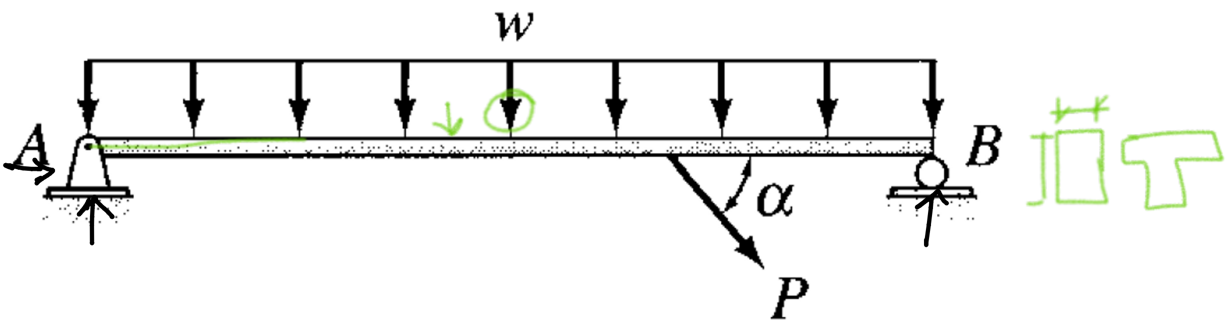
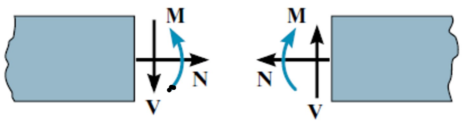
Analysis of Beams - specific point

Monday, March 15, 2021 5:50 PM



Internal Loads in Beams ?



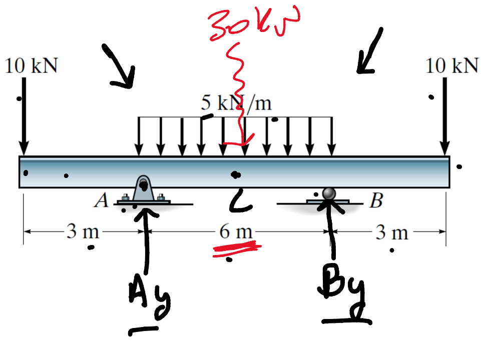
sign convension

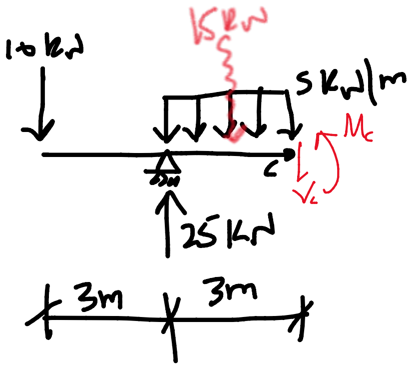
Procedure of Analysis

- Get support reactions
- Make a section cut at the point of interest
- Draw Free Body diagram and Indicate the positive direction of internal loadings at that point
- Calculate the equivalent forces for distributed loads (uniform, linear, etc.)
- Apply equations of equilibrium

Example:

Determine the moment in the beam at point C ← mid-span





$\Sigma M_A = 0$

$10(3) - 30(3) + B_y(6) - 10(9) = 0$

$30 - 90 + 6B_y - 90 = 0$

$B_y = \frac{150}{6} = 25\ kN$

$\Sigma F_y = 0$

$-10 - 30 - 10 + 25 + A_y = 0$

$A_y = 25\ kN$

$\Sigma M_c = 0$

$10(6) - 25(3) + 15(1.5) + M_c = 0$ (kN, m)

$M_c = -7.5\ kN \cdot m$

$10(6) - 25(3) + 15(1.5) + M_c = 0$

$M_c = -7.5$ kN·m

$\Sigma F_y = 0$

$-10 + 25 - 15 - V_c = 0$

$V_c = 0$ kN

Analysis of Beams - Functions

Monday, March 15, 2021 6:05 PM

For most cases, we can not predict the exact location of the maximum internal load.

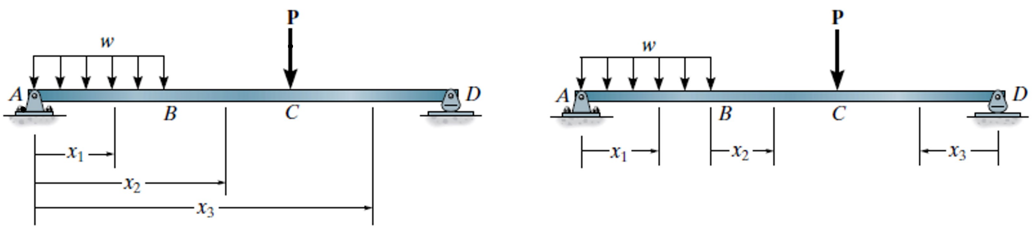
Therefore, we can express the internal load as a function of the location on the beam , V(x) or M(x)

We can use these equations to find the Value of the maximum internal load and its location

The variations of V and M as a function of the position x of an arbitrary point along the beam's axis can be obtained by using the method of sections

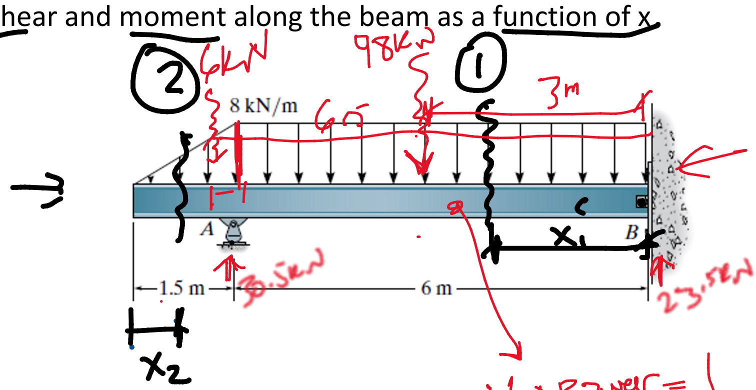
In general, the internal shear and moment functions will be discontinuous

It is necessary to indicate our reference axes and its applicable range for each section of the beam



Example:

Express the internal shear and moment along the beam as a function of x



V ⇒ power = 1

M ⇒ power = 2

$$\Sigma M_B = 0$$

$$6(6.5) - A_y(6) + 48(3) = 0$$

$$A_y = 30.5\,kN$$

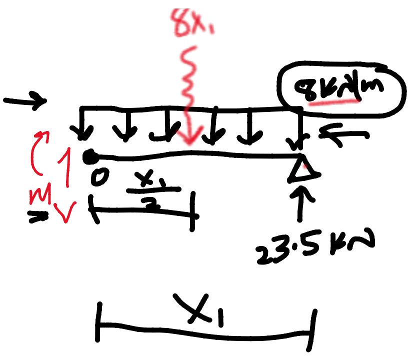
$$\Sigma F_y = 0$$

$$-6 + 30.5 - 48 + B_y = 0$$

$$B_y = 23.5\,kN$$

Section 1:

Section 1:



$\Sigma M = 0$

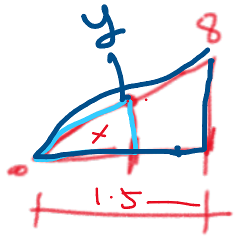
$-M - 8x_1\left(\frac{x_1}{2}\right) + 23.5(x_1) = 0$

$M = -4x_1^2 + 23.5x_1$ kN·m → power = 2

$\Sigma F_y = 0$

$V - 8x_1 + 23.5 = 0$

$V = 8x_1 - 23.5$ kN → slope



$\frac{y}{x} = \frac{8}{1.5}$

$y = 5.3x$

$\frac{8}{1.5} = \frac{y}{z - 1.5}$

$y = \frac{8z - 12}{1.5}$

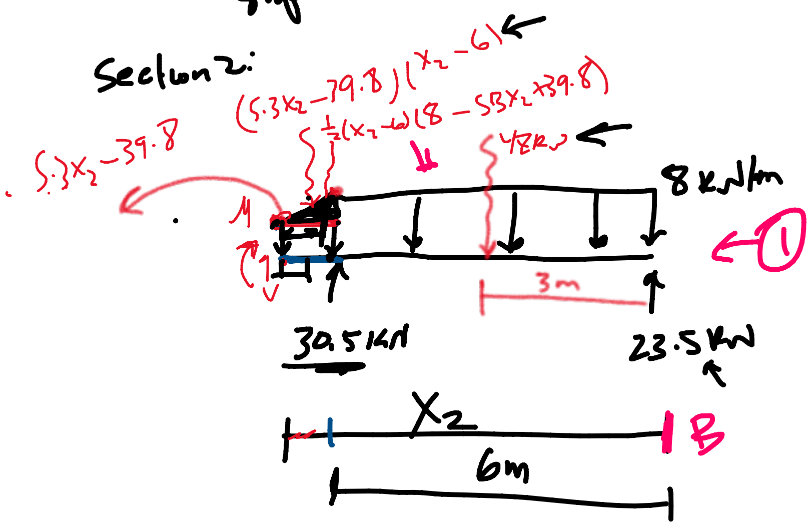
$y = 5.3z - 8$

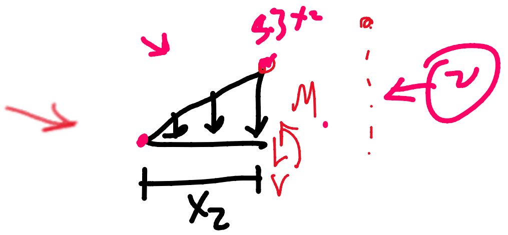
$y = 5.3(x_2 - 6) - 8$

$y = 5.3x_2 - 31.8 - 8$

$y = 5.3x - 39.8$

Section 2:





part (i):

$\Sigma M_o = 0$

$-M - (5.3x_2 - 39.8)(x_2 - 6)\frac{(x_2 - 6)}{2} - \frac{1}{2}(8 - 5.3x_2 + 39.8)(x_2 - 6) \cdot \frac{2}{3}(x_2 - 6)$

$$-M-(5.3x_2-39.8)(x_2-6)\frac{(x_2-6)}{2}-\frac{1}{2}(8-5.3x_2+39.8)(x_2-6)\cdot\frac{2}{3}(x_2-6)$$

$$+30.5(x_2-6)-48(x_2-3)+23.5(x_2)=0$$

HW

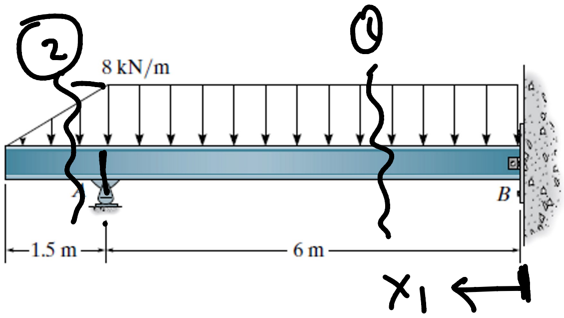
$M=$

$\Sigma f_y=0$

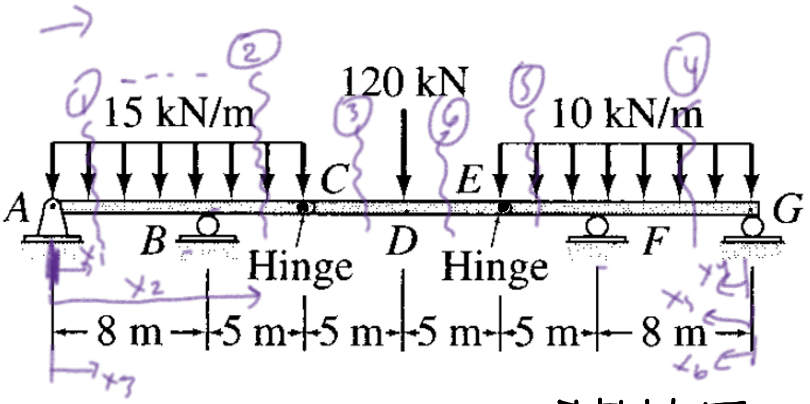
HW

$V=$

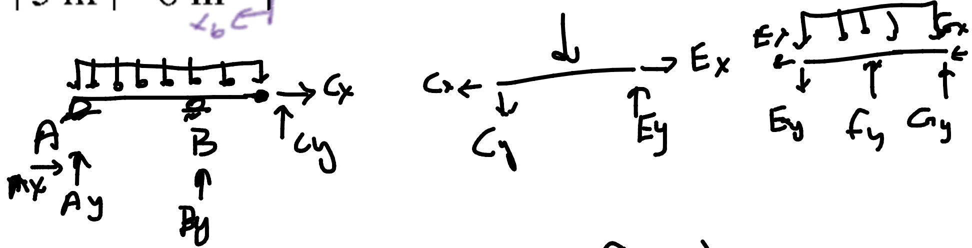
$$M=\begin{cases} M_1(x_1) & 0<x_1<6 \\ M_2(x_2) & 0<x_2<1.5 \end{cases}$$



$$V=\begin{cases} V_1(x_1) & 0<x_1<6 \\ V_2(x_2) & 0<x<1.5 \end{cases}$$



Step 1: reactions →



$3n = 9$ Eq → 9 unknowns

det. Beam

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M_o = 0$ →

Analysis of beams - diagrams

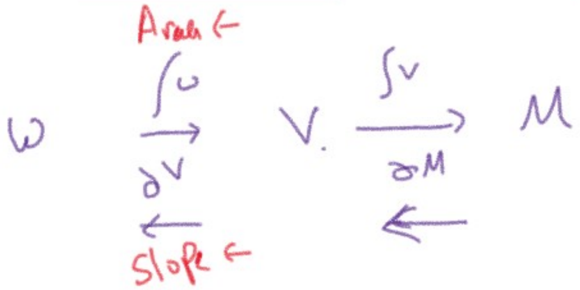
Tuesday, March 23, 2021 8:27 AM

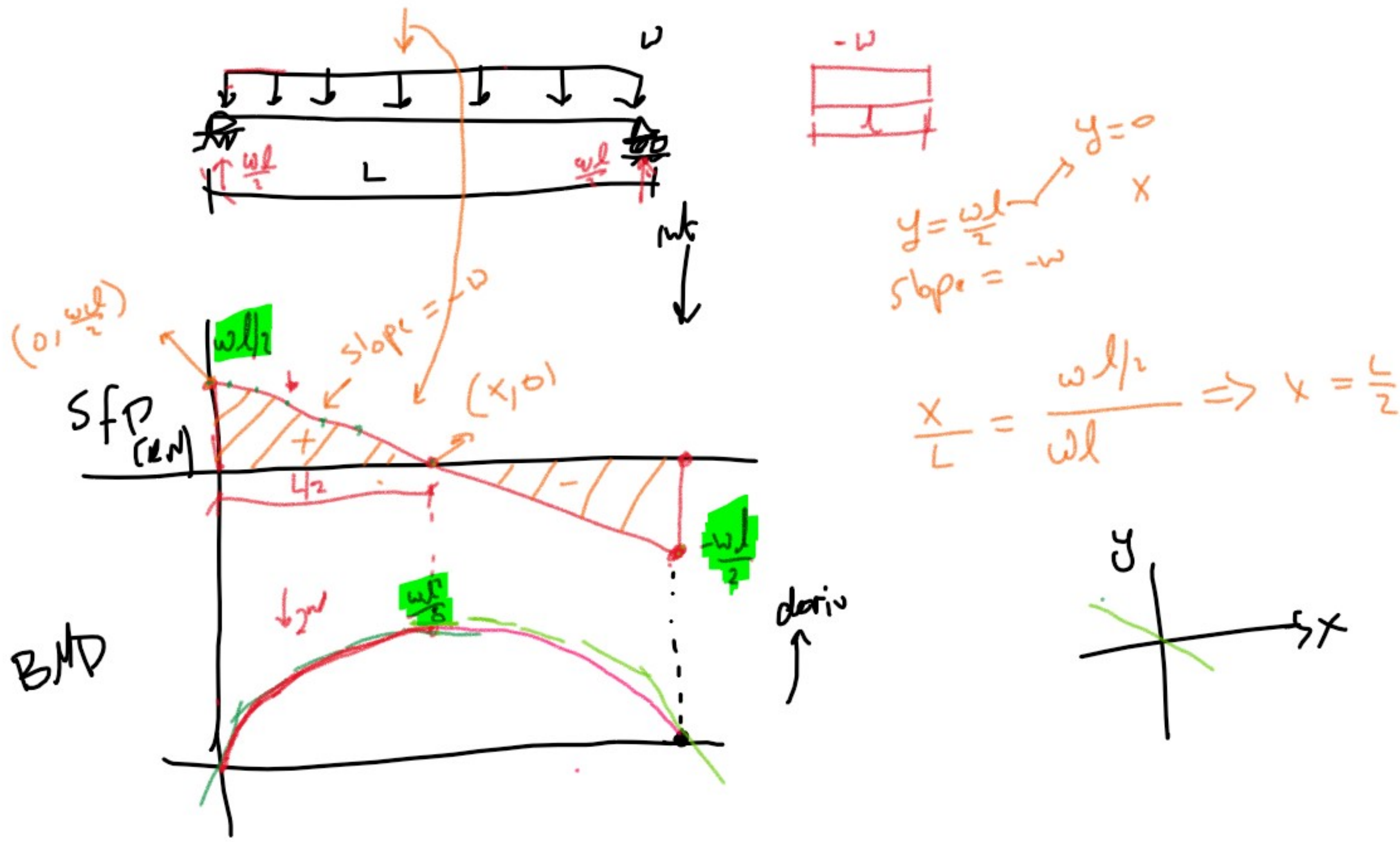
$$\frac{dV}{dx} = w$$

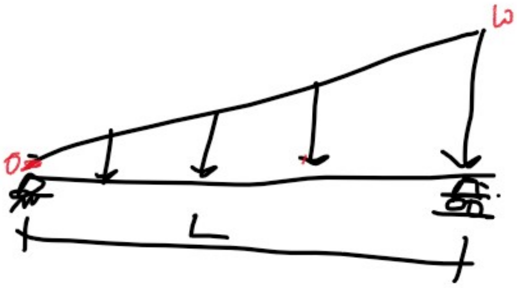
$$\left.\begin{array}{r}\text{Slope of}\\\text{Shear Diagram}\end{array}\right\} = \left\{\begin{array}{l}\text{Intensity of}\\\text{Distributed Load}\end{array}\right.$$

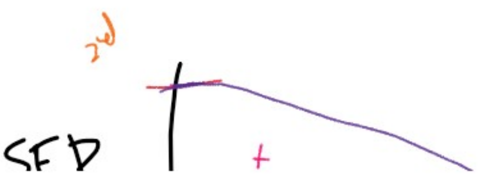
$$\frac{dM}{dx} = V$$

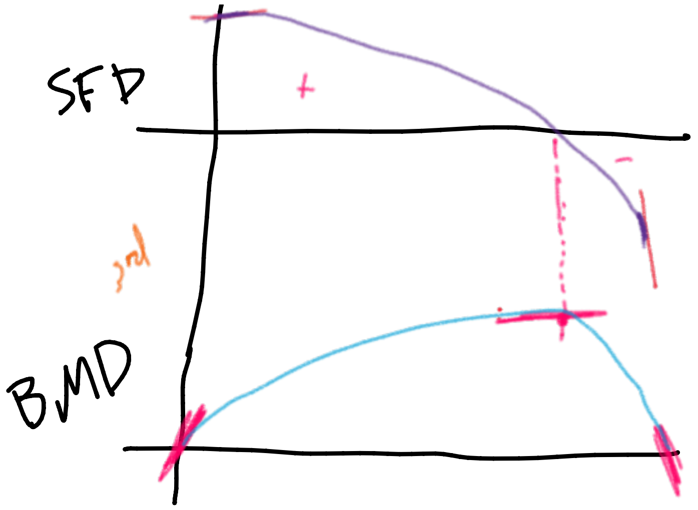
$$\left.\begin{array}{r}\text{Slope of}\\\text{Moment Diagram}\end{array}\right\} = \{\text{Shear}$$

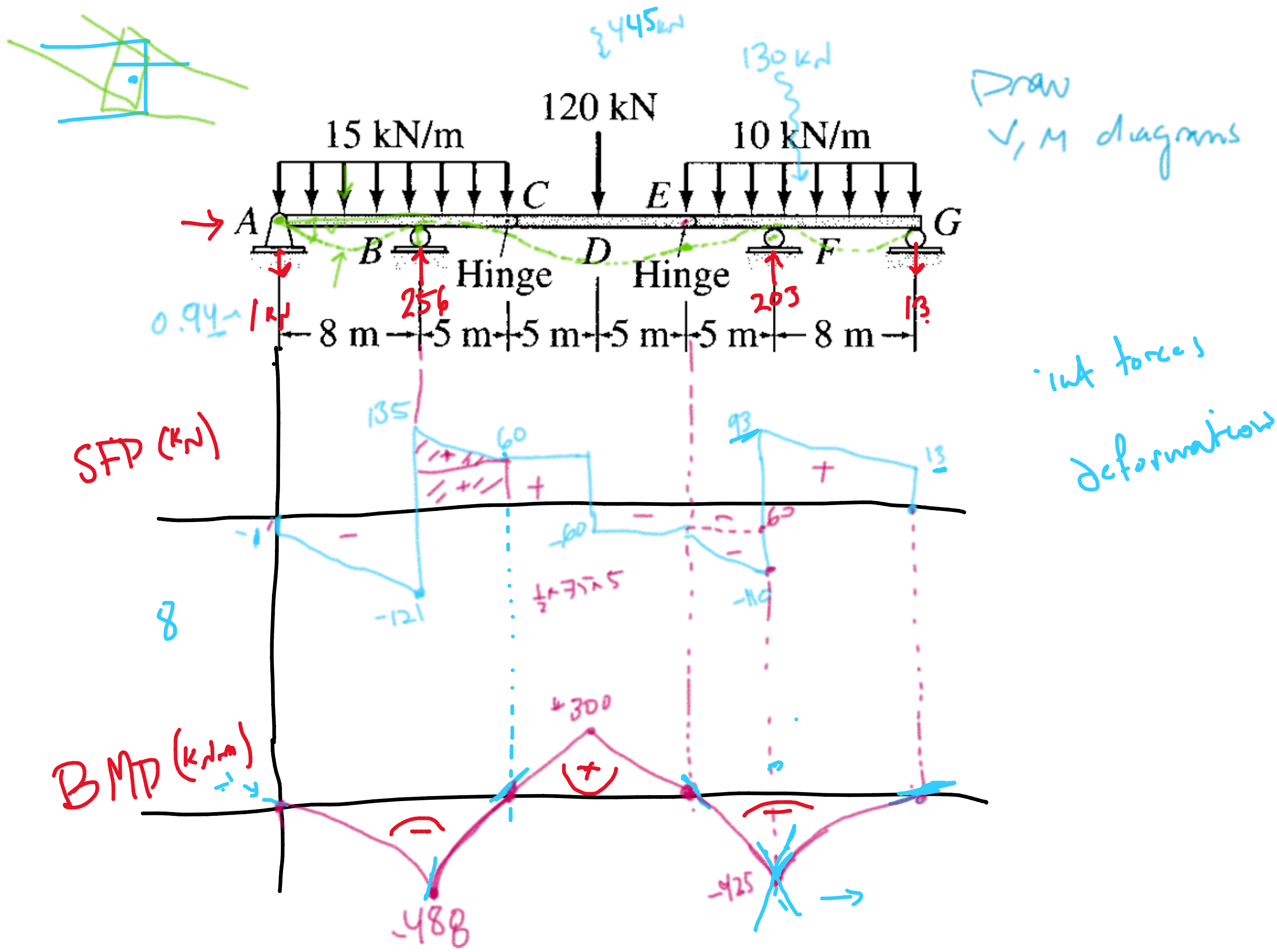


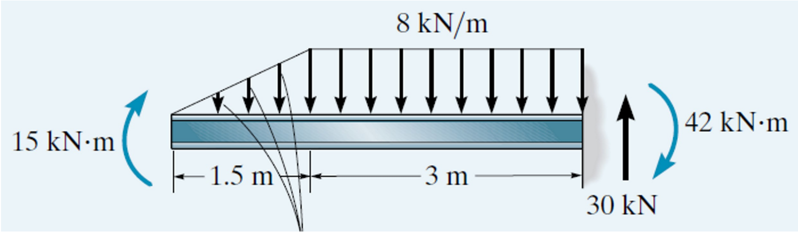




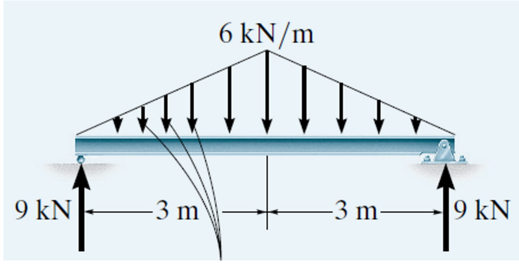



SFD
+
-
3rd
BMD


445kN
130kN
Draw
V, M diagrams
15 kN/m
120 kN
10 kN/m
A
B
C
D
E
F
G
Hinge
Hinge
0.94~1kN
256
203
13
8 m
5 m
5 m
5 m
5 m
8 m
int forces
deformation
SFD (kN)
135
60
+
-
-121
-60
-110
93
13
BMD (kNm)
+300
-488
-425



8 kN/m
15 kN·m
42 kN·m
1.5 m
3 m
30 kN



6 kN/m
9 kN
3 m
3 m
9 kN

Elastic beam theory

Thursday, March 25, 2021 7:58 AM

elastic, small deformation

$\epsilon = \frac{\Delta L}{L} = \frac{ds' - ds}{ds}$

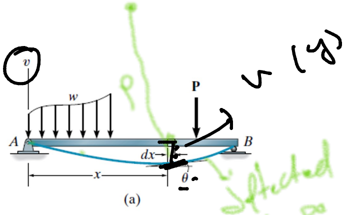
$\epsilon = \frac{(\rho - y)d\theta - \rho d\theta}{\rho d\theta}$

$\epsilon = \frac{-y}{\rho} \rightarrow \sigma = E\epsilon, \ \sigma = \frac{-My}{I}$

$\frac{-My}{I} = \frac{E * -y}{\rho}$

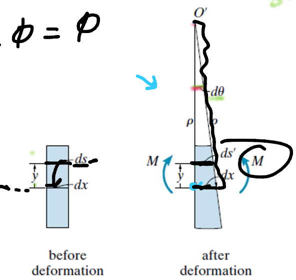
$k \quad \frac{1}{\rho} = \frac{M}{EI}$

Calc 3 $\rightarrow \quad \frac{1}{\rho} = \frac{d^2v/dx^2}{[1 + (dv/dx)^2]^{3/2}}$, $(dv/dx)^2 \approx 0$ small



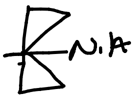
$\frac{\Delta y}{\Delta x}$

$\tan\phi = \rho$



ρ: radius of curvature

$M \rightarrow \sigma \rightarrow \epsilon$



$ds' = (\rho - y)d\theta$

$ds = \rho\, d\theta$

$\frac{1}{\rho} = \frac{d^2v}{dx^2}$

$\frac{d^2}{dx^2}$ = concavity

$\frac{d}{dx}$ = slope

$$\boxed{\frac{d^2v}{dx^2} = \frac{M}{EI}}$$

$\frac{dv}{dx}$

$\theta = \frac{dv}{dx} = \quad \frac{M}{EI} = \quad \theta$

Int. forces · deformations

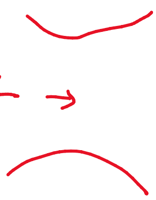
$w \xrightarrow{\int} V \xrightarrow{\int} M \xrightarrow{\int / EI} \theta \xrightarrow{\int} v$

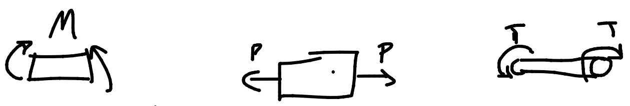
$w \xleftarrow{d} V \xleftarrow{d} M \xleftarrow{d} \theta \xleftarrow{d} v$

$\frac{d^2v}{dx^2} \rightarrow$ Concavity

$M+ \rightarrow$

$M- \rightarrow$





$\frac{d^2v}{dx^2} = \frac{M}{EI}$ — M: int. forces; EI: material; dx^2: length

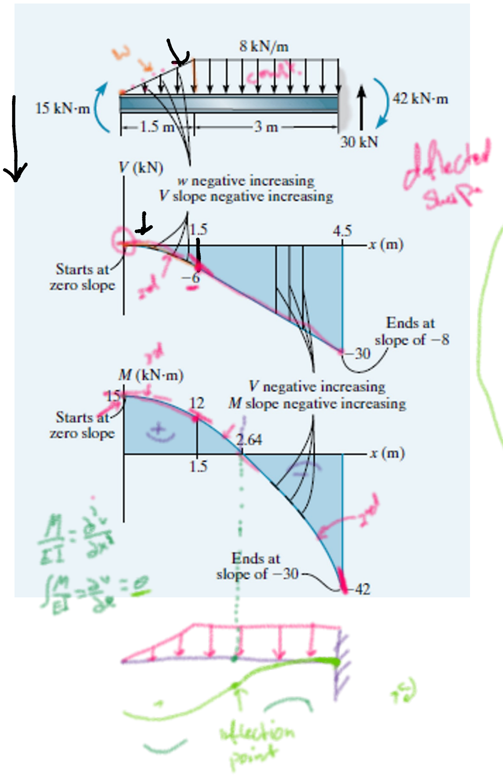
$\Delta = \frac{PL}{AE}$

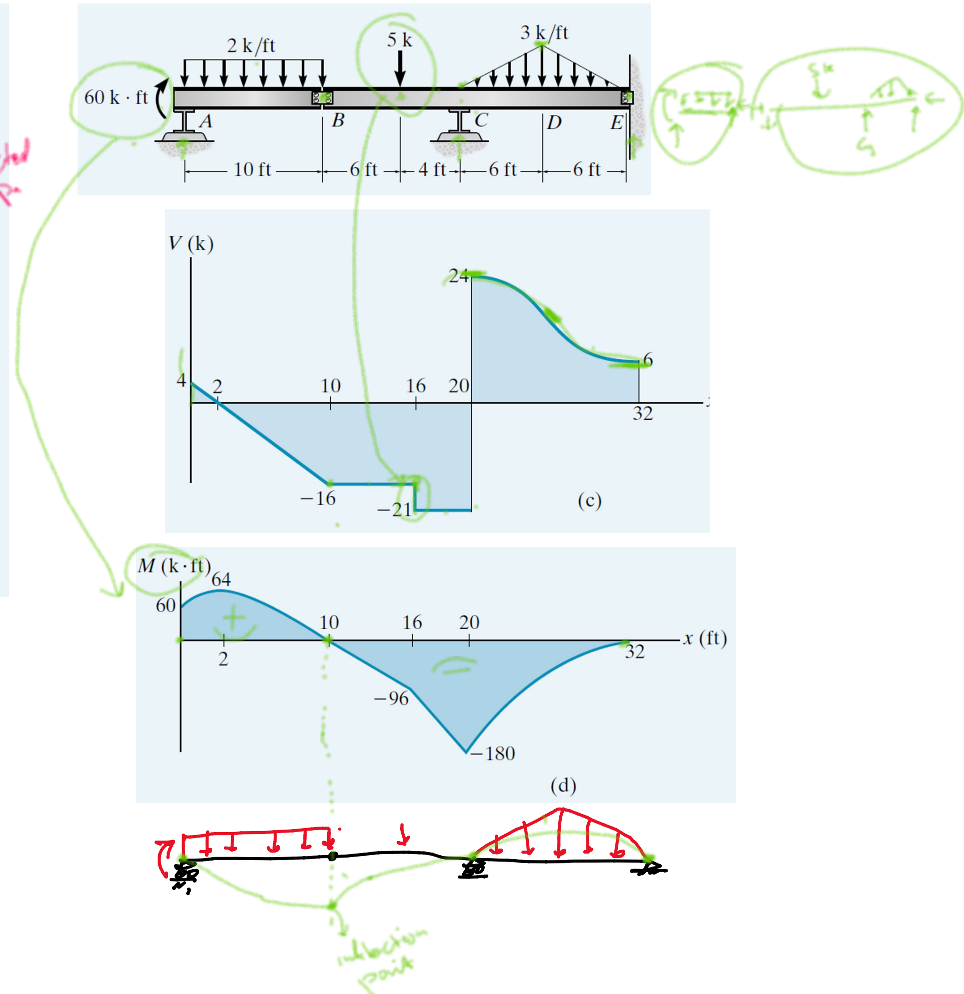
$\theta = \frac{TL}{JG}$

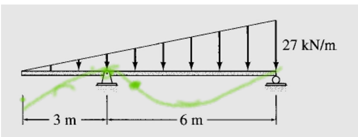
$w \xrightarrow{\int} V$, $V \xrightarrow{d} w$ (slope)

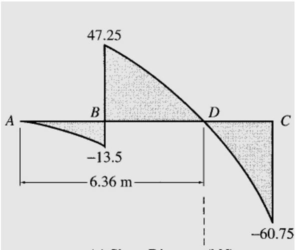
$\int x \rightarrow x^2$

Sketch the deflected shape for the following beams

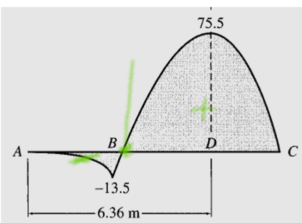








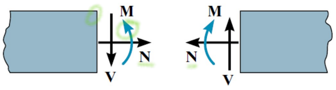
(c) Shear Diagram (kN)



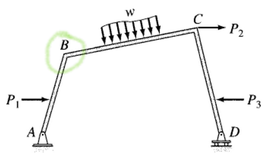
Analysis of Frames

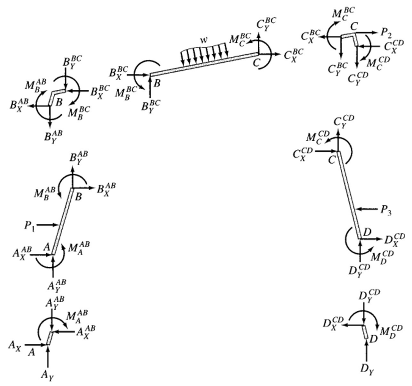
Monday, March 29, 2021 6:34 PM

Types of internal loads



Load transfer between joints



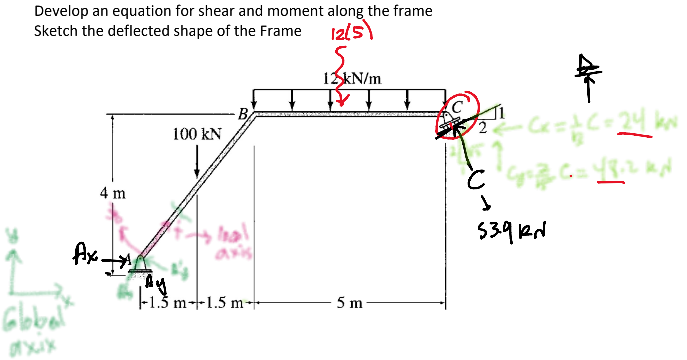


Example

Draw shear and moment and axial force diagrams

Develop an equation for shear and moment along the frame

Sketch the deflected shape of the Frame



Reactions:

$\Sigma F_y = 0$

$A_y + \frac{2}{\sqrt{5}} C - 100 - 12(5) = 0$

$\Sigma F_x = 0$

$A_x - \frac{1}{\sqrt{5}} C = 0$

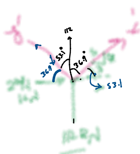
$+\circlearrowleft \Sigma M_A = 0$

$-100(1.5) - 12(5)(5.5) + \frac{2}{\sqrt{5}} C(8) + \frac{1}{\sqrt{5}} C(4) = 0$

$C = 53.9$ kN

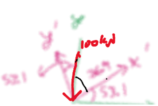
$A_x = 24.1$ kN

$A_y = 112$ kN



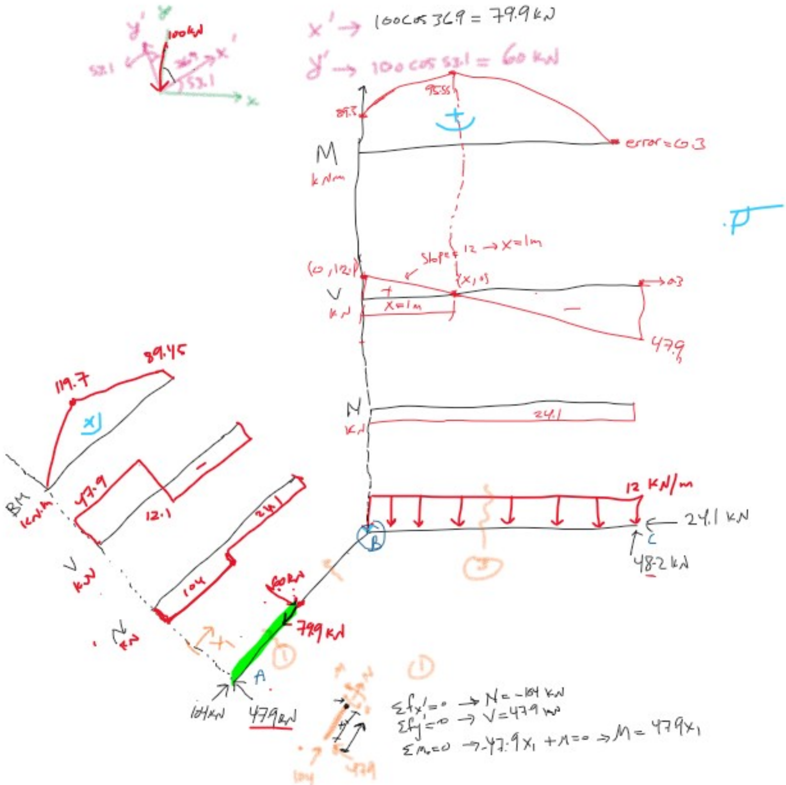
$x' \rightarrow 112\cos 36.9 + 24.1\cos 53.1 = 104$ kN

$y' \rightarrow 112\cos 53.1 - 24.1\cos 36.9 = 47.9$ kN



$x' \rightarrow 100\cos 36.9 = 79.9$ kN

$y' \rightarrow 100\cos 53.1 = 60$ kN



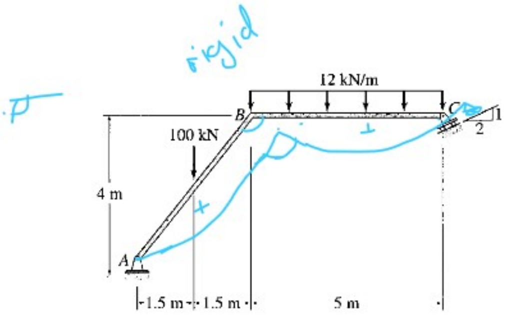
$x' \rightarrow 100\cos 36.9 = 79.9\ kN$

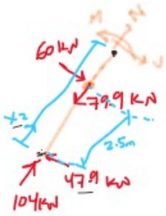
$y' \rightarrow 100\cos 53.1 = 60\ kN$

$\Sigma f_{x'} = 0 \rightarrow N = -104\ kN$

$\Sigma f_{y'} = 0 \rightarrow V = 47.9\ kN$

$\Sigma M_o = 0 \rightarrow -47.9 x_1 + M = 0 \Rightarrow M = 47.9 x_1$





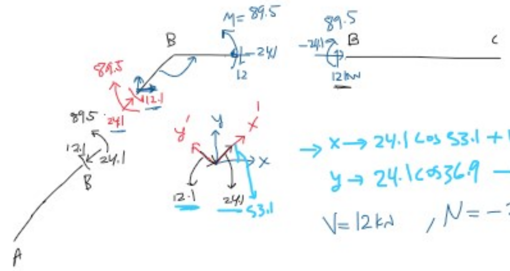
$\Sigma f_{x'} = 0 \rightarrow N = -24.1\ kN$

$\Sigma f_{y'} = 0 \rightarrow V = -12.1\ kN$

$\Sigma M_o = 0 \rightarrow -47.9 x_1 + 60(x_2 - 2.5) + M = 0$

$M = 47.9 x_2 - 60 x_2 + 150$

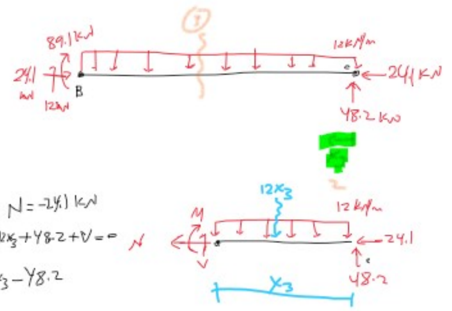
$M = 150 - 12.1 x_2$



$\rightarrow x \rightarrow 24.1\cos 53.1 + 12.1\cos 36.9 = 24.1\ kN$

$y \rightarrow 24.1\cos 36.9 - 12.1\cos 53.1 = 12\ kN$

$V = 12\ kN$, $N = -24.1$



$\Sigma f_x = 0 \rightarrow N = -24.1\ kN$

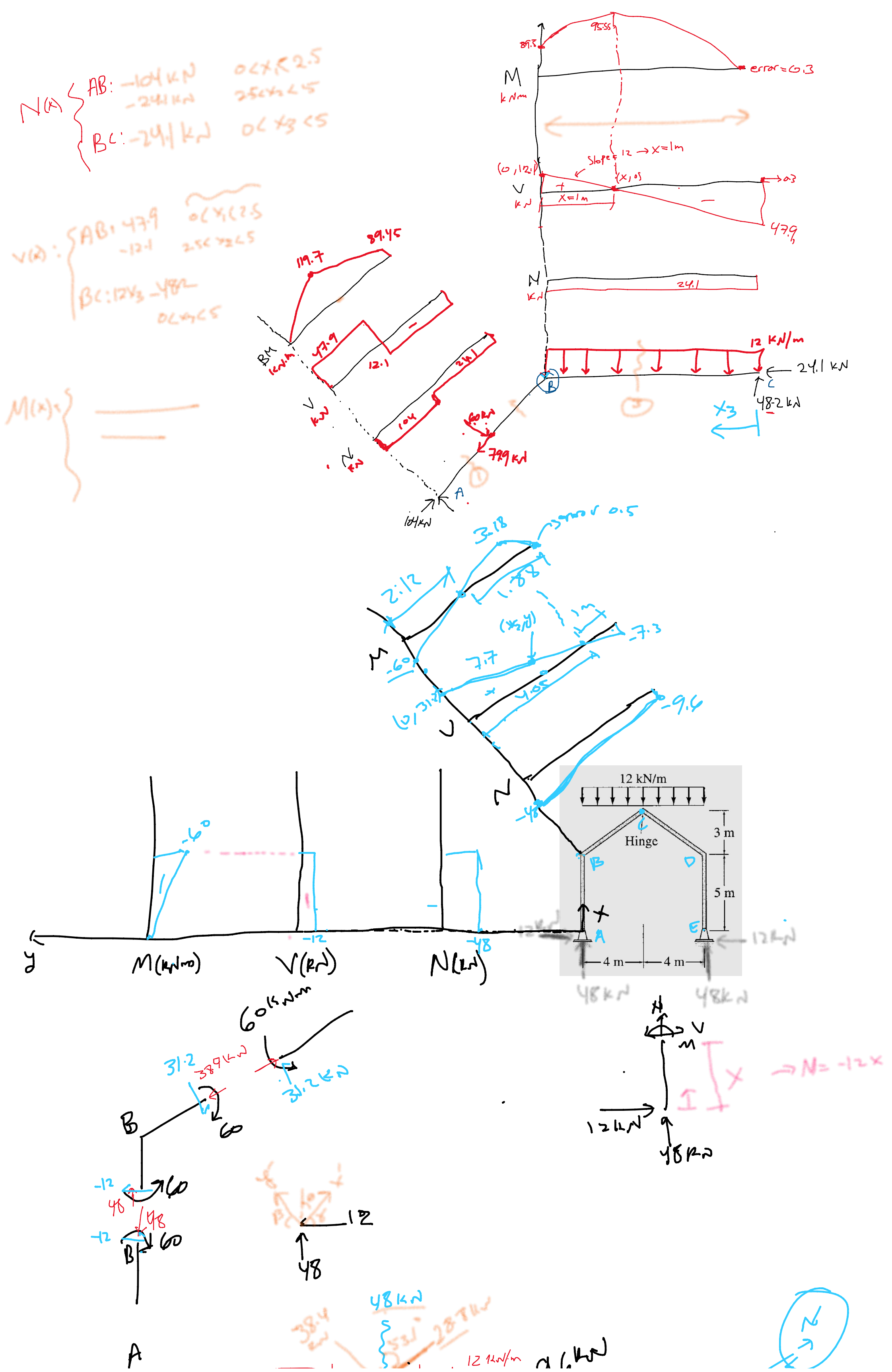
$\Sigma f_y = 0 \rightarrow -12x_3 + 48.2 + V = 0$

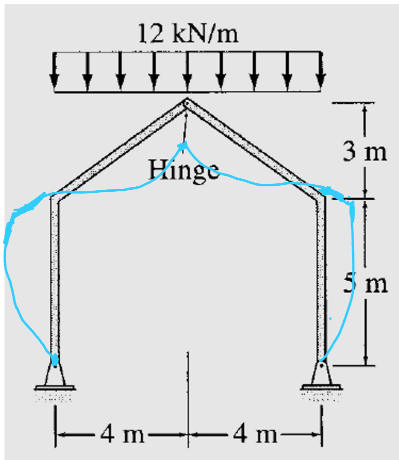
$V = 12x_3 - 48.2$

$\Sigma M_o = 0$

$48.2 x_3 - 12x_3\left(\frac{x_3}{2}\right) - M = 0$

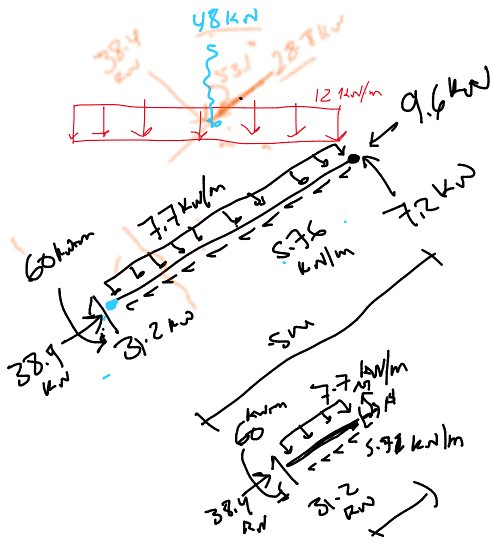
$M = 48.2 x_3 - 6x_3^2$

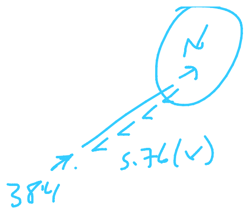

N(x) { AB: −104 kN 0<x<2.5
−24.1 kN 2.5<x<5
BC: −24.1 kN 0<x<5
V(x): { AB: 47.9 0<x<2.5
−12.1 2.5<x<5
BC: 12x − 48.2 0<x<5
M(x):
M
kNm
V
kN
N
kN
24.1
12 kN/m
24.1 kN
48.2 kN
119.7
89.45
47.9
12.1
104
60 kN
77.9 kN
104 kN
12 kN/m
3 m
Hinge
5 m
4 m
4 m
M(kNm)
V(kN)
N(kN)
48 kN
12 kN
N = −12x

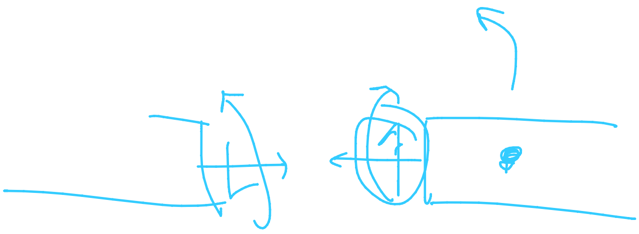

12 kN/m
3 m
Hinge
5 m
4 m
4 m

1

A


48 kN
38.4 kN
28.8 kN
12 kN/m
9.6 kN
7.7 kN/m
5.76 kN/m
7.2 kN
60 kNm
38.4 kN
31.2 kN
5m
60 kNm
7.7 kN/m
38.4 kN
31.2 kN


384
5.76 (V)



Deflections - double integration

Friday, April 2, 2021 5:23 PM

Elastic Beam Theory:

$$\int \frac{d^2v}{dx^2} = \int \frac{M}{EI}$$

axial load

$$\Delta = \frac{PL}{AE}$$

Torsion

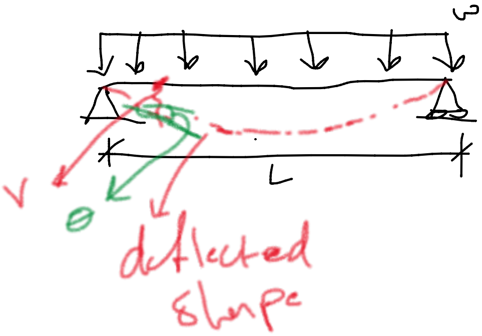
$$\phi = \frac{TL}{JG}$$

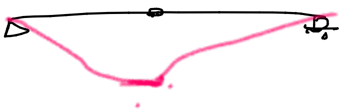
$$\int \frac{M(x)}{EI} = \frac{dv}{dx} = \theta(x) \quad \left(\frac{x^2}{2} + C_1\right)$$

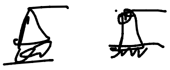
$$\iint \frac{M(x)}{EI} = v(x) \quad \left(\frac{x^3}{6} + C_1 x + C_2\right)$$

→ B.C.

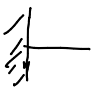
$v(x)$ → max, at specific point

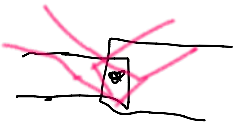




 → $\theta = ??$ $v = 0$

$v = 0$

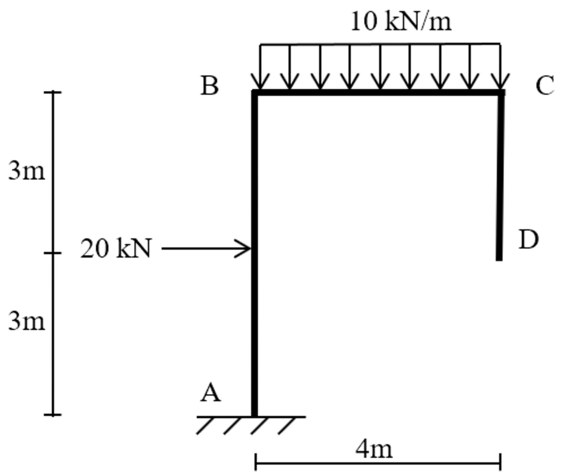
 → $\theta = 0$

 → $v^- = v^+$

FRAME ??

For the given frame Determine:

- The maximum horizontal displacement (sway) of the frame
- The vertical deflection of point D.



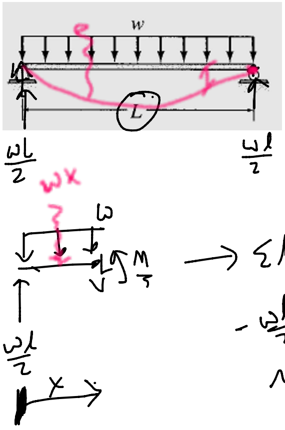
Example 1

Saturday, April 3, 2021 12:25 AM

Find the rotation and the deflection of the beam along the beam as a function of x

Find the maximum deflection in the beam

$EI = const.$



$M(x)$

B.C.

pin → $v(0) = 0$

roller → $v(L) = 0$

$\rightarrow \sum M_o = 0$

$-\frac{wl}{2}x + wx\left(\frac{x}{2}\right) + M = 0$

$M = \frac{wl}{2}x - \frac{wx^2}{2}$

$EI\theta = \int \frac{M}{EI} = \int \left(\frac{wl}{2}x - \frac{w}{2}x^2\right)dx = \frac{wl}{4}x^2 - \frac{wx^3}{6} + C_1$

$EI\,v = \int \theta dx = \frac{wl}{12}x^3 - \frac{wx^4}{24} + C_1 x + C_2$

$v(0) = 0 \rightarrow C_2 = 0$

$v(L) = 0 \rightarrow \frac{wl^4}{12} - \frac{wl^4}{24} + C_1 L = 0 \rightarrow C_1 L = \frac{-wl^4}{24}$

$C_1 = -\frac{wl^3}{24}$

$EI\,\theta(x) = \frac{wl}{4}x^2 - \frac{wx^3}{6} - \frac{wl^3}{24}$

$EI\,v(x) = \frac{wl}{12}x^3 - \frac{wx^4}{24} - \frac{wl^3}{24}x$

$\frac{wl}{12}\left(\frac{l^3}{8}\right) - \frac{w(l/2)^4}{24} - \frac{wl^4}{48} = -\frac{5wl^4}{384EI}$

$Max \rightarrow \frac{\partial v}{\partial x} = 0 \rightarrow x_1 \rightarrow v(x_1)$

$\theta(x) = 0 \rightarrow v = max$

$EI\,\theta(x) = 0 \rightarrow \frac{wl}{4}x^2 - \frac{wx^3}{6} - \frac{wl^3}{24} = 0$

$6wlx^2 - 4wx^3 - wl^3$

$-4x^2 + 4Lx + 2L^2$

$(\ldots 6Lx^2 - L^3) = 0$

$$
\begin{array}{r}
-4x^2+4Lx+2L^2 \\
x-\frac{L}{2}\,\overline{)\,-4x^3+6Lx^2-L^3} \\
+4x^3-2Lx^2 \\
\hline
0+4Lx^2-L^3 \\
-4Lx^2+2L^2x \\
\hline
0+2L^2x-L^3 \\
-2L^2x+L^3 \\
\hline
0+0
\end{array}
$$

$$(-4x^3+6Lx^2-L^3)=0$$

$$(-4x^2+4Lx+2L^2)\left(x-\frac{L}{2}\right)=0$$

$$\boxed{X=\frac{L}{2}}$$

$$-x^2+Lx+\frac{L^2}{2}=0$$

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-L\pm\sqrt{L^2-(4\times-1\times\frac{L^2}{2})}}{-2}=\frac{-L\pm\sqrt{L^2+2L^2}}{-2}$$

$(0, L)$ ↑

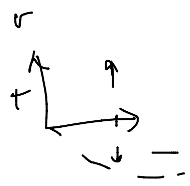
$$=\frac{-L\pm\sqrt{3L^2}}{-2}=\frac{-L\pm\sqrt{3}L}{-2}=\frac{L}{2}\pm\frac{\sqrt{3}}{2}L=\frac{1\mp\sqrt{3}}{2}L$$

$x_1=(\frac{1}{2}+\frac{\sqrt{3}}{2})L$

$x_2=(\frac{1}{2}-\frac{\sqrt{3}}{2})L$ ---

$EI\theta(\frac{L}{2})=0 \rightarrow$ Max v

$$v\left(\frac{L}{2}\right)=\frac{-5}{96}\,\frac{wl^4}{EI} \leftarrow$$



Example 2

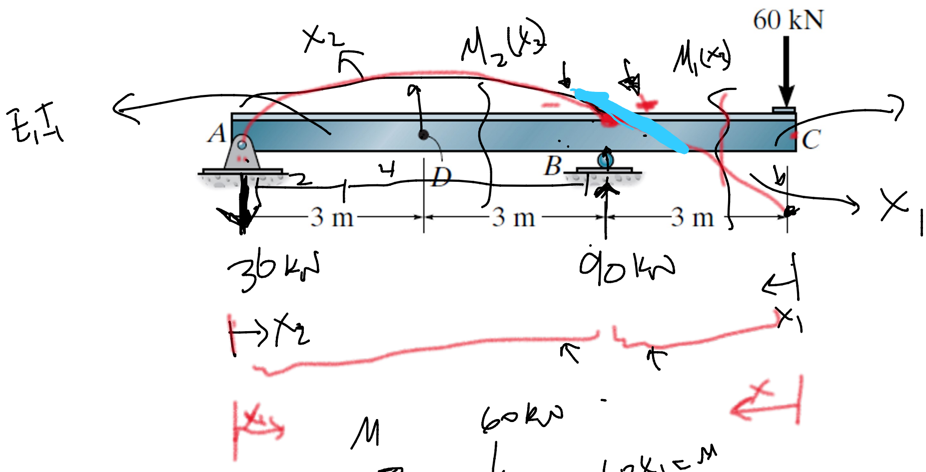
Saturday, April 3, 2021 12:26 AM

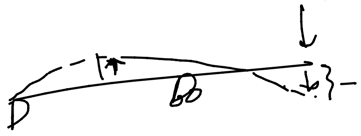
Calculate the deflection at point C, D

, EI →

$$\iint \frac{d^2v}{dx^2} = \iint \frac{M}{EI}$$

v(x) = ... + C_1 + C_2 → B.C.





① $M_1(x_1)$ →

60 kN, M, x_1: $-60x_1 = M$

$$M_1(x_1) = -60x_1 \qquad C_1, C_2$$

② 30 kN, M, x_2

$$\rightarrow M_2(x_2) = -30x_2 \qquad C_3, C_4$$

B.C.

at A: $v_A(0) = 0$ ← x_2

at B: $v_B(x_1 = 3) = 0$ ← x_1

$v_B(x_2 = 6) = 0$ ← x_2

$\rightarrow \theta_{B^-} = \theta_{B^+}$

$\theta_B^-(x_1 = 3) = \theta_B^+(x_2 = 6)$ ✓

$$EI\theta = \begin{cases} -30x_1^2 + C_1 & 0 < x_1 < 3 \\ -15x_2^2 + C_3 & 0 < x_2 < 6 \end{cases}$$

D ← $x_2 = 3$

E ← $x_2 = 2$

C ← $x_1 = 0$

$$EIv = \begin{cases} -10x_1^3 + C_1x_1 + C_2 & 0 < x_1 < 3 \\ -5x_2^3 + C_3x_2 + C_4 & 0 < x_2 < 6 \end{cases}$$

B.C.

at A: $v_A(0) = 0$ ← x_2

at B: $v_B(x_1 = 3) = 0$ ← x_1

$v_B(x_2 = 6) = 0$ ← x_2

$\rightarrow \theta_{B^-} = \theta_{B^+}$

$\theta_B^-(x_1 = 3) = \theta_B^+(x_2 = 6)$ ✓

$C_1 = 0$ ① ②

$\theta_B^-(x_1=3) = \theta_B^+(x_2=6)$ ✓

ref axis $\theta^- = -\theta^+$

$v_A(x_2=0)=0 \rightarrow C_4=0$ — (1)

$v_B(x_1=3)=0 \rightarrow -270+3C_1+C_2=0$ — (2)

$v_B(x_2=6)=0 \rightarrow -1080+6C_3=0 \rightarrow C_3=180$ — (3)

$\theta_B^r(x_1=3) = \theta_B^{left}(x_2=6) \Rightarrow -270+C_1=-540+180 \rightarrow C_1=-90$ — (4)

Eq (2) $\rightarrow$ $-270+3\times 90+C_2=0 \rightarrow C_2=540$ (−1620)

$$EI\theta = \begin{cases} -30x_1^2-90 & 0<x_1<3 \\ -15x_2^2+180 & 0<x_2<6 \end{cases}$$

$$EIv = \begin{cases} -10x_1^3 - 90x_1+540 & 0<x_1<3 \\ -5x_2^3+180x_2 & 0<x_2<6 \end{cases}$$

kN, m

$E=200$ GPa $\rightarrow$ kPa (kN/m²)

$I=5\times 10^8$ mm⁴ $\rightarrow$ m⁴ $(10^{-3})^4$

kN/m

$EIv_C = 540$

$EIv_D = 405$??

giga $\rightarrow 10^9$

Mega $\rightarrow 10^6$

kilo $\rightarrow 10^3$

$$v_C = \frac{540}{200\times 10^6 \times 5\times 10^8 \times 10^{-12}} = 5.4\times 10^{-3}\text{ m} \rightarrow 5.4\text{ mm}$$

(corrected: -16.2×10^{-3}, 16.2 mm ↓)

$$v_D = \frac{405}{200\times 10^6\times 5\times 10^8\times 10^{-12}} = 4.05\text{ mm} \uparrow$$

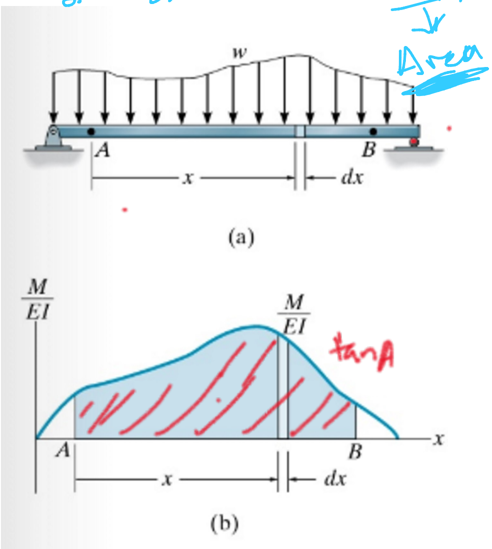
$EI\,\theta_A = 180 \longrightarrow \theta_A = \dfrac{180}{200\times10^{6}\times45\times10^{8}\times10^{-2}} = 2\times10^{-3}\ rad$

Deflections - Moment area

Saturday, April 3, 2021 12:12 AM

$$\frac{d^2v}{dx^2} = \frac{M}{EI}$$

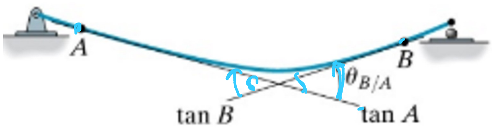
$$\int \frac{d^2v}{dx^2} = \theta = \int \frac{M}{EI} = \text{area} \rightarrow \frac{M}{EI}$$



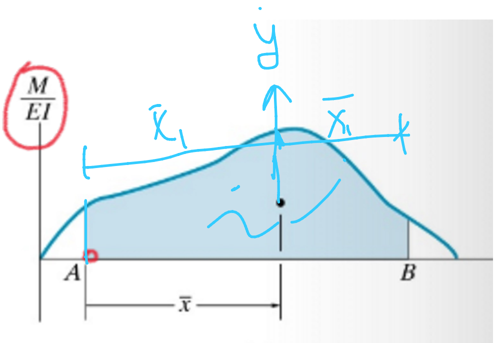
Theorem 1: The change in slope between any two points on the elastic curve equals the area of the M/EI diagram between these two points.

$\theta_{B/A}$ = area under $\frac{M}{EI}$

$\theta_{A/B}$ = " " " = $\theta_{B/A}$

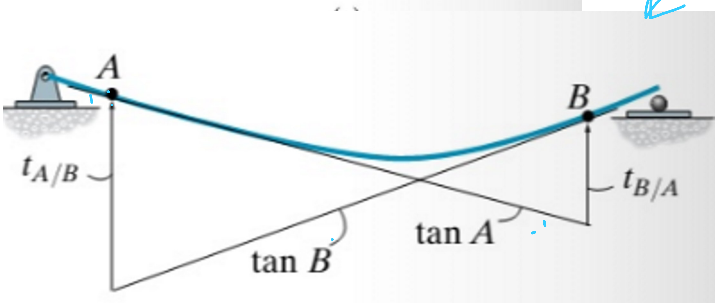


$$\iint \frac{M}{EI} = \int \theta = \text{Moment of area}$$



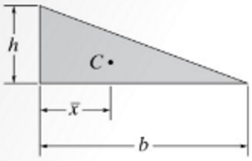
$$v = \iint \frac{M}{EI}$$

Moment of area



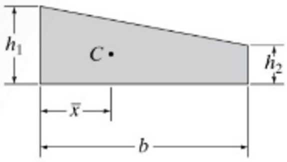
Theorem 2: The vertical deviation of the tangent at a point (A) on the elastic curve with respect to the tangent extended from another point (B) equals the "moment" of the area under the M/EI diagram between the two points (A and B). This moment is computed about point A (the point on the elastic curve), where the deviation $t_{A/B}$ is to be determined.

Geometric Properties of Areas



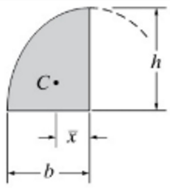
Triangle

$$A = \frac{1}{2}bh$$
$$\bar{x} = \frac{1}{3}b$$



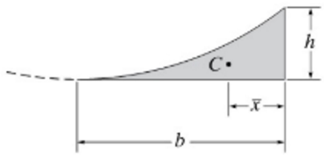
Trapezoid

$$A = \frac{1}{2}b(h_1 + h_2)$$
$$\bar{x} = \frac{b(2h_2 + h_1)}{3(h_1 + h_2)}$$



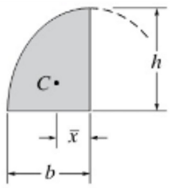
Semi Parabola

$$A = \frac{2}{3}bh$$
$$\bar{x} = \frac{3}{8}b$$



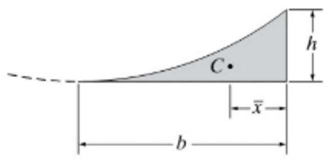
Parabolic spandrel

$$A = \frac{1}{3}bh$$
$$\bar{x} = \frac{1}{4}b$$



Semi-segment of *n*th degree curve

$$A = bh\left(\frac{n}{n + 1}\right)$$
$$\bar{x} = \frac{b(n + 1)}{2(n + 2)}$$



Spandrel of *n*th degree curve

$$A = bh\left(\frac{1}{n + 1}\right)$$
$$\bar{x} = \frac{b}{n + 2}$$

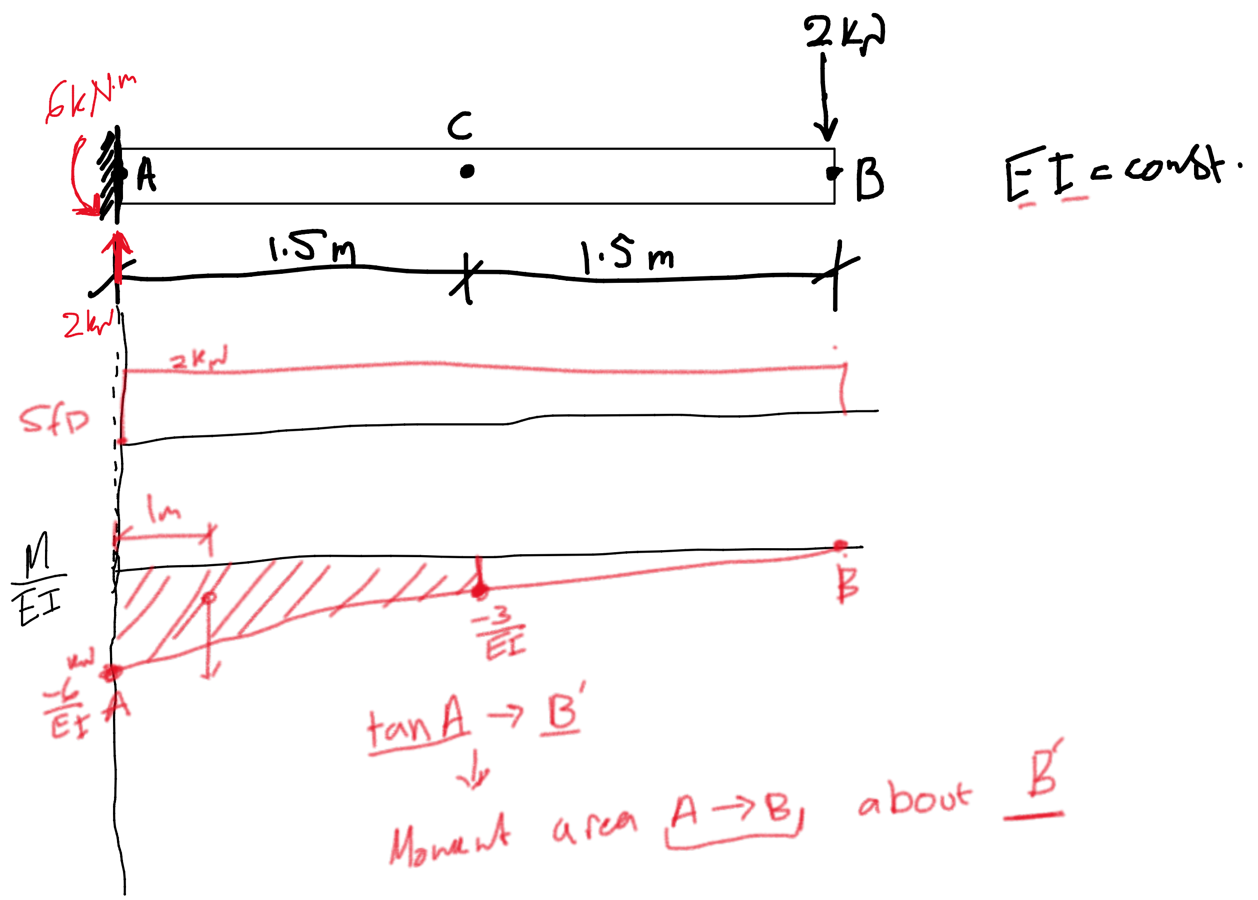
Example 1

Saturday, April 3, 2021 12:22 AM

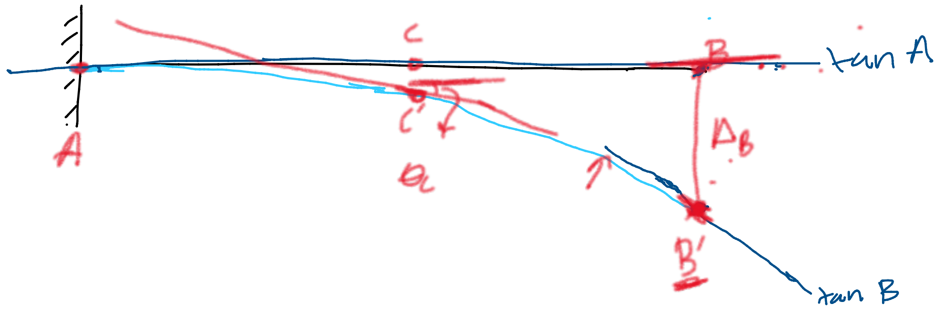
find

Δ_B ??

θ_C ??



ΔB



$\Delta_B = \tan A \to B' \to$ moment of area $A \to B$ about B ($\frac{M}{EI}$)

GPa → kPa

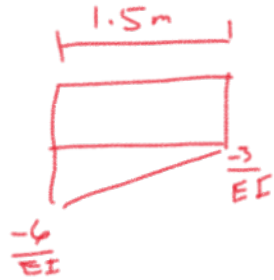
$10^9 \to 10^3$ Pa

kPa

$\Delta_B = \underbrace{\frac{1}{2} \times 3 \times \frac{-6}{EI}}_{\text{area}} \times (2\,m) = \frac{-18}{EI}$ kN, m

$E = 70\,GPa$ $I = 2 \times 10^7\,mm^4$ → $\Delta_B = \frac{-18}{70 \times 10^6 \times 2 \times 10^7 \times 10^{-12}} = -12.85\,mm$ ↓ downward

$$\theta_C = \theta_{A/C} = \text{Area } \frac{M}{EI} \; C \to A = \frac{1}{2}\left(\frac{-6}{EI} + \frac{-3}{EI}\right) \times 1.5$$



$$\theta_C = \frac{-6.75}{EI}$$

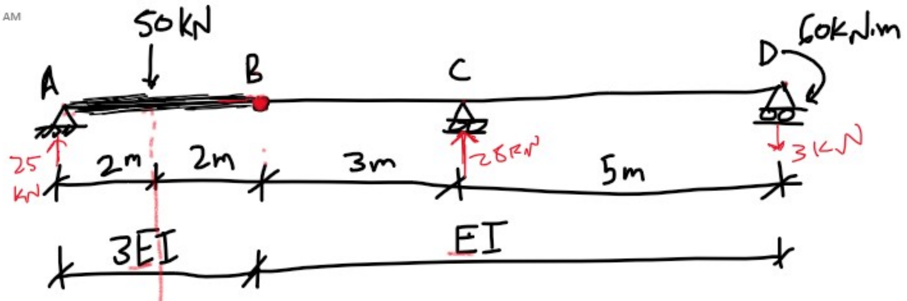
$$\theta_C = \frac{-6.75}{70\times10^{6} \times 2\times10^{7} \times 10^{-12}} = -4.82\times10^{-3} \text{ rad}$$

CW

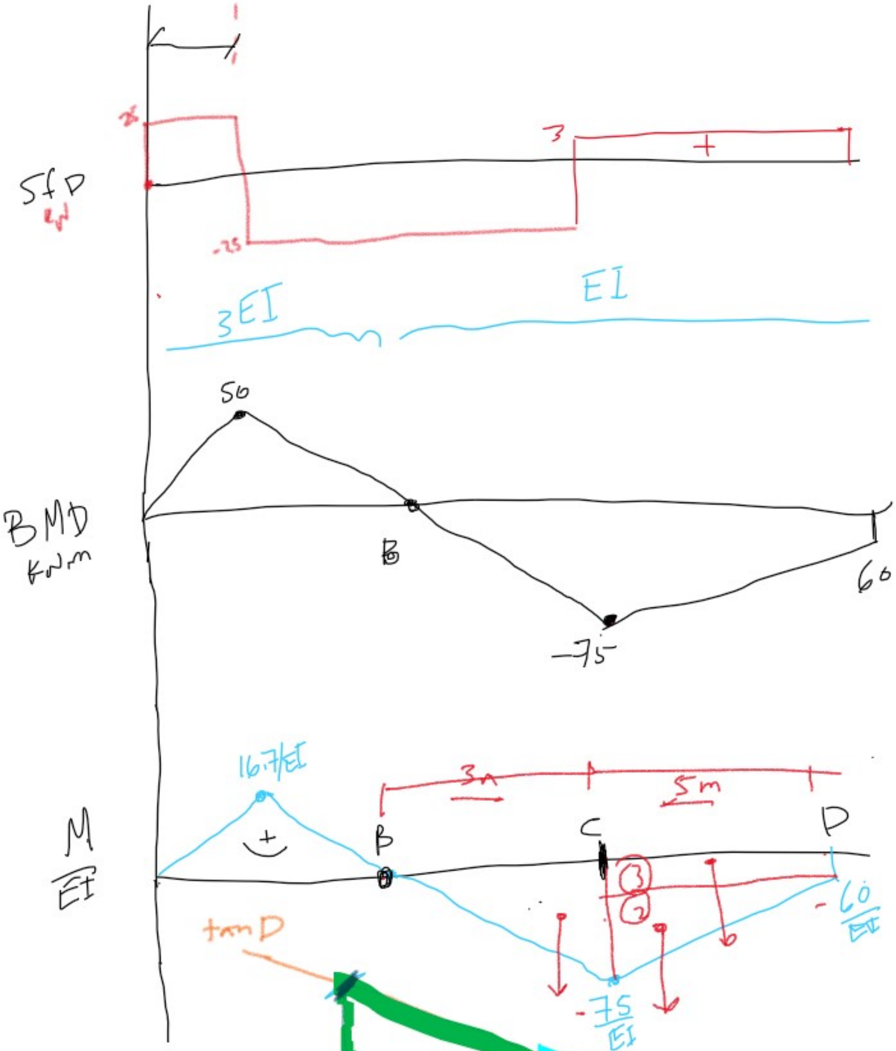
Example 2

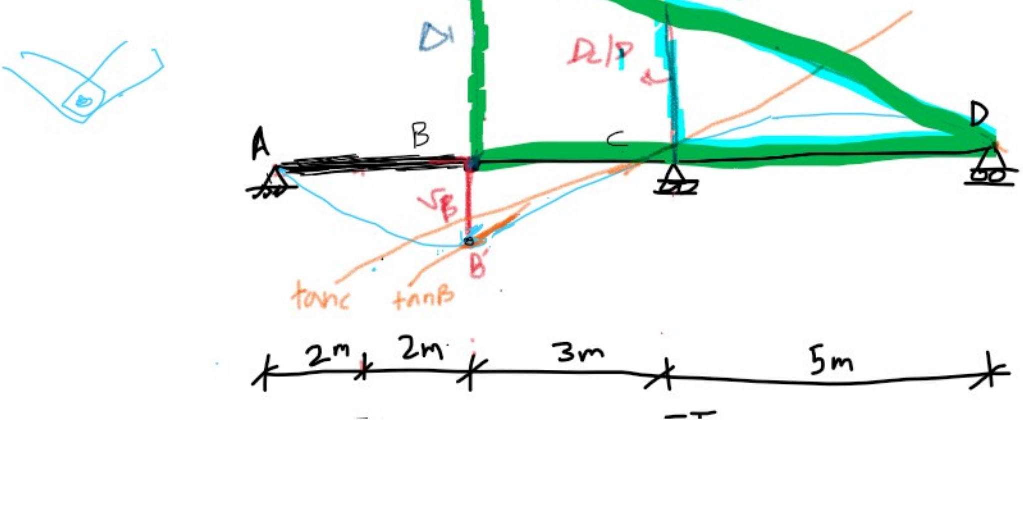
Saturday, April 3, 2021 12:22 AM

find Δ_B



discont. Elastic curve → Moment area ← A → B, B → D



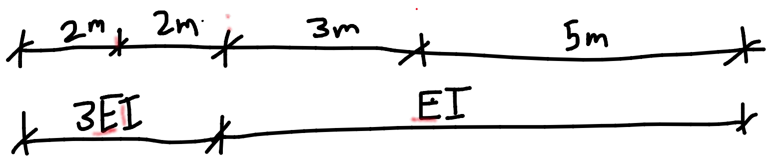


$$v_B = \Delta_{B/D} - \Delta_1$$

$$\frac{\Delta_{C/D}}{5} = \frac{\Delta_1}{8}$$

$$\Delta_1 = \frac{8}{5}\Delta_{C/D}$$

$$v_B = \Delta_{B/D} - \frac{8}{5}\Delta_{C/D}$$



$v_B = \Delta_{B/D} - \frac{8}{5}\Delta_{C/D}$

① ② ③

$\Delta_{B/D}$ = Moment (Area B → D) about B $= \frac{1}{2}\times 3\times\frac{-75}{EI}\times(2) + \frac{1}{2}\times 5\times\frac{-75+60}{EI}\times\left(3+\frac{5}{3}\right) + 5\times\frac{-60}{EI}\times\left(3+\frac{5}{2}\right)$

$= \frac{225}{EI} + \frac{175}{EI} + \frac{1650}{EI} = \frac{-2050}{EI}$

$\Delta_{C/D}$ = Moment (C → D) about C $= \frac{1}{2}\times 5\times\frac{75-60}{EI}\left(\frac{5}{3}\right) + 5\times\frac{60}{EI}(2.5) = \frac{-812.5}{EI}$

E = —

I = —

$v_B = \Delta_{B/D} - \frac{8}{5}\Delta_{C/D} = \frac{2050}{EI} - \frac{8}{5}\left(\frac{812.5}{EI}\right) = \frac{-750}{EI}$ ↓

Deflections - Conjugate beam

Monday, April 19, 2021 11:34 AM

Load

Shear

Moment

Slope

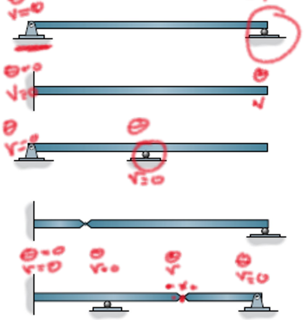
Deflection

conjugate beam

Match Support conditions to B.C

TABLE 8.2

	Real Beam			Conjugate Beam	
1)	θ $\Delta = 0$	pin	V $M = 0$	pin
2)	θ $\Delta = 0$	roller	V $M = 0$	roller
3)	$\theta = 0$ $\Delta = 0$	fixed	$V = 0$ $M = 0$	free
4)	θ Δ	free	V M	fixed
5)	θ $\Delta = 0$	internal pin	V $M = 0$	hinge
6)	θ $\Delta = 0$	internal roller	V $M = 0$	hinge
7)	θ Δ	hinge	V M	internal roller



real beam

Theorem 1: The slope at a point in the real beam is numerically equal to the shear at the corresponding point in the conjugate beam.

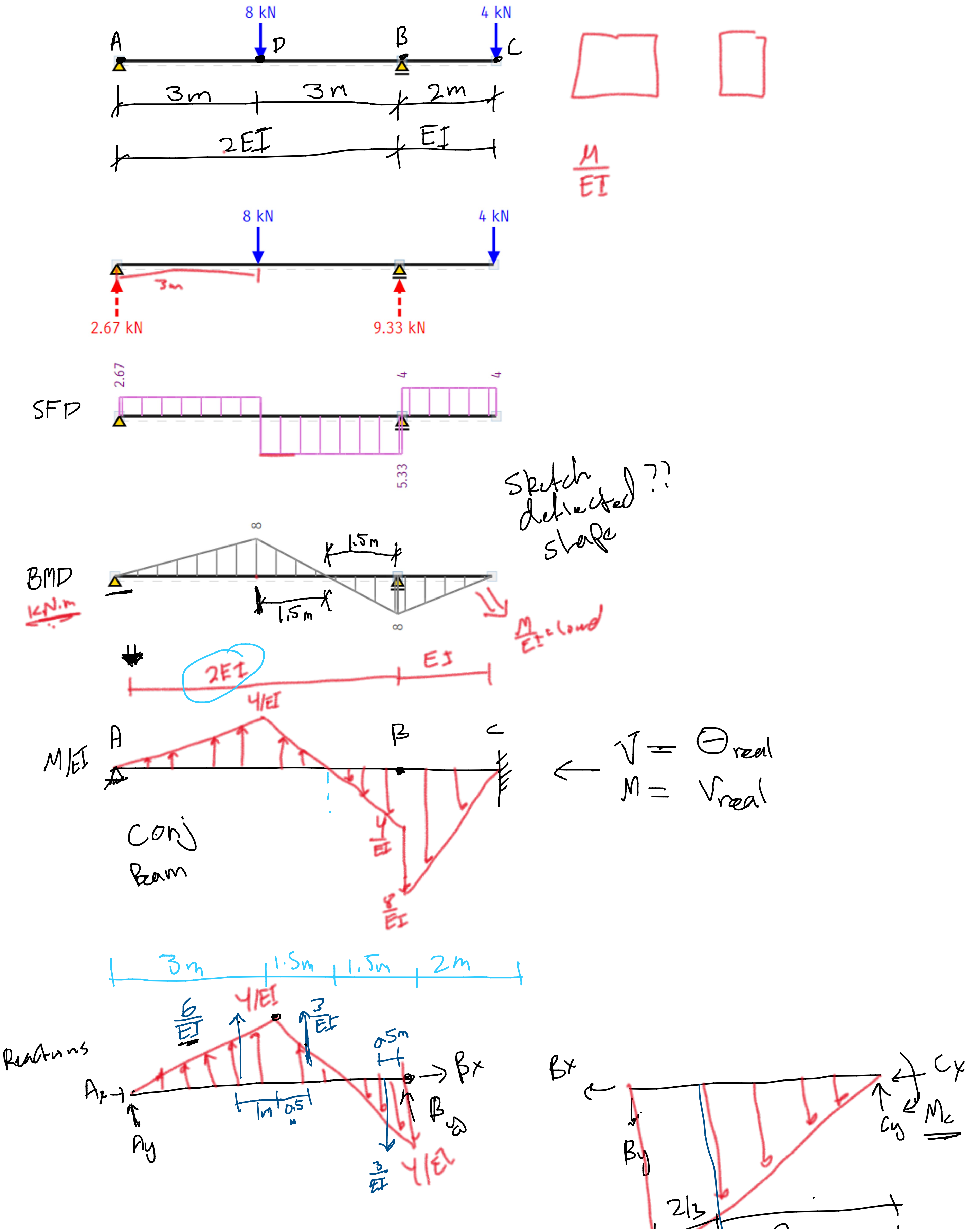
Theorem 2: The displacement of a point in the real beam is numerically equal to the moment at the corresponding point in the conjugate beam.

Example 1

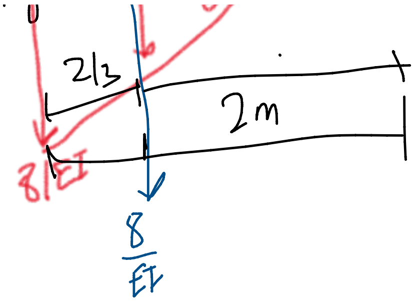
Monday, April 19, 2021 11:55 AM

find v_C, v_D, θ_D

$E = 200\,GPa$
$I = 7\times10^{7}\,mm^4$



$\frac{3}{EI}$ 4/EI



$\Sigma f_x = 0 \Rightarrow A_x = -B_x$

$\Sigma f_y = 0 \Rightarrow A_y + \frac{6}{EI} + \frac{3}{EI} - \frac{3}{EI} + B_y = 0$

$A_y + B_y = -\frac{6}{EI}$

12 10.5

$\Sigma M_A = 0 \Rightarrow \frac{6}{EI}(2) + \frac{3}{EI}(3.5) - \frac{3}{EI}(5.5) + B_y(6) = 0$

$B_y = \frac{-1}{EI}$

$A_y = \frac{-5}{EI}$

$\Sigma f_x = B_x = -C_x$

$\Sigma f_y = 0$

$-B_y - \frac{8}{EI} + C_y = 0$

$C_y = \frac{7}{EI}$

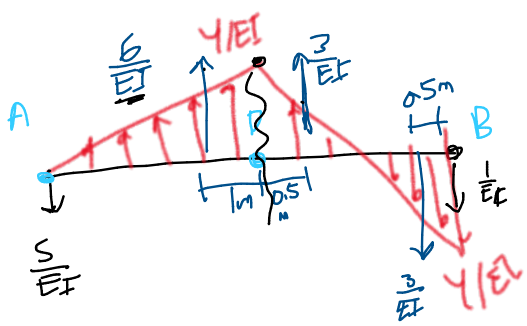
$\Sigma M_c = 0$

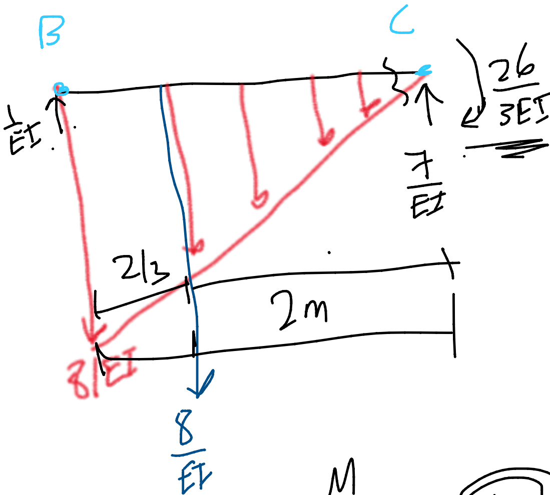
$B_y(2) + \frac{8}{EI}\left(\frac{4}{3}\right) - M_c = 0$

$\frac{-1}{EI}$

$M_c = \frac{26}{3EI}$

v_c, v_D, θ_D





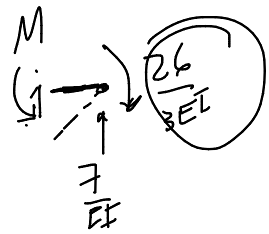
kN / m

Real conj

$v_c = M_c = \frac{-26}{3EI} = \frac{-26}{3 \times 200 \times 10^6 \times 7 \times 10^{3} \times 10^{-12}}$

↑ int

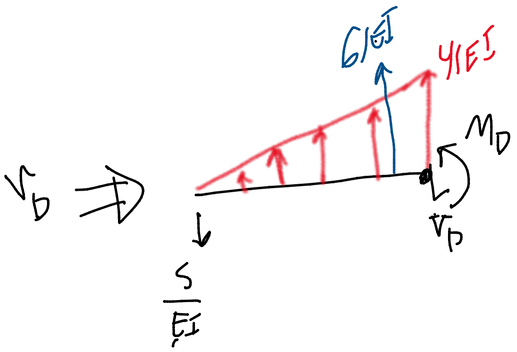
GPa → kPa



$\ldots \times 10^{-4}$ m = 0.62 mm

$\frac{-1}{EI}$

$v_c = -6.2 \times 10^{-4}\ m = \underline{0.62\ mm}$



real v_D = conj M_D

$\Sigma M_D = 0 \Rightarrow \frac{5}{EI}(3) - \frac{6}{EI}(1) + M_D = 0$

$M_D = \frac{-9}{EI} = v_D = \frac{-9}{200 \times 10^{6} \times 7 \times 10^{-3} \times 10^{-2}} = 0.642\ mm$

real conj

$\theta_D = V_D \Rightarrow \frac{-5}{EI} + \frac{6}{EI} - V_D = 0$

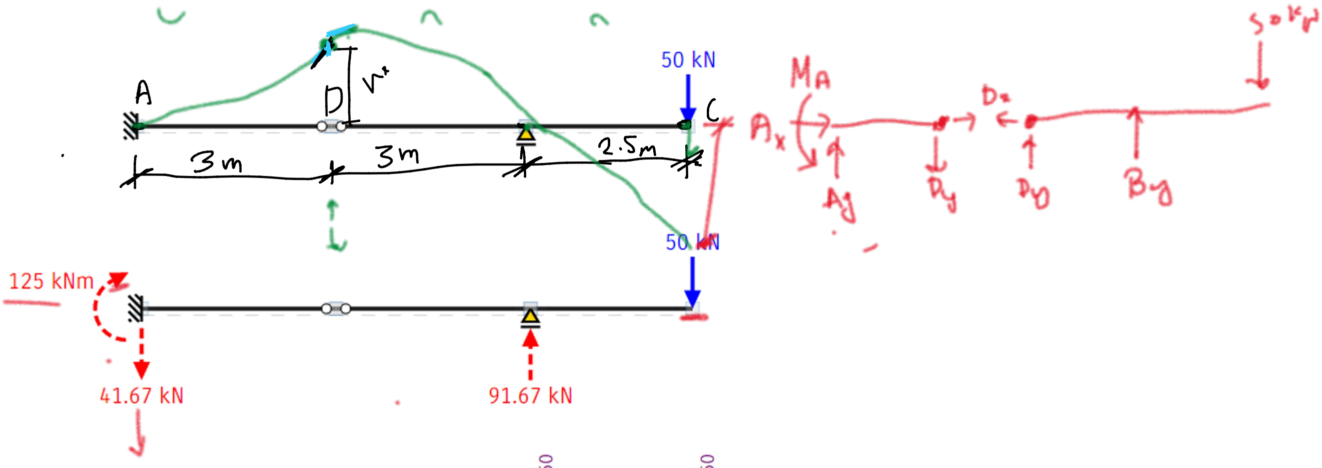
$V_D = \frac{1}{EI} = \theta_D$

Example 2

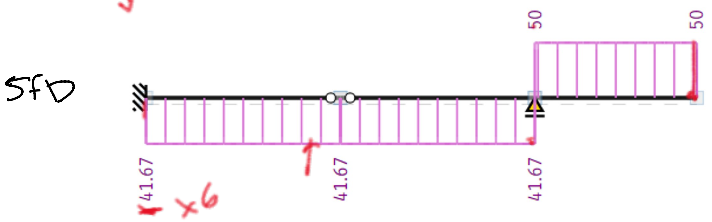
Monday, April 19, 2021 12:00 PM

find θ_D^-, θ_D^+, v_C

EI = const.

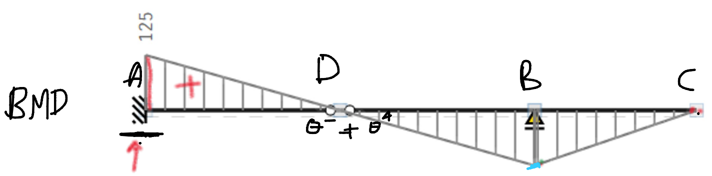


SFD

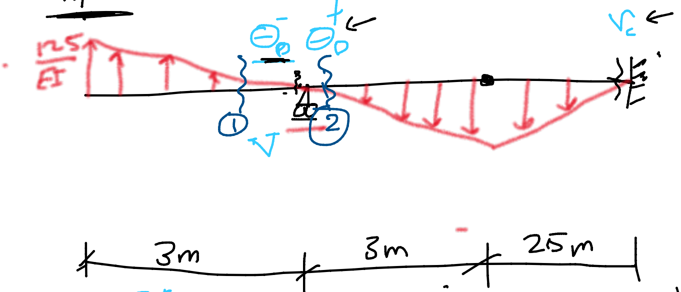


Sketch deflected shape??

BMD



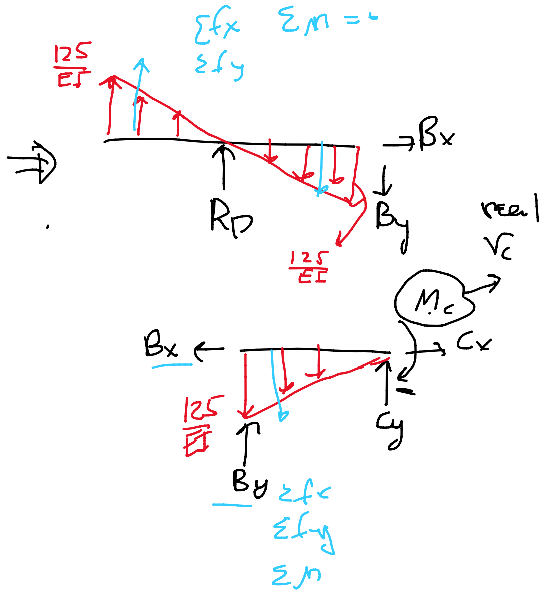
A	D	B	C
$\theta = 0$, $v = 0$	$\theta = ??$, $v = ??$ Real	$\theta = ??$, $v = 0$	$\theta \neq 0$, $v \neq 0$
⇓	⇓	⇓	⇓
$V = 0$?, $M = 0$	$V \neq 0$, $M = 0$ Conj	$V \neq 0$, $M = 0$	$V \neq 0$, $M \neq 0$

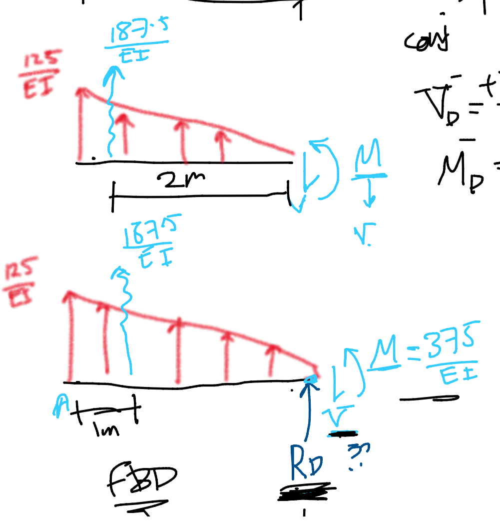


Conj

$V \rightarrow \theta_{real}$

$M \rightarrow v_{real}$





conj real

$$V_D^- = \frac{+187.5}{EI} = \theta_D^-$$

$$M_D^- = \frac{+375}{EI} = v_D^-$$

$$\Sigma F_y = 0 \Rightarrow \frac{187.5}{EI} + R_D - V = 0$$

'Im'

FBD → RD ??

[redacted]

b

conj

Mc → Vt

Deflections - Energy methods

Monday, April 19, 2021 12:01 PM

Analysis
- Truss → Axial int. Loads
- Beam → V, M
- Frame → N, V, M

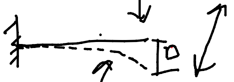
Deflection

$$\frac{\partial^2 v}{\partial x^2} = \frac{M}{EI}$$ → v → geometric methods: ∫∫, Moment area, conjugate Beam

Beams

Energy methods → Principle → conservation of Energy

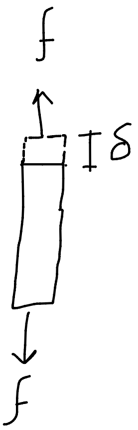
↓



→ Ext. work = P * Δ

$$\frac{\partial^2 v}{\partial x^2} = \frac{M}{EI}$$

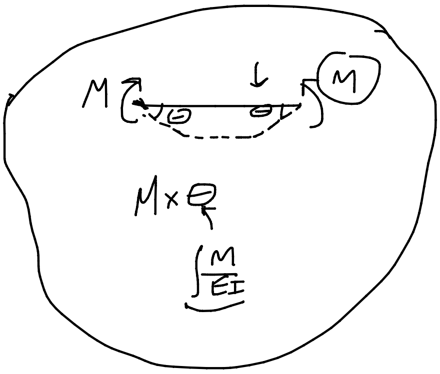
$$\frac{\partial v}{\partial x} = \int$$



internal Energy ↓

$$f * \frac{fL}{AE}$$

Axial



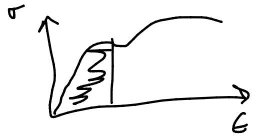
$M \times \theta$

$$\int \frac{M}{EI}$$

Strain Energy

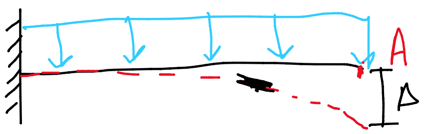
$$P \cdot \Delta = M \int \frac{M}{EI}$$

Ext. Int.

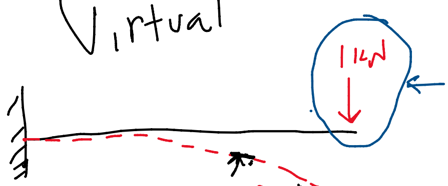


Virtual Work method

Real



Virtual



External Virtual work = Internal Virtual work

External Virtual work = Internal Virtual work

$1 \times \Delta$ ← Ext. disp. = $u \times dL$ ← int. disp

1 → Virtual ← Ext. force

Δ → real

u → virtual → int. force

dL → real

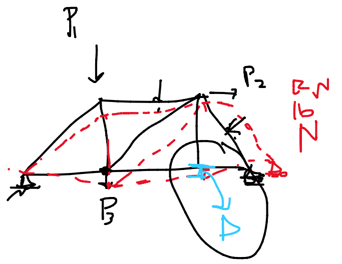
int. force →

- Truss → $n \times \delta = \frac{N \times L}{AE}$
- Beam: $m \times \int \frac{M}{EI}$
- frame: $m \times \int \frac{M}{EI}$ ←

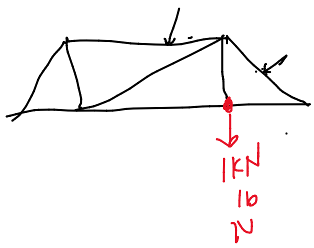
Truss

Thursday, May 20, 2021 9:01 AM

Main internal force → Axial force (N)



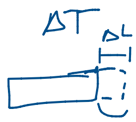
Real → N → disp. ←



Virtual → n forces

Ext. Virtual work = Int. Virtual Work

$$1 * \Delta = \sum n \times \frac{NL}{AE} \leftarrow \text{system}$$

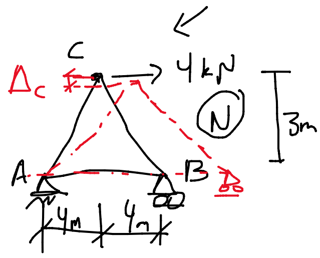


Temp. change: $1 \times \Delta = \sum n * \alpha \Delta T L$

fabrication Error: $1 \times \Delta = \sum n \times \Delta L_{\text{Error}}$

Example:

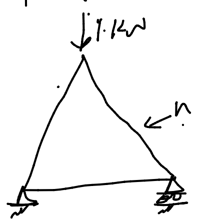
find Vertical disp. at C



Real

all members
$A = 400\ mm^2$
$E = 70\ GPa$

$$1 * \Delta_C = \sum n \times \frac{NL}{AE}$$



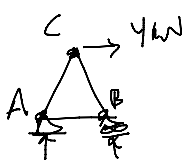
Virtual

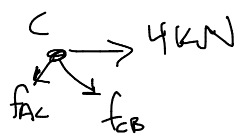
	n	N	L	nNL
AC	-0.83	2.5	5	-10.375
AB	0.67	2	8	10.72
CB	-0.83	-2.5	5	10.375
			Σ	10.72

$$1 * \Delta_C = \frac{10.72}{AE} = \frac{10.72}{400 \times 10^{-6} \times 70 \times 10^{6}}$$

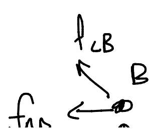
(A: m^2, E: kPa)

$$\Delta_C = 3.825 \times 10^{-4}\ m = 0.3825\ mm$$



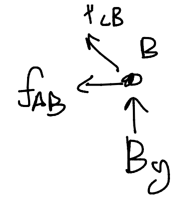


$\sum f_x = 0$
$\sum f_y = 0$ → f_{CB}, f_{AC}



$\sum F_x = 0$ → f_{AB}

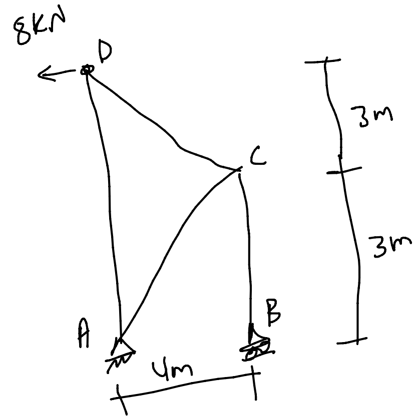
$\Delta_C = 3.825 \times 10^{-?} m = 0.00\ldots$



$\Sigma f_x = 0 \rightarrow f_{AB}$

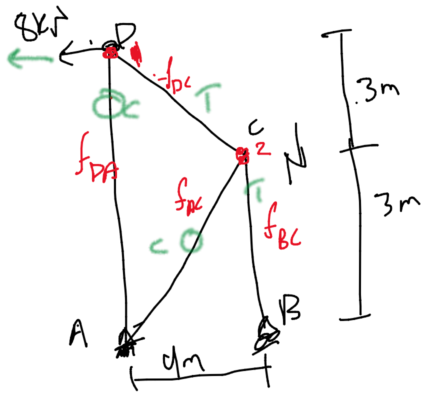
Example 2: find Horizontal disp at D due to:
① loading
② Temp. change AD, DC — $T_1 = 30°C$, $T_2 = 120°C$
③ member DC was fabricated $L = 5.05$ m
④ all

$A = 200$ mm^2
$E = 200$ GPa

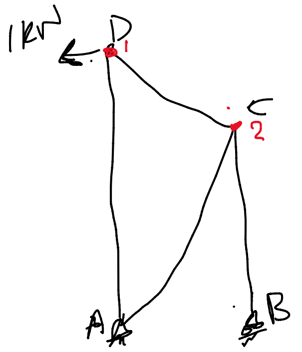


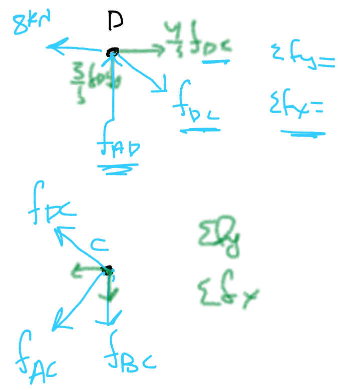
$\alpha = 1.2 \times 10^{-5}$ /°C

Real



Virtual





① Δ_D due to loading ⇐ real int. disp $= \frac{NL}{AE} \times n$

	N kN	n kN	L m	nNL	$\alpha \Delta T L$ m	$n \times \alpha \Delta T L$	ΔL m	$n \times \Delta L$
AD	-6	-0.75	6	27	6.48×10^{-3}	-4.86×10^{-3}	0.05	-0.0375
AC	-10	-1.25	5	62.5	0	0	0	0
BC	12	1.5	3	54	0	0	0	0
CD	10	1.25	5	62.5	5.4×10^{-3}	6.75×10^{-3}	0	0
			Σ	206 kN²m		1.89×10^{-3}		-0.0375

$1 \times \Delta_D = \frac{206 \text{ kN}^2 \cdot m}{200 \times 10^{-6} \cdot 200 \times 10^{6}} \Rightarrow \Delta_D = 5.15$ mm ←

$$1 \times \Delta_D = \frac{206\ kN^2.m}{200\times10^{-6} \times 200\times10^{6}} \Rightarrow \Delta_D = 5.15\,mm \leftarrow$$

(2) Temp. change ← real int disp $= \alpha \Delta T L \times n$

$$1 \times \Delta_D = 1.89\times10^{-3} \Rightarrow \Delta_D = 1.89\ mm \leftarrow$$

(3) fab. error (AD = 5.05 m) → real int. disp. ⇒ $\Delta_{AD} = 0.05\,m$

$$1 \times \Delta_D = -0.0375 \Rightarrow \Delta_D = 3.75\ cm \rightarrow$$

n × real int. disp

(4) ALL

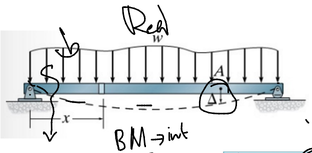
$$\Delta_D = 37.5 - 1.89 - 5.15 = 30.46\,mm \rightarrow$$

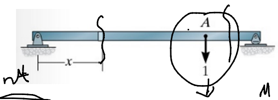
HW

Tuesday, May 25, 2021 8:01 AM

Beams

Tuesday, May 25, 2021 7:51 AM

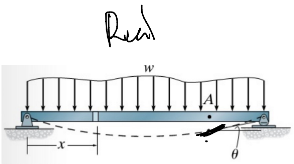




Work

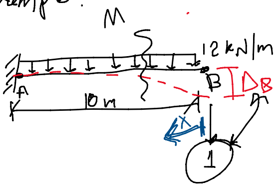
Ext. $1 \cdot \Delta = \int_0^L \frac{mM}{EI} dx$ — int

m ... M ↔ rotation $\int \frac{M}{EI}$



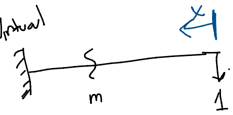
Ext. $1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI} dx$

Example:

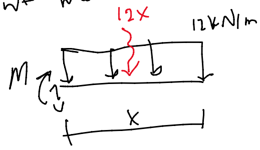


find Δ_B

$E = 200\,GPa$
$I = 560 \times 10^6\ mm^4$

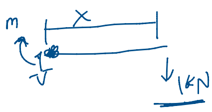


M: real int moment



$M = -6x^2$

m: virtual int moment



$m = -x$

$$1 \times \Delta_B = \int_0^{10} \frac{mM}{EI} dx$$

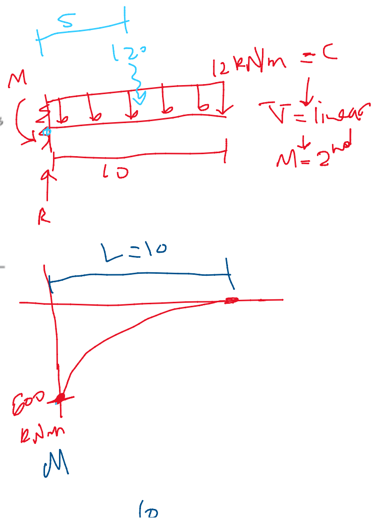
$$1 \times D_B = \int_0^{10} \frac{-6x^2(-x)}{EI} dx$$

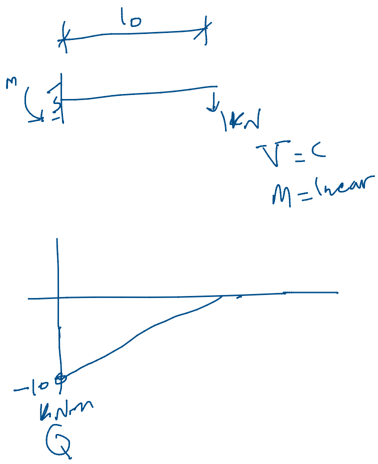
$$= \int_0^{10} \frac{6x^3}{EI} dx = \frac{1}{EI} \times \frac{6}{4} (x)^4 \Big|_0^{10}$$

$$= \frac{1}{EI} \times \frac{6}{4} \times (10)^4 = \frac{15000}{EI} = \frac{15000}{200 \times 10^6 \times 560 \times 10^6 \times 10^{-12}} = 0.615\ m = \Delta_B$$

Table for: $\int_0^L M\,Q\,dx$ — The values in the table represent the integration of the product of the two shapes with a common length L.

	A: Rectangle (Q, L)	B: Triangle (Q at right end, L)	C: Triangle (Q at left end, L)	D: Trapezoid (Q_a, Q_b, L)
① Rectangle (M, L)	LMQ	$\frac{LMQ}{2}$	$\frac{LMQ}{2}$	$\frac{LM}{2}(Q_a + Q_b)$
② Triangle (M at right end, L)	$\frac{LMQ}{2}$	$\frac{LMQ}{3}$	$\frac{LMQ}{6}$	$\frac{LM}{6}(Q_a + 2Q_b)$
③ Triangle (M at left end, L)	$\frac{LMQ}{2}$	$\frac{LMQ}{6}$	$\frac{LMQ}{3}$	$\frac{LM}{6}(2Q_a + Q_b)$
④ Triangle (M at peak, a, b, L)	$\frac{LMQ}{2}$	$\frac{MQ}{6}(L + a)$	$\frac{MQ}{6}(L + b)$	$\frac{M}{6}[Q_a(L + b) + Q_b(L + a)]$





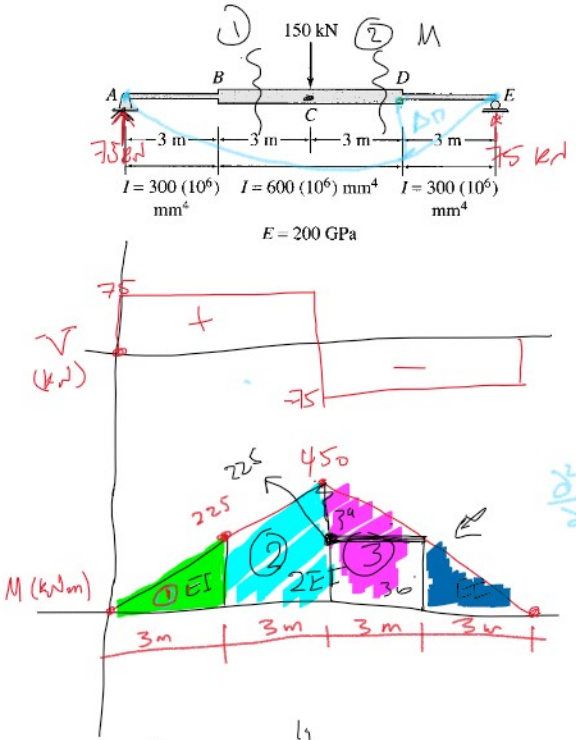
(4) Triangle, M, a, b, L	$\frac{LMQ}{2}$	$\frac{MQ}{6}(L+a)$	$\frac{MQ}{6}(L+b)$	$\frac{M}{6}\left[Q_a(L+b) + Q_b(L+a)\right]$
(5) Trapezoid, M_a, M_b, L	$\frac{LQ}{2}(M_a+M_b)$	$\frac{LQ}{6}(M_a+2M_b)$	$\frac{LQ}{6}(2M_a+M_b)$	$\frac{L}{6}\left[Q_a(2M_a+M_b) + Q_b(M_a+2M_b)\right]$
Parabola, slope = 0, M, L	$\frac{2LMQ}{3}$	$\frac{5LMQ}{12}$	$\frac{LMQ}{4}$	$\frac{LM}{12}(3Q_a+5Q_b)$
Parabola, slope = 0, M, L	$\frac{2LMQ}{3}$	$\frac{LMQ}{4}$	$\frac{5LMQ}{12}$	$\frac{LM}{12}(5Q_a+3Q_b)$
Parabola, slope = 0, M, L	$\frac{LMQ}{3}$	$\frac{LMQ}{4}$	$\frac{LMQ}{12}$	$\frac{LM}{12}(Q_a+3Q_b)$
Parabola, M, slope = 0, L	$\frac{LMQ}{3}$	$\frac{LMQ}{12}$	$\frac{LMQ}{4}$	$\frac{LM}{12}(3Q_a+Q_b)$

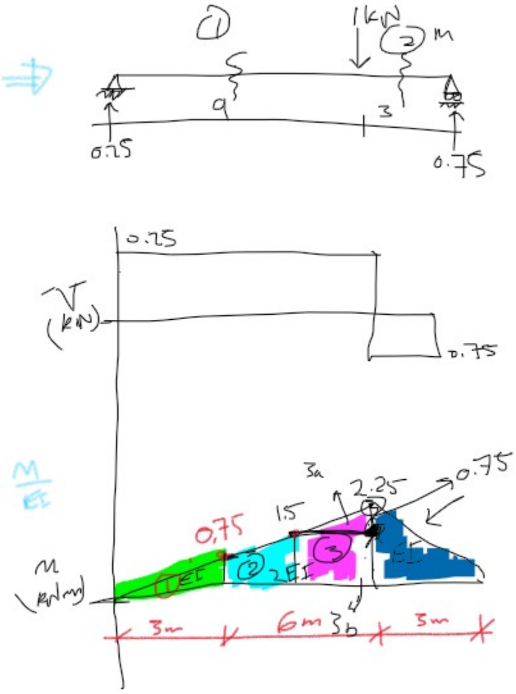
kN·m M

Q

$$\int_0^{10} \frac{mM}{EI}dx = \frac{LMQ}{4} = \frac{10\times600\times10}{4} = \frac{15000}{EI}$$

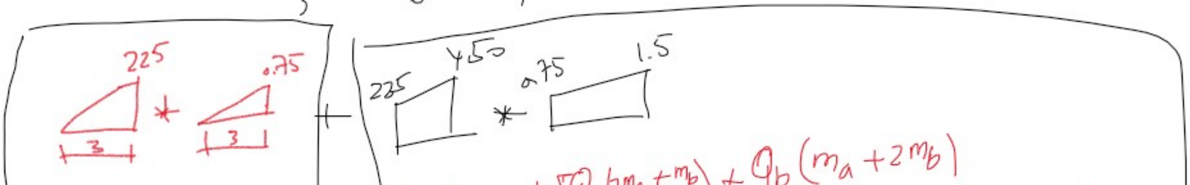
find Δ_D, virtual work Method





$$\int_0^6 \frac{m_1M_1}{EI} + \int_6^{12} \frac{mM}{EI}$$

$$\int_0^3 + \int_3^6 + \int_6^9 + \int_9^{12}$$



$\int Mm\,dx = \frac{L}{6}\left[Q_a(2m_a+m_b) + Q_b(m_a+2m_b)\right]$

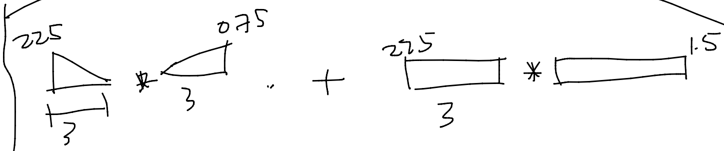
$$\int \frac{Mm\,dx}{EI} = \frac{3 \times 0.75 \times 225}{3EI}$$

①

$$\int \frac{Mm\,dx}{EI} = \frac{L}{6}\left[Q_a(2m_a + m_b) + Q_b(m_a + 2m_b)\right]$$

$$= \frac{\frac{3}{6}\left[0.75(2 \times 225 + 450) + 1.5(225 + 2 \times 450)\right]}{2EI}$$

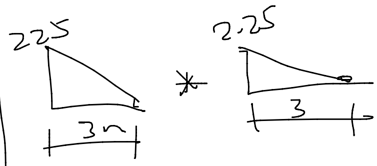
②



$$\frac{LMQ}{6} \qquad LMQ$$

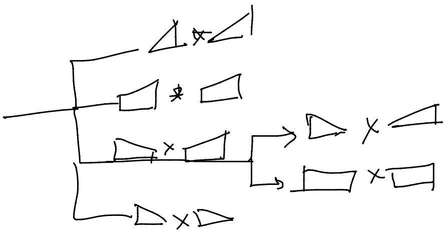
$$\frac{3 \times 225 \times 0.75}{6 \times 2EI} + \frac{3 \times 225 \times 1.5}{2EI}$$

③



$$\frac{LMQ}{3}$$

$$\frac{3 \times 225 \times 2.25}{3EI}$$



① $\frac{168.75}{EI}$

② $\frac{759.375}{EI}$

③ $\frac{548.44}{EI}$

④ $\frac{506.25}{EI}$

$$\int \frac{Mm}{EI} dx = \frac{1982.815}{EI}$$

$$1 \times \Delta_D = \frac{1982.815}{EI} \longrightarrow \underline{\underline{\Delta_D = 0.033\ m}}$$

↑ Vr ↑ real

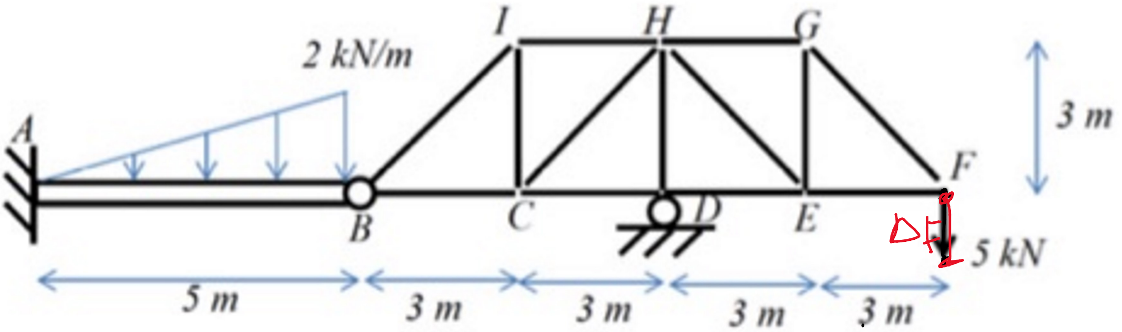
EI: $200\,GPa$ → 200×10^9; $300 \times 10^6\ mm^4$ → $300 \times 10^6 \times 10^{-12}$

discussion

Thursday, May 27, 2021 7:57 AM

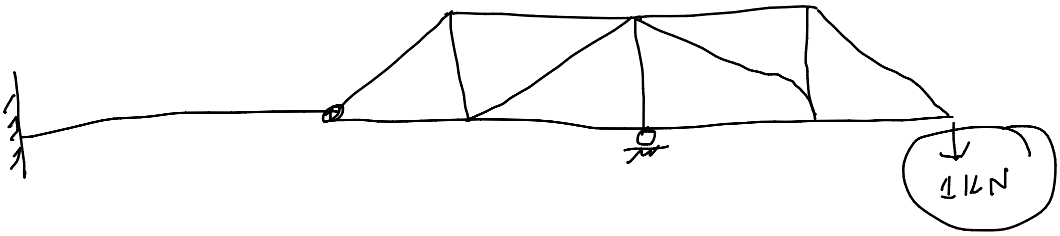
Δ_F

Real



N

Virtual



Virtual Work

Ext. = int.

$$1 * \Delta_f = \int \frac{mM}{EI} + \sum \frac{nNL}{AE}$$

rotation elongation

HW

Thursday, May 27, 2021 7:56 AM

Frame and other types of strain energy

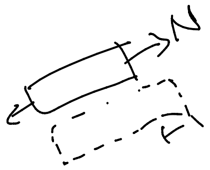
Thursday, May 27, 2021 8:03 AM

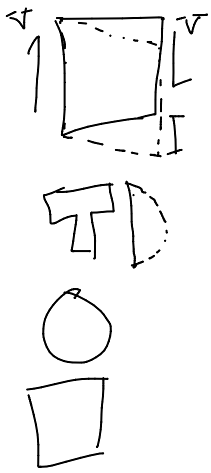
frames → int forces → N ← int disp $\frac{NL}{AE}$

→ V

→ M ← $\theta = \int \frac{M}{EI}$

$\tau = \frac{VQ}{It} \Rightarrow \text{avg } \tau = \frac{V}{A}$





$\gamma = k\frac{V}{GA}$

↑ Shear Modulus

rect. $k = 1.2$

circle $k = \frac{10}{9}$

T $k = 1$, A_{web}

real → virtual

$\frac{NL}{AE}$ → n

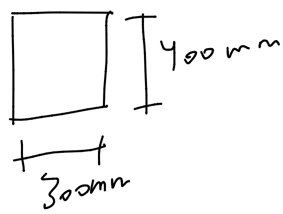
$\frac{kV}{GA}$ → v

$\int \frac{M}{EI}$ → m

Example:

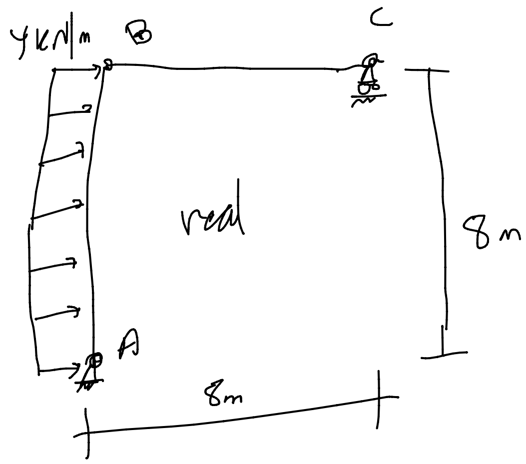
rect. section

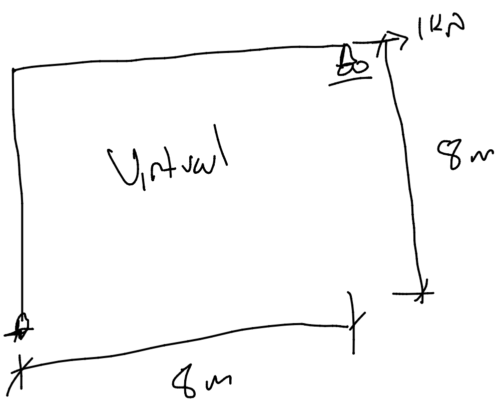
$E = 25\text{ GPa}$



Horizontal disp of point C

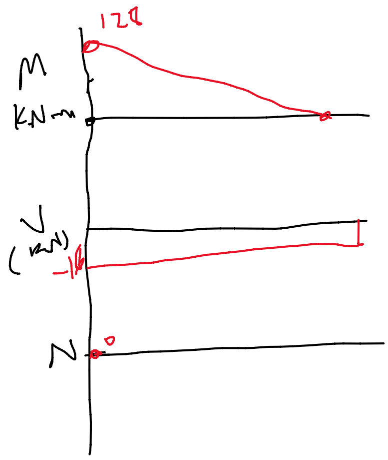
Consider All int disp.



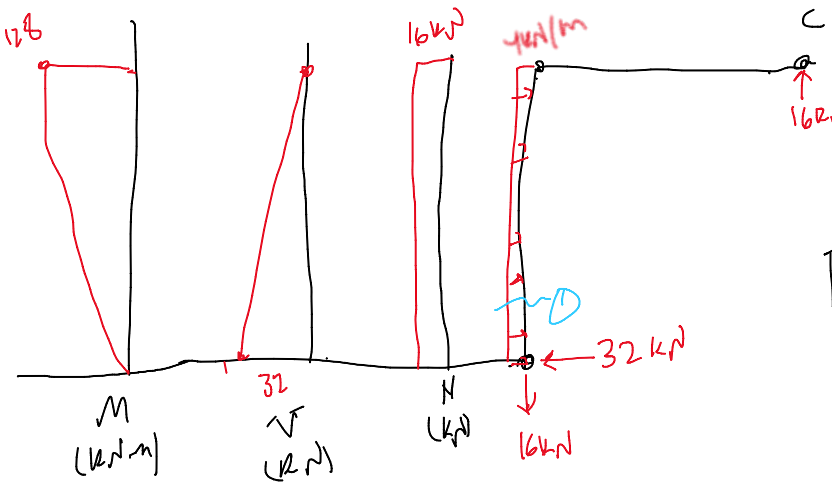


128

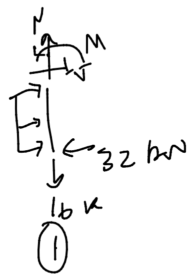
/1 8 m



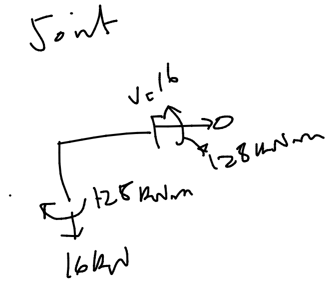
128
M
KN·m
V
(kN)
-16
N
0



128
16kN
4kN/m
C
16kN
32kN
16kN
M
(kN·m)
32
V
(kN)
N
(kN)



N
M
V
32 kN
16 k
1



Joint
V=16
0
128 kN·m
128 kN·m
16kN