084	0.00042						
0.2	0.00419	0.00167	0.00083	0.00048	0.00030	0.00020			
0.3	0.00622	0.00250							
0.4									
0.5	0.01013	0.00407	0.00209	0.00118	0.00071	0.00053	0.00025	0.00014	0.00009
0.6									
0.7									
0.8									
0.9									
1	0.01742	0.00761	0.00393	0.00226	0.00143	0.00097	0.00050	0.00029	0.00018
1.2	0.01935	0.00871	0.00459	0.00269	0.00171	0.00115			
1.5	0.02142	0.01013	0.00548	0.00325	0.00210	0.00141	0.00073	0.00043	0.00027
2	0.02221	0.01160	0.00659	0.00399	0.00264	0.00180	0.00094	0.00056	0.00036
2.5	0.02143	0.01221	0.00732	0.00463	0.00308	0.00214	0.00115	0.00068	0.00043
3	0.01980	0.01220	0.00770	0.00505	0.00346	0.00242	0.00132	0.00079	0.00051
4	0.01592	0.01109	0.00768	0.00536	0.00384	0.00282	0.00160	0.00099	0.00065
5	0.01249	0.00949	0.00708	0.00527	0.00394	0.00298	0.00179	0.00113	0.00075
6	0.00983	0.00795	0.00628	0.00492	0.00384	0.00299	0.00188	0.00124	0.00084
7	0.00784	0.00661	0.00548	0.00445	0.00360	0.00291	0.00193	0.00130	0.00091
8	0.00635	0.00554	0.00472	0.00398	0.00332	0.00276	0.00189	0.00134	0.00094
9	0.00520	0.00466	0.00409	0.00353	0.00301	0.00256	0.00184	0.00133	0.00096
10	0.00438	0.00397	0.00352	0.00326	0.00273	0.00241			

Vertical Stress at Any Point below a Uniformly Loaded Circular Area

- For this two-dimensional loading condition, the vertical stress increase may be expressed as

$$\Delta\sigma_z = q(A' + B')$$

where A' and B' are functions of z/R and r/R.

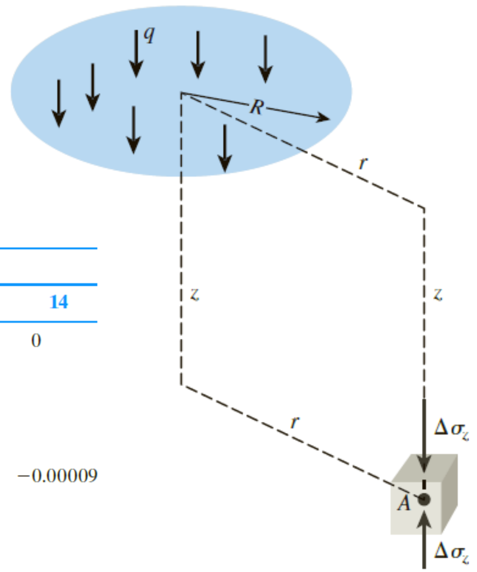


Table 10.9 Variation of B' with z/R and r/R*

z/R	r/R = 0	0.2	0.4	0.6	0.8	1	1.2	1.5	2
0	0	0	0	0	0	0	0	0	0
0.1	0.09852	0.10140	0.11138	0.13424	0.18796	0.05388	−0.07899	−0.02672	−0.00845
0.2	0.18857	0.19306	0.20772	0.23524	0.25983	0.08513	−0.07759	−0.04448	−0.01593
0.3	0.26362	0.26787	0.28018	0.29483	0.27257	0.10757	−0.04316	−0.04999	−0.02166
0.4	0.32016	0.32259	0.32748	0.32273	0.26925	0.12404	−0.00766	−0.04535	−0.02522
0.5	0.35777	0.35752	0.35323	0.33106	0.26236	0.13591	0.02165	−0.03455	−0.02651
0.6	0.37831	0.37531	0.36308	0.32822	0.25411	0.14440	0.04457	−0.02101	
0.7	0.38487	0.37962	0.36072	0.31929	0.24638	0.14986	0.06209	−0.00702	−0.02329
0.8	0.38091	0.37408	0.35133	0.30699	0.23779	0.15292	0.07530	0.00614	
0.9	0.36962	0.36275	0.33734	0.29299	0.22891	0.15404	0.08507	0.01795	
1	0.35355	0.34553	0.32075	0.27819	0.21978	0.15355	0.09210	0.02814	−0.01005
1.2	0.31485	0.30730	0.28481	0.24836	0.20113	0.14915	0.10002	0.04378	0.00023
1.5	0.25602	0.25025	0.23338	0.20694	0.17368	0.13732	0.10193	0.05745	0.01385
2	0.17889	0.18144	0.16644	0.15198	0.13375	0.11331	0.09254	0.06371	0.02836
2.5	0.12807	0.12633	0.12126	0.11327	0.10298	0.09130	0.07869	0.06022	0.03429
3	0.09487	0.09394	0.09099	0.08635	0.08033	0.07325	0.06551	0.05354	0.03511
4	0.05707	0.05666	0.05562	0.05383	0.05145	0.04773	0.04532	0.03995	0.03066
5	0.03772	0.03760				0.03384			0.02474
6	0.02666					0.02468			0.01968
7	0.01980					0.01868			0.01577
8	0.01526					0.01459			0.01279
9	0.01212					0.01170			0.01054
10								0.00924	0.00879

Table 10.9 (*continued*)

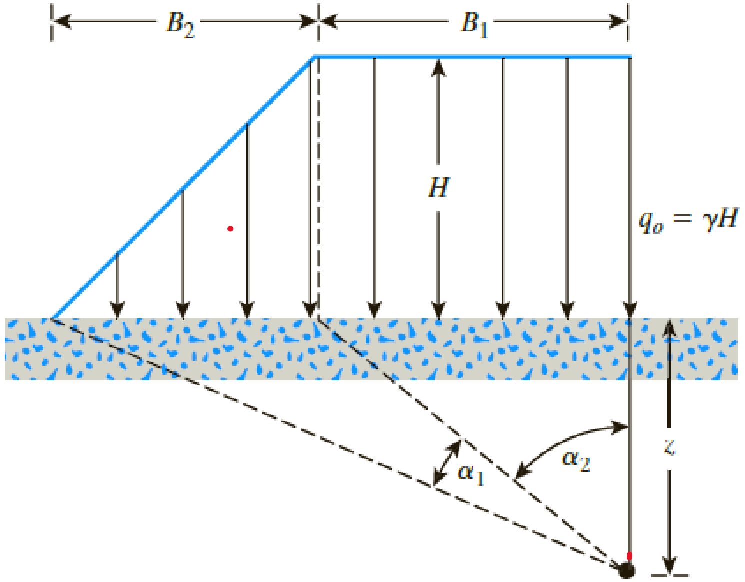
z/R	r/R = 3	4	5	6	7	8	10	12	14
0	0	0	0	0	0	0	0	0	0
0.1	−0.00210	−0.00084	−0.00042						
0.2	−0.00412	−0.00166	−0.00083	−0.00024	−0.00015	−0.00010			
0.3	−0.00599	−0.00245							
0.4									
0.5	−0.00991	−0.00388	−0.00199	−0.00116	−0.00073	−0.00049	−0.00025	−0.00014	−0.00009
0.6									
0.7									
0.8									
0.9									
1	−0.01115	−0.00608	−0.00344	−0.00210	−0.00135	−0.00092	−0.00048	−0.00028	−0.00018
1.2	−0.00995	−0.00632	−0.00378	−0.00236	−0.00156	−0.00107			
1.5	−0.00669	−0.00600	−0.00401	−0.00265	−0.00181	−0.00126	−0.00068	−0.00040	−0.00026
2	0.00028	−0.00410	−0.00371	−0.00278	−0.00202	−0.00148	−0.00084	−0.00050	−0.00033
2.5	0.00661	−0.00130	−0.00271	−0.00250	−0.00201	−0.00156	−0.00094	−0.00059	−0.00039
3	0.01112	0.00157	−0.00134	−0.00192	−0.00179	−0.00151	−0.00099	−0.00065	−0.00046
4	0.01515	0.00595	0.00155	−0.00029	−0.00094	−0.00109	−0.00094	−0.00068	−0.00050
5	0.01522	0.00810	0.00371	0.00132	0.00013	−0.00043	−0.00070	−0.00061	−0.00049
6	0.01380	0.00867	0.00496	0.00254	0.00110	0.00028	−0.00037	−0.00047	−0.00045
7	0.01204	0.00842	0.00547	0.00332	0.00185	0.00093	−0.00002	−0.00029	−0.00037
8	0.01034	0.00779	0.00554	0.00372	0.00236	0.00141	0.00035	−0.00008	−0.00025
9	0.00888	0.00705	0.00533	0.00386	0.00265	0.00178	0.00066	0.00012	−0.00012
10	0.00764	0.00631	0.00501	0.00382	0.00281	0.00199			

Vertical Stress Due to Embankment Loading

- For this two-dimensional loading condition the vertical stress increase may be expressed as

$$\Delta\sigma_z = \frac{q_o}{\pi}\left[\left(\frac{B_1 + B_2}{B_2}\right)(\alpha_1 + \alpha_2) - \frac{B_1}{B_2}(\alpha_2)\right]$$

$$\alpha_1\,(\text{radians}) = \tan^{-1}\left(\frac{B_1 + B_2}{z}\right) - \tan^{-1}\left(\frac{B_1}{z}\right) \qquad \alpha_2 = \tan^{-1}\left(\frac{B_1}{z}\right)$$



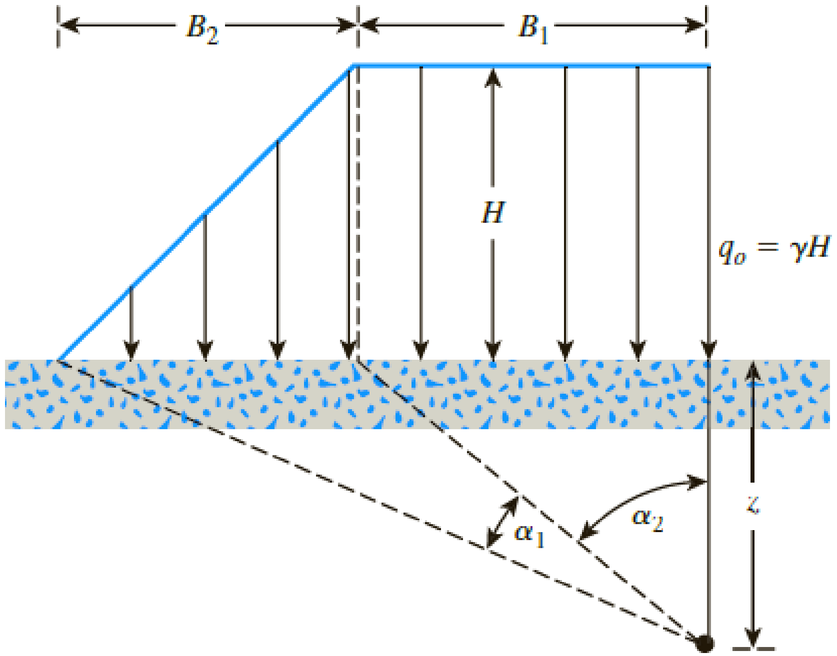
- Re-arranging this equation and a contour plot for the variation of influence factor I_2

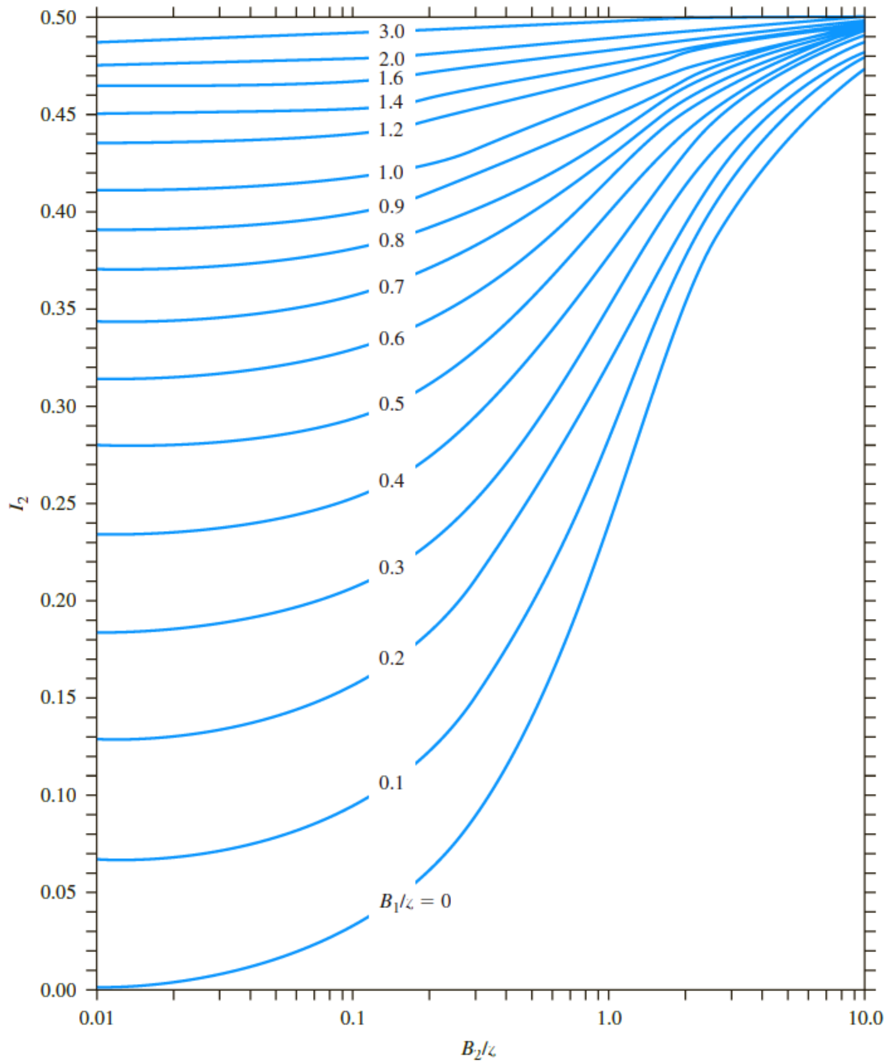
$$\Delta\sigma_z = q_o I_2$$

Vertical Stress Due to Embankment Loading

- Re-arranging this equation and a contour plot for the variation of influence factor I_2

$$\Delta\sigma_z = q_o I_2$$





Vertical Stress Due to Embankment Loading

- Example:
  - An embankment is shown in the Figure. Determine the stress increase under the embankment at points A2.