

# Solved Problems in Soil Mechanics

Based on “Principles of Geotechnical Engineering, 8<sup>th</sup> Edition”



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February -2015

**Chapter (3) & Chapter (6)**

**Soil Properties**

**&**

**Soil Compaction**

## Useful Formulas:

You should know the following formulas:

$$V_{\text{total}} = V_{\text{solid}} + V_{\text{voids}} \rightarrow V_{\text{total}} = V_{\text{solid}} + V_{\text{air}} + V_{\text{water}}$$

$$W_{\text{total}} = W_{\text{solid}} + W_{\text{water}} \rightarrow (W_{\text{air}} = 0, W_{\text{solid}} = W_{\text{dry}})$$

$$\gamma_{\text{dry}} = \frac{G_s \times \gamma_w}{1 + e}, \quad \gamma_{\text{dry}} = \frac{\gamma_{\text{moist}}}{(1 + \%w)}, \quad \gamma_{\text{dry}} = \frac{W_{\text{dry}}}{V_{\text{total}}}, \quad \gamma_{\text{solid}} = \frac{W_{\text{dry}}}{V_{\text{solid}}}$$

$$\gamma_{\text{moist}} = \frac{G_s \times \gamma_w (1 + \%w)}{1 + e}, \quad \gamma_{\text{sat}} = \frac{G_s \times \gamma_w \left(1 + \frac{e}{G_s}\right)}{1 + e} \rightarrow (S = 1)$$

$$\gamma_{\text{Z.A.V}} = \frac{G_s \times \gamma_w}{1 + G_s w} \rightarrow (S = 1 \rightarrow e = e_{\text{min}} = G_s w / 1)$$

$$S \cdot e = G_s \cdot w, \quad S = \frac{V_{\text{water}}}{V_{\text{voids}}}, \quad \left(\text{at saturation} \rightarrow S = 1 \rightarrow w_{\text{sat}} = \frac{e}{G_s}\right)$$

$$w = \frac{\text{Weight of water}}{\text{Weight of solid}} = \frac{W_w}{W_s} = \frac{W_{\text{wet}} - W_{\text{dry}}}{W_{\text{dry}}} \times 100\%$$

$$e = \frac{V_{\text{voids}}}{V_{\text{solid}}} = \frac{V_T - V_s}{V_s}, \quad n = \frac{e}{1 + e}, \quad n = \frac{V_{\text{voids}}}{V_{\text{total}}}$$

$$G_s = \frac{\gamma_{\text{solid}}}{\gamma_{\text{water}}}, \quad \gamma_{\text{solid}} = \frac{W_{\text{dry}}}{V_{\text{solid}}}, \quad \gamma_{\text{water}} = \frac{W_{\text{water}}}{V_{\text{water}}}$$

$$\gamma_{\text{water}} = 9.81 \text{KN/m}^3 = 62.4 \text{lb/ft}^3, \quad 1 \text{ton} = 2000 \text{lb}, \quad 1 \text{yd}^3 = 27 \text{ft}^3$$

$$\text{Air content (A)} = \frac{V_{\text{air}}}{V_{\text{total}}}$$

$$D_r = \frac{e_{\text{max}} - e}{e_{\text{max}} - e_{\text{min}}}$$

$$\text{Relative Compaction (R.C)} = \frac{\gamma_{\text{dry,max,field}}}{\gamma_{\text{dry,max,proctor}}} \times 100\%$$

$V_{\text{solid}}$  must be constant if we want to use the borrow pit soil in a construction site or on earth dam or anywhere else.

**1. (Mid 2014):**

a) Show the saturated moisture content is:  $W_{\text{sat}} = \gamma_w \left[ \frac{1}{\gamma_d} - \frac{1}{\gamma_s} \right]$

**Hint:**  $\gamma_s$  = solid unit weight

**Solution**

S.  $e = G_s \cdot w$  , at saturation  $\rightarrow S = 1 \rightarrow w_{\text{sat}} = \frac{e}{G_s} \rightarrow \rightarrow$  Eq. (1)

$$\gamma_d = \frac{G_s \times \gamma_w}{1 + e} \rightarrow e = \frac{G_s \times \gamma_w}{\gamma_d} - 1, \text{ substitute in Eq. (1)} \rightarrow \rightarrow$$

$$w_{\text{sat}} = \frac{\frac{G_s \times \gamma_w}{\gamma_d} - 1}{G_s} = \frac{\gamma_w}{\gamma_d} - \frac{1}{G_s} \quad \text{but } G_s = \frac{\gamma_s}{\gamma_w} \rightarrow \frac{1}{G_s} = \frac{\gamma_w}{\gamma_s} \rightarrow \rightarrow$$

$$w_{\text{sat}} = \frac{\gamma_w}{\gamma_d} - \frac{\gamma_w}{\gamma_s} = \gamma_w \left[ \frac{1}{\gamma_d} - \frac{1}{\gamma_s} \right] \checkmark .$$

b) A geotechnical laboratory reported these results of five samples taken from a single boring. Determine which are not correctly reported, if any, show your work.

**Hint:** take  $\gamma_w = 9.81 \text{ kN/m}^3$

Sample #1:  $w = 30\%$ ,  $\gamma_d = 14.9 \text{ kN/m}^3$ ,  $\gamma_s = 27 \text{ kN/m}^3$  , clay

Sample #2:  $w = 20\%$ ,  $\gamma_d = 18 \text{ kN/m}^3$ ,  $\gamma_s = 27 \text{ kN/m}^3$  , silt

Sample #3:  $w = 10\%$ ,  $\gamma_d = 16 \text{ kN/m}^3$ ,  $\gamma_s = 26 \text{ kN/m}^3$  , sand

Sample #4:  $w = 22\%$ ,  $\gamma_d = 17.3 \text{ kN/m}^3$ ,  $\gamma_s = 28 \text{ kN/m}^3$  , silt

Sample #5:  $w = 22\%$ ,  $\gamma_d = 18 \text{ kN/m}^3$ ,  $\gamma_s = 27 \text{ kN/m}^3$  , silt

**Solution**

For any type of soil, the moisture content ( $w$ ) must not exceed the saturated moisture content, so for each soil we calculate the saturated moisture content from the derived equation in part (a) and compare it with the given water content.

Sample #1: (Given water content = 30%)

$$w_{\text{sat}} = 9.81 \left[ \frac{1}{14.9} - \frac{1}{27} \right] = 29.5\% < 30\% \rightarrow \text{not correctly reported} \checkmark .$$

Sample #2: (Given water content = 20%)

$$w_{\text{sat}} = 9.81 \left[ \frac{1}{18} - \frac{1}{27} \right] = 18.16\% < 20\% \rightarrow \text{not correctly reported} \checkmark .$$

Sample #3: (Given water content= 10%)

$$w_{\text{sat}} = 9.81 \left[ \frac{1}{16} - \frac{1}{26} \right] = 23.58\% > 10\% \rightarrow \text{correctly reported} \checkmark .$$

Sample #4: (Given water content= 22%)

$$w_{\text{sat}} = 9.81 \left[ \frac{1}{17.3} - \frac{1}{28} \right] = 21.67\% < 22\% \rightarrow \text{not correctly reported} \checkmark .$$

Sample #5: (Given water content= 22%)

$$w_{\text{sat}} = 9.81 \left[ \frac{1}{18} - \frac{1}{27} \right] = 18.16\% < 22\% \rightarrow \text{not correctly reported} \checkmark .$$

## 2. (Mid 2013):

If a soil sample has a dry unit weight of  $19.5 \text{ KN/m}^3$ , moisture content of 8% and a specific gravity of solids particles is 2.67. Calculate the following:

- The void ratio.
- Moisture and saturated unit weight.
- The mass of water to be added to cubic meter of soil to reach 80% saturation.
- The volume of solids particles when the mass of water is 25 grams for saturation.

### Solution

#### Givens:

$$\gamma_{\text{dry}} = 19.5 \text{ KN/m}^3, \quad \%w = 8\%, \quad G_s = 2.67$$

a)

$$\gamma_{\text{dry}} = \frac{G_s \times \gamma_w}{1 + e} \rightarrow 19.5 = \frac{2.67 \times 9.81}{1 + e} \rightarrow e = 0.343 \checkmark$$

b)

$$* \gamma_{\text{moist}} = \gamma_{\text{dry}}(1 + \%w) = 19.5 \times (1 + 0.08) = 21.06 \text{ KN/m}^3 \checkmark$$

$$* \gamma_{\text{sat}} = \gamma_{\text{dry}}(1 + \%w_{\text{sat}}) \rightarrow \%w_{\text{sat}} \text{ means } \%w @ S = 100\%$$

$$S.e = G_s.w \rightarrow \%w_{\text{sat}} = \frac{S.e}{G_s} = \frac{1 \times 0.343}{2.67} \times 100\% = 12.85\%$$

$$\text{So, } \gamma_{\text{sat}} = 19.5(1 + 0.1285) = 22 \text{ KN/m}^3 \checkmark$$

c)

$$\gamma_{\text{moist}} = 21.06 \text{ KN/m}^3 \rightarrow \text{وهي القيمة الأصلية الموجودة في المسألة}$$

Now we want to find  $\gamma_{\text{moist}} @ 80\%$  Saturation so, firstly we calculate  $\%w @ 80\%$  saturation:

$$\%w_{80\%} = \frac{S.e}{G_s} = \frac{0.8 \times 0.343}{2.67} \times 100\% = 10.27\%$$

$$\gamma_{\text{moist},80\%} = 19.5(1 + 0.1027) = 21.5 \text{ KN/m}^3$$

$$\text{Weight of water to be added} = 21.5 - 21.06 = 0.44 \text{ KN/m}^3 \checkmark$$

$$\text{Mass of water to be added} = 0.44 \times \frac{1000}{9.81} = 44.85 \text{ Kg/m}^3 \checkmark$$

**Another solution:**

$V_T = 1\text{m}^3 \rightarrow$  وهي الكمية التي يجب إضافة الماء إليها والموجودة في نص المطلوب

The water content before adding water ( $\%w_1$ ) = 8%

The water content after adding water ( $\%w_2$ ) = 10.27% @80%saturation

$$w = \frac{\text{Weight of water}}{\text{Weight of solid}} = \frac{W_w}{W_s}$$

**ملاحظة هامة:** لأي عينة تربة دائما وأبدا قيمة  $W_s$  تكون ثابتة ولا تتغير

$$\gamma_{\text{dry}} = \frac{W_s}{V_T} \rightarrow W_s = 19.5 \times 1 = 19.5\text{KN}$$

$$W_w = W_s \times w \rightarrow W_{w,1} = W_s \times w_1, \text{ and } W_{w,2} = W_s \times w_2$$

$$\text{Then, } W_{w,1} = 19.5 \times 0.08 = 1.56 \text{ KN}$$

$$W_{w,2} = 19.5 \times 0.1027 = 2 \text{ KN}$$

$$\text{Weight of water to be added} = 2 - 1.56 = 0.44 \text{ KN } \checkmark$$

$$\text{Mass of water to be added} = 0.44 \times \frac{1000}{9.81} = 44.85 \text{ Kg } \checkmark$$

d)

$$M_w = 25\text{grams for saturation} \rightarrow S = 100\% \rightarrow \%w_{\text{sat}} = 12.85\%$$

$$W_w = (25 \times 10^{-3})\text{Kg} \times \frac{9.81}{1000} = 24.525 \times 10^{-5}\text{KN}$$

$$W_s = \frac{W_w}{w} = \frac{24.525 \times 10^{-5}}{0.1285} = 190.85 \times 10^{-5} \text{ KN}$$

$$\text{Now, } G_s = \frac{\gamma_{\text{solid}}}{\gamma_{\text{water}}} \rightarrow \gamma_{\text{solid}} = 2.67 \times 9.81 = 26.2\text{KN/m}^3$$

$$\gamma_{\text{solid}} = \frac{W_s}{V_s} \rightarrow V_s = \frac{W_s}{\gamma_{\text{solid}}} = \frac{190.85 \times 10^{-5}}{26.2} = 7.284 \times 10^{-5} \text{ m}^3 = 72.84 \text{ cm}^3 \checkmark$$

### 3. (Mid 2013):

An earth dam require **one hundred cubic meter** of soil compacted with unit weight of  $20.5 \text{ KN/m}^3$  and moisture content of 8%, choose two from the three borrow pits given in the table below, **knowing that the first must be one of the two borrow pits**, the specific gravity of solid particles is 2.7. Choose the most economical choice.

Borrow pit No.	Void ratio	Cost(\$/m <sup>3</sup> )	Available volume (m <sup>3</sup> )
1	0.6	1	80
2	1	1.5	100
3	0.75	1.7	100

#### Some Explanations about the problem:

##### ✓ Borrow pits:

هي عينات من التربة تكون متواجدة بكميات معينة ولها خواص معينة وبالتالي تختلف في أسعارها حسب خواصها.

##### ✓ Available Volume:

هو الحجم المتوفر من كل عينة من العينات الموجودة في الجدول.

✓

**شرح السؤال:** يوجد سد رملي بحجم 100 متر مكعب يتطلب إنشاءه من تربة معينة ولها خواص معينة, ويوجد لدينا ثلاث أنواع من التربة كل نوع له خواص معينة وسعر معين ومتوفر بكميات محددة كما هو موجود في الجدول, المطلوب هو اختيار مزيج نوعين من هذه الثلاث أنواع ليتم وضعها في تربة السد الرملي وبالتالي كما نرى أن الشرط الأساسي هو يجب الحفاظ على قيمة  $V_s$  لتربة السد في جميع العينات المتوفرة وأيضا بشرط أن يكون النوع الأول أحد هذين النوعين بالإضافة إلى تحقيق أقل سعر ممكن .

#### Solution

The first step is to find the value of  $V_s$  for earth dam that must be maintained in borrow pits.

$$\gamma_{\text{dry}} = \frac{\gamma_{\text{moist}}}{1 + \%w} = \frac{G_s \times \gamma_w}{1 + e} \rightarrow \frac{20.5}{1 + 0.08} = \frac{2.7 \times 9.81}{1 + e} \rightarrow e = 0.395$$

$$e = \frac{V_T - V_s}{V_s} \rightarrow 0.395 = \frac{100 - V_s}{V_s} \rightarrow V_s = 71.68 \text{ m}^3$$

The value of  $V_s = 71.68 \text{ m}^3$  must be maintained on each borrow pit.



Now we calculate the total volume of each type that required for the dam:

**For borrow pit #1:**

$$e = \frac{V_T - V_s}{V_s} \rightarrow 0.6 = \frac{V_{T,1} - 71.68}{71.68} \rightarrow V_{T,1} = 114.68\text{m}^3$$

**For borrow pit #2:**

$$e = \frac{V_T - V_s}{V_s} \rightarrow 1 = \frac{V_{T,2} - 71.68}{71.68} \rightarrow V_{T,2} = 143.36\text{m}^3$$

**For borrow pit #3:**

$$e = \frac{V_T - V_s}{V_s} \rightarrow 0.75 = \frac{V_{T,3} - 71.68}{71.68} \rightarrow V_{T,3} = 125.44\text{m}^3$$

الآن من معطيات السؤال أنه يجب أن تكون العينة الأولى إحدى العينتين, بالتالي يجب أخذ المتوفر منها كلها والمتبقي يتم إكماله من أحد النوعين الآخرين.

**Total** required volume from borrow pit#1 = 114.68m<sup>3</sup>

The **available** volume from borrow pit #1 = 80m<sup>3</sup>

The **rest** required volume from borrow pit #1 = 114.68 – 80 = 34.68m<sup>3</sup>

الآن نلاحظ أننا حققنا الشرط الأول وهو أن النوع الأول هو من إحدى العينتين, والآن يجب تحقيق الشرط الثاني وهو إيجاد الكمية المطلوبة من العينة الأخرى وبأقل تكلفة ممكنة, وبالتالي يجب إيجاد الحجم الكلي المطلوب من العينتين الباقيتين والذي يكافئ 34.68m<sup>3</sup> من العينة الأولى وبالتالي يجب الحفاظ على حبيبات solid المتبقي من العينة الأولى في العينتين المتبقيتين :

For the rest required from borrow pit #1:

$$e = \frac{V_T - V_s}{V_s} \rightarrow 0.6 = \frac{34.68 - V_{s,rest}}{V_{s,rest}} \rightarrow V_{s,rest} = 21.675\text{m}^3$$

Now, we calculate the required volume from borrow pits 2&3 and calculate the cost of each volume and take the lowest cost soil.

**For borrow pit #2:**

$$e = \frac{V_T - V_s}{V_s} \rightarrow 1 = \frac{V_{T,2} - 21.675}{21.675} \rightarrow V_{T,2} = 43.35\text{m}^3$$

Required cost = 43.35 × 1.5 = 65.025\$

**For borrow pit #3:**

$$e = \frac{V_T - V_s}{V_s} \rightarrow 0.75 = \frac{V_{T,3} - 21.675}{21.675} \rightarrow V_{T,3} = 37.93\text{m}^3$$

Required cost = 37.93 × 1.7 = 64.48\$ → Choose the borrow pit #3 (lowest cost)

So, the two required soils are: 80 m<sup>3</sup> from **borrow pit #1** and 37.93 m<sup>3</sup> from **borrow pit #3** ✓.

#### 4. (Mid 2012):

A soil sample has void ratio of 0.72, moisture content = 12% and  $G_s = 2.72$  determine the following:

- Dry unit weight, moist unit weight ( $\text{KN/m}^3$ ).
- Weight of water in  $\text{KN/m}^3$  to be added for 80% degree of saturation.
- Is it possible to reach a water content of 30% **without change** the present void ratio.
- Is it possible to compact the soil sample to a dry unit weight of  $23.5 \text{ KN/m}^3$ .

#### Solution

#### Givens:

$$e = 0.72 \quad , \quad \%w = 12\% \quad , \quad G_s = 2.72$$

a)

$$* \gamma_{\text{dry}} = \frac{G_s \times \gamma_w}{1 + e} = \frac{2.72 \times 9.81}{1 + 0.72} = 15.51 \text{ KN/m}^3 \quad \checkmark .$$

$$* \gamma_{\text{moist}} = \gamma_{\text{dry}}(1 + \%w) = 15.51 \times (1 + 0.12) = 17.374 \text{ KN/m}^3 \quad \checkmark .$$

b)

The original value of  $\gamma_{\text{moist}} = 17.374 \text{ KN/m}^3$

The value of  $\gamma_{\text{moist}}$  @80% degree of saturation can be calculated as following:

$$S \cdot e = G_s \cdot w \rightarrow \%w_{80\%} = \frac{0.8 \times 0.72}{2.72} = 0.2117$$

$$\rightarrow \gamma_{\text{moist},80\%} = \gamma_{\text{dry}}(1 + \%w) = 15.51 \times (1 + 0.2117) = 18.8 \text{ KN/m}^3 .$$

$$\text{So, the of water added} = 18.8 - 17.374 = 1.428 \text{ KN/m}^3 \quad \checkmark .$$

c)

$$e = 0.72 \quad , \quad \%w = 30\% \quad , \quad G_s = 2.72 \quad , \quad S_{30\%} = ??$$

We know that the max. value of  $S=1$  so, if the value of  $S_{30\%} > 1 \rightarrow$  it's not possible, but if  $S_{30\%} \leq 1 \rightarrow$  it's possible.

$$S \cdot e = G_s \cdot w \rightarrow S_{30\%} = \frac{2.72 \times 0.3}{0.72} \times 100\% = 1.133 > 1 \rightarrow \text{Not possible} \quad \checkmark .$$

d)

$\gamma_{\text{dry,new}} = 23.5 \text{ KN/m}^3 \rightarrow$  Can we reach to this value after compaction???, to

Know this, we find the maximum possible value of  $\gamma_{\text{dry}} = \gamma_{\text{Z.A.V}}$  (Zero Air Voids)

$$\gamma_{\text{Z.A.V}} = \frac{G_s \times \gamma_w}{1 + e_{\text{min}}} \rightarrow e_{\text{min}} = \frac{G_s \cdot w}{S_{\text{max}}} = \frac{2.72 \times 0.12}{1} = 0.3264$$

$$\rightarrow \gamma_{\text{Z.A.V}} = \frac{G_s \times \gamma_w}{1 + e_{\text{min}}} = \frac{2.72 \times 9.81}{1 + 0.3264} = 20.12 < 23.5 \rightarrow \text{Not possible} \quad \checkmark$$

### 5. (Mid 2011):

An undisturbed sample of clayey soil is found to have a wet weight of 285 N, a dry weight of 250 N, and a total volume of  $14 \times 10^3 \text{ cm}^3$  if the specific gravity of soil solids is 2.70, determine the water content, void ratio, and the degree of saturation.

#### Solution

#### Givens:

$$W_{\text{wet}} = 285 \text{ N} , W_{\text{dry}} = 250 \text{ N} , V_T = 14 \times 10^3 \text{ cm}^3 , G_s = 2.7$$

$$* \%W = \frac{W_{\text{wet}} - W_{\text{dry}}}{W_{\text{dry}}} \times 100\% = \frac{285 - 250}{250} \times 100\% = 14\% \checkmark .$$

$$* \gamma_{\text{dry}} = \frac{G_s \times \gamma_w}{1 + e} , \text{ but } \gamma_{\text{dry}} = ??? \rightarrow \gamma_{\text{dry}} = \frac{W_{\text{dry}}}{V_T} = \frac{250 \times 10^{-3}}{(14 \times 10^3) \times 10^{-6}} = 17.86 \text{ kN/m}^3$$

$$\rightarrow 17.86 = \frac{2.7 \times 9.81}{1 + e} \rightarrow e = 0.483 \checkmark .$$

$$* S.e = G_s.w \rightarrow S = \frac{2.7 \times 0.14}{0.483} = 0.7812 = 78.12\% \checkmark .$$

### 6. (Mid 2011):

A proposed earth dam requires  $7500 \text{ m}^3$  of compacted soil with relative density of 94% , maximum void ratio of 0.73, minimum void ratio of 0.4 and specific gravity ( $G_s$ )=2.67. Four borrow pits are available as described in the following table. **Choose the best borrow pit with minimum cost.**

Borrow Pit	Degree of saturation %	Moisture content %	Cost (\$/m <sup>3</sup> )
A	82	18.43	10
B	100	24.34	5

## Solution

## Givens:

$$D_r = 94\% , e_{\max} = 0.73 , e_{\min} = 0.4 , G_s = 2.67$$

فكرة الحل أنه يجب الحفاظ على قيمة  $V_s$  للسد من أي عينة تربة نريد إحضارها للسد (العينات الموجودة في الجدول)

So, firstly we calculate the value of  $V_s$  that required for earth dam as following:

$$e = \frac{V_v}{V_s} = \frac{V_T - V_s}{V_s}, \text{ but } e = ??? \rightarrow D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}} \rightarrow 0.94 = \frac{0.73 - e}{0.73 - 0.4} \rightarrow e = 0.42$$

$$\rightarrow 0.42 = \frac{7500 - V_s}{V_s} \rightarrow V_s = 5281.7 \text{ m}^3 \text{ that must be maintained.}$$

الآن, حتى نجد سعر كل عينة يجب تحديد الحجم الكلي لكل عينة والذي يحقق قيمة  $V_s = 5281.7$

## For sample " A " :

$$S = 82\% , \%w = 18.43\%$$

$$e = \frac{V_T - V_s}{V_s}, \text{ but } e = ??? \rightarrow e = \frac{G_s \times w}{S} = \frac{2.67 \times 0.1843}{0.82} = 0.6$$

$$\rightarrow 0.6 = \frac{V_T - 5281.7}{5281.7} \rightarrow V_T = 8450.72 \text{ m}^3.$$

So, the total cost for sample " A " =  $8450.72 \text{ m}^3 \times 10 \frac{\$}{\text{m}^3} = 84,507\$$ .

## For sample " B " :

$$S = 100\% , \%w = 24.34\%$$

$$e = \frac{V_T - V_s}{V_s}, \text{ but } e = ??? \rightarrow e = \frac{G_s \times w}{S} = \frac{2.67 \times 0.2434}{1} = 0.65$$

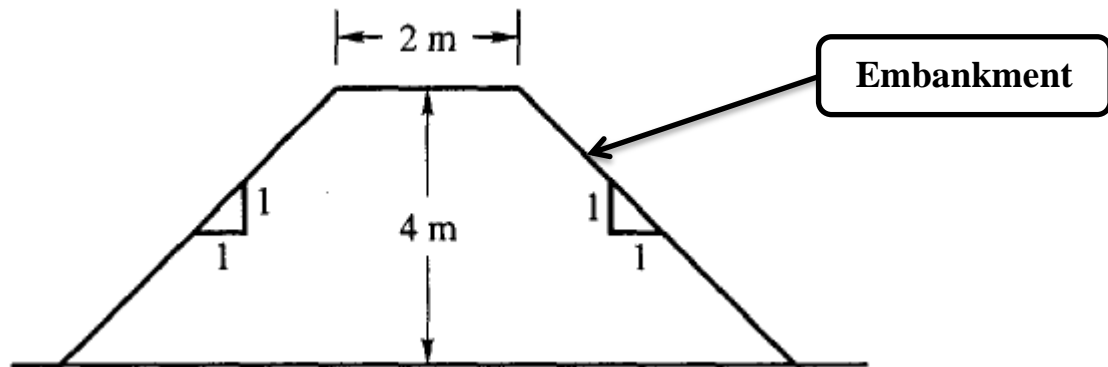
$$\rightarrow 0.65 = \frac{V_T - 5281.7}{5281.7} \rightarrow V_T = 8714.8 \text{ m}^3.$$

So, the total cost for sample " B " =  $8714.8 \text{ m}^3 \times 5 \frac{\$}{\text{m}^3} = 43,574\$$ .

So, we choose the sample " B " because it has the lowest cost ✓.

### 7. (Mid 2010):

Earth is required to be excavated from borrow pits for building an embankment as shown in the figure below. The moisture unit weight of the borrow pits is  $18 \text{ kN/m}^3$  and its water content is 8%. Estimate the quantity of earth required to be excavated per meter length of embankment. The dry unit weight required for the embankment is  $15 \text{ kN/m}^3$  with a moisture content of 10%. Assume the specific gravity of solids as 2.67. Also determine the degree of saturation of the embankment soil and the volume of water in the embankment. (hint: Volume of embankment per meter length)



### Solution

**شرح السؤال:** توجد منطقة معينة فيها تربة معينة (borrow pit) بمواصفات محددة... يراد إنشاء سد رملي (embankment) في هذه المنطقة حيث أن شكل السد موضح في الشكل أعلاه، لكن هذا السد يتطلب تربة بمواصفات معينة... وبناء على ذلك توجد عدة مطالبات ولكن الفكرة كما تعودنا أنه يجب الحفاظ على نفس قيمة  $V_s$  لكل من التربة الموجودة وتربة السد.

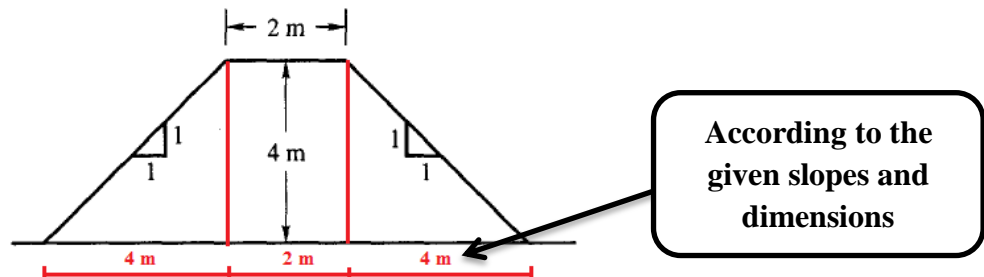
### Givens:

#### -For borrow pit

$$\gamma_{\text{moist}} = 18 \text{ kN/m}^3, \quad \%w = 8\%, \quad G_s = 2.67$$

#### -For Soil of embankment

$$\gamma_{\text{dry}} = 15 \text{ kN/m}^3, \quad \%w = 10\%, \quad G_s = 2.67, \quad V_T = (\text{From the given figure})$$



**Required:**

a)

Now, for embankment  $\rightarrow V_T =$  area of the embankment(trapezoidal)/meter length

$$V_T = \frac{1}{2} \times (2 + 10) \times 4 \times 1 = 24 \text{ m}^3/\text{m}.$$

$$\gamma_{\text{dry}} = \frac{G_s \times \gamma_w}{1+e} \rightarrow 15 = \frac{2.67 \times 9.81}{1+e} \rightarrow e = 0.746$$

$$e = \frac{V_T - V_s}{V_s} \rightarrow 0.746 = \frac{24 - V_s}{V_s} \rightarrow V_s = 13.74 \text{ m}^3/\text{m} \text{ (that must be maintained) .}$$

Now, for borrow pit  $\rightarrow \gamma_{\text{moist}} = 18 \text{ kN/m}^3$ ,  $\%w = 8\%$ ,  $G_s = 2.67$ ,  $V_T = ??$

$$\gamma_{\text{dry}} = \frac{\gamma_{\text{moist}}}{1+w} = \frac{18}{1+0.08} = 16.67 \text{ kN/m}^3 .$$

$$\gamma_{\text{dry}} = \frac{G_s \times \gamma_w}{1+e} \rightarrow 16.67 = \frac{2.67 \times 9.81}{1+e} \rightarrow e = 0.57$$

$$e = \frac{V_T - V_s}{V_s} \rightarrow 0.57 = \frac{V_T - 13.74}{13.74} \rightarrow V_T = 21.6 \text{ m}^3/\text{m} \checkmark .$$

b)

$$\%w = 10\% \quad , \quad G_s = 2.67 \quad , \quad e = 0.746$$

$$S.e = G_s.w \rightarrow S = \frac{2.67 \times 0.1}{0.746} = 0.358 = 35.8\% \checkmark .$$

c)

$$S = \frac{V_w}{V_v} \text{ , but } V_v = ???$$

$$n = \frac{V_v}{V_T} \text{ , also } n = \frac{e}{1+e} = \frac{0.746}{1+0.746} = 0.427$$

$$\rightarrow V_v = n \times V_T = 0.427 \times 24 = 10.25 \text{ m}^3/\text{m} \checkmark$$

$$V_w = S \times V_v = 0.358 \times 10.25 = 3.67 \text{ m}^3/\text{m} \checkmark .$$

**8. (Mid 2010):**

The results of a standard compaction test for a soil having a value of ( $G_s = 2.5$ ) are shown in the table below.

<b>Water Content (%)</b>	6.2	8.1	9.8	11.5	12.3	13.2
<b>Unit Weight (KN/m<sup>3</sup>)</b>	16.9	18.7	19.5	20.5	20.4	20.1

**Find:**

- The optimum water content.
- The maximum dry unit weight.
- The void ratio ( $e$ ).
- Degree of saturation ( $S$ ).
- Moisture unit weight.
- Find the weight of water need to be added to  $1\text{m}^3$  to reach 100% degree of saturation.

**Solution**

Firstly, we calculate the value of dry unit weight in the following table:

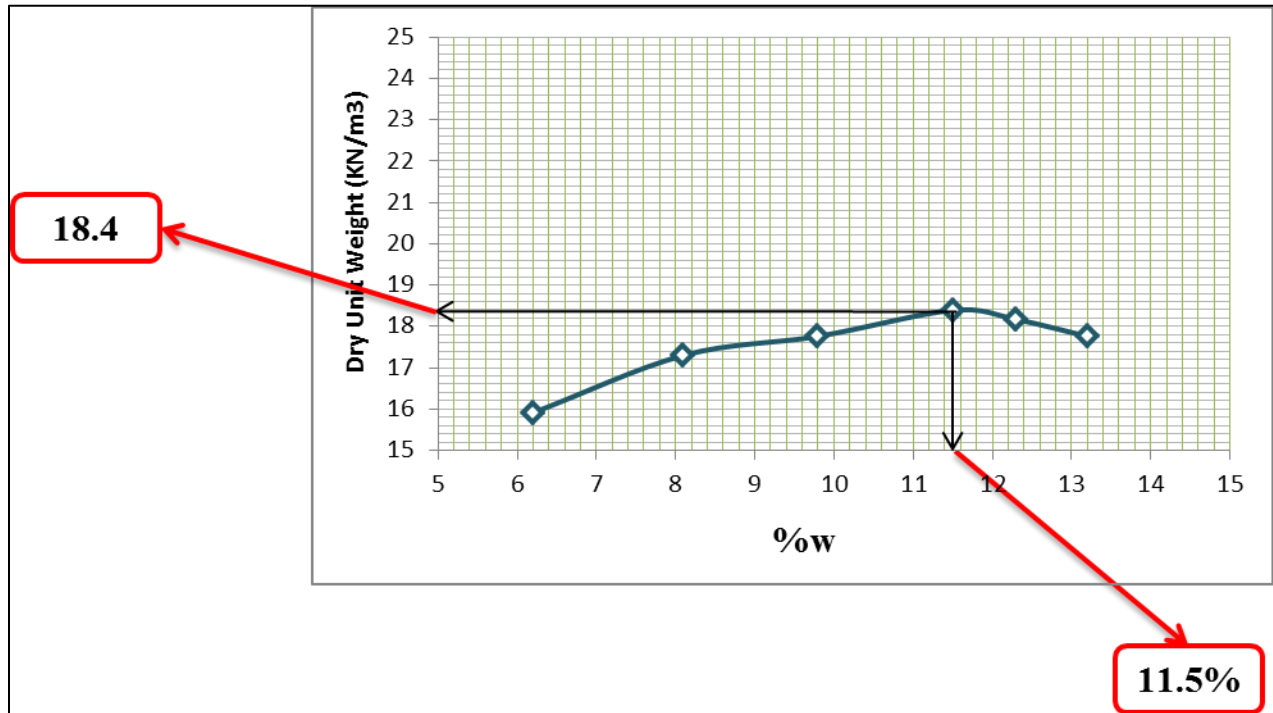
<b>Water Content (%)</b>	6.2	8.1	9.8	11.5	12.3	13.2
<b>Water Content (value)</b>	0.062	0.081	0.098	0.115	0.123	0.132
<b>Unit Weight (KN/m<sup>3</sup>)</b>	16.9	18.7	19.5	20.5	20.4	20.1
<b>Dry unit Weight (KN/m<sup>3</sup>)</b>	15.91	17.3	17.76	18.4	18.17	17.76

$\frac{\%w}{100}$

$\gamma_{dry} = \frac{\gamma_{moist}}{1 + w}$

From the above table we note that the optimum water content = 11.5% ✓

And the maximum dry density = 18.4 kN/m<sup>3</sup> ✓ and the following graph will ensure these results:



Now, we have the following:

$$\gamma_{\text{dry}} = 18.4 \text{ kN/m}^3, \quad \%w = 11.5\%, \quad G_s = 2.5$$

$$\gamma_{\text{dry}} = \frac{G_s \times \gamma_w}{1 + e} \rightarrow 18.4 = \frac{2.5 \times 9.81}{1 + e} \rightarrow e = 0.334 \checkmark.$$

$$S \cdot e = G_s \cdot w \rightarrow S = \frac{2.5 \times 0.115}{0.334} = 0.86 = 86\% \checkmark.$$

Moisture unit weight exist in the given table and equal 20.5 (this part just for confusion so, trust by yourself 😊) ✓.

**The last required :**

$$S = 100\% \rightarrow w_{100\%} = \frac{1 \times 0.334}{2.5} = 0.134 = 13.4\%$$

$$\gamma_{\text{moist, 100\% (sat)}} = 18.4(1 + 0.134) = 20.86 \text{ kN/m}^3$$

$$\text{Weight of water to be added} = 20.86 - 20.5 = 0.36 \text{ kN/m}^3 \checkmark.$$



**9. (Mid 2009):**

A sample of **saturated** clay was placed in a container and weighed. The weight was 6N. The clay in its container was placed in an oven dry for 24 hours @ 105°C. The weight reduced to a constant weight of 5N. The weight of the container is 1N. If  $G_s = 2.7$ , determine:

- Water content.
- Void ratio.
- Moist unit weight.
- Dry unit weight.
- Effective unit weight (Submerged unit weight).

**Solution****Givens:**

$$W_{\text{wet}} + W_{\text{Container}} = 6\text{N} , W_{\text{dry}} + W_{\text{Container}} = 5\text{N} , W_{\text{Container}} = 1\text{N} , G_s = 2.7$$

(a).

$$W_{\text{wet}} = W_{\text{wet}} + W_{\text{Container}} - W_{\text{Container}} = 6 - 1 = 5\text{N}.$$

$$W_{\text{dry}} = W_{\text{dry}} + W_{\text{Container}} - W_{\text{Container}} = 5 - 1 = 4\text{N}.$$

$$\%w = \frac{W_{\text{wet}} - W_{\text{dry}}}{W_{\text{dry}}} \times 100\% = \frac{5 - 4}{4} \times 100\% = 25\% \checkmark.$$

(b).

Saturated clay  $\rightarrow S=1$ 

$$S \cdot e = G_s \cdot w \rightarrow e = \frac{2.7 \times 0.25}{1} = 0.675 \checkmark.$$

(c).

$$\gamma_{\text{moist,100\%(sat)}} = \frac{G_s \times \gamma_w \times (1+w@S=1)}{1+e} = \frac{2.7 \times 9.81 \times (1+0.25)}{1+0.675} = 19.76 \text{ KN/m}^3 \checkmark.$$

(d).

$$\gamma_{\text{dry}} = \frac{\gamma_{\text{moist}}}{1+w} = \frac{19.76}{1+0.25} = 15.8 \text{ KN/m}^3 \checkmark.$$

(e).

Effective unit weight : هي الكثافة الفعلية لحبيبات التربة نفسها بدون أي فراغات وبدون أي كمية من الماء

$$\gamma_{\text{Eff.}} = \gamma_{\text{sat}} - \gamma_{\text{water}} = 19.76 - 9.81 = 9.95 \text{ KN/m}^3 \checkmark.$$

**10. (Mid 2009):**

An earth dam requires one million cubic meter of soil compacted to a void ratio of 0.8. In the vicinity (بالقرب) of the proposed dam, three borrow pits were identified as having suitable materials. The cost of purchasing the soil and the cost of excavation are the same for each borrow pit. The only difference of the cost for each borrow pit is the cost of transportation.

Which borrow pit would be the most economical?

Borrow pit No.	Void ratio	Transportation cost (\$/m <sup>3</sup> )
1	1.8	0.6
2	0.9	1
3	1.5	0.75

**Solution**

**شرح السؤال:** يراد إنشاء سد بحيث يتطلب 1 مليون متر مكعب من التربة المدموكة والتي لها نسبة فراغات تساوي 0.8 , وبالتالي توجد قيمة معينة ل  $V_s$  يجب الحفاظ عليها... بحيث يوجد 3 أنواع من التربة موجودة بالقرب من مكان إنشاء السد , وبالتالي مطلوب إيجاد أرخص نوع من هذه الأنواع مع تحقيقه شرط الحفاظ على قيمة  $V_s$  المطلوبة للسد.

Firstly, we calculate the value of  $V_s$  that required for earth dam as following:

$$e = \frac{V_v}{V_s} = \frac{V_T - V_s}{V_s} \rightarrow 0.8 = \frac{1,000,000 - V_s}{V_s} \rightarrow V_s = 555,555.5 \text{ m}^3 \text{ that must be maintained.}$$

**For borrow pit #1 :**

$$e=1.8$$

$$e = \frac{V_T - V_s}{V_s} \rightarrow 1.8 = \frac{V_T - 555,555.5}{555,555.5} \rightarrow V_T = 1,555,555.4 \text{ m}^3.$$

So, the total cost for borrow pit #1 =  $1,555,555.4 \text{ m}^3 \times 0.6 \frac{\$}{\text{m}^3} = 933,333\$$ .

**For borrow pit #2 :**

$$e=0.9$$

$$e = \frac{V_T - V_s}{V_s} \rightarrow 0.9 = \frac{V_T - 555,555.5}{555,555.5} \rightarrow V_T = 1,055,555.4 \text{ m}^3.$$

So, the total cost for borrow pit #2 =  $1,055,555.4 \text{ m}^3 \times 1 \frac{\$}{\text{m}^3} = 1,055,555.4\$$ .

**For borrow pit #3 :**

$$e=1.5$$

$$e = \frac{V_T - V_s}{V_s} \rightarrow 1.5 = \frac{V_T - 555,555.5}{555,555.5} \rightarrow V_T = 1,388,888.8 \text{ m}^3.$$

So, the total cost for borrow pit #2 =  $1,388,888.8 \text{ m}^3 \times 0.75 \frac{\$}{\text{m}^3} = 1,041,666.6\$$ .

Choose the **borrow pit #1** which has the lowest cost ✓.

**11. (Mid 2007):**

A dry sand is placed in a container having a volume of  $0.3 \text{ ft}^3$ . The dry weight of the sample is 31 Ib. Water is carefully added to the container so as not to disturb the condition of the sand. When the container is **filled**, the combined weight of soil plus water is 38.2 Ib. From these data, compute the void ratio of soil in the container, and the specific gravity of the soil particles.

[Hint: water density =  $62.4 \text{ Ib/ft}^3$ ]

**Solution**

**شرح السؤال:** تم وضع عينة من التربة الجافة داخل وعاء له حجم معين... مع ملاحظة أن الوعاء لم يمتلئ. وبعد ذلك تم إضافة الماء إلى الوعاء بحيث أن التربة التي في الوعاء تبدأ في البداية بامتصاص المياه ثم يخف ذلك الامتصاص تدريجياً إلى أن تنتشع التربة بالكامل ويمتلئ الوعاء.. وبالتالي فإن التربة وصلت إلى حالة التشبع وبالتالي فإن درجة التشبع تكون 100% .

**Givens:**

$$V_T = 0.3 \text{ ft}^3, W_{\text{dry}} = 31 \text{ Ib}, S=1, W_{\text{sat}} = 38.2 \text{ Ib}$$

$$\%w = \frac{W_{\text{sat}} - W_{\text{dry}}}{W_{\text{dry}}} \times 100\% = \frac{38.2 - 31}{31} \times 100\% = 23.2\%$$

$$\gamma_{\text{dry}} = \frac{W_{\text{dry}}}{V_T} = \frac{31}{0.3} = 103.33 \text{ Ib/ft}^3$$

$$\text{Also, } \gamma_{\text{dry}} = \frac{G_s \times \gamma_w}{1+e} \rightarrow 103.33 = \frac{62.4 G_s}{1+e} \rightarrow \text{Eqn.1}$$

$$S \cdot e = G_s \cdot w \rightarrow 1 \times e = 0.232 G_s \rightarrow \text{Eqn.2}$$

$$\text{Substituting from 2 to 1} \rightarrow 103.33 = \frac{62.4 G_s}{1 + 0.232 G_s} \rightarrow G_s = 2.69 \checkmark .$$

$$e = 0.232 G_s = 0.232 \times 2.69 = 0.624 \checkmark .$$

**12. (Mid 2007):**

The moist densities and degree of saturation of a soil sample are given in the following table:

Soil Density (Kg/m <sup>3</sup> )	Degree of Saturation (%)
1690	45
1808	75

Determine the weight of water in Kg, that will be in 7 m<sup>3</sup> of the soil **when it saturated**.

**Solution**

Firstly, we must find the values of  $G_s$ ,  $e$

we know that  $\rho_{\text{moist}} = \frac{G_s \times \rho_w \times (1+w)}{1+e}$ , and  $w = \frac{S \cdot e}{G_s}$

So, the eqn. will be in this form :  $\rho_{\text{moist}} = \frac{G_s \times \rho_w \times \left(1 + \frac{S \cdot e}{G_s}\right)}{1+e}$  → Eqn.\*

**Case#1:**

$\rho_{\text{moist}} = 1690 \text{ Kg/m}^3$ ,  $S = 45\%$ , substitute in Eqn.\*

$$1690 = \frac{G_s \times 1000 \times \left(1 + \frac{0.45e}{G_s}\right)}{1+e} \rightarrow \text{Eqn.1}$$

**Case#2:**

$\rho_{\text{moist}} = 1808 \text{ Kg/m}^3$ ,  $S = 75\%$ , substitute in Eqn.\*

$$1808 = \frac{G_s \times 1000 \times \left(1 + \frac{0.75e}{G_s}\right)}{1+e} \rightarrow \text{Eqn.2}$$

Now, by solving the two equations (solve by your self) the results are :

$$G_s = 2.49, \quad e = 0.648$$

Now, the required is the weight of water in Kg will be added in  $V_T = 7 \text{ m}^3$  when  $S=100\%$ (saturated)

$$w_{100\%} = \frac{S \times e}{G_s} = \frac{1 \times 0.648}{2.49} = 0.26 = 26\%.$$

$$w = \frac{M_{\text{water}}}{M_{\text{solid}}}, \rightarrow w = \checkmark, \quad M_{\text{water}}(\text{required}), \quad \text{but } M_{\text{solid}} = ???$$

$$\text{Now, } G_s = \frac{\rho_{\text{solid}}}{\rho_w} \rightarrow \rho_{\text{solid}} = 1000 \times 2.49 = 2490 \text{ Kg/m}^3$$

$$\rho_{\text{solid}} = \frac{M_{\text{solid}}}{V_s} \rightarrow M_{\text{solid}} = 2490 V_s.$$

$$n = \frac{e}{1 + e} = \frac{0.648}{1 + 0.648} = 0.393.$$

$$n = \frac{V_v}{V_T} \rightarrow V_v = 0.393 \times 7 = 2.751 \text{ m}^3$$

$$V_s = V_T - V_v = 7 - 2.751 = 4.25 \text{ m}^3.$$

$$\text{So, } M_{\text{solid}} = 2490 \times 4.25 = 10582.5 \text{ Kg.}$$

$$\text{Finally, } M_{\text{water}} = w \times M_{\text{solid}} = 0.26 \times 10582.5 = 2751.45 \text{ Kg } \checkmark.$$

**13. (Mid 2006):**

A borrow material has a volume of 191,000 m<sup>3</sup> and void ratio of 1.2. After compaction its new void ratio is 0.7, **find the corresponding volume?**

**Solution****Givens:**

$$V_{T,1} = 191,000 \text{ m}^3, \quad e_1 = 1.2, \quad V_{T,2} = ??, \quad e_2 = 0.7$$

The main idea of this problem that the value of  $V_s$  is constant.

**Before compaction:**

$$e_1 = \frac{V_{T,1} - V_s}{V_s} \rightarrow 1.2 = \frac{191,000 - V_s}{V_s} \rightarrow V_s = 86818.18 \text{ m}^3$$

**After compaction:**

$$e_2 = \frac{V_{T,2} - V_s}{V_s} \rightarrow 0.7 = \frac{V_{T,2} - 86818.18}{86818.18} \rightarrow V_{T,2} = 147,590.9 \text{ m}^3 \checkmark.$$

**14. (Mid 2006):**

The total volume of a soil specimen is 80,000 mm<sup>3</sup> and it weighs 150 grams. The dry weight of the specimen is 130 grams and the density of the soil solids is 2680 Kg/m<sup>3</sup>. **Find the following:**

- The water content.
- Void Ratio.
- Porosity.
- Degree of saturation.
- Saturated unit weight.
- Dry unit weight.

**Solution****Givens:**

$$V_T = 80,000 \text{ mm}^3, \quad M_{\text{moist}} = 150 \text{ gm}, \quad M_{\text{dry}} = 130 \text{ gm}, \quad \rho_s = 2680 \text{ Kg/m}^3$$

a)

$$\%W = \frac{M_{\text{moist}} - M_{\text{dry}}}{M_{\text{dry}}} \times 100\% = \frac{150 - 130}{130} \times 100\% = 15.38\% \checkmark.$$

b)

$$G_s = \frac{\rho_s}{\rho_{\text{water}}} = \frac{2680}{1000} = 2.68$$

$$V_T = 80,000 \text{ mm}^3 = 80,000 \times 10^{-9} = 80 \times 10^{-6} \text{ m}^3$$

$$\rho_{\text{dry}} = \frac{M_{\text{dry}}}{V_T} = \frac{130 \times 10^{-3}}{80 \times 10^{-6}} = 1625 \text{ Kg/m}^3$$

$$\rho_{\text{dry}} = \frac{G_s \times \rho_w}{1 + e} \rightarrow 1625 = \frac{2.68 \times 1000}{1 + e} \rightarrow e = 0.649 \checkmark.$$

c)

$$n = \frac{e}{1 + e} = \frac{0.649}{1 + 0.649} = 0.393 \checkmark.$$

d)

$$S.e = G_s.w \rightarrow S = \frac{G_s.w}{e} = \frac{2.68 \times 0.1538}{0.649} = 0.635 = 63.5\% \checkmark.$$

e)

$$S=1 \rightarrow w_{100\%} = \frac{1 \times 0.649}{2.68} = 0.242 = 24.2$$

$$\gamma_{\text{moist,100\%(sat)}} = \frac{G_s \times \gamma_w \times (1 + w_{@s=1})}{1 + e} = \frac{2.68 \times 9.81 \times (1 + 0.242)}{1 + 0.649} = 19.8 \text{ KN/m}^3 \checkmark.$$

f)

$$\rho_{\text{dry}} = 1625 \text{ Kg/m}^3 \rightarrow \gamma_{\text{dry}} = 1625 \times 9.81 \times 10^{-3} = 15.94 \text{ KN/m}^3 \checkmark.$$



**15. (Mid 2005):**

A sample of moist soil was found to have the following characteristics:

**Volume** 0.01456 m<sup>3</sup> (as sampled)

**Mass** 25.74 Kg (as sampled)

22.10 Kg (after oven drying)

**Specific gravity of solids:** 2.69

- Find the density, dry unit weight, void ratio, porosity, degree of saturation for the soil.
- What would be the moist unit weight when the degree of saturation is 80%?

**Solution****Givens:**

$$V_T = 0.01456 \text{ m}^3, \quad M_{\text{moist}} = 25.74 \text{ Kg}, \quad M_{\text{dry}} = 22.1 \text{ Kg}, \quad G_s = 2.69$$

•

(The first required is density that means **moist and dry densities**)

$$\rho_{\text{moist}} = \frac{M_{\text{moist}}}{V_T} = \frac{25.74}{0.01456} = 1767.56 \text{ Kg/m}^3 \checkmark.$$

$$\rho_{\text{dry}} = \frac{M_{\text{dry}}}{V_T} = \frac{22.1}{0.01456} = 1517.86 \text{ Kg/m}^3 \checkmark.$$

$$\rho_{\text{dry}} = 1517.86 \text{ Kg/m}^3 \rightarrow \gamma_{\text{dry}} = 1517.86 \times 9.81 \times 10^{-3} = 14.89 \text{ KN/m}^3 \checkmark.$$

$$\gamma_{\text{dry}} = \frac{G_s \times \gamma_w}{1 + e} \rightarrow 14.89 = \frac{2.69 \times 9.81}{1 + e} \rightarrow e = 0.772 \checkmark.$$

$$S \cdot e = G_s \cdot w \rightarrow w = ??$$

$$\rho_{\text{moist}} = \rho_{\text{dry}}(1 + w) \rightarrow 1767.56 = 1517.86(1 + w) \rightarrow w = 0.1645 = 16.45\%$$

$$S = \frac{G_s \cdot w}{e} = \frac{2.69 \times 0.1645}{0.772} = 0.573 = 57.3\% \checkmark.$$

•

$$w = \frac{S \cdot e}{G_s} = \frac{0.8 \times 0.772}{2.69} = 0.229 = 22.9\%$$

$$\gamma_{\text{moist}} = \frac{G_s \times \gamma_w (1 + w)}{1 + e} = \frac{2.69 \times 9.81 \times (1 + 0.229)}{1 + 0.772} = 18.3 \text{ KN/m}^3 \checkmark.$$

**16. (Final 2009):**

Dry soil with  $G_s = 2.7$  is mixed with water to produce **20% water content** and compacted to produce a cylindrical sample of 40 mm diameter and 80mm long with 5% air content. Calculate the following:

- A- Mass of the mixed soil that will be required.
- B- Void ratio of the sample.
- C- Dry, moisture and saturated unit weight.
- D- Amount of water to be added for full saturation.

**Solution****Givens:**

$$V_T = \text{volume of the cylindrical sample} = \frac{\pi}{4} \times (0.04)^2 \times 0.08 = 1.005 \times 10^{-4} \text{ m}^3$$

$$\%w=20\% \quad , \quad \text{air content} = 5\% \quad , \quad G_s = 2.7$$

$$\text{Important Note: Air content} = \frac{V_{\text{air}}}{V_T} \rightarrow 0.05 = \frac{V_{\text{air}}}{V_T} \rightarrow V_{\text{air}} = 0.05V_T.$$

A-

( $M_{\text{mixed soil}} = M_{\text{solid}}$ ) because the mixed soil is a dry soil and  $M_{\text{dry}} = M_{\text{solid}}$

$$w = \frac{M_{\text{water}}}{M_{\text{solid}}} \rightarrow M_{\text{solid}} = \frac{M_{\text{water}}}{w} = \frac{M_{\text{water}}}{0.2} \rightarrow \boxed{M_{\text{solid}} = 5 M_{\text{water}}} \rightarrow \boxed{\text{Eqn.1}}$$

$$G_s = \frac{\rho_{\text{solid}}}{\rho_{\text{water}}} \rightarrow \rho_{\text{solid}} = 1000 \times 2.7 = 2700 \text{ Kg/m}^3$$

$$\rho_{\text{solid}} = \frac{M_{\text{solid}}}{V_{\text{solid}}} \rightarrow \boxed{M_{\text{solid}} = 2700 V_s} \rightarrow \boxed{\text{Eqn.2}}$$

$$V_v = V_{\text{air}} + V_{\text{water}} \quad , \quad \text{and} \quad \rho_{\text{water}} = \frac{M_{\text{water}}}{V_{\text{water}}} \rightarrow V_{\text{water}} = \frac{M_{\text{water}}}{\rho_{\text{water}}} \quad , \quad \text{and} \quad V_v = V_T - V_s$$

$$\rightarrow V_T - V_s = 0.05V_T + \frac{M_{\text{water}}}{\rho_{\text{water}}} \quad \text{but,} \quad V_T = 1.005 \times 10^{-4} \quad \text{and} \quad \rho_{\text{water}} = 1000 \text{ Kg/m}^3$$

$$\text{So, } V_s = 0.95V_T - \frac{M_{\text{water}}}{\rho_{\text{water}}} \rightarrow V_s = 9.5 \times 10^{-5} - 0.001 M_{\text{water}}$$

Now, substitute in Eqn.2:  $\rightarrow M_{\text{solid}} = 0.2565 - 2.7 M_{\text{water}} \rightarrow$  Substitute in Eq. 1

$$\rightarrow 0.2565 - 2.7 M_{\text{water}} = 5 M_{\text{water}} \rightarrow M_{\text{water}} = 0.0333 \text{ Kg.}$$

$$M_{\text{solid}} = 5 \times 0.0333 = 0.1665 \text{ Kg} \checkmark.$$

B-

$$e = \frac{V_V}{V_S} \rightarrow V_S = 9.5 \times 10^{-5} - 0.001 M_{\text{water}} = V_S = 9.5 \times 10^{-5} - 0.001 \times 0.0333$$

$$V_S = 6.17 \times 10^{-5} \text{ m}^3$$

$$V_V = V_T - V_S = 1.005 \times 10^{-4} - 6.17 \times 10^{-5} = 3.88 \times 10^{-5} \text{ m}^3$$

$$\text{Then, } e = \frac{3.88 \times 10^{-5}}{6.17 \times 10^{-5}} = 0.628 \checkmark.$$

C-

$$\gamma_{\text{dry}} = \frac{G_s \times \gamma_w}{1+e} = \frac{2.7 \times 9.81}{1+0.628} = 16.27 \text{ KN/m}^3 \checkmark.$$

$$\gamma_{\text{moist}} = \frac{G_s \times \gamma_w (1+w)}{1+e} = \frac{2.7 \times 9.81 (1+0.2)}{1+0.628} = 19.52 \text{ KN/m}^3 \checkmark.$$

$$\gamma_{\text{sat}} = \frac{G_s \times \gamma_w (1 + \frac{1 \times e}{G_s})}{1+e} \quad (\text{Saturated} \rightarrow S = 1 \rightarrow w = \frac{1 \times e}{G_s})$$

$$\gamma_{\text{sat}} = \frac{2.7 \times 9.81 (1 + \frac{1 \times 0.628}{2.7})}{1+0.628} = 20.05 \text{ KN/m}^3 \checkmark.$$

D-

$$\text{Amount (KN/m}^3) = \gamma_{\text{sat}} - \gamma_{\text{moist}} \rightarrow \text{Amount (KN)} = (\gamma_{\text{sat}} - \gamma_{\text{moist}}) \times V_T$$

$$\text{Amount (KN)} = (20.05 - 19.52) \times 1.005 \times 10^{-4} = 5.3265 \times 10^{-5} \text{ KN} \checkmark.$$

$$\text{Amount (Kg)} = 5.3265 \times 10^{-5} \times \frac{1000}{9.81} = 5.4296 \times 10^{-3} \text{ Kg} \checkmark.$$

## 17.

Moist clayey soil has initial void ratio of 1.5, dry mass of 80gm, and specific gravity of solid particles of 2.5. The sample is exposed to atmosphere so that the sample volume decrease to **one half** of its initial volume . Calculate the following:

- The new void ratio.
- Mass of water if degree of saturation became 25 %.

## Solution

## Givens:

$$e_1 = 1.5, M_{\text{dry}} = M_{\text{solid}} = 80\text{gm}, G_s = 2.5, V_{T,2} = 0.5V_{T,1}$$

a)

Firstly, we must calculate the value of  $V_T$  that must be the same in each case.

$$e_1 = \frac{V_{T,1} - V_s}{V_s} \rightarrow 1.5 = \frac{V_{T,1} - V_s}{V_s}, V_{T,1} = ???$$

$$\rho_{\text{dry}} = \frac{M_{\text{dry}}}{V_{T,1}} = \frac{G_s \times \rho_w}{1 + e_1} \rightarrow \frac{0.08}{V_{T,1}} = \frac{2.5 \times 1000}{1 + 1.5} \rightarrow V_{T,1} = 8 \times 10^{-5} \text{m}^3.$$

$$\text{So, } 1.5 = \frac{8 \times 10^{-5} - V_s}{V_s} \rightarrow V_s = 3.2 \times 10^{-5} \text{m}^3.$$

$$\text{Now, } V_{T,2} = 0.5 \times 8 \times 10^{-5} = 4 \times 10^{-5} \text{m}^3.$$

$$e_2 = \frac{V_{T,2} - V_s}{V_s} = \frac{4 \times 10^{-5} - 3.2 \times 10^{-5}}{3.2 \times 10^{-5}} = 0.25 \checkmark.$$

b)

$$e = 0.25, S = 25\%, G_s = 2.5$$

$$S \cdot e = G_s \cdot w \rightarrow w = \frac{0.25 \times 0.25}{2.5} = 0.025 = 2.5\%.$$

$$w = \frac{M_{\text{water}}}{M_{\text{solid}}} \rightarrow M_{\text{water}} = 0.025 \times 0.08 = 2 \times 10^{-3} \text{Kg} = 2\text{gm} \checkmark.$$

## 18.

Soil has been compacted in an embankment at a bulk unit weight of  $2.15 \text{ t/m}^3$  And water content of 12% , the solid particles of soil having specific gravity of 2.65.

- Calculate the dry unit weight, degree of saturation, and air content.
- Would it possible to **compact** the above soil at a water content of 13.5% to a dry unit weight of  $2 \text{ t/m}^3$ .

## Solution

## Givens:

$$\gamma_{\text{bulk}} = \gamma_{\text{moist}} = 2.15 \text{ t/m}^3 = 2.15 \times 9.81 = 21.0915 \text{ KN/m}^3 \text{ (assume } g=9.81\text{m/s}^2\text{)}$$

$$\%w=12\% \quad , \quad G_s = 2.65$$

a)

$$\gamma_{\text{dry}} = \frac{\gamma_{\text{moist}}}{1+w} = \frac{21.0915}{1+0.12} = 18.83 \text{ KN/m}^3 \checkmark.$$

$$\gamma_{\text{dry}} = \frac{G_s \times \gamma_w}{1+e} \rightarrow 18.83 = \frac{2.65 \times 9.81}{1+e} \rightarrow e = 0.38.$$

$$S.e = G_s \cdot w \rightarrow S = \frac{2.65 \times 0.12}{0.38} = 0.837 = 83.7\% \checkmark.$$

$$\text{Air content} = \frac{V_{\text{air}}}{V_T} = ??$$

$$V_v = V_{\text{air}} + V_{\text{water}} \rightarrow \text{Eqn.*}$$

$$V_v = V_T - V_s$$

$$w = \frac{W_{\text{water}}}{W_{\text{solid}}} \rightarrow W_{\text{water}} = 0.12 W_{\text{solid}}$$

$$G_s = \frac{\gamma_{\text{solid}}}{\gamma_{\text{water}}} \rightarrow \gamma_{\text{solid}} = \frac{W_s}{V_s} \rightarrow G_s = \frac{W_s}{V_s \times \gamma_{\text{water}}} \rightarrow W_s = G_s \times V_s \times \gamma_{\text{water}}$$

$$\text{Substitute in Eqn. 1} \rightarrow W_{\text{water}} = 0.12 \times 2.65 \times 9.81 \times V_s \rightarrow W_{\text{water}} = 3.12 V_s$$

$$V_{\text{water}} = \frac{W_{\text{water}}}{\gamma_{\text{water}}} \rightarrow V_{\text{water}} = \frac{3.12 V_s}{9.81} = 0.318 V_s$$

$$\gamma_{\text{dry}} = 18.83 = \frac{W_s}{V_T} = \frac{G_s \times V_s \times \gamma_{\text{water}}}{V_T} \rightarrow V_s = 0.7243 V_T$$

$$\rightarrow V_{\text{water}} = 0.318 \times 0.7243 V_T = 0.23 V_T$$

Substitute in Eqn.\*

$$V_T - V_S = V_{\text{air}} + V_{\text{water}}$$

$$\rightarrow V_T - 0.7243 V_T = V_{\text{air}} + 0.23 V_T \quad (\text{Dividing by } V_T)$$

$$\rightarrow 1 - 0.7243 = \frac{V_{\text{air}}}{V_T} + 0.23 \rightarrow \frac{V_{\text{air}}}{V_T} = 0.0457 = 4.57\% \quad (\text{Air content}) \checkmark.$$

b)

$$\%w = 13.5\% \quad , \quad \gamma_{\text{dry}} = 2 \text{ t/m}^3 = 2 \times 9.81 = 19.62 \text{ KN/m}^3 (\text{need to check})$$

If  $\gamma_{Z.A.V} > 19.62 \rightarrow \text{Ok}$ , else  $\rightarrow \text{Not Ok}$ .

$$\gamma_{Z.A.V} = \frac{G_s \times \gamma_w}{1 + e_{\text{min}}} \rightarrow e_{\text{min}} = \frac{G_s \cdot w}{S_{\text{max}}} = \frac{2.65 \times 0.135}{1} = 0.3577$$

$$\gamma_{Z.A.V} = \frac{2.65 \times 9.81}{1 + 0.3577} = 19.147 \text{ KN/m}^3 < 19.62 \rightarrow \text{Not possible} \checkmark.$$

Because the value of  $\gamma_{Z.A.V}$  is the maximum value of dry unit weight can be reach.

**Another solution:**

It's supposed that the value of (e) must be greater than the value of ( $e_{\text{min}}$ )

$$e_{\text{min}} = \frac{G_s \cdot w}{S_{\text{max}}} = \frac{2.65 \times 0.135}{1} = 0.3577$$

$$\gamma_{\text{dry}} = \frac{G_s \times \gamma_w}{1 + e} \rightarrow 19.62 = \frac{2.65 \times 9.81}{1 + e} \rightarrow e = 0.325 < e_{\text{min}} \rightarrow \text{Not possible} \checkmark.$$

**19.**

A specimen of soil was immersed in mercury. The mercury which came out was collected and its weight was 290gm. The sample was oven dried and its weight became 30.2gm. If the specific gravity was 2.7 and weight of soil in natural state was 34.6gm. Determine :

- Void ratio, and porosity.
- Water content of the original sample.
- Degree of saturation of the original sample.

[Hint: the density of mercury is 13.6 gm/cm<sup>3</sup>]

**Solution****Givens:**

$$M_{\text{mer. (came out)}} = 290\text{gm}, \quad M_{\text{dry}} = 30.2\text{gm}, \quad M_{\text{wet}} = 34.6\text{gm}, \quad G_s = 2.7$$

**Archimedes Law:** The volume of specimen **equal** the volume of liquid came out.

$$V_T = \frac{M_{\text{mer}}}{\rho_{\text{mer}}} = \frac{290}{13.6} = 21.32 \text{ cm}^3 = 21.32 \times 10^{-6} \text{ m}^3.$$

a)

$$\rho_{\text{dry}} = \frac{M_{\text{dry}}}{V_T} = \frac{30.2 \times 10^{-3}}{21.32 \times 10^{-6}} = 1416.51 \text{ Kg/m}^3$$

$$\rho_{\text{dry}} = \frac{G_s \times \rho_w}{1 + e} \rightarrow 1416.51 = \frac{2.7 \times 1000}{1 + e} \rightarrow e = 0.906 \checkmark.$$

$$n = \frac{e}{1 + e} = \frac{0.906}{0.906 + 1} = 0.475 \checkmark.$$

b)

$$\%w = \frac{M_{\text{moist}} - M_{\text{dry}}}{M_{\text{dry}}} \times 100\% = \frac{34.6 - 30.2}{30.2} \times 100\% = 14.57\% \checkmark.$$

c)

$$S.e = G_s \cdot w \rightarrow S = \frac{2.7 \times 0.1457}{0.906} = 0.4342 = 43.42\% \checkmark.$$

**20. (Important)**

The in-situ(field) moisture content of a soil is 18% and its moisture unit weight is 105 pcf (lb/ft<sup>3</sup>). The specific gravity of soil solids is 2.75. This soil is to be excavated and transported to a construction site, and then compacted to a minimum dry weight of 103.5 pcf at a moisture content of 20 %.

- a) How many **cubic yards** of excavated soil are needed to produce 10,000 yd<sup>3</sup> of compacted fill?  
 b) How many truckloads are needed to transport the excavated soil if each truck can carry 20 tons?

[ **Hint:** 1ton = 2000lb , 1yd<sup>3</sup>=27ft<sup>3</sup> ,  $\gamma_w = 62.4$ pcf ] (I advise you to remember these units)

**Solution****Givens:**

- For excavated soil (in-situ soil)  
 $\%w=18\%$  ,  $\gamma_{\text{moist}} = 105$  pcf ,  $G_s = 2.75$
- For soil in the construction site  
 $\%w=20\%$  ,  $\gamma_{\text{dry}} = 103.5$  pcf ,  $G_s = 2.75$

**شرح السؤال:** توجد لدينا عينة تربة في موقع معين وبمواصفات معينة.. حيث أنه يراد استخدام هذه التربة في موقع معين لأعمال الإنشاءات.. بالتالي سوف يتم حفر هذه التربة ونقلها في عربات وعند وصولها لموقع الإنشاء سوف يتم دمكها وبالتالي سوف تتغير بعض خصائصها... لكن دائما وأبدا كما ذكرنا سابقا أن قيمة  $V_s$  تبقى ثابتة وأيضا قيمة  $W_s$  تبقى ثابتة.. لأن الحبيبات الصلبة لا يتغير حجمها أبدا .

a)

$$V_{T,\text{excavated soil}} = ?? , V_{T,\text{construction site soil}} = 10,000 \text{ yd}^3$$

For construction site soil :

$$\gamma_{\text{dry}} = \frac{G_s \times \gamma_w}{1 + e} \rightarrow 103.5 = \frac{2.75 \times 62.4}{1 + e} \rightarrow e = 0.658$$

$$e = \frac{V_T - V_s}{V_s} \rightarrow 0.658 = \frac{10,000 - V_s}{V_s} \rightarrow V_s = 6031.36 \text{ yd}^3 \text{ (That must be maintained)}$$

Now, for excavated soil :

$$\gamma_{\text{moist}} = \frac{G_s \times \gamma_w(1 + w)}{1 + e} \rightarrow 105 = \frac{2.75 \times 62.4 \times (1 + 0.18)}{1 + e} \rightarrow e = 0.9284$$

$$e = \frac{V_T - V_s}{V_s} \rightarrow 0.9284 = \frac{V_T - 6031.36}{6031.36} \rightarrow V_T = 11,631.4 \text{ yd}^3 \checkmark .$$



b)

To find the number of trucks to transport the excavated soil we need two things:

- The total volume of excavated soil (in part **a** we calculate it =11,631.4 yd<sup>3</sup>)
- The total volume of each truck.

Each truck can carry 20 tons of excavated soil ...we want to convert this weight to volume as following:

For each truck:

$$\gamma_{\text{moist}} = \frac{W_{\text{moist}}}{V_T} \rightarrow V_{T,\text{truck}} = \frac{W_{\text{moist,truck}}}{\gamma_{\text{moist,excavated soil}}} = \frac{(20\text{ton} \times 2000)\text{lb}}{105} = 380.95 \text{ ft}^3.$$

$$V_{T,\text{truck}} = 380.95 \text{ ft}^3 = \frac{380.95}{27} = 14.1 \text{ yd}^3$$

$$\text{So, \# of trucks} = \frac{V_{T,\text{excavated soil}}}{V_{T,\text{truck}}} = \frac{11,631.4}{14.1} = 824.9 \text{ truck } \checkmark.$$

Don't say 825 because you have only 90 % (0.9) of the truck.

**21. (Important)**

An embankment for a highway 30 m wide and 1.5 m thick is to be constructed from sandy soil, trucked in from a borrow pit. The water content of the sandy soil in the borrow pit is 15% and its void ratio is 0.69. Specifications require the embankment to compact to a dry weight of 18 KN/m<sup>3</sup>. Determine- for 1 km length of embankment-the following:

- The dry unit weight of sandy soil from the borrow pit to construct the embankment, assuming that  $G_s = 2.7$ .
- The number of 10 m<sup>3</sup> truckloads of sandy soil required to construct the embankment.
- The weight of water per truck load of sandy soil.
- The degree of saturation of the in-situ sandy soil.

**Solution****Givens:**

- For borrow pit (in-situ soil)  
%w=15% ,  $e = 0.69$  ,  $G_s = 2.7$
- For embankment soil

$$V_T = 30 \times 1.5 \times 1000 = 45,000 \text{ m}^3 , \gamma_{\text{dry}} = 18 \text{ KN/m}^3 , G_s = 2.7$$

**شرح السؤال:** يوجد لدينا سد رملي مراد إنشائه بترربة لها خواص معينة حيث أن هذه التربة يتم إحضارها من أماكن مخصصة لها (borrow pit) والتي لها خواص معينة حيث سيتم نقلها الى مكان السد عن طريق شاحنات..ومن المعروف أن قيمة  $V_s$  تكون نفسها في كلا الحالتين وذلك لأنه مهما تم دمك التربة فان حجم الحبيبات الصلبة لا يتغير.

a)

$$\gamma_{\text{dry}} = \frac{G_s \times \gamma_w}{1+e} = \frac{2.7 \times 9.81}{1+0.69} = 15.67 \text{ KN/m}^3.$$

b)

$$V_{T,\text{truck}} = 10 \text{ m}^3 \text{ but } V_{T,\text{borrow pit soil}} = ??$$

For embankment soil:

$$\gamma_{\text{dry}} = \frac{G_s \times \gamma_w}{1+e} \rightarrow 18 = \frac{2.7 \times 9.81}{1+e} \rightarrow e = 0.4715.$$

$$e = \frac{V_T - V_s}{V_s} \rightarrow 0.4715 = \frac{45,000 - V_s}{V_s} \rightarrow V_s = 30,581 \text{ m}^3 \text{ (That must be maintained)}$$

Now, for borrow pit soil:

$$e = \frac{V_T - V_s}{V_s} \rightarrow 0.69 = \frac{V_T - 30,581}{30,581} \rightarrow V_{T, \text{borrow pit soil}} = 51,682 \text{ m}^3.$$

$$\text{So, \# of trucks} = \frac{V_{T, \text{borrow pit soil}}}{V_{T, \text{truck}}} = \frac{51,682}{10} = 5168.2 \text{ truck } \checkmark.$$

c)

$$\text{For each truck} \rightarrow V_{T, \text{truck}} = 10 \text{ m}^3$$

$$\gamma_{\text{dry}} = 15.67 \text{ KN/m}^3 \text{ for borrow pit soil}$$

$$\text{Now, } \gamma_{\text{dry}} = \frac{W_{\text{dry}}}{V_T} \rightarrow \text{for each truck} \rightarrow W_{\text{dry, truck}} = \gamma_{\text{dry}} \times V_{T, \text{truck}}$$

$$\rightarrow W_{\text{dry, truck}} = 10 \times 15.67 = 156.7 \text{ KN.}$$

$$\%w = \frac{W_{\text{water}}}{W_{\text{dry}}} \rightarrow W_{\text{water, truck}} = 156.7 \times 0.15 = 23.5 \text{ KN } \checkmark.$$

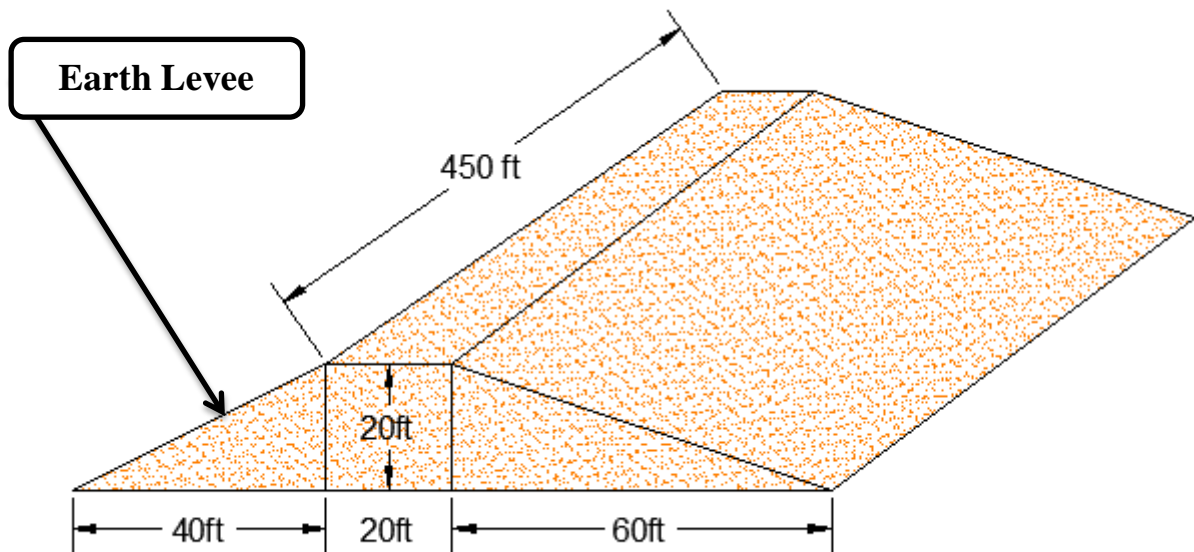
d)

$$S.e = G_s \cdot w \rightarrow S = \frac{2.7 \times 0.15}{0.69} = 0.587 = 58.7\% \checkmark.$$

## 22. ( Very Important)

Your company has won a contract to provide and compact the fill material for an earth levee (سد رملي لمنع الفيضانات) with the dimensions shown below. The levee fill is a silty clay soil to be compacted to at least 95% of maximum standard proctor of  $\gamma_{dry} = 106\text{pcf}$  at an OMC of 18%. Your borrow pit has a silty clay with an in-situ moist density of  $112.1\text{pcf}$  at 18% moisture content, and  $G_s = 2.68$ . When the soil is excavated and loaded on to your trucks, the void ratio of material is  $e=1.47$ . Your trucks can haul  $15\text{ yd}^3$  of materials per trip.

- Determine the volume of fill required for levee.
- Determine the volume required from the borrow pit.
- Determine the required number of trucks.



### Solution

#### Givens:

- For borrow pit (in-situ soil)  
 $\gamma_{moist} = 112.1\text{pcf}$  ,  $\%w=18\%$  ,  $G_s = 2.68$
- For earth levee fill  
 $V_T(\text{Dimensions})$  ,  $R.C = 95\%$  (Relative Compaction) ,  $\gamma_{dry,max,proctor} = 106\text{pcf}$   
 $\%w = \text{OMC} = 18\%$  (Optimum Moisture Content)  $G_s = 2.68$
- For excavated soil (that will be loaded):  $e = 1.47$  ,  $G_s = 2.68$  ,  $V_{T,truck} = 15\text{yd}^3$

**شرح السؤال:** نلاحظ في هذه المسألة أنه يوجد لدينا 3 حالات للتربة .. الحالة الأولى عندما تكون التربة مدفونة في الأرض حيث تكون لها خواص معينة ... والحالة الثانية: نلاحظ أن التربة تغيرت خواصها بعد أن تم تحضيرها للنقل في الشاحنات .. والحالة الثالثة: هي حالة التربة في موقع السد المراد إنشاؤه بخواص معينة. نلاحظ أيضا أن الحالة الثانية من المنطقي أن تحدث لأنه خلال نقل التربة من مكانها إلى الشاحنات فإن حبيبات التربة سوف تتناثر وتتغير خواصها وهذا واضح من خلال القيمة الكبيرة لـ  $e=1.47$  ... مع أن هذه الحالة لم توجد في السؤالين السابقين لكنها الأكثر واقعية وبالتالي يجب أخذها بالإعتبار..

a)

Volume of Levee= (Area of trapezoidal) X (Length of Levee)

$$V_{T,Levee} = \left[ \left( \frac{1}{2} \times (20 + (20 + 40 + 60)) \right) \times 20 \right] \times [450] = 630,000 \text{ ft}^3 \checkmark.$$

b)

For the earth levee:

$$R. C = \frac{\gamma_{dry,field}}{\gamma_{dry,proctor}} \rightarrow \gamma_{dry,field} = 0.95 \times 106 = 100.7 \text{ pcf. (That's the true value in site)}$$

$$\gamma_{dry} = \frac{G_s \times \gamma_w}{1 + e} \rightarrow 100.7 = \frac{2.68 \times 62.4}{1 + e} \rightarrow e = 0.66$$

$$e = \frac{V_T - V_s}{V_s} \rightarrow 0.66 = \frac{630,000 - V_s}{V_s} \rightarrow V_s = 379,518 \text{ ft}^3 \text{ (That must be maintained)}$$

Now, for borrow pit soil:

$$\gamma_{moist} = \frac{G_s \times \gamma_w (1+w)}{1+e} \rightarrow 112.1 = \frac{2.68 \times 62.4 \times (1+0.18)}{1+e} \rightarrow e = 0.76$$

$$e = \frac{V_T - V_s}{V_s} \rightarrow 0.76 = \frac{V_T - 379,518}{379,518} \rightarrow V_{T,borrow \text{ pit soil}} = 667,951.7 \text{ ft}^3 \checkmark.$$

c)

$$\# \text{ of trucks} = \frac{V_{T,excavated(loaded) \text{ soil}}}{V_{T,truck}}, \quad e = 1.47 \text{ for excavated soil}$$

$$e = \frac{V_T - V_s}{V_s} \rightarrow 1.47 = \frac{V_T - 379,518}{379,518} \rightarrow V_{T,excavated \text{ soil}} = 937,409.5 \text{ ft}^3$$

$$V_{T,truck} = 15 \text{ yd}^3 = 15 \times 27 = 405 \text{ ft}^3$$

$$\# \text{ of trucks} = \frac{937,409.5}{405} = 2314.6 \text{ truck} \checkmark.$$

## **Chapter (5)**

# **Classification of Soil**

## How to classify soil according Unified Soil Classification System (USCS)

### 1. Finding Group Symbol from (Table 5.2 P.131) According the following procedures:

#### a. Determine whether the soil is Coarse-grained or Fine-grained:

- If  $R_{200} > 50\%$  → Coarse – grained soil.
  - If  $R_{200} \leq 50\%$  → Fine – grained soil
- Where:  $R_{200}$  = Cumulative % retain on sieve # 200 (0.075 mm sieve opening).

#### b. If the soil is Coarse-grained soil → Follow the following:

- If  $\frac{R_4}{R_{200}} > 50\%$  → The soil is Gravel.
  - If  $\frac{R_4}{R_{200}} \leq 50\%$  → The soil is Sand.
- Where:  $R_4$  = Cumulative % retain on sieve # 4 (4.75 mm sieve opening).
- $$\frac{R_4}{R_{200}} = \text{Coarse fraction retained on \#4 seive.}$$

#### - If the soil is gravel → Follow the following:

- ✓ If  $F_{200} < 5\%$  → Choose Group Symbol according to values of ( $C_u$  &  $C_c$ )
- ✓ If  $F_{200} > 12\%$  → Choose Group Symbol according to values of (PI) and you may use the footnote (d) below table (5.2) if  $4 \leq PI \leq 7$ .
- ✓ If  $5\% \leq F_{200} \leq 12\%$  → Choose Group Symbol according to footnote (a) below table (5.2).

#### - If the soil is sand → Follow the following:

- ✓ If  $F_{200} < 5\%$  → Choose Group Symbol according to values of ( $C_u$  &  $C_c$ )
- ✓ If  $F_{200} > 12\%$  → Choose Group Symbol according to values of (PI) and you may use the footnote (d) below table (5.2) if  $4 \leq PI \leq 7$ .
- ✓ If  $5\% \leq F_{200} \leq 12\%$  → Choose Group Symbol according to footnote (b) below table (5.2).

Where:  $F_{200}$  = Cumulative % passing from sieve # 200 (% Fines)

#### Important Notes:

- If you have to go to footnote (d) below the table you must choose GC-GM → if the soil is gravel, and choose SC-SM → if the soil is sand
- “A” line equation →  $PI_{A\text{-line}} = 0.73(LL - 20)$   
If  $PI = PI_{A\text{-line}}$  (on), If  $PI > PI_{A\text{-line}}$  (above), If  $PI < PI_{A\text{-line}}$  (below)

- If you have to go to footnote (a) or (b) below the table → you have more than one choice. **How to find the correct symbol:**

Assume that you have a gravel soil (for example) and  $5\% \leq F_{200} \leq 12\%$

You have to go to footnote (a) and you have the following choices:

(GW-GM, GW-GC, GP-GM, GP-GC) only one of them is true.

You will take each symbol and check it whether achieve the conditions or not

Firstly, you take GW-GM and check it ...you must check each part of this dual symbol as following:

GW: check it from table 5.2 according to values of ( $C_u$  &  $C_c$ )

GM: check it from table 5.2 according to value of (PI)

If one of them doesn't achieve the condition you will reject the symbol

(GW-GM) and apply the same procedures on other symbols **till one of the symbols achieve the conditions (each part (from the 2 parts)) achieve the conditions ...only one symbol will achieve the conditions.**

If you check GW (for example) in GW-GM, you don't need to check it another time in GW-GC ..Because it is the same check.

The same procedures above will apply if you have to go to footnote (b).

**c. If the soil is Fine-grained soil → Follow the following:**

- According to value of liquid limit (LL) either  $LL < 50$  or  $LL \geq 50$ .
- Always we deal with **inorganic** soil and don't deal with organic soil.
- If  $LL < 50$  , **Choose Group Symbol according to values of (PI) and you may use the footnote (e) below table (5.2) if  $4 \leq PI \leq 7$  .**
- If  $LL \geq 50$ , **Choose Group Symbol according to comparison between PI of soil and PI from "A" line.**

**2. Finding group name:**

- From (**figure 5.4 P.133**) for **Coarse** (gravel & sand) and from (**figure 5.5 P.143**) for **Fines** (silt & clay).
- To find group name easily you should know the following:
  - $\% \text{ Sand} = R_{200} - R_4$  ,  $\% \text{ Gravel} = R_4$
  - The value of (PI) >>>( in what range).
  - Comparison between PI &  $PI_{A\text{-line}}$  to know (on, above or below A-line).
  - $\% \text{ Plus \#200 sieve} = \% \text{ cumulative retained on \#200 sieve} = R_{200}$  .

**Important Note:** All values of  $R_4$ ,  $R_{200}$  and  $F_{200}$  must depend on sieve analysis >> must be cumulative ("R" increase with opening decrease and "F" decrease with opening decrease).



**23. (Mid 2013):**

Classify the following soils using the unified soil classification system.  
give group symbol & group name.

- a. A 55% of soil sample retained on sieve No.4 and 89% retained on sieve No.200. The soil has a liquid limit (LL) =28% and plastic limit (PL) =18% ,  
 $C_u=4.2$ ,  $C_c=1.4$ , (A-line:  $PI=0.73(LL-20)$ ).

**Solution****Givens:**

$R_4 = 55\%$  ,  $R_{200} = 89\%$  (R increase >> so, Comulative) ,  
 $F_{200} = 100\% - R_{200} = 11\%$  ,  $LL=28\%$  ,  $PL=18\%$  ,  $PI = LL-PL=28-18=10\%$   
 $C_u=4.2$  ,  $C_c=1.4$

**1. Finding Group Symbol from Table (5.2):**

- a) Determine whether the soil is **Coarse-grained** or **Fine-grained**:

$R_{200} = 89\% > 50\% \rightarrow$  Coarse – grained soil.

- b) Determine whether the soil is gravel or sand:

$\frac{R_4}{R_{200}} \times 100\% = \frac{55}{89} \times 100\% = 61.8\% > 50\% \rightarrow$  The soil is Gravel.

- $F_{200} = 11\% \rightarrow 5\% \leq F_{200} \leq 12\% \rightarrow$  **Choose Group Symbol according the footnote (a) below table (5.2).**

- The group symbol must be one of the following :

GW-GM, GW-GC, GP-GM, GP-GC >> Now we check each symbol

GW  $\rightarrow$  according the values of  $C_u$  &  $C_c$ :

$C_u=4.2 > 4$  **AND**  $C_c=1.4$  ( $1 \leq C_c \leq 3$ )  $\rightarrow$  **GW is Ok.**

GM  $\rightarrow$  according the values of PI or A-line equation:

$PI=10\% > 4 \rightarrow$  Not ok **OR** PI plots below "A"-line  $\rightarrow$

$PI_{A-line} = 0.73(28 - 20) = 5.84 < 10 \rightarrow$  above (not below)  $\rightarrow$  **GM is Not Ok**

So, each symbol having GM must be canceled (GW-GM & GP-GM)

Now we want to check only one of (GC and GP) to know the correct symbol because one of them is true and the other is false

Try GP  $\rightarrow$  according the values of  $C_u$  &  $C_c$ :

$C_u=4.2 > 4$  not ok **AND/OR**  $C_c=1.4$  ( $1 \leq C_c \leq 3$ ) not ok  $\rightarrow$  **GP is**

**Not Ok**  $\rightarrow$  GP-GC **Not Ok**  $\rightarrow$  so, the group Symbol is **GW-GC** ✓.

**Note:** ( $1 > C_c > 3$ ) means  $C_c > 3$  OR  $C_c < 1$

2. **Finding Group Name from Figure (5.4):**

% Sand =  $R_{200} - R_4 = 89\% - 55\% = 34\% > 15\% \rightarrow \rightarrow$  The group name is:  
**Well-graded gravel with clay and sand (or silty clay and sand) ✓.**

b. 200 grams of a soil sample has a grain size distribution as shown in the table below with a liquid limit of 32% and plastic limit of 15%.

Particle Size (mm)	4.75	2.36	1.3	0.6	0.2	0.075
Retained (grams)	12	15	13	28	20	2

**Solution**

**Givens:**

LL=32% , PL=15% , PI = LL-PL= 32-15=17% ,  $M_{\text{soil}} = 200$  gram,

From the given table we note the following:

- ✓ The given is the mass retained (not percent).
- ✓ The given mass is **not cumulative**.
- ✓ So, we must calculate the % cumulative Retained as shown in the following table :

	#4					#200
Particle Size (mm)	4.75	2.36	1.3	0.6	0.2	0.075
Retained (grams)	12	15	13	28	20	2
Cumulative Retained (grams)	12	27	40	68	88	90
% Com. Retained (%)	6	13.5	20	34	44	45
% Com. Passing (%)	94	86.5	80	66	56	55

Calculation in the above table was done according the following:

Cumulative Retained (grams)  $\rightarrow 12 \rightarrow 12+15=27 \rightarrow 27+13= 40 \rightarrow 40+28=68$  and so on.

$$\% \text{ Com. Retained (\%)} = \frac{\text{Cumulative Retained (grams)}}{M_{\text{soil}}=200 \text{ gram}} \times 100\%$$

$$\% \text{ Com. Passing (\%)} = 100\% - \% \text{ Com. Retained (\%)}$$

Now, from the above table  $\rightarrow R_4 = 6\%$  ,  $R_{200} = 45\%$  ,  $F_{200} = 55\%$

**1. Finding Group Symbol from Table (5.2):**

a) Determine whether the soil is **Coarse-grained** or **Fine-grained**:

$$R_{200} = 45\% < 50\% \rightarrow \text{Fine - grained soil.}$$

b) Finding group symbol from the lower part of table (5.2):

$LL = 32 < 50$  & Inorganic soil  $\rightarrow$  Classify according PI and "A"-line

$\rightarrow PI = 17\% > 7$

$\rightarrow PI_{A\text{-line}} = 0.73(32 - 20) = 8.76 < 17 \rightarrow$  above

So,  $PI > 7$  **AND** Plots above "A"-line  $\rightarrow$  Group Symbol is **CL** ✓.

## 2. Finding Group Name from Figure (5.5):

The following parameters will be used:

$LL=32$  ,  $PI=17$  and Plots above "A"-line , %plus No.200 =  $R_{200} = 45\%$

% Sand =  $R_{200} - R_4 = 45 - 6 = 39\%$  , %Gravel =  $R_4 = 6\%$

Now, we find group name as following:

$LL = 32 < 50 \rightarrow$  Inorganic  $\rightarrow PI > 7$  and Plots above "A"-line

$\rightarrow CL \rightarrow R_{200} = 45\% > 30\% \rightarrow$  %Sand =  $39\% >$  %Gravel =  $6\%$

$\rightarrow$  %Gravel =  $6\% < 15\% \rightarrow \rightarrow$  Group Name is **Sandy Lean Clay** ✓.

**24. (Mid 2012):**

Using the unified soil classification system to classify the following soils.  
(Group Symbol & Group Name).

- a. A 65% of the soil sample retained on No.4 Sieve, 30% retained on No.200 Sieve,  $C_u=3$  and  $C_c=0.9$ . Also the  $LL = 28\%$  and  $PL = 24.5\%$ , (“A”-line:  $PI = 0.73(LL-20)$ ).

**Solution****Givens:**

$R_4 = 65\%$  ,  $R_{200} = 65\% + 30\% = 95\% \rightarrow F_{200} = 5\%$  (Why)  $\rightarrow$  Because the values must be cumulative and we know that %Retain must increase ..but in this case %Retain decrease thus, it must be cumulative by adding 65% to 30% .

$LL=28\%$  ,  $PL=24.5\%$  ,  $PI = LL-PL= 28-24.5=3.5\%$

$PI_{A-line} = 0.73(28 - 20) = 5.84 > 3.5 \rightarrow$  below.

**1. Finding Group Symbol from Table (5.2):**

- a) Determine whether the soil is **Coarse-grained** or **Fine-grained**:

$R_{200} = 95\% > 50\% \rightarrow$  Coarse – grained soil.

- b) Determine whether the soil is gravel or sand:

$\frac{R_4}{R_{200}} \times 100\% = \frac{65}{95} \times 100\% = 68.4\% > 50\% \rightarrow$  The soil is Gravel.

- $F_{200} = 5\% \rightarrow 5\% \leq F_{200} \leq 12\% \rightarrow$  Choose Group Symbol according the footnote (a) below table (5.2).

- The group symbol must be one of the following :

GW-GM, GW-GC, GP-GM, GP-GC>> Now we check each symbol

GW $\rightarrow$  according the values of  $C_u$  &  $C_c$ :

$C_u = 3 < 4$  (not ok) **AND**  $C_c = 0.9$  not in  $(1 \leq C_c \leq 3) \rightarrow$  **GW is Not Ok.**

So, each symbol having GW must be canceled(GW-GM & GW-GC)

GM $\rightarrow$  according the values of PI or A-line equation:

$PI = 3.5 < 4 \rightarrow$  (Ok) **OR** PI plots below “A”-line(Ok) $\rightarrow$  **GM is Ok**

Now we want to check only one of (GC and GP) to know the correct symbol because one of them is true and the other is false

Try GP $\rightarrow$  according the values of  $C_u$  &  $C_c$ :

$C_u = 3 < 4$  (Ok) **AND/OR**  $C_c = 0.9 < 1$  (Ok)  $\rightarrow$  **GP is OK**

$\rightarrow$  So, the group Symbol is **GP-GM** ✓ .

**2. Finding Group Name from Figure (5.4):**

$\% \text{ Sand} = R_{200} - R_4 = 95\% - 65\% = 30\% > 15\% \rightarrow \rightarrow$  The group name is:  
**Poorly graded gravel with silt and sand ✓.**

Now, as you see the classification is so easy, just you need some focusing, so I will finish solving problems completely in this chapter, but I will explain some ideas, and I give you the answer to solve the problems by yourself and check your solution.

**25. (Mid 2011):**

- a. 86% of a soil sample passed Sieve No.4 and retained on Sieve No.200. Also given that: Coefficient of gradation=1, Uniformity Coefficient = 3

**Solution**

**شرح السؤال:** نلاحظ أن 86% من العينة مرت من المنخل رقم 4 وبالتالي تبقى عليه 14% من العينة، وأن الـ 86% التي مرت من المنخل رقم 4 كلها بقيت على المنخل رقم 200.. وبالتالي فإن الـ 14% التي بقيت على المنخل رقم 4 سوف تبقى أيضا على المنخل رقم 200 وبالتالي يصبح المتبقي على المنخل رقم 200 يساوي  $100\% = 14 + 86$  (لأن التجربة يجب أن تكون تراكمية) أما المار منه فهو صفرا.

$$F_4 = 86\% \rightarrow R_4 = 14\% , F_{200} = 0.0 , R_{200} = 100\%$$

$$\text{Coefficient of gradation} = C_c = 1 , \text{Uniformity Coefficient} = C_u = 3$$

The group symbol is: **SP ✓.**

The group name is: **Poorly graded sand ✓.**

- b. A sieve analysis of a soil show that 97% of the soil passed sieve No.4 and 73% passed sieve No.200. If the liquid limit of the soil is 62 and its plastic limit =34.

**Solution**

$$F_4 = 97\% \rightarrow R_4 = 3\% , F_{200} = 73\% , R_{200} = 27\%$$

$$LL=62\% , PL=34\% , PI = LL-PL= 62-34=28\%$$

No idea in this question but you should be attention for the large value of LL because it will be used when you finding group symbol.

The group symbol is: **MH ✓.**

The group name is: **Elastic silt with sand ✓.**

**26. (Mid 2009):**

Use Unified Soil Classification System to classify the given soil knowing that the liquid limit is 30% and plastic limit is 27%. (give group Symbol and group name)

Particle Size (mm)	% Finer than
6	100
4.75	90
2.36	84
( $D_{60}$ )1.3	60
( $D_{30}$ )0.6	30
( $D_{10}$ )0.2	10
0.075	8

**Solution**

$$LL=30\% , PL=27\% , PI = LL-PL= 30-27=3 \%$$

The idea is how to calculate  $C_c$  and  $C_u$

From the given table we note the following :

- % Finer than = % Passing
- % Passing is cumulative (if not you must make it cumulative)
- After insuring the values are cumulative we can take the values of  $D_{60}$  ,  $D_{30}$  ,  $D_{10}$  from the table as

following :  $D_{60}=1.3$  ,  $D_{30}=0.6$  ,  $D_{10}=0.2$  thus, we can calculate  $C_c$  and  $C_u$  :

$$C_u = \frac{D_{60}}{D_{10}} = \frac{1.3}{0.2} = 6.5 , C_c = \frac{(D_{30})^2}{D_{60} \times D_{10}} = \frac{0.6^2}{1.3 \times 0.2} = 1.38$$

$$R_4 = 100 - 90 = 10\% , R_{200} = 100 - 8 = 92\% , F_{200} = 8\%$$

The group symbol is: **SW-SM** ✓ .

The group name is: **Well graded sand with silt** ✓ .

**See AASHTO Classification System (Page 126 in your textbook)**

## **Chapter (7)**

# **Soil Permeability**

## Bernoulli's Equation (for Soil):

Total Head = Pressure Head + Velocity Head + Elevation Head

$$h_{\text{total}} = \frac{u}{\gamma_w} + \frac{v^2}{2g} + Z$$

u: pore water pressure

v: velocity of water through the soil

Z: vertical distance of a given point above or below a datum plane.

### Notes:

- ✓ Pressure head is also called "piezometric head".
- ✓  $u = h_{\text{pressure}} \times \gamma_w$ .
- ✓ The velocity of water through soil is very small (about 0.01 → 0.001)m/sec and when the velocity is squared the value will be very small so, the velocity head in Bernoulli's equation should be canceled and the final form of Bernoulli's

equation will be :  $h_{\text{total}} = \frac{u}{\gamma_w} + Z$

- ✓ The head loss that result from the movement of water through the soil ( $\Delta H$ ) can be expressed in a nondimensional form as:

$$i = \frac{\Delta H}{L}, \quad \text{where:}$$

$i$  = Hydraulic gradient (The head loss per unit length)

وهي تعبر عن مقدار الفقدان في ال head لكل متر (أو أي وحدة طول) تتحركه الماء في التربة

$L$  = Total length of soil or **soils** that the water passes through it

## Darcy's Law:

Darcy found that, there are proportional relationship between velocity ( $v$ ) and hydraulic gradient ( $i$ ), this relationship still valid if the flow still laminar, and in soil the velocity is small so, the flow is always laminar.

$$v \propto i \rightarrow v = k \cdot i$$

K: Hydraulic conductivity of soil = Permeability of soil (m/sec)

Now, we know that:  $Q = V \times A$  and  $v = k \cdot i$  (Darcy's law)  $\rightarrow \rightarrow \rightarrow$

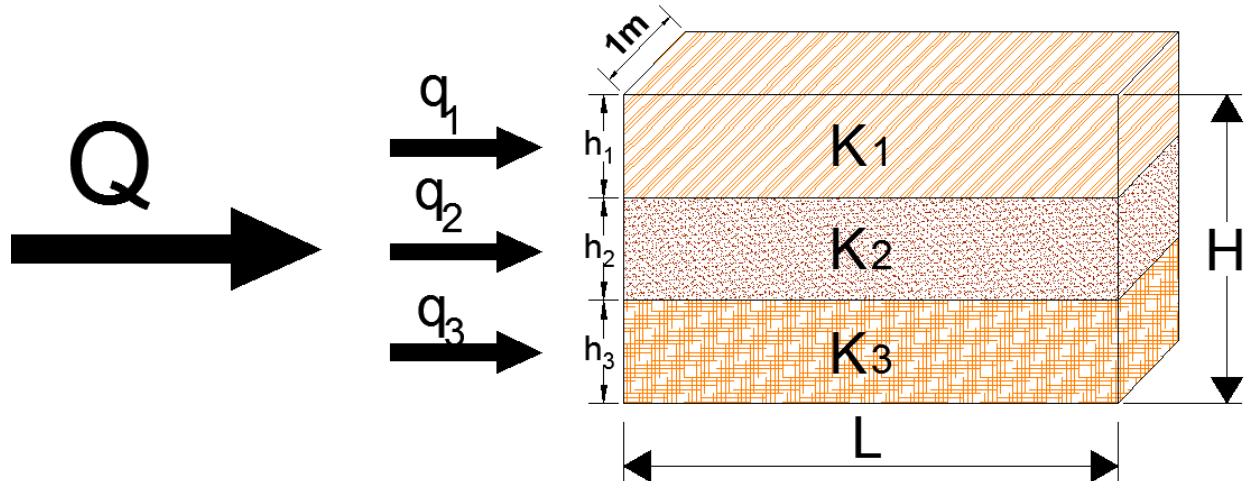
$$Q = K \cdot i \cdot A$$

A: Cross sectional area that **perpendicular** to flow direction.



## Directional Variation of Permeability:

### 1. Horizontal Permeability (Flow parallel to layers):



From the above graph we note the following:

- ✓ All values of H and h are perpendicular to the flow .
- ✓  $H_{\text{total}} = H = h_1 + h_2 + h_3 + h_n$
- ✓  $Q_{\text{total}} = Q_{\text{eq.}} = q_1 + q_2 + q_3 + \dots + q_n$
- ✓  $A_{\text{total}} = A_{\text{eq.}} = H \times 1$  ,  $A_1 = h_1 \times 1$  ,  $A_2 = h_2 \times 1$  ,  $A_3 = h_3 \times 1$  , and  $A_n = h_n \times 1$
- ✓  $\Delta H_{\text{total}} = \Delta h_1 = \Delta h_2 = \Delta h_3 = \Delta h_n$  (because all layers have the same length)  $\rightarrow \rightarrow$
- ✓  $i_{\text{total}} = i_{\text{eq.}} = i_1 = i_2 = i_3 = \dots = i_n$

Now,

$$Q_{\text{total}} = K_{\text{eq.}} \times i_{\text{eq.}} \times A_{\text{eq.}} \quad , \quad q_1 = K_1 \times i_1 \times A_1 \text{ and so on}$$

$$K_{\text{eq.}} \times i_{\text{eq.}} \times A_{\text{eq.}} = K_1 \times i_1 \times A_1 + K_2 \times i_2 \times A_2 + K_3 \times i_3 \times A_3 + \dots + K_n \times i_n \times A_n$$

$$i_{\text{eq.}} = i_1 = i_2 = i_3 = i_n \text{ So we cancel it}$$

$$K_{\text{eq.}} \times A_{\text{eq.}} = K_1 \times A_1 + K_2 \times A_2 + K_3 \times A_3 + \dots + K_n \times A_n$$

$$K_{\text{eq.}} \times H \times 1 = K_1 \times h_1 \times 1 + K_2 \times h_2 \times 1 + K_3 \times h_3 \times 1 + \dots + K_n \times h_n \times 1$$

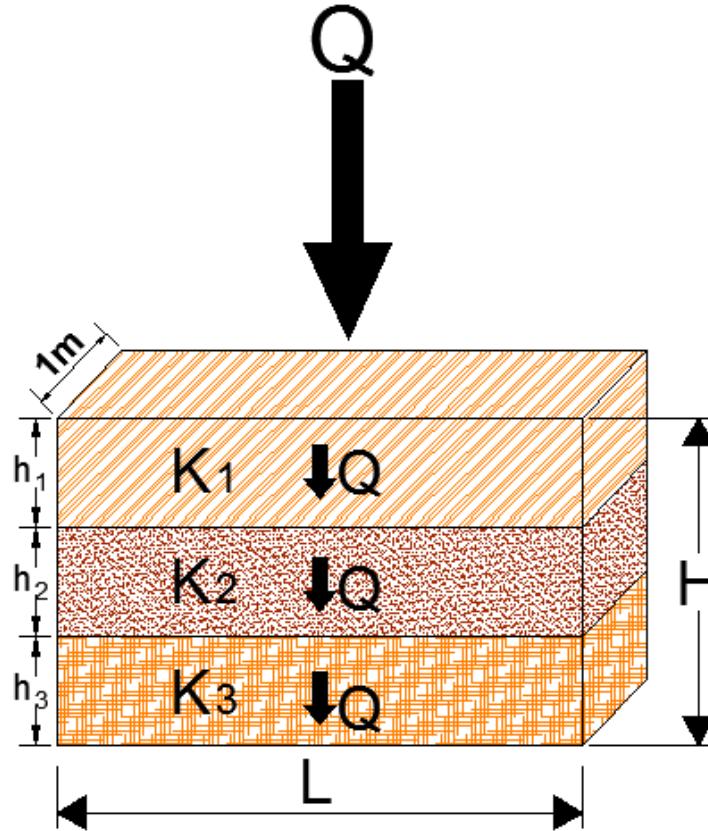
Rearrange the last equation:

$$K_{\text{eq.}} = \frac{K_1 \times h_1 \times 1 + K_2 \times h_2 \times 1 + K_3 \times h_3 \times 1 + \dots + K_n \times h_n \times 1}{H}$$

The final equation:

$$K_{\text{eq.}} = \frac{\sum(K_n \times h_n)}{H}$$

## 2. Vertical Permeability (Flow perpendicular to layers):



From the above graph we note the following:

- ✓  $H_{\text{total}} = H = h_1 + h_2 + h_3 + h_n$
- ✓  $Q_{\text{total}} = Q_{\text{eq.}} = q_1 = q_2 = q_3 = \dots = q_n$
- ✓  $A_{\text{total}} = A_{\text{eq.}} = L \times 1 = A_1 = A_2 = A_3 = \dots = A_n$
- ✓  $\Delta H_{\text{total}} = \Delta h_1 + \Delta h_2 + \Delta h_3 + \Delta h_n$  (because flow pass through each layer)  $\rightarrow \rightarrow$
- ✓  $i_{\text{total}} = i_{\text{eq.}} + i_1 + i_2 + i_3 + \dots + i_n$

Now,

$$q_n = K_n \times \frac{\Delta h_n}{h_n} \times A_n \rightarrow \Delta h_n = \frac{q_n \times h_n}{K_n \times A_n}$$

$$\frac{Q_{\text{eq.}} \times H}{K_{\text{eq.}} \times A_{\text{eq.}}} = \frac{q_1 \times h_1}{K_1 \times A_1} + \frac{q_2 \times h_2}{K_2 \times A_2} + \frac{q_3 \times h_3}{K_3 \times A_3} + \dots + \frac{q_n \times h_n}{K_n \times A_n} \quad (Q \text{ and } A \text{ are the same} \rightarrow \text{cancel them})$$

$$\frac{H}{K_{\text{eq.}}} = \frac{h_1}{K_1} + \frac{h_2}{K_2} + \frac{h_3}{K_3} + \dots + \frac{h_n}{K_n} \rightarrow \rightarrow \text{The final equation:}$$

$$K_{\text{eq.}} = \frac{H}{\frac{h_1}{K_1} + \frac{h_2}{K_2} + \frac{h_3}{K_3} + \dots + \frac{h_n}{K_n}}$$

**Note:** All values of  $H$  and  $h$  are parallel to the flow

## Permeability Test in the Field by Pumping from Wells:

### 1. Un Confined Aquifer:

The equation of un confined aquifer:

$$K = \frac{2.303q \times \log_{10} \left( \frac{r_1}{r_2} \right)}{\pi \times (h_1^2 - h_2^2)}$$

### 2. Confined Aquifer:

The equation of confined aquifer:

$$K = \frac{q \times \log_{10} \left( \frac{r_1}{r_2} \right)}{2.727 \times H \times (h_1 - h_2)}$$

### Important Notes about the above two cases:

- ✓ Impervious layer = Impermeable layer.
- ✓ Clay layer considered impermeable layer.
- ✓ Always take the max.value of r as  $r_1 \rightarrow$  the value of  $h_1$  must be larger than  $h_2$
- ✓ r is the distance from the center of the well to the center of observation well.
- ✓ h is the elevation of water in the observation well.
- ✓ The following graph explain “h” :

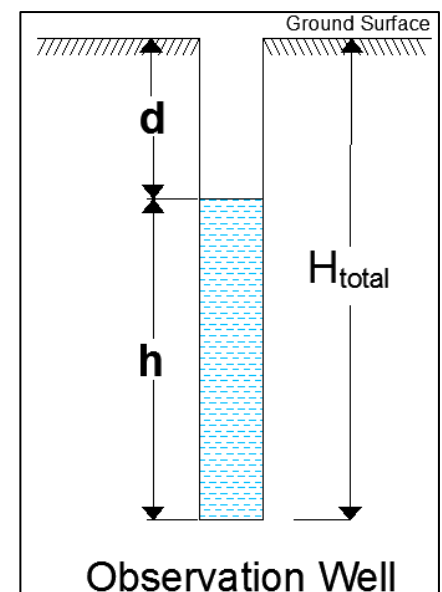
In all Questions usually the value of d is given to make confusion.

d: Draw-down (**Level of water below the ground surface**)

(مقدار السحب الذي تم ملاحظته في البئر )

$$\text{So, } h = H_{\text{total}} - d$$

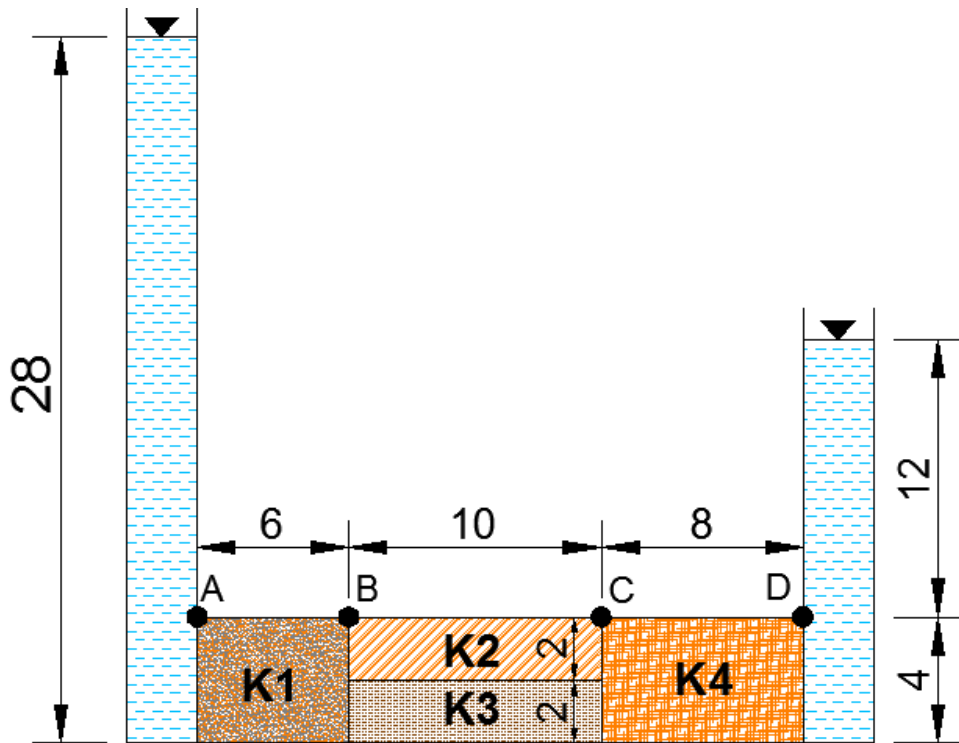
- ✓ H in equation of confined aquifer is the thickness of soil layer that confined between two impermeable layers.



**27. (Mid 2013):**

1. Find **The total head and Pressure head** at points(A-B-C-D) with respect to given datum, Assume  $3K_1 = K_2 = 1.5K_3 = 2K_4$ .
2. Find the flow rate ( $K_1 = 3.5 \times 10^{-2}(\text{cm}/\text{sec})$ )

**Note: All dimensions are in cm**

**Solution**

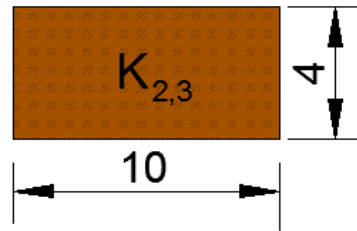
The first step is to making the soil 2 and soil 3 as one soil by calculating the equivalent value of K for these two layers (because all layers perpendicular to flow except these two layers)...we want to make them as one layer perpendicular to flow.

$$K_2 = 3K_1, \quad K_3 = 2K_1, \quad K_4 = 1.5K_1$$

$$K_{\text{eq.}} = K_{2,3} = \frac{\sum(K_n \times h_n)}{H} \rightarrow K_{2,3} = \frac{K_2 \times h_2 + K_3 \times h_3}{h_2 + h_3}$$

$$\rightarrow K_{2,3} = \frac{3K_1 \times 2 + 2K_1 \times 2}{2 + 2} = 2.5K_1$$

The new soil layer is shown in the figure below:



Now we can calculate  $K_{eq.}$  for whole system (all layers perpendicular to the flow).

$$K_{eq.} = \frac{H}{\frac{h_1}{K_1} + \frac{h_2}{K_2} + \frac{h_3}{K_3} + \dots + \frac{h_n}{K_n}} = \frac{6 + 10 + 8}{\frac{6}{K_1} + \frac{10}{K_{2,3}} + \frac{8}{K_4}}, K_{2,3} = 2.5K_1, K_4 = 1.5K_1$$

$$\rightarrow K_{eq.} = \frac{6 + 10 + 8}{\frac{6}{K_1} + \frac{10}{2.5K_1} + \frac{8}{1.5K_1}} \rightarrow K_{eq.} = 1.565K_1$$

$$\Delta H_{total} = 28 - (12 + 4) = 12 \text{ cm.}$$

$$i_{eq.} = \frac{\Delta H_{total}}{H} = \frac{12}{6 + 10 + 8} = 0.5$$

$$Q_{eq.} = q_1 = q_{2,3} = q_4, \text{ and } A_{eq.} = 4 \times 1 = A_1 = A_{2,3} = A_4 \text{ (Solution Key)}$$

**Point "A":**

$$h_{total,A} = 28 \text{ (no losses because no soil) (Total head) } \checkmark.$$

$$h_{pressure,A} = h_{total,A} - h_{elevation,A} = 28 - 4 = 24 \text{ cm } \checkmark. \text{ (Pressure = piezometric head)}$$

$h_{elevation,A}$  هو بعد النقطة المطلوبة عن المرجع المأخوذ منه الإحداثيات الرأسية وهنا يساوي 4 سم

**Point "B":** (the soil pass through soil "1" then reach point B) So,

$$Q_{eq.} = q_1 \rightarrow K_{eq.} \times i_{eq.} \times A_{eq.} = K_1 \times i_1 \times A_1 \text{ but, } A_{eq.} = A_1 = 4 \rightarrow \rightarrow \rightarrow$$

$$K_{eq.} \times i_{eq.} = K_1 \times i_1 \rightarrow 1.565K_1 \times 0.5 = K_1 \times i_1 \rightarrow i_1 = 0.7825$$

$$h_{total,B} = h_{total,A} - i_1 \times h_1 \rightarrow h_{total,B} = 28 - 0.7825 \times 6 = 23.305 \text{ cm } \checkmark.$$

$$h_{pressure,B} = h_{total,B} - h_{elevation,B} = 23.305 - 4 = 19.305 \text{ cm } \checkmark.$$

**Point “C”:** (the soil pass through soil “2,3” then reach point C) So,

$$Q_{eq.} = q_{2,3} \rightarrow K_{eq.} \times i_{eq.} \times A_{eq.} = K_{2,3} \times i_{2,3} \times A_{2,3} \text{ but, } A_{eq.} = A_{2,3} = 4 \rightarrow \rightarrow \rightarrow$$

$$K_{eq.} \times i_{eq.} = K_{2,3} \times i_{2,3} \rightarrow 1.565K_1 \times 0.5 = 2.5K_1 \times i_{2,3} \rightarrow i_{2,3} = 0.313$$

$$h_{total,C} = h_{total,B} - i_{2,3} \times h_{2,3} \rightarrow h_{total,C} = 23.305 - 0.313 \times 10 = 20.175\text{cm} \checkmark.$$

$$h_{pressure,C} = h_{total,C} - h_{elevation,C} = 20.175 - 4 = 16.175\text{cm} \checkmark.$$

**Point “D”:** (the soil pass through soil “4” then reach point D) So,

$$Q_{eq.} = q_4 \rightarrow K_{eq.} \times i_{eq.} \times A_{eq.} = K_4 \times i_4 \times A_4 \text{ but, } A_{eq.} = A_4 = 4 \rightarrow \rightarrow \rightarrow$$

$$K_{eq.} \times i_{eq.} = K_4 \times i_4 \rightarrow 1.565K_1 \times 0.5 = 1.5K_1 \times i_4 \rightarrow i_4 = 0.5216$$

$$h_{total,D} = h_{total,C} - i_4 \times h_4 \rightarrow h_{total,D} = 20.175 - 0.5216 \times 8 = 16\text{cm} \checkmark.$$

$h_{total,D} = 16$  must be checked ( $h_{total,D} = 12 + 4 = 16$ )OK , if not equal >> you must revise your solution because some error exist.

$$h_{pressure,D} = h_{total,D} - h_{elevation,D} = 16 - 4 = 12\text{cm} \checkmark.$$

**The second required** (Don't forget it)

$$Q_{eq.} = q_1 = q_2 = q_3 \quad K_1 = 3.5 \times 10^{-2}(\text{cm/sec})$$

$$Q_{eq.} = K_{eq.} \times i_{eq.} \times A_{eq.}$$

$$Q_{eq.} = 1.565K_1 \times 0.5 \times 4 \times 1 = 1.565 \times 3.5 \times 10^{-2} \times 0.5 \times 4 = 10.955\text{cm}^3/\text{sec}$$

To check it:

$$q_1 = K_1 \times i_1 \times A_1 \quad i_1 = 0.7825$$

$$= 3.5 \times 10^{-2} \times 0.7825 \times 4 = 10.955\text{cm}^3/\text{sec} \checkmark.$$

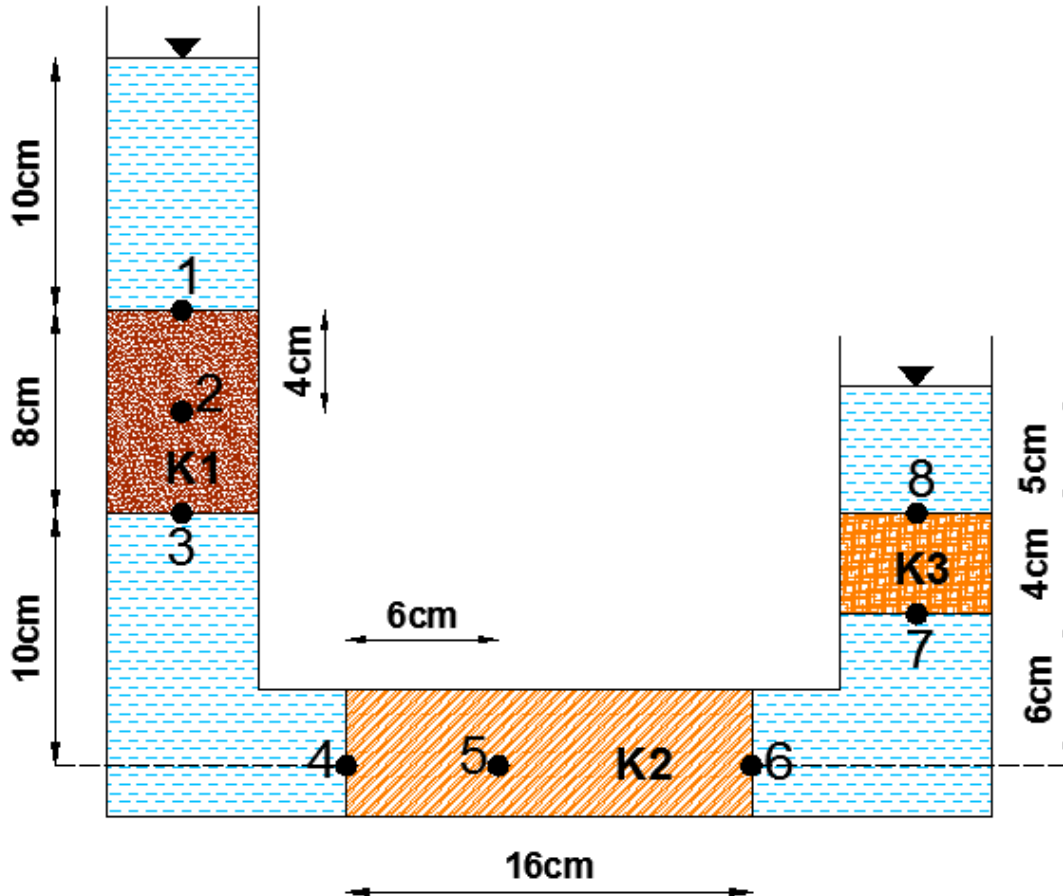
**Note:**

If the pore water pressure (u) is required at each point >>> do the following:

At each point:  $u = \text{pressure head} \times \gamma_{\text{water}}$

**28. (Mid 2013):**

Find the **Total head** at points (1 to 8), if the cross section of the system is  $10 \text{ cm}^2$ .  
Knowing that:  $K_1 = 0.5K_2 = 3K_3$

**Solution**

It is noted that all layers are perpendicular to flow (Vertical Permeability)

Now we can calculate  $K_{eq.}$  for the whole system

$$K_{eq.} = \frac{H}{\frac{h_1}{K_1} + \frac{h_2}{K_2} + \frac{h_3}{K_3} + \dots + \frac{h_n}{K_n}} = \frac{8 + 16 + 4}{\frac{8}{K_1} + \frac{16}{K_2} + \frac{4}{K_3}}, \quad K_2 = 2K_1, \quad K_3 = 0.333K_1$$

$$\rightarrow K_{eq.} = \frac{8 + 16 + 4}{\frac{8}{K_1} + \frac{16}{2K_1} + \frac{4}{0.333K_1}} \rightarrow K_{eq.} = 0.9996K_1 \rightarrow K_{eq.} \cong K_1$$

$$\Delta H_{\text{total}} = (10 + 8 + 10) - (6 + 4 + 5) = 13\text{cm.}$$

$$i_{\text{eq.}} = \frac{\Delta H_{\text{total}}}{H} = \frac{13}{8 + 16 + 4} = 0.4643$$

$$Q_{\text{eq.}} = q_1 = q_2 = q_3, \text{ and } A_{\text{eq.}} = 10\text{cm}^2 = A_1 = A_2 = A_3 \text{ (Solution Key)}$$

**Point “1”:**

$$h_{\text{total},1} = 28 \text{ (no losses because no soil) (Total head) } \checkmark.$$

**Point “2”:** ( the soil pass through the mid of soil “1” then reach point 2) So,

$$Q_{\text{eq.}} = q_1 \rightarrow K_{\text{eq.}} \times i_{\text{eq.}} \times A_{\text{eq.}} = K_1 \times i_1 \times A_1 \text{ but, } A_{\text{eq.}} = A_1 = 10 \rightarrow \rightarrow \rightarrow$$

$$K_{\text{eq.}} \times i_{\text{eq.}} = K_1 \times i_1 \rightarrow K_1 \times 0.4643 = K_1 \times i_1 \rightarrow i_1 = 0.4643$$

$$h_{\text{total},2} = h_{\text{total},1} - i_1 \times 4 \rightarrow h_{\text{total},2} = 28 - 0.4643 \times 4 = 26.1428\text{cm} \checkmark.$$

**Point “3”:** ( the soil pass through all of soil “1” then reach point 3) So,

$$Q_{\text{eq.}} = q_1 \rightarrow K_{\text{eq.}} \times i_{\text{eq.}} \times A_{\text{eq.}} = K_1 \times i_1 \times A_1 \text{ but, } A_{\text{eq.}} = A_1 = 10 \rightarrow \rightarrow \rightarrow$$

$$K_{\text{eq.}} \times i_{\text{eq.}} = K_1 \times i_1 \rightarrow K_1 \times 0.4643 = K_1 \times i_1 \rightarrow i_1 = 0.4643$$

$$h_{\text{total},3} = h_{\text{total},2} - i_1 \times 4 \rightarrow h_{\text{total},3} = 26.1428 - 0.4643 \times 4 = 24.2856\text{cm}$$

$$\text{Or, } h_{\text{total},3} = h_{\text{total},1} - i_1 \times 8 \rightarrow h_{\text{total},3} = 28 - 0.4643 \times 8 = 24.2856\text{cm} \checkmark.$$

**Point “4”:**

$$h_{\text{total},4} = h_{\text{total},3} = 24.2856 \text{ cm (no losses because no soil) } \checkmark.$$

**Point “5”:** ( the soil pass through 6cm of soil “2” then reach point 5) So,

$$Q_{\text{eq.}} = q_2 \rightarrow K_{\text{eq.}} \times i_{\text{eq.}} \times A_{\text{eq.}} = K_2 \times i_2 \times A_2 \text{ but, } A_{\text{eq.}} = A_2 = 10 \rightarrow \rightarrow \rightarrow$$

$$K_{\text{eq.}} \times i_{\text{eq.}} = K_2 \times i_2 \rightarrow K_1 \times 0.4643 = 2K_1 \times i_2 \rightarrow i_2 = 0.23215$$

$$h_{\text{total},5} = h_{\text{total},4} - i_2 \times 6 \rightarrow h_{\text{total},5} = 24.2856 - 0.23215 \times 6 = 22.8927\text{cm}$$

**Point “6”:** ( the soil pass through all of soil “2” then reach point 6) So,

$$Q_{\text{eq.}} = q_2 \rightarrow K_{\text{eq.}} \times i_{\text{eq.}} \times A_{\text{eq.}} = K_2 \times i_2 \times A_2 \text{ but, } A_{\text{eq.}} = A_2 = 10 \rightarrow \rightarrow \rightarrow$$

$$K_{\text{eq.}} \times i_{\text{eq.}} = K_2 \times i_2 \rightarrow K_1 \times 0.4643 = 2K_1 \times i_2 \rightarrow i_2 = 0.23215$$

$$h_{\text{total},6} = h_{\text{total},5} - i_2 \times (16 - 6) \rightarrow h_{\text{total},6} = 22.8927 - 0.23215 \times 10 = 20.5712\text{cm}$$

Or,

$$h_{\text{total},6} = h_{\text{total},4} - i_2 \times 16 \rightarrow h_{\text{total},6} = 24.2856 - 0.23215 \times 16 = 20.5712\text{cm} \checkmark.$$



**Point “7”:**

$$h_{\text{total},7} = h_{\text{total},6} = 20.5712 \text{ cm (no losses because no soil)} \checkmark .$$

**Point “8”:** ( the soil pass through all of soil “3” then reach point 8) So,

$$Q_{\text{eq.}} = q_3 \rightarrow K_{\text{eq.}} \times i_{\text{eq.}} \times A_{\text{eq.}} = K_3 \times i_3 \times A_3 \text{ but, } A_{\text{eq.}} = A_3 = 10 \rightarrow \rightarrow \rightarrow$$

$$K_{\text{eq.}} \times i_{\text{eq.}} = K_3 \times i_3 \rightarrow K_1 \times 0.4643 = 0.333K_1 \times i_3 \rightarrow i_3 = 1.3929$$

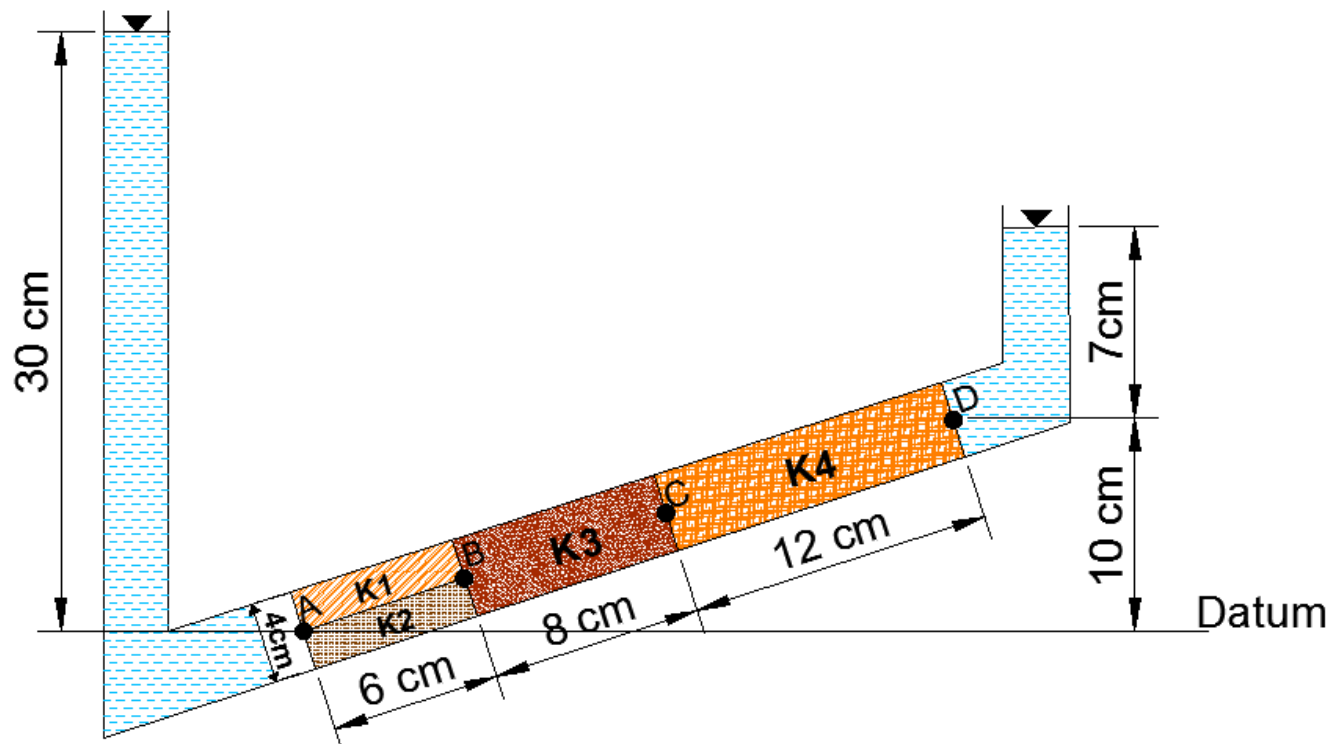
$$h_{\text{total},8} = h_{\text{total},7} - i_3 \times h_3 \rightarrow h_{\text{total},8} = 20.5712 - 1.3929 \times 4 \cong 15 \text{ cm} \checkmark .$$

$h_{\text{total},8} = 15$  must be checked ( $h_{\text{total},8} = 6 + 4 + 5 = 15$ )OK , if not equal>> you must revise your solution because some error exist.

## 29. (Mid 2009):

An inclined permeameter tube is filled with layers of soil of different permeability as shown below.

Find the **total head**, **elevation head** and **pore water pressure** at points (A-B-C-D) with respect to the given datum, Assume:  $3K_1 = K_2 = 2K_3 = 1.5K_4$



## Solution

The only different between this problem and problem "26" is to finding elevation head.

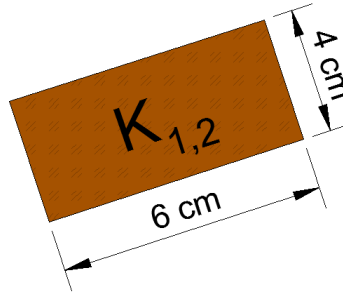
The first step is to making the soil 1 and soil 2 as one soil by calculating the equivalent value of  $K$  for these two layers (because all layers perpendicular to flow except these two layers)...we want to make them as one layer perpendicular to flow.

$$K_2 = 3K_1, \quad K_3 = 1.5K_1, \quad K_4 = 2K_1$$

$$K_{eq.} = K_{1,2} = \frac{\sum(K_n \times h_n)}{H} \rightarrow K_{1,2} = \frac{K_1 \times h_1 + K_2 \times h_2}{h_1 + h_2}$$

$$\rightarrow K_{1,2} = \frac{K_1 \times 2 + 3K_1 \times 2}{2 + 2} = 2K_1$$

The new soil layer is shown in the figure below :



Now we can calculate  $K_{eq.}$  for whole system (all layers perpendicular to the flow).

$$K_{eq.} = \frac{H}{\frac{h_1}{K_1} + \frac{h_2}{K_2} + \frac{h_3}{K_3} + \dots + \frac{h_n}{K_n}} = \frac{6 + 8 + 12}{\frac{6}{K_{1,2}} + \frac{8}{K_3} + \frac{12}{K_4}}$$

$$\rightarrow K_{eq.} = \frac{6 + 8 + 12}{\frac{6}{2K_1} + \frac{8}{1.5K_1} + \frac{12}{2K_1}} \rightarrow K_{eq.} = 1.814K_1$$

**Firstly**, we calculate the **total head** at each point.

$$\Delta H_{\text{total}} = 30 - (10 + 7) = 13\text{cm.}$$

$$i_{\text{eq.}} = \frac{\Delta H_{\text{total}}}{H} = \frac{13}{6 + 8 + 12} = 0.5$$

$$Q_{\text{eq.}} = q_{1,2} = q_3 = q_4, \text{ and } A_{\text{eq.}} = 4 \times 1 = A_{1,2} = A_3 = A_4 \text{ (Solution Key)}$$

**Point “A”:**

$$h_{\text{total,A}} = 30\text{cm (no losses because no soil) (Total head) } \checkmark .$$

**Point “B”:** (the soil pass through soil “1,2” then reach point B) So,

$$Q_{\text{eq.}} = q_{1,2} \rightarrow K_{\text{eq.}} \times i_{\text{eq.}} \times A_{\text{eq.}} = K_{1,2} \times i_{1,2} \times A_{1,2} \text{ but, } A_{\text{eq.}} = A_{1,2} = 4 \rightarrow \rightarrow \rightarrow$$

$$K_{\text{eq.}} \times i_{\text{eq.}} = K_{1,2} \times i_{1,2} \rightarrow 1.814K_1 \times 0.5 = 2K_1 \times i_{1,2} \rightarrow i_{1,2} = 0.4535$$

$$h_{\text{total,B}} = h_{\text{total,A}} - i_{1,2} \times h_{1,2} \rightarrow h_{\text{total,B}} = 30 - 0.4535 \times 6 = 27.279\text{cm} \checkmark .$$

**Point “C”:** (the soil pass through soil “3” then reach point C) So,

$$Q_{\text{eq.}} = q_3 \rightarrow K_{\text{eq.}} \times i_{\text{eq.}} \times A_{\text{eq.}} = K_3 \times i_3 \times A_3 \text{ but, } A_{\text{eq.}} = A_3 = 4 \rightarrow \rightarrow \rightarrow$$

$$K_{\text{eq.}} \times i_{\text{eq.}} = K_3 \times i_3 \rightarrow 1.814K_1 \times 0.5 = 1.5K_1 \times i_3 \rightarrow i_3 = 0.6046$$

$$h_{\text{total,C}} = h_{\text{total,B}} - i_3 \times h_3 \rightarrow h_{\text{total,C}} = 27.279 - 0.6064 \times 8 = 22.4278\text{cm} \checkmark .$$

**Point “D”:** (the soil pass through soil “4” then reach point D) So,

$$Q_{\text{eq.}} = q_4 \rightarrow K_{\text{eq.}} \times i_{\text{eq.}} \times A_{\text{eq.}} = K_4 \times i_4 \times A_4 \text{ but, } A_{\text{eq.}} = A_4 = 4 \rightarrow \rightarrow \rightarrow$$

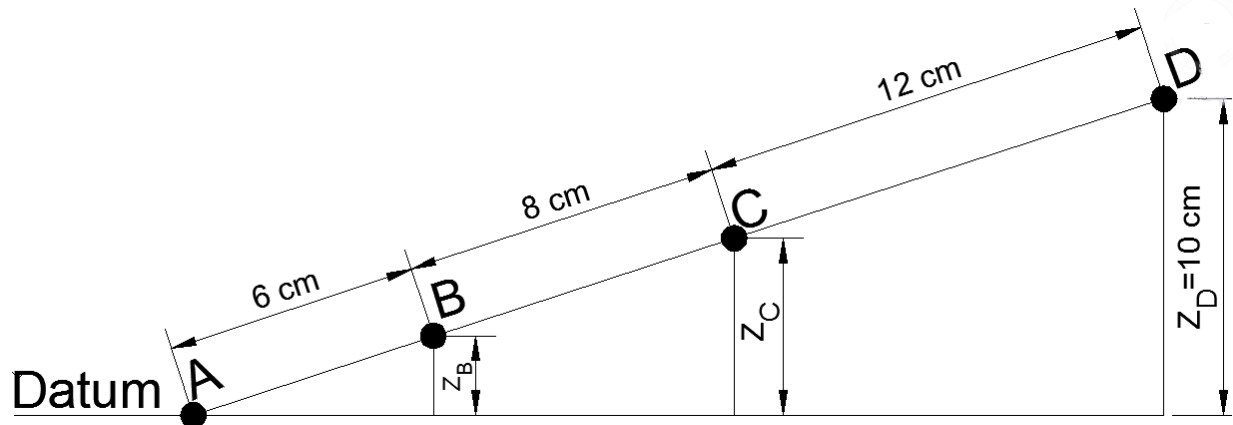
$$K_{\text{eq.}} \times i_{\text{eq.}} = K_4 \times i_4 \rightarrow 1.814K_1 \times 0.5 = 2K_1 \times i_4 \rightarrow i_4 = 0.4535$$

$$h_{\text{total,D}} = h_{\text{total,C}} - i_4 \times h_4 \rightarrow h_{\text{total,D}} = 22.4278 - 0.4535 \times 12 \cong 17\text{cm} \checkmark .$$

$h_{\text{total,D}} = 17$  must be checked ( $h_{\text{total,D}} = 10 + 7 = 17$ )OK , if not equal >> you must revise your solution because some error exist.

**Secondly**, we calculate the **elevation head** at each point with respect to the given datum.

By **interpolation** we calculate the elevation of each point from the datum as shown in the following graph :



From this graph we can calculate elevation head ( $Z$ ) at each point:

$$Z_A = 0.0 \checkmark.$$

$$\frac{Z_B}{6} = \frac{10}{6 + 8 + 12} \rightarrow Z_B = 2.307 \text{ cm} \checkmark.$$

$$\frac{Z_C}{6 + 8} = \frac{10}{6 + 8 + 12} \rightarrow Z_C = 5.384 \text{ cm} \checkmark.$$

$$Z_D = 10 \text{ cm} \checkmark.$$

Finally, we calculate the pore water pressure at each point:

$$u = \text{pressure head} \times \gamma_{\text{water}}$$

**Point "A":**

$$h_{\text{pressure,A}} = h_{\text{total,A}} - h_{\text{elevation,A}} = 30 - 0 = 30 \text{ cm}$$

$$u_A = \frac{30}{100} \times 9.81 = 2.943 \text{ KN/m}^2 \checkmark.$$

**Point "B":**

$$h_{\text{pressure,B}} = h_{\text{total,B}} - h_{\text{elevation,B}} = 27.279 - 2.307 = 24.972 \text{ cm}$$

$$u_B = \frac{24.972}{100} \times 9.81 = 2.449 \text{ KN/m}^2 \checkmark.$$

**Point "C":**

$$h_{\text{pressure,C}} = h_{\text{total,C}} - h_{\text{elevation,C}} = 22.4278 - 5.384 = 17.0438 \text{ cm}$$

$$u_C = \frac{17.0438}{100} \times 9.81 = 1.672 \text{ KN/m}^2 \checkmark.$$

**Point "D":**

$$h_{\text{pressure,D}} = h_{\text{total,D}} - h_{\text{elevation,D}} = 17 - 10 = 7 \text{ cm}$$

$$u_D = \frac{7}{100} \times 9.81 = 0.6867 \text{ KN/m}^2 \checkmark.$$

**30. (Mid 2005):**

A pumping well test was made in sands extending to a depth of 15 m where an impermeable stratum was encountered. The initial ground-water level was at the ground surface. Observation wells were sited at distances of 3 and 7.5 m from the pumping well. A steady state was established at about 20 hours when the discharge was 3.8 L/s. The drawdowns at the two observation well were 1.5 m and 0.35 m.

Calculate the coefficient of permeability.

$$K = \frac{q \times \log_e \left( \frac{r_1}{r_2} \right)}{\pi \times (h_1^2 - h_2^2)} \quad K = \frac{q \times \log_{10} \left( \frac{r_1}{r_2} \right)}{2.727 \times H \times (h_1 - h_2)}$$

**Solution**

It's clear that the problem about (**un confined aquifer**) you should know this because there exist impermeable layer below sands and no any impermeable layer above sand.

$$r_1 = 7.5\text{m (large value)} , r_2 = 3\text{m (small value)} , H_{\text{total}} = 15\text{ m}$$

$$d_1 = 0.35\text{m} , d_2 = 1.5\text{m (drawdowns)}$$

نلاحظ أن قيمة  $r_1$  هي الكبيرة وهي القيمة الثانية المعطاة في السؤال .. بالتالي قيمة  $d_1$  يجب أن تكون القيمة الثانية أيضا

$$h_1 = H_{\text{total}} - d_1 = 15 - 0.35 = 14.65\text{ (large value)}$$

$$h_2 = H_{\text{total}} - d_2 = 15 - 1.5 = 13.5\text{ (small value)}$$

$$Q = 3.8 \frac{\text{L}}{\text{sec}} \times \frac{3600 \text{ sec}}{\text{hr}} \times \frac{\text{m}^3}{1000\text{L}} = 13.68\text{m}^3/\text{hr}$$

Now, you are given two equations you must choose the correct equation for un confined aquifer (you must know the form of the equation of the two types)

$$K = \frac{q \times \log_e \left( \frac{r_1}{r_2} \right)}{\pi \times (h_1^2 - h_2^2)} \quad \text{note: } \log_e \left( \frac{r_1}{r_2} \right) = \frac{\text{Ln} \left( \frac{r_1}{r_2} \right)}{\text{Ln}(e)} = \text{Ln} \left( \frac{r_1}{r_2} \right) \text{ so the equation will be :}$$

$$K = \frac{q \times \text{Ln} \left( \frac{r_1}{r_2} \right)}{\pi \times (h_1^2 - h_2^2)} = \frac{13.68 \times \text{Ln} \left( \frac{7.5}{3} \right)}{\pi \times (14.65^2 - 13.5^2)} = 0.123\text{m/hr} \checkmark .$$

**31.**

A layer of sand 6 m thick lies beneath a **clay** stratum 5 m thick and above a bed of thick shale (نوع من الصخور). In order to determine the permeability of sand, a well was driven to the top of the shale and water pumped out at a rate of  $0.01\text{m}^3/\text{sec}$ . Two observation wells driven through the clay at 15m and 30 m from the pumping well and water was found to rise to levels of 3m and 2.4 m **below** the ground water surface. Calculate the permeability of soil.

**Solution**

It's clear that the problem is about a **(confined aquifer)** because the clay layer is considered an impermeable layer and a shale is a type of rock, and the sand layer exists between these two layers.

$$r_1 = 30\text{m (large value)}, \quad r_2 = 15\text{m (small value)}, \quad H_{\text{total}} = 5 + 6 = 11 \text{ m}$$

$$d_1 = 2.4\text{m}, \quad d_2 = 3\text{m} \quad [\text{drawdowns, (below the ground water surface)}]$$

$$h_1 = H_{\text{total}} - d_1 = 11 - 2.4 = 8.6 \text{ (large value)}$$

$$h_2 = H_{\text{total}} - d_2 = 11 - 3 = 8 \text{ (small value)}$$

$$H_{\text{sand}} = 6\text{m}$$

$$Q = 0.01\text{m}^3/\text{sec} = 36\text{m}^3/\text{hr}$$

$$K = \frac{q \times \log_{10} \left( \frac{r_1}{r_2} \right)}{2.727 \times H \times (h_1 - h_2)} = \frac{36 \times \log_{10} \left( \frac{30}{15} \right)}{2.727 \times 6 \times (8.6 - 8)} = 1.1 \text{ m/hr} \checkmark.$$

# **Chapter (8)**

# **Seepage**





## Calculating the amount of flow (Rate of seepage):

$$\Delta q = k \times i \times A$$

$$\Delta q = k \times i \text{ (m}^3 \cdot \text{sec/m)}$$

$$\Delta q = k \times \frac{\Delta H}{N_{d,number}} \text{ (for one flow channel)}$$

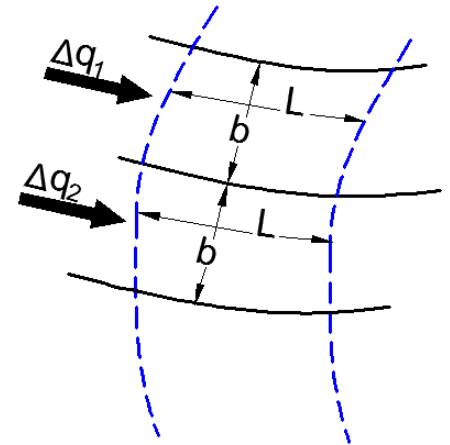
$$q_{total} = \Delta q_1 + \Delta q_2 + \Delta q_3 + \dots + \Delta q_{N_f}$$

$$\text{But, } \Delta q_1 = \Delta q_2 = \Delta q_3 = \dots = \Delta q_{N_f}$$

$$\text{So, } q_{total} = \Delta q \times N_f = k \times \frac{\Delta H}{N_{d,number}} \times N_f$$

$$\text{Finally, } q_{total} = k \times \frac{\Delta H}{N_{d,number}} \times N_f \times n \text{ (m}^3 \cdot \text{sec/m)}$$

$$n = \frac{b(\text{perpendicular to flow})}{L(\text{parallel to flow})}$$



قيمة  $n$  هي النسبة بين البعد العمودي على اتجاه السريان إلى البعد الموازي لاتجاه السريان لكل عنصر (element) من عناصر شبكة التسريب وهذه النسبة يجب أن تكون ثابتة في كل عنصر في الشبكة حتى تكون الشبكة سليمة وهذه النسبة في الغالب تساوي 1 وبالتالي لا تكون معطاة في المسألة.. وإذا كانت معطاة يكون لها حالتان:

**الحالة الأولى:** أن يعطيك قيمة ل  $n$  في المسألة وبالتالي سوف تدخلها في المعادلة كما هي.

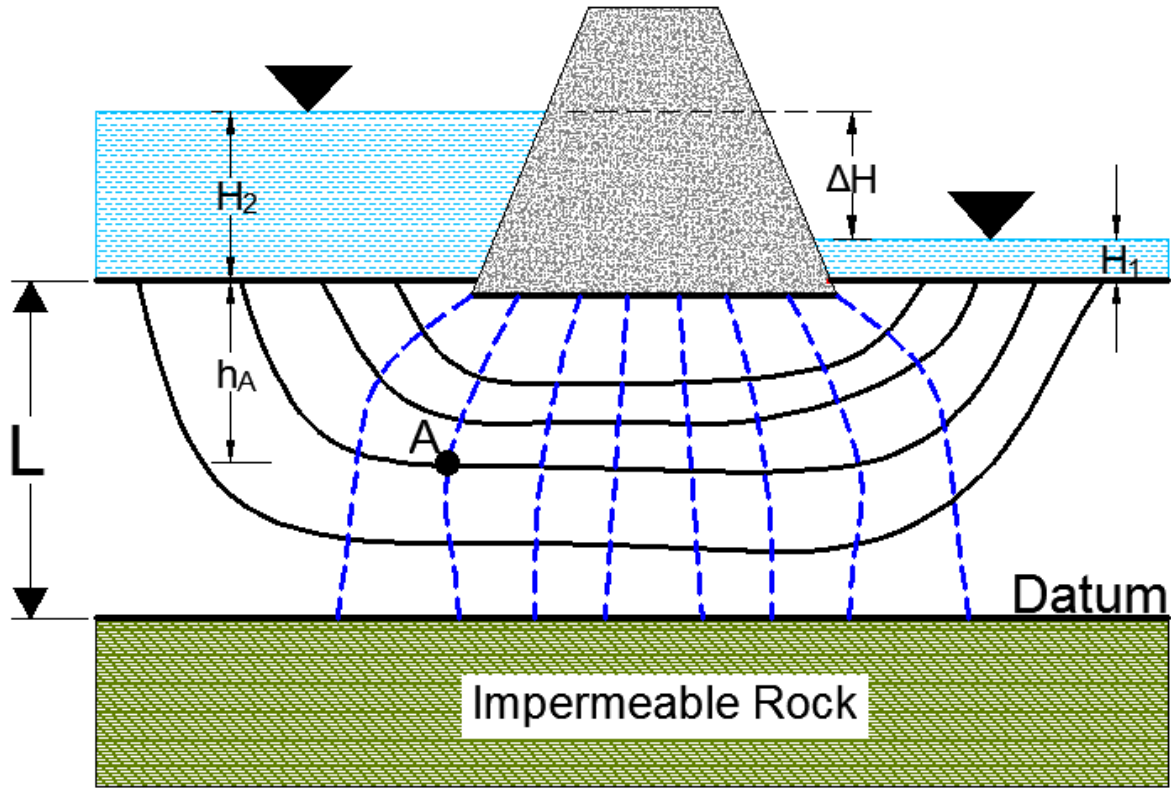
**الحالة الثانية:** في غالب شبكات التسريب تكون النسبة في كل عنصر تساوي 1 أي أن العناصر مربعة.. باستثناء العناصر الموجودة في آخر قناة تسريب وهي  $N_{f,5}$  في الشبكة الموجودة في الأعلى حيث أن

النسبة في هذه الحالة لا تساوي باقي النسب وذلك لأن آخر خط تسريب هو طبقة الصخر الموجودة بالتالي لا يمكن التحكم بها لضبط النسبة.. وهنا يوجد حلان: الأول كما في الحالة الأولى يتم أخذ هذه النسبة لقناة

التسريب الأخيرة وتعميمها على جميع الشبكة وبالتالي تدخل القيمة في المعادلة بشكل طبيعي. والثاني أن يعطيك قيمة  $b$  لآخر قناة تسريب وبالتالي فإن التغيير يحدث على قيمة  $N_f$  بحيث لا يمكن اعتبار آخر قناة هي قناة كاملة.. وهذا تماما مثل مثال 8.2 في الكتاب.

لكن: في غالب المسائل لا يعطيك قيمة  $n$  وبالتالي في هذه الحالة نعتبرها 1 على أساس أن جميع النسب متساوية.

## Head calculations and the uplift force under the dam:



Calculate **total**, **pressure** and **driving** head at point A

### Total Head:

هو المنسوب الكلي بناء على مرجع معين مطروحا منه الفقدان في ال head

$$h_{\text{total},A} = L + H_2 - \text{losses}_A$$

$$\text{losses}_A = N_{d,\text{value}} \times N_{d,A}$$

هو مقدار الفقدان في ال head إلى أن وصلت المياه إلى النقطة A ونلاحظ هنا أن المياه مرت بعدد 2 من ال  $N_{d,A}$  حتى وصلت إلى النقطة A وبالتالي فإن:  $N_{d,A} = 2$  بالتالي المنسوب التي فقدته المياه حتى وصلت للنقطة A هو مقدار 2 potential drop. وهكذا عند أي نقطة أخرى .

### Pressure(piezometric) Head:

$$h_{\text{pressure},A} = h_{\text{total},A} - h_{\text{elevation},A}$$

$$h_{\text{pressure},A} = (L + H_2 - \text{losses}_A) - (L - h_A) \quad \text{Rearrange the equation} \rightarrow$$

$$h_{\text{pressure},A} = H_2 + h_A - \text{losses}_A$$

نلاحظ أنه إذا أردنا أن نجد  $h_{\text{pressure}}$  عند أي نقطة لا تلزم قيمة  $L$ .. وإنما يلزم فقط بعد النقطة عن سطح الأرض لأنه في غالب المسائل لا يعطيك قيمة  $L$

**Driving Head:**

هو مقدار الهبوط عند أي نقطة والذي يسبب حركة وسريان الماء, أو بمعنى آخر هو الكمية المتبقية من ال head الكلي ( $\Delta H$ ) عند أي نقطة والتي تسبب سريان الماء حتى تصل المياه إلى المنسوب النهائي  $H_1$ .

$$h_{\text{Driving,A}} = \Delta H - \text{losses}_A$$
**Uplift force under the dam:**

هي مقدار القوة المؤثرة لأعلى على أسفل السد والناجمة عن ضغط الماء الموجود أسفل السد (ك رد فعل لوزن السد) وبالتالي فهي تعبر عن قيمة ال pore water pressure أسفل السد ولكن على شكل قوة لكل وحدة طول, وبالتالي إذا كانت قاعدة السد (سطح التقاء السد مع التربة) مستوية ومنتظمة نجد الضغط عند أول نقطة وآخر نقطة أسفل السد ومن ثم نجد المساحة المتكونة من هذا الضغط. أما إذا لم تكن القاعدة مستوية ومنتظمة فإننا نجد ضغط الماء عند كل تغير في شكل القاعدة ومن ثم نجد المساحة.

$$u = \text{pressure head} \times \gamma_{\text{water}}$$

عند أي نقطة نريد إيجاد ضغط الماء عليها يجب أولاً إيجاد  $h_{\text{pressure}}$  عند تلك النقطة ومن ثم إيجاد  $u$  وسوف يتم حل مسائل بأفكار مختلفة لتوضيح الفكرة

**Effective Stress at any point (chapter 9):**

هذا الموضوع سوف يتم تناوله بشكل مفصل في شبر 9 لكن في غالب الأحيان يكون أحد المطالب هو إيجاد قيمة الاجهاد الفعال عند نقطة معينة وهو أمر بسيط كما يلي:

الاجهاد الفعال: هو الضغط الرأسى الناتج على نقطة معينة نتيجة وزن حبيبات التربة فقط (أي الحبيبات نفسها بدون ماء)

At any point we can calculate the effective stress as following:

$$\sigma_{\text{total}} = \sigma_{\text{effectiv}} + u \rightarrow \sigma_{\text{effectiv}} = \sigma_{\text{total}} - u$$

$u$  = pore water pressure at any point (we calculate it as mentioned above)

$\sigma_{\text{total}}$  = The total (vertical) stress applied on a point and can be calculated as following:  
هو الضغط الرأسى الكلي الناتج على نقطة معينة من وزن التربة ووزن الماء معا وأي أوزان أخرى (إن وجدت)

For example at point A in the flow net on the last graph

$$\sigma_{\text{total,A}} = (H_2 \times \gamma_{\text{water}}) + (h_A \times \gamma_{\text{sat}})$$

نلاحظ هنا لا يتم خصم أي losses وذلك لأنه متعلق بالأوزان الموجودة فوق النقطة نفسها

Now, we can calculate the effective stress easily.

$$\sigma_{\text{effectiv}} = \sigma_{\text{total}} - u$$

## Flow Nets in Anisotropic soil:

من المعروف أنه حتى يتم رسم flow net صحيحة بخطوط متعامدة على بعضها البعض يجب أن تكون التربة isotropic أي أن التربة متوحدة الخواص وأن  $k_x = k_z$  للتربة وهو الشرط الأساسي لمعادلة Laplace التي يتم من خلالها رسم ال flow net .

### Laplace's Equation:

$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$  , For isotropic soil  $\rightarrow k_x = k_z \rightarrow$  The equation will be:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \rightarrow \text{That is the required form to draw a true flow net}$$

(Lines perpendicular to each other).

Now, for anisotropic soil  $\rightarrow k_x \neq k_z$  so, the **first step** is to convert the two soils for one **isotropic** soil ( $k_{eq,x} = k_{eq,z} = k_{eq}$ ).

Laplace's equation doesn't written as the above form and the lines are not perpendicular to each other so, the **second step** that we must rewrite the equation to be in the required form to draw true flow net.

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \quad (\text{Divided by } k_z) \rightarrow \frac{\partial^2 h}{(k_x/k_z) \partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

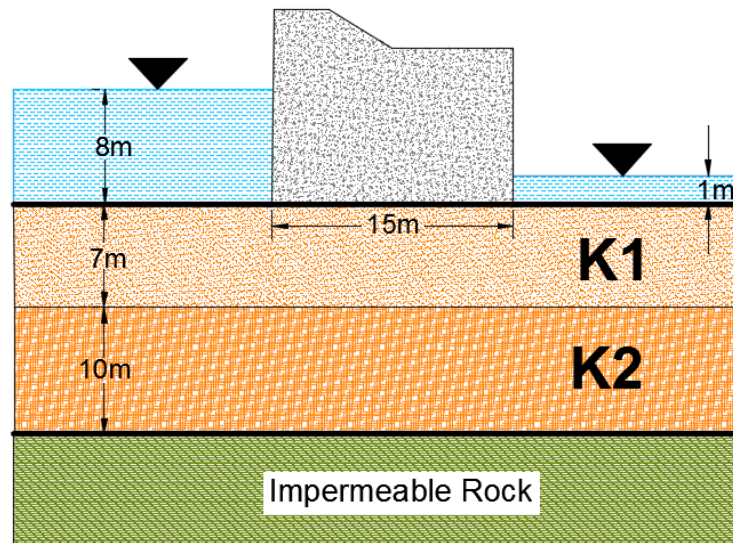
Now let  $x' = \sqrt{k_z/k_x} x \rightarrow$  the equation will be:

$$\frac{\partial^2 h}{\partial x'^2} + \frac{\partial^2 h}{\partial z^2} = 0 \rightarrow \text{That is the required form to draw a true flow net}$$

(Lines perpendicular to each other)

### Example:

For the shown soil profile under the dam. **Make the soil isotropic.**



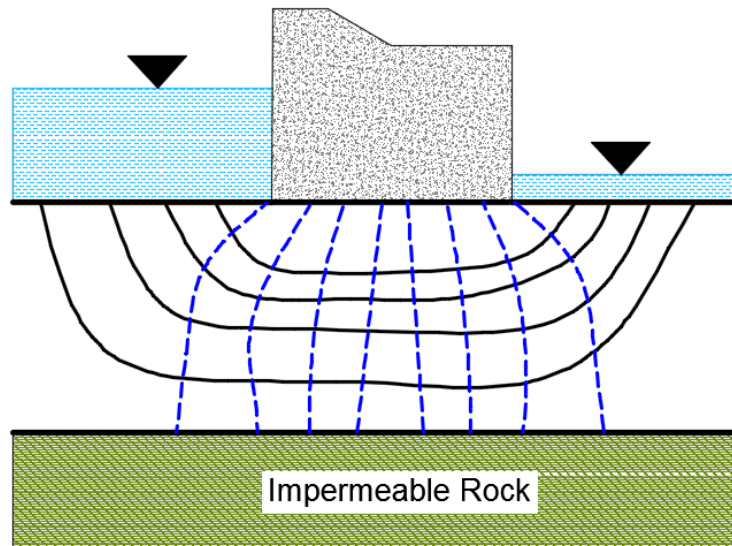
Soil will be isotropic if:

$$k_{eq,x} = k_{eq,z} = k_{eq}$$

$$k_x = \frac{7 \times k_1 + 10 \times k_2}{7 + 10}, \quad k_z = \frac{7 + 10}{\frac{7}{k_1} + \frac{10}{k_2}}$$

$$k_{eq,x} = k_{eq,z} = k_{eq} = \sqrt{k_x k_z} \rightarrow \text{(That make the soil isotropic)}$$

After making the soil isotropic the flow net under the dam was drawn and given in the figure below. **Find** all dimensions on the graph and then calculate the rate of seepage.



كما أثبتنا سابقاً أنه حتى يتم رسم ال flow net طبقاً لمعادلة Laplace فإن جميع قيم  $x$  يجب إستبدالها بقيم  $x'$  حيث أن:  $x' = \sqrt{k_z/k_x} x$  بالتالي جميع قيم  $x$  يجب تغييرها إلى  $x'$  , أما الأبعاد الرأسية تبقى كما هي ولا تتغير مطلقاً, حيث أن الأبعاد الرأسية في الرسمة الأولى يتم وضعها كما هي في هذه الرسمة أما الأبعاد الأفقية يجب تحويلها كلها إلى قيم  $x'$  . أما بالنسبة لقيمة  $q$  يتم حسابها كما يلي:

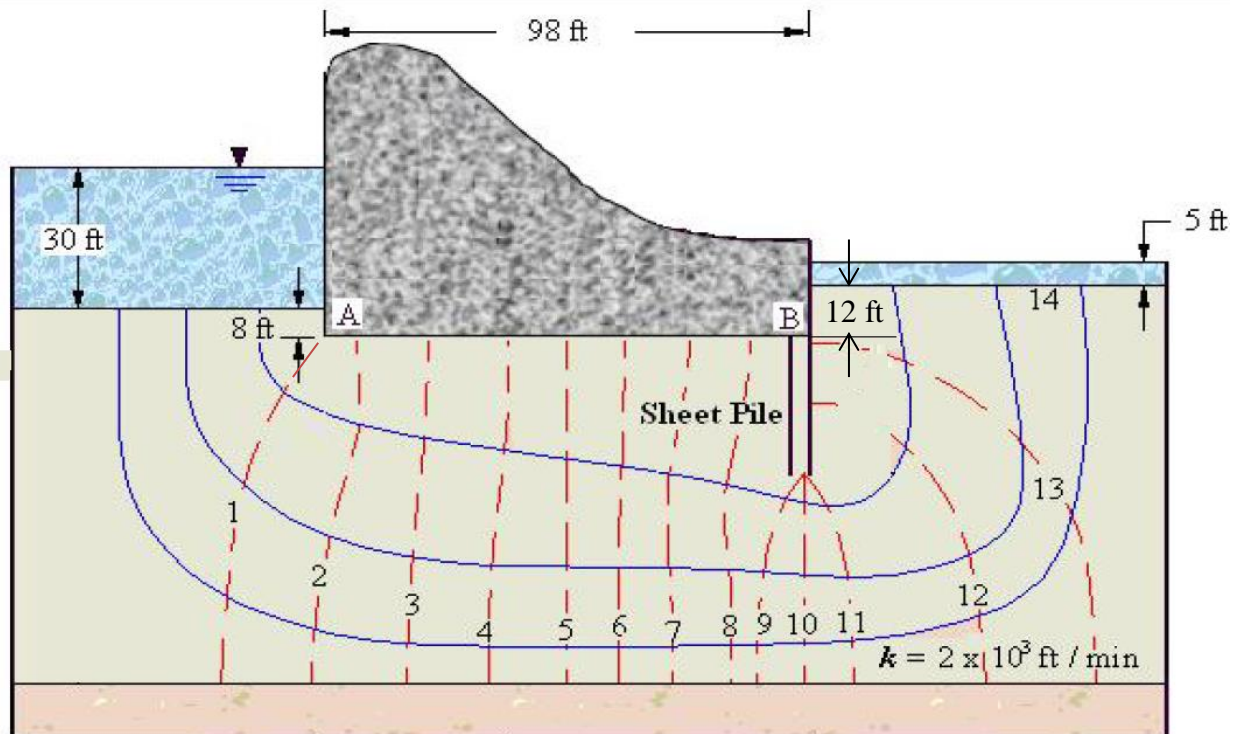
$$q_{total} = \sqrt{k_x k_z} \times \frac{\Delta H}{N_{d,number}} \times N_f \times n$$

الآن بعد وضع جميع الأبعاد الجديدة على الرسم, وبعد إيجاد قيمة التدفق, تصبح المسألة مثل أي مسألة على موضوع ال seepage بأي مطالب كما سنرى في الأسئلة التالية.

### 32. (Mid 2012):

For the shown flow net:

- Calculate the uplift force at the base of the weir , per foot of length  
Point A and B at the corners of the concrete dam.
- Calculate the driving head at lines 5 and 12.



### Solution

#### Givens:

$$N_{d,\text{number}} (\# \text{ of potential drops}) = 14, \quad N_f = 4$$

$$\Delta H = (30 + 8) - (12 + 5) = 21 \text{ ft}$$

$$i = N_{d,\text{value}} = \frac{\Delta H}{N_{d,\text{number}}} = \frac{21}{14} = 1.5 \text{ ft (value of one potential drop)}$$

(a).

من الواضح أن قاعدة السد منتظمة وبالتالي سوف نجد ضغط الماء عند أول نقطة (A) وآخر نقطة (B) في السد ونجد المساحة بينهما, لكن يجب الإنتباه أن النقطة B يتم إيجاد ضغط الماء عندها قبل ال sheet pile .



**Point “A”:**

نلاحظ أن الماء حتى وصلت إلى النقطة A فقد مرت ب potential drop واحدة فقط.

$$h_{\text{pressure,A}} = (30 + 8) - 1 \times 1.5 = 36.5 \text{ ft.}$$

$$u_A = h_{\text{pressure,A}} \times \gamma_{\text{water}} = 36.5 \times 62.4 = 2277.6 \text{ lb/ft}^2$$

**Point “B”:**

كما قلنا أن النقطة B سوف نجد ضغط الماء لها قبل ال sheet pile وبالتالي فإن عدد ال potential drops التي مرت بها الماء حتى وصلت النقطة B تساوي 8.5 لأنها وصلت تقريبا النصف عند النقطة B .

$$h_{\text{pressure,B}} = (30 + 8) - 8.5 \times 1.5 = 25.25 \text{ ft.}$$

$$\text{Or, } h_{\text{pressure,B}} = (5 + 12) + (5.5 \times 1.5) = 25.25$$

هنا تم حسابه من الجهة الأخرى وبالتالي هنا سوف يتم الجمع وليس الطرح لأن المنسوب يزداد كلما تحركنا من المستوى المنخفض إلى المرتفع, ونلاحظ أن عدد ال potential drops=5.5 من الجهة الأخرى.

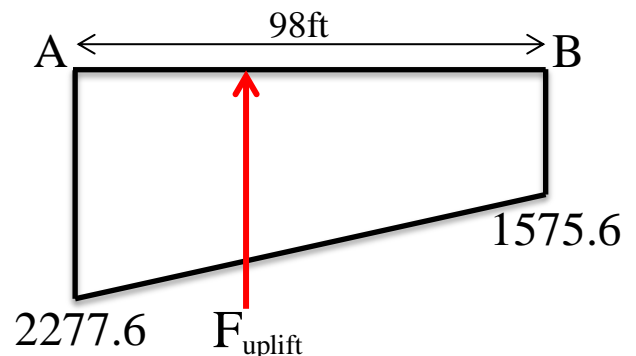
$$u_B = h_{\text{pressure,B}} \times \gamma_{\text{water}} = 25.25 \times 62.4 = 1575.6 \text{ lb/ft}^2$$

Now, the stress distribution under the dam as shown in figure below:

And the uplift force is calculated as following:

$$F_{\text{uplift}} = \frac{1}{2} (2277.6 + 1575.6) \times 98$$

$$F_{\text{uplift}} = 188806.8 \text{ flb/ft} \checkmark .$$



(b).

$$h_{\text{Driving}} = \Delta H - \text{losses}$$

At line 5, there are 5 potential drops were lost

$$h_{\text{Driving,line5}} = \Delta H - \text{losses}_{\text{line5}}$$

$$h_{\text{Driving,line5}} = 21 - 5 \times 1.5 = 13.5 \text{ ft} \checkmark .$$

At line 12, there are 12 potential drops were lost

$$h_{\text{Driving,line12}} = \Delta H - \text{losses}_{\text{line12}}$$

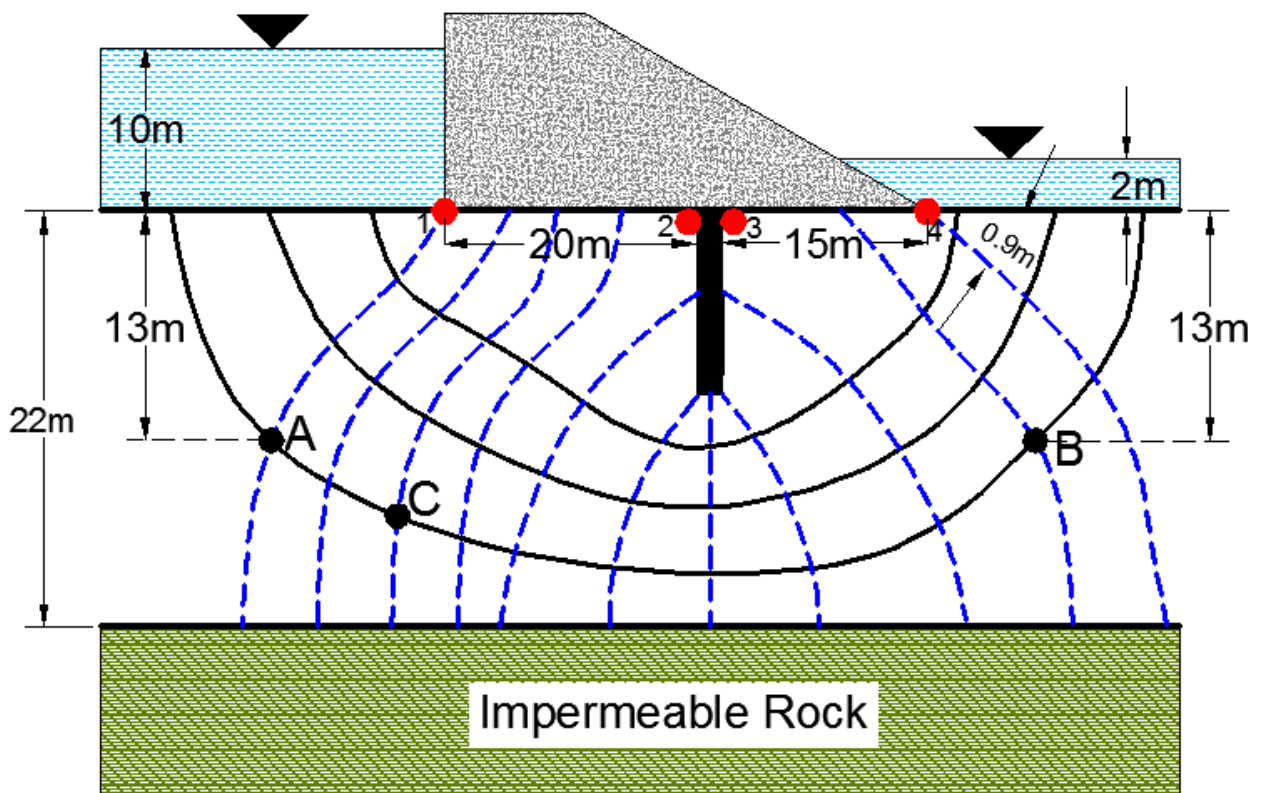
$$h_{\text{Driving,line12}} = 21 - 12 \times 1.5 = 3 \text{ ft} \checkmark .$$

### 33. (Mid 2011):

For the flow net shown below, calculate:

- Flow Rate per unit length.
- Uplift force per unit length under the dam.
- Effective stress at points A and B.
- The seepage loss for 8 m length of sheet pile (additional)
- Pressure head at point C (**from ground surfac**) (additional).
- Total head at point A (additional).
- The exit gradient (additional).

$$(k = 1 \times 10^{-5} \text{m/s} , \gamma_{\text{sat}} = 19 \text{ KN/m}^3)$$





## Solution

## Givens:

$$N_{d,number}(\text{\#of potential drops}) = 12 , N_f = 4 , k = 1 \times 10^{-5} \text{m/s}$$

$$\Delta H = (10) - (2) = 8 \text{ m} , \gamma_{sat} = 19 \text{ KN/m}^3$$

$$i = N_{d,value} = \frac{\Delta H}{N_{d,number}} = \frac{8}{12} = 0.667 \text{ m (value of one potential drop)}$$

a.

$$q = k \times \frac{\Delta H}{N_{d,number}} \times N_f \times n = 1 \times 10^{-5} \times 0.667 \times 4 \times 1 = 2.668 \times 10^{-5} \text{ m}^3/\text{s.m} \checkmark .$$

b.

لإيجاد قوة ضغط الماء لأعلى يجب إيجاد ضغط الماء عند كل تغير أسفل السد وبالتالي هنا يجب إيجاد الضغط عند أول نقطة وآخر نقطة بالإضافة إلى النقطتين قبل وبعد ال sheet pile لأنه وكما سنرى أنه يوجد فرق كبير بينهما.

## Point “1”:

نلاحظ أن الماء حتى وصلت إلى النقطة 1 فقد مرت ب potential drop واحدة فقط.

$$h_{pressure,1} = 10 - 1 \times 0.667 = 9.333 \text{m.}$$

$$u_1 = h_{pressure,1} \times \gamma_{water} = 9.333 \times 9.81 = 91.55 \text{ KN/m}^2$$

## Point “2”:

نلاحظ أن الماء حتى وصلت إلى النقطة 2 فقد مرت ب 4.5 potential drop وليس 5.

$$h_{pressure,2} = 10 - 4.5 \times 0.667 = 6.998 \text{m.}$$

$$u_2 = h_{pressure,2} \times \gamma_{water} = 6.998 \times 9.81 = 68.65 \text{ KN/m}^2$$

## Point “3”:

نلاحظ أن الماء حتى وصلت إلى النقطة 3 فقد مرت ب 9.5 potential drop وليس 10.

$$h_{pressure,3} = 10 - 9.5 \times 0.667 = 3.663 \text{m.}$$

$$u_3 = h_{pressure,3} \times \gamma_{water} = 3.663 \times 9.81 = 35.93 \text{ KN/m}^2$$

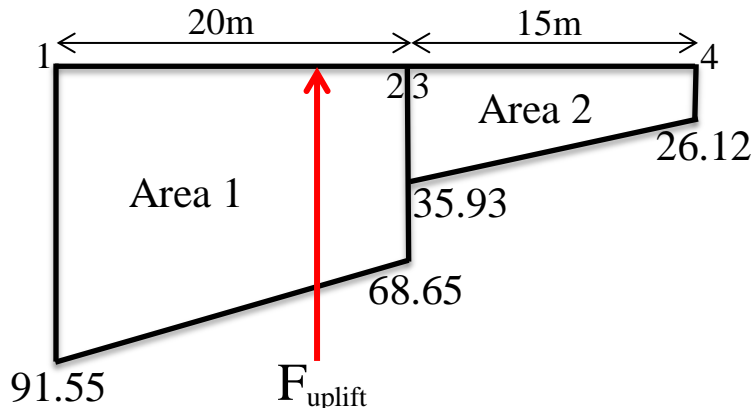
## Point “4”:

نلاحظ أن الماء حتى وصلت إلى النقطة 4 فقد مرت ب 11 potential drop.

$$h_{pressure,4} = 10 - 11 \times 0.667 = 2.663 \text{m.}$$

$$u_4 = h_{pressure,4} \times \gamma_{water} = 2.663 \times 9.81 = 26.12 \text{ KN/m}^2$$

Now, the stress distribution under the dam as shown in figure below:



The uplift force can be calculated as following:

$$F_{uplift} = \text{Area 1} + \text{Area 2}$$

$$F_{uplift} = \frac{1}{2}(91.55 + 68.65) \times 20 + \frac{1}{2} \times (35.93 + 26.12) \times 15 = 2067.38 \text{ KN/m}^2 \checkmark .$$

نلاحظ من الرسم أعلاه أن وجود ال sheet pile قد قلل الضغط على السد في المنطقة ما بعد ال sheet pile وهذا هو الهدف الأساسي من وضعها.

c.

**Point “A”:**

$$\sigma_{total} = \sigma_{effectiv} + u \rightarrow \sigma_{effectiv} = \sigma_{total} - u$$

$$\sigma_{total,A} = 10 \times \gamma_{water} + 13 \times \gamma_{sat} = 10 \times 9.81 + 13 \times 19 = 345.1 \text{ KN/m}^2$$

نلاحظ أن الماء حتى وصلت إلى النقطة A فقد مرت ب 1 potential drop

$$h_{pressure,A} = (10 + 13) - 1 \times 0.667 = 22.333\text{m}..$$

$$u_A = 22.333 \times 9.81 = 219.08 \text{ KN/m}^2$$

$$\sigma_{effectiv,A} = 345.1 - 219.08 = 126 \text{ KN/m}^2 \checkmark .$$

**Point “B”:**

$$\sigma_{total,B} = 2 \times \gamma_{water} + 13 \times \gamma_{sat} = 2 \times 9.81 + 13 \times 19 = 266.62 \text{ KN/m}^2$$

نلاحظ أن الماء حتى وصلت إلى النقطة B فقد مرت ب 10 potential drop

$$h_{pressure,B} = (10 + 13) - 10 \times 0.667 = 16.333\text{m}..$$

$$u_B = 16.33 \times 9.81 = 160.2 \text{ KN/m}^2$$

$$\sigma_{effectiv,B} = 266.62 - 160.2 = 106.42 \text{ KN/m}^2 \checkmark .$$

d.

$$\text{Seepage loss} = q \times \text{length of sheet pile} = 2.668 \times 10^{-5} \times 8 = 21.344 \times 10^{-5} \text{ m}^3/\text{s} \checkmark.$$

e.

نلاحظ أن المعلومات غير كافية لحساب قيمة  $h_{\text{pressure},c}$  لكن في كثير من الأحيان ما يكون أحد المطالبين وبالتالي يجب إيجاد هذا الضغط بالنسبة لمرجع معين، مثلاً حدد في السؤال مستوى سطح الأرض وقد لا يحدد مرجع أنت الذي تقرر.  
الآن نفرض أن المسافة بين النقطة C و سطح الأرض (المرجع) هي  $h_c$  وبالتالي:

We note that, there are **3** potential drops to reach point C

$$h_{\text{pressure},c} = (10 + h_c) - 3 \times 0.667 = 7.99 + h_c = 7.99 \text{ (above ground surface)}$$

If the value 7.99 is negative value, that means (below the ground surface)  $\checkmark$ .

f.

$$h_{\text{total},A} = (10 + 22) - \text{losses}_A \rightarrow 1 \text{ potential drop to reach A}$$

$$h_{\text{total},A} = (10 + 22) - 0.667 \times 1 = 31.333 \text{ m} \checkmark.$$

We note that, also we can calculate pressure head from total head

$$h_{\text{pressure},A} = h_{\text{total},A} - h_{\text{elevation},A}$$

$$h_{\text{pressure},A} = 31.333 - (22 - 13) = 22.333 \text{ m (the same as calculated above).}$$

g.

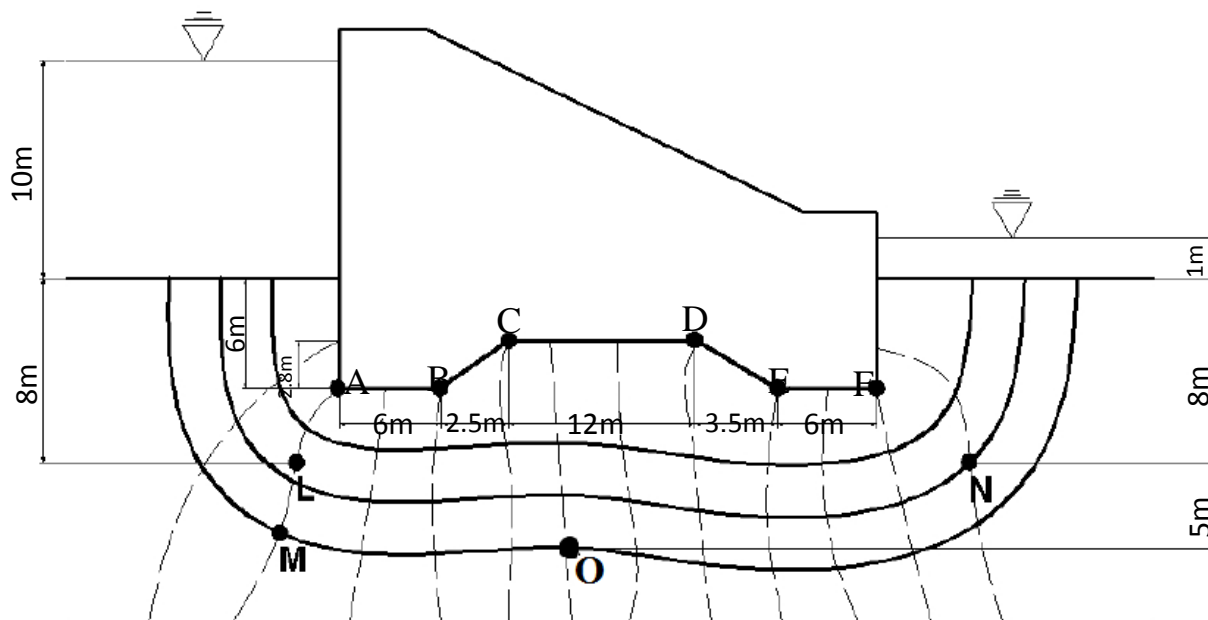
المطلوب هنا هي قيمة  $i$  للمخرج أي لآخر potential drop موجودة في الشبكة ويتضح من الرسم أن مسافتها تساوي 0.9 بالتالي إذا قسمنا الفقدان في المنسوب في هذه الـ potential drop على مسافتها فسنحصل على قيمة  $i$  للمخرج

$$i_{\text{exit}} = \frac{\text{exit losse}}{\text{exit distance}} = \frac{0.667}{0.9} = 0.741 \checkmark.$$

### 34. (Final 2009)

For the flow net shown below, if the coefficient of permeability of the soil below the dam is  $(5.3 \times 10^{-5} \text{ cm/sec})$  and  $\gamma_{\text{sat}} = 17.7 \text{ KN/m}^3$ , find the following:

- A- The **rate of seepage** ( $\text{m}^3/\text{sec}/\text{m}$ ).
- B- The **uplift force** underneath the dam.
- C- The **driving head** at point M.
- D- The **effective stress** at points L and N.
- E- If the weight of the dam is  $120 \text{ KN/m}^2$ , calculate the **effective stress** at point O  
(Assume the weight of the dam is constant with depth)



### Solution

#### Givens:

$$N_{d,\text{number}} (\text{\#of potential drops}) = 13, \quad N_f = 4, \quad k = 5.3 \times 10^{-5} \text{ m/s}$$

$$\Delta H = (10) - (1) = 9 \text{ m}, \quad \gamma_{\text{sat}} = 17.7 \text{ KN/m}^3$$

$$i = N_{d,\text{value}} = \frac{\Delta H}{N_{d,\text{number}}} = \frac{9}{13} = 0.6923 \text{ m (value of one potential drop)}$$

A-

$$q = k \times \frac{\Delta H}{N_{d,number}} \times N_f \times n = 5.3 \times 10^{-5} \times 0.6923 \times 4 = 14.67 \times 10^{-5} \text{ m}^3/\text{s.m} \checkmark.$$

B-

نلاحظ هنا أن قاعدة السد غير منتظمة وبالتالي يجب إيجاد ضغط الماء عند كل تغير (A to F)

**Point “A”:**

We note that, there are 2 potential drops to reach point A

$$h_{\text{pressure,A}} = (10 + 6) - 2 \times 0.6923 = 14.6154\text{m.}$$

$$u_A = h_{\text{pressure,A}} \times \gamma_{\text{water}} = 14.6154 \times 9.81 = 143.377 \text{ KN/m}^2$$

**Point “B”:**

We note that, there are 4 potential drops to reach point B

$$h_{\text{pressure,B}} = (10 + 6) - 4 \times 0.6923 = 13.2308\text{m.}$$

$$u_B = h_{\text{pressure,B}} \times \gamma_{\text{water}} = 13.2308 \times 9.81 = 129.794 \text{ KN/m}^2$$

**Point “C”:**

We note that, there are 5 potential drops to reach point C

$$h_{\text{pressure,C}} = (10 + 6 - 2.8) - 5 \times 0.6923 = 9.7385\text{m.}$$

$$u_C = h_{\text{pressure,C}} \times \gamma_{\text{water}} = 9.7385 \times 9.81 = 95.534 \text{ KN/m}^2$$

**Point “D”:**

We note that, there are 8 potential drops to reach point D

$$h_{\text{pressure,D}} = (10 + 6 - 2.8) - 8 \times 0.6923 = 7.6616\text{m.}$$

$$u_D = h_{\text{pressure,D}} \times \gamma_{\text{water}} = 7.6616 \times 9.81 = 75.16 \text{ KN/m}^2$$

**Point “E”:**

We note that, there are 9 potential drops to reach point E

$$h_{\text{pressure,E}} = (10 + 6) - 9 \times 0.6923 = 9.7693\text{m.}$$

$$u_E = h_{\text{pressure,E}} \times \gamma_{\text{water}} = 9.7693 \times 9.81 = 95.836 \text{ KN/m}^2$$

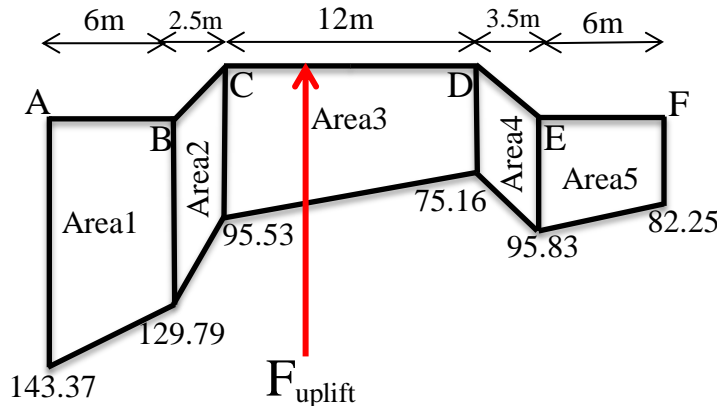
**Point “F”:**

We note that, there are 11 potential drops to reach point F

$$h_{\text{pressure,F}} = (10 + 6) - 11 \times 0.6923 = 8.3847\text{m.}$$

$$u_F = h_{\text{pressure,F}} \times \gamma_{\text{water}} = 8.3847 \times 9.81 = 82.254 \text{ KN/m}^2$$

The stress distribution under the dam is shown in figure below:



The uplift force can be calculated as following:

$$F_{\text{uplift}} = \text{Area1} + \text{Area2} + \text{Area3} + \text{Area4} + \text{Area5}$$

$$F_{\text{uplift}} = \frac{1}{2} (143.37 + 129.79) \times 6 + \frac{1}{2} \times (129.79 + 95.53) \times 2.5$$

$$+ \frac{1}{2} \times (95.53 + 75.16) \times 12 + \frac{1}{2} \times (75.16 + 95.83) \times 3.5 + \frac{1}{2} \times (95.83 + 82.25) \times 6$$

$$F_{\text{uplift}} = 2959.51 \text{ KN/m} \checkmark.$$

C-

$$h_{\text{Driving}} = \Delta H - \text{losses}$$

At point M, there are 2 potential drops were lost

$$h_{\text{Driving,point M}} = \Delta H - \text{losses}_{\text{point M}}$$

$$h_{\text{Driving,point M}} = 9 - 2 \times 0.6923 = 7.6154 \text{ m} \checkmark.$$

D-

**Point “L”:**

$$\sigma_{\text{total}} = \sigma_{\text{effectiv}} + u \rightarrow \sigma_{\text{effectiv}} = \sigma_{\text{total}} - u$$

$$\sigma_{\text{total,L}} = 10 \times \gamma_{\text{water}} + 8 \times \gamma_{\text{sat}} = 10 \times 9.81 + 8 \times 17.7 = 239.7 \text{ KN/m}^2$$

We note that, there are 2 potential drops to reach point L

$$h_{\text{pressure,L}} = (10 + 8) - 2 \times 0.6923 = 16.6154 \text{ m.}$$

$$u_L = 16.6154 \times 9.81 = 163 \text{ KN/m}^2$$

$$\sigma_{\text{effectiv,L}} = 239.7 - 163 = 76.7 \text{ KN/m}^2 \checkmark.$$

**Point “N”:**

$$\sigma_{\text{total}} = \sigma_{\text{effectiv}} + u \rightarrow \sigma_{\text{effectiv}} = \sigma_{\text{total}} - u$$

$$\sigma_{\text{total,N}} = 1 \times \gamma_{\text{water}} + 8 \times \gamma_{\text{sat}} = 1 \times 9.81 + 8 \times 17.7 = 151.41 \text{ KN/m}^2$$

We note that, there are 12 potential drops to reach point N

$$h_{\text{pressure,N}} = (10 + 8) - 12 \times 0.6923 = 9.6924 \text{ m.}$$

$$u_N = 9.6924 \times 9.81 = 95.08 \text{ KN/m}^2$$

$$\sigma_{\text{effectiv,N}} = 151.41 - 95.08 = 56.32 \text{ KN/m}^2 \checkmark.$$

E- (Additional)

نلاحظ أن النقطة O تقع أسفل السد وبالتالي عند حساب الإجهادات الكلية عند هذه النقطة يجب أخذ كل الأوزان التي فوقها وبالتالي يجب إضافة وزن السد إلى الإجهادات الكلية عند هذه النقطة.

نلاحظ أيضا أن ارتفاع التربة فوق النقطة O حتى الوصول إلى قاع السد يساوي  $(5+8-6+2.8) = 9.8 \text{ m}$

**Point “O”:**

$$\sigma_{\text{total}} = \sigma_{\text{effectiv}} + u \rightarrow \sigma_{\text{effectiv}} = \sigma_{\text{total}} - u$$

$$\sigma_{\text{total,O}} = 9.8 \times \gamma_{\text{sat}} + \text{dam weight} = 9.8 \times 17.7 + 120 = 293.46 \text{ KN/m}^2$$

We note that, there are 6 potential drops to reach point O

$$h_{\text{pressure,O}} = (10 + 8 + 5) - 6 \times 0.6923 = 18.846 \text{ m.}$$

$$u_O = 18.846 \times 9.81 = 184.88 \text{ KN/m}^2$$

$$\sigma_{\text{effectiv,O}} = 293.46 - 184.88 = 108.578 \text{ KN/m}^2 \checkmark.$$





a)

$$q = k \times \frac{\Delta H}{N_{d,number}} \times N_f \times n = 4 \times 10^{-7} \times 0.45 \times 6 = 1.08 \times 10^{-6} \text{ m}^3/\text{s.m} \checkmark .$$

b)

$$h_{total,C} = (2.25 + 6 + 2.5) - \text{losses}_C \rightarrow 4 \text{ potential drop to reach C}$$

$$h_{total,C} = (2.25 + 6 + 2.5) - 0.45 \times 4 = 8.95 \text{ m} \checkmark .$$

c)

نلاحظ أنه لا توجد معلومات كافية لحساب  $h_{pressure,D}$  أي أن المسافة بين المرجع والنقطة D ( $h_D$ ) غير موجودة بالتالي يتم حساب  $h_{pressure,D}$  فوق ذلك المرجع أي بالنسبة لذلك المرجع.

We note that, there are **8** potential drops to reach point D

$$h_{pressure,D} = (2.5 + 2 + h_D) - 8 \times 0.45 = 0.9 + h_D = 0.9 \text{ m (above Datum)} \checkmark .$$

If the value 0.9 is negative value, that means (below Datum).

d) (From chapter 9)

$$F. S_{boiling} = \frac{i_{critical}}{i_{exit}}$$

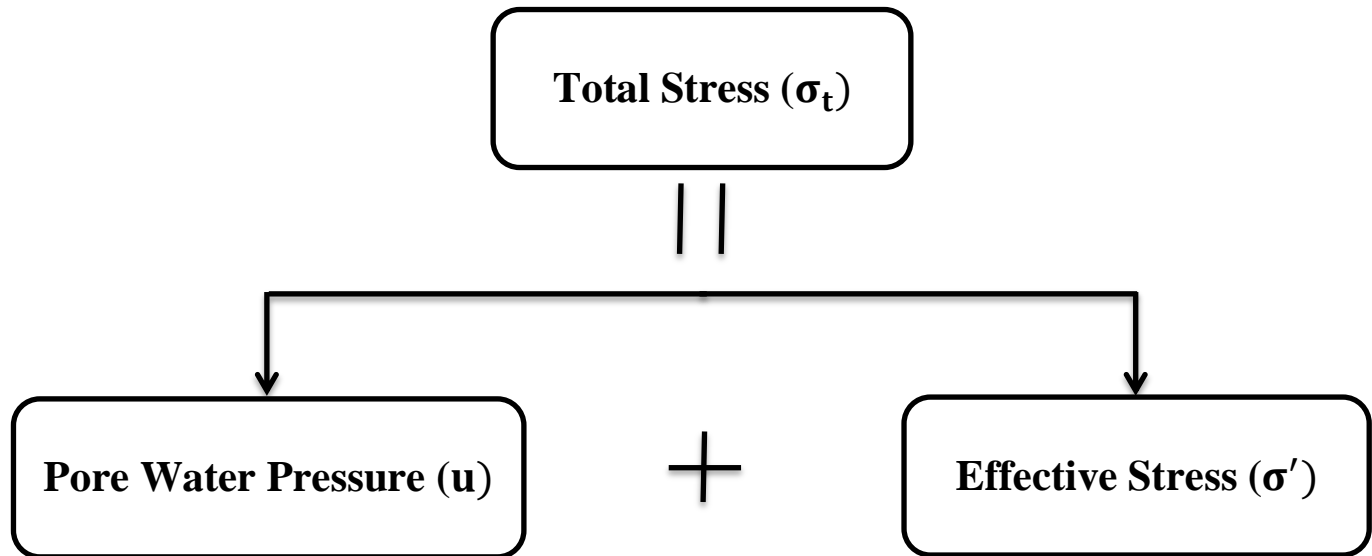
$$i_{critical} = \frac{\gamma_{sat} - \gamma_{water}}{\gamma_{water}} = \frac{20 - 9.81}{9.81} = 1.038$$

$$i_{exit} = \frac{\text{exit losse}}{\text{exit distance}} = \frac{0.45}{0.9} = 0.5$$

$$F. S_{boiling} = \frac{1.038}{0.5} = 2.076 \checkmark .$$

## **Chapter (9)**

# **In Situ Stresses**



**Total Stress ( $\sigma_t$ ):** Is the total weight of all layers above a certain point divided by the area of the layers, and it is independent on seepage of water.

هو الوزن الكلي لجميع الطبقات الموجودة فوق نقطة معينة (طبقات التربة بالإضافة إلى طبقات الماء) مقسوما على مساحة هذه الطبقات.

**Pore Water Pressure (Hydrostatic Pressure) ( $u$ ):** Is the total weight of the column of water (Pressure head) divided by the area of the layers, and it is dependent on the seepage of water (Upward or Downward).

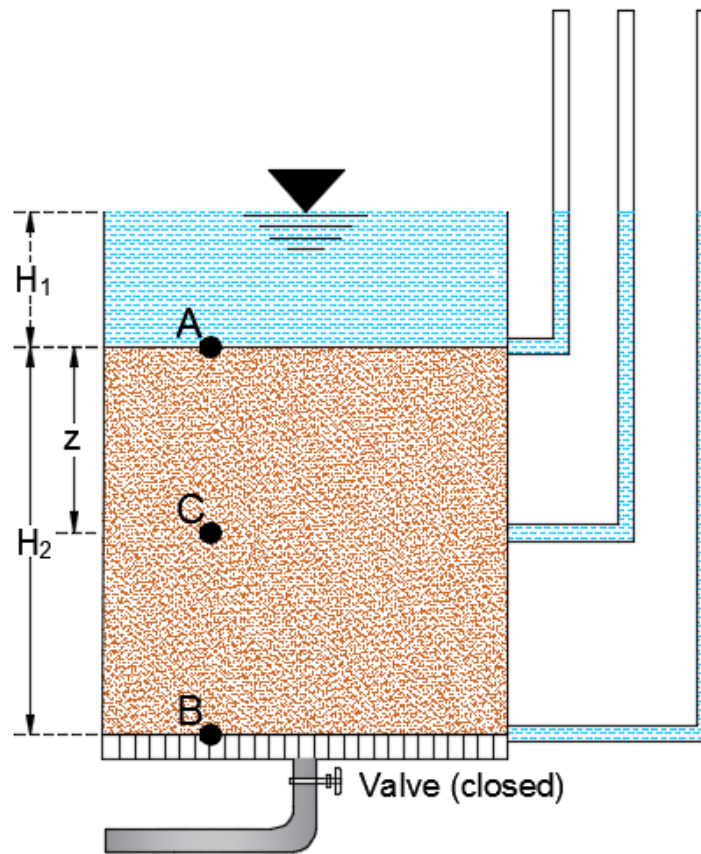
**Effective Stress ( $\sigma'$ ):** The sum of the vertical components of all intergranular contact forces divided by unit gross cross-sectional area.

هو مجموع الأوزان الرأسية (أوزان حبيبات التربة نفسها، أي التربة فقط بدون أي ماء) عند نقطة معينة مقسوما على المساحة الكلية لطبقة التربة.

$$\sigma_t = \sigma' + u \rightarrow \sigma' = \sigma_t - u$$

The above equation is the most important equation in geotechnical engineering, because the compressibility of soil and shearing resistance of a soil depend to a great extent on the effective stress. Thus, the concept of effective stress is significant in solving geotechnical engineering problems, such as the lateral earth pressure on retaining structures, settlement of foundations, and the stability of earth slopes.

## Stresses in Saturated Soil without Seepage:



Calculate the **total stress**, **pore water pressure**, and **effective stress** at points A, B, and C.

Firstly, you should note that, there are no losses in head (piezometric head) due to the movement of water through the soil, because there is **no seepage of water**.

### Point A:

$$\sigma_{t,A} = H_1 \times \gamma_W$$

$$u_A = H_1 \times \gamma_W$$

$$\sigma'_{A} = \sigma_{\text{total}} - u = H_1 \times \gamma_W - H_1 \times \gamma_W = 0.0$$

### Point B:

$$\sigma_{t,B} = H_1 \times \gamma_W + H_2 \times \gamma_{\text{sat}}$$

$$u_B = (H_1 + H_2) \times \gamma_W$$

$$\sigma'_{B} = (H_1 \times \gamma_W + H_2 \times \gamma_{\text{sat}}) - (H_1 + H_2) \times \gamma_W = H_2(\gamma_{\text{sat}} - \gamma_W).$$

The term of  $(\gamma_{\text{sat}} - \gamma_W)$  is called effective unit weight or submerged unit weight ( $\gamma'$ ). So, the effective stress at **point B** equal:  $H_2 \times \gamma'$

**Point C:**

$$\sigma_{t,C} = H_1 \times \gamma_W + z \times \gamma_{\text{sat}}$$

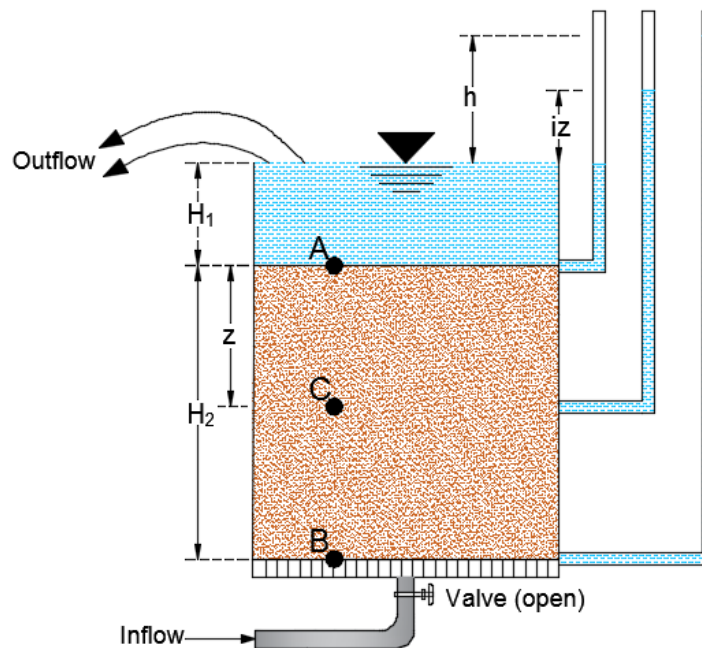
$$u_C = (H_1 + z) \times \gamma_W$$

$$\sigma'_{C} = (H_1 \times \gamma_W + z \times \gamma_{\text{sat}}) - (H_1 + z) \times \gamma_W = z(\gamma_{\text{sat}} - \gamma_W) = z \times \gamma'$$

**Note:**

All soils under the ground water table are saturated soils, and soils above the ground water table are dry soils unless if there exist capillary rise on the layer above the ground water table and we will discuss this case later.

### Stresses in Saturated Soil with Upward Seepage:



We note that, there is upward seepage from upstream (point B) to downstream (point A), also we note that, the elevation of water on the tank is remained constant by controlling the amount of flow in and flow out.

So, due to the upward seepage through the soil, there will be losses in piezometric head as the water move upward, and the total losses is “h” due to the movement of water in length”  $H_2$  “ through the soil, thus, the hydraulic gradient “ i “ will be :

$$i = \frac{h}{H_2}$$

Calculate the **total stress**, **pore water pressure**, and **effective stress** at points A, B, and C

Firstly, you should note that, as the depth increase the piezometric (pressure) head increase due to the upward seepage, so, pore water pressure **increase** with depth.

**Point A:**

$$\sigma_{t,A} = H_1 \times \gamma_W$$

$$u_A = H_1 \times \gamma_W$$

$$\sigma'_{A} = \sigma_{total} - u = H_1 \times \gamma_W - H_1 \times \gamma_W = 0.0$$

**Point B:**

$$\sigma_{t,B} = H_1 \times \gamma_W + H_2 \times \gamma_{sat} \text{ (Not dependent on seepage of water)}$$

Due to the upward seepage the pressure head at point B will increase by “h”

$$u_B = (H_1 + H_2 + iH_2) \times \gamma_W \rightarrow iH_2 = \frac{h}{H_2} \times H_2 = h \rightarrow u_B = (H_1 + H_2 + h) \times \gamma_W$$

$$\sigma'_{B} = (H_1 \times \gamma_W + H_2 \times \gamma_{sat}) - (H_1 + H_2 + h) \times \gamma_W = H_2 \times \gamma' - h \times \gamma_W$$

**Point C:**

$$\sigma_{t,C} = H_1 \times \gamma_W + z \times \gamma_{sat} \text{ (Not dependent on seepage of water)}$$

Due to the upward seepage the pressure head at point B will increased by “iz”

$$u_C = (H_1 + z + iz) \times \gamma_W$$

$$\sigma'_{C} = (H_1 \times \gamma_W + z \times \gamma_{sat}) - (H_1 + z + iz) \times \gamma_W = z \times \gamma' - iz \times \gamma_W$$

### Limiting Conditions:

Due to the upward seepage, as the depth increase the pore water pressure increase, so, the effective stress will decreases with depth, and this is very dangerous because at certain depth the effective stress **may** equal zero ( $\sigma_{total} = u$ ) and this is a limiting conditions.

Now, suppose that the effective stress will equal zero at depth z:

$$\sigma' = z \times \gamma' - iz \times \gamma_W = 0.0$$

The hydraulic gradient at this point is called the critical hydraulic gradient ( $i_{cr}$ )

$$\text{So, } z \times \gamma' = i_{cr} z \times \gamma_W \rightarrow i_{cr} = \frac{\gamma'}{\gamma_W} = \frac{\gamma_{sat} - \gamma_W}{\gamma_W} \text{ range : ( 0.9 to 1.1)}$$

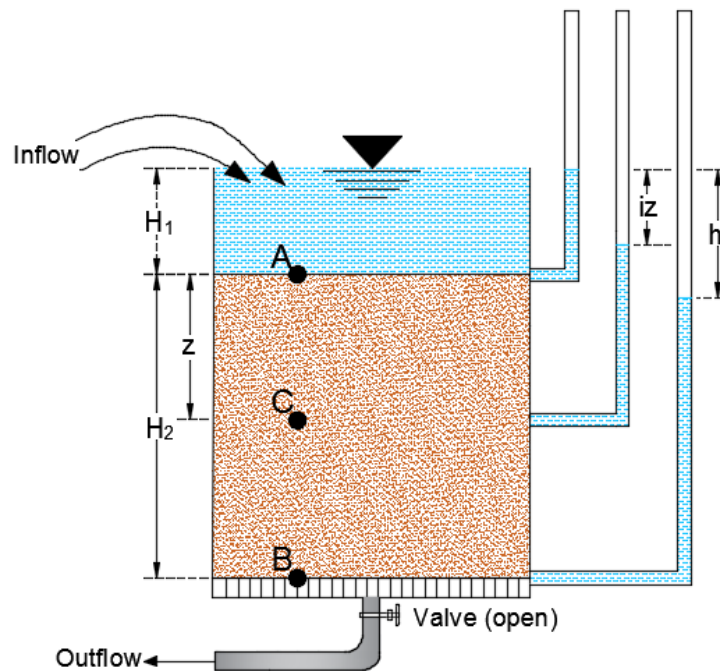
Now, if the value of “u” is larger than the value of  $\sigma_{total}$  this means the effective stress is **negative** and if we reach to this state the soil stability will lost and this situation generally is referred to as **boiling** or **quick condition**.

So, the value of  $z$  (the depth having  $\sigma' = 0$ ) is the maximum depth can be reach to prevent **boiling** or **quick conditions** or to **prevent heave** (الرفع) of soil

### Boiling or Quick conditions or Heave of soil:

هذه الظاهرة تحدث في التربة كما ذكرنا عندما تصبح قيمة الإجهاد الفعال عند نقطة معينة سالبة وهنا يكون ضغط الماء عند هذه النقطة أكبر من الضغط الكلي على هذه النقطة (حيث أن، هذه الحالة تحدث فقط في حالة التسرب لأعلى) حيث يقوم الماء برفع التربة إلى أعلى وتحدث حالة عدم تجانس في التربة وتصبح التربة وحل (Mud) حيث أنها تكون غير مستقرة وهي حالة خطيرة جدا لا نريد الوصول إليها أبدا.

### Stresses in Saturated Soil with Downward Seepage:



We note that, there is downward seepage from upstream (point A) to downstream (point B), also we note that, the elevation of water on the tank is remained constant by controlling the amount of flow in and flow out.

So, due to the downward seepage through the soil, there will be losses in piezometric head as the water move downward, and the total losses is “ $h$ ” due to the downward movement of water in length”  $H_2$  “ through the soil, thus, the hydraulic gradient “  $i$  “ will be :

$$i = \frac{h}{H_2}$$

Calculate the **total stress**, **pore water pressure**, and **effective stress** at points A, B, and C

Firstly, you should note that, as the depth increase the piezometric (pressure) head decrease due to the downward seepage, so, pore water pressure **decrease** with depth.

**Point A:**

$$\sigma_{t,A} = H_1 \times \gamma_W$$

$$u_A = H_1 \times \gamma_W$$

$$\sigma'_A = \sigma_{total} - u = H_1 \times \gamma_W - H_1 \times \gamma_W = 0.0$$

**Point B:**

$$\sigma_{t,B} = H_1 \times \gamma_W + H_2 \times \gamma_{sat} \text{ (Not dependent on seepage of water)}$$

Due to the downward seepage the pressure head at point B will decrease by “h”

$$u_B = (H_1 + H_2 - iH_2) \times \gamma_W \rightarrow iH_2 = \frac{h}{H_2} \times H_2 = h \rightarrow u_B = (H_1 + H_2 - h) \times \gamma_W$$

$$\sigma'_B = (H_1 \times \gamma_W + H_2 \times \gamma_{sat}) - (H_1 + H_2 - h) \times \gamma_W = H_2 \times \gamma' + h \times \gamma_W$$

**Point C:**

$$\sigma_{t,C} = H_1 \times \gamma_W + z \times \gamma_{sat} \text{ (Not dependent on seepage of water)}$$

Due to the upward seepage the pressure head at point B will decreased by “iz”

$$u_C = (H_1 + z - iz) \times \gamma_W$$

$$\sigma'_C = (H_1 \times \gamma_W + z \times \gamma_{sat}) - (H_1 + z - iz) \times \gamma_W = z \times \gamma' + iz \times \gamma_W$$

It is clear that no limiting conditions exist in case of downward seepage because the pore water pressure decreased by depth, thus, the effective stress will increase with depth and never reach zero.

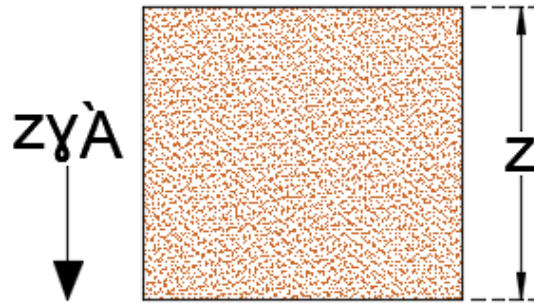
**ملاحظة:** إذا كان ارتفاع عمود الماء عند أي نقطة في التربة لا يساوي منسوب المياه الجوفية فهذا يعني أنه يوجد seepage, حيث إذا كان ارتفاع عمود الماء أعلى من منسوب المياه الجوفية فإن الحالة تكون upward seepage وخطر حدوث ال boiling موجود أما إذا كان ارتفاع عمود الماء أقل من منسوب المياه الجوفية فإن الحالة تكون downward seepage ولا يوجد خطر boiling.. لكن إذا كان ارتفاع المياه الجوفية نفس ارتفاع عمود الماء.. فإنه لا يوجد seepage ولا يوجد خطر boiling.



## Seepage Force:

Due to the seepage of water through the soil, the effective stress may increase or decrease according to the direction of seepage, so, this concept can be expressed by seepage force by unit volume as following:

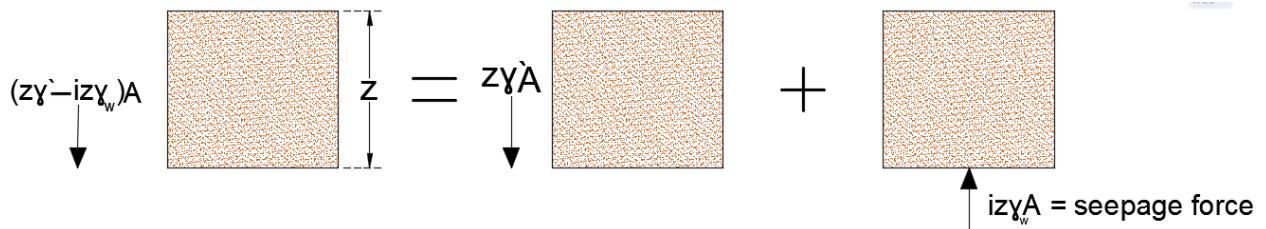
**In case of no seepage:**



It is noted that, there is no seepage force because there is no seepage of water in the soil. So, the only force exerted on the soil is the weight of soil layer:

$$P = Z \gamma' A$$

**In case of Upward Seepage:**

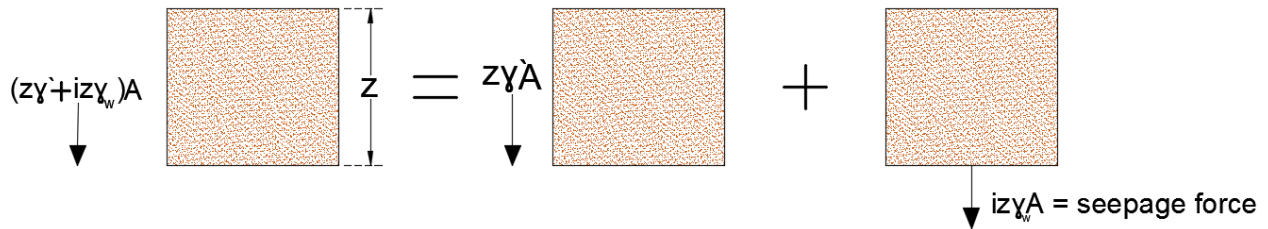


It is noted that, there is a seepage force directed upward (in the direction of flow) and we also note that there are two forces exerted on the soil, the weight of the soil directed downward, and the seepage force directed upward:

$$P = Z \gamma' A - iz \gamma_w A$$

Also we note that, if the seepage force ( $iz \gamma_w A$ ) smaller than the weight of the soil ( $Z \gamma' A$ ), the soil will be safe and the boiling will not occur, and the **critical case** when the weight of the soil equal the seepage force, but if the seepage force is larger than the weight of the soil, the soil stability will lost and the boiling will occur.

**In case of Downward Seepage:**



It is noted that, there is a seepage force directed downward (in the direction of flow) and we also note that there are two forces exerted on the soil, the weight of the soil directed downward, and the seepage force directed downward:

$$P = Z \gamma' A + iz \gamma_w A$$

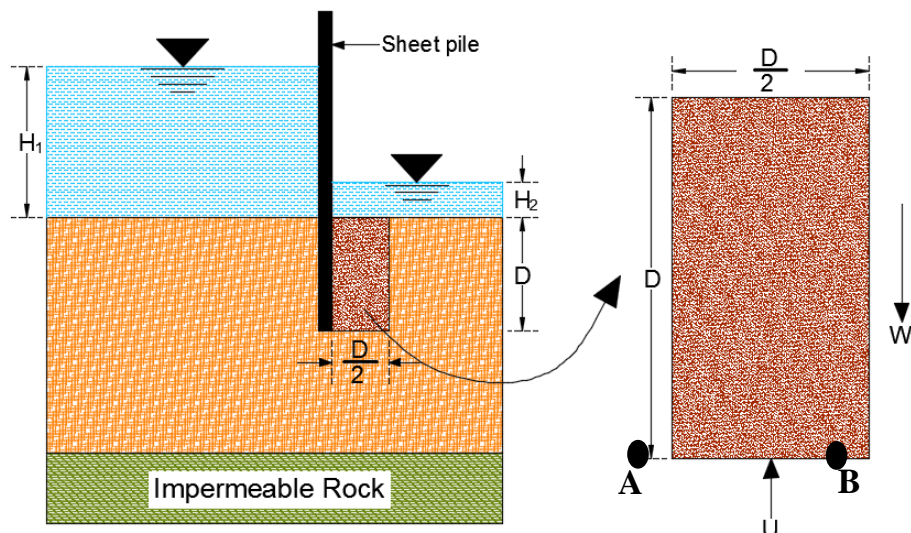
It is clear that, the force exerted on the soil is always downward and the soil is always safe.

The seepage force in the two cases having the same magnitude ( $iz \gamma_w A$ ) and the volume of the soil is ( $zA$ ), so, the seepage force can be expressed by the form of: seepage force =  $i\gamma_w$  (per unit volume)

**Heaving in Soil Due to Flow around Sheet Pile:**

If there is a sheet pile separated two levels of water (downstream and upstream), the soil on the downstream side may be heave (ترتفع) due to the uplift seepage force on the bottom of the heave zone. Terzaghi found that the heaving generally occurred within a distance of  $\left(\frac{D}{2}\right)$  from the sheet piles.

Such that  $D$  = the depth of embedment (العمق المدفون) of sheet piles into the permeable layer. This can be explained on the figure shown below:



Now, we want to calculate the **factor of safety against heaving** of soil:

$$F.S = \frac{W'}{U}$$

$W'$  = Submerged weight of soil in the heave zone per unit length of sheet pile.

$W'$  = Unit weight of soil  $\times$  Volume per unit length.

$$W' = (\gamma_{sat} - \gamma_w) \times \frac{D}{2} \times D = \frac{1}{2} D^2 \times \gamma'$$

$U$  = The uplift force caused by seepage on the same volume of soil.

$$U = \frac{1}{2} D^2 \times i_{av} \times \gamma_w$$

$i_{av}$  is the average hydraulic gradient between points A&B

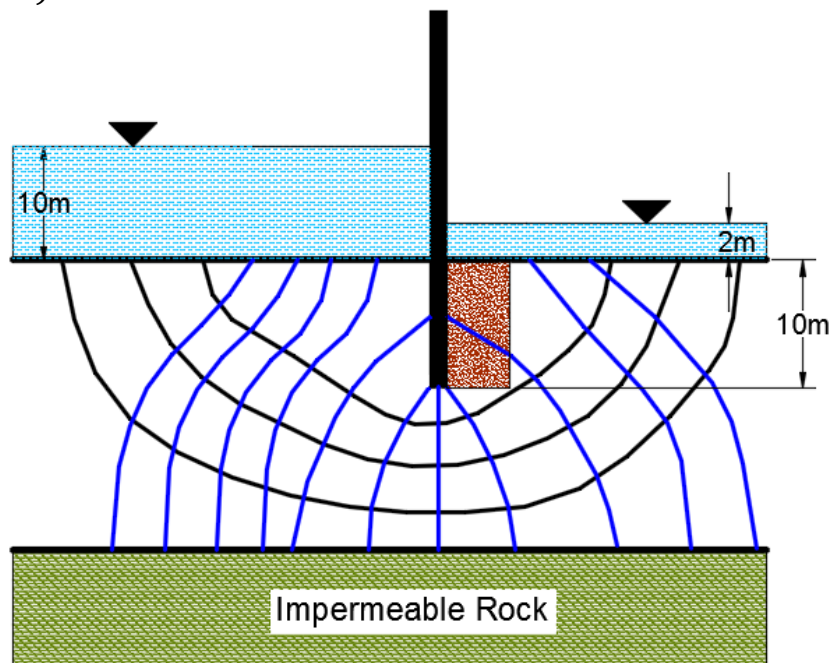
$$F.S = \frac{\frac{1}{2} D^2 \times \gamma_{eff}}{\frac{1}{2} D^2 \times i_{av} \times \gamma_w} = \frac{\gamma'}{i_{av} \times \gamma_w}$$

How to calculate  $i_{av}$ :

$$i_{av} = \frac{h_{av}}{D} \quad , \quad h_{av} = \frac{h_{driving,A} + h_{driving,B}}{2} \quad (\text{Driving head at points A\&B})$$

**Example:**

In the figure shown below, calculate the factor of safety against heave ( $\gamma_{sat} = 19\text{KN/m}^3$ ).



The first step is to draw the heave zone on the flow net (if it's not drawn) and then calculate the factor of safety:

$$\gamma' = \gamma_{\text{sat}} - \gamma_w = 19 - 9.81 = 9.19 \text{ KN/m}^3$$

The heave zone can be enlarged as shown in figure below:

You should know how to draw this heave zone  
(with dimensions)

$$\Delta H = 10 - 2 = 8 \text{ m} , N_{d,\text{number}} = 12 \quad D = 10 \text{ m}$$

$$i = N_{d,\text{value}} = \frac{\Delta H}{N_{d,\text{number}}} = \frac{8}{12} = 0.667$$

There are 8 potential drops to reach point "A"

$$h_{\text{driving,A}} = 8 - 0.667 \times 8 = 2.664 \text{ m}$$

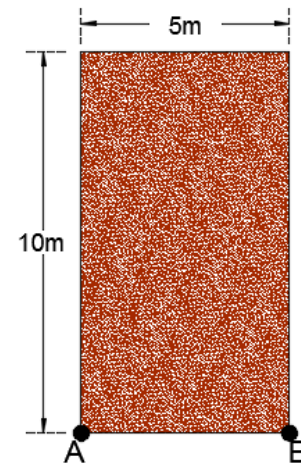
There are 8.8 potential drops to reach point "B"

$$h_{\text{driving,B}} = 8 - 0.667 \times 8.8 = 2.13 \text{ m}$$

$$h_{\text{av}} = \frac{h_{\text{driving,A}} + h_{\text{driving,B}}}{2} = \frac{2.664 + 2.13}{2} = 2.397 \text{ m}$$

$$i_{\text{av}} = \frac{h_{\text{av}}}{D} = \frac{2.397}{10} = 0.2397$$

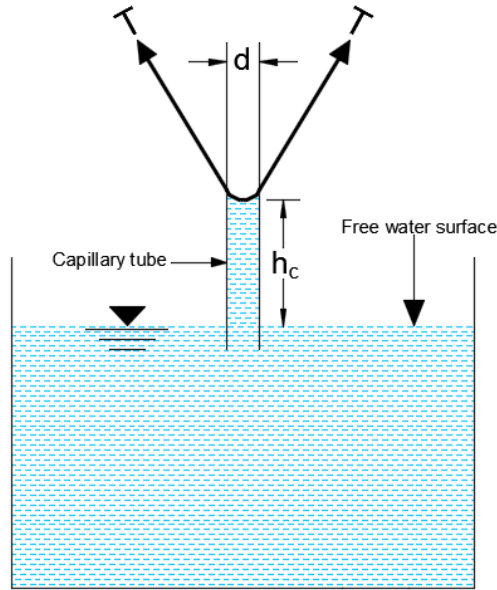
$$F.S = \frac{\gamma'}{i_{\text{av}} \times \gamma_w} = \frac{9.19}{0.2397 \times 9.81} = 3.9$$



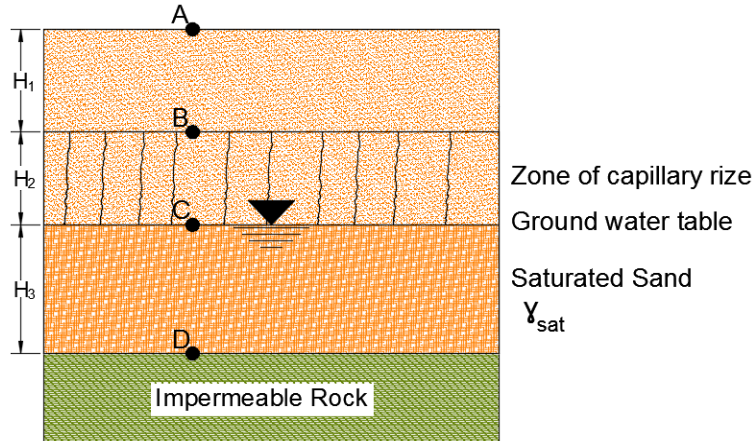
## Capillary Rise in Soil & the Effective Stress in the Zone of Capillary Rise:

### Capillary Rise in Soil:

تحدث الخاصية الشعرية بشكل عام نتيجة وضع أنبوب شعيري رقيق في حوض من الماء حيث يقوم الشد السطحي الموجود على سطح الماء في الحوض برفع الماء إلى أعلى بينما يقاوم وزن الماء إلى أسفل بحيث يرتفع منسوب الماء في هذا الأنبوب الشعيري مسافة معينة ( $h_c$ ) كما يوضح الشكل التالي:



نفس المفهوم تماما يوجد في التربة حيث أنه في بعض الأحيان تحتوي الطبقة التي فوق منسوب المياه الجوفية على شعيرات رقيقة من الفراغات على شكل أنابيب شعيرية بحيث ترتفع الماء في هذه الشعيرات عن طريق مفهوم الخاصية الشعرية , وهنا هذه الطبقة التي تكون فوق منسوب المياه الجوفية وتحتوي على فراغات شعيرية لا تكون جافة وذلك بسبب انتقال الماء إليها بواسطة الخاصية الشعرية وبالتالي فهي إما أن تكون مشبعة بشكل كامل ( $S=100\%$ ) أو مشبعة بشكل جزئي ( $0<S<100\%$ ) وبالتالي يجب أخذ ذلك بالاعتبار عند حساب الإجهادات الناتجة من التربة . **الشكل التالي يوضح الفراغات الشعرية في التربة والتي تنقل الماء للطبقة:**





To calculate **total stress** result from zone of capillary rise you should calculate the unit weight of this layer [saturated or moist (partially saturated)] and then multiply this unit weight by the height of this layer.

To calculate the **pore water pressure** from the zone of capillary rise:

$$u = - \left( \frac{S}{100} \right) \gamma_w h$$

S = Degree of saturation (%)

h = The height of the point under consideration measured from the groundwater table.

Now, to find the pore water pressure at point B on the above graph you **must** calculate it just before point (B) and just after point (B), because the degree of saturation(S) is different before and after point (B)

So, to find effective stress at point (B) you must calculate it just before and just after point (B).

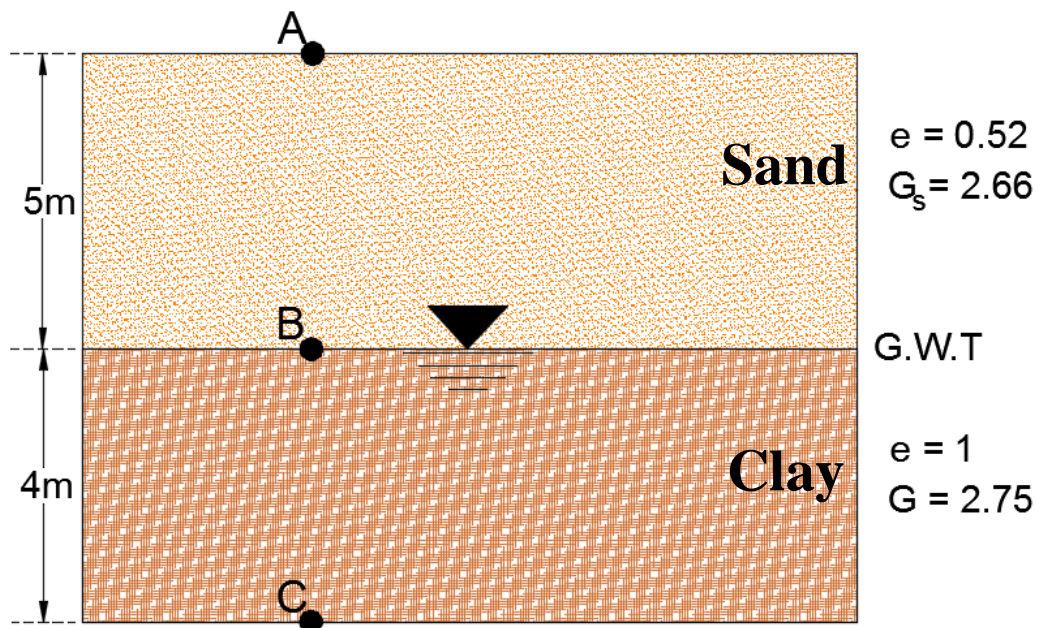
The negative sign in the equation is due to the points always exist above the G.W.T.

**ملاحظة:** بالنسبة لضغط الماء ... دائما أي نقطة توجد على نفس منسوب المياه الجوفية يكون ضغط الماء عندها صفر, وأي نقطة تكون أسفل منسوب المياه الجوفية يكون ضغط الماء لها مساويا بُعد هذه النقطة عن منسوب المياه الجوفية مضروبا في كثافة الماء النوعية, وإذا كانت النقطة فوق منسوب المياه الجوفية يكون ضغط الماء لها يساوي صفر إلا إذا كانت هذه النقطة موجودة في طبقة حدثت فيها الخاصية الشعرية بحيث يكون لهذه النقطة ضغط ماء ولكن ضغط سالب لأنها فوق منسوب المياه الجوفية.

**36. (Final 2012):**

Refer to the soil profile shown in figure below:

- Calculate the variation of  $\sigma$ ,  $u$ , and  $\sigma'$  with depth.
- If the water table rises to the top of the ground surface, what is the change in the effective stress at the bottom of the clay layer?
- How many meters must the ground water table rise to decrease the effective stress by  $15 \text{ KN/m}^2$  at the bottom of the clay layer?

**Solution**

We note that the G.W.T is below the sand layer and above clay layer so the sand layer is **dry** (No capillary rise) and the clay layer is **saturated**.

For Dry Sand Layer:

$$\gamma_{\text{dry}} = \frac{G_s \times \gamma_w}{1 + e} = \frac{2.66 \times 9.81}{1 + 0.52} = 17.17 \text{ KN/m}^3$$

For Saturated (S=1) Clay Layer:

$$S \cdot e = G_s w \rightarrow w = \frac{S \cdot e}{G_s} = \frac{1 \times 1}{2.75} = 0.363$$

$$\gamma_{\text{sat}} = \frac{2.75 \times 9.81(1 + 0.363)}{1 + 1} = 18.4 \text{ KN/m}^3$$

a.

Let the point **A** at depth “0”, point **B** at depth “5m”, and point **C** at depth 9m.

**Point “A”** ( $z = 0.0$ ):

$$\sigma_{t,A} = 0.0 \checkmark .$$

$$u_A = 0.0 \checkmark .$$

$$\sigma'_{A} = 0.0 \checkmark .$$

**Point “B”** ( $z = 5\text{m}$ ):

$$\sigma_{t,B} = 5 \times 17.17 = 85.85 \text{ KN/m}^2 \checkmark .$$

$$u_B = 0.0 \checkmark .$$

$$\sigma'_{B} = 85.85 - 0.0 = 85.85 \text{ KN/m}^2 \checkmark .$$

**Point “C”** ( $z = 9\text{m}$ ):

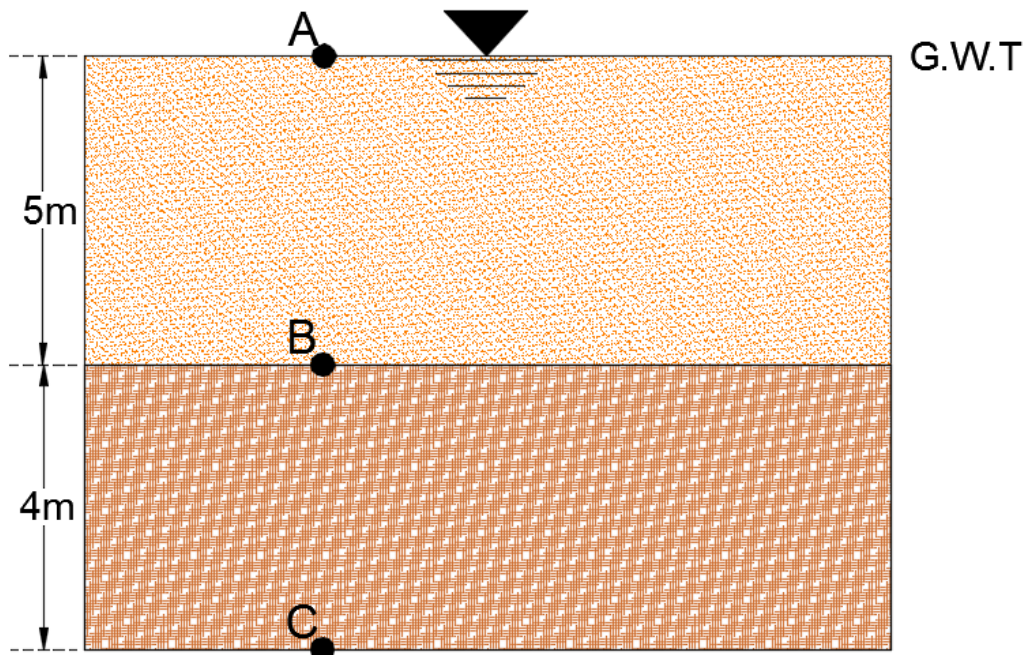
$$\sigma_{t,C} = 5 \times 17.17 + 4 \times 18.4 = 159.45 \text{ KN/m}^2 \checkmark .$$

$$u_C = 4 \times 9.81 = 39.24 \text{ KN/m}^2 \checkmark .$$

$$\sigma'_{C} = 159.45 - 39.24 = 120.21 \text{ KN/m}^2 \checkmark .$$

b.

نلاحظ هنا أن منسوب المياه الجوفية ارتفع إلى سطح الأرض وهذا ليس معناه أنه يوجد seepage لأنه لا يوجد تدفق داخل وتدفق خارج ولا يوجد فرق في مناسيب المياه، وإنما هذا الارتفاع قد ينتج عن وجود مطر على هذه المنطقة فتتسبع التربة تدريجياً بالماء حتى يصل منسوب الماء إلى السطح وبالتالي فإن طبقة الرمل في هذه الحالة تصبح مشبعة تماماً وبالتالي يجب إيجاد  $\gamma_{\text{sat}}$  لها. **الرسم التالي يوضح هذه الحالة:**





We note that from the above graph both sand and clay layers are saturated

For Saturated (S=1) Sand Layer:

$$S.e = G_s w \rightarrow w = \frac{S.e}{G_s} = \frac{1 \times 0.52}{2.66} = 0.1955$$

$$\gamma_{sat} = \frac{2.66 \times 9.81(1 + 0.1955)}{1 + 0.52} = 20.52 \text{ KN/m}^3$$

For Saturated (S=1) Clay Layer:

$$\gamma_{sat} = 18.4 \text{ KN/m}^3 \text{ (Calculated above)}$$

The required is to calculate the **change** of effective stress at depth of clay layer (point C).

$$\text{From the first case: } \sigma'_C = 120.21 \text{ KN/m}^2$$

Now, for this case:

$$\sigma_{t,C} = 5 \times 20.52 + 4 \times 18.4 = 176.2 \text{ KN/m}^2$$

$$u_C = 5 \times 9.81 + 4 \times 9.81 = 88.29 \text{ KN/m}^2$$

$$\sigma'_C = 176.2 - 88.29 = 87.91 \text{ KN/m}^2$$

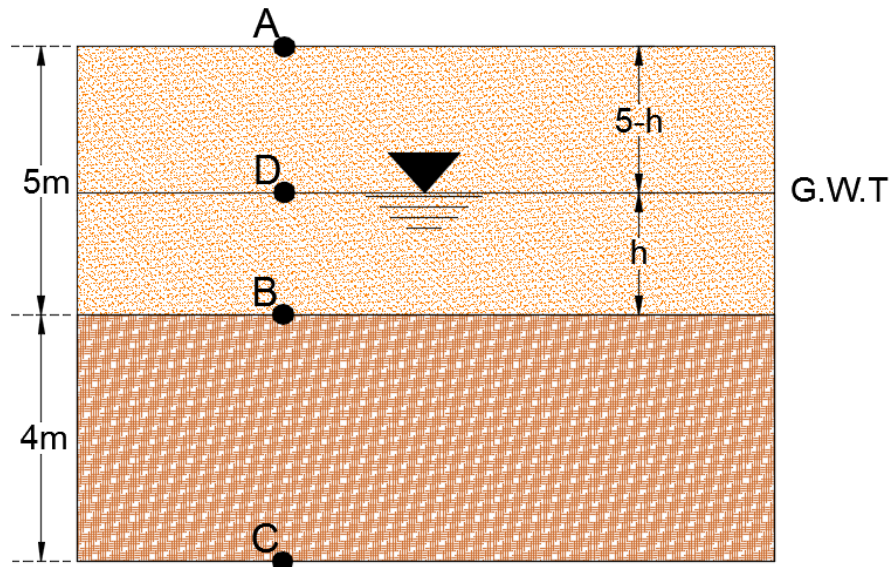
Now, the change in effective stress at point C:

$$\Delta \sigma'_C = 120.21 - 87.91 = 32.3 \text{ KN/m}^2 \text{ (Decrease) } \checkmark .$$

نلاحظ أنه عند ارتفاع منسوب المياه الجوفية فإن ضغط الماء يزداد عند النقطة C وبالتالي فإن الإجهاد الفعال يقل.

C.

في هذه الحالة معطى قيمة الانخفاض في الإجهاد الفعال عند النقطة C نتيجة ارتفاع منسوب المياه الجوفية مسافة h عن المنسوب في الحالة الأصلية الحالة الأولى وبالتالي فإن جزء من طبقة التربة سوف يكون جاف فوق منسوب المياه الجوفية والجزء الآخر مشبع بالماء مثل طبقة الطين. **الرسم التالي يوضح هذه الحالة:**



From the first case:  $\sigma'_c = 120.21 \text{ KN/m}^2$  and  $\Delta\sigma'_c = 15 \text{ KN/m}^2$  (Given)

Now, for this case:

$$\sigma_{t,c} = (5 - h) \times 17.17 + h \times 20.52 + 4 \times 18.4 = 159.45 + 3.35h$$

$$u_c = h \times 9.81 + 4 \times 9.81 = 9.81h + 39.24$$

$$\sigma'_c = (159.45 + 3.35h) - (39.24 + 9.81h) = 120.21 - 6.46h$$

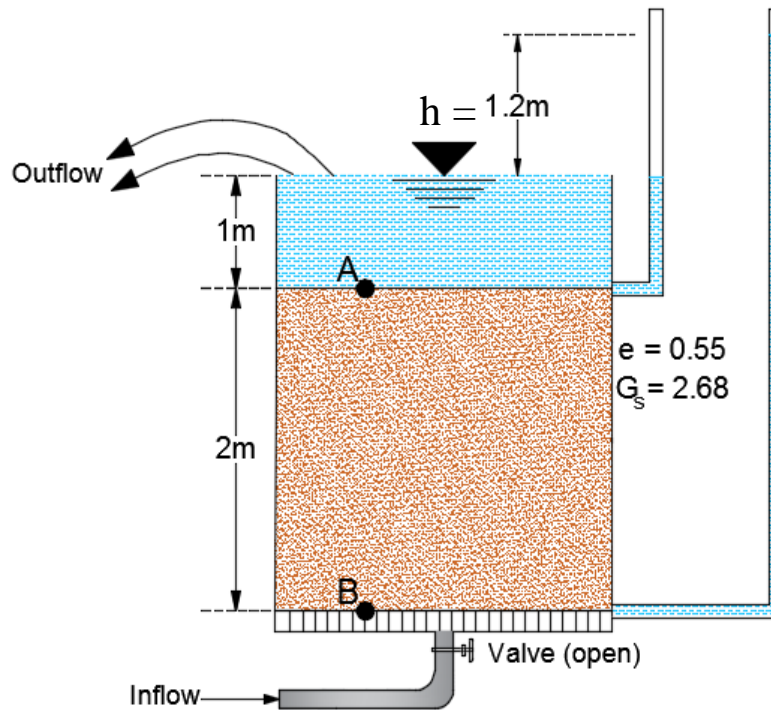
$$\Delta\sigma'_c = 15 = 120.21 - (120.21 - 6.46h) \rightarrow 15 = 6.46h \rightarrow h = 2.322 \text{ m} \checkmark.$$

نلاحظ أن قيمة  $h$  أقل من 5 وهذا يؤكد أن القيمة صحيحة أما إذا كانت أكبر من 5 فيجب مراجعة الحل.

### 37.

For the figure shown below, if the area of the tank is  $0.5 \text{ m}^2$  and hydraulic conductivity of sand is  $0.1 \text{ cm/sec}$ .

- What is the magnitude and direction of the rate of seepage?
- If the value of  $h=1.2\text{m}$ , will boiling occur? Why?
- What should be the value of “ $h$ ” to cause boiling?



### Solution

It is clear that the direction of seepage is upward, so, boiling may be occurred.

a.

$$q = KiA, \quad K = 0.1 \times 10^{-2} \text{ m/sec}, \quad A = 0.5\text{m}^2$$

$$i = \frac{\text{Total Losses}}{\text{Length of Soil}} = \frac{1.2}{2} = 0.6$$

$$q = 0.1 \times 10^{-2} \times 0.6 \times 0.5 = 3 \times 10^{-4} \text{ m}^3/\text{sec} \text{ (Upward)} \checkmark.$$

b.

Always we check stability at the point having maximum pressure head because it have maximum pore water pressure and minimum effective stress.

So, we want to calculate the effective stress at point **(B)**, if the value is negative, the boiling will occur, otherwise, the boiling will not occur.

Firstly, we calculate the saturated unit weight for the soil:

$$S.e = G_s w \rightarrow w = \frac{S.e}{G_s} = \frac{1 \times 0.55}{2.68} = 0.2052$$

$$\gamma_{\text{sat}} = \frac{2.68 \times 9.81(1 + 0.2052)}{1 + 0.55} = 20.44 \text{ KN/m}^3$$

$$\sigma_{t,B} = 1 \times 9.81 + 2 \times 20.44 = 50.69 \text{ KN/m}^2$$

$$u_B = 1 \times 9.81 + 2 \times 9.81 + 1.2 \times 9.81 = 41.202 \text{ KN/m}^2$$

$$\sigma'_{B} = 50.69 - 41.202 = 9.488 \text{ KN/m}^2 > 0 \rightarrow \text{boiling will not occur} \checkmark.$$

c.

The value of (h) that causes boiling must be greater than the critical value of h ( $\sigma'_{B} = 0$ )

$$\sigma_{t,B} = 1 \times 9.81 + 2 \times 20.44 = 50.69 \text{ KN/m}^2$$

$$u_B = 1 \times 9.81 + 2 \times 9.81 + h \times 9.81 = 29.43 + 9.81 h$$

$$\text{Critical case occur when } \sigma'_{B} = 0 \rightarrow \sigma_{t,B} = u_B$$

$$50.69 = 29.43 + 9.81 h \rightarrow h = 2.167 \text{ m}$$

So, the value of “h” that cause boiling must be **larger than** 2.167 m

**38. (Mid 2006):**

A layer of sand 5 meters deep overlies a thick bed of clay. Draw diagrams indicating the total and effective stresses and pore water pressure on horizontal planes to a depth of 9 meters below the ground surface in the following cases:

- If the water table is at the ground level.
- If the water table is at 2 meters below ground level and the sand above remains saturated with capillary moisture.
- If the water table is at the top of the clay and the sand dry.

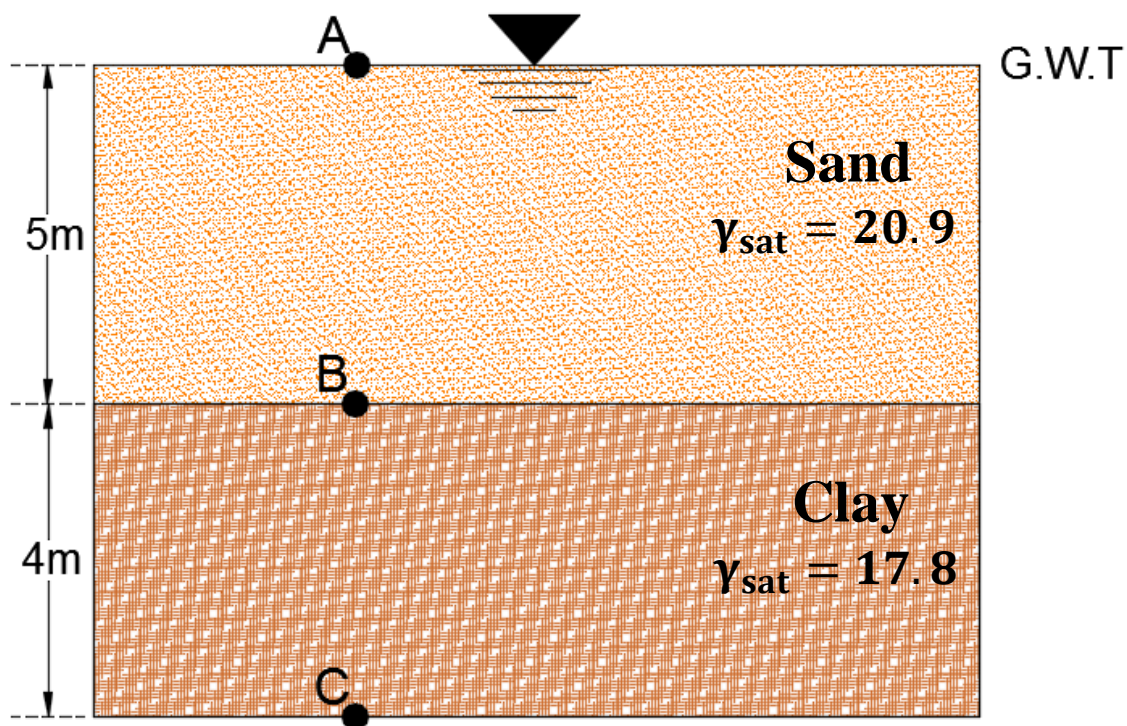
**For Sand:**  $\gamma_{\text{sat}} = 20.9 \text{ KN/m}^3$  ,  $\gamma_{\text{dry}} = 17.4 \text{ KN/m}^3$

**For Clay:**  $\gamma_{\text{sat}} = 17.8 \text{ KN/m}^3$

**Solution**

The graph of the problem is not given so you should know how to draw it.

a.



Let the point **A** at depth “0”, point **B** at depth “5m”, and point **C** at depth 9m

**Point “A”** ( $z = 0.0$ ):

$$\sigma_{t,A} = 0.0$$

$$u_A = 0.0$$

$$\sigma'_{A} = 0.0$$

**Point “B”** ( $z = 5\text{m}$ ):

$$\sigma_{t,B} = 5 \times 20.9 = 104.5 \text{ KN/m}^2$$

$$u_B = 5 \times 9.81 = 49.05 \text{ KN/m}^2$$

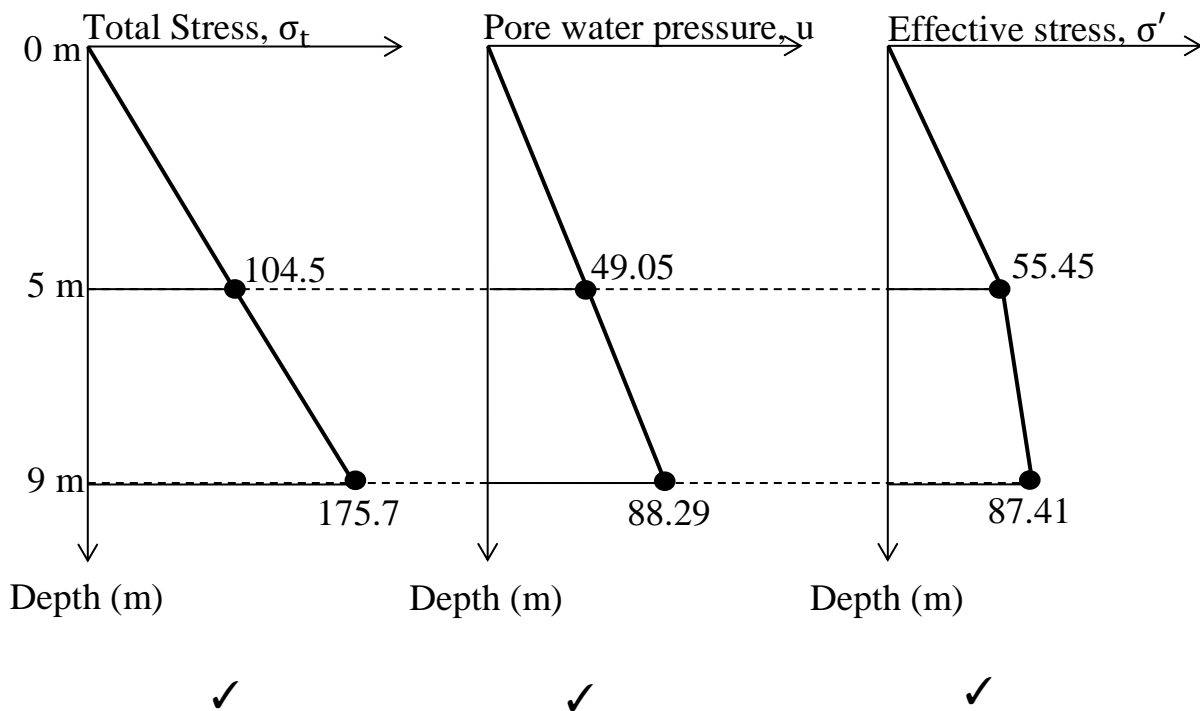
$$\sigma'_{B} = 104.5 - 49.05 = 55.45 \text{ KN/m}^2$$

**Point “C”** ( $z = 9\text{m}$ ):

$$\sigma_{t,C} = 5 \times 20.9 + 4 \times 17.8 = 175.7 \text{ KN/m}^2$$

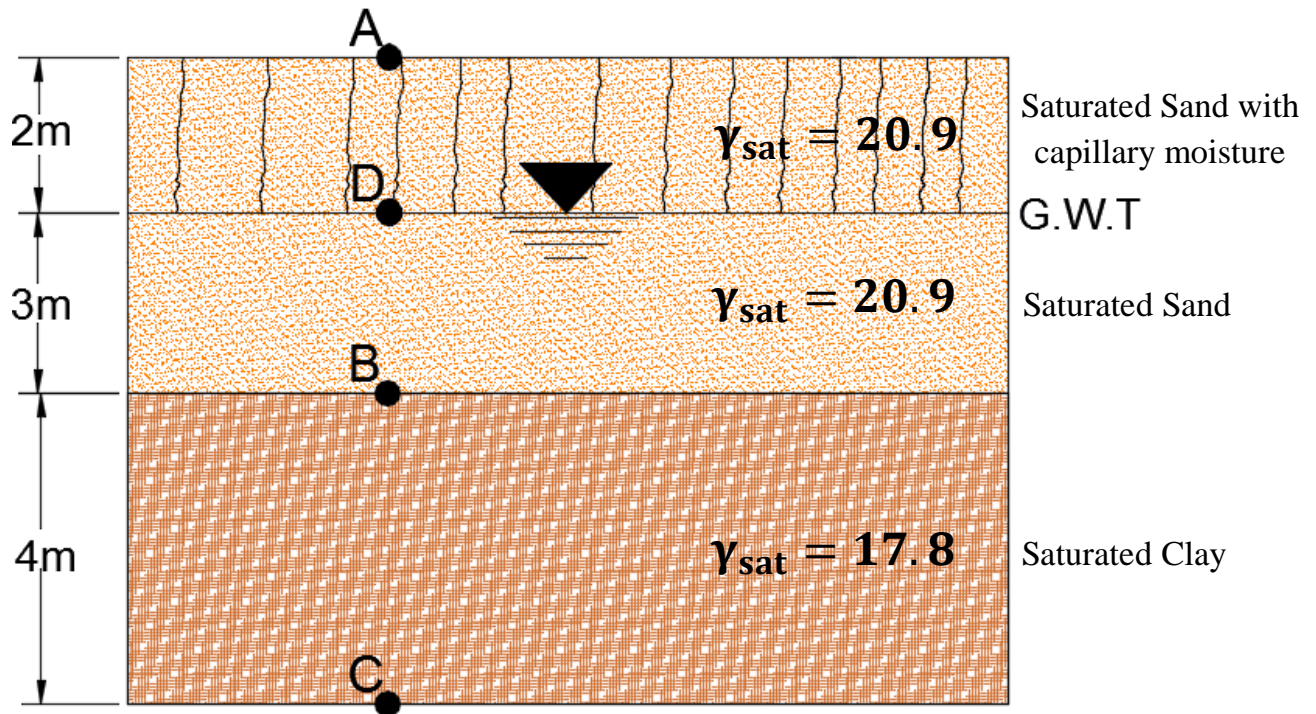
$$u_C = (5 + 4) \times 9.81 = 88.29 \text{ KN/m}^2$$

$$\sigma'_{C} = 175.7 - 88.29 = 87.41 \text{ KN/m}^2$$





b.



Let the point **D** at depth “2m” (at the G.W.T.)

**Point “A”** ( $z = 0.0$ ):

$$\sigma_{t,A} = 0.0$$

$$u_A = -\left(\frac{S}{100}\right) \gamma_w h, (S = 100\% \rightarrow \text{Given}) \quad (\text{لم نأخذ قبل وبعد لأنه لا يوجد شيء قبلها})$$

$$u_A = -\left(\frac{100}{100}\right) 9.81 \times 2 = -19.62 \text{ KN/m}^2$$

$$\sigma'_A = 0.0 - (-19.62) = 19.62 \text{ KN/m}^2$$

**Point “D”** ( $z = 2\text{m}$ ):

$$\sigma_{t,D} = 2 \times 20.9 = 41.8 \text{ KN/m}^2 \quad (\text{Because the sand above point D is saturated})$$

$$u_D = 0.0 \quad (\text{at the G. W. T} \rightarrow \text{No head})$$

$$\sigma'_D = 41.8 - 0.0 = 41.8 \text{ KN/m}^2$$

**Point “B”** ( $z = 5\text{m}$ ):

$$\sigma_{t,B} = (2 + 3) \times 20.9 = 104.5 \text{ KN/m}^2$$

$$u_B = 3 \times 9.81 = 29.43 \text{ KN/m}^2$$

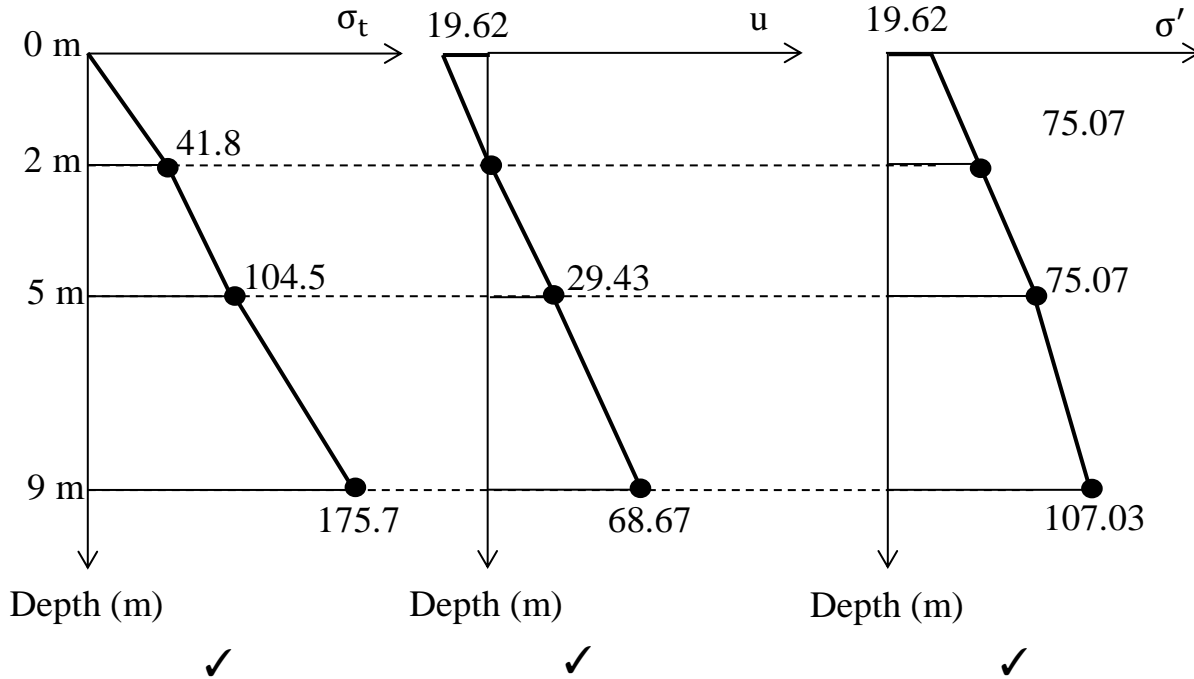
$$\sigma'_B = 104.5 - 29.43 = 75.07 \text{ KN/m}^2$$

**Point “C”** ( $z = 9\text{m}$ ):

$$\sigma_{t,C} = (2 + 3) \times 20.9 + 4 \times 17.8 = 175.7 \text{ KN/m}^2$$

$$u_C = (3 + 4) \times 9.81 = 68.67 \text{ KN/m}^2$$

$$\sigma'_{C} = 175.7 - 68.67 = 107.03 \text{ KN/m}^2$$



c.

**Point “A”** ( $z = 0.0$ ):

$$\sigma_{t,A} = 0.0$$

$$u_A = 0.0$$

$$\sigma'_{A} = 0.0$$

**Point “B”** ( $z = 5\text{m}$ ):

$$\sigma_{t,B} = 5 \times 17.4 = 87 \text{ KN/m}^2$$

$$u_B = 0.0$$

$$\sigma'_{B} = 87 - 0.0 = 87 \text{ KN/m}^2$$

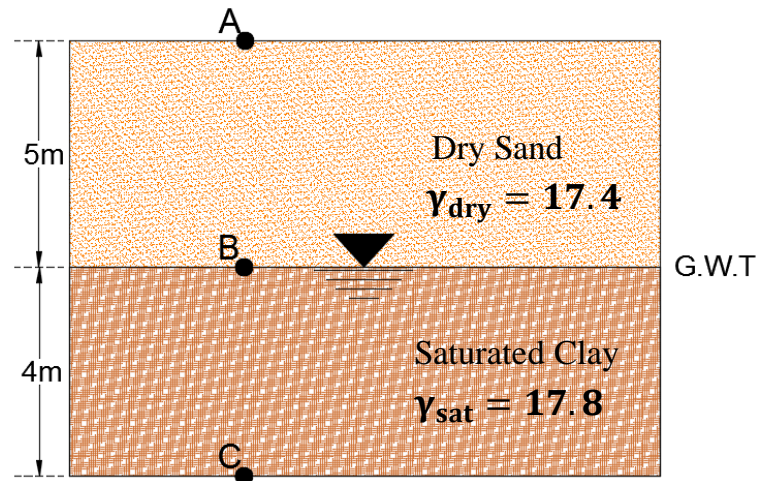
**Point “C”** ( $z = 9\text{m}$ ):

$$\sigma_{t,C} = 5 \times 17.4 + 4 \times 17.8 = 158.2 \text{ KN/m}^2$$

$$u_C = 4 \times 9.81 = 39.24 \text{ KN/m}^2$$

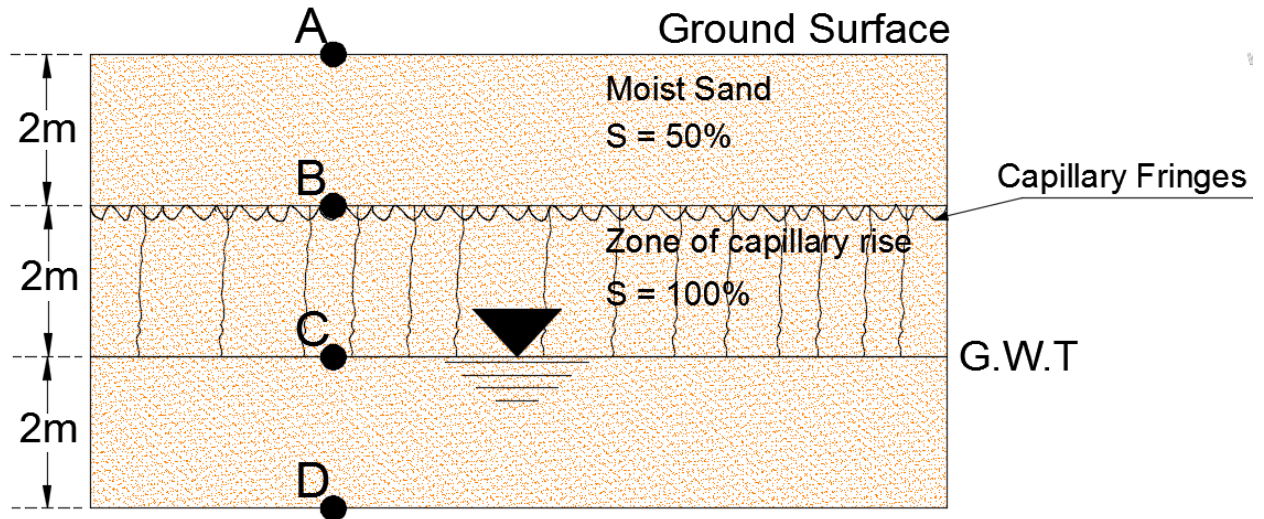
$$\sigma'_{C} = 158.2 - 39.24 = 118.96 \text{ KN/m}^2$$

Draw the stress diagrams by yourself ☺.



## 39.

For sand in the shown profile below, if  $e = 0.7$  and  $G_s = 2.65$ . Draw diagrams of total and effective stresses and pore water pressure with depth.



## Solution

## Capillary Fringes:

هي الأهداب أو النهايات للأنايبب الشعيرية الموجودة في التربة وهي تُعبر عن انتهاء هذه الأنايبب الشعيرية وبالتالي انتهاء ارتفاع الماء بالخاصية الشعيرية.

For moist sand:

$$S = 0.5, \quad e = 0.7, \quad G_s = 2.65$$

$$S \cdot e = G_s w \rightarrow w = \frac{S \cdot e}{G_s} = \frac{0.5 \times 0.7}{2.65} = 0.132$$

$$\gamma_{\text{moist}} = \frac{2.65 \times 9.81(1 + 0.132)}{1 + 0.7} = 17.31 \text{ KN/m}^3$$

For saturated sand:

$$S = 1, \quad e = 0.7, \quad G_s = 2.65$$

$$S \cdot e = G_s w \rightarrow w = \frac{S \cdot e}{G_s} = \frac{1 \times 0.7}{2.65} = 0.264$$

$$\gamma_{\text{moist}} = \frac{2.65 \times 9.81(1 + 0.264)}{1 + 0.7} = 19.33 \text{ KN/m}^3$$



**Point “A”** ( $z = 0.0$ ):

$$\sigma_{t,A} = 0.0$$

$$u_A = 0.0$$

$$\sigma'_{A} = 0.0$$

**Point “B”** ( $z = 2\text{m}$ ):

$$\sigma_{t,B} = 2 \times 17.31 = 34.62 \text{ KN/m}^2$$

$$u_{B,\text{just before}} = -\left(\frac{50}{100}\right) 9.81 \times 2 = -9.81 \text{ KN/m}^2$$

$$u_{B,\text{just after}} = -\left(\frac{100}{100}\right) 9.81 \times 2 = -19.62 \text{ KN/m}^2$$

$$\sigma'_{B,\text{just before}} = 34.62 - (-9.81) = 44.43 \text{ KN/m}^2$$

$$\sigma'_{B,\text{just after}} = 34.62 - (-19.62) = 54.24 \text{ KN/m}^2$$

**Point “C”** ( $z = 4\text{m}$ ):

$$\sigma_{t,C} = 2 \times 17.31 + 2 \times 19.33 = 73.28 \text{ KN/m}^2$$

$$u_C = 0.0$$

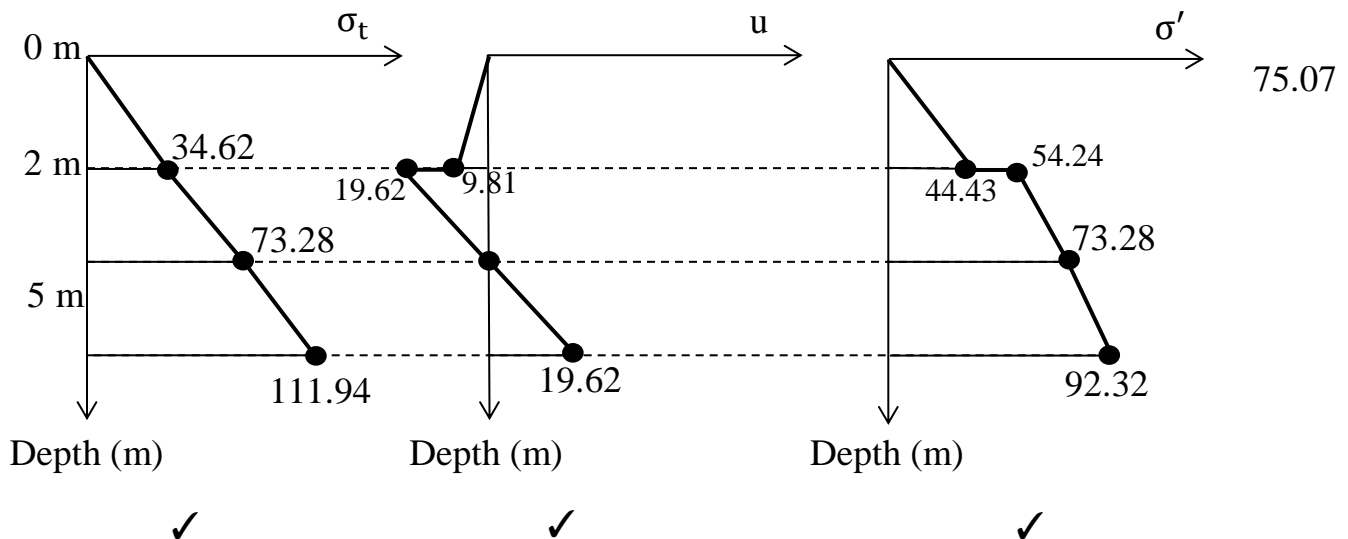
$$\sigma'_{C} = 73.28 - 0.0 = 73.28 \text{ KN/m}^2$$

**Point “D”** ( $z = 6\text{m}$ ):

$$\sigma_{t,D} = 2 \times 17.31 + 2 \times 19.33 + 2 \times 19.33 = 111.94 \text{ KN/m}^2$$

$$u_D = 2 \times 9.81 = 19.62$$

$$\sigma'_{D} = 111.94 - 19.62 = 92.32 \text{ KN/m}^2$$

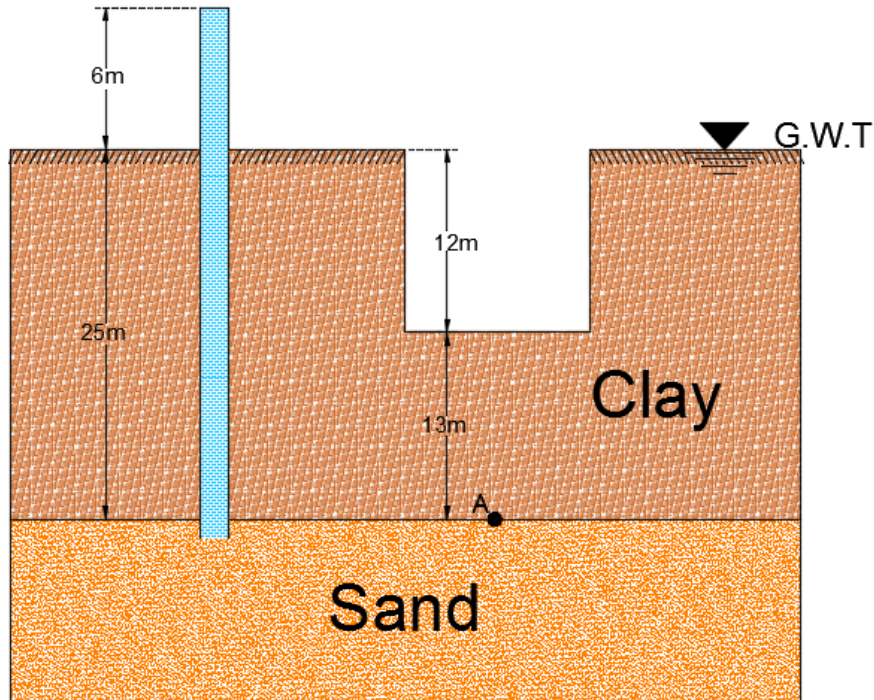


40.

A lay layer of 25 m thickness with a saturated unit weight of  $21 \text{ KN/m}^3$  is underlain by a sand layer. Ground water table is at the ground surface. An artesian pressure amounting to a water level of 6 m above the ground surface is known to exist in the sand layer. A 12 m deep, wide, open excavation, which is to be made in the dry is planned in the clay layer. Determine whether the bottom of the excavation will heave or not under the given conditions.

**Solution**

**شرح السؤال:** يوجد طبقة من الطين بسمك 25 متر أسفلها طبقة من الرمل, بحيث أن منسوب المياه الجوفية يوجد على سطح الأرض (أي سطح طبقة الطين) ويوجد بئر ارتوازي محفور في الأرض يصل إلى طبقة الرمل وارتفاع المياه فيه 6 متر فوق منسوب المياه الجوفية (وهذا معناه أنه يوجد لدينا حالة Upward Seepage). يُراد عمل حفرة عميقة في طبقة الطين بعمق 12 متر في وقت الجفاف أي في الوقت الذي لا يكون منسوب المياه الجوفية على سطح الأرض حتى يتمكنوا من الحفر. هل قاع هذه الحفرة سوف يفقد استقراره (يقوم الماء برفع التربة ويصبح القاع كالوحد) في حالة ارتفاع منسوب المياه الجوفية الى سطح الأرض ومنسوب البئر الارتوازي 6 متر فوق سطح الأرض؟؟. في البداية يتم الرسم للتسهيل:



Consider point (A) is at the bottom of the clay layer below the excavation.

If  $u_A > \sigma_{t,A} \rightarrow$

The bottom of excavation will heave, other wise, the bottom will not heave

$$\sigma_{t,A} = (25 - 12) \times 21 = 273 \text{ KN/m}^2$$

$$u_A = (25 + 6) \times 9.81 = 304.11 \text{ KN/m}^2$$

$u_A > \sigma_{t,A}$  ( $\sigma'_A =$  negative value) so the of excavation will heave ✓.

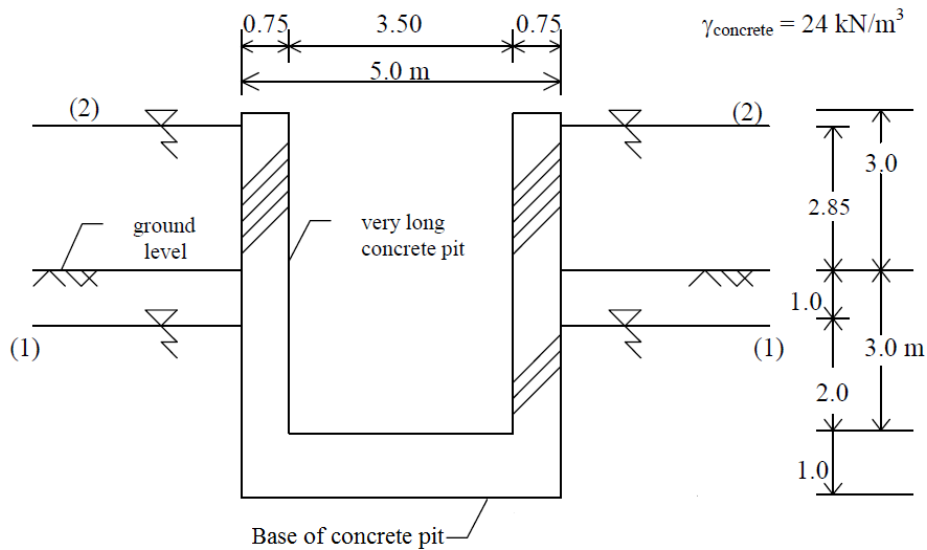
### 41.

The figure below shows a concrete structure (concrete pit) of a canal. Calculate (a) the factor of safety against uplift at the base of the concrete pit (b) the effective stress at the base of the concrete pit.

Do the above required calculations for the following two cases:

- If the water table is at level (1).
- If the water table is at level (2).

**Hint:** [State whether the design is good or not for the above two cases]



### Solution

#### Case I: Water table at level (1)

1.

$$\text{F. S. against uplift} = \frac{\text{structure weight}}{\text{uplift force}} \geq 1.5$$

$$\text{structure weight} = V_{\text{structure}} \times \gamma_{\text{concrete}}$$

$$V_{\text{structure}} = V_{\text{box}} - V_{\text{space}} = (7 \times 5) - (6 \times 3.5) = 14 \text{ m}^3$$

$$\rightarrow \text{structure weight} = 14 \times 24 = 336 \text{ kN}$$

$$\text{uplift force} = u_{\text{@base}} \times \text{base area}$$

$$u_{\text{@base}} = \gamma_w \times h_{w,\text{above base}} = 10 \times 3 = 30 \text{ kN/m}^2$$

$$\text{uplift force} = 30 \times (5 \times 1) = 150 \text{ kN.}$$

$$\text{F. S.} = \frac{336}{150} = 2.24 > 1.5 \rightarrow \text{The design is good in this case } \checkmark.$$

2.

$$\sigma'_{\text{@base}} = q_{\text{structure}} - u_{\text{@base}} = \frac{336}{(5 \times 1)} - 30 = 37.2 \text{ kN/m}^2 \checkmark.$$

### Case II: Water table at level (2)

1.

$$\text{structure weight} = 336 \text{ kN}$$

$$\text{uplift force} = u_{\text{@base}} \times \text{base area}$$

$$u_{\text{@base}} = \gamma_w \times h_{w,\text{above base}} = 10 \times 6.85 = 68.5 \text{ kN/m}^2$$

$$\text{uplift force} = 68.5 \times (5 \times 1) = 342.5 \text{ kN.}$$

$$\text{F. S.} = \frac{336}{342.5} = 0.98 < 1.5 \rightarrow \text{The design is not good in this case } \checkmark.$$

2.

$$\sigma'_{\text{@base}} = q_{\text{structure}} - u_{\text{@base}} = \frac{336}{(5 \times 1)} - 68.5 = -1.3 \cong 0.0 \checkmark.$$

→ this means the structure weight equals uplift force

→ Soil does not carry any load and the structure will float

## **Chapter (10)**

# **Stresses in a Soil Mass**

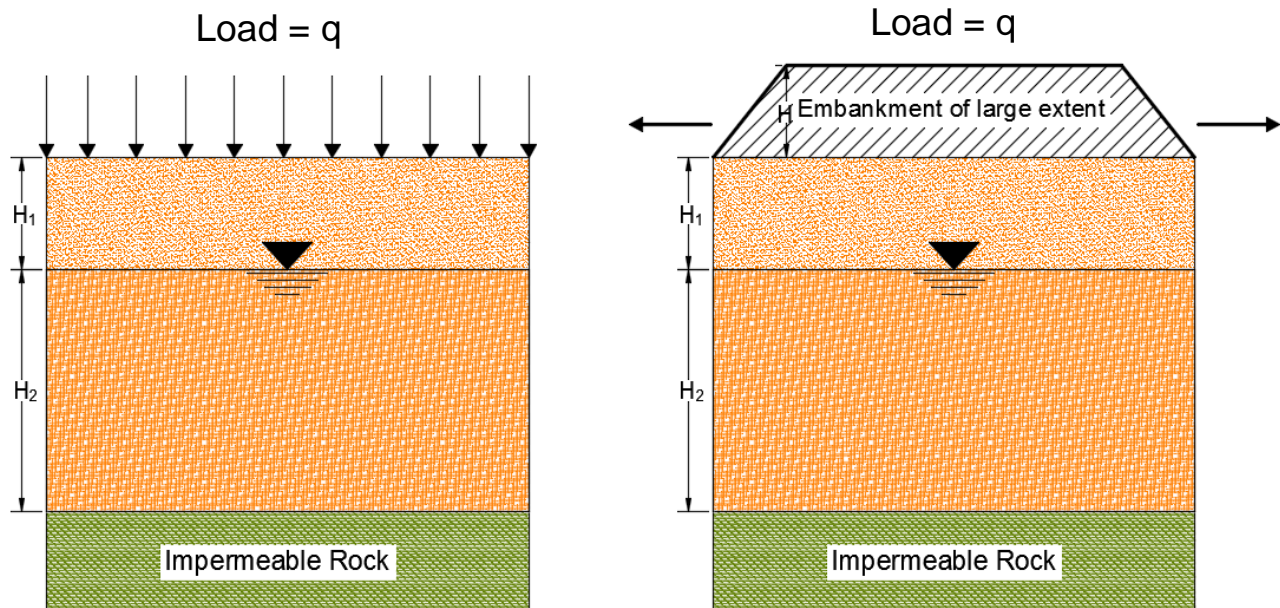
## Important Introduction:

تعلمنا في (CH.9) كيفية حساب قيم الإجهاد الكلي وضغط الماء والإجهاد الفعال عند أي نقطة على أي عمق معين في مقطع معين من التربة يتكون من طبقات معينة لها خواص معينة. لكن في حالة وجود أحمال إضافية على سطح التربة مثل: حمل عمود كهربائي على التربة "point load" أو حمل حائط جدار على التربة "Line Load" أو حمل قاعدة شريطية "Strip Load" أو حمل سد رمل "Embankment Load" أو حمل قاعدة دائرية الشكل أو ربما حمل خزان مياه دائري الشكل أو حمل قاعدة مستطيلة أو مربعة الشكل بما فيها اللبشة الخرسانية وغيرها من أنواع الأحمال. أي نوع من هذه الأحمال إذا كان موجود على مقطع معين من التربة فإنه سوف يزيد من قيمة الإجهاد الفعال (Effective Stress) عند أي نقطة على أي عمق من مقطع التربة وبالتالي فإن هذه الزيادة تسمى (Increase in Vertical Stresses) وهو موضوع هذا الـ (Chapter).

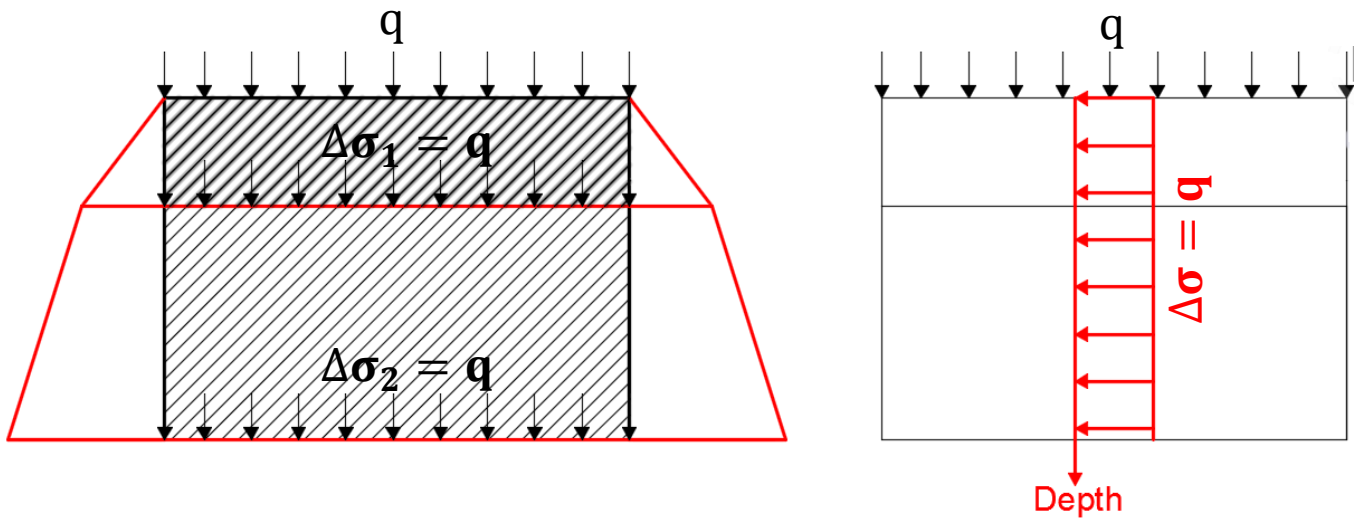
قبل الشروع في كيفية حساب هذه الزيادة نتيجة الأحمال المختلفة من التربة لا بد من توضيح نقطة هامة جدا: أن هذه الزيادة تلزم في حساب القيمة الكلية للإجهاد الفعال عند أي نقطة وعلى أي عمق معين وقيمة الإجهاد الفعال هذه تلزم بشكل أساسي في حساب قيم الهبوط في التربة (Primary Consolidation Settlement) والذي سوف نتناوله بشكل مفصل في (CH.11), ومن الجدير ذكره هنا أن أهم أنواع الأحمال والتي نهتم بدراستها لحساب قيم الهبوط في التربة هي أحمال السدود الرملية وأحمال خزانات المياه الدائرية وأحمال القواعد المستطيلة والمربعة أو القواعد الشريطية وأحمال لبشة خرسانية (قاعدة تحمل المبنى بالكامل) أي باختصار الأحمال بوحدة (Force/unit area) وهنا لا بد من الإشارة إلى كيفية انتقال هذه الأنواع من الأحمال إلى التربة وهي على نوعين:

1. Loads are applied on the whole soil profile, or loads have a large lateral extent.

لنعتبر أن الحمل الموجود على التربة بقيمة "q" وأن هذا الحمل موجود على كامل مقطع التربة أو أن الحمل ممتد إلى مسافة جانبية كبيرة (Large lateral Extent) كما يوضح الشكل التالي:



بشكل عام، طريقة إنتقال الأحمال إلى التربة هي أنه كلما زاد العمق كلما زادت المساحة التي تتوزع عليها تلك الأحمال وبالتالي تقل قيمة هذا الحمل مع زيادة العمق. لكن في هذه الحالة كما هو واضح من الشكلين السابقين أن الحمل موزع على كامل مقطع التربة وبالتالي فإنه كلما نزلنا إلى أسفل فإن المساحة التي يتوزع عليها هذا الحمل تزداد خارج مقطع التربة الموجود وبالتالي نحن غير معنيين بها لأننا معنيين بالمساحة من مقطع التربة والتي يتوزع عليها هذا الحمل، لذلك فإن المساحة التي يتوزع عليها هذا الحمل تبقى ثابتة مع زيادة العمق وهي نفس مساحة مقطع التربة الموجود وبالتالي فإن قيمة الحمل سوف تبقى ثابتة كلما نزلنا إلى أسفل لأنها تتوزع على نفس المساحة. الشكل التالي يوضح: (1) كيفية انتقال الحمل إلى مقطع التربة وأن المساحة المظللة هي المساحة التي نهتم بها لأنها داخل مقطع التربة. (2) قيمة الحمل "q" مع زيادة العمق تبقى ثابتة.



So, we conclude that, for this type of loading:

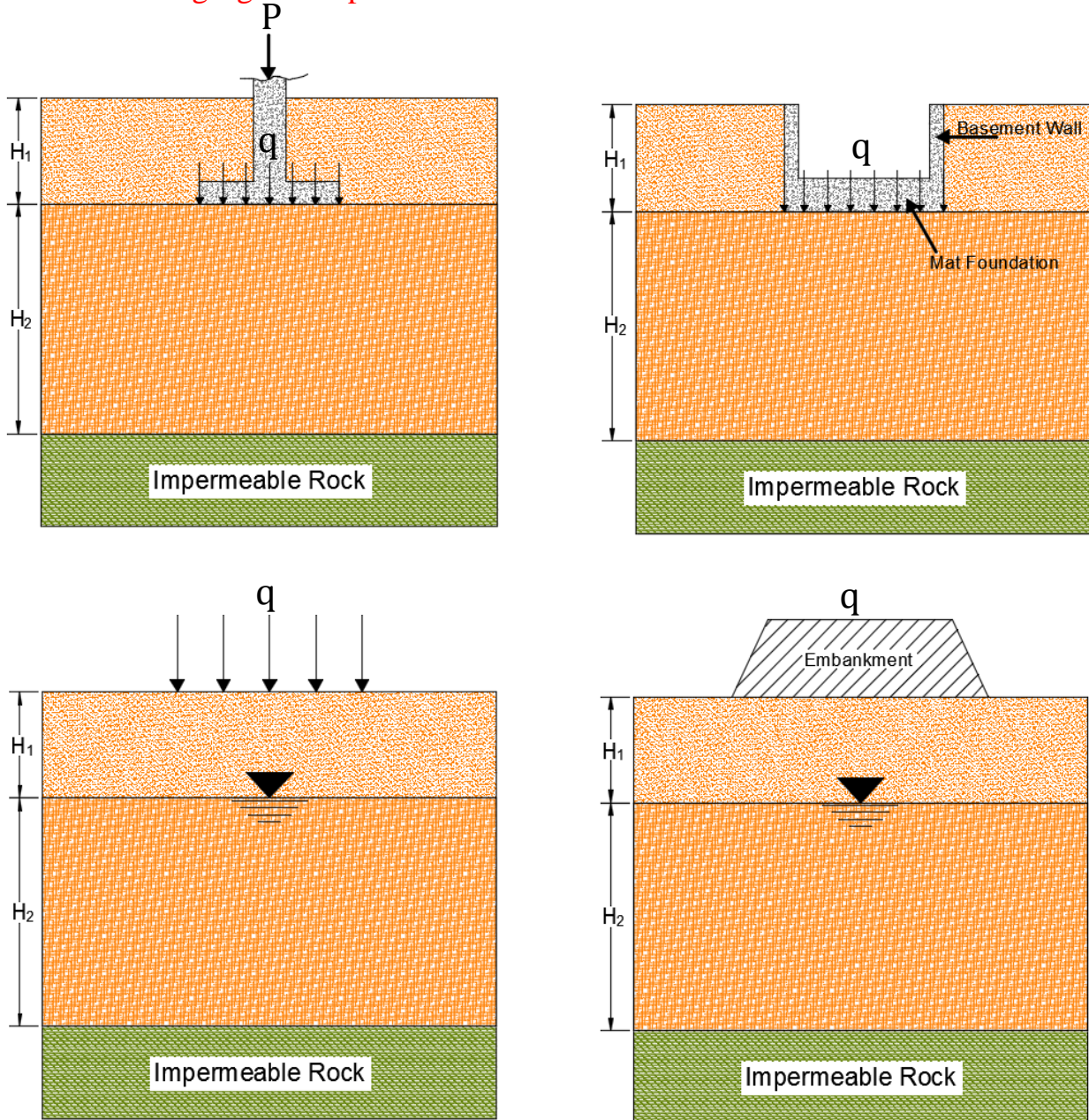
$$q = \Delta\sigma_1 = \Delta\sigma_2 = \Delta\sigma_z \rightarrow \rightarrow$$

Increase in vertical stresses at any depth " $\Delta\sigma_z$ " equal the load applied on the soil surface " $q$ ".



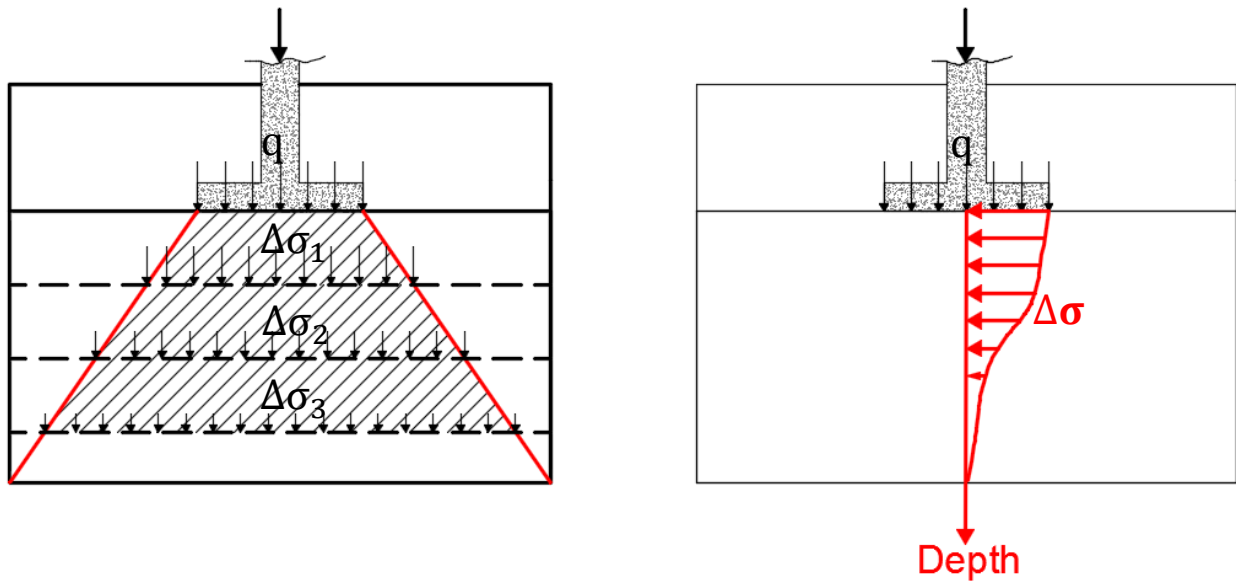
2. Loads are applied on a specific part of the soil profile:

The following figures explain this case:



نلاحظ هنا أن الأحمال الناتجة عن جميع الأنواع السابقة تؤثر على جزء معين من مقطع التربة وبالتالي كما ذكرنا سابقاً أن هذه الأحمال تنتزع على مساحات أكبر كلما زاد العمق وهذه المساحات هي جزء من مقطع التربة (في هذه الحالات) وبالتالي فإن هذه الأحمال تقل مع زيادة العمق لأن المساحات (المساحات المظلمة في الشكل أدناه) التي تنتزع عليها هذه الأحمال تزداد. **الشكل التالي يوضح كيفية انتقال هذا النوع من الأحمال إلى التربة:**





So, we conclude that, for this type of loading:

$$q > \Delta\sigma_1 > \Delta\sigma_2 > \Delta\sigma_3 > \Delta\sigma_z \rightarrow\rightarrow$$

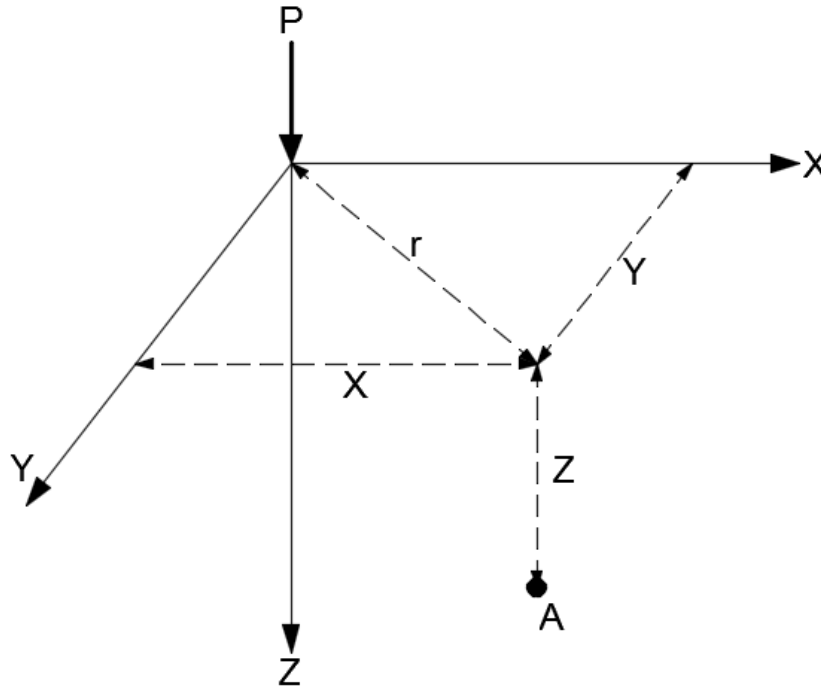
Increase in vertical stresses at any depth " $\Delta\sigma_z$ " is **less than** the load applied on the soil surface " $q$ ". And we will learn how to calculate it later in this chapter.

Now, we want to calculate the increase in vertical stresses at any depth due to different types of loading.

## Stresses Caused by a Point Load:

We calculate the vertical stress increase at any point at any depth due to the applied point load as following:

Consider we want to calculate the vertical stress increase at point **A** in figure below:



$$\Delta\sigma_{z,A} = \frac{3 \cdot P \cdot Z^3}{2\pi(r^2 + Z^2)^{\frac{5}{2}}}, \text{ and the same at any point.}$$

$$r = \sqrt{X^2 + Y^2}$$

X, Y and Z are measured from the point of applied load as shown in figure above.

**Note(1):** We can calculate the vertical stress increase due to point load alternatively as following: (Preferable)

$$\Delta\sigma_{z,A} = \frac{P}{Z^2} I_1$$

The value of  $I_1$  can be taken from (**Table 10.1 P.313**) according to the value of  $\frac{r}{Z}$

(Do interpolation “on table” **if required**).

**Note(2):** If there are more than one point load applied on the soil profile at different positions, you should calculate  $\Delta\sigma_z$  for each load and then :

$$\Delta\sigma_{z,t} = \Delta\sigma_{z,1} + \Delta\sigma_{z,2} + \Delta\sigma_{z,3} + \dots + \Delta\sigma_{z,n}$$

## Vertical Stress Caused by a Vertical Line Load:

Consider we want to calculate the vertical stress increase at point A in figure below:

$$\Delta\sigma_{z,A} = \frac{2q \cdot Z^3}{\pi(X^2 + Z^2)^2}$$

And the same for any point.

X and Z are measured from the start point of the vertical line load as shown in figure.

**Note(1):** We can calculate the vertical stress increase due to vertical line load alternatively as following: (Preferable)

$$\frac{\Delta\sigma_z}{(q/z)} = f\left(\frac{X}{Z}\right)$$

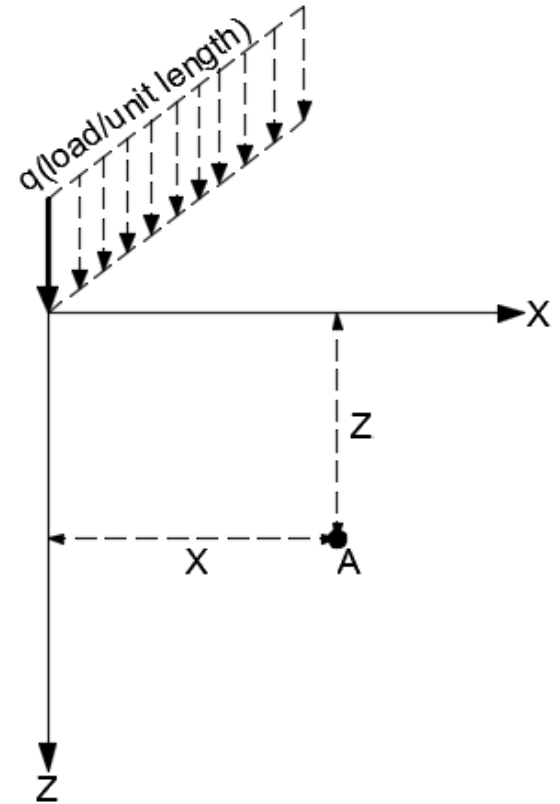
So, for any point, the value of  $\frac{\Delta\sigma_z}{(q/z)}$  can be taken

from (**Table 10.2 P.315**) according to the values of  $\frac{X}{Z}$  (Do interpolation "if required")

And if  $\frac{\Delta\sigma_z}{(q/z)}$  is known  $\rightarrow$  the value of  $\Delta\sigma_z$  will be known (because q and z are known).

**Note(2):** If there are more than one vertical line load applied on the soil profile at different positions, you should calculate  $\Delta\sigma_z$  for each load and then :

$$\Delta\sigma_{z,t} = \Delta\sigma_{z,1} + \Delta\sigma_{z,2} + \Delta\sigma_{z,3} + \dots + \Delta\sigma_{z,n}$$



## Vertical Stress Caused by a Horizontal Line Load:

Consider we want to calculate the vertical stress increase at point A in figure below:

$$\Delta\sigma_{z,A} = \frac{2q.X.Z^2}{\pi(X^2+Z^2)^2}$$

And the same for any point.

X and Z are measured from the start point of the horizontal line load as shown in figure.

**Note(1):** We can calculate the vertical stress increase due to horizontal line load alternatively as following: (Preferable)

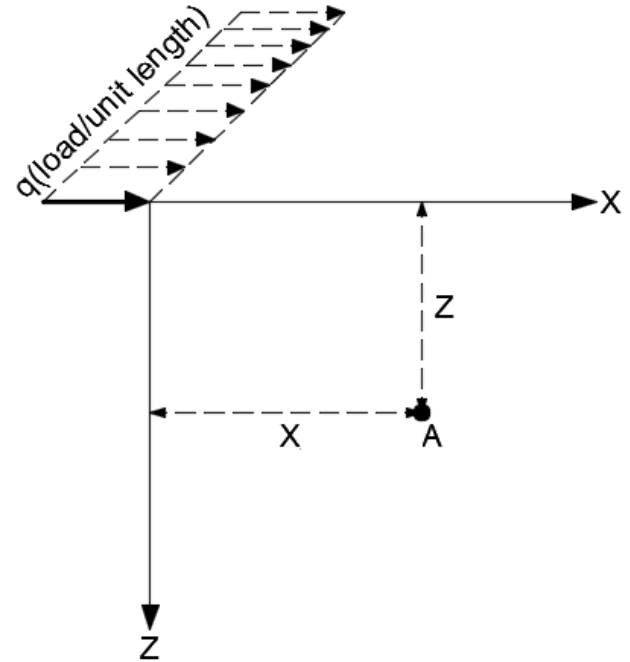
$$\frac{\Delta\sigma_z}{(q/z)} = f\left(\frac{X}{Z}\right)$$

So, for any point, the value of  $\frac{\Delta\sigma_z}{(q/z)}$  can be taken from (**Table 10.3 P.317**) according to the values of  $\frac{X}{Z}$ . (Do interpolation “if required”)

And if  $\frac{\Delta\sigma_z}{(q/z)}$  is known  $\rightarrow$  the value of  $\Delta\sigma_z$  will be known (because q and z are known).

**Note(2):** If there are more than one horizontal line load applied on the soil profile at different positions, you should calculate  $\Delta\sigma_z$  for each load and then :

$$\Delta\sigma_{z,t} = \Delta\sigma_{z,1} + \Delta\sigma_{z,2} + \Delta\sigma_{z,3} + \dots + \Delta\sigma_{z,n}$$



## Vertical Stress Caused by a Vertical Strip Load: (Finite Width and Infinite Length)

### Strip Load:

يتمثل بشكل أساسي عند وجود قاعدة شريطية (وظيفة هذه القاعدة هي حمل جدار بديوم مثلا) بحيث يكون لها عرض محدد B وطول غير محدد.

Consider we want to calculate the vertical stress increase at point A in figure below:

X and Z are measured from the midpoint of the strip load as shown in figure.

And we calculate the vertical stress increase at any point due to the strip load as following:

$$\frac{\Delta\sigma_z}{q} = f\left(\frac{2Z}{B}, \frac{2X}{B}\right)$$

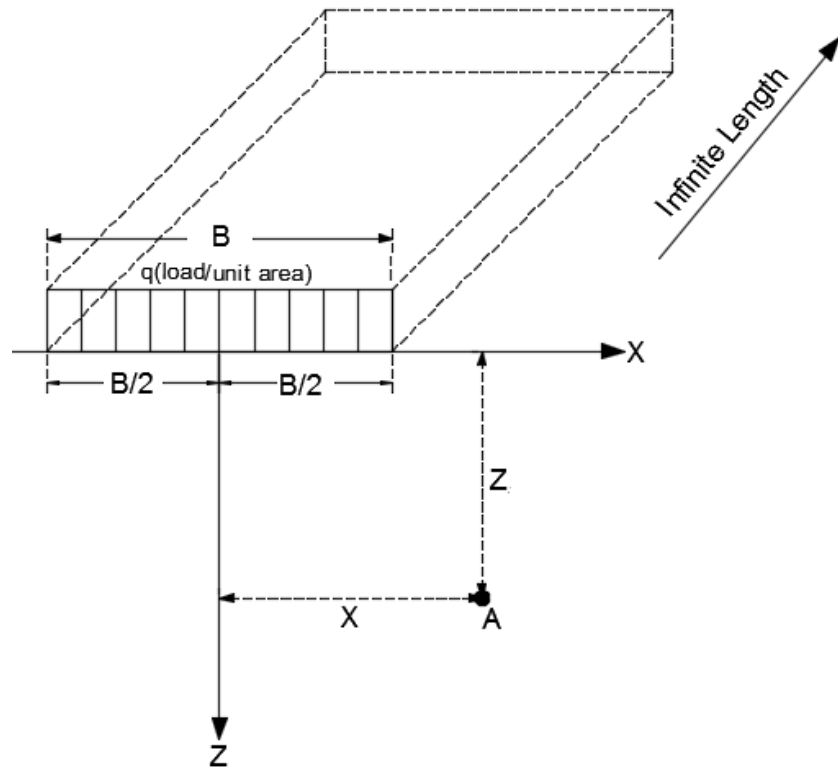
So, for any point, the value of  $\frac{\Delta\sigma_z}{q}$  can

be taken from (Table 10.4 P.320)

according to the values of  $\frac{2Z}{B}$  and  $\frac{2X}{B}$ .

And if  $\frac{\Delta\sigma_z}{q}$  is known  $\rightarrow$  the value of

$\Delta\sigma_z$  will be known (because q is known)



**Note:** in table 10.4 you may need to do interpolation 3 times to get the value of  $\frac{\Delta\sigma_z}{q}$

**Example:** consider  $\frac{2Z}{B} = 0.833$  and  $\frac{2X}{B} = 1.37$

From table: 1.37 is between (1.3 and 1.4), and 0.833 is between (0.8 and 0.9)

For  $\frac{2Z}{B} = 0.8 \rightarrow \frac{\Delta\sigma_z}{q}$  (at  $\frac{2X}{B} = 1.37$ ) = 0.2345 (interpolation between 1.3 and 1.4)

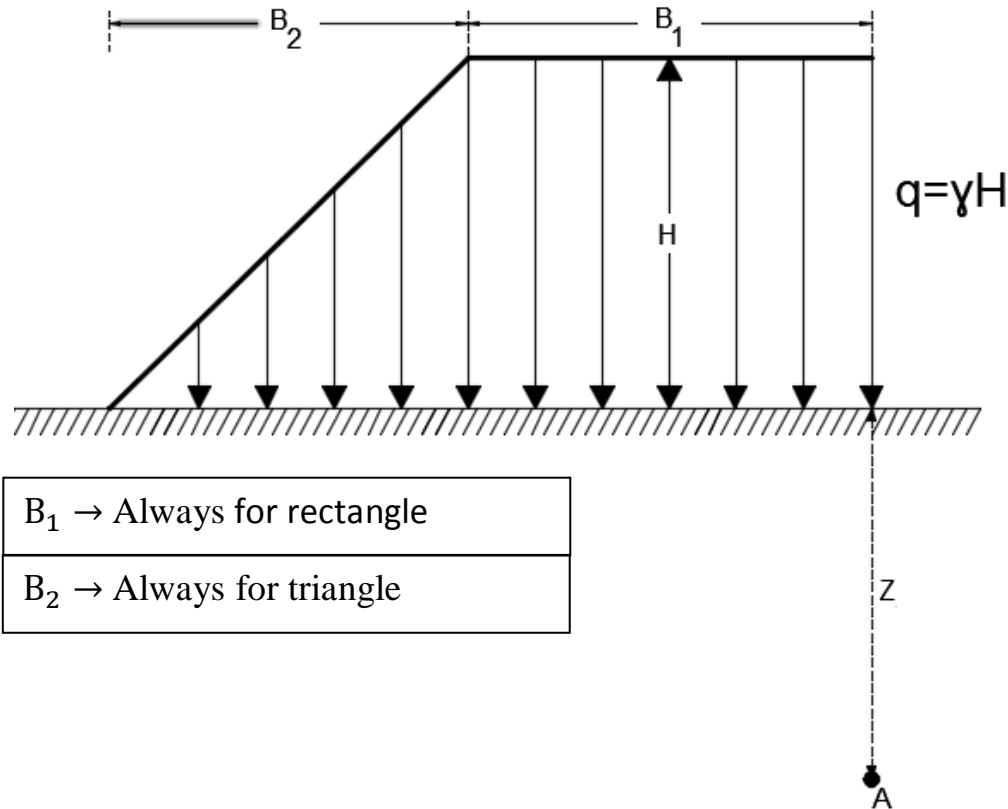
For  $\frac{2Z}{B} = 0.9 \rightarrow \frac{\Delta\sigma_z}{q}$  (at  $\frac{2X}{B} = 1.37$ ) = 0.2546 (interpolation between 1.3 and 1.4)

Now, For  $\frac{2Z}{B} = 0.833 \rightarrow \frac{\Delta\sigma_z}{q}$  (at  $\frac{2X}{B} = 1.37$ ) = **0.241** (interpolation bet. 0.8 and 0.9)

## Vertical Stress Due to Embankment Loading:

### Embankment Loading:

قد يكون سد رملي فوق مقطع من التربة أو قد يكون عبارة عن طمم (fill) موجود فوق مقطع تربة معين.  
Consider we want to calculate the vertical stress increase at point A in figure below:



We can calculate the vertical stress increase at any point due to the embankment load as following:

$$\Delta\sigma_z = qI_2$$

The value of  $I_2$  can be taken from (Figure 10.20 P.327) according to the values of  $\frac{B_1}{Z}$  and  $\frac{B_2}{Z}$

### Note:

نلاحظ من شكل 10.20 أن قيمة  $\frac{B_2}{Z}$  لا تساوي صفر أبداً، أما قيمة  $\frac{B_1}{Z}$  قد تساوي صفر، وبالتالي فإنه في بعض الحالات يمكن حساب  $\Delta\sigma_z$  أسفل طمم على شكل مثلث فقط ( $\frac{B_1}{Z} = 0$ )، وقد تكون أسفل متوازي مستطيلات (مستطيل + مثلث)، لكن لا يمكن حساب  $\Delta\sigma_z$  أسفل مستطيل فقط. لذلك، لحساب  $\Delta\sigma_z$  عند أي نقطة يجب أن تكون هذه النقطة أسفل متوازي مستطيلات أو أسفل مثلث، أما إذا كانت الحالة غير ذلك فقد يتم إضافة مثلث وطرحه أو إضافة متوازي مستطيلات وطرحه حتى تتوفر الشروط كما سنرى في الأسئلة.

## Vertical Stress below the Center of a Uniformly Loaded Circular Area:

Consider we want to calculate the vertical stress increase at point **A** in figure below:

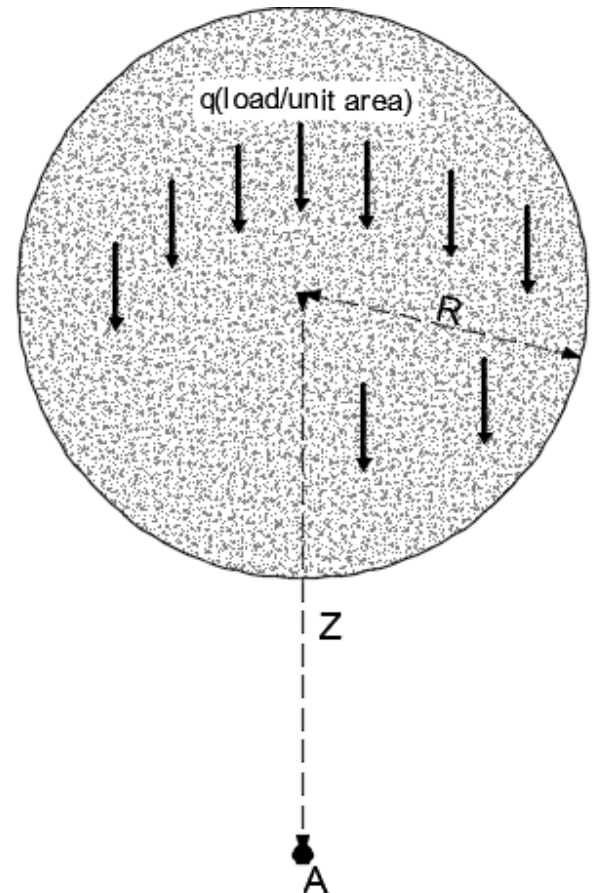
We calculate the vertical stress increase at any point below the center of uniformly loaded circular area as following:

$$\frac{\Delta\sigma_z}{q} = f\left(\frac{Z}{R}\right)$$

R: Radius of the circular area.

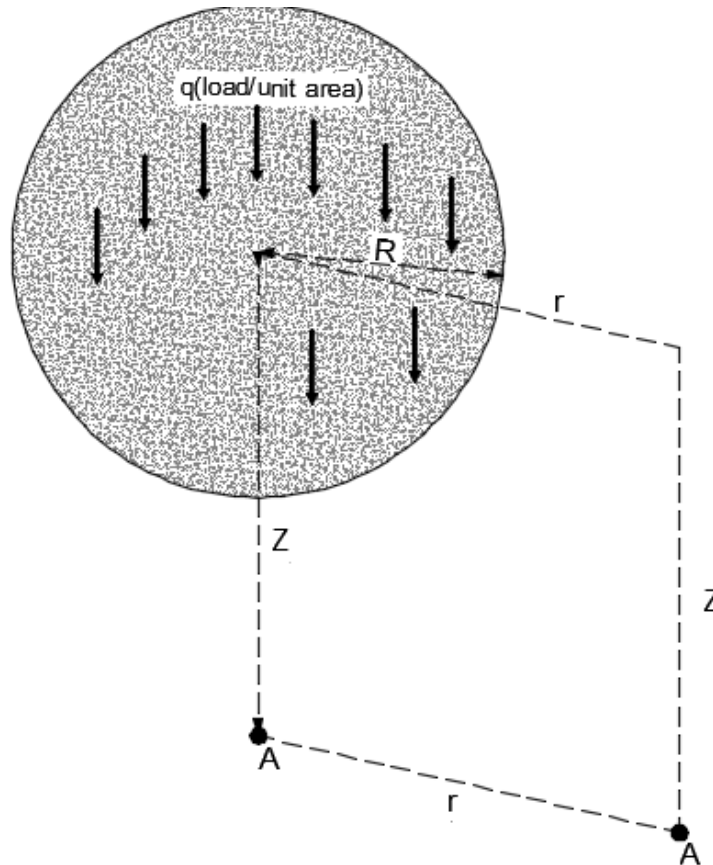
So, for any point, the value of  $\frac{\Delta\sigma_z}{q}$  can be taken from (**Table 10.6 P.331**) according to the value of  $\frac{Z}{R}$ . (Do interpolation “if required”)

And if  $\frac{\Delta\sigma_z}{q}$  is known  $\rightarrow$  the value of  $\Delta\sigma_z$  will be known (because q is known)



## Vertical Stress at Any Point below a Uniformly Loaded Circular Area:

Consider we want to calculate the vertical stress increase at point **A** in figure below:



We calculate the vertical stress increase at any point below a uniformly loaded circular area as following:

$$\Delta\sigma_z = q(A' + B')$$

$A'$  and  $B' = f\left(\frac{Z}{R}, \frac{r}{R}\right)$  ( $A' \rightarrow$  From **Table 10.7**), ( $B' \rightarrow$  From **Table 10.8**)

$R$ : Radius of the circular area.

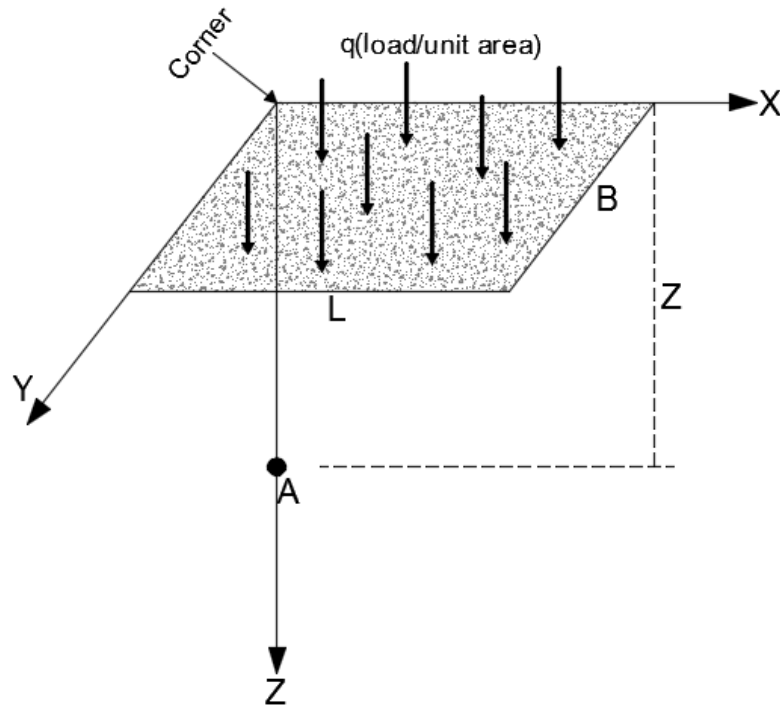
$r$ : Distance from the center of the circle to the point of consideration.

Note that, you may need to do interpolation (3 times) to get the value of  $A'$  or the value of  $B'$  (as we explain above “in strip load in table 10.4”).



## Vertical Stress Caused by a Rectangularly Loaded Area:

Consider we want to calculate the vertical stress increase at point A in figure below:



We calculate the vertical stress increase at point below the **corner** of rectangular loaded area as following:

$$\Delta\sigma_z = qI_3$$

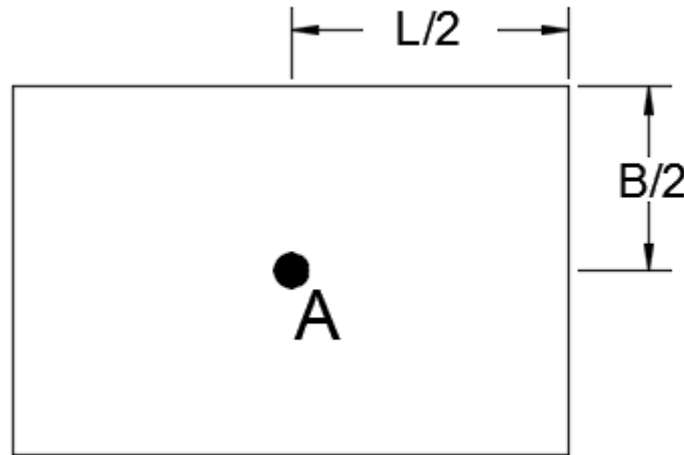
$$I_3 = f(m, n) \text{ (From Table 10.9 P.337 or Figure 10.26 P.338)}$$

$$m = \frac{B}{Z}, \quad n = \frac{L}{Z}$$

B: Smaller dimension , L: Larger dimension

**ملاحظة هامة:** يجب أن تكون النقطة المراد إيجاد  $\Delta\sigma_z$  عندها أسفل ركن (corner) لمنطقة مستطيلة, بالتالي قد يكون مطلوب إيجاد  $\Delta\sigma_z$  لعدد من النقاط لا توجد أسفل ركن منطقة مستطيلة, وهنا لا بد من تقسيم المسألة إلى أجزاء بحيث يمكن إضافة مستطيل أو مربع وطرحه مرة أخرى وستعرف على ذلك من خلال الأسئلة.

If we want to calculate  $\Delta\sigma_z$  below the center of rectangular area there are two methods:



1. Divide this area into 4 areas to make point "A" under the corner of each area:

We note that, point "A" is under the corner of each rectangular area, so:

$$\Delta\sigma_{z,t} = q(I_{(3)1} + I_{(3)2} + I_{(3)3} + I_{(3)4})$$

Because the total area is rectangular and divided into 4 areas it is clear that the four areas are equal so:

$$I_{(3)1} = I_{(3)2} = I_{(3)3} = I_{(3)4} = I_{(3)}$$

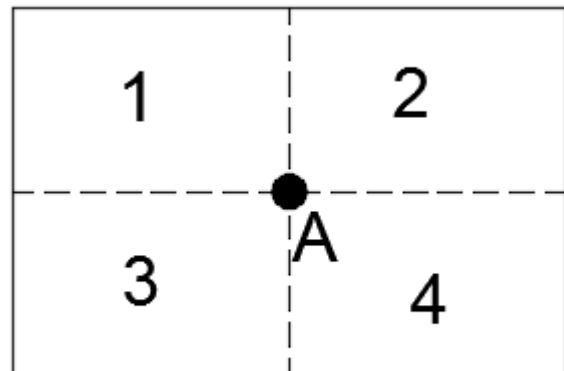
$$\Delta\sigma_{z,t} = q(4I_{(3)})$$

2.

$$\Delta\sigma_{z,t} = qI_4$$

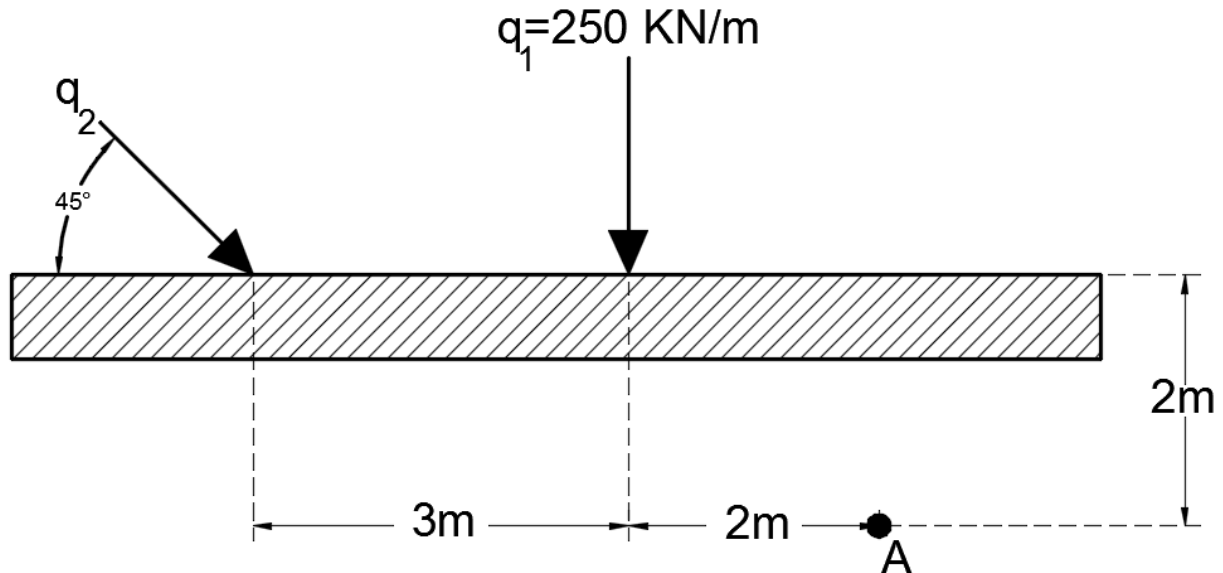
$I_4 = f(m_1, n_1)$  (From Table 10.10 P.341 or Figure 10.29 P.340)

$$m_1 = \frac{L}{B}, \quad n_1 = \frac{Z}{b} = \frac{2Z}{B}$$



42.

For the shown figure below, if the value of  $\Delta\sigma_z$  at point "A" is  $30 \text{ KN/m}^2$ . Determine the value of  $q_2$ .

**Solution**

It is clear that the given loads are **line** loads.

$$\Delta\sigma_{z,t} = \Delta\sigma_{z,1} + \Delta\sigma_{z,2} = 30 \text{ KN/m}^2$$

**Calculating the value of  $\Delta\sigma_{z,1}$** 

It is clear that the value of  $q_1$  is vertical line load.

$$q = 250 \text{ KN/m} \quad , \quad X = 2 \text{ m} \quad , \quad Z = 2 \text{ m} \rightarrow \frac{X}{Z} = \frac{2}{2} = 1$$

So, we can get the value of  $\frac{\Delta\sigma_z}{(q/z)}$  from "Table 10.2" according the value of  $\frac{X}{Z} = 1$

$$\frac{\Delta\sigma_{z,1}}{(q/z)} = 0.159 \rightarrow \Delta\sigma_{z,1} = 0.159 \times \frac{q}{z} = 0.159 \times \frac{250}{2} = 19.875 \text{ KN/m}^2$$

**Calculating the value of  $\Delta\sigma_{z,2}$** 

It is noted that, the line load  $q_2$  can be divided into two components (Horiz.&Vert.)

$$\Delta\sigma_{z,2} = \Delta\sigma_{z,2V} + \Delta\sigma_{z,2H}$$

Calculating the value of  $\Delta\sigma_z$  due to the vertical component of  $q_2$  ( $\Delta\sigma_{z,2V}$ ):

$$q = q_2 \times \sin(45) = 0.707q_2 \quad , \quad X = 5 \text{ m} \quad , \quad Z = 2 \text{ m} \rightarrow \frac{X}{Z} = \frac{5}{2} = 2.5$$

So, we can get the value of  $\frac{\Delta\sigma_z}{(q/z)}$  from “**Table 10.2**” according the value of  $\frac{X}{Z} = 2.5$

$$\frac{\Delta\sigma_{z,2V}}{(q/z)} = \frac{0.011 + 0.014}{2} = 0.0125 \text{ (average value between 2.4 and 2.6)}$$

$$\Delta\sigma_{z,2V} = 0.0125 \times \frac{q}{Z} = 0.0125 \times \frac{0.707q_2}{2} = 4.419 \times 10^{-3}q_2$$

Calculating the value of  $\Delta\sigma_z$  due to the horizontal component of  $q_2$  ( $\Delta\sigma_{z,2H}$ ):

$$q = q_2 \times \cos(45) = 0.707q_2, \quad X = 5\text{m}, \quad Z = 2\text{m} \rightarrow \frac{X}{Z} = \frac{5}{2} = 2.5$$

So, we can get the value of  $\frac{\Delta\sigma_z}{(q/z)}$  from “**Table 10.3**” according the value of  $\frac{X}{Z} = 2.5$

$$\frac{\Delta\sigma_{z,2H}}{(q/z)} = \frac{0.051 + 0.019}{2} = 0.035 \text{ (average value between 2 and 3)}$$

$$\Delta\sigma_{z,2H} = 0.035 \times \frac{q}{Z} = 0.035 \times \frac{0.707q_2}{2} = 0.01237q_2$$

So, the total value of  $\Delta\sigma_{z,2}$  will be:

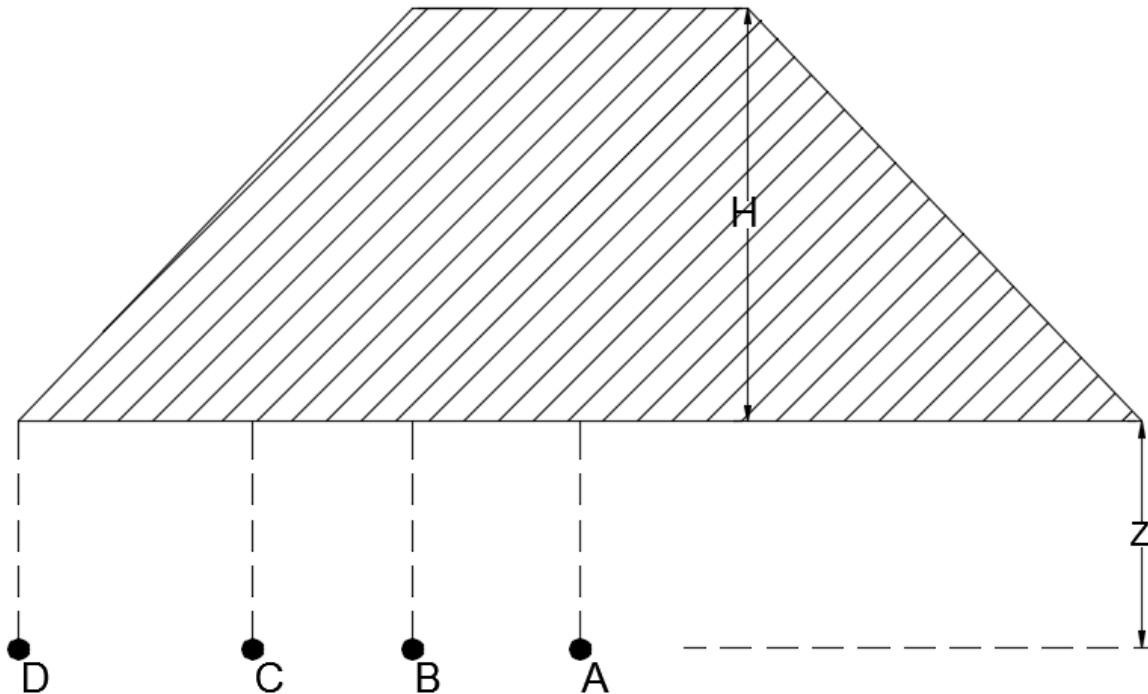
$$\Delta\sigma_{z,2} = 4.419 \times 10^{-3}q_2 + 0.01237q_2 = 0.01679q_2$$

$$\Delta\sigma_{z,t} = \Delta\sigma_{z,1} + \Delta\sigma_{z,2} = 30 \text{ KN/m}^2$$

$$30 = 19.875 + 0.01679q_2 \rightarrow q_2 = 603 \text{ KN/m} \checkmark.$$

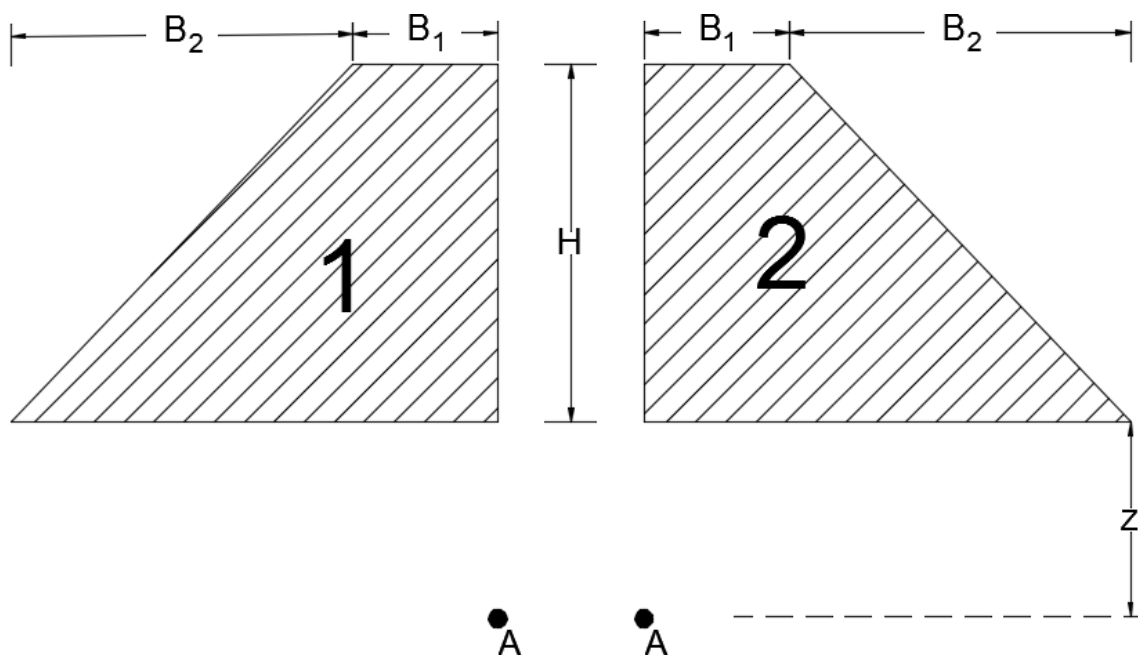
43.

For the shown figure below, assume all required data are given. Calculate the increase in vertical stresses ( $\Delta\sigma_z$ ) at points A, B, C, and D.



Solution

Point "A":



As shown in figure above, to calculate  $\Delta\sigma_z$  at point A we must divide the given area into two areas (trapezoidal) to meet the conditions (point must be under trapezoidal or under triangle).

For area “1” (Trapezoidal  $\rightarrow B_1 = \checkmark, B_2 = \checkmark$ )

$$q_{o(1)} = \gamma \times H, \quad \frac{B_1}{Z} = \checkmark, \quad \frac{B_2}{Z} = \checkmark \rightarrow I_{2(1)} = \checkmark \text{ (From Figure 10.20)}$$

$$\Delta\sigma_{z(1)} = q_{o(1)} \times I_{2(1)} = \checkmark.$$

For area “2” (Trapezoidal  $\rightarrow B_1 = \checkmark, B_2 = \checkmark$ )

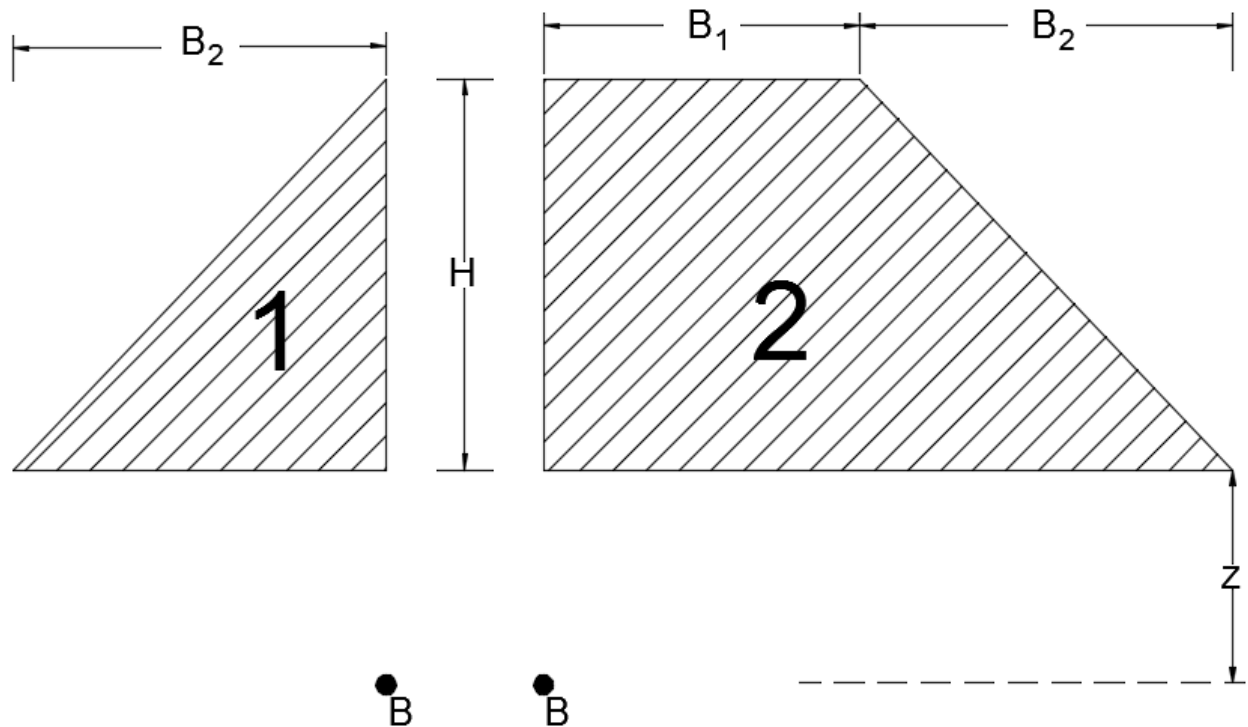
$$q_{o(2)} = \gamma \times H, \quad \frac{B_1}{Z} = \checkmark, \quad \frac{B_2}{Z} = \checkmark \rightarrow I_{2(2)} = \checkmark \text{ (From Figure 10.20)}$$

$$\Delta\sigma_{z(2)} = q_{o(2)} \times I_{2(2)} = \checkmark.$$

So,  $\Delta\sigma_{z(A)} = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} = \checkmark.$

**Note:** If the embankment is symmetry, and point A under the middle of the embankment, you can calculate  $\Delta\sigma_z$  for one area and then multiply the value by 2.

**Point “B”:**



As shown in figure above, to calculate  $\Delta\sigma_z$  at point B we must divide the given area into two areas ( triangle"1" and trapezoidal"2") to meet the conditions (point must be under trapezoidal or under triangle).

For area "1" (Triangle  $\rightarrow B_1 = 0.0, B_2 = \checkmark$ )

$$q_{o(1)} = \gamma \times H \quad , \quad \frac{B_1}{Z} = 0.0 \quad , \quad \frac{B_2}{Z} = \checkmark \rightarrow I_{2(1)} = \checkmark \quad (\text{From Figure 10.20})$$

$$\Delta\sigma_{z(1)} = q_{o(1)} \times I_{2(1)} = \checkmark .$$

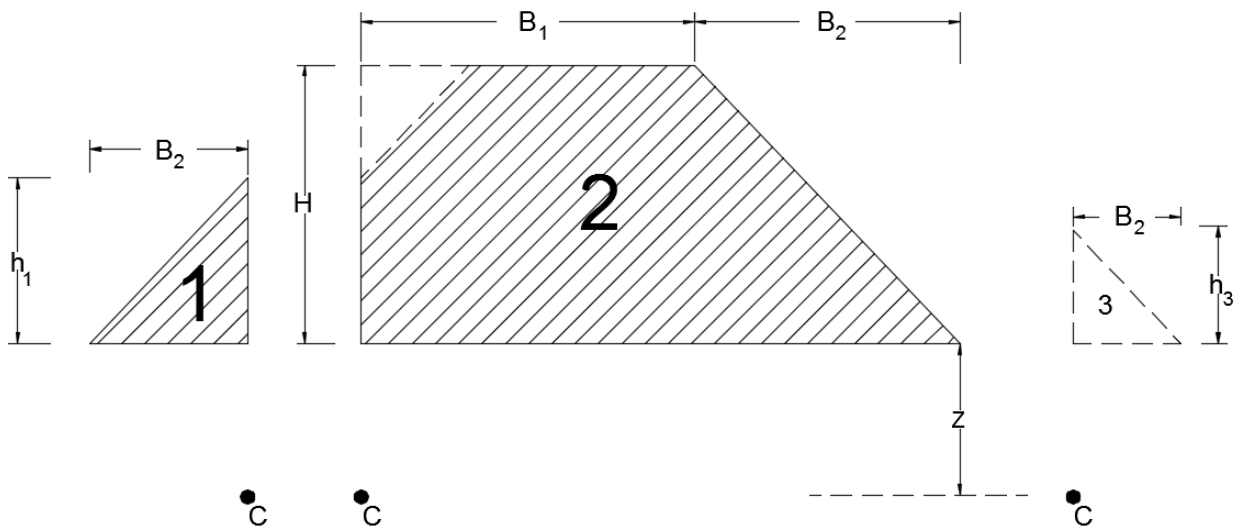
For area "2" (Trapezoidal  $\rightarrow B_1 = \checkmark, B_2 = \checkmark$ )

$$q_{o(2)} = \gamma \times H \quad , \quad \frac{B_1}{Z} = \checkmark \quad , \quad \frac{B_2}{Z} = \checkmark \rightarrow I_{2(2)} = \checkmark \quad (\text{From Figure 10.20})$$

$$\Delta\sigma_{z(2)} = q_{o(2)} \times I_{2(2)} = \checkmark .$$

$$\text{So, } \Delta\sigma_{z(B)} = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} = \checkmark .$$

Point "C":



As shown in figure above, to calculate  $\Delta\sigma_z$  at point C we must divide the given area into two areas ( triangle"1" and trapezoidal"2") to meet the conditions (point must be under trapezoidal or under triangle).

For area "1" (Triangle  $\rightarrow B_1 = 0.0, B_2 = \checkmark$ ) ( $h_1$  is given)

$$q_{o(1)} = \gamma \times h_1 \quad , \quad \frac{B_1}{Z} = 0.0 \quad , \quad \frac{B_2}{Z} = \checkmark \rightarrow I_{2(1)} = \checkmark \quad (\text{From Figure 10.20})$$

$$\Delta\sigma_{z(1)} = q_{o(1)} \times I_{2(1)} = \checkmark .$$

For area “2” (Trapezoidal  $\rightarrow B_1 = \checkmark, B_2 = \checkmark$ )

We note that, area “2” it is not trapezoidal, so, to make it trapezoidal we must **add** the triangle (dashed lines) to make area “2” trapezoidal.

$$q_{o(2)} = \gamma \times H, \quad \frac{B_1}{Z} = \checkmark, \quad \frac{B_2}{Z} = \checkmark \rightarrow I_{2(2)} = \checkmark \quad (\text{From Figure 10.20})$$

$$\Delta\sigma_{z(2)} = q_{o(2)} \times I_{2(2)} = \checkmark.$$

For area “3” (Triangle  $\rightarrow B_1 = 0.0, B_2 = \checkmark$ ) ( $h_3$  is given)

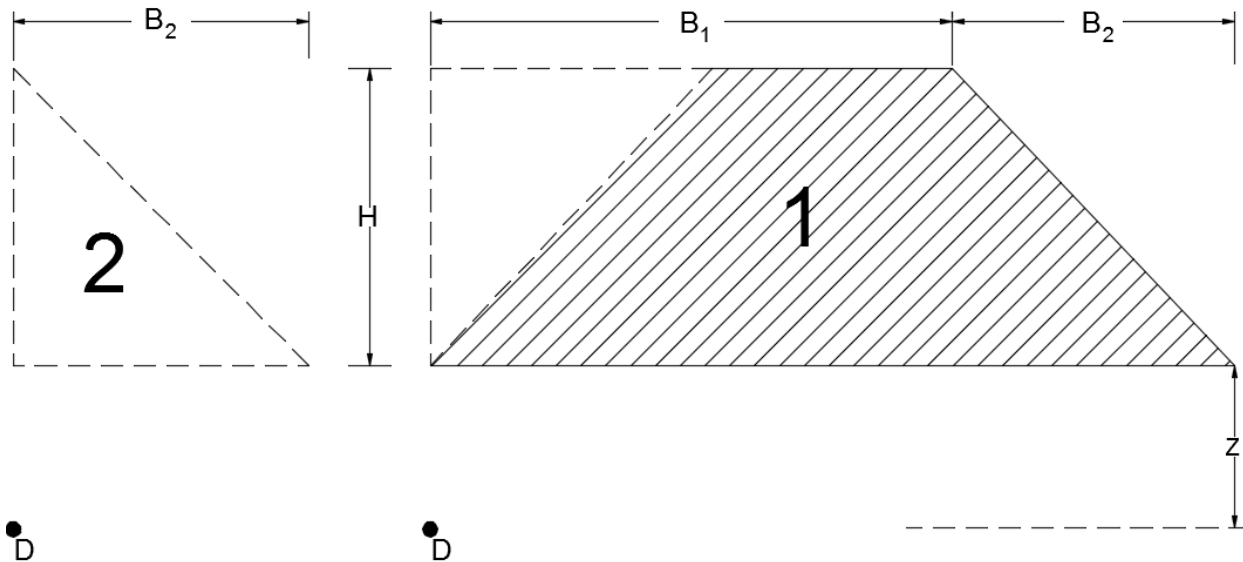
The triangle that added to area “2” to be a trapezoidal area must be **subtract**, because it is not from the total embankment area.

$$q_{o(3)} = \gamma \times h_3, \quad \frac{B_1}{Z} = 0.0, \quad \frac{B_2}{Z} = \checkmark \rightarrow I_{2(3)} = \checkmark \quad (\text{From Figure 10.20})$$

$$\Delta\sigma_{z(3)} = q_{o(3)} \times I_{2(3)} = \checkmark.$$

$$\text{So, } \Delta\sigma_{z(C)} = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} - \Delta\sigma_{z(3)} = \checkmark.$$

**Point “D”:**



For area “1” (Trapezoidal  $\rightarrow B_1 = \checkmark, B_2 = \checkmark$ )

We note that, area “1” it is not trapezoidal, so, to make it trapezoidal we must **add** the triangle (dashed lines) to make area “1” trapezoidal.

$$q_{o(1)} = \gamma \times H, \quad \frac{B_1}{Z} = \checkmark, \quad \frac{B_2}{Z} = \checkmark \rightarrow I_{2(1)} = \checkmark \quad (\text{From Figure 10.20})$$

$$\Delta\sigma_{z(1)} = q_{o(1)} \times I_{2(1)} = \checkmark.$$



For area "2" (Triangle  $\rightarrow B_1 = 0.0, B_2 = \checkmark$ )

The triangle that added to area "1" to be a trapezoidal area must be **subtract**, because it is not from the total embankment area.

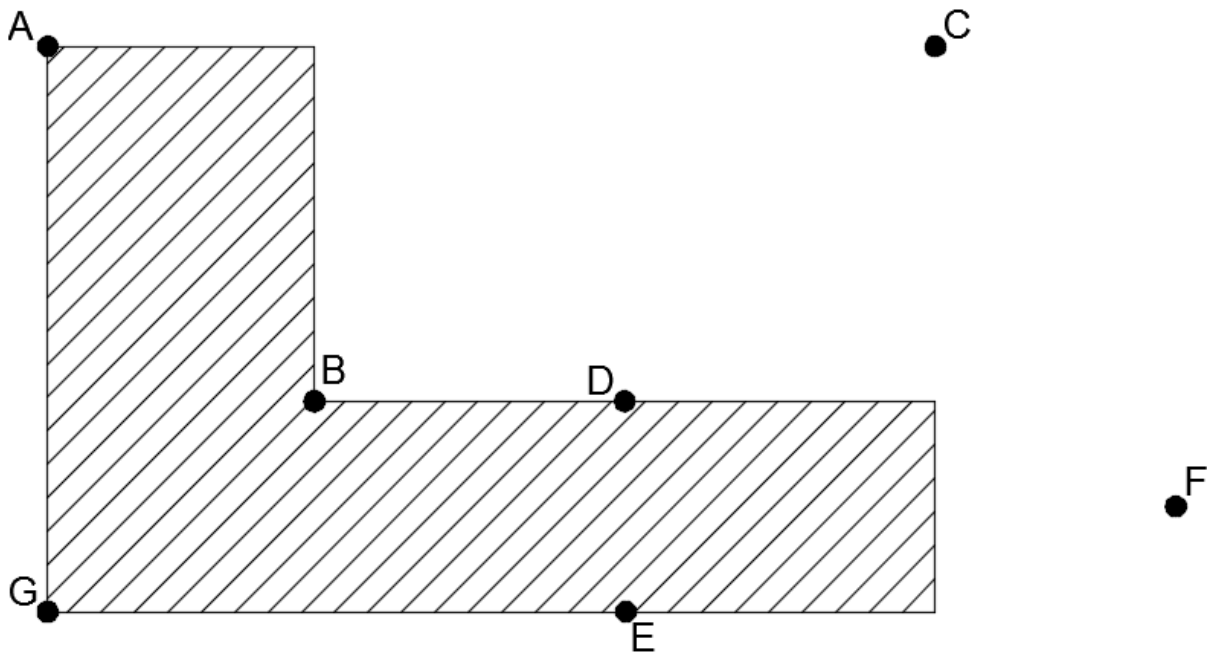
$$q_{o(2)} = \gamma \times H, \quad \frac{B_1}{Z} = 0.0, \quad \frac{B_2}{Z} = \checkmark \rightarrow I_{2(2)} = \checkmark \text{ (From Figure 10.20)}$$

$$\Delta\sigma_{z(2)} = q_{o(2)} \times I_{2(2)} = \checkmark.$$

$$\text{So, } \Delta\sigma_{z(D)} = \Delta\sigma_{z(1)} - \Delta\sigma_{z(2)} = \checkmark.$$

#### 44.

For the shown figure below, assume all required data are given. Calculate the increase in vertical stresses ( $\Delta\sigma_z$ ) at points A, B, C, D, E, F, and G.



#### Solution

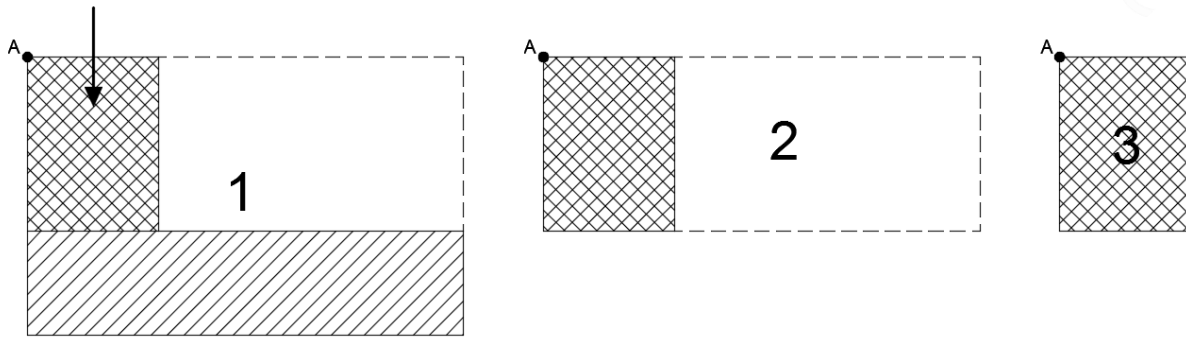
For each area  $q = \checkmark$ ,  $m = \frac{B}{Z} = \checkmark$ ,  $n = \frac{L}{Z} = \checkmark \rightarrow I_3 = \checkmark$

The value of  $I_3$  can be taken from (Table 10.9 or Figure 10.26)

Always the main condition that must be satisfied that the point must be at the corner of the rectangle, thus, sometimes we need to add an area to meet conditions and then subtract it, and there are many techniques will be known after this problem

**Point “A”:**

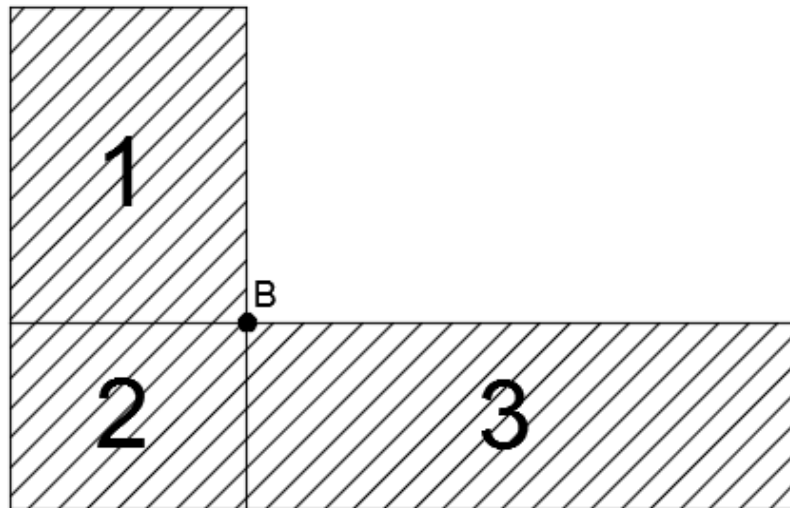
Overlapping Area

**Overlapping Area:**

هي منطقة تداخل حيث تشترك في أكثر من عملية (طرح أو جمع) حيث تكون جزء من الشكل المطلوب، فأحيانا يتم جمعها ثم نضطر لطرحها لطرح مساحة أخرى معينة وبالتالي في هذه الحالة يجب جمعها مرة أخرى كما في النقطة (A) وأحيانا يتم جمعها مرتين لأننا نريد تحقيق الشرط أن النقطة يجب أن تكون في ركن المستطيل أو المربع وهنا يجب طرحها لأنه تم جمعها لمرتين.

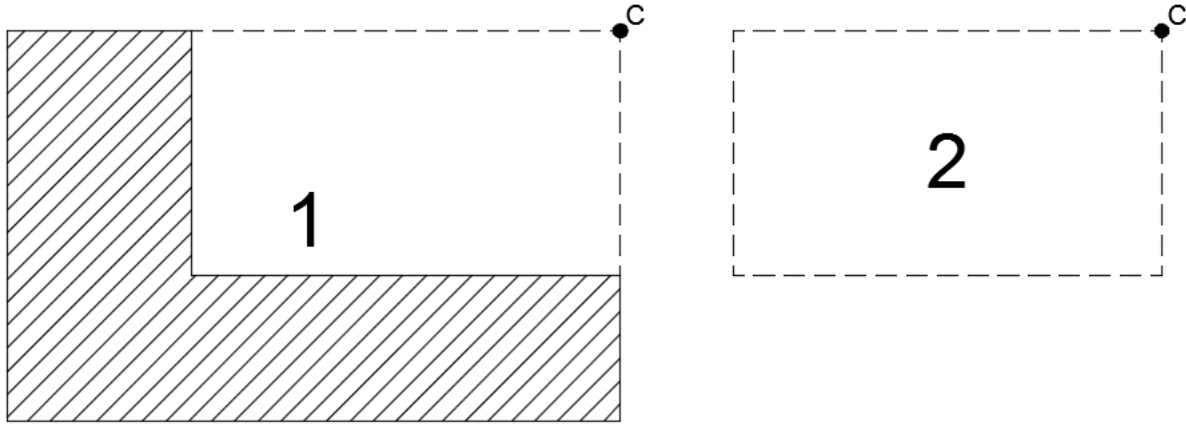
$$\Delta\sigma_{z(A)} = q \times (I_{3(1)} - I_{3(2)} + I_{3(3)})$$

نلاحظ أن المساحة رقم 3 تم إضافتها لأنه تم حذفها ضمن المساحة رقم 2 لتحقيق الشرط.

**Point “B”:**

$$\Delta\sigma_{z(B)} = q \times (I_{3(1)} + I_{3(2)} + I_{3(3)})$$

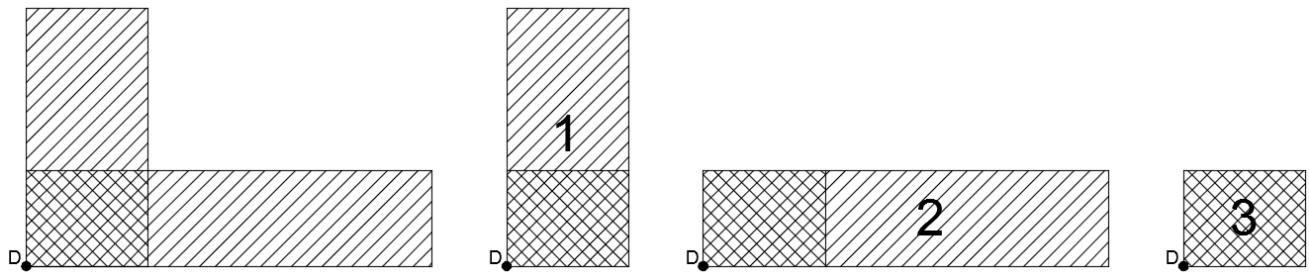
Point “C”:



تم إضافة المساحة 2 للمساحة 1 لتحقيق الشرط وبالتالي يجب حذفها لأنها ليست من ضمن الشكل المطلوب.

$$\Delta\sigma_{z(C)} = q \times (I_{3(1)} - I_{3(2)})$$

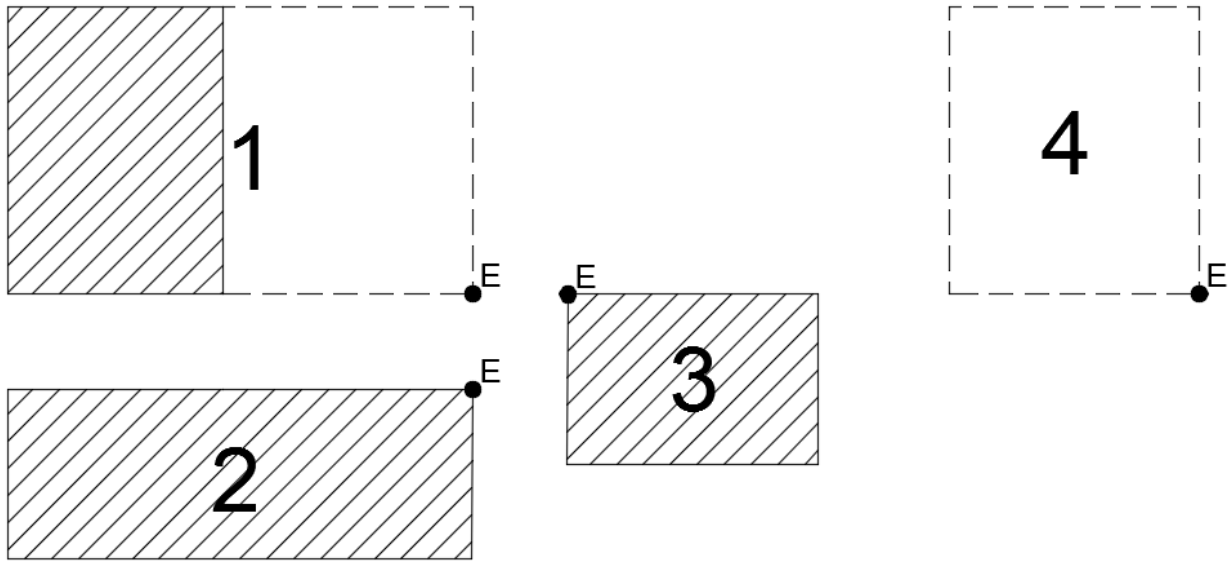
Point “D”:



نلاحظ هنا أن المساحة 3 مشتركة ضمن المساحتين 1 و2 وبالتالي تم اضافتها لمرتين لذلك يجب طرحها حتى تكون مضافة لمرة واحدة.

$$\Delta\sigma_{z(D)} = q \times (I_{3(1)} + I_{3(2)} - I_{3(3)})$$

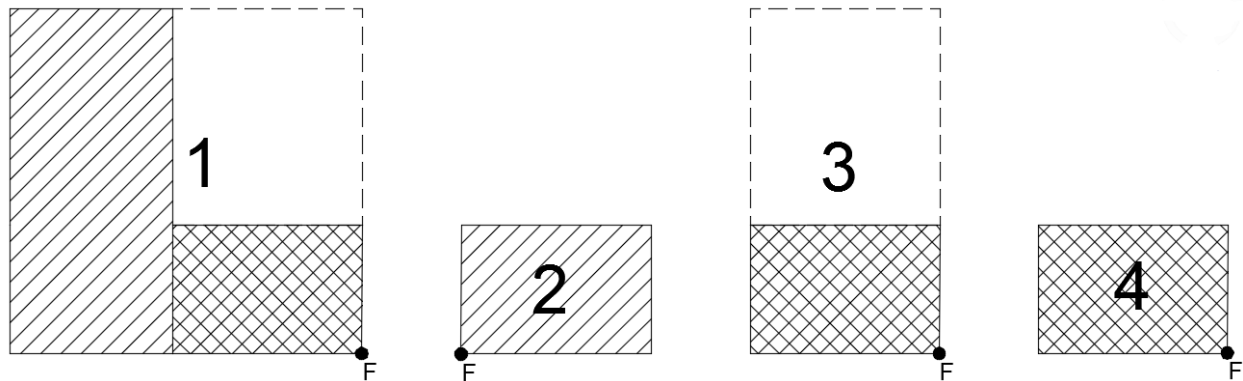
Point “E”:



تم إضافة المساحة 4 للمساحة 1 لتحقيق الشرط وبالتالي يجب حذفها لأنها ليست من ضمن الشكل المطلوب.

$$\Delta\sigma_{z(E)} = q \times (I_{3(1)} + I_{3(2)} + I_{3(3)} - I_{3(4)})$$

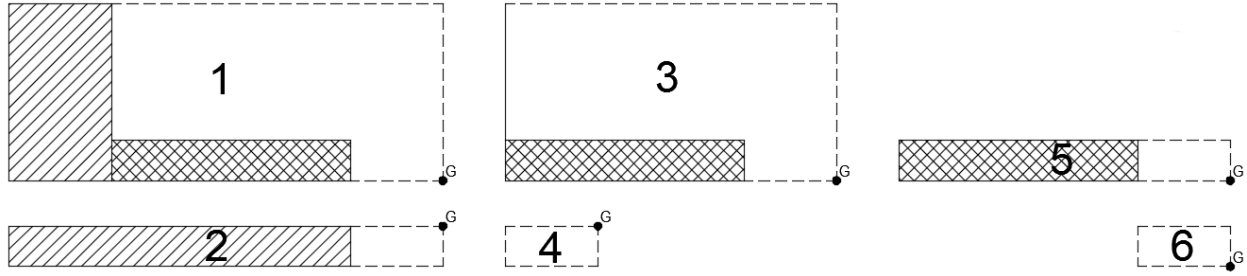
Point “F”:



نلاحظ هنا أنه تم إضافة المستطيل (بالخطوط المنقطة) للمساحة 1 لتحقيق الشرط وبالتالي يجب حذفه , لكن عند حذفه يجب إضافة المساحة المتداخلة (4) إليه لتحقيق الشرط وبالتالي فإن المساحة المتداخلة سوف يتم حذفها لأنها ضمن المساحة رقم 3 لذلك يجب إضافتها مرة أخرى (مساحة 4) لأنه تم حذفها في مساحة (3) .

$$\Delta\sigma_{z(F)} = q \times (I_{3(1)} + I_{3(2)} - I_{3(3)} + I_{3(4)})$$

Point "G":



نلاحظ أنه تم إضافة مستطيل للمساحة رقم 1 لتحقيق الشرط, لكن ضمن هذا المستطيل توجد مساحة متداخلة وتم حذف المستطيل مع المساحة المتداخلة في المساحة رقم 3 وبالتالي فإن المساحة المتداخلة قد تم طرحها لذلك يجب إضافتها ولكن لإضافتها يجب إضافة مستطيل آخر إليها (6) لتحقيق الشرط ومن ثم طرحه مرة أخرى. بالنسبة للمساحة رقم 2 تم إضافة مستطيل (4) إليها لتحقيق الشرط وبالتالي يجب حذفه.

$$\Delta\sigma_{z(G)} = q \times (I_{3(1)} + I_{3(2)} - I_{3(3)} - I_{3(4)} + I_{3(5)} - I_{3(6)})$$

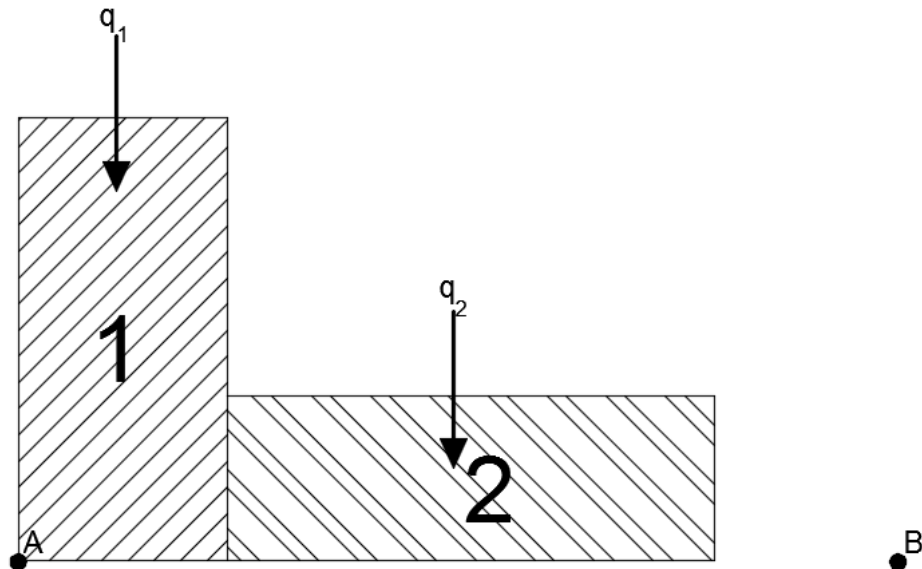
### Important Note:

في بعض الأحيان قيمة (q) لا تكون ثابتة لجميع المساحات بالتالي هنا لكل مساحة يجب أن يتم ضرب قيمة (q) بقيمة (I<sub>3</sub>) لهذه المساحة أي أنه يجب التجزئة كالآتي:

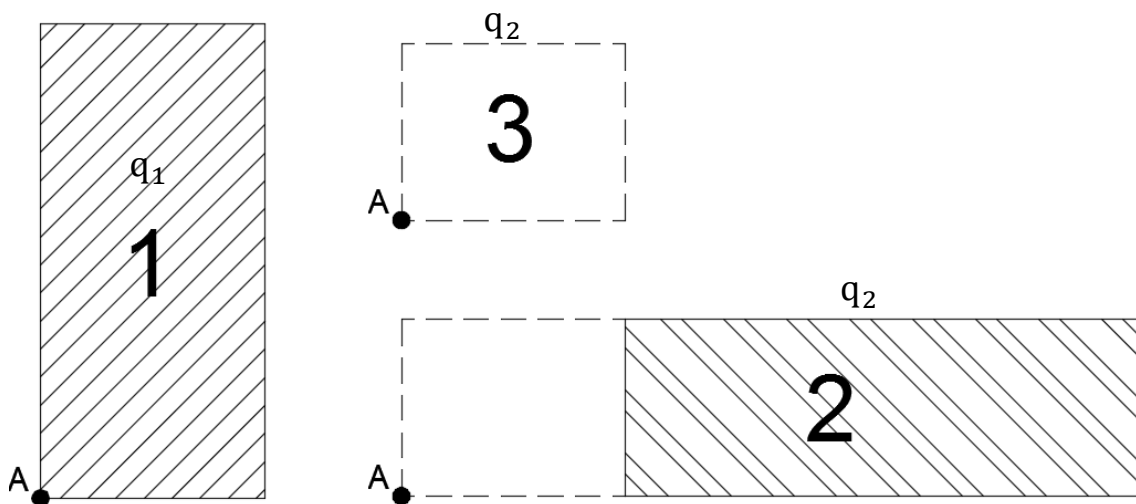
$$\Delta\sigma_{z(\text{any point})} = q_1 \times I_{3(1)} + q_2 \times I_{3(2)} \dots$$

45.

For the shown figure below, calculate the increase in vertical stresses at points A and B.

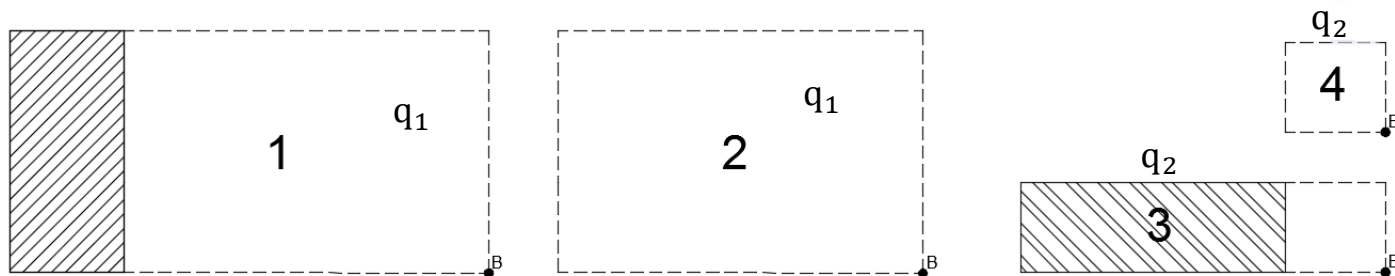


Point "A":



$$\Delta\sigma_{z(A)} = q_1 \times I_{3(1)} + q_2 \times I_{3(2)} - q_2 \times I_{3(3)}$$

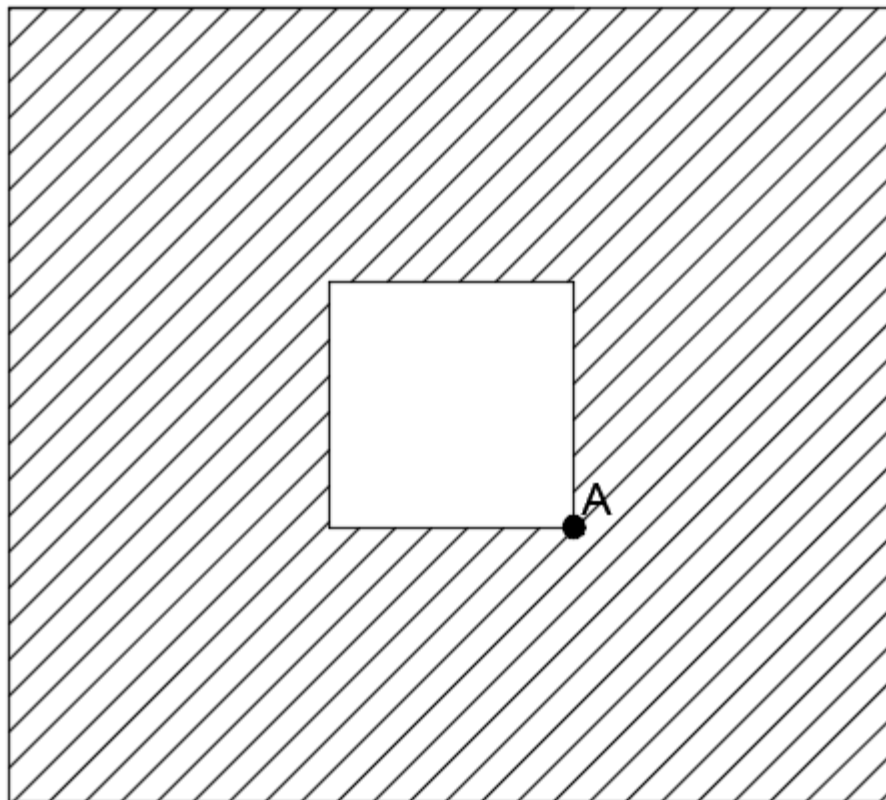
Point "B":



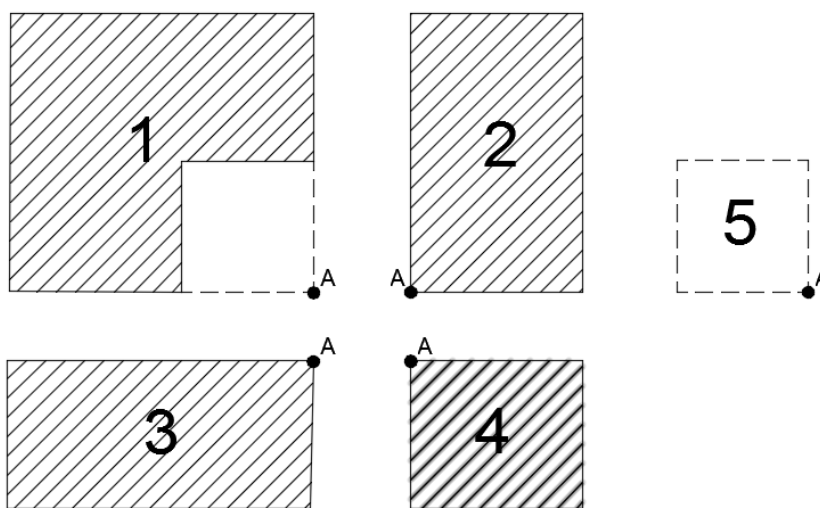
$$\Delta\sigma_{z(B)} = q_1 \times I_{3(1)} - q_1 \times I_{3(2)} + q_2 \times I_{3(3)} - q_2 \times I_{3(4)}$$

46.

For the shown figure below, calculate the increase in vertical stresses at point A.



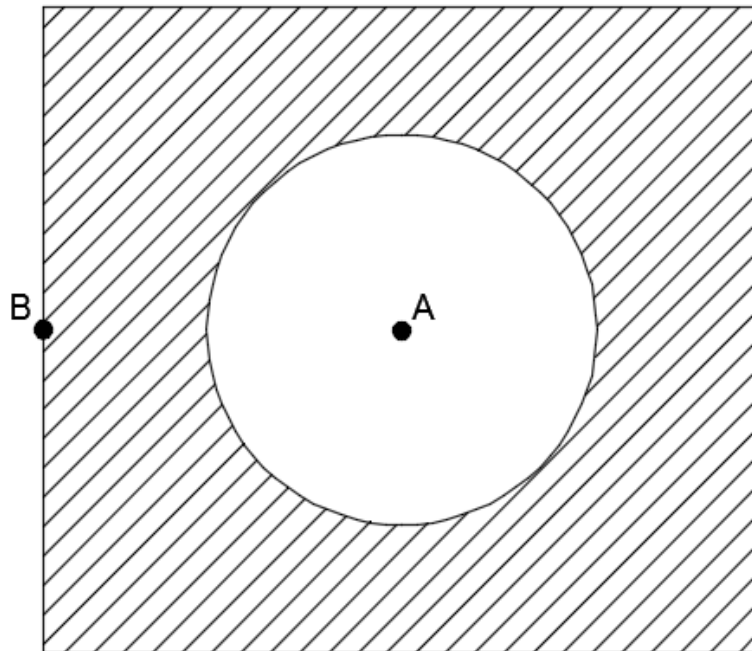
Point "A":



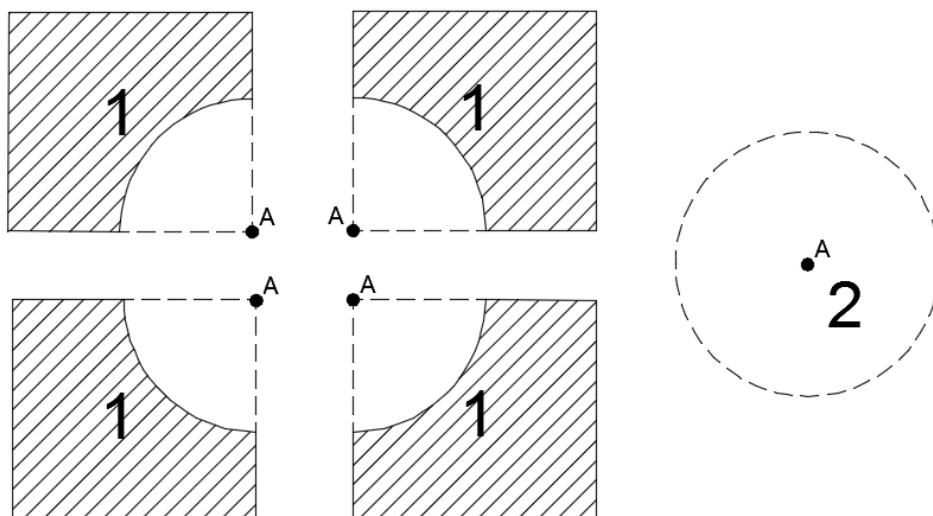
$$\Delta\sigma_{z(A)} = q \times (I_{3(1)} + I_{3(2)} + I_{3(3)} + I_{3(4)} - I_{3(5)})$$

47.

For the shown figure below, calculate the increase in vertical stresses at points A and B.



Point "A":



نلاحظ أن النقطة A تقع في منتصف المربع أو المستطيل وبالتالي فهي تقسم المساحة إلى أربع مساحات متساوية جميعها لها نفس الأبعاد وبالتالي جميعها لها نفس قيمة  $(I_3)$ . ونلاحظ أيضا أن الدائرة ليست ضمن المنطقة المطلوبة بالتالي يجب طرحها حيث أن النقطة (A) توجد أسفل مركز الدائرة.



For Area "1":

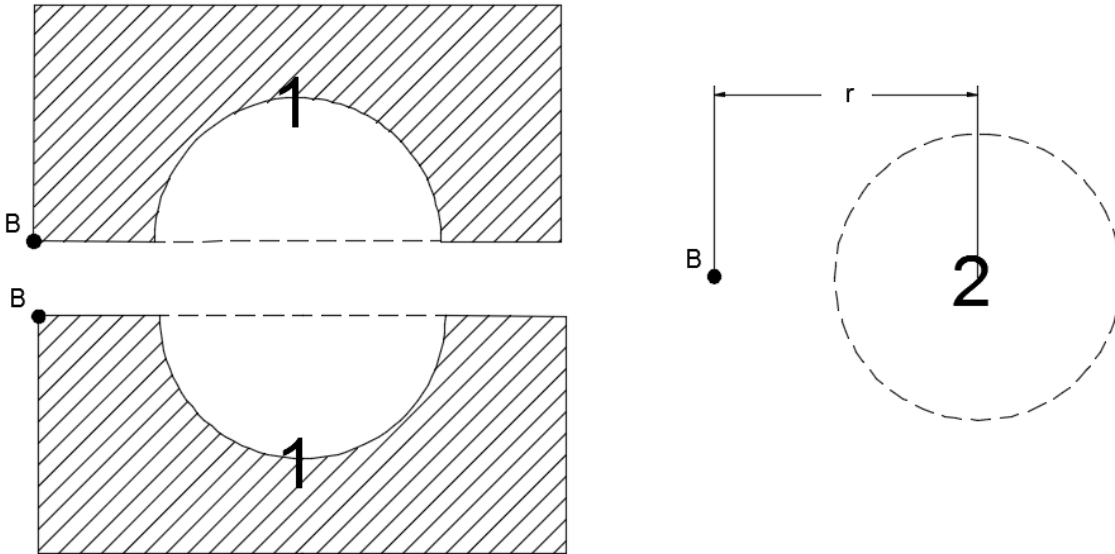
$$\Delta\sigma_{z(A,1)} = q \times 4 \times I_{3(1)} \quad (\text{تم الضرب في 4 لأنه يوجد 4 مساحات متماثلة})$$

For Area "2":

$$R = \checkmark, \quad Z = \checkmark \rightarrow \frac{Z}{R} = \checkmark \rightarrow \frac{\Delta\sigma_{z(A,2)}}{q} = \checkmark \quad (\text{Table 10.6}) \rightarrow \Delta\sigma_{z(A,2)} = \checkmark$$

$$\text{So, } \Delta\sigma_{z(A)} = \Delta\sigma_{z(A,1)} - \Delta\sigma_{z(A,2)}$$

Point "B":



For Area "1":

$$\Delta\sigma_{z(B,1)} = q \times 2 \times I_{3(1)} \quad (\text{تم الضرب في 2 لأنه يوجد مساحتين متماثلتين})$$

For Area "2":

$$r = \checkmark, \quad R = \checkmark, \quad Z = \checkmark \rightarrow \frac{Z}{R} = \checkmark \text{ and } \frac{r}{R} = \checkmark \rightarrow A' \text{ and } B' = \checkmark \quad (\text{Table 10.7 and 10.8})$$

$$\rightarrow \Delta\sigma_{z(B,2)} = q \times (A' + B')$$

$$\text{So, } \Delta\sigma_{z(B)} = \Delta\sigma_{z(B,1)} - \Delta\sigma_{z(B,2)}$$

## **Chapter (11)**

# **Compressibility of Soil**

## Introduction:

إن وجود أحمال إضافية على التربة مثل حمل قاعدة أو حمل مبنى أو حمل سد رملي وغيره من الأحمال يؤدي إلى انضغاط حبيبات التربة وبالتالي يؤدي إلى هبوط التربة (Settlement). هذا الهبوط له عدة أسباب وهي (1) تشوه حبيبات التربة نتيجة الأحمال, (2) إعادة تموضع حبيبات التربة, (3) خروج الماء والهواء من الفراغات الموجودة بين حبيبات التربة.

In general, there are three basic types of settlement:

### 1. Elastic Settlement (Immediate Settlement):

هو الهبوط الذي يحدث نتيجة خروج الهواء من الفراغات الموجودة في التربة حيث أنه يحدث أثناء وضع الأحمال على التربة (بناية المنشأة) وينتهي بانتهاء وضع الأحمال ولا يحدث خلاله أي تغير في نسبة المحتوى المائي للطبقات المختلفة من التربة (لأنه عبارة عن خروج الهواء فقط من بين حبيبات التربة) وهو يحدث في جميع أنواع التربة سواء رمل أو طين وسواء كانت تربة جافة أو مشبعة بالماء أو مشبعة جزئياً.

### 2. Primary Consolidation Settlement:

هو الهبوط الذي يحدث في التربة نتيجة خروج الماء من بين حبيبات التربة, وبالتالي ونتيجة لتطبيق الأحمال فإن الفراغات التي يخرج منها الماء سوف تتلاشى وبالتالي سوف تهبط التربة, لذلك هذا النوع من الهبوط يحدث في الطبقات المشبعة كلياً بالماء.

### 3. Secondary Consolidation Settlement:

بعد خروج كل الماء من طبقة التربة وانتهاء مرحلة ال(primary consolidation) يبدأ هذا النوع من الهبوط والناج عن إعادة ترتيب حبيبات التربة وبسبب التشوه البلاستيكي الذي يحدث للتربة مع الزمن وتأثير الأحمال.

So, the total settlement due to the applied load can be expressed as:

$$S_T = S_e + S_c + S_s$$

$$S_T = \text{Total Settlement}$$

$$S_e = \text{Elastic Settlement (Immediate Settlement)}$$

$$S_c = \text{Primary Consolidation Settlement}$$

$$S_s = \text{Secondary Consolidation Settlement}$$

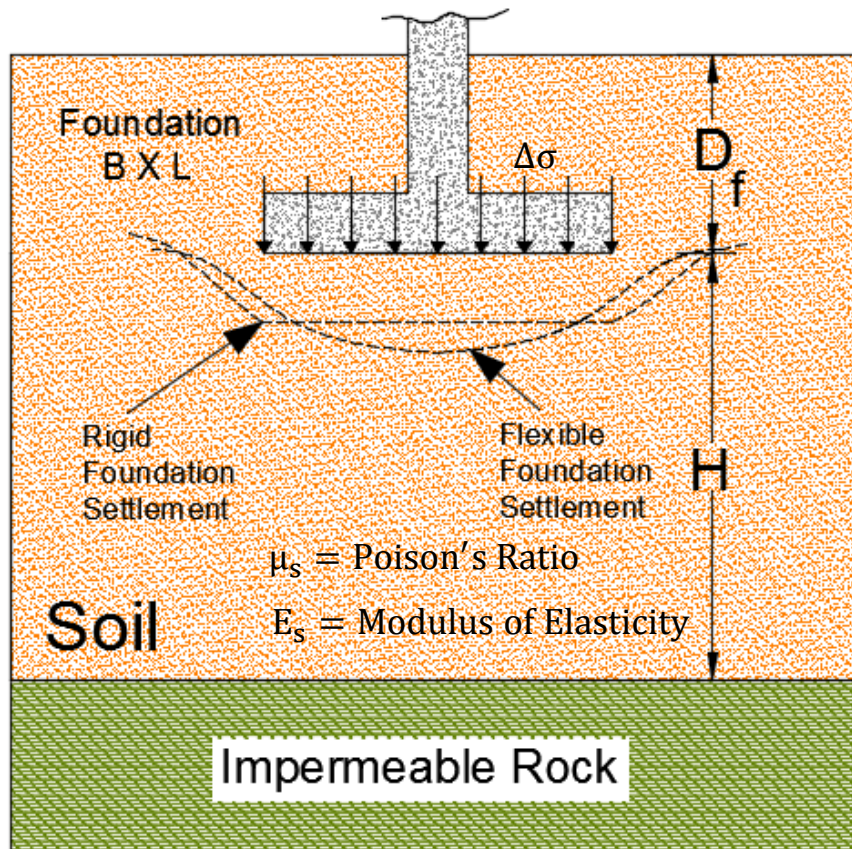
In this chapter, the three types of settlement will be discussed completely.

## Elastic Settlement:

As mentioned above, elastic or immediate settlement ( $S_e$ ) occurs directly after the application of the load without change in the moisture content of the soil. The magnitude of the elastic settlement will depend on the **flexibility** of the foundation (flexible or rigid), and on the type of material (soil) that the foundation will rest on it.

Elastic Settlement under a **flexible** foundation can be calculated from the following equation:

$$S_e = \Delta\sigma(\alpha B') \frac{1 - \mu_s^2}{E_s} I_s I_f$$



$L$  = Larger dimension of the foundation

$B$  = Smaller dimension of the foundation

$\Delta\sigma$  = Applied Net Pressure = Load/Area

$E_s$  = Modulus of Elasticity of the soil through the depth H.

If there are more than one soil layer through the depth H we can find the modulus of elasticity for all layers by weighted average method:

$$E_s = \frac{\sum E_{s(i)} \times \Delta Z}{Z}$$

$E_{s(i)}$  = Modulus of elasticity of each layer.

$\Delta Z$  = Depth of each layer.

$Z$  = H or 5B whichever is **smaller**

H = Distance from the face of the footing to the nearest rock layer (as shown in figure above)

Now, as shown in figure above, the elastic settlement under a flexible foundation at the center is larger than at the corner, thus, there are some differences in calculating ( $S_e$ ) under the **center** and under the **corner** of the footing.

This difference can be considered in the values of ( $B'$  and  $\alpha$ ).

**For calculating  $S_e$  under the center of the foundation:**

$$B' = \frac{B}{2} \quad \text{and} \quad \alpha = 4$$

**For calculating  $S_e$  under the corner of the foundation:**

$$B' = B \quad \text{and} \quad \alpha = 1$$

Where,  $\alpha$  = factor depends on the location on the foundation where settlement is being calculated.

$I_s$  = shape factor.

$$I_s = F_1 + \frac{1-2\mu_s}{1-\mu_s} F_2$$

$F_1$  (can be calculated from **Table 11.1 P. 358**)

$F_2$  (can be calculated from **Table 11.2 P. 360**)

To get the values of  $F_1$  and ( $F_2$ ) from tables we need the values of  $m'$  and  $n'$

$$m' = \frac{L}{B} \quad , \quad n' = \frac{H}{B'}$$

$I_f$  = factor depends on depth of excavation, footing dimensions and soil type

The value of ( $I_f$ ) can be calculated from (**Table 11.3 P.362**).

**Note:** If  $D_f = 0.0 \rightarrow I_f = 1$

Elastic Settlement under a **rigid** foundation:

From the figure above (page 138) it is noted that the elastic settlement under a rigid foundation is **constant** and **less than**  $S_e$  for flexible foundation (at center).

So, the value of  $S_e$  under a rigid foundation can be estimated as following:

$$S_{e(\text{rigid})} = 0.93S_{e(\text{flexible,center})}$$

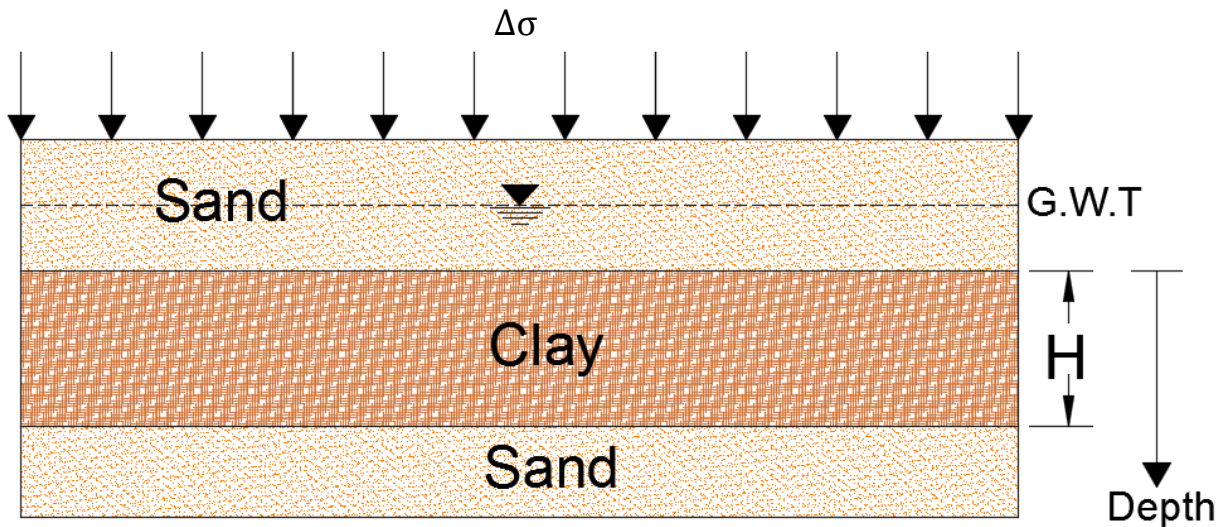
See (**example 11.1 P.362**) in your textbook.

## Consolidation Settlement

### Fundamentals of Consolidation:

هذا النوع من الهبوط يحدث فقط في طبقات التربة المشبعة كلياً بالماء لأنه في الأصل عبارة عن عملية خروج الماء من بين حبيبات التربة نتيجة التعرض للأحمال الخارجية. حيث أنه إذا تعرضت طبقة من التربة المشبعة بالماء إلى أحمال إضافية فإن ضغط الماء في هذه الطبقة سوف يزداد بشكل مفاجئ. فإذا كانت هذه الطبقة من الرمل (saturated sandy soil) حيث أن نفاذيتها للماء كبيرة وبالتالي فإن خروج الماء منها أثناء تطبيق الأحمال يكون بشكل سريع وينتهي بانتهااء إنشاء المبنى ويتم الهبوط في هذه التربة الرملية نتيجة خروج الماء (consolidation settlement). وبالتالي يمكن القول أنه في التربة الرملية فإن خروج الماء من هذه التربة (consolidation settlement) وخروج الهواء منها (elastic settlement) يحدثان بشكل متزامن وينتهيان بمجرد انتهاء وضع الأحمال. أما في حالة التربة الطينية (saturated clay) فإن خروج الهواء من هذه التربة نتيجة وجود الأحمال الإضافية عليها (elastic settlement) يكون أيضاً بشكل لحظي وسريع. لكن معامل النفاذية للتربة الطينية صغير جداً مقارنة بالتربة الرملية، وبالتالي فإن خروج الماء منها نتيجة الأحمال الإضافية (consolidation settlement) يكون بطيء وبشكل تدريجي وبالتالي فهو يأخذ فترة زمنية كبيرة أكبر بكثير من فترة خروج الهواء (elastic settlement). وبالتالي فإن الهبوط الذي يحدث نتيجة خروج الماء من التربة الطينية (consolidation settlement) أكبر بكثير من الهبوط الذي يحدث نتيجة خروج الهواء منها (elastic settlement). مما سبق يتضح أننا نهتم بحساب (consolidation settlement) للتربة الطينية المشبعة بالماء (saturated clay soil).

### Consolidation Process:





نلاحظ من الشكل في الأعلى أن طبقة الـ (clay) محصورة بين طبقتين من الـ (permeable sand) وهذا يعني أنه نتيجة تطبيق الحمل ( $\Delta\sigma$ ) فإن المياه سوف تخرج من الطبقة عن طريق حركة المياه من منتصف الطبقة إلى أعلى وأيضا حركة المياه من منتصف الطبقة إلى أسفل. عند وضع الحمل الإضافي ( $\Delta\sigma$ ) فإنه سوف ينتقل إلى الماء الموجود داخل حبيبات التربة وإلى حبيبات التربة نفسها وهذا يعني أن قيمة ( $\Delta\sigma$ ) سوف تنقسم إلى قسمين (Effective Stress and Pore Water Pressure). وبالتالي:

$$\Delta\sigma = \Delta\sigma' + \Delta u$$

$\Delta\sigma'$  = increase in effective stress due to application of  $\Delta\sigma$

$\Delta u$  = increase in pore water pressure due to application of  $\Delta\sigma$

or  $\Delta u$  = excess in pore water pressure

So, if there exist an additional load ( $\Delta\sigma$ ), the total effective stress and the total pore water pressure at any depth in the clay layer are:

$$\sigma' = \sigma'_o + \Delta\sigma'$$

$$u = u_o + \Delta u$$

$\sigma'_o$  = present effective stress at any point due to the weight of soils above this point (exactly as CH. 9)(also called present effective overburden pressure)

$u_o$  = pore water pressure due to the height of G.W.T above a certain point (as ch.9)

The values of ( $\Delta u$ ) and ( $\Delta\sigma'$ ) are varies with **time** during the consolidation process as following:

1. قبل وضع أي أحمال على المنشأ, أي أنه لا يوجد أي أحمال إضافية فإنه لا توجد أي زيادة في كل من ضغط الماء والإجهاد الفعال وبالتالي:

Before applying any additional loading:

$$\Delta\sigma = 0.0 \rightarrow \Delta u = 0.0 \text{ and } \Delta\sigma' = 0.0 \rightarrow \sigma' = \sigma'_o \text{ and } u = u_o$$

2. بعد وضع الأحمال الإضافية ( $\Delta\sigma$ ) مباشرة أي عند زمن ( $t = 0.0$ ) فإن كل قيمة الحمل الإضافي ( $\Delta\sigma$ ) سوف تنتقل إلى الماء ولا يوجد أي جزء منه تحمله التربة وهذا يرجع إلى أن التربة الطينية معامل النفاذية لها صغير جدا وبالتالي أثناء تطبيق الحمل مباشرة فإن الماء لم يبدأ بالخروج بعد من طبقة الطين وبالتالي فإن الماء هو الذي يتحمل كل الحمل الإضافي ( $\Delta\sigma$ ) وبالتالي:

Just after (at  $t = 0.0$ ) applying additional loading ( $\Delta\sigma$ ):

$$\Delta u = \Delta\sigma \text{ and } \Delta\sigma' = 0 \rightarrow \sigma' = \sigma'_o \text{ and } u = u_o + \Delta\sigma$$



3. مع مرور الزمن على تطبيق الأحمال الإضافية ( $\Delta\sigma$ ) أي عند زمن ( $0 < t < \infty$ ) فإن الماء يبدأ بالخروج من بين حبيبات التربة نتيجة تطبيق الأحمال حيث يتسرب الماء من أعلى ومن أسفل لأنه يوجد طبقات منفذة أعلى وأسفل طبقة الطين وبالتالي، نتيجة لخروج الماء فإن الزيادة في ضغط الماء ( $\Delta u$ ) تبدأ بالتناقص تدريجياً وبالمقابل فإن الزيادة في الإجهاد الفعال ( $\Delta\sigma'$ ) تبدأ بالزيادة تدريجياً نتيجة لخروج الماء من بين حبيبات التربة ونتيجة لانتقال الحمل إلى حبيبات التربة تدريجياً وبالتالي:

At any time ( $0 < t < \infty$ ) after applying additional loading ( $\Delta\sigma$ ):

$$\Delta\sigma = \Delta\sigma' + \Delta u \rightarrow \Delta\sigma' > 0.0 \text{ and } \Delta u < \Delta\sigma$$

$$\text{And } \sigma' = \sigma'_o + \Delta\sigma' \text{ and } u = u_o + \Delta u$$

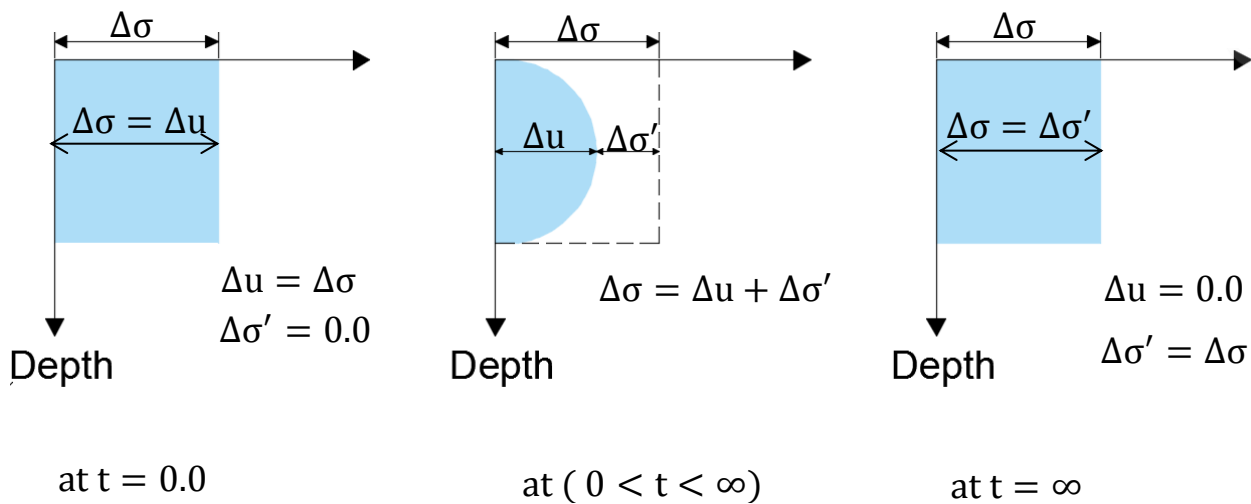
The values of  $\Delta u$  and  $\Delta\sigma'$  at any time at any depth in the clay will be calculated later in (**Time Rate of Consolidation**) section.

4. بعد مرور زمن كبير على تطبيق الأحمال الإضافية أي نظرياً عند زمن ( $t = \infty$ ) والمقصود بعد عدد كبير من السنوات فإن الماء الموجود داخل حبيبات التربة يكون قد خرج بالكامل من طبقة الطين وأن عملية الـ (consolidation) قد اكتملت وبالتالي فإن قيمة الزيادة في ضغط الماء ( $\Delta u$ ) في هذه الحالة تساوي صفر لأنه لا يوجد ماء في الأصل وبالتالي فإن جميع الحمل الإضافي انتقل إلى حبيبات التربة وبالتالي فإن قيمة الزيادة في الإجهاد الفعال ( $\Delta\sigma'$ ) تساوي قيمة الأحمال الإضافية على التربة ( $\Delta\sigma$ ):

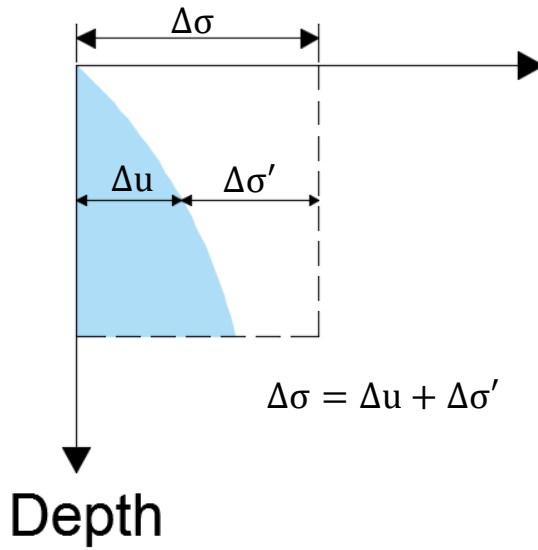
After many years ( $t = \infty$ ) of applying additional loading ( $\Delta\sigma$ ):

$$\Delta u = 0.0 \text{ and } \Delta\sigma' = \Delta\sigma \rightarrow \sigma' = \sigma'_o + \Delta\sigma' \text{ and } u = u_o$$

The following figure summarized the above explanations:

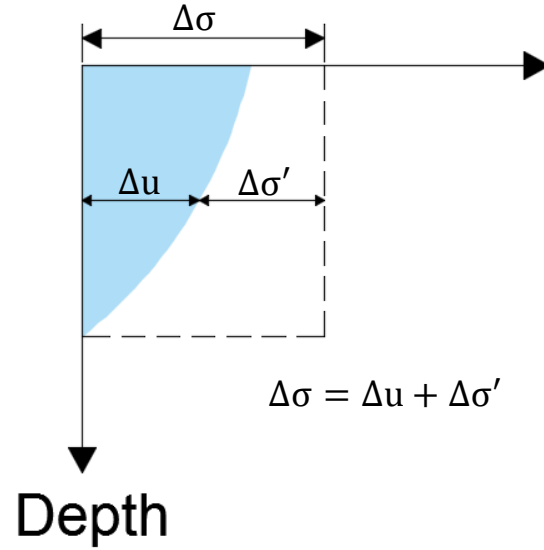


الآن عند الزمن ( $0 < t < \infty$ ) نلاحظ أن الزيادة في ضغط الماء في البداية صفر وفي النهاية صفر وذلك لأن الماء يتسرب إلى أعلى وإلى أسفل طبقة الطين لأن الطبقتين أعلى وأسفل طبقة الطين منفذتين للماء. حيث أن الماء يبدأ بالتحرك من منتصف طبقة الطين إلى أعلى وإلى أسفل. لكن إذا كانت الطبقة العلوية منفذة للماء والطبقة السفلية غير منفذة للماء فإن الماء سيتحرك من منتصف الطبقة إلى أعلى ومن ثم يخرج، وأيضا الماء سوف يتحرك من منتصف طبقة الطين إلى أسفل ولكن لن يستطيع الخروج بسبب وجود طبقة غير منفذة في الأسفل وبالتالي سوف تزداد قيمة الزيادة في ضغط الماء كلما نزلنا إلى أسفل وبالتالي فإن الماء سيرتد من أسفل إلى أعلى (باتجاه المخرج) وبالتالي تقل الزيادة في ضغط الماء كلما صعد الماء لأعلى. وإذا كانت الطبقة الغير منفذة في الأعلى والطبقة المنفذة للماء في الأسفل يحدث العكس تماما. والشكل التالي يوضح هاتين الحالتين:



at ( $0 < t < \infty$ )

Permeable Layer above  
And  
Impermeable Layer below



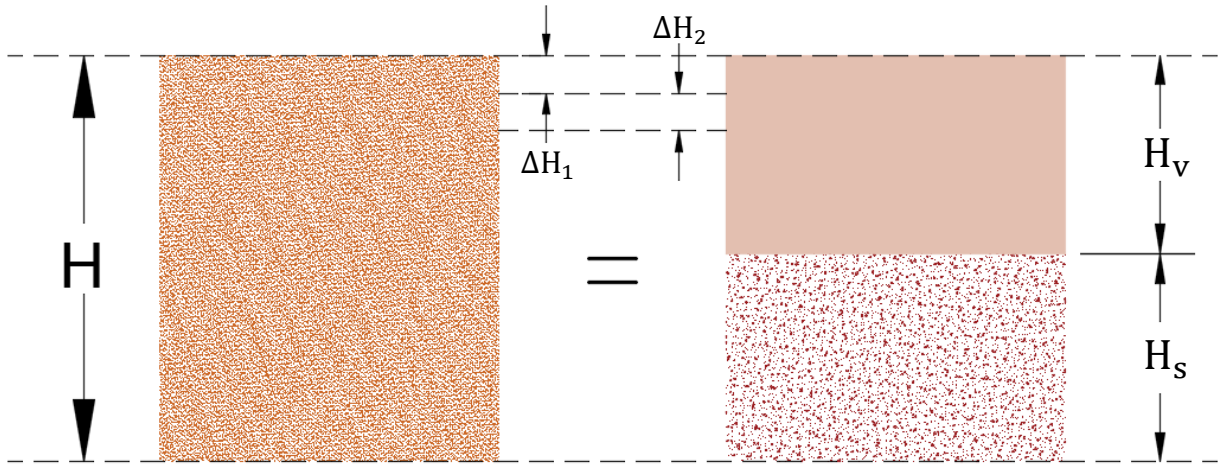
at ( $0 < t < \infty$ )

Permeable Layer below  
And  
Impermeable Layer above

هذه التغيرات (اتجاه سريان الماء) يتم أخذها بالاعتبار عند حساب قيم  $\Delta u$  وقيم  $\Delta\sigma'$  وأيضا عند حساب قيم الهبوط عند زمن معين وعمق معين كما سنتعلم لاحقا.

### Void Ratio - Pressure Plots:

عندما تتعرض عينة من التربة لضغوط خارجية فإن الفراغات في هذه التربة سوف تقل تدريجياً مع زيادة الضغط وبالتالي فإن حجم هذه العينة يقل تدريجياً مع زيادة الضغط، ومن المعروف أن حجم الحبيبات الصلبة يكون دائماً ثابت ولا يتغير مع الضغط وبالتالي فإن الحجم الذي يتغير مع الضغط هو حجم الفراغات الموجودة في التربة. لذلك لا بد من رسم منحنى يوضح العلاقة بين الضغط (Effective Stress) ونسبة الفراغات (Void Ratio). هذا المنحنى مهم جداً حيث يستخدم في تحديد معاملات تلزم في حساب قيم الهبوط في التربة كما سنتناوله لاحقاً. **الخطوات التالية توضح كيفية رسم المنحنى:**



1. Calculating the value of ( $H_s$ ) in the soil specimen:

$$A_T = A_V = A_S = A$$

$$W_s = \text{Dry weight of the specimen} = \checkmark$$

$$G_s = \frac{\gamma_s}{\gamma_w} \text{ and } \gamma_s = \frac{W_s}{V_s} \rightarrow W_s = G_s \times \gamma_w \times V_s$$

$$\text{But, } V_s = A_s \times H_s = A \times H_s \rightarrow W_s = G_s \times \gamma_w \times A \times H_s$$

$$\text{So, } H_s = \frac{W_s}{G_s \times \gamma_w \times A} = \frac{M_s}{G_s \times \rho_w \times A}$$

2. Calculate the initial height of voids ( $H_v$ ):

$$H_v = H - H_s$$

$H$  = initial height of the specimen.

3. Calculate the initial void ratio ( $e_o$ ) of the specimen:

$$e_o = \frac{V_v}{V_s} = \frac{A \times H_v}{A \times H_s} = \frac{H_v}{H_s}$$

4. The loading will start and at the first increment loading ( $\sigma_1$ ) will causes a deformation ( $\Delta H_1$ ), so, the void ratio will reduced to ( $e_1$ ), thus, the height of voids is reduced by ( $\Delta H_1$ ) because the volume of solid is constant.

$\sigma_1 =$  (total load / unit area of specimen)

$$H_{v(1)} = H_v - \Delta H_1 \rightarrow e_1 = \frac{H_{v(1)}}{H_s}$$

At the end of consolidation the total stress ( $\sigma_1$ ) is equal the effective stress ( $\sigma'_1$ )

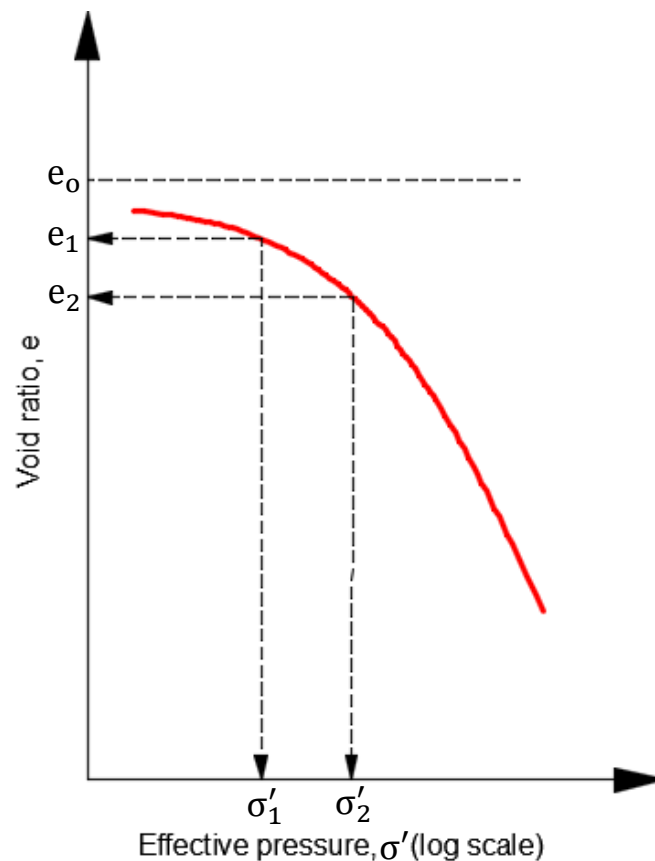
5. For the next loading ( $\sigma_2$ ) will causes additional deformation ( $\Delta H_2$ ), and void ratio will reduced to ( $e_2$ ).

$\sigma_2 =$  (commulative load / unit area of specimen)

$$H_{v(2)} = H_{v(1)} - \Delta H_2 \rightarrow e_2 = \frac{H_{v(2)}}{H_s}$$

At the end of consolidation the total stress ( $\sigma_2$ ) is equal the effective stress ( $\sigma'_2$ )

6. Continue the test for more increments and when consolidation is complete, the effective stress ( $\sigma'$ ) and the void ratio ( $e$ ) are plotted on semi logarithmic graph paper as shown in figure below.



## Normally Consolidated and Overconsolidated Clays:

**Normally Consolidated Clay:** The present effective overburden pressure ( $\sigma'_0$ ) is the maximum pressure that the soil was subjected to in the past (this means, the soil has never subjected to a vertical effective overburden pressure greater than the present effective vertical pressure)

**مثال على هذا النوع من التربة:** عند إنشاء مبنى معين يتم وضع القواعد مباشرة على سطح الأرض دون حفر أي كمية من التربة وبالتالي فإن طبقة الطين الموجودة على عمق معين يكون عليها حمل كل التربة التي فوقها وهو أقصى حمل ممكن أن تؤثره التربة على هذه الطبقة حيث يبدأ التحميل مباشرة من سطح الأرض أي أنه يتم وضع القواعد على سطح الأرض مباشرة, وهنا نتيجة الأحمال الكبيرة فإن الهبوط يكون كبير نوعا ما.

**Overconsolidated Clay:** The present effective overburden pressure ( $\sigma'_0$ ) is less than the maximum effective overburden pressure that the soil was subjected to in the past.

**مثال على هذا النوع من التربة:** عند إنشاء مبنى معين من المعروف أنه بدون حفر غالبا ما تكون قيمة الإجهاد الفعال الحالي ( $\sigma'_0$ ) هي أقصى قيمة تعرضت طبقة طين على بعد معين من التربة وهذه هي الحالة الأولى المذكورة أعلاه. لكن إذا كان هناك مبنى وفيه طابق بدروم فإنه لا بد من الحفر داخل التربة إلى عمق معين وبالتالي فإن الإجهاد الفعال الحالي ( $\sigma'_0$ ) الموجود على طبقة الطين والموجودة على عمق معين سوف يكون أقل من قيمة أقصى إجهاد فعال تعرضت له في الماضي (قبل الحفر) وبالتالي نلاحظ أنه حتى نصل إلى حالة ال(Overconsolidated Clay) قد تم حفر كمية معينة من التربة (أي تقليل الأحمال على طبقة الطين) وهذه العملية تسمى (Unloading) ثم يتم وضع المنشأة وتطبيق الأحمال وهذه العملية تسمى (Loading or Recompression). نلاحظ أيضا في هذا النوع من الطين تكون الأحمال على طبقة الطين قليلة نوعا ما وبالتالي تكون قيم الهبوط أقل منها في حالة (Normally Consolidated Clay).

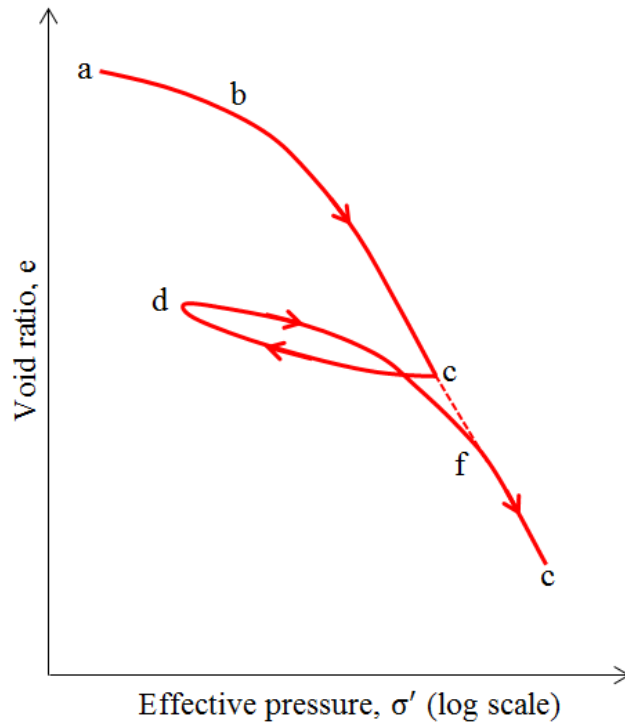
الكلام الموضح أعلاه يبين ما يحدث في الموقع (in field). لكن لحساب قيم الهبوط التي تنتج عن كل نوع من التربة لا بد من تمثيل ما يحدث في الواقع في المختبر لحساب بعض المعاملات الهامة والتي تلزم في حساب قيم الهبوط. حيث يتم تمثيل نوعي التربة في المختبر في تجربة واحدة كما يلي:

1. يتم وضع عينة من الطين تحت ضغط معين ويبدأ عليها الضغط تدريجيا حيث يكون الضغط في البداية صغير نوعا ما وبالتالي فإن قيم الهبوط (النقصان في قيمة "e") تكون صغيرة في البداية وهذا يتمثل في المسافة "ab" على المنحنى الموجود في الأسفل وهذه هي المرحلة الأولى والتي تتمثل في بداية بناء المنشأة في الموقع (على سطح التربة بدون حفر).

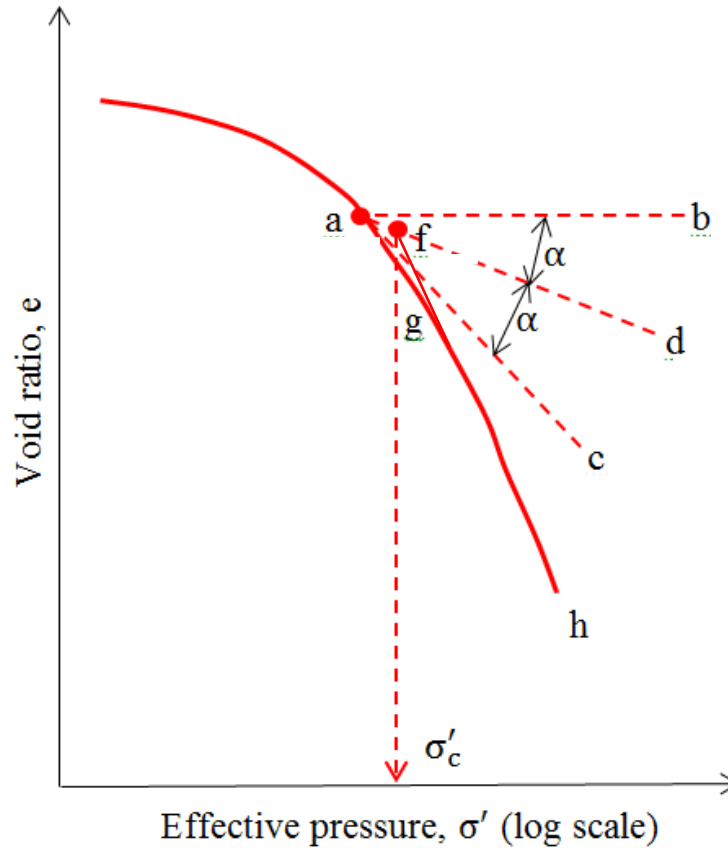
2. يبدأ الحمل بالزيادة تدريجيا على العينة وبالتالي فإن قيم الهبوط (النقصان في قيمة "e") تزداد بشكل ملحوظ مع زيادة الأحمال وهذا يتمثل في المسافة "bc" على المنحنى الموجود في الأسفل حيث أن التربة في هذه الحالة تكون (normally consolidated clay) وتتمثل في الموقع (بعد بداية تنفيذ المنشأة بفترة معينة إلى الانتهاء من تنفيذ المنشأة ووضع كافة الأحمال).

3. لتمثيل التربة (overconsolidated clay) في المختبر فإنه لا بد من وجود تربة مطبق عليها أحمال كبيرة ومن ثم البداية في تقليل هذه الأحمال عليها (unloading) وهذا ما سوف يحدث في المختبر حيث أن نفس العينة التي تم التحميل عليها في البداية وعند الوصول للنقطة "c" يتم إزالة الضغط وإزالة الأحمال تدريجياً عن العينة (unloading) حتى نصل إلى النقطة "d" ويتم إجراء هذه الخطوة لأن العينة عليها ضغط كبير وهو أقصى ضغط تعرضت له هذه العينة (الضغط المقابل للنقطة "c") وبالتالي حتى تكون التربة "overconsolidated" لا بد أن تقل قيمة الضغط عن أقصى قيمة تعرضت لها العينة وبالتالي يتم عمل (unloading) للعينة حيث تقل قيم الضغط عليها تدريجياً وبالتالي فإن قيم "e" تبدأ بالزيادة بشكل طفيف جداً وتنتفخ العينة بشكل بسيط (Swelling) وهذه العملية تتمثل في المسافة "cd" في الشكل في الأسفل، أما في الواقع فهي تتمثل في عملية الحفر في الموقع وإزالة التربة حتى منسوب التأسيس.

4. بعد انتهاء عملية رفع الحمل عند النقطة "d" تبدأ عملية التحميل (Recompression) على العينة وبالتالي فإن قيم الهبوط تبدأ بالزيادة تدريجياً ولكن نلاحظ من المنحنى أن قيم الهبوط في منطقة إعادة التحميل في المسافة "df" تزداد ولكن بشكل قليل جداً مقارنة بقيم الهبوط في المسافة "bc" وهذا يؤكد أن التربة "overconsolidated clay" قيم الهبوط فيها أقل من التربة "normal consolidated clay" حيث تتمثل هذه المرحلة في الموقع في عملية بدأ وضع المنشأة في الموقع على عمق التأسيس وبالتالي فإن قيم الهبوط تكون قليلة مع زيادة الحمل حتى نصل إلى النقطة "f" حيث أن الحمل بدأ يزداد بشكل كبير والمنشأ قد اقترب الانتهاء من بناؤه وبالتالي هنا تبدأ قيم الهبوط في الزيادة بشكل أكبر لكنها فترة صغيرة جداً حيث تتمثل في المسافة "fg" والتي تعد أصغر بكثير من المسافة "bc". نلاحظ أن المسافة "ab" هي المسافة في بداية التحميل على العينة وقيم الهبوط فيها صغيرة نسبياً وهي تشبه إلى حد كبير المسافة "df" حيث أن لهما نفس قيم الهبوط ونفس الميل تقريباً ونحن نعرف أن المسافة "df" تعبر عن مرحلة (recompression) في المختبر وبالتالي يمكن أيضاً تسمية المسافة "ab" (recompression curve) في المختبر لأن لها نفس قيم الهبوط ونفس الميل تقريباً للمسافة "df".



The value that used to determine whether the soil is normally consolidated or overconsolidated is the value of maximum effective overburden pressure that the soil was subjected in the past and it is called “**Preconsolidation Pressure ( $\sigma'_c$ )**”. The value of the preconsolidation pressure ( $\sigma'_c$ ) can be determined for a clay specimen in the lab using the laboratory ( $e$ -log  $\sigma'$ ) plot according the following procedures:



1. نرسم رسمة “e-log  $\sigma'$  plot” كما تعلمنا سابقاً.
2. نحدد النقطة على المنحنى والتي عندها أقصى قيمة انحناء ونسميها “a” ومن ثم نرسم منها خط أفقي ونسميه “ab”.
3. نرسم الخط “ac” والذي يعتبر مماس للمنحنى عند النقطة “a” ومن ثم نرسم الخط “ad” والذي ينصف الزاوية “cab”.
4. من المعروف أن سلوك المنحنى “e-log  $\sigma'$  plot” يكون قريب من الخط المستقيم في نهايته وهذا يتمثل في الخط “gh” حيث نقوم برسم امتداد هذا الخط ليقطع الخط “ad” عند النقطة “f” ومن ثم نرسم خط رأسي من النقطة “f” حتى يتقاطع مع محور “x” وعند نقطة التقاطع هذه تكون قيمة ( $\sigma'_c$ ).





$$A_T = A_v = A_s = A$$

The initial void ratio is  $e_o$  and the initial volume is  $V_o = A \times H$  (H is initial height)

The final Volume after consolidation is  $V_f = (H - S_c) \times A$

$$V_o - V_f = A \times H - (A \times H - A \times S_c) = A \times S_c \rightarrow \Delta V = A \times S_c$$

But, we know that the change in volume will occur in voids volume and volume of solid will remain constant, so,  $\Delta V = \Delta V_v = A \times S_c \longrightarrow 1$

$$e = \frac{V_v}{V_s} \rightarrow V_v = e \times V_s \rightarrow \Delta V_v = \Delta e \times V_s \longrightarrow 2$$

$$V_o = V_s + V_v \rightarrow V_o = V_s + e_o V_s \rightarrow V_o = V_s(1 + e_o) \rightarrow V_s = \frac{V_o}{1 + e_o} \text{ (substitute in eqn. 2)}$$

$$\Delta V_v = \Delta e \times \frac{V_o}{1 + e_o} \text{ (substitute in eqn. 1)}$$

$$\Delta e \times \frac{V_o}{1 + e_o} = A \times S_c \text{ but, } V_o = A \times H \rightarrow \Delta e \times \frac{A \times H}{1 + e_o} = A \times S_c \rightarrow \rightarrow \rightarrow$$

$$S_c = H \frac{\Delta e}{1 + e_o}$$

→ The basic equation for calculating consolidation settlement.

To calculate the value of ( $S_c$ ), firstly we should calculate the value of  $\Delta e$ .

In general, the value of  $\Delta e$  can be calculated from ( $e$ - $\log \sigma'$  plot) as following:

$$\Delta e = e_1 - e_2$$

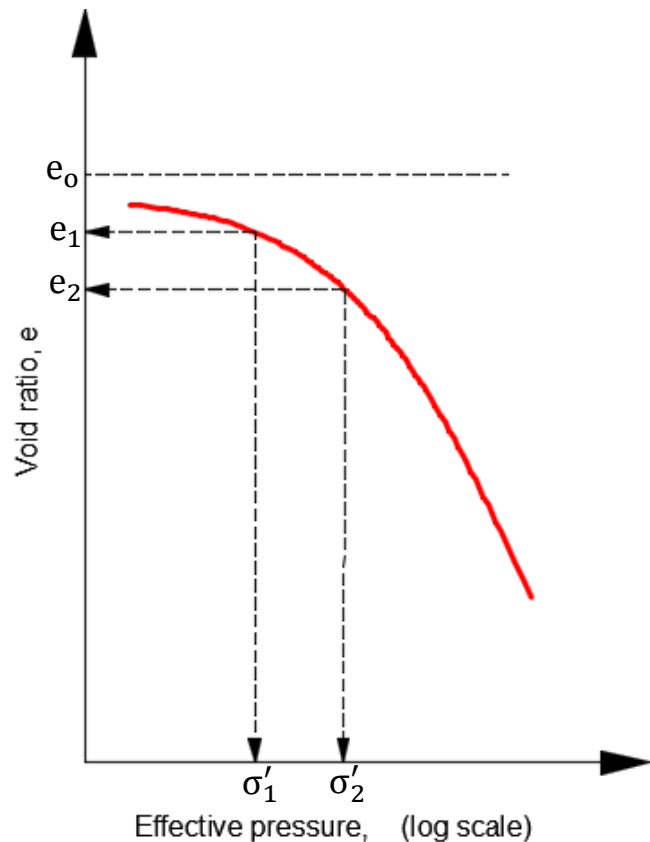
$$\text{Slope} = \frac{\Delta e}{\log \frac{\sigma_2'}{\sigma_1'}}$$

$$\text{So, } \Delta e = \text{Slope} \times \log \frac{\sigma_2'}{\sigma_1'}$$

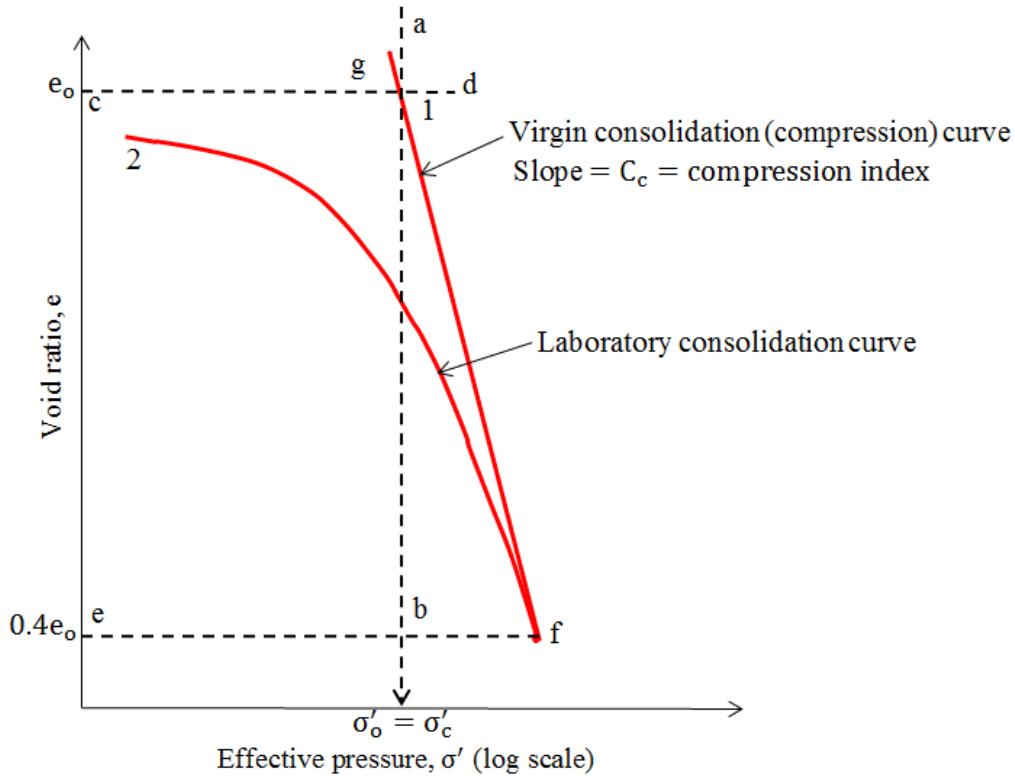
$\sigma_2'$  = large value

$\sigma_1'$  = small value

The value of the Slope is depend on the type of the clay (Normally Consolidated or Overconsolidated) and now we will calculate the value of slope for both types of clay:



## Calculating the value of slope for normally consolidated clay:



According to the above plot, follow the following procedures:

1. بما أن التربة من نوع "normally consolidated clay" فإن  $\sigma'_c = \sigma'_o$
2. من المنحنى رقم "2" والذي يمثل منحنى المختبر نحدد قيمة  $\sigma'_c$  " كما تعلمنا سابقا ومن ثم نرسم خط رأسي من هذه القيمة "ab".
3. نحسب قيمة  $e_o = H_v/H_s$  " ومن ثم نرسم منها خط أفقي "cd".
4. نحسب قيمة  $0.4e_o$  " ومن ثم نرسم منها خط أفقي "ef" " حيث أن النقطة "f" تقع على المنحنى رقم 2.
5. نحدد النقطة "g" وهي نقطة تقاطع الخطين "ab&cd" " ومن ثم نصل بينها وبين النقطة "f" بخط مستقيم فنحصل على المنحنى رقم "1" والذي يمثل نوع التربة "normally consolidated clay" في المختبر ويسمى "Virgin Compression Curve" " حيث أننا نستفيد من هذا المنحنى في تحديد قيمة معامل الهبوط للتربة "normally consolidated clay" والذي يسمى " $C_c$  (Compression Index)" حيث أن هذا المعامل يمثل ميل المنحنى رقم 1 وبالتالي:

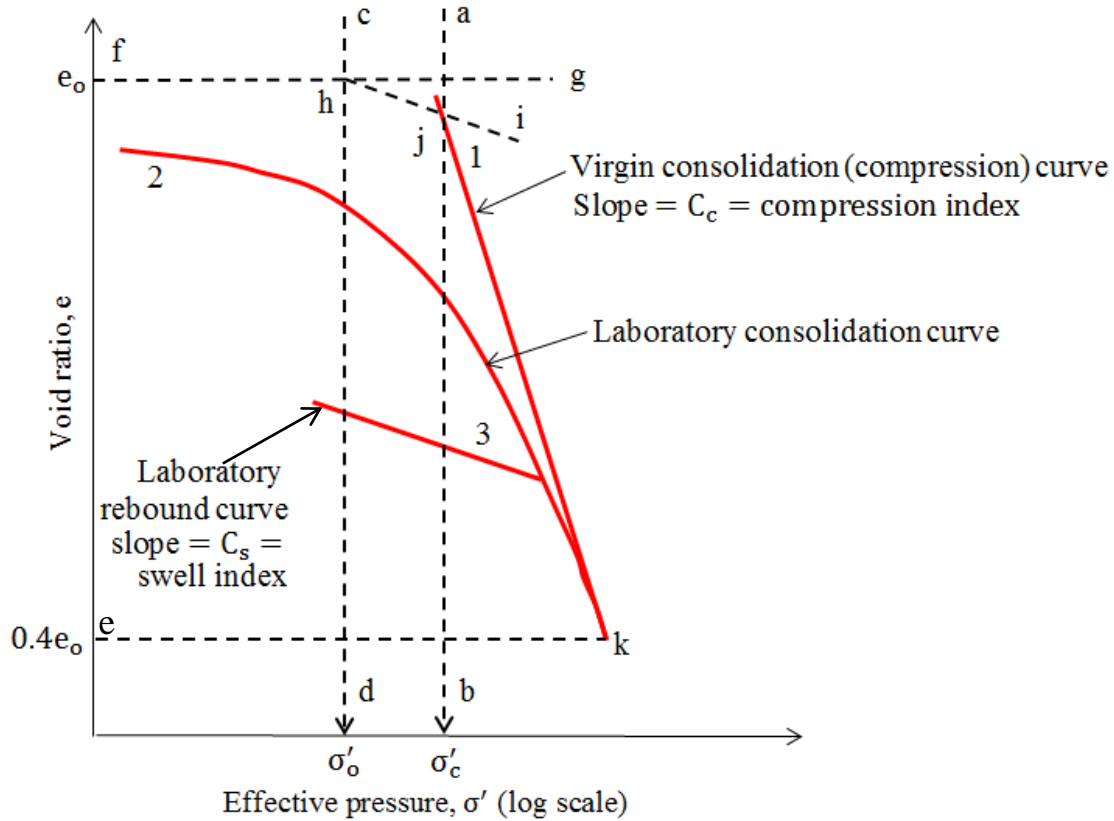
Slope of virgin compression curve = compression index =  $C_c$

Slope =  $C_c = \frac{\Delta e}{\log \frac{\sigma'_2}{\sigma'_1}}$  →→ For normally consolidated clay →→

$$\Delta e = C_c \times \log \frac{\sigma'_2}{\sigma'_1}$$

$\sigma'_2$  and  $\sigma'_1$  are horizontal component of Virgin Curve

## Calculating the value of slope for overconsolidated clay:



According to the above plot, follow the following procedures:

1. بما أن التربة من نوع "overconsolidated clay" فإن  $\sigma'_c > \sigma'_o$
  2. من المنحنى رقم "2" والذي يمثل منحنى المختبر نحدد قيمة  $\sigma'_c$  كما تعلمنا سابقا ومن ثم نرسم خط رأسي من هذه القيمة "ab" ونرسم خط آخر من قيمة  $\sigma'_o$  "cd".
  3. نحسب قيمة  $e_o$  ومن ثم نرسم منها خط أفقي "fg" والذي يقطع الخط "cd" في نقطة "h".
  4. أثناء عمل التجربة من المعروف أنه يتم تحميل العينة (Loading) ثم يتم رفع الحمل عنها (Unloading) ومن ثم يتم إعادة التحميل (Reloading or Recompression) ونتيجة لذلك يتم رسم المنحنى رقم "3" والذي يعتبر منحنى متوسط بين منحنى رفع التحميل (Unloading) ومنحنى إعادة التحميل (Reloading or Recompression) (أي أنه يتم رسم خط مستقيم بينهما فينتج الخط رقم 3) حيث أن هذا الخط يعبر عن التربة "overconsolidated clay" في المختبر ويسمى "Rebound Curve or Swelling Curve" حيث أننا نستفيد من هذا المنحنى في تحديد قيمة معامل الهبوط للتربة "overconsolidated clay" والذي يسمى " $C_s$  (Swell Index)"
- حيث أن هذا المعامل يمثل ميل المنحنى رقم 3 وبالتالي:

$$\text{Slope of rebound/swelling curve} = \text{swell index} = C_s$$

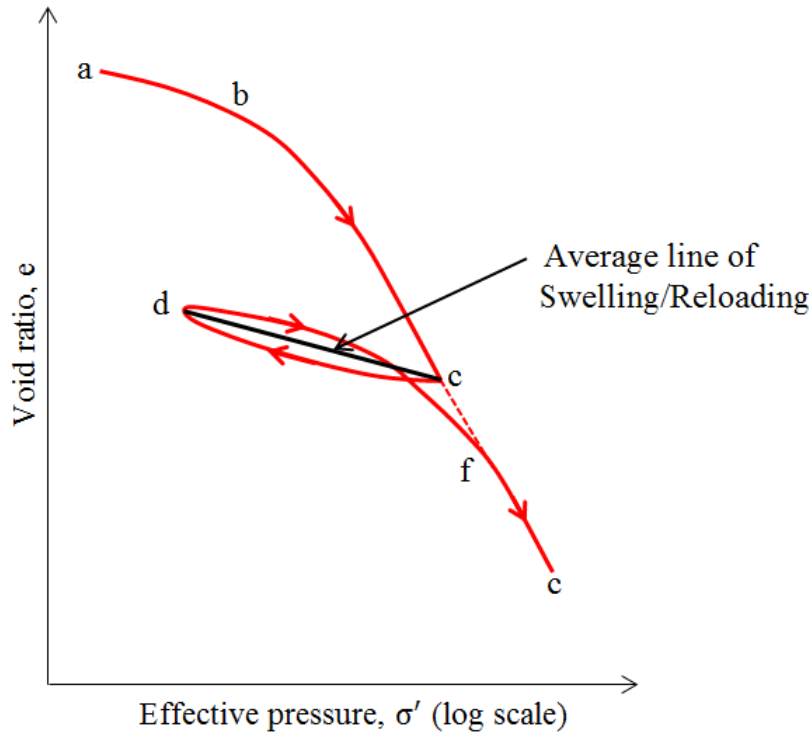
5. نرسم الخط "hi" والذي يوازي المنحنى رقم 3 حيث يقطع الخط "ab" في نقطة "j".
6. نحسب قيمة "0.4e<sub>o</sub>" ومن ثم نرسم منها خط أفقي "ek" حيث أن النقطة "k" تقع على المنحنى رقم 2.
7. نصل بين النقطتين "k&j" فنحصل على الخط رقم "1" والذي يسمى "Virgin Compression Curve" وله ميل "C<sub>c</sub> (Compression Index)" كما تم شرحه في الأعلى.
8. نلاحظ أن الخط "hi" له نفس ميل الخط رقم "3" وذلك لأنه موازي للخط رقم "3" حيث أن الخط "hi" يمثل مسار "Recompression Path in field" أي أنه في الموقع وبعد حفر التربة (Unloading) وعند بداية تنفيذ المنشأة ووضع الأحمال (Recompression or Reloading) فإن تصرف عملية وضع هذه الأحمال في الموقع يتمثل في الخط "hi" وبالتالي يمكن التعبير عن تربة "overconsolidated clay" من خلال الخط "hi" والذي يمثل نفس سلوك الخط رقم "3" في المختبر ولهما نفس الميل:
- Slope of swelling curve(3) = Slope of recompression curve(hi) = C<sub>s</sub> = C<sub>r</sub> = Swell index

$$\text{Slope} = C_s = \frac{\Delta e}{\log \frac{\sigma'_2}{\sigma'_1}} \rightarrow \rightarrow \text{For overconsolidated clay} \rightarrow \rightarrow$$

$$\Delta e = C_s \times \log \frac{\sigma'_2}{\sigma'_1}$$

$\sigma'_2$  and  $\sigma'_1$  are horizontal coordinates of recompression(rebound) curve

كما ذكرنا في الأعلى أن الخط رقم "3" يتم تحديده كخط متوسط بين منحنى رفع التحميل ومنحنى إعادة التحميل والشكل التالي يوضح كيفية الحصول عليه: حيث يتم التوصيل بين النقطتين "c&d" بخط مستقيم

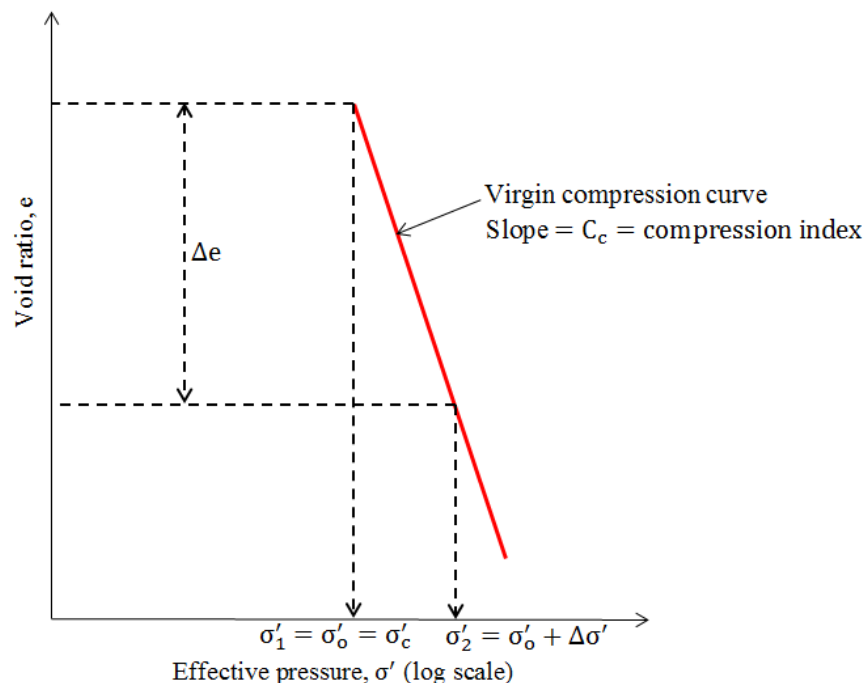


Now, we can calculate the consolidation settlement but, before starting calculations you should know the following:

1. We calculate the total value of  $S_c$  at the end of consolidation process (at  $t = \infty$ ), therefore, the value of external load ( $\Delta\sigma$ ) will be carried by the soil solids  $\rightarrow\rightarrow$   $\Delta\sigma = \Delta\sigma'$  and  $\Delta u = 0.0$  (at the end of consolidation process).
2. Always the initial value of effective stress in the clay layer is the present effective pressure ( $\sigma'_1 = \sigma'_0$ ), and the final value of effective stress at the end of consolidation process ( $\sigma'_2 = \sigma'_0 + \Delta\sigma'$ ), and the reduction in volume of soil which indicates by ( $\Delta e$ ) is due to the application of loads from ( $\sigma'_1 = \sigma'_0$ ) to ( $\sigma'_2 = \sigma'_0 + \Delta\sigma'$ ).
3. The value of preconsolidation pressure ( $\sigma'_c$ ) is used to classify weather the clay is normally consolidated or overconsolidated also is used to calculating the value of total consolidation settlement( $S_c$ ).

### Calculation of primary consolidation settlement for normally consolidated clay:

It is preferable to draw this simple graph when you want to calculate ( $S_c$ ) for (N.C.Clay)



For(N.C.Clay) we know that,  $\sigma'_c = \sigma'_0$  thus, ( $\sigma'_1 = \sigma'_0 = \sigma'_c$ ) and( $\sigma'_2 = \sigma'_0 + \Delta\sigma'$ ) Also we know the normally consolidated clay can be represents by virgin compression curve which have a slope of ( $C_c = \text{compression index}$ ).

The basic equation for calculating primary consolidation settlement is:

$$S_c = \frac{H}{1 + e_0} \times \Delta e \quad (\text{As we derive it Previously})$$

For (N.C.Clay) as shown in figure above, the value of ( $\Delta e$ ) can be calculated as following:

$$\text{Slope} = C_c = \frac{\Delta e}{\log\left(\frac{\sigma'_0 + \Delta\sigma'}{\sigma'_0}\right)} \rightarrow \Delta e = C_c \times \log\left(\frac{\sigma'_0 + \Delta\sigma'}{\sigma'_0}\right)$$

Substitute by  $\Delta e$  in the equation of  $S_c$  :

$$S_c = \frac{C_c \times H}{1 + e_0} \times \log\left(\frac{\sigma'_0 + \Delta\sigma'}{\sigma'_0}\right) \rightarrow \text{Primary Consolidation Settlement for (N.C.Clay)}$$

### Calculation of primary consolidation settlement for

#### Overconsolidated clay:

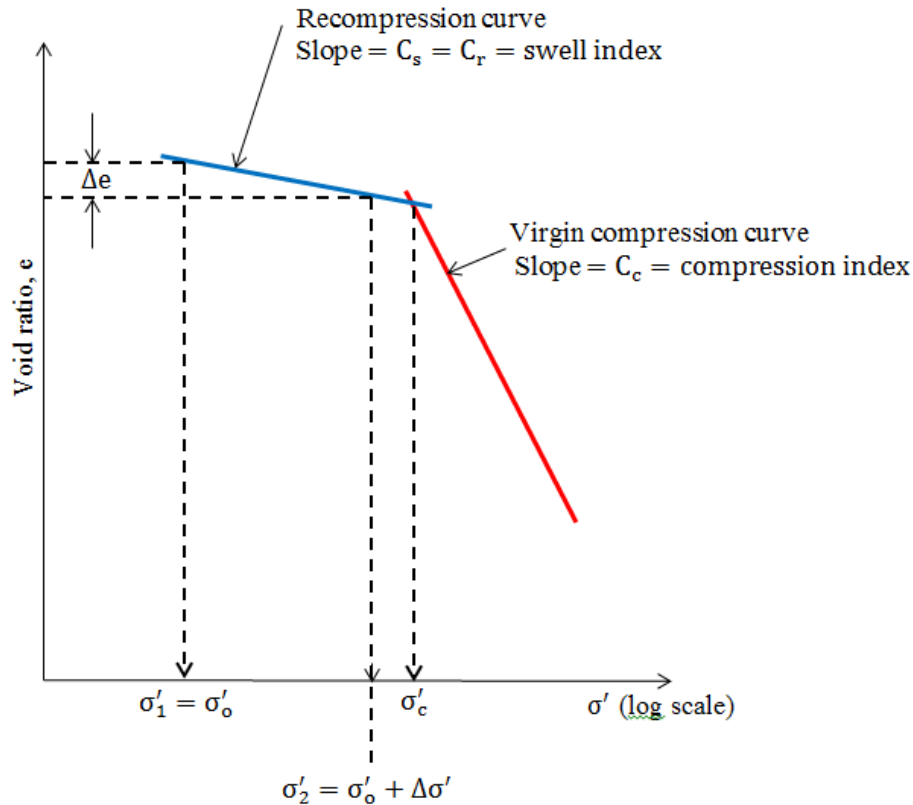
For (O.C.Clay) we know that,  $\sigma'_c > \sigma'_0$  thus, ( $\sigma'_1 = \sigma'_0$ ) and  $\sigma'_2 = \sigma'_0 + \Delta\sigma'$

Also we know the overconsolidated clay can be represented by recompression curve which have a slope of ( $C_s =$  well index).

But, there are two cases, the first case is ( $\sigma'_c \geq \sigma'_0 + \Delta\sigma'$ ) and the second case is ( $\sigma'_c < \sigma'_0 + \Delta\sigma'$ ) and now we will calculate the value of ( $S_c$ ) for each case:

#### Case One ( $\sigma'_c \geq \sigma'_0 + \Delta\sigma'$ ):

It is preferable to draw this simple graph when you want to calculate ( $S_c$ ) for (O.C.Clay ; 1<sup>st</sup> case):



$$\sigma'_1 = \sigma'_o \quad \text{and} \quad \sigma'_2 = \sigma'_o + \Delta\sigma'$$

We note from the above graph, the consolidation settlement occur in the overconsolidated curve (recompression curve) before reaching the normally consolidated curve (virgin curve), this is case one.

The value of  $(\Delta e)$  can be calculated as following:

$$\text{Slope} = C_s = \frac{\Delta e}{\log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)} \rightarrow \Delta e = C_s \times \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$$

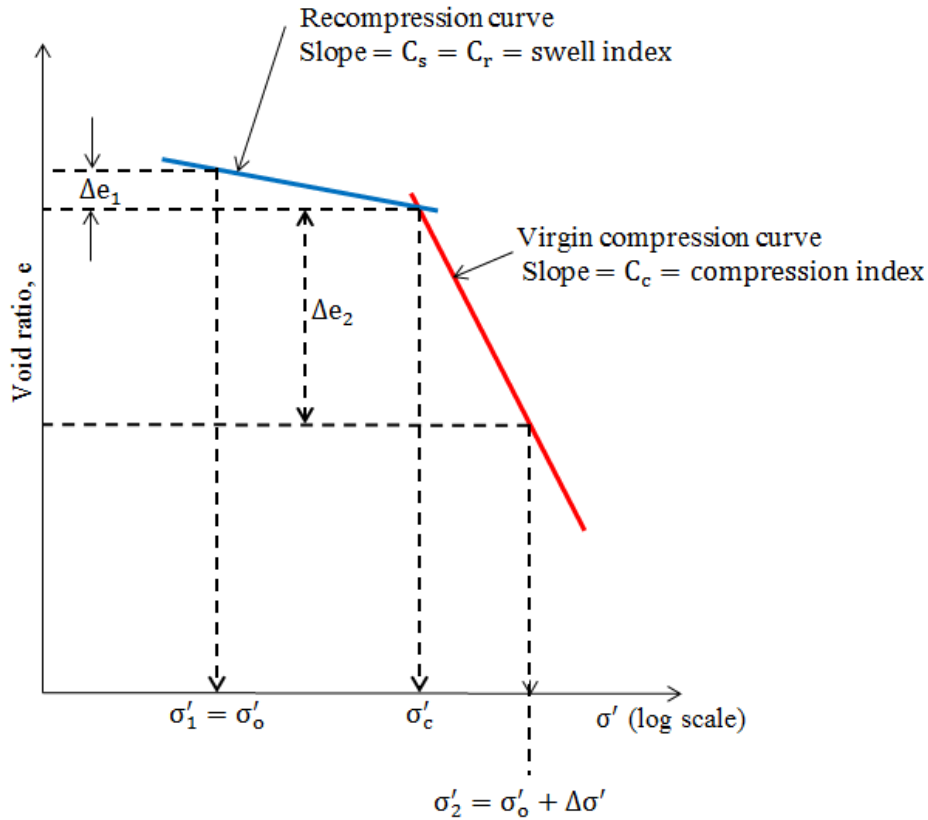
Substitute by  $\Delta e$  in the equation of  $S_c$ :

$$S_c = \frac{C_s \times H}{1 + e_o} \times \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right) \rightarrow \text{Primary Consolidation Settlement for (O.C. Clay 1st case)}$$

We note that, the value of  $(\Delta e)$  is very small if we compare it with  $(\Delta e)$  for (N.C. Clay).

**Case Two ( $\sigma'_c < \sigma'_o + \Delta\sigma'$ ):**

It is preferable to draw this simple graph when you want to calculate ( $S_c$ ) for (O.C.Clay ; 2<sup>nd</sup> case):



We note that, the consolidation in this case occur in two steps, step (1) is due the application of ( $\sigma'_1 = \sigma'_o$ ) firstly and start applying the external load ( $\Delta\sigma$ ) until reaching the value of ( $\sigma'_c$ ), the reduction in void ratio in this step is ( $\Delta e_1$ ), this step is done in the recompression curve (overconsolidated curve).

Step (2) is start from the value of ( $\sigma'_c$ ) until the end of consolidation process and reaching the value of ( $\sigma'_2 = \sigma'_o + \Delta\sigma'$ ), the reduction in void ratio in this step is ( $\Delta e_2$ ), and this step is done in virgin compression curve (normally consolidated curve).

So,  $\Delta e = \Delta e_1 + \Delta e_2$

**Calculating of ( $\Delta e_1$ ):** [Stresses causes ( $\Delta e_1$ ) are ( $\sigma'_1 = \sigma'_o$ ) and ( $\sigma'_2 = \sigma'_c$ ):]

$$\text{Slope} = C_s = \frac{\Delta e_1}{\log\left(\frac{\sigma'_c}{\sigma'_o}\right)} \rightarrow \Delta e_1 = C_s \times \log\left(\frac{\sigma'_c}{\sigma'_o}\right)$$



**Calculating of ( $\Delta e_2$ ):** [Stresses causes( $\Delta e_2$ ) are ( $\sigma'_1 = \sigma'_c$ ) and ( $\sigma'_2 = \sigma'_o + \Delta\sigma'$ ):

$$\text{Slope} = C_c = \frac{\Delta e_2}{\log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c}\right)} \rightarrow \Delta e_2 = C_c \times \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c}\right)$$

So, the value of total reduction in void ratio ( $\Delta e$ ) is:

$$\Delta e = C_s \times \log\left(\frac{\sigma'_c}{\sigma'_o}\right) + C_c \times \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c}\right)$$

Substitute by  $\Delta e$  in the equation of  $S_c$ :

$$S_c = \frac{C_s \times H}{1 + e_o} \times \log\left(\frac{\sigma'_c}{\sigma'_o}\right) + \frac{C_c \times H}{1 + e_o} \times \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c}\right)$$

This is the primary consolidation settlement for (O.C.Clay 2<sup>nd</sup> case).

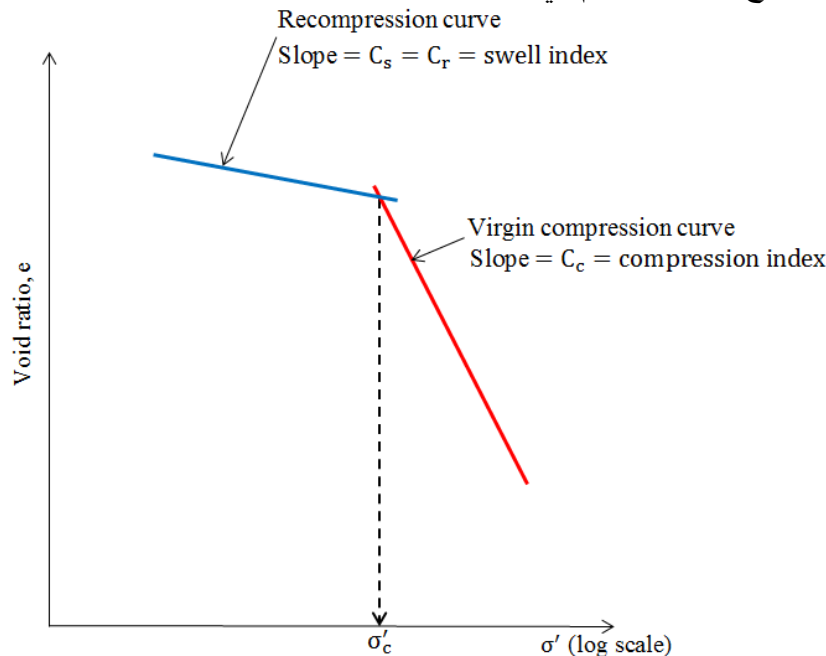
**Empirical Equations to calculate the values of  $C_c$  and  $C_s$ :**

$$C_c = 0.009(LL - 10) \quad (LL = \text{Liquid Limit})$$

$$C_s = (0.1 \rightarrow 0.2)C_c$$

**ملاحظات هامة جدا أثناء الحل:**

1. دائما عند حساب قيمة ( $\sigma'_o$ ) يتم حسابها في وسط طبقة الـ "clay" لأن التأثير الأكبر في منتصف الطبقة والمياه تبدأ بالتسرب من منتصف الطبقة إلى أسفل وإلى أعلى.
2. عند حساب قيمة الهبوط ( $S_c$ ) لأي نوع من التربة يفضل دائما رسم هذه الرسمة ومنها يتم استنتاج العلاقة التي تناسب الحالة عن طريق وضع قيم ( $\sigma'_o$ ) and ( $\sigma'_c$ ) and ( $\sigma'_o + \Delta\sigma'$ ) على هذه الرسمة ومن ثم استنتاج العلاقة كما تم في الأعلى.



حيث نرى من الشكل في الأعلى أن قيمة ( $\sigma'_c$ ) تكون دائما بين المنحنيين لأنها الحد الفاصل بين نوعي التربة (N.C.Clay and O.C.Clay) ويتم حساب قيم  $[(\sigma'_o) \text{ and } (\sigma'_o + \Delta\sigma')]$  ووضعها على المنحنى ومن ثم استنتاج العلاقة كما تم شرحه في الأعلى.

3. إذا كان يوجد لدينا حمل غير متوزع على كامل مقطع التربة (مثل قاعدة منفصلة أو لبشة خرسانية أو أي حمل آخر) فإن ذلك الحمل يتناقص كلما زاد العمق (انظر مقدمة CH.10). لكن عند حساب قيم الهبوط فإن قيمة ( $\Delta\sigma'$ ) يجب أن تكون كمتوسط لكل طبقة الطين, لذلك في هذه الحالة سوف نجد ( $\Delta\sigma'$ ) في بداية الطبقة وفي منتصف الطبقة وفي نهاية الطبقة (أسفل منتصف القاعدة كما تم دراسته في CH.10) ومن ثم إيجاد متوسط هذه القيم كما يلي:

Using Simpson's rule, the value of  $\Delta\sigma'_{av}$  can be estimated as following:

$$\Delta\sigma'_{av} = \frac{\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b}{6}$$

$\Delta\sigma'_t$  = Increase in effective stress at the top of clay layer.

$\Delta\sigma'_m$  = Increase in effective stress at the middle of clay layer.

$\Delta\sigma'_b$  = Increase in effective stress at the bottom of clay layer.

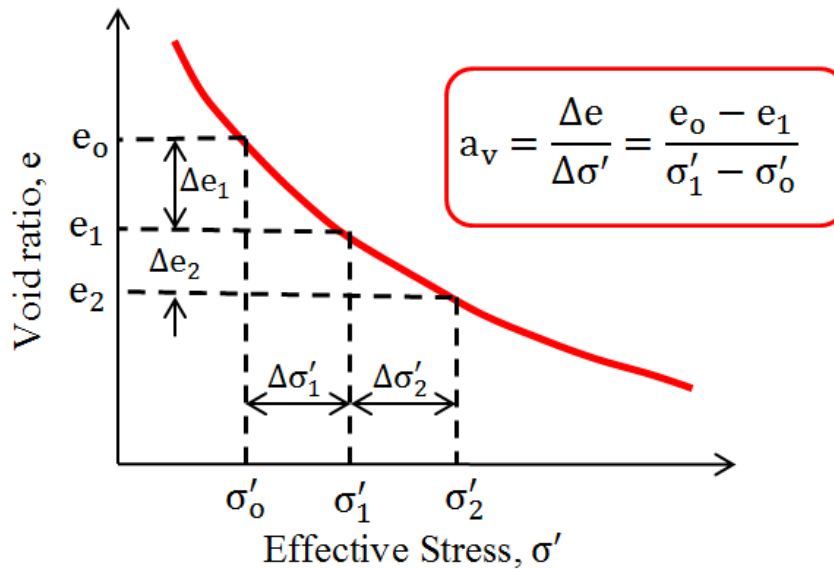
وسوف يتضح ذلك عند حل الأسئلة. أما في حالة كون الأحمال موزعة على كامل التربة فإن قيمة الحمل تنتقل ثابتة تقريبا إلى التربة ولا تتناقص مع زيادة العمق وبالتالي نأخذها كما هي. (انظر مقدمة CH.10)

### Time Rate of Consolidation:

فيما سبق، تعلمنا كيفية حساب قيمة الهبوط الكلي الذي يحدث في التربة نتيجة تعرضها لأحمال خارجية وذلك بعد انتهاء خروج الماء واكتمال عملية الهبوط للمبنى (at the end of consolidation process) لكن أحيانا نكون مهتمين بمعرفة قيمة الهبوط التي سوف تحدث في التربة بعد فترة معينة من الزمن على سبيل المثال بعد سنة أو بعد سنتين حيث أننا سنتعرف على كيفية حساب قيمة الهبوط عند أي فترة زمنية بعد تنفيذ المنشأة (at  $0 < t < \infty$ ). أيضا درسنا في مقدمة هذا الموضوع (Consolidation Settlement) أن الأحمال الخارجية التي يتم تطبيقها على التربة يقاومها كل من الماء الموجود داخل حبيبات التربة وحببيبات التربة نفسها وبالتالي فإن تطبيق الحمل الخارجي ( $\Delta\sigma$ ) يؤدي إلى زيادة ضغط الماء بقيمة ( $\Delta u$ ) وزيادة الإجهاد الفعال بقيمة ( $\Delta\sigma'$ ) بحيث تختلف هذه القيم مع الزمن، ففي البداية يكون الزيادة في ضغط الماء مساويا للحمل الخارجي المطبق لأن الحمل لم ينتقل إلى حبيبات التربة بعد، ومن ثم يبدأ الماء بالخروج من التربة مع الزمن وتقل الزيادة في ضغط الماء تدريجيا مع الزمن وبالتالي تزداد الزيادة في قيمة الإجهاد الفعال من الزمن حيث سنتعلم كيفية الحصول على قيمة ( $\Delta u$ ) وقيمة ( $\Delta\sigma'$ ) عند أي فترة من الزمن. وسنتعرف أيضا على بعض المفاهيم المهمة مثل (Coefficient of Consolidation) وغيره.

### Coefficient of Compressibility ( $a_v$ ):

It is refer to change in volume of soil ( $\Delta e$ ) due to the increase in stresses and it is the slope of the (void ratio versus effective stress) plot as shown in figure below:



**Note:**  $a_v$  can be considered **constant** for a narrow range of pressure increase.

This means,  $a_v = \frac{e_0 - e_1}{\sigma'_1 - \sigma'_0} = \frac{e_1 - e_2}{\sigma'_2 - \sigma'_1}$  ( $\frac{m^2}{KN}$ ) or ( $\frac{ft^2}{lb}$ )

**Coefficient of Volume Compressibility ( $m_v$ ):**

It is the ratio of change in volume of a soil ( $\Delta e$ ) to the **unit initial volume** due to the increase in stresses (effective stresses) and is given by:

$$m_v = \frac{e_0 - e_1}{1 + e_0} \times \frac{1}{\sigma'_1 - \sigma'_0} = \frac{a_v}{1 + e_0}$$

Now, when we want to calculate the value of ( $m_v$ ) we will use this formula:

$$m_v = \frac{a_v}{1 + e_{av}}$$

$$e_{av} = \frac{e_0 + e_1}{2} = \frac{e_1 + e_2}{2}$$

$$m_v = \frac{\left(\frac{e_0 - e_1}{\sigma'_1 - \sigma'_0}\right)}{1 + \left(\frac{e_0 + e_1}{2}\right)} = \frac{\left(\frac{e_1 - e_2}{\sigma'_2 - \sigma'_1}\right)}{1 + \left(\frac{e_1 + e_2}{2}\right)} \left(\frac{m^2}{KN}\right) \text{ or } \left(\frac{ft^2}{lb}\right)$$

**Coefficient of Consolidation ( $c_v$ ):**

Is the parameter used to describe the rate at which saturated clay undergoes consolidation, when subjected to an increase in pressure and can be calculated as following:

$$c_v = \frac{K}{\gamma_w \times m_v} \left(\frac{m^2}{\text{unit time}}\right) \text{ or } \left(\frac{ft^2}{\text{unit time}}\right)$$

$K$  = coefficient of permeability

There is another formula to calculate the value of ( $c_v$ ):

$$T_v = \frac{c_v \times t}{H_{dr}^2} \rightarrow c_v = \frac{T_v \times H_{dr}^2}{t}$$

$T_v$  = time factor (nondimensional number).

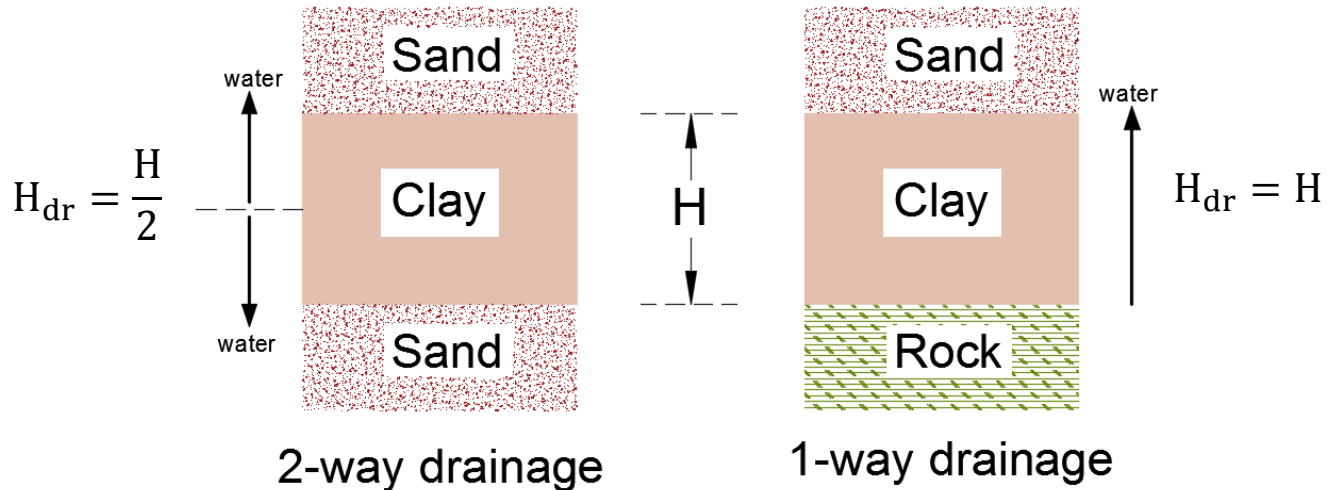
$t$  = time required to reach a specific **degree of consolidation**.

$H_{dr}$  = Drainage path of water through the clay layer (during consolidation)

**Important Note:**

The value of ( $c_v$ ) is always constant for the same soil, this means, the value of ( $c_v$ ) is the same in lab and in field (for the same soil).

The value of ( $H_{dr}$ ) can be determined according the following graph:



**Calculating the increase in pore water pressure ( $\Delta u$ ) and the increase in effective stress ( $\Delta \sigma'$ ) at distance ( $z$ ) at any time ( $t$ ):**

Because consolidation progresses by the dissipation of excess pore water pressure ( $\Delta u$ ), the degree of consolidation at a distance ( $z$ ) at any time ( $t$ ) is:

$$U_z = \frac{\Delta \sigma - \Delta u_{(z,t)}}{\Delta \sigma} = 1 - \frac{\Delta u_{(z,t)}}{\Delta \sigma}$$

This is (eqn. 11.57) in text book should be at this form to make sense

$\Delta \sigma$  = initial excess in pore water pressure at ( $t = 0.0$ )  $\rightarrow \Delta u = \Delta \sigma$

$\Delta u_{(z,t)}$  = excess in pore water pressure at distance ( $z$ ) at any time ( $t$ )

Note that, the term  $\left(\frac{\Delta u_{(z,t)}}{\Delta \sigma}\right)$  refer to the percent of water exist in the clay layer at any time ( $t$ ) during consolidation process and at distance ( $z$ ), so, the degree of consolidation (percent of water dissipate) at the same time and depth is the total percent of water (100%) minus the percent of water exist in the clay layer.

Now, how we can calculate the value of ( $\Delta u_{(z,t)}$ ):

According the above equation, to calculate ( $\Delta u_{(z,t)}$ ) we want the values of ( $\Delta \sigma$ ) and ( $U_z$ ). the value of ( $\Delta \sigma$ ) is always known because it is the applied load.

The value of ( $U_z$ ) can be taken from (**Figure 11.25**) according the values of ( $T_v$ ) and  $\left(\frac{z}{H_{dr}}\right)$ . The values of ( $T_v$ ) and ( $H_{dr}$ ) will be calculated as explained above.

**Note:**

The value of ( $z$ ) is the depth starting from the **top** of clay layer. (i.e. At the top of clay layer ( $z = 0.0$ ) and at the middle ( $z = H/2$ ) and so on.)

Now, we can get the value of  $(U_z)$  from **(Figure 11.25)** easily and we can calculate the value of  $(\Delta u_{(z,t)})$  such that:  $\left(U_z = 1 - \frac{\Delta u_{(z,t)}}{\Delta \sigma}\right) \rightarrow \Delta \sigma'_{(z,t)} = \Delta \sigma - \Delta u_{(z,t)}$ .

**Calculating the value of consolidation settlement at any time (t) during consolidation process ( $S_{c(t)}$ ):**

Previously, we learn how to calculate the total amount of consolidation settlement at the end of consolidation process ( $S_{c(\infty)}$ ) and now we want to calculate the value of consolidation settlement before completion of consolidation process ( $S_{c(t)}$ ), so, the value of  $S_{c(t)}$  is percent of  $S_{c(\infty)}$  such that:  $\frac{S_{c(t)}}{S_{c(\infty)}} = \text{percent}$ , this percent called

**[Average degree of consolidation (U)].**

$$\text{So, } U = \frac{S_{c(t)}}{S_{c(\infty)}}$$

Degree of consolidation ( $U_z$ ) is the percent of water dissipates from the clay layer at a certain depth (z) and at any time (t), But, the average degree of consolidation (U) is the total amount of water dissipates from the whole clay layer at time (t).

The value of (U) is dependent on the value of ( $T_v$ ), thus if one of them is known, we can calculate the other value by one of the following three ways:

1. By equations (Not preferable):

$$\text{For } U = 0 \text{ to } 60\%, \quad T_v = \frac{\pi}{4} \left(\frac{U\%}{100}\right)^2$$

$$\text{For } U > 60\%, \quad T_v = 1.781 - 0.933 \text{LOG}(100 - U\%)$$

2. By **(Figure 11.26)** (Not preferable)

3. By **(Table 11.7)** (Preferable and recommended).

Now, if we get the value of (U), we can calculate the value of ( $S_{c(t)}$ ) as following:

$$(S_{c(t)}) = U \times S_{c(\infty)}$$

### **Important Notes:**

$t_{50}$  = time required to reach  $U = 50\%$ ,  $t_{60}$  = time required to reach  $U = 60\%$  ....

All the following values are different in lab from the field for the same soil:

$$H_{dr(\text{lab})} \neq H_{dr(\text{field})}, \quad t_{\text{lab}} \neq t_{(\text{field})}, \quad \text{and } U_{\text{lab}} \neq U_{(\text{field})}$$

لأنه في المختبر تتم التجربة على عينة صغيرة من التربة مقارنة بالطبقات الكبيرة الموجودة في الواقع وأيضاً تتم عملية ال(consolidation) في فترة زمنية قصيرة مقارنة بالواقع الذي قد تستمر فيه العملية لسنوات.

But, the values of ( $c_v$ ,  $m_v$ , and  $K$ ) are the same in field and lab because they relates to type of soil.

## Secondary Consolidation Settlement ( $S_s$ ):

كما ذكرنا سابقاً، أن هذا النوع من الهبوط يبدأ في نهاية مرحلة (primary consolidation) حيث أنه يحدث نتيجة إعادة ترتيب حبيبات التربة وبسبب التشوه البلاستيكي الذي يحدث للتربة مع الزمن وتأثير الأحمال.

### Calculation of Secondary Consolidation Settlement:

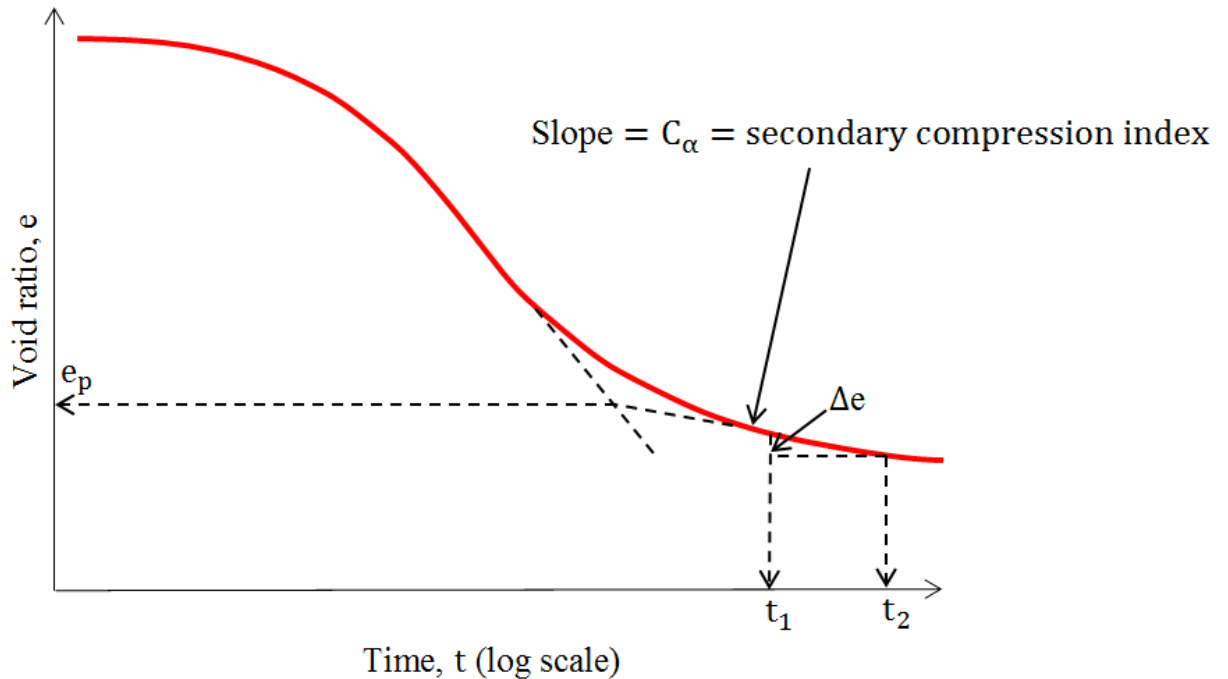
Previously, we derived the basic equation of consolidation settlement in general:

$$S_c = H \frac{\Delta e}{1+e_o}$$

But, the value of ( $e_o$ ) is the void ratio at the beginning of primary consolidation settlement so must be replaced by void ratio at the beginning of secondary consolidation settlement (or at the end of primary consolidation settlement) and this void ratio will be termed as: ( $e_p = e_{\text{primary}}$ ), so, the equation will be:

$$S_s = H \frac{\Delta e}{1+e_p}$$

Now, the values of ( $\Delta e$ ) and ( $e_p$ ) can be determined from the ( $e$  with  $\log t$ ) plot as following:



يتم تحديد قيمة ( $e_p$ ) عن طريق مد مماسين من أعلى ومن أسفل (حيث أن المنحنى يقترب من الخط المستقيم من أعلى ومن أسفل) وعند نقطة تقاطع هذين الخطين يتم مد خط أفقي حتى يقطع محور ( $y$ ) وتكون هي قيمة ( $e_p$ ).

We note from the above graph, the value of ( $\Delta e$ ) can be calculated as following:

$$\text{Slope} = \frac{\Delta e}{\log\left(\frac{t_2}{t_1}\right)} \rightarrow \Delta e = \text{Slope} \times \log\left(\frac{t_2}{t_1}\right)$$

Slope =  $C_\alpha$  = Secondary Compression Curve  $\rightarrow \rightarrow$

$$\Delta e = C_\alpha \times \log\left(\frac{t_2}{t_1}\right) \rightarrow \rightarrow$$

$$S_s = \frac{H \times C_\alpha}{1 + e_p} \times \log\left(\frac{t_2}{t_1}\right)$$

$t_1$  = time at the end of primary consolidation settlement (start secondary consolidation settlement).

$t_2$  = any time after beginning secondary consolidation settlement.

the value of  $C_\alpha$  is depend on type of clay (N.C.Clay or O.C.Clay or Organic clay)

And there is a typical values for each type.

(See Example 11.6) in your textbook.

### Finally 😊 : Why we concerned about settlement:

نهتم بدراسة وتحديد قيم الهبوط في التربة لما لها من أهمية كبيرة في التأثير على المنشآت المختلفة حيث أننا نهتم بشكل كبير في حساب الفرق في الهبوط أسفل منتصف البناية وأسفل طرفها على سبيل المثال حيث أنه كلما زاد ذلك الفرق (Differential Settlement) كلما زاد الخطر على المبنى وإذا زادت القيم إلى حد معين فإنه قد يؤدي إلى إنهيار المبنى بالكامل وبالتالي لا بد من دراسة وتحديد قيم الهبوط الناتجة عن وضع المبنى مع مرور الزمن لمراعاتها في عمليات التصميم للمنشآت. لكن إذا كان الهبوط أسفل جميع أجزاء المبنى متساوي أو متقارب فإنه لا يكون هناك خطر كبير ولا يؤدي إلى حدوث إنهيارات ولكن قد يؤدي إلى ظهور تشققات سطحية في المبنى خاصة في أماكن الأبواب والشبابيك، بالتالي دائما يجب محاولة تقليل الفرق في الهبوط (Differential Settlement) لما لها من خطر كبير.



**48. (Final 2009 and 2012)** A tower is to be constructed on the site shown below, it was decided that the expected settlement will be too large and so the site is to be preloaded before the construction of the tower. A consolidation test was prepared on a sample of **normal consolidated** clay in the laboratory (**height of specimen = 0.8 inch**) **double drained**, and the following data are given:

$$t_{50} = 5 \text{ minutes.}$$

A long **Virgin compression curve**:

$$\sigma_1 = 500 \text{ psf} \rightarrow e_1 = 0.6$$

$$\sigma_2 = 1000 \text{ psf} \rightarrow e_2 = 0.54$$

A- Calculate the coefficient of consolidation  $C_v(\text{ft}^2/\text{yr})$ .

B- Calculate the compression index  $C_c$ .

C- Find the permeability coefficient  $k(\text{ft}/\text{yr})$ .

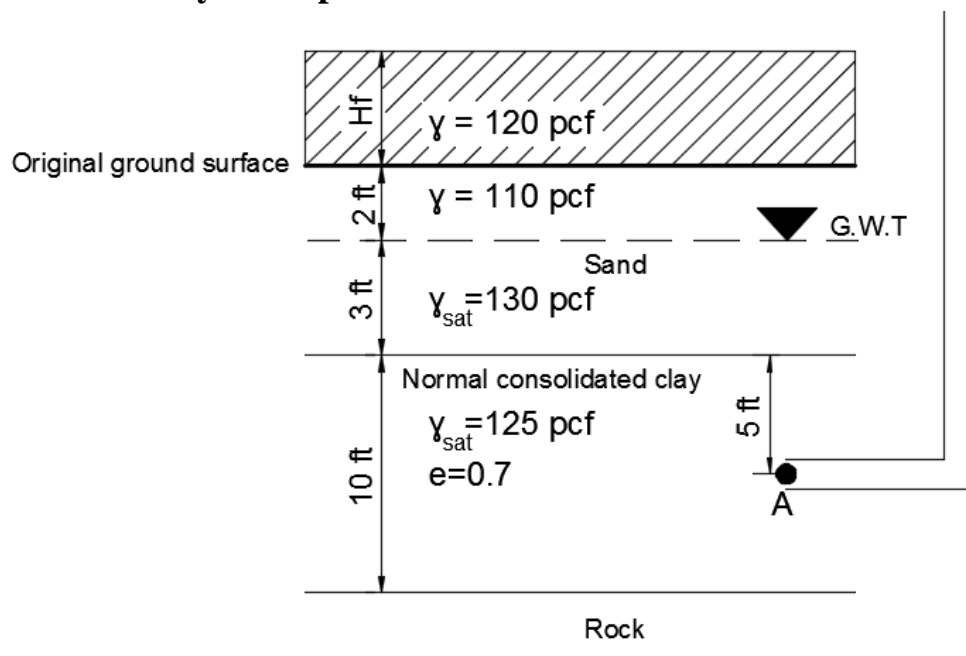
D- For preloading, a fill is to be placed for **one year**, what is the average degree of consolidation after one year.

E- We want this fill to result in 2 inches of settlement at the end of one year, what is required height of fill (**Hint**;  $\Delta\sigma$  will be due to the fill weight).

F- If OCR becomes 1.3, what is the consolidation settlement after one year ( $C_s = 0.04$ ).

G- If a piezometer is to be placed at point A, what will be the height of water in the piezometer in the following cases:

- **Before placement of fill.**
- **Immediately after placement of fill.**
- **After one year of placement of fill.**



## Solution

**شرح السؤال:** يوجد برج يُراد إنشاؤه فوق مقطع من التربة كما يتضح في الشكل, ولكن المصممون توقعوا أن يكون الهبوط كبير جدا نتيجة أحمال البرج الكبيرة وبالتالي قد يشكل خطر كبير على البرج في المستقبل حيث أنه تم وضع حل أن يتم تحميل هذه التربة قبل الشروع في إنشاء البرج عن طريق وضع طمم من التربة الرملية فوق مقطع التربة ولمدة سنة وذلك حتى تهبط التربة نتيجة الحمل المطبق عليها من الطمم وتستقر حالة التربة, وبالتالي عند إنشاء البرج تكون التربة في حالة استقرار وتكون قيم الهبوط صغيرة في التربة لأن الهبوط الأكبر قد حدث نتيجة وضع الطمم وبالتالي لا يوجد خطر على البرج.

A- ( $C_v(\text{ft}^2/\text{yr}) = ???$ )

$$T_v = \frac{C_v \times t}{H_{dr}^2} \rightarrow C_v = \frac{T_v \times H_{dr}^2}{t}$$

$$H_{dr} = \frac{H}{2} \text{ (double drained)} = \frac{0.8}{2} = 0.4 \text{ inch} = 0.4 \text{ inch} \times \frac{1 \text{ ft}}{12 \text{ inch}} = 0.0333 \text{ ft.}$$

$$t = t_{50} = 5 \text{ minutes} \times \frac{1 \text{ hr}}{60 \text{ minutes}} \times \frac{1 \text{ day}}{24 \text{ hr}} \times \frac{1 \text{ yr}}{365 \text{ day}} = 9.513 \times 10^{-6} \text{ yr.}$$

$t_{50}$  means  $U = 50\% \rightarrow T_v = 0.197$  (from **Table 11.7**).

$$C_v = \frac{0.197 \times 0.0333^2}{9.513 \times 10^{-6}} \cong 23 \text{ ft}^2/\text{yr} \checkmark.$$

B- ( $C_c = ???$ )

$C_c$  = Slope of virgin compression curve

You must know that, the virgin curve is (e-log  $\sigma'$ ) plot.

$$C_c = \text{Slope} = \frac{e_1 - e_2}{\log\left(\frac{\sigma'_2}{\sigma'_1}\right)} = \frac{0.6 - 0.54}{\log\left(\frac{1000}{500}\right)} = 0.1993 \checkmark.$$

C- ( $k(\text{ft}/\text{yr}) = ???$ )

$$C_v = \frac{k}{\gamma_w \times m_v} \rightarrow k = C_v \times \gamma_w \times m_v$$

$$C_v = 23 \text{ ft}^2/\text{yr} \text{ (As calculated in part A)}, \gamma_w = 62.4 \text{ lb}/\text{ft}^3$$

Previously, the formula of calculating ( $m_v$ ) was derived as following:

$$m_v = \frac{\left(\frac{e_1 - e_2}{\sigma'_2 - \sigma'_1}\right)}{1 + \left(\frac{e_1 + e_2}{2}\right)} = \frac{\left(\frac{0.6 - 0.54}{1000 - 500}\right)}{1 + \left(\frac{0.6 + 0.54}{2}\right)} = 7.64 \times 10^{-5} \text{ ft}^2/\text{lb}$$

$$\text{So, } k = 23 \times 62.4 \times 7.64 \times 10^{-5} = 0.1097 \text{ ft}/\text{yr} \checkmark.$$

**D- (In site, if  $t = 1\text{ yr} \rightarrow U = ???$ )**

$U$  is related to  $T_v$ , so firstly we want to calculate  $T_v$  in site

$$T_v = \frac{C_v \times t}{H_{dr}^2}, \quad t = 1 \text{ yr}, \quad C_v = 23 \text{ ft}^2/\text{yr} \text{ (the same in field and lab)}$$

$H_{dr} = H = 10\text{m}$  (Because there is a rock layer under clay layer).

$$T_v = \frac{23 \times 1}{10^2} = 0.23 \text{ (unit less)}$$

Now from (Table 11.7) at  $T_v = 0.23 \rightarrow U = 54\% \checkmark$ .

**E- (If  $S_{c(t=1\text{yr})} = 2\text{inch} \rightarrow H_f = ???$ )**

We know that,  $U = \frac{S_{c(t)}}{S_{c(t=\infty)}}$

$S_{c(t=1\text{yr})} = 2\text{inch}$ , at  $t = 1\text{year} \rightarrow U = 54\%$  (As calculated in part D)

$$0.54 = \frac{2}{S_{c(t=\infty)}} \rightarrow S_{c(t=\infty)} = 3.703\text{inch} = 0.3086 \text{ ft.}$$

Now, we want to calculate the value of  $S_{c(t=\infty)}$ , for normally consolidated clay the formula of calculating  $S_{c(t=\infty)}$  is:

$$S_{c(t=\infty)} = \frac{C_c \times H}{1 + e_0} \times \log \left( \frac{\sigma'_0 + \Delta\sigma'}{\sigma'_0} \right)$$

$C_c = 0.1993$  (As calculated in part B),  $H = 10\text{m}$ ,  $e_0 = 0.7$  (from soil profile)

Now we want to calculate the value of  $(\sigma'_0)$  (always at the **middle** of clay layer)

$$\sigma_t = 110 \times 2 + 130 \times 3 + 125 \times \left( \frac{10}{2} \right) = 1235 \text{ lb/ft}^2$$

$$u_0 = 62.4 \times 3 + 62.4 \times \left( \frac{10}{2} \right) = 499.2 \text{ lb/ft}^2$$

$$\sigma'_0 = 978 - 499.2 = 735.8 \text{ lb/ft}^2$$

Note that, the fill is distributed on the whole soil profile, thus, the value of  $(\Delta\sigma)$  will be constant at any depth in soil profile (See introduction of CH.10), so:

$$\text{at } t = \infty \rightarrow \Delta\sigma' = \Delta\sigma = \gamma \times H_f \rightarrow \Delta\sigma' = 120 \times H_f$$

$$S_{c(t=\infty)} = 0.3086 = \frac{0.1993 \times 10}{1 + 0.7} \times \log \left( \frac{735.8 + \Delta\sigma'}{735.8} \right) \rightarrow \Delta\sigma' \cong 613 \text{ lb/ft}^2$$

$$\Delta\sigma' = 613 = 120 \times H_f \rightarrow 5.1 \text{ ft } \checkmark.$$

F- (If  $OCR = 1.3$  and  $C_s = 0.04 \rightarrow S_{c(t=1yr)} = ???$ )

$$\text{At } t = 1\text{year} \rightarrow U = 54\% \rightarrow \frac{S_{c(t=1yr)}}{S_{c(t=\infty)}} = 0.54 \rightarrow S_{c(t=1yr)} = 0.54 \times S_{c(t=\infty)}$$

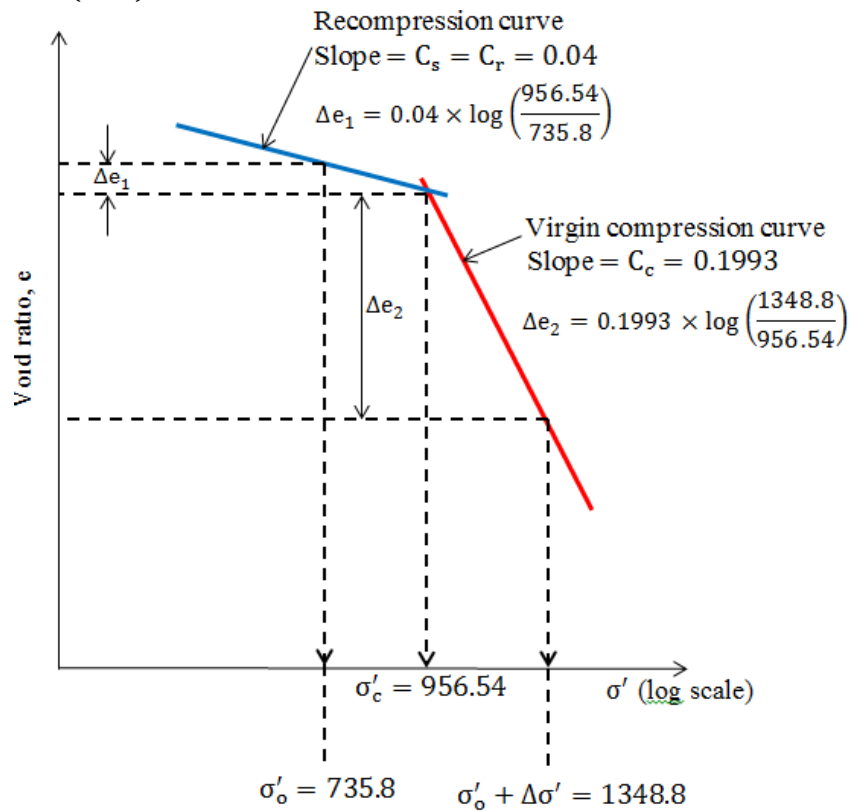
Now, we want to calculate the value of  $S_{c(t=\infty)}$ :

$OCR = 1.3 > 1 \rightarrow$  Overconsolidated clay

$$OCR = 1.3 = \frac{\sigma'_c}{\sigma'_o} \rightarrow \sigma'_c = 1.3 \times 735.8 = 956.54.$$

$$\Delta\sigma' = 613 \text{ (From part E)} \rightarrow \Delta\sigma' + \sigma'_o = 613 + 735.8 = 1348.8 \text{ lb/ft}^2$$

The formula of  $S_{c(t=\infty)}$  at this case can be derived from the following graph:



$$S_{c(t=\infty)} = H \frac{\Delta e}{1 + e_o} \text{ (Basic equation)}$$

So, according to the above graph and the value of  $(\Delta e = \Delta e_1 + \Delta e_2)$  :

$$S_{c(t=\infty)} = \frac{10 \times 0.04}{1 + 0.7} \times \log\left(\frac{956.54}{735.8}\right) + \frac{10 \times 0.1993}{1 + 0.7} \times \log\left(\frac{1348.8}{956.54}\right) = 0.2017\text{ft}$$

$$\text{So, } S_{c(t=1yr)} = 0.54 \times S_{c(t=\infty)} = 0.54 \times 0.2017 = 0.109\text{ft} \checkmark.$$

**G- ( $h_p = ???$  in the following cases)**

In general, the piezometric height is the pressure head and ( $h_p = \frac{u}{\gamma_w}$ )

And the general formula to calculate ( $u$ ) is: ( $u = u_o + \Delta u$ )

The value of ( $u_o$ ) is the present pore water pressure due to the rise of G.W.T over any point and is calculated in (part E) ( $u_o = 499.2 \text{ lb/ft}^2$ ).

As we mentioned previously the value of ( $\Delta u$ ) is varies with time and we will calculate it at the different required three cases.

- **Case 1 (Before placement of fill)**

Before placement of fill means ( $\Delta\sigma = 0.0 \rightarrow \Delta u = 0.0$ )  $\rightarrow\rightarrow\rightarrow$

$$u = 499.2 + 0.0 = 499.2 \rightarrow h_p = \frac{499.2}{62.4} = 8 \text{ ft} \checkmark.$$

- **Case 2 (Immediately after placement of fill) ( $t = 0.0$ ):**

In this case the whole value of ( $\Delta\sigma$ ) will be carried by water because the load doesn't transfer to the soil yet  $\rightarrow (\Delta u = \Delta\sigma = 613)$   $\rightarrow\rightarrow$

$$u = 499.2 + 613 = 1112.2 \rightarrow h_p = \frac{1112.2}{62.4} = 17.82 \text{ ft} \checkmark.$$

- **Case 3 (after one year of placement the fill) ( $0 < t < \infty$ ):**

After completion of placement the loads the value of ( $\Delta\sigma$ ) start to transfer to clay and the water start to dissipate from the clay ( $\Delta u$  decreases with time)  $\rightarrow (\Delta u < \Delta\sigma)$

To calculate the value of ( $\Delta u_{(z,t)}$ ) at ( $t = 1\text{yr}$ ) we will use the formula:

$$U_z = \frac{\Delta\sigma - \Delta u_{(z,t)}}{\Delta\sigma}$$

$\Delta\sigma =$  initial value of  $\Delta u = 613 \text{ lb/ft}^2$

$\Delta u_{(z,t)} = ???$  at  $t = 1\text{yr}$  and  $z_A = 5 \text{ ft}$  ("z" start from the top of clay layer)

To calculate ( $\Delta u_{(5\text{ft},1\text{yr})}$ ) we must calculate the value of ( $U_z$ )  $\rightarrow$

The value of ( $U_z$ ) can be calculated from (**Figure 11.25**) according the values of ( $\frac{z}{H_{dr}}$ ) and  $T_v$ . ( $H_{dr} = 10$ ,  $T_{v(1\text{yr})} = 0.23$ ) as calculated above

$$\frac{z}{H_{dr}} = \frac{5}{10} = 0.5, \quad T_{v(1\text{yr})} = 0.23 \rightarrow U_z \cong 0.54 \rightarrow\rightarrow$$

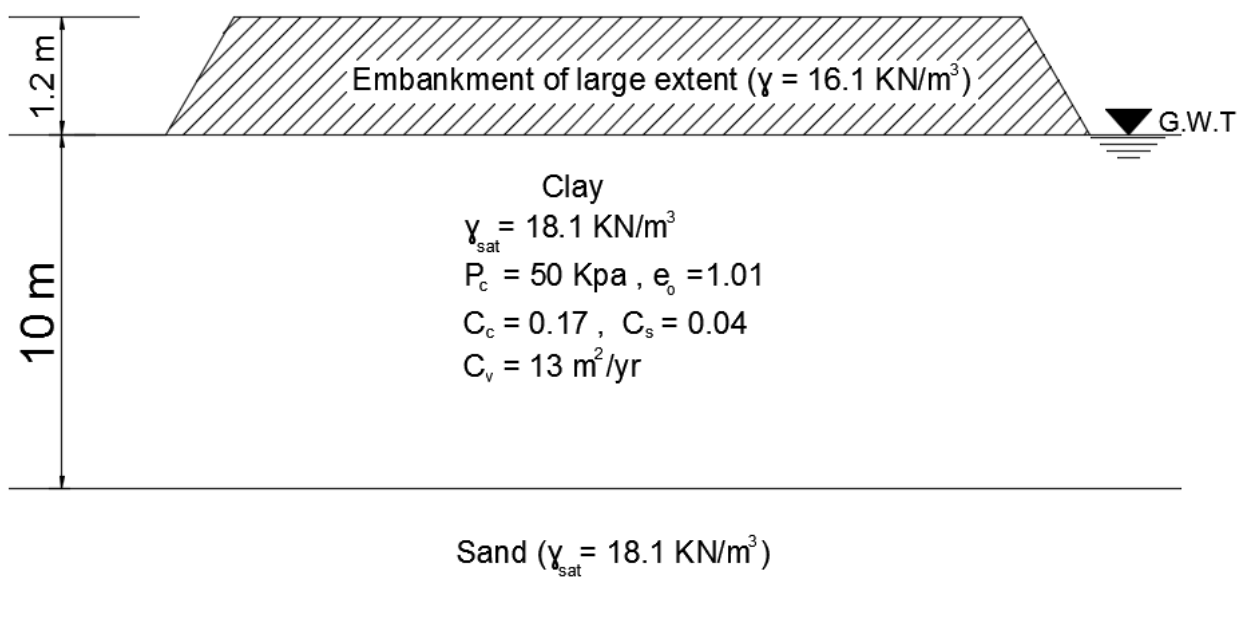
$$0.54 = \frac{613 - \Delta u_{(5\text{ft},1\text{yr})}}{613} \rightarrow \Delta u_{(5\text{ft},1\text{yr})} = 281.98 \text{ lb/ft}^2 \rightarrow\rightarrow\rightarrow$$

$$u = 499.2 + 281.98 = 781.18 \rightarrow h_p = \frac{781.18}{62.4} = 12.52 \text{ ft} \checkmark.$$

## 49.

A 1.2 m height highway embankment of **large lateral extent** is to be placed over a surface clay deposit of 10 m thickness which is underlain by sand as shown in the figure below.

- Compute the ultimate settlement expected under the embankment.
- If the maximum tolerable settlement of the embankment surface pavement is 3cm, **when** should the pavement be placed after completion of embankment placement?
- What is the average excess pore water pressure at the completion of embankment placement?
- What is the total effective stress (at mid of clay layer) at the **time of paving**?



## Solution

**شرح السؤال:** كما في السؤال السابق، يراد إنشاء طريق سريع فوق مقطع التربة الموضح في الشكل، لكن قبل إنشاء هذا الطريق تم وضع سد رملي فوق هذه التربة حتى تهبط وتستقر التربة وبعد فترة زمنية معينة من وجود هذا السد فإن التربة يحدث فيها نوع من الاستقرار وتكون قد هبطت واستقرت وبالتالي يمكن رصف الطريق في ذلك الوقت دون مواجهة أي مخاطر مستقبلية كبيرة.

A- ( $S_{c(t=\infty)}$  = ???)

Firstly, we want to know the type of clay (N.C.Clay or O.C.Clay):

Calculating the effective stress at the **middle** of clay layer:

$$\sigma_t = 18.1 \times \left(\frac{10}{2}\right) = 90.5 \text{ KN/m}^2$$

$$u_o = 9.81 \times \left(\frac{10}{2}\right) = 49.05 \text{ KN/m}^2$$

$$\sigma'_o = 90.5 - 49.05 = 41.45 \text{ lb/ft}^2$$

The preconsolidation pressure ( $\sigma'_c = 50\text{kpa} = 50\text{KN/m}^2$ )(given in graph)

$$\text{OCR} = \frac{50}{41.45} = 1.206 > 1 \rightarrow (\text{O. C. Clay})$$

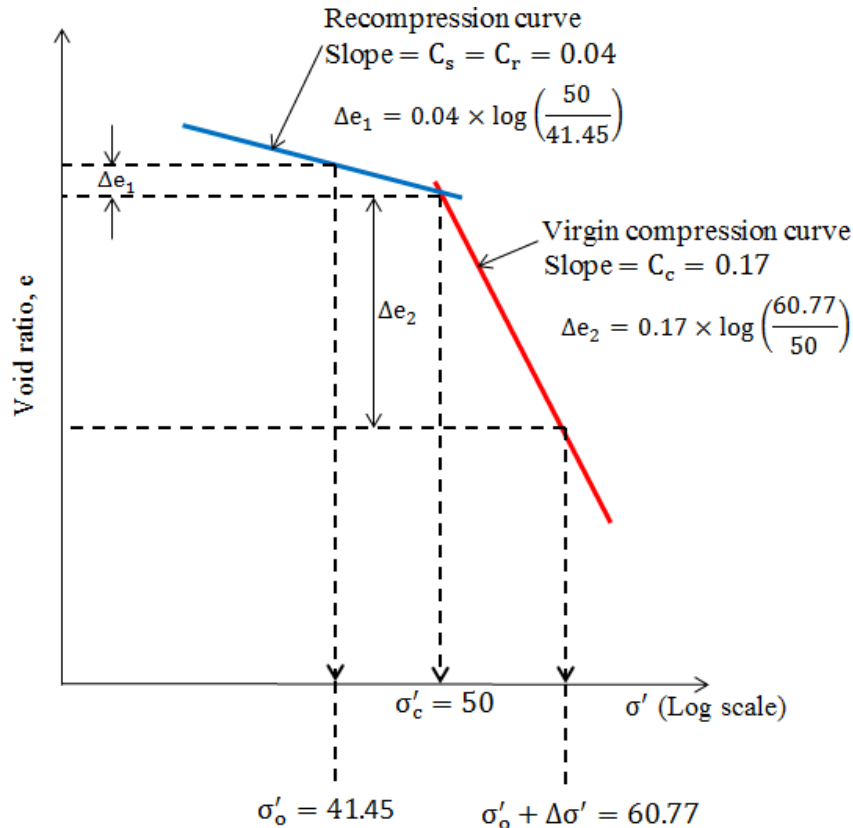
Because the embankment has large lateral extent, the value of ( $\Delta\sigma$ ) will be constant with depth (See introduction of CH.10)

$$\text{at } (t = \infty) \rightarrow \Delta\sigma = \Delta\sigma' = 1.2 \times 16.1 = 19.32 \text{ KN/m}^2$$

$$\sigma'_o + \Delta\sigma' = 41.45 + 19.32 = 60.77 \text{ KN/m}^2$$

$C_c = 0.17$  ,  $C_s = 0.04$  ,  $e_o = 1.01$  (Givens on above graph)

The formula of  $S_{c(t=\infty)}$  at this case can be derived from the following graph:



$$S_{c(t=\infty)} = H \frac{\Delta e}{1 + e_0} \text{ (Basic equation)}$$

So, according the above graph and the value of ( $\Delta e = \Delta e_1 + \Delta e_2$ ):

$$S_{c(t=\infty)} = \frac{10 \times 0.04}{1 + 1.01} \times \log\left(\frac{50}{41.45}\right) + \frac{10 \times 0.17}{1 + 1.01} \times \log\left(\frac{60.77}{50}\right) \rightarrow \rightarrow$$

$$S_{c(t=\infty)} = 0.0878\text{m} = 8.78\text{cm} \checkmark .$$

**B- (t = ??? if  $S_{c(t)} = 3\text{cm}$ ):**

$$\text{We know that, } U = \frac{S_{c(t)}}{S_{c(t=\infty)}} = \frac{3}{8.78} \times 100 = 34.17\%$$

(From **Table 11.7**  $\rightarrow$  at  $U = 34.16\% \rightarrow T_v \cong 0.091$ )

$$T_v = \frac{C_v \times t}{H_{dr}^2} \quad (C_v = 13\text{m}^2/\text{yr}, \quad H_{dr} = H = 10\text{m}) \quad H_{dr} = H \rightarrow \rightarrow$$

(Beacuse there is sand below the clay and fill pavement(prevent drainage) above)

$$0.091 = \frac{13 \times t}{10^2} \rightarrow t = 0.7 \text{ yr} = 255.5 \text{ day} \checkmark .$$

**C- ( $\Delta u_{(5,0.0)} = ???$ )**

Average excess (means at the mid of clay layer “at  $z = 5\text{m}$ ”)

At the completion of embankment placement means ( $t = 0.0$ )

$$\text{So, } \Delta u_{(5,0.0)} = \Delta \sigma = 19.32 \text{ KN/m}^2 \checkmark .$$

**D- ( $\sigma'_t = ???$  at t(time of pavement “part B”) = 0.7 yr):**

$$\sigma'_t = \sigma'_o + \Delta \sigma'_{(z,t)} \quad (z = 5\text{m}, t = 0.7 \text{ yr})$$

To calculate ( $\Delta \sigma'_{(5\text{m},0.7\text{yr})}$ ), we must calculate( $\Delta u_{(5\text{m},0.7\text{yr})}$ )

$$U_z = \frac{\Delta \sigma - \Delta u_{(5\text{m},0.7\text{yr})}}{\Delta \sigma}, \quad \Delta \sigma = 19.32 \text{ KN/m}^2$$

$$\frac{z}{H_{dr}} = \frac{5}{10} = 0.5, \quad T_{v(0.7\text{yr})} = 0.091 \rightarrow U_z \cong 0.23 \text{ (from Figure 11.25)} \rightarrow \rightarrow$$

$$0.23 = \frac{19.32 - \Delta u_{(5\text{m},0.7\text{yr})}}{19.32} \rightarrow \Delta u_{(5\text{m},0.7\text{yr})} = 14.87 \text{ KN/m}^2 \rightarrow$$

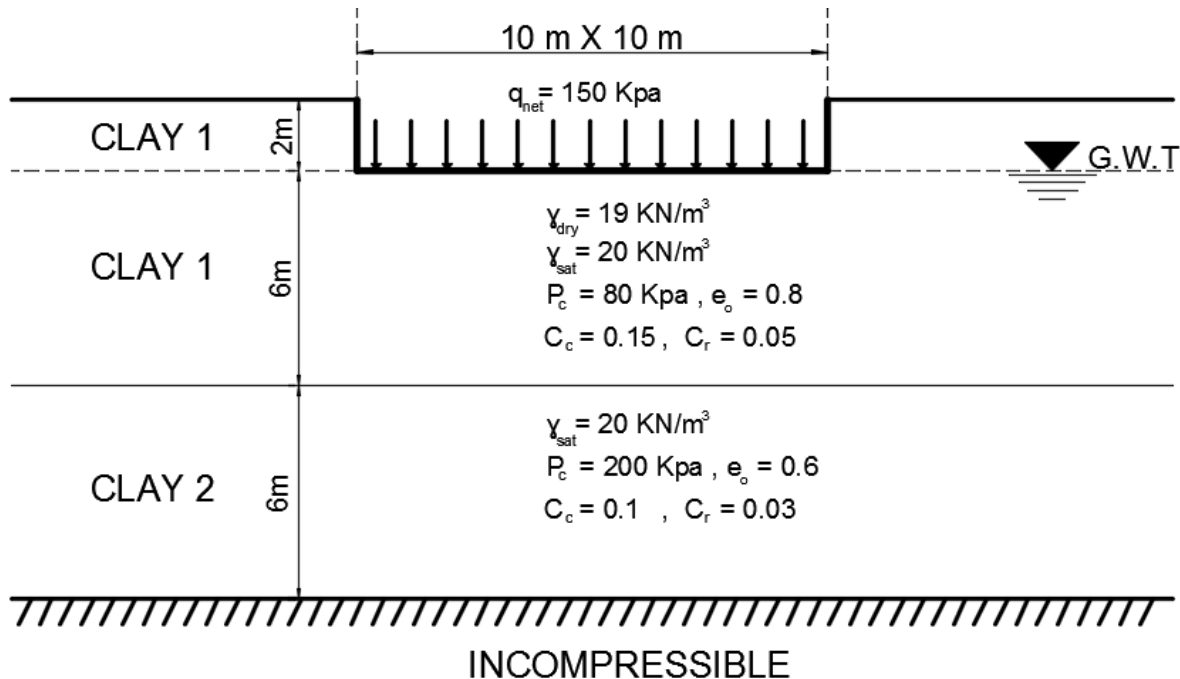
$$\Delta \sigma'_{(5\text{m},0.7\text{yr})} = \Delta \sigma - \Delta u_{(5\text{m},0.7\text{yr})} = 19.32 - 14.87 = 4.45 \text{ KN/m}^2$$

$$\sigma'_t = 41.45 + 4.45 = 45.9 \text{ KN/m}^2 \checkmark .$$



**50.**

In the soil profile shown below, calculate the total primary consolidation settlement due to 150 KPa net foundation loading. (Take  $\gamma_w = 10 \text{ KN/m}^3$ ).

**Solution**

As shown, there are two clay layers, so firstly we must calculate primary consolidation settlement for each clay layer individually then summation the values for the two layers to get the total settlement.

**For CLAY (1):**

$\sigma'_c = 80 \text{ KPa}$  ,  $C_c = 0.15$  ,  $C_s = 0.05$  ,  $e_o = 0.8$  (Givens on above graph)

Now, we want to calculate the value of  $(\sigma'_o)$  at the **middle** of clay layer.

$$\sigma_t = 19 \times 2 + 20 \times \left(\frac{6}{2}\right) = 98 \text{ KN/m}^2$$

$$u_o = 10 \times \left(\frac{6}{2}\right) = 30 \text{ KN/m}^2$$

$$\sigma'_o = 98 - 30 = 68 \text{ KN/m}^2 \rightarrow \rightarrow$$

Note that the value of  $(\sigma'_o)$  is always calculated from the **ground surface** to the middle of clay layer.

$$\text{OCR} = \frac{\sigma'_c}{\sigma'_o} = \frac{80}{68} = 1.17 > 1 \rightarrow \rightarrow (\text{Overconsolidated Clay})$$

Now, we want to calculate the value of  $(\Delta\sigma')$  at the **middle** of clay layer:

It is clear that, the foundation does not distribute on the whole soil profile so, the value of  $(\Delta\sigma')$  will **decrease** with depth (See introduction of CH.10) and we will calculate the average value of  $(\Delta\sigma' = \Delta\sigma'_{av})$ , thus we must calculate  $(\Delta\sigma')$  at the top, middle, and bottom of clay layer to use Simpson's rule such that:

$$\Delta\sigma'_{av} = \frac{\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b}{6}$$

To calculate  $(\Delta\sigma')$  at any depth under the **center** of foundation we will use the formula that discussed previously in (CH.10):

$$\Delta\sigma_{z,t} = qI_4 \quad (q = 150 \text{ KN/m}^2)$$

$$I_4 = f(m_1, n_1) \text{ (From Table 10.10)}$$

$$m_1 = \frac{L}{B}, \quad n_1 = \frac{Z}{b} = \frac{2Z}{B}$$

(At the top of clay one "z = 0.0")

Because  $z = 0.0 \rightarrow$  the value of  $(\Delta\sigma)$  will not change.

$$\Delta\sigma'_{top} = 150 \text{ KN/m}^2$$

(At the middle of clay one "z = 3 m")

$$m_1 = \frac{10}{10} = 1, \quad n_1 = \frac{2 \times 3}{10} = 0.6 \rightarrow I_4 = 0.892$$

$$\rightarrow \Delta\sigma'_{middle} = 0.892 \times 150 = 133.8 \text{ KN/m}^2$$

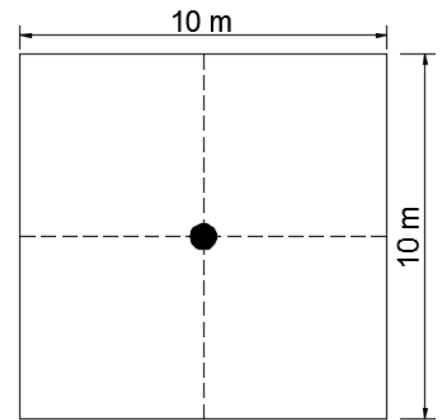
(At the bottom of clay one "z = 6 m")

$$m_1 = \frac{10}{10} = 1, \quad n_1 = \frac{2 \times 6}{10} = 1.2 \rightarrow I_4 = 0.606$$

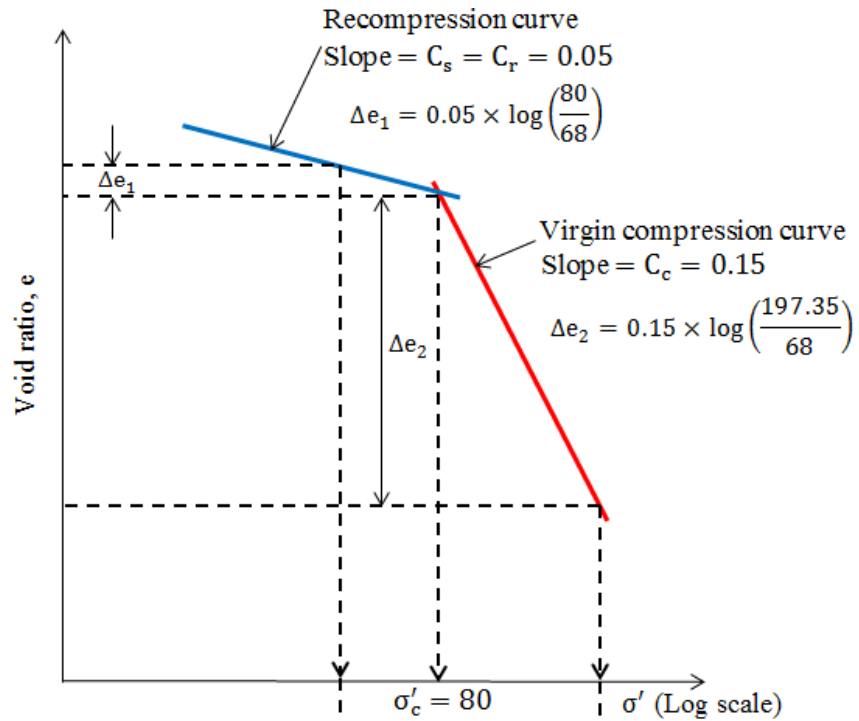
$$\rightarrow \Delta\sigma'_{bottom} = 0.606 \times 150 = 90.9 \text{ KN/m}^2 \rightarrow \rightarrow$$

$$\Delta\sigma'_{av} = \frac{150 + 4 \times 133.8 + 90.9}{6} = 129.35 \text{ KN/m}^2$$

$$\sigma'_o + \Delta\sigma'_{av} = 68 + 129.35 = 197.35 \text{ KN/m}^2$$



The formula of  $S_{c(t=\infty)}$  at this case can be derived from the following graph:



$$S_{c(t=\infty)} = H \frac{\Delta e}{1 + e_0} \text{ (Basic equation)} \quad \sigma'_o = 68 \quad \sigma'_o + \Delta\sigma' = 197.35$$

So, according to the above graph and the value of  $(\Delta e = \Delta e_1 + \Delta e_2)$ :

$$S_{c(t=\infty)} = \frac{6 \times 0.05}{1 + 0.8} \times \log\left(\frac{80}{68}\right) + \frac{6 \times 0.15}{1 + 0.8} \times \log\left(\frac{197.35}{80}\right) \rightarrow \rightarrow$$

$$S_{c(t=\infty)} = 0.207 \text{ m} = 20.7 \text{ cm} \checkmark.$$

**For CLAY (2):**

$\sigma'_c = 200 \text{ KPa}$  ,  $C_c = 0.1$  ,  $C_s = 0.03$  ,  $e_o = 0.6$  (Givens on above graph)

Now, we want to calculate the value of  $(\sigma'_o)$  at the **middle** of clay layer.

$$\sigma_t = 19 \times 2 + 20 \times 6 + 20 \times \left(\frac{6}{2}\right) = 218 \text{ KN/m}^2$$

$$u_o = 10 \times 6 + 10 \times \left(\frac{6}{2}\right) = 90 \text{ KN/m}^2$$

$$\sigma'_o = 218 - 90 = 128 \text{ KN/m}^2 \rightarrow \rightarrow$$

$$\text{OCR} = \frac{\sigma'_c}{\sigma'_o} = \frac{200}{128} = 1.56 > 1 \rightarrow \rightarrow \text{(Overconsolidated Clay)}$$

(At the top of clay two “z = 6m” )

Is the same value at the bottom of CLAY (1)

$$\Delta\sigma'_{top} = 90.9 \text{ KN/m}^2$$

(At the middle of clay two “z = 9 m” )

$$m_1 = \frac{10}{10} = 1 \quad , \quad n_1 = \frac{2 \times 9}{10} = 1.8 \rightarrow I_4 = 0.388$$

$$\rightarrow \Delta\sigma'_{middle} = 0.388 \times 150 = 58.2 \text{ KN/m}^2$$

(At the bottom of clay two “z = 12 m” )

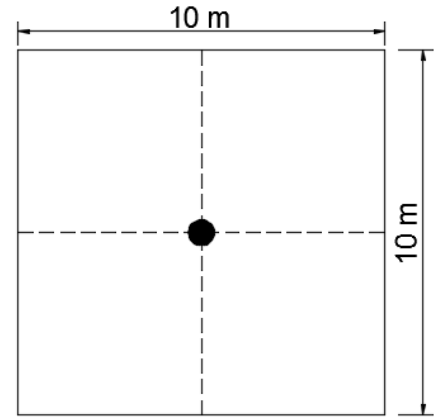
$$m_1 = \frac{10}{10} = 1 \quad , \quad n_1 = \frac{2 \times 12}{10} = 2.4 \rightarrow I_4 \cong 0.245$$

$$\rightarrow \Delta\sigma'_{bottom} = 0.245 \times 150 = 36.75 \text{ KN/m}^2 \rightarrow \rightarrow$$

$$\Delta\sigma'_{av} = \frac{90.9 + 4 \times 58.2 + 36.75}{6} \cong 60 \text{ KN/m}^2$$

$$\sigma'_o + \Delta\sigma'_{av} = 128 + 60 = 188 \text{ KN/m}^2$$

The formula of  $S_{c(t=\infty)}$  at this case can be derived from the following graph:



$$S_{c(t=\infty)} = H \frac{\Delta e}{1 + e_o} \text{ (Basic equation)}$$

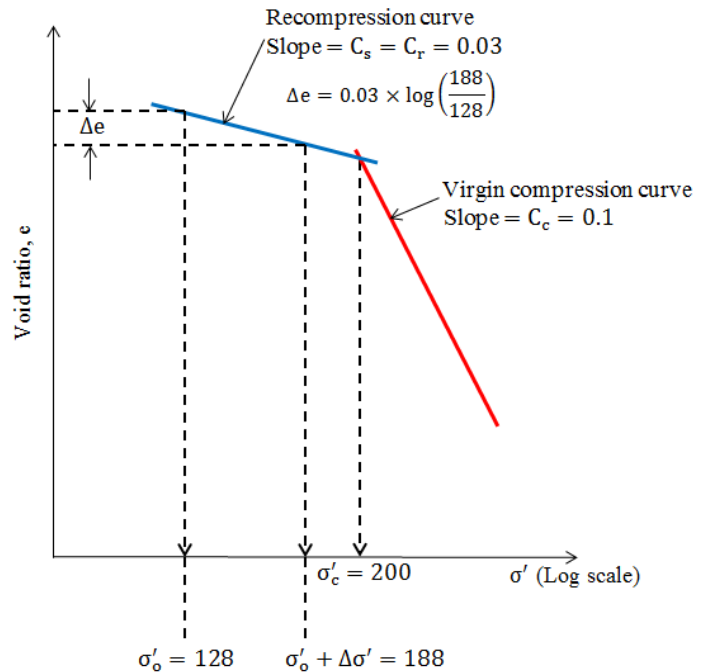
So, according the above graph and the value of ( $\Delta e$ ):

$$S_{c(t=\infty)} = \frac{6 \times 0.03}{1 + 0.6} \times \log\left(\frac{188}{128}\right) \rightarrow$$

$$S_{c(t=\infty)} = 0.0187 \text{ m} = 1.87 \text{ cm}$$

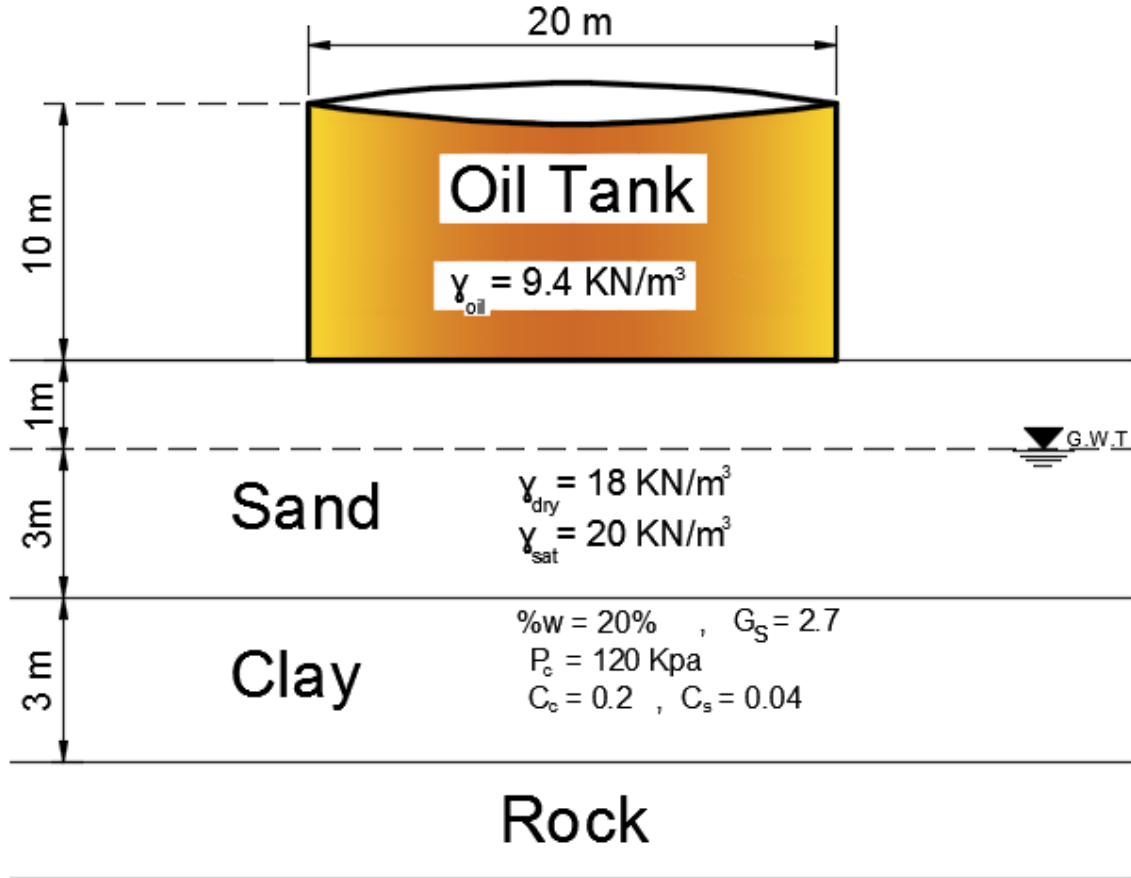
So, the total consolidation settlement is the summation of settlement for CLAY(1) and CLAY(2):

$$S_{c(total)} = 20.7 + 1.87 = 22.57 \text{ cm} \checkmark .$$



## 51.

An oil tank, 20 m in diameter, is built over a site shown below. A specimen of clay in this site (**20 mm thickness**) **double drained** was to be tested in an oedometer apparatus, the time required to reach 60% degree of consolidation is 15 minutes. Estimate the differential settlement between the **center** and **perimeter** of the tank after 1 year after building the tank.



## Solution

**شرح السؤال:** يراد إنشاء خزان نفط فوق مقطع التربة الموضح أعلاه, حيث تم أخذ عينة من طبقة الطين وفحصها في المختبر بهدف تحديد المعاملات التي تلزم في حساب (Coefficient of consolidation) والمطلوب هو حساب فرق الهبوط بين وسط وطرف الخزان بعد سنة من بناء الخزان, حيث نلاحظ أن الخزان دائري الشكل وأنه غير موزع على كامل التربة, لذلك فإن قيمة الضغط الناتج عن وزن الخزان تقل تدريجياً مع زيادة العمق كما تم توضيحه في (CH.10) وبالتالي فإن الضغط المؤثر على طبقة الطين أسفل منتصف الخزان يختلف عنه أسفل طرف الخزان وبالتالي فإن قيم الهبوط تختلف أسفل الوسط وأسفل الطرف, حيث نحسب قيمة الهبوط أسفل منتصف الخزان وأسفل طرف الخزان ونجد الفرق بينهما وهو المطلوب.

The first step is to find the average stress increase at the middle of clay layer under the center of the tank and under the edge (perimeter) of the tank (As CH.10):

$$\Delta\sigma = \text{weight of the oil tank} = \gamma_{\text{oil}} \times h_{\text{tank}}$$

$$\gamma_{\text{oil}} = 9.4 \text{ KN/m}^3, \quad h_{\text{tank}} = 10 \text{ m} \rightarrow \Delta\sigma = q = 9.4 \times 10 = 94 \text{ KN/m}^2$$

**Calculating the value of  $(\Delta\sigma'_{\text{av}})$  at the middle of clay under the center of the tank:**

At top of clay layer:

$$R = 10 \text{ m}, \quad Z = 4 \text{ m} \rightarrow \frac{Z}{R} = 0.4 \rightarrow (\text{Table 10.6})$$

$$\frac{\Delta\sigma'_{\text{top}}}{q} = 0.9488 \rightarrow \Delta\sigma'_{\text{top}} = 94 \times 0.9488 = 89.18 \text{ KN/m}^2$$

At middle of clay layer:

$$R = 10 \text{ m}, \quad Z = 5.5 \text{ m} \rightarrow \frac{Z}{R} = 0.55 \rightarrow (\text{Table 10.6})$$

$$\frac{\Delta\sigma'_{\text{middle}}}{q} \cong 0.8848 \text{ (interpolation)}$$

$$\rightarrow \Delta\sigma'_{\text{middle}} = 94 \times 0.8848 = 83.17 \text{ KN/m}^2$$

At bottom of clay layer:

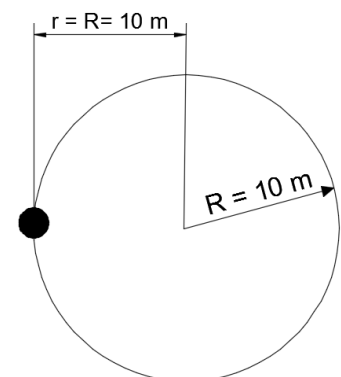
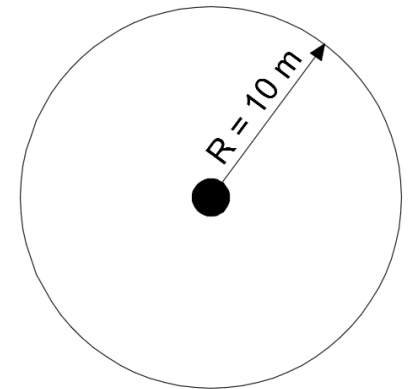
$$R = 10 \text{ m}, \quad Z = 7 \text{ m} \rightarrow \frac{Z}{R} = 0.7 \rightarrow (\text{Table 10.6})$$

$$\frac{\Delta\sigma'_{\text{bottom}}}{q} \cong 0.807 \text{ (interpolation)}$$

$$\rightarrow \Delta\sigma'_{\text{bottom}} = 94 \times 0.807 = 75.85 \text{ KN/m}^2$$

$$\Delta\sigma'_{\text{av}} = \frac{89.18 + 4 \times 83.17 + 75.85}{6} \cong 83 \text{ KN/m}^2$$

**Calculating the value of  $(\Delta\sigma'_{\text{av}})$  at the middle of clay under the center of the tank:**



At top of clay layer:

$$r = 10 \text{ m}, R = 10 \text{ m}, Z = 4 \text{ m} \rightarrow \frac{Z}{R} = \frac{4}{10} = 0.4 \text{ and } \frac{r}{R} = \frac{10}{10} = 1$$

$$\rightarrow A' = 0.31048 \text{ and } B' = 0.12404 \text{ (Table 10.7 and Table 10.8)}$$

$$\Delta\sigma'_{\text{top}} = q \times (A' + B') = 94 \times (0.31048 + 0.12404) = 40.84 \text{ KN/m}^2$$

At middle of clay layer:

$$r = 10 \text{ m}, R = 10 \text{ m}, Z = 5.5 \text{ m} \rightarrow \frac{Z}{R} = \frac{5.5}{10} = 0.55 \text{ and } \frac{r}{R} = \frac{10}{10} = 1$$

$$\rightarrow A' = 0.26872 \text{ and } B' = 0.140155 \text{ (average) (Table 10.7 and Table 10.8)}$$

$$\Delta\sigma'_{\text{middle}} = q \times (A' + B') = 94 \times (0.26872 + 0.140155) = 38.43 \text{ KN/m}^2$$

At bottom of clay layer:

$$r = 10 \text{ m}, R = 10 \text{ m}, Z = 7 \text{ m} \rightarrow \frac{Z}{R} = \frac{7}{10} = 0.7 \text{ and } \frac{r}{R} = \frac{10}{10} = 1$$

$$\rightarrow A' = 0.21727 \text{ and } B' = 0.14986 \text{ (Table 10.7 and Table 10.8)}$$

$$\Delta\sigma'_{\text{bottom}} = q \times (A' + B') = 94 \times (0.21727 + 0.14986) = 34.51 \text{ KN/m}^2$$

$$\Delta\sigma'_{\text{av}} = \frac{40.84 + 4 \times 38.43 + 34.51}{6} \cong 38 \text{ KN/m}^2$$

Note that, the value of  $(\Delta\sigma'_{\text{av}})$  under the edge of the tank is much smaller than under the center, this is realistic because under the edge the weight of the tank is much smaller than under the center.

This difference, will make also a different in settlement between the center and the edge of the tank, and it is expected the settlement under the center much larger than the settlement under the edge.

$$\text{The required is } (S_{c(t=1\text{yr})}) \rightarrow U = \frac{S_{c(t=1\text{yr})}}{S_{c(t=\infty)}} \rightarrow S_{c(t=1\text{yr})} = U \times S_{c(t=\infty)}$$

**Firstly, we want to calculate the value of (U) at t = 1yr:**

From lab results we can calculate the coefficient of consolidation ( $C_v$ ):

$$t_{60} = 15 \text{ minutes} = 2.85 \times 10^{-5} \text{ yr} \rightarrow (t_{60} \text{ means}) \rightarrow U = 60\% \rightarrow T_v = 0.286$$

$$H_{\text{dr}} = \frac{H}{2} \text{ (Double drained)} = \frac{20}{2} = 10 \text{ mm} = 0.01\text{m}$$

$$C_v = \frac{T_v \times H_{dr}^2}{t} = \frac{0.286 \times 0.01^2}{2.85 \times 10^{-5}} \cong 1 \text{m}^2/\text{yr}$$

Now in field:

$$t = 1 \text{yr} \quad , \quad C_v = 1 \text{m}^2/\text{yr} \quad , \quad H_{dr} = H = 3 \text{m} \text{ (because the rock under clay)}$$

$$T_v = \frac{C_v \times t}{H_{dr}^2} = \frac{1 \times 1}{3^2} = 0.111 \rightarrow U \cong 38\% \text{ (Table 11.7)}$$

$$\rightarrow S_{c(t=1\text{yr})} = 0.38 \times S_{c(t=\infty)}$$

To calculate ( $S_{c(t=\infty)}$ ) firstly we want to calculate ( $\sigma'_o$ ) at the **mid** of clay layer:

For clay layer we want to calculate ( $\gamma_{sat}$  and  $e_o$ ):

$$Se = G_s w \rightarrow e = e_o = 2.7 \times 0.2 = 0.54 \text{ (S = 1 because the clay is saturated)}$$

$$\gamma_{sat} = \frac{G_s \times \gamma_w (1 + w)}{1 + e} = \frac{2.7 \times 9.81 \times (1 + 0.2)}{1 + 0.54} = 20.64 \text{ KN/m}^3$$

$$\sigma_t = 18 \times 1 + 20 \times 3 + 20.64 \times \left(\frac{3}{2}\right) = 108.96 \text{ KN/m}^2$$

$$u_o = 9.81 \times 3 + 9.81 \times \left(\frac{3}{2}\right) = 44.14 \text{ KN/m}^2$$

$$\sigma'_o = 108.96 - 44.14 = 64.8 \text{ KN/m}^2$$

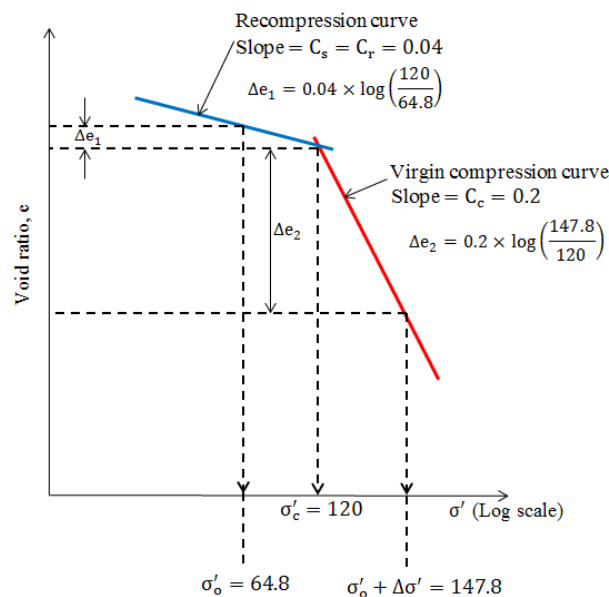
$$\sigma'_c = 120 \text{ KPa} \quad , \quad C_c = 0.2 \quad , \quad C_s = 0.04 \quad , \quad e_o = 0.54$$

**Calculating ( $S_{c(t=\infty)}$ ) under the center of tank:**

$$\text{OCR} = \frac{\sigma'_c}{\sigma'_o} = \frac{120}{64.8} = 1.85 > 1 \rightarrow \rightarrow \text{(Overconsolidated Clay)}$$

$$\sigma'_o + \Delta\sigma'_{av} = 64.8 + 83 = 147.8 \text{ KN/m}^2$$

The formula of  $S_{c(t=\infty)}$  at this case can be derived from the following graph:





$$S_{c(t=\infty)} = H \frac{\Delta e}{1 + e_0} \text{ (Basic equation)}$$

So, according to the above graph and the value of  $(\Delta e = \Delta e_1 + \Delta e_2)$ :

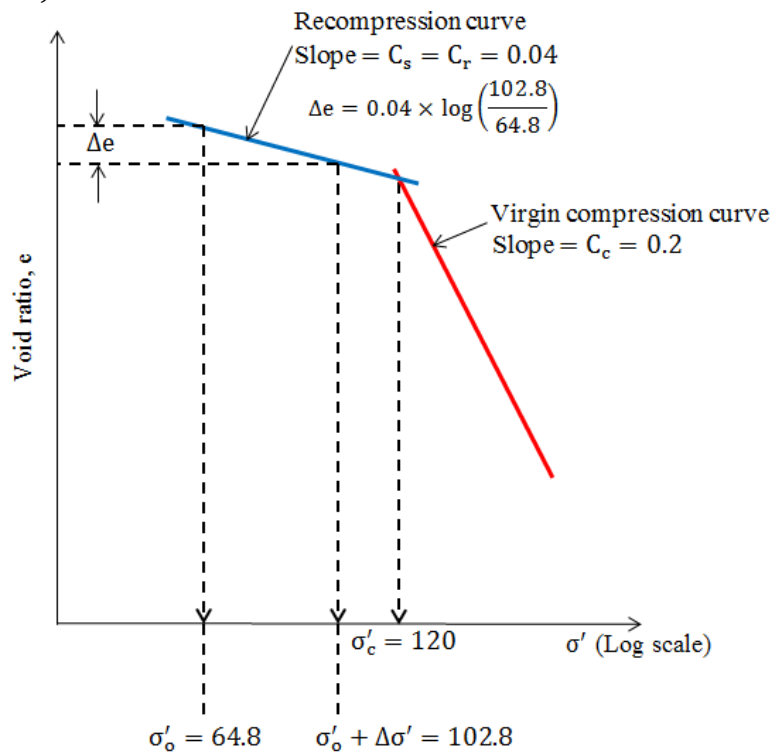
$$S_{c(t=\infty)} = \frac{3 \times 0.04}{1 + 0.54} \times \log\left(\frac{120}{64.8}\right) + \frac{3 \times 0.2}{1 + 0.54} \times \log\left(\frac{147.8}{120}\right) = 0.056 \text{ m} = 5.6 \text{ cm}$$

**Calculating  $(S_{c(t=\infty)})$  under the edge of tank:**

$$\text{OCR} = \frac{\sigma'_c}{\sigma'_o} = \frac{120}{64.8} = 1.85 > 1 \rightarrow \rightarrow \text{(Overconsolidated Clay)}$$

$$\sigma'_o + \Delta\sigma'_{av} = 64.8 + 38 = 102.8 \text{ KN/m}^2$$

The formula of  $S_{c(t=\infty)}$  at this case can be derived from the following graph:



$$S_{c(t=\infty)} = H \frac{\Delta e}{1 + e_0} \text{ (Basic equation)}$$

So, according to the above graph and the value of  $(\Delta e)$

$$S_{c(t=\infty)} = \frac{3 \times 0.04}{1 + 0.54} \times \log\left(\frac{102.8}{64.8}\right) = 0.0156 \text{ m} = 1.56 \text{ cm}$$

$$\text{So, } \Delta S_{c(t=\infty)} = S_{c(t=\infty), \text{center}} - S_{c(t=\infty), \text{edge}} = 5.6 - 1.56 = 4.04 \text{ cm} \rightarrow \rightarrow$$

$$\Delta S_{c(t=1\text{yr})} = \Delta S_{c(t=\infty)} \times U_{1\text{yr}} = 4.04 \times 0.38 = 1.535 \text{ cm} \checkmark .$$

**52.** (Solve problem 11.15 in your textbook)