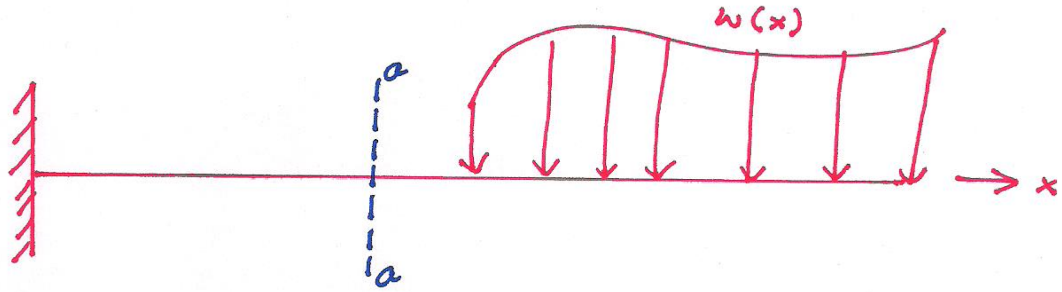
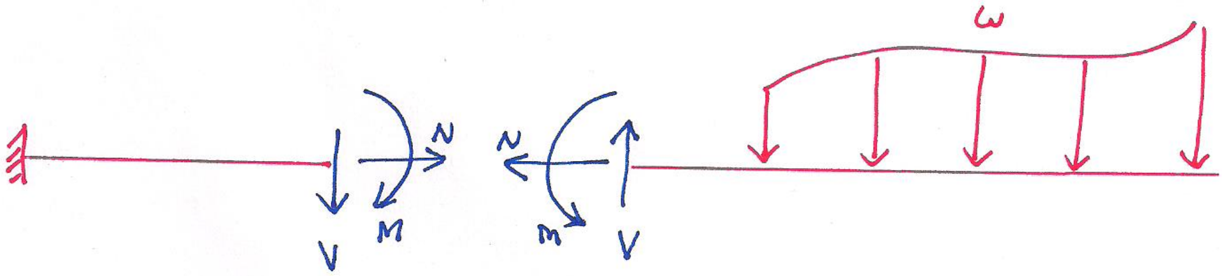
Flexural Systems
"Beams and Frames"

When a beam or frame is subjected to transverse loadings, the three possible internal forces that are developed are the normal or axial force, the shearing force, and the bending moment, as shown in section a-a of the cantilever beam.



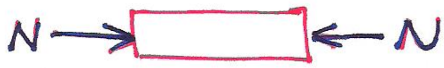


Internal forces in a beam

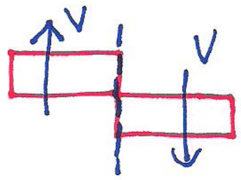
To predict the behavior of structures, the magnitudes of these forces must be known.

Sign Convention

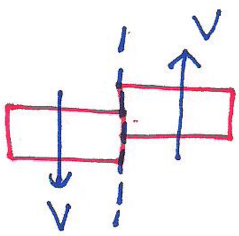
Positive axial force "Tension"



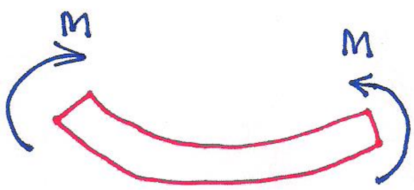
Negative axial force "Compression"



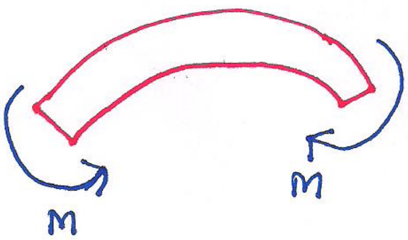
Positive shear force



Negative shear force



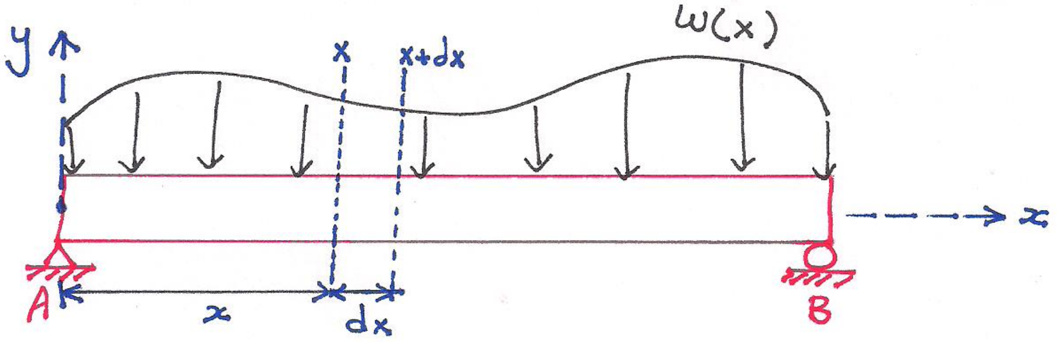
Positive bending moment



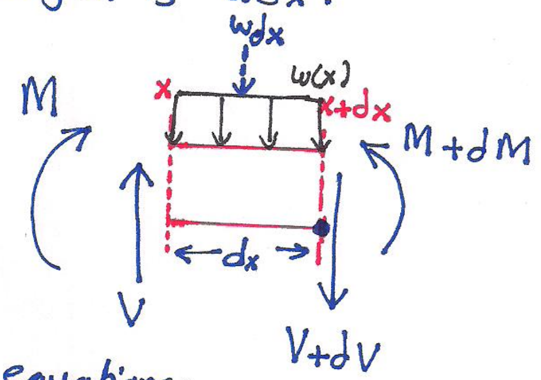
Negative bending moment

Relation Among distributed load, shearing force, and bending moment

Consider the simply supported beam (AB) which is subjected to the load $w(x)$



Let the shear force and bending moment at a section located at a distance of x from the left support be V and M, respectively, and at a section $x+dx$ be $V+dV$ and $M+dM$, respectively. The total load acting through the center of the infinitesimal length is $w\,dx$.



* Apply positive sign convention

Apply the equilibrium equations:

$$+\uparrow \Sigma F_y = 0: \quad +V - w.dx - (V + dV) = 0 \rightarrow \frac{dV}{dx} = -w$$

$$\Delta V = -\int w\,dx$$

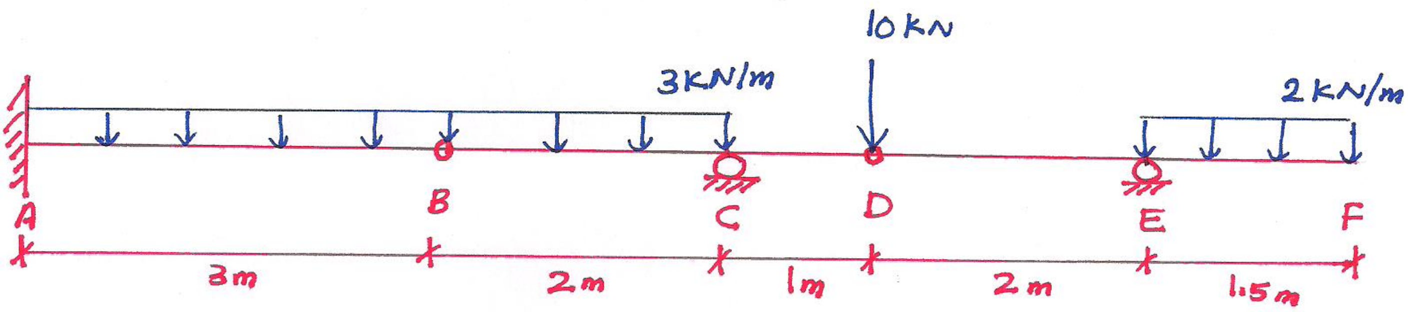
$$\circlearrowleft + \Sigma M_{\substack{right \\ corner}} = 0: \quad +(M+dM) + w\,dx\left(\frac{dx}{2}\right) - V.dx - M = 0$$

$$\rightarrow \frac{dM}{dx} = V$$

$$\Delta M = \int V\,dx$$

Example: Shear force "SFD" and bending moment "BMD" diagrams

Beam AF has a fixed support at A, internal hinges at B and D, rollers at C and E, and free end F. It is subjected to the transverse loadings shown. Determine the reactions and draw SFD & BMD.



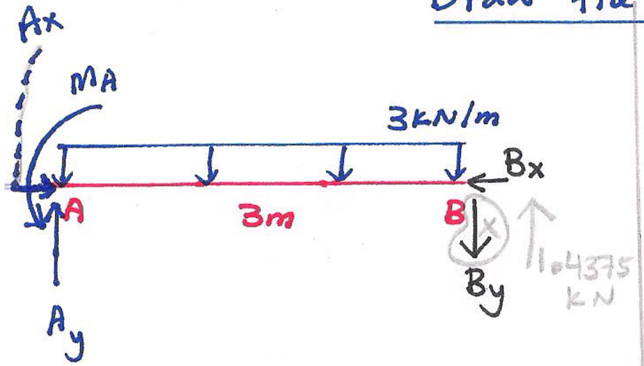
Solution

The beam is statically determinate because the number of unknown reactions = number of equilibrium equations

No. of reactions = 9

No. of equations = 9

Draw the FBD and determine the reactions



$$+\circlearrowleft \Sigma M_A = 0$$

$$+M_A + 1.4375(3) - 3(3)(1.5) = 0$$

$$M_A = +9.1875\ \text{kN}\cdot\text{m}$$

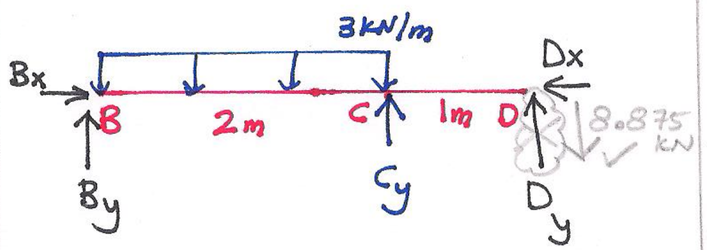
$$+\uparrow \Sigma F_y = 0$$

$$+A_y - 3(3) + 1.4375 = 0$$

$$A_y = +7.5625\ \text{kN}$$

$$\Sigma F_x = 0$$

$$A_x = 0$$



$$+\circlearrowleft \Sigma M_B = 0$$

$$-3(2)(1) + 2C_y - 3(8.875) = 0$$

$$C_y = 16.3125\ \text{kN}$$

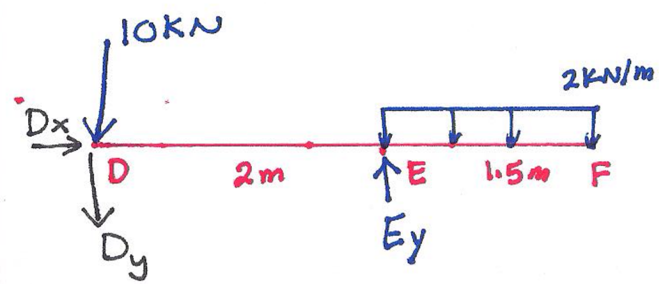
$$+\uparrow \Sigma F_y = 0$$

$$+B_y - 3(2) + 16.3125 - 8.875 = 0$$

$$B_y = -1.4375\ \text{kN}\ (\downarrow)$$

$$\Sigma F_x = 0$$

$$B_x = 0$$



Only 3-unknowns → start from here.

$$+\circlearrowleft \Sigma M_D = 0 : +2E_y - 2(1.5)(2.75) = 0$$

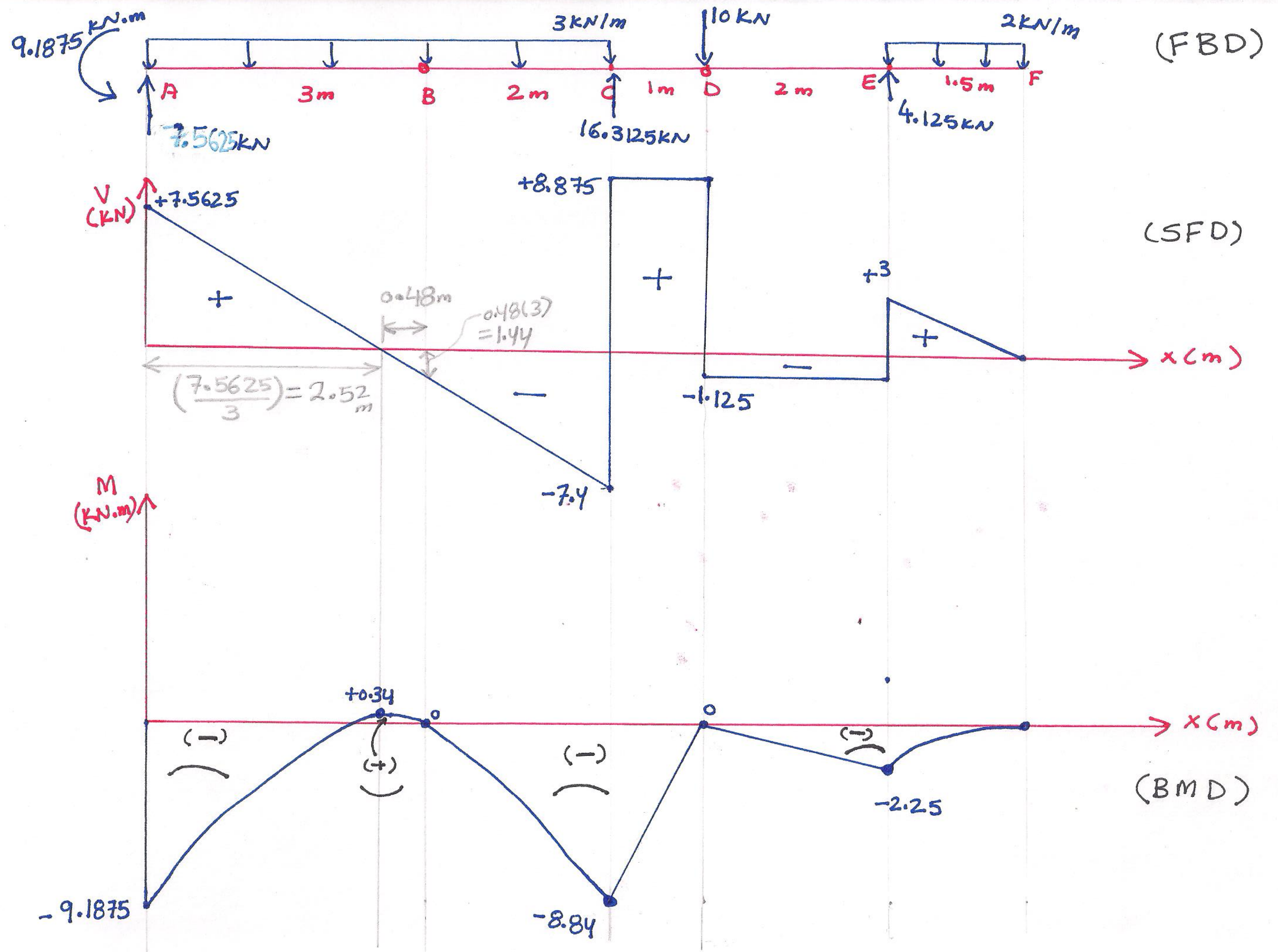
$$E_y = 4.125\ \text{kN}\ (\uparrow)$$

$$+\uparrow \Sigma F_y = 0$$

$$-10 - D_y + 4.125 - 2(1.5) = 0$$

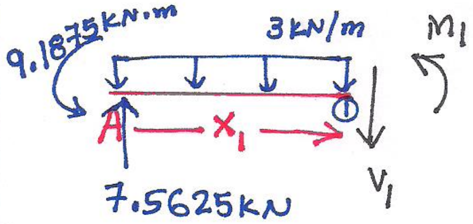
$$D_y = -8.875\ \text{kN}\ (\uparrow)$$

$$\Sigma F_x = 0 : \quad D_x = 0$$


(FBD)
9.1875 kN.m
3 kN/m
10 kN
2 kN/m
A
3m
B
2m
C
1m
D
2m
E
1.5 m
F
7.5625 kN
16.3125 kN
4.125 kN
(SFD)
V (kN)
+7.5625
+8.875
+3
0.48m
0.48(3) =1.44
$\left(\frac{7.5625}{3}\right) = 2.52\,m$
-1.125
-7.4
x (m)
M (kN.m)
(BMD)
+0.34
0
0
(−)
(+)
(−)
(−)
-9.1875
-8.84
-2.25
x (m)

Write the equations of shear force and bending moment

*** Part ABC:**



$0 \leq x_1 \leq 5m$

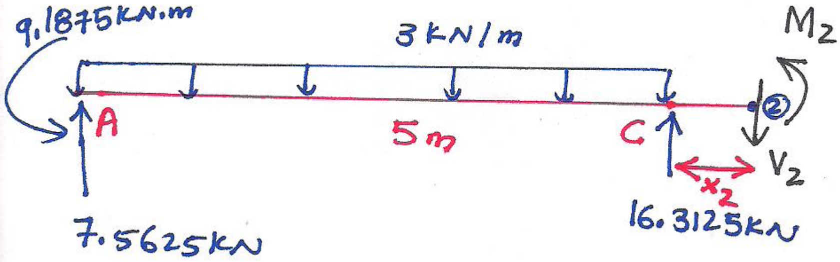
$+\uparrow \Sigma F_y = 0: \; +7.5625 - 3x_1 - V_1 = 0$

$V_1 = 7.5625 - 3x_1$

$\circlearrowleft + \Sigma M_{①} = 0: \; +M_1 + 3x_1\,(x_1/2) + 9.1875 - 7.5625\, x_1 = 0$

$M_1 = -1.5\, x_1^2 + 7.5625\, x_1 - 9.1875$

*** Part CD:** $0 \leq X_2 \leq 1m$



$+\uparrow \Sigma F_y = 0$

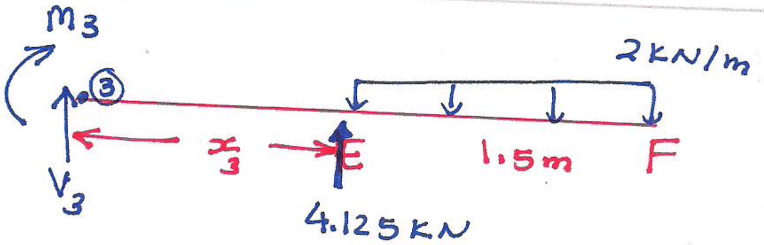
$+7.5625 - 3(5) + 16.3125 - V_2 = 0$

$V_2 = +8.875 \text{ kN}$

$\circlearrowleft + \Sigma M_{②} = 0: \; +M_2 - 16.3125\,(x_2) + 3(5)\left(x_2 + \frac{5}{2}\right) + 9.1875 - 7.5625\,(5 + x_2) = 0$

$M_2 = +8.875\, X_2 - 8.875$

*** Part DE:**



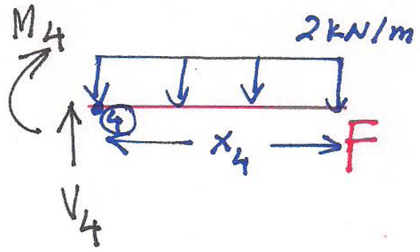
$0 \leq x_3 \leq 2\,m$

$+\uparrow \Sigma F_y = 0: \; +V_3 + 4.125 - 2(1.5) = 0 \rightarrow V_3 = -1.125 \text{ kN}$

$\circlearrowleft + \Sigma M_{③} = 0: \; -M_3 + 4.125\, x_3 - 2(1.5)(x_3 + 1.5/2) = 0$

$M_3 = +1.125\, x_3 - 2.25$

* Part EF :



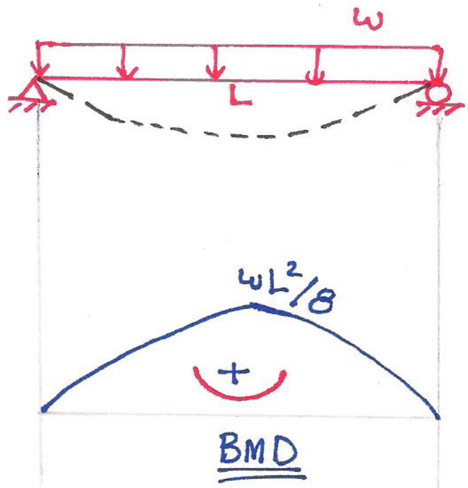
$0 \leq x_4 \leq 1.5m$

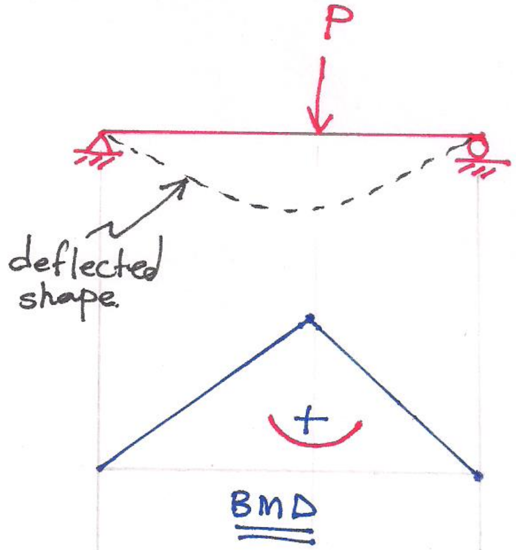
$+\uparrow \sum F_y = 0 :$ $\quad + V_4 - 2x_4 = 0 \rightarrow V_4 = +2x_4$

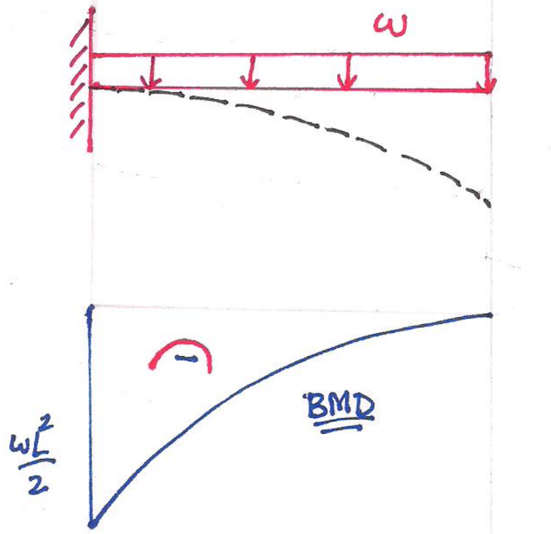
$\circlearrowleft + \sum M_{④} = 0 :$ $\quad -M_4 - 2x_4 (x_4/2) = 0$

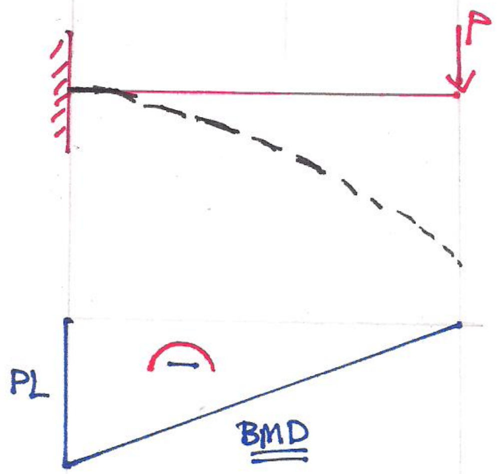
$$M_4 = -x_4^2$$

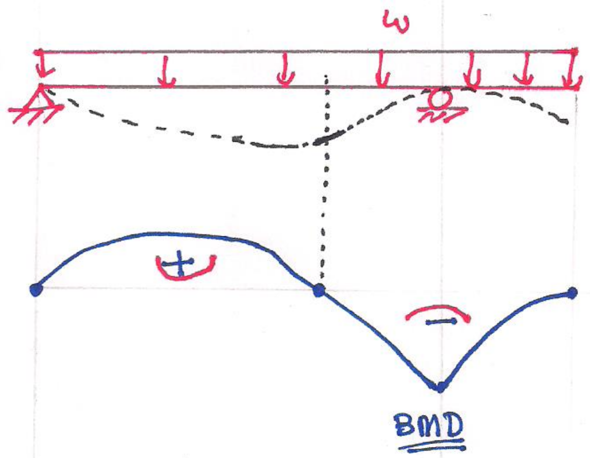
Deformed shape "Qualitative"

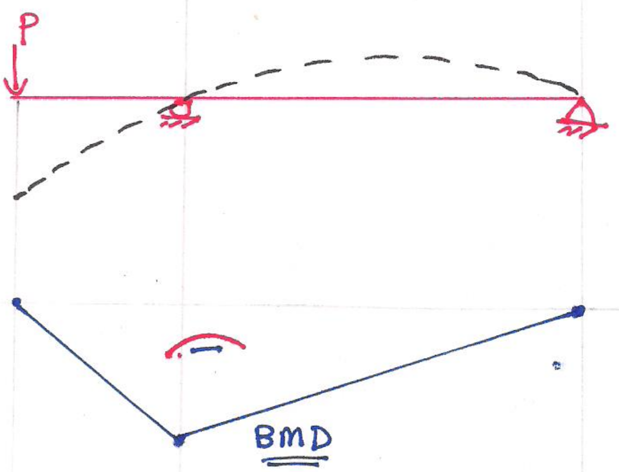


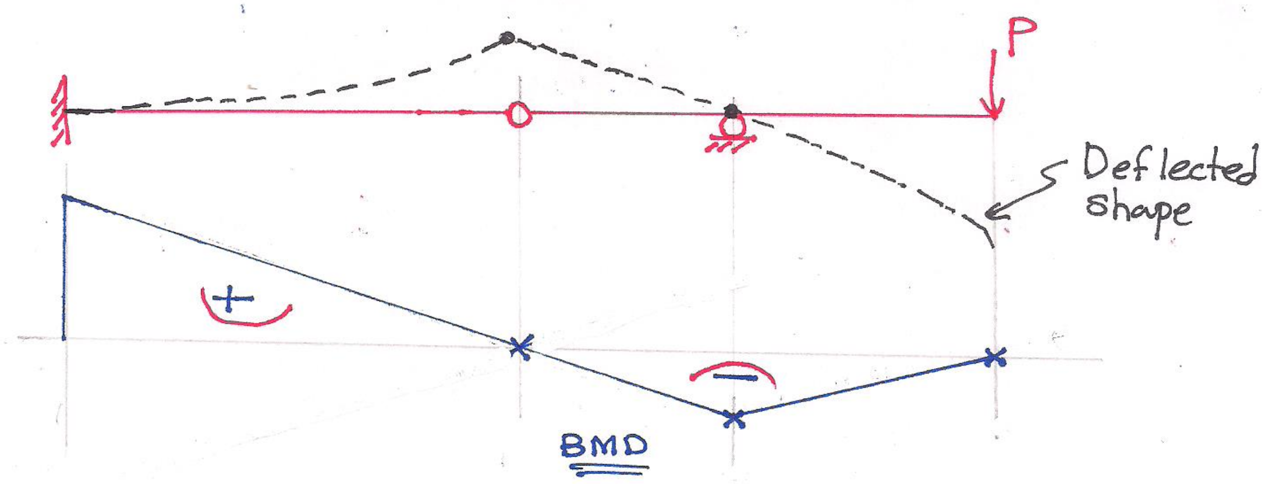


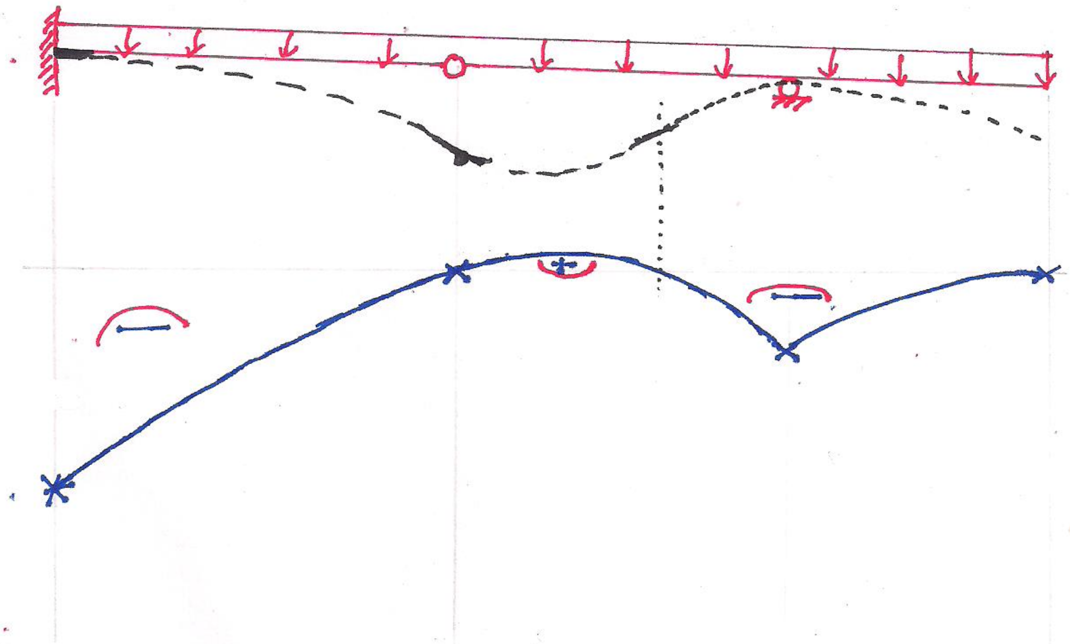









P
Deflected shape
+
−
BMD


−
+
−