BIRZEIT UNIVERSITY

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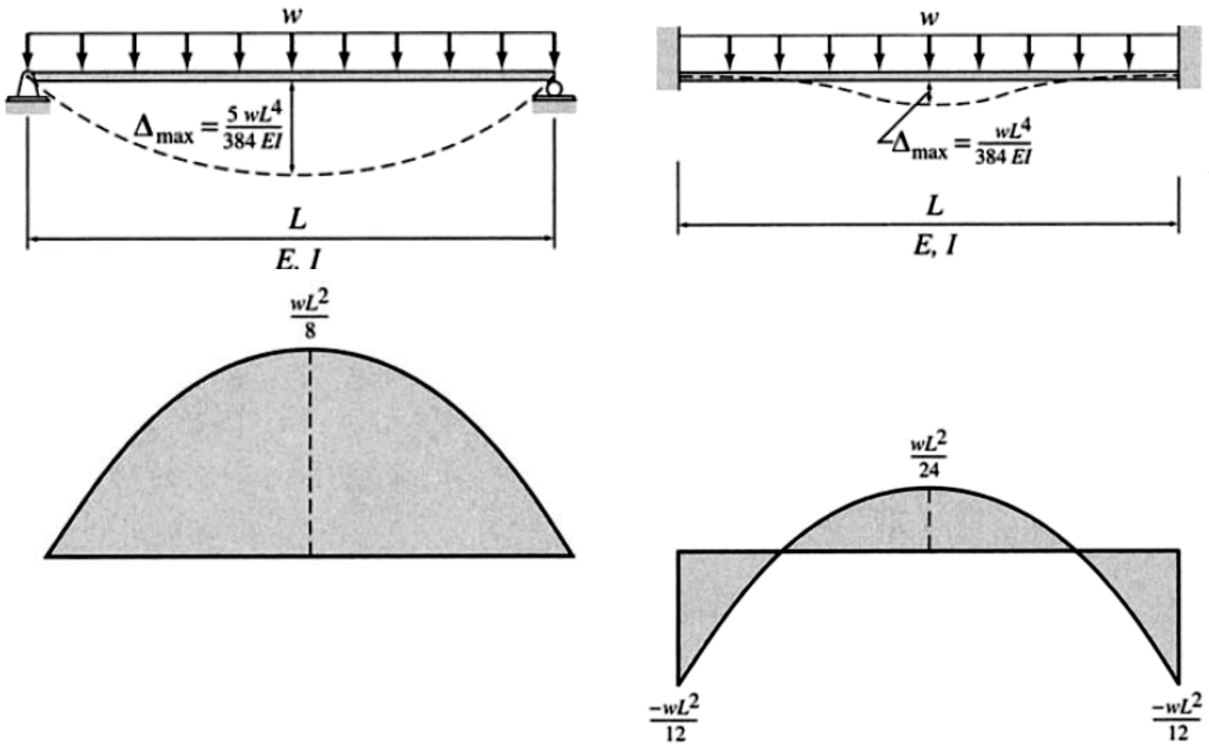
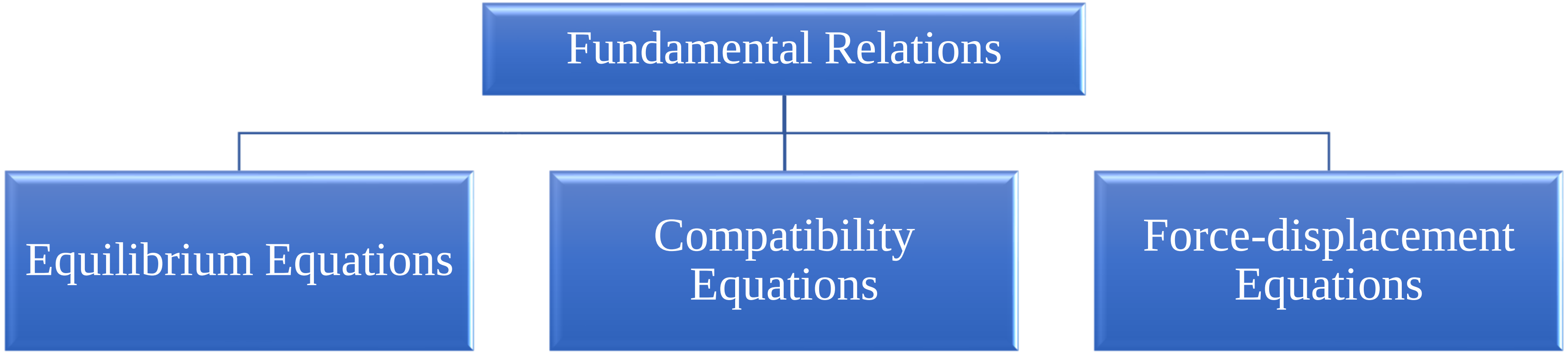
Analysis of indeterminate structures

Indeterminate structures

Advantages	Disadvantages
Less stresses , deflections due to loading	More stresses due to temperature and fabrication errors
Cost (member size)	Cost (support condition)
Redundancy	



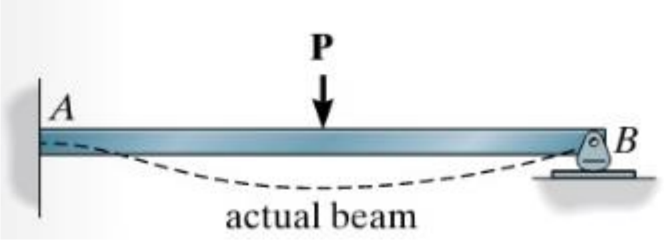
Analysis of Indeterminate structures



	Unknowns	Equations Used for Solution	Coefficients of the Unknowns
Force Method	Forces	Compatibility and Force Displacement	Flexibility Coefficients
Displacement Method	Displacements	Equilibrium and Force Displacement	Stiffness Coefficients

Force Method
Indeterminate to 1^{st} degree

Force Method



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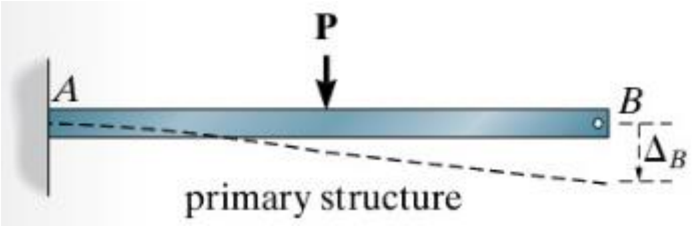
Step 1: Remove a redundant force



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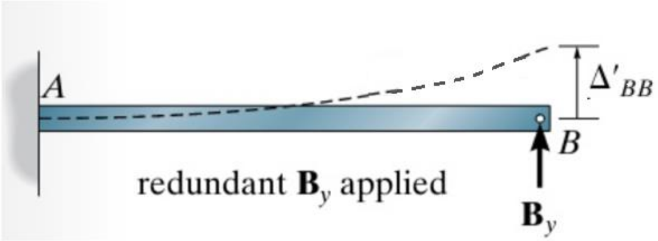
Step 2: Reapply redundant force



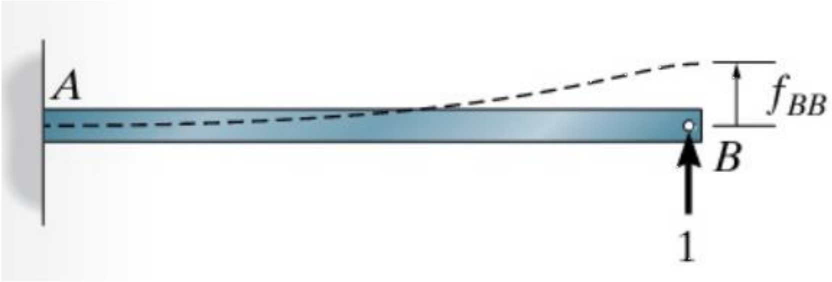
Step 3: Write Compatibility Equation

$$0 = -\Delta_B + \Delta'_{BB}$$

$$\Delta'_{BB} = B_y f_{BB}$$

$$0 = -\Delta_B + B_y f_{BB}$$

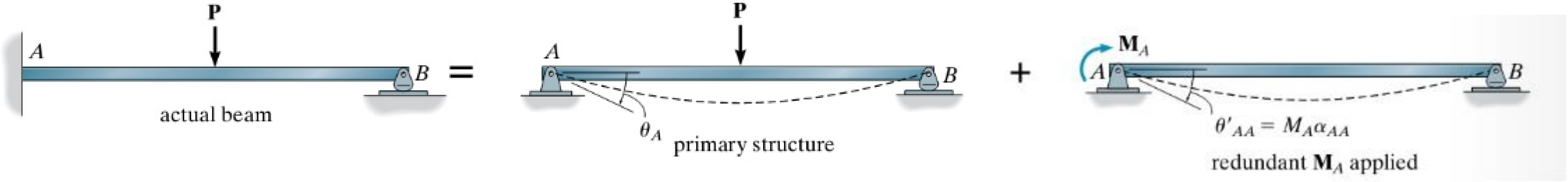
Step 4: Find Displacements



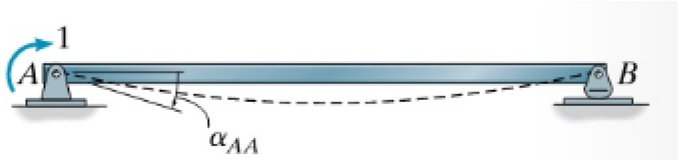
$\Delta_B =$

$f_{BB} =$

Force Method



$$0 = \theta_A + M_A\alpha_{AA}$$



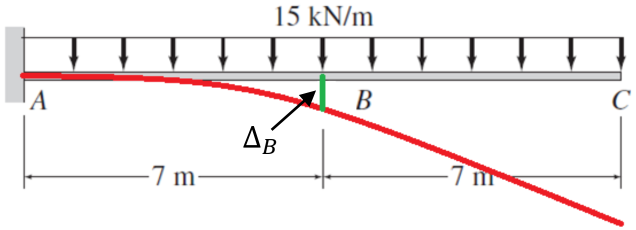
$\theta_A =$

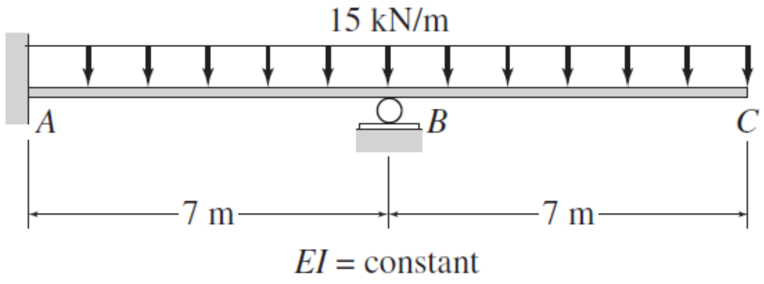
$\alpha_{AA} =$

Beam Example

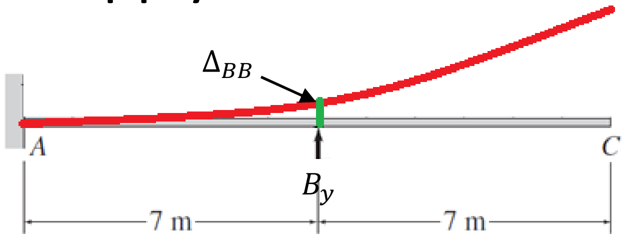
Draw shear and moment diagrams for the Given Beam

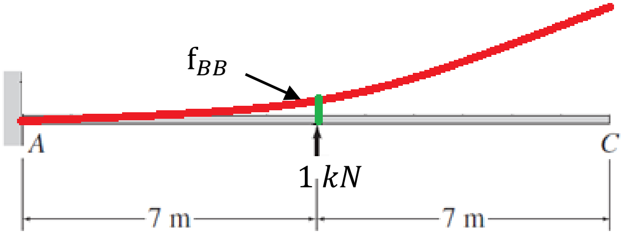
- Step 1: remove redundant support reaction (By)





- Step 2: Reapply redundant reaction (By)



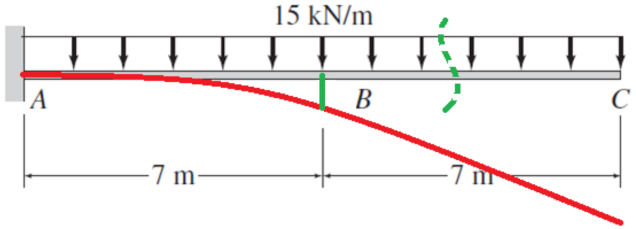


- Step 3: Compatibility equation (deflection upward is positive)

$$-\Delta_B + \Delta_{BB} = 0 \rightarrow \quad -\Delta_B + f_{BB}\, B_y = 0$$

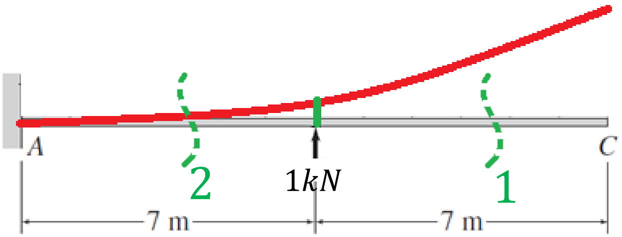
Beam Example

- Step 4: Find deflections (Δ_B, Δ_{BB})
  - Find Δ_B, f_{BB} using virtual work method



Real Structure

$$M = -\frac{15}{2}x^2$$
$$0 > x > 14$$



Virtual Structure

$$m_1 = 0 \qquad 0 < x < 7$$
$$m_2 = x - 7 \qquad 7 < x < 14$$

$$f_{BB} = \int_0^7 \frac{mm}{EI} + \int_7^{14} \frac{mm}{EI} = 0 + \int_7^{14} \frac{(x-7)^2}{EI} dx = \frac{1}{EI} \int_7^{14} (x^2 - 14x + 49) dx = \frac{114.3}{EI}$$

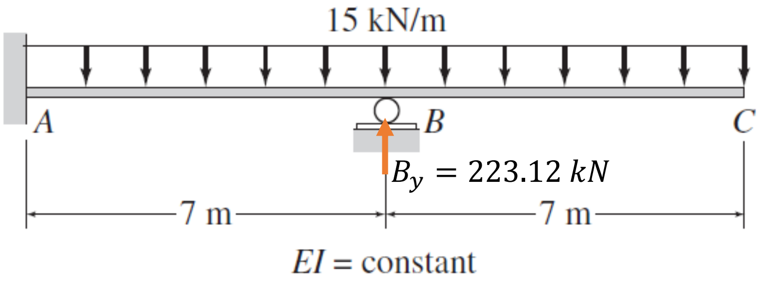
$$\Delta_B = \int_0^7 \frac{Mm}{EI} dx + \int_7^{14} \frac{Mm}{EI} dx = 0 + \int_7^{14} \frac{\left(-\frac{15}{2}x^2\right)(x-7)}{EI} dx = \frac{1}{EI} \int_7^{14} \left(-\frac{15}{2}x^3 + 52.5x^2\right) dx = -\frac{25510}{EI}$$

Beam Example

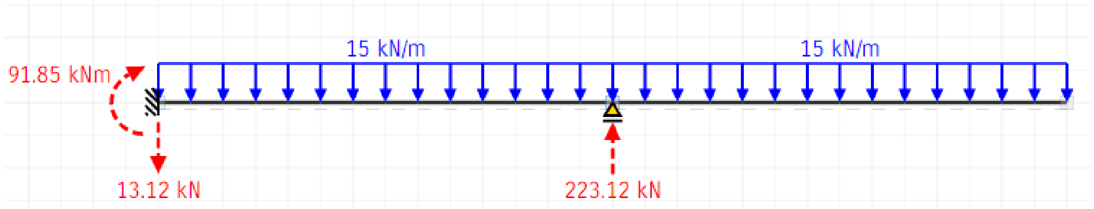
- Step 5: Apply compatibility and solve for B_y

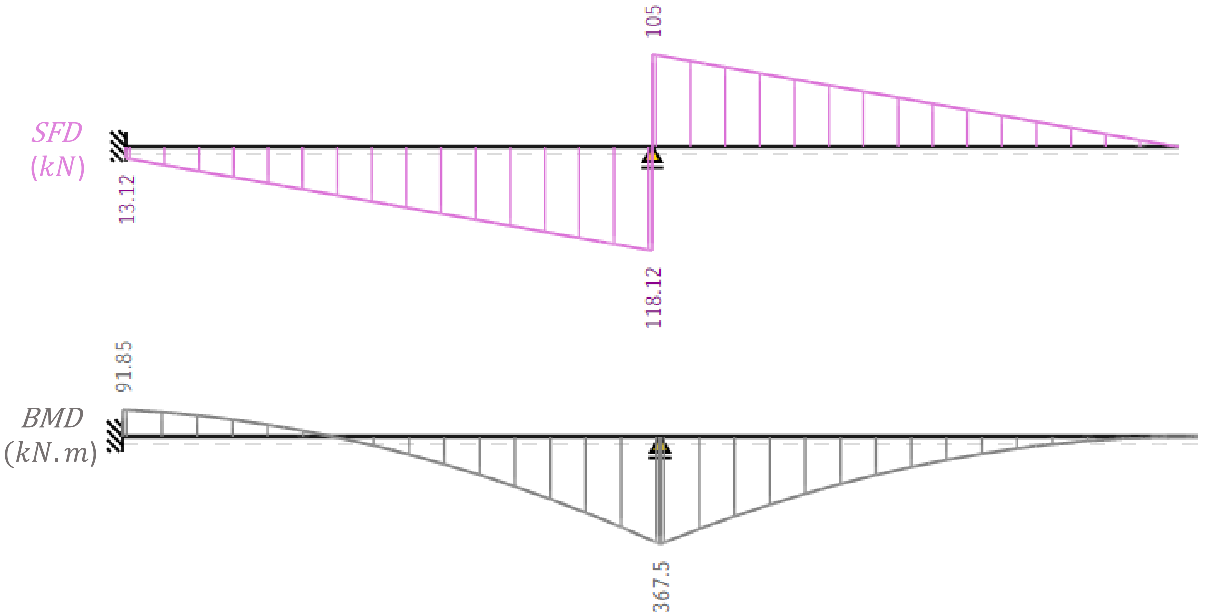
$$compatability \rightarrow -\frac{25510.6}{EI} + \frac{114.3}{EI} B_y = 0$$

$$B_y = 223.2\ kN$$



- Step 6: Find reactions at A and draw shear and moment diagrams

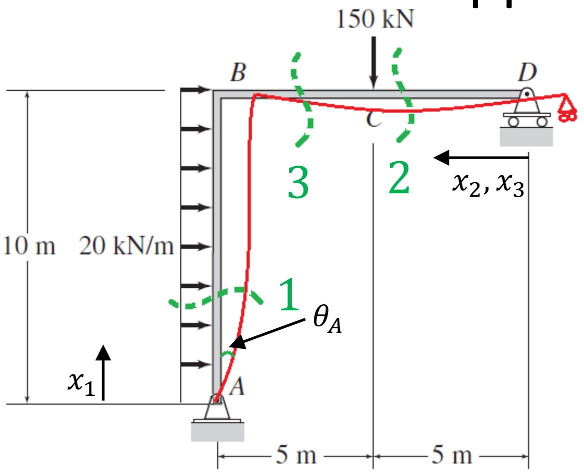




Frame Example

Draw axial, shear and moment diagrams for the Given Frame

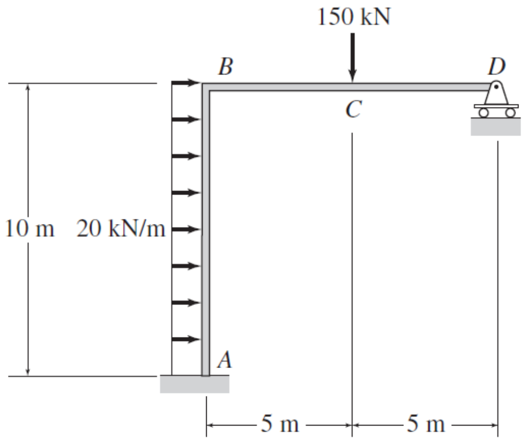
- Step 1: remove redundant support reaction (M_A)



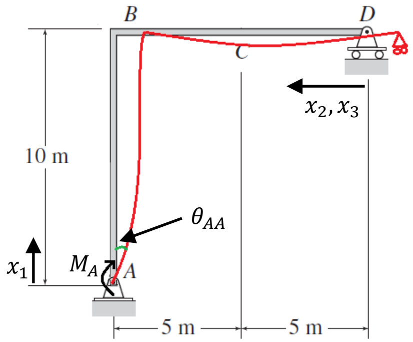
$M_1 = 200x - 10x^2$
$0 > x_1 > 10$

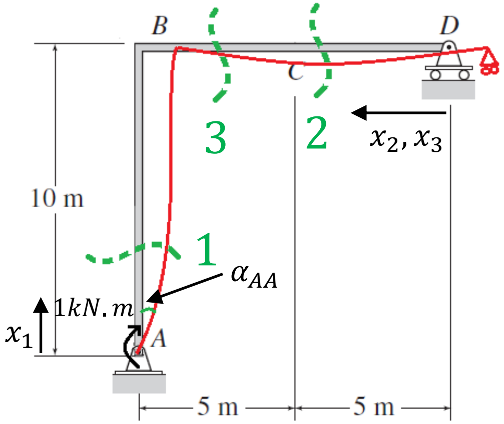
$M_2 = 175x$
$0 > x_2 > 5$

$M_3 = 25x + 750$
$5 > x_2 > 10$



- Step 2: Reapply redundant reaction (M_A)





$m_1 = 1$
$0 > x_1 > 10$

$m_2 = 0.1x$
$0 > x_2 > 5$

$m_3 = 0.1x$
$5 > x_2 > 10$

Frame example

- Step 3: Write compatibility equation (c.w. rotation positive)

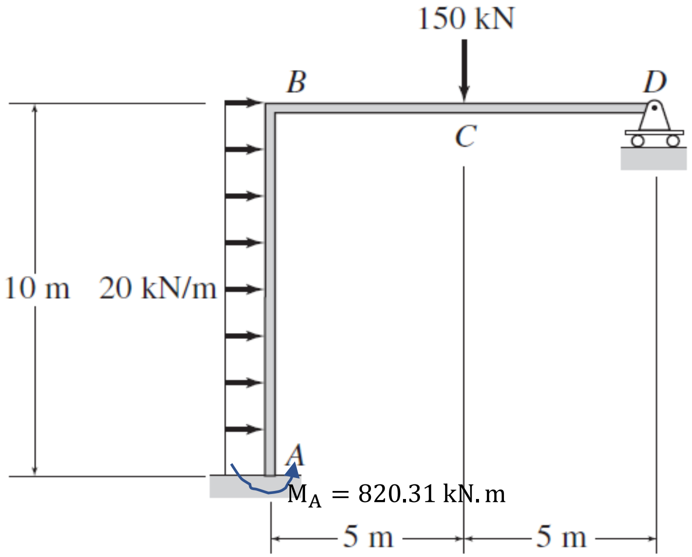
$$\theta_A + \theta_{AA} = 0 \;\rightarrow \theta_A + \alpha_{AA} M_A = 0$$

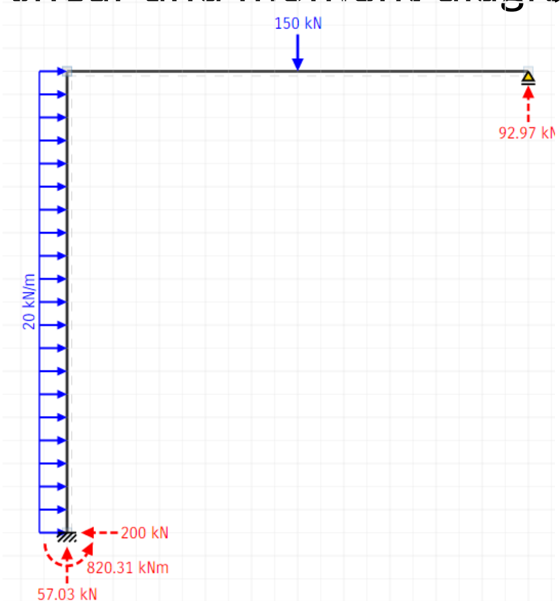
- Step 4: Find deflections
  - Find θ_A and α_{AA} using virtual work method

$$\theta_{AA} = \int_0^{10} \frac{M_1 m_1}{EI} dx_1 + \int_0^{5} \frac{M_2 m_2}{EI} dx_2 + \int_5^{10} \frac{M_3 m_3}{EI} dx_3 = \frac{10943}{EI}$$

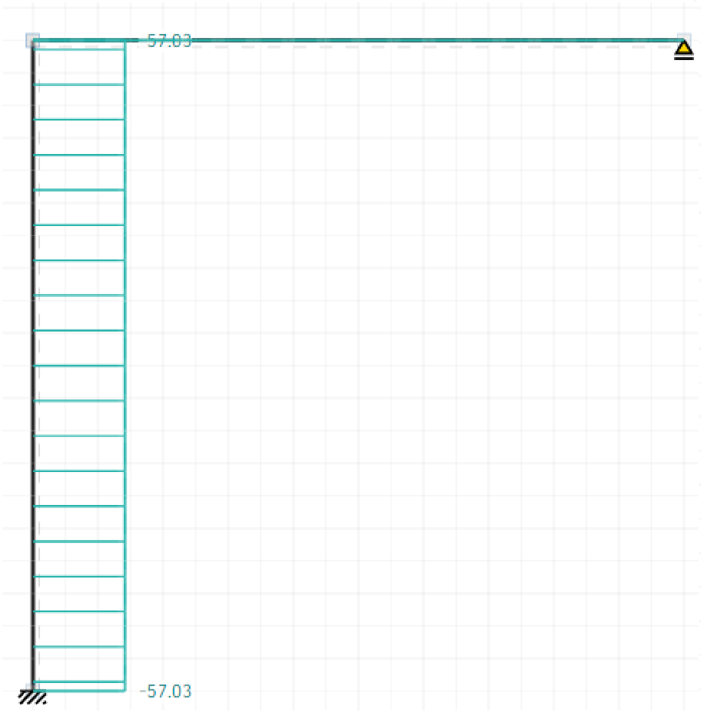
$$\alpha_{AA} = \int_0^{10} \frac{m_1 m_1}{EI} dx_1 + \int_0^{5} \frac{m_2 m_2}{EI} dx_2 + \int_5^{10} \frac{m_3 m_3}{EI} dx_3 = \frac{13.34}{EI}$$

- Step 5: Apply compatibility and solve for $\mathrm{M_A} \Rightarrow \mathrm{M_A} = -820.31\ \mathrm{kN.m}$ (opposite to assumption)
- Step 6: Find the rest of the reactions and draw shear and moment diagrams

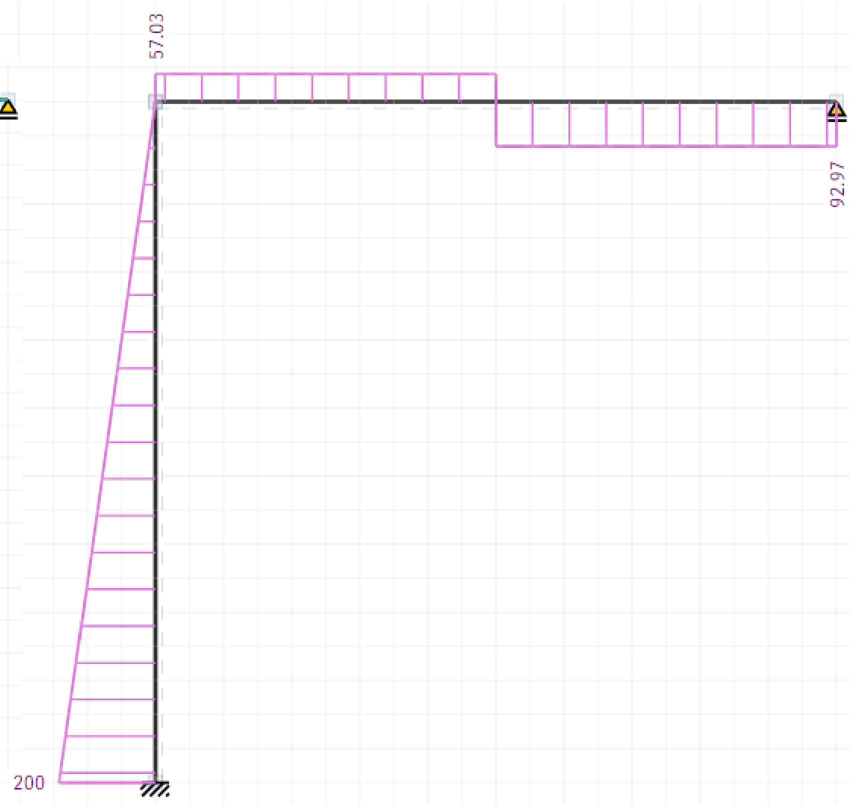




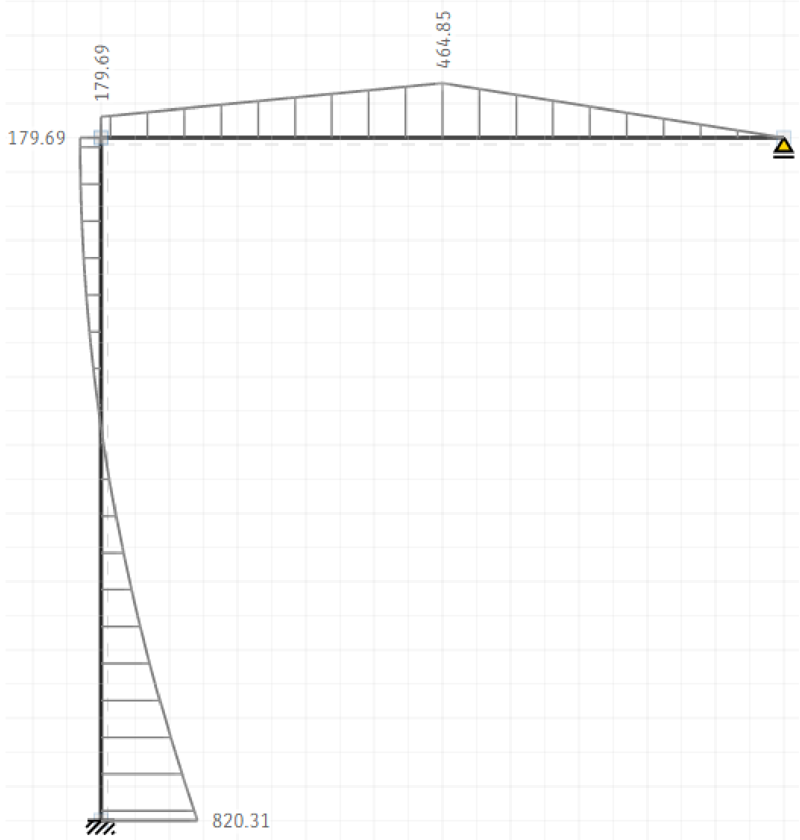
Frame example



Axial (kN)



SFD (kN)



BMD (kN.m)