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- 118 1401

- Final Exam (ENCE 3341)

Question 1

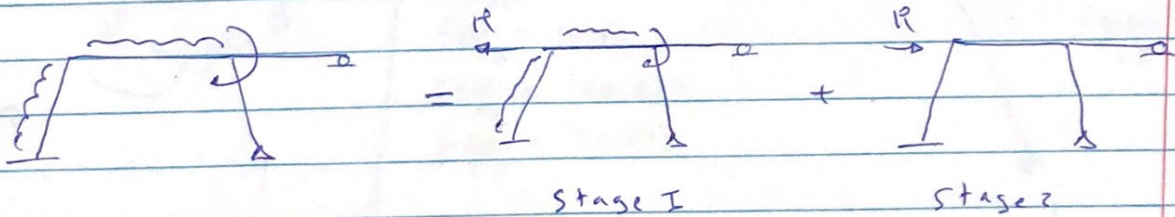
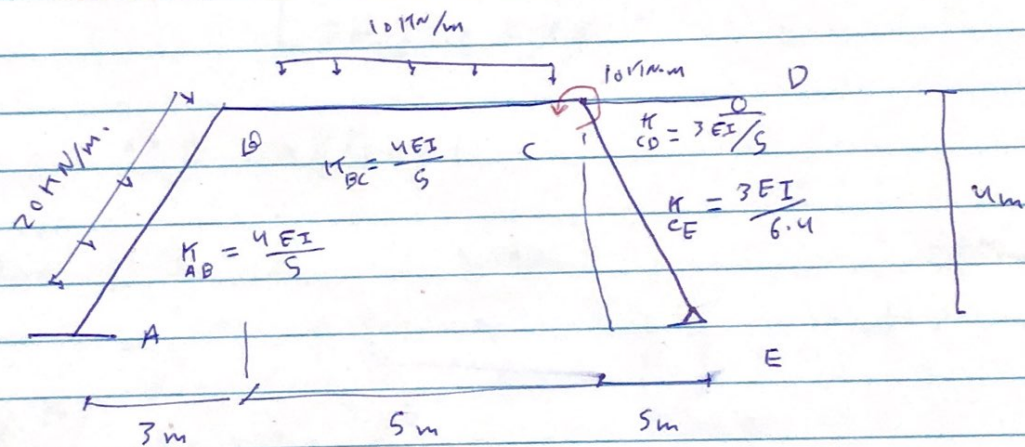
$h = 4\text{m}$, $S_1 = 3\text{m}$

$S_2 = 5\text{m}$

$m = 10\text{ kN}\cdot\text{m}$, $W = 10\text{ kN/m}$, $L_{CE} = 6.4\text{m}$

$L_{AB} = 5\text{m}$

& Sway frame



Stage 1

DF

at A $\rightarrow DF_{AB} = 0$, at D $\rightarrow DF_{DC} = 1$

at B $\rightarrow DF_{BA} = \frac{1/BA}{1/BA + 1/BC} = 0.5$

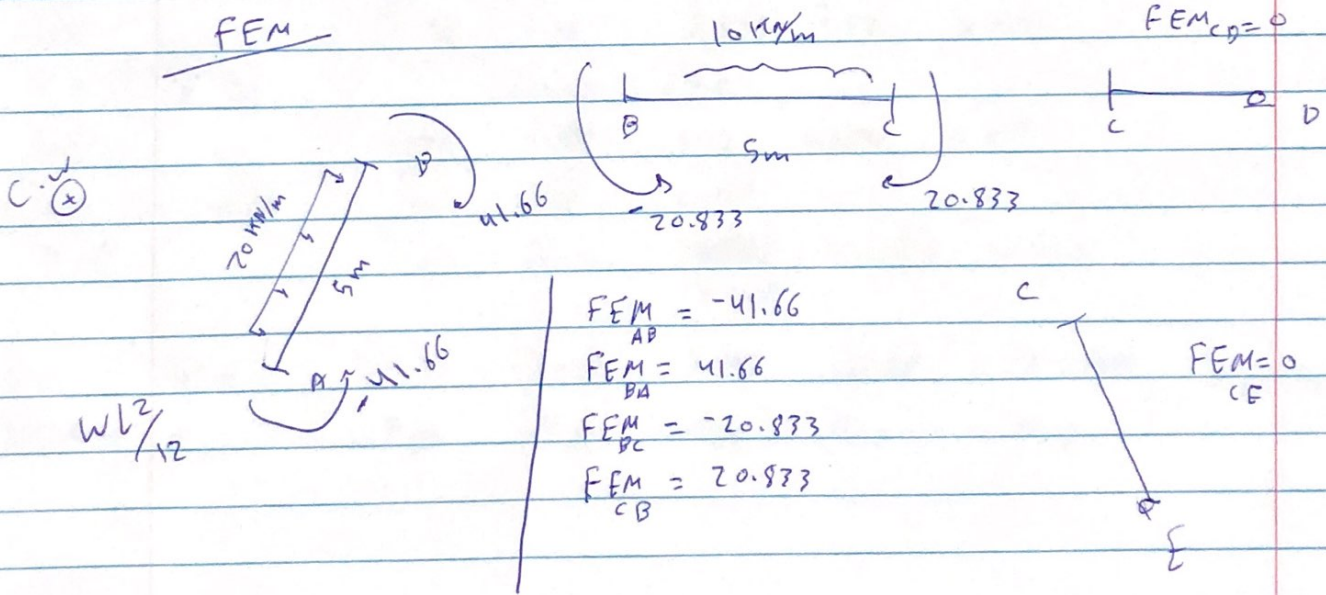
$\rightarrow DF_{BC} = 0.5$

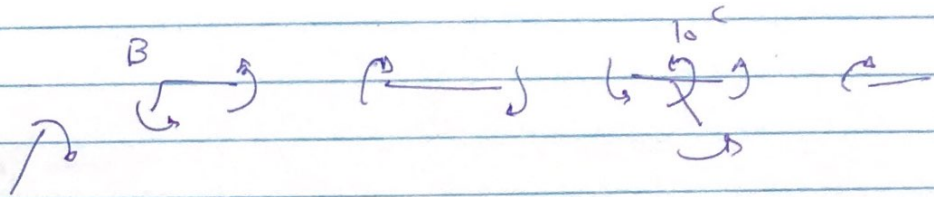
at C $\rightarrow DF_{CB} = 0.43$

$\rightarrow DF_{CD} = 0.32$

$\rightarrow DF_{CE} = 0.25$

at E $\rightarrow DF_{EC} = 1$





$$\sum M_B = 0$$

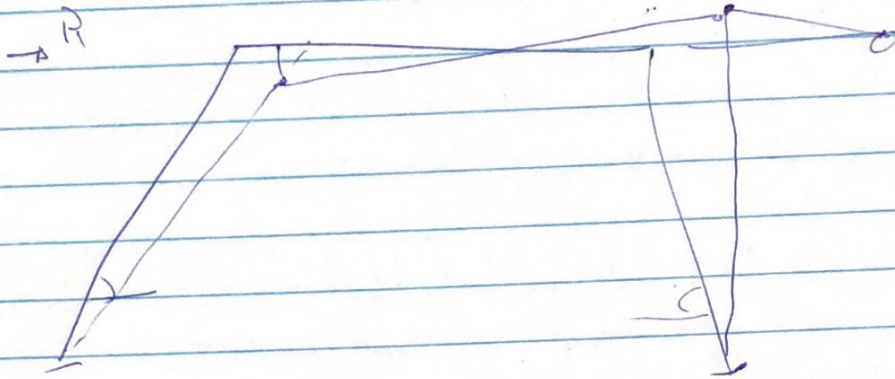
$$\sum M_C = 0 \rightarrow$$

$$\boxed{M_{CD} + M_{CB} + M_{CE} = -10}$$

$$M_{CB} + M_{BC} = 0$$

Joint	A	B	C	D	E			
Member	AB	BA	BC	CB	CD	CE	DC	EC
DF	0	0.5	0.5	0.43	0.32	0.25	1	1
FEM	-41.66	41.66	-20.833	20.833	0	0	0	0
Dist		10.4135	10.4135	13.26	9.866	-7.7		
Co	-5.2		6.63	-5.2				
Dist		3.315	3.315	2.236	1.664	1.3		
Co	1.658		1.118	1.658				
Dist		0.56	0.56	0.713	0.53	0.4145		
Co	0.28		0.3565	0.28				
Dist		0.178	0.178	0.12	0.0896	0.07		
Co	0.089		0.06	0.089				
Dist		0.03	0.03	0.0383	0.02848	0.02225		
				-0.0383				
$\sum M$	45.4	34.15	34.15	5.445	8.67	-6.76675		$M_{DC} = M_{EC} = 0$
End Moment	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{CE}		

Stage II



FEM \circ

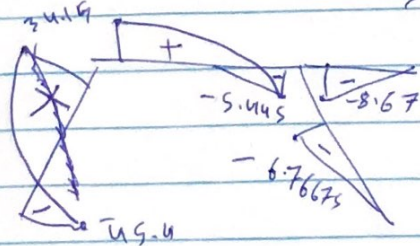
I should find END moments from Stage II and R' and determine END moment for real Sway System

$$M_{i,j} = M_{\text{Stage I, ij}} + \left(\frac{R}{R'}\right) M_{2,ij}$$

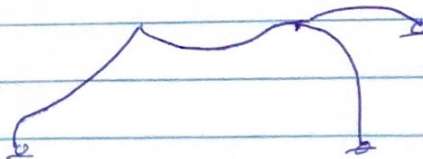
and find reactions

but no enough time

determine as Non-sway systems \circ

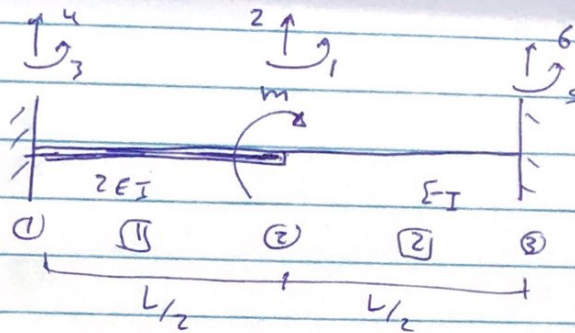


def Shape \circ

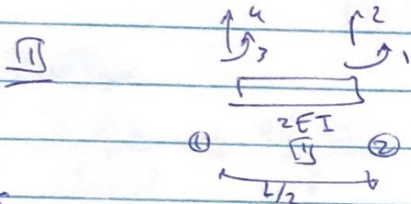


Question 2

m, L



of DOF = 6



$$\frac{12(2EI)}{(L/2)^3} = \frac{192 EI}{L^3}$$

$$\frac{6(2EI)}{(L/2)^2} = \frac{48 EI}{L^2}$$

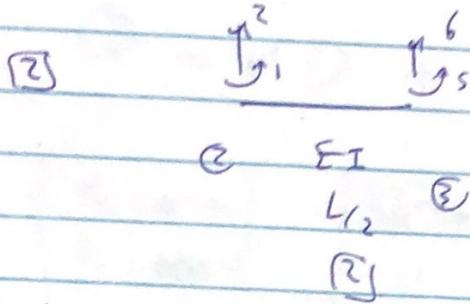
$$\frac{4(2EI)}{L/2} = \frac{16 EI}{L}$$

$$\frac{2(2EI)}{L/2} = \frac{8 EI}{L}$$

No FEF

$$K_{II} = EI$$

	4	3	2	1	
192/L ³	48/L ²	-192/L ³	48/L ²	4	①
48/L ²	16/L	-48/L ²	8/L	3	
-192/L ³	-48/L ²	192/L ³	-48/L ²	2	②
48/L ²	8/L	-48/L ²	16/L	1	



~~12 EI / (L/2)^3 = 96 EI / L^3~~

$$\frac{12 EI}{(L/2)^3} = \frac{96 EI}{L^3}$$

No FEF

$$\frac{6 EI}{(L/2)^2} = \frac{24 EI}{L^2}$$

$$\frac{4 EI}{L/2} = \frac{8 EI}{L}$$

$$\frac{2 EI}{L/2} = \frac{4 EI}{L}$$

$$K_{23} = \begin{bmatrix} 96/L^3 & 24/L^2 & -96/L^3 & 24/L^2 \\ 24/L^2 & 8/L & -24/L^2 & 4/L \\ -96/L^3 & 24/L^2 & 96/L^3 & -24/L^2 \\ 24/L^2 & 4/L & -24/L^2 & 8/L \end{bmatrix} EI$$

$$\Theta = HD + Q_0$$

	1	2	3	4	5	6			
$Q_1 = -m$	$\frac{16+8-24}{L^2} \frac{-24}{L^2}$						1	D_1	$Q_1 = 0$
$Q_2 = 0$	$\frac{24}{L^2}$	$\frac{288}{L^3}$					2	D_2	$Q_2 = 0$
Q_3	$\frac{8}{L}$	$-\frac{48}{L^2}$					3	$D_3 = 0$	$\vdots = 0$
Q_4	$\frac{48}{L^2}$	$-\frac{192}{L^3}$					4	$D_4 = 0$	$\vdots = 0$
Q_5	$\frac{4}{L}$	$\frac{24}{L^2}$					5	$D_5 = 0$	$\vdots = 0$
Q_6	$\frac{24}{L^2}$	$-\frac{96}{L^3}$					6	$D_6 = 0$	$\vdots = 0$

$$\Rightarrow -m = \frac{24}{L} EI D_1 - \frac{24}{L^2} EI D_2 + 0$$

$$0 = \frac{-24}{L^2} EI D_1 + \frac{288}{L^3} EI D_2 + 0$$

$$0 = -24 D_1 + \frac{288}{L} D_2 \Rightarrow D_1 = \frac{12}{L} D_2$$

$$\Rightarrow -m = \frac{24}{L} EI \left(\frac{12}{L}\right) D_2 - \frac{24}{L^2} EI D_2$$

$$-m = \frac{24}{L^2} EI D_2 (12) - \frac{24}{L^2} EI D_2$$

$$-m = (11) \frac{24}{L^2} EI D_2 \Rightarrow D_2 = \frac{-m(L^2)}{EI(264)}$$

$$\Rightarrow D_1 = \frac{-m(L)}{EI(22)}$$

$$Q_3 = \frac{8}{L} EI \left(\frac{-m(L)}{EI(22)}\right) - \frac{48}{L^2} EI \left(\frac{-m(L^2)}{EI(264)}\right) = -m \left(\frac{8}{22}\right) + m \left(\frac{48}{264}\right)$$

$$Q_3 = \left(-\frac{4}{22}\right) m$$

$$Q_4 = \frac{48}{L^2} \frac{m(L)}{22} + \frac{192}{L^3} \frac{m(L^2)}{264} = -\frac{32}{22} m$$

$$Q_5 = -m \frac{4}{22} - m \frac{(24)}{264} = -\frac{6}{22} m$$

$$Q_6 = \frac{24 m}{L (22)} + \frac{96 m}{L (264)} = \frac{32 m}{L}$$

$$(1) \begin{pmatrix} q_4 \\ q_3 \\ q_2 \\ q_1 \end{pmatrix} = \begin{bmatrix} | & | \\ \hline 192/L^3 & -48/L^2 \\ \hline -48/L^2 & 96/L \\ | & | \end{bmatrix} \begin{pmatrix} d_2 \\ d_1 \end{pmatrix}$$

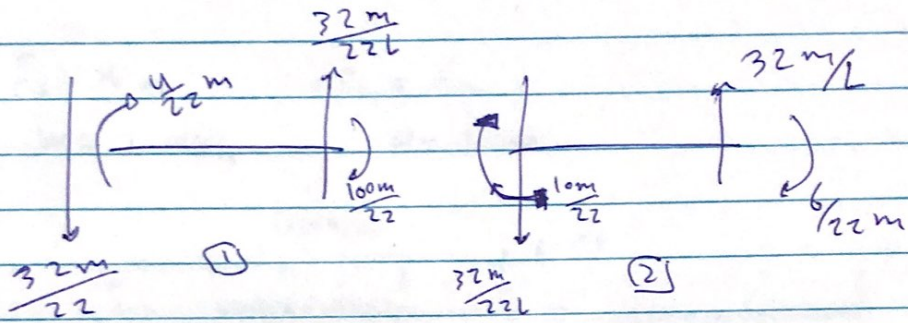
$$q_{t_2} = -\frac{m 16}{L (22)} + \frac{m 48}{L (22)} = \frac{32 m}{(22) L}$$

$$q_{t_1} = \frac{-4 m}{22} - \frac{m(96)}{22} = -\frac{100 m}{22}$$

$$(2) \begin{pmatrix} q_{t_2} \\ q_{t_1} \\ q_6 \\ q_5 \end{pmatrix} = \begin{bmatrix} | & | \\ \hline 96/L^3 & 24/L^2 \\ \hline 24/L^2 & 8/L \\ | & | \end{bmatrix} \begin{pmatrix} d_2 \\ d_1 \\ 0 \\ 0 \end{pmatrix}$$

$$q_{t_2} = \frac{-8 m}{(22) L} - \frac{24 m}{22 L} = -\frac{32 m}{(22) L}$$

$$q_{t_1} = \frac{-2 m}{22} - \frac{8 m}{22} = -\frac{10 m}{22}$$



Question 3

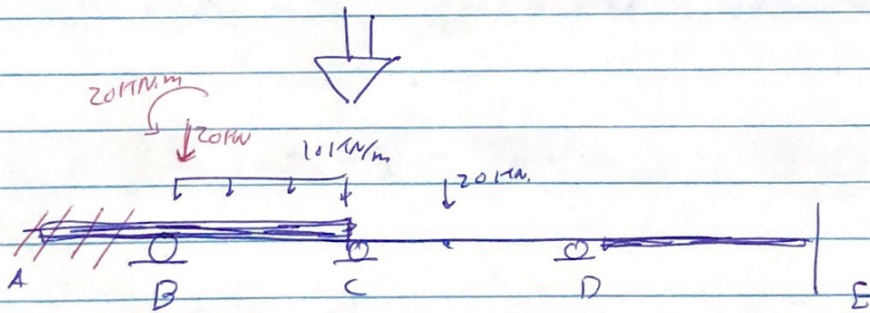
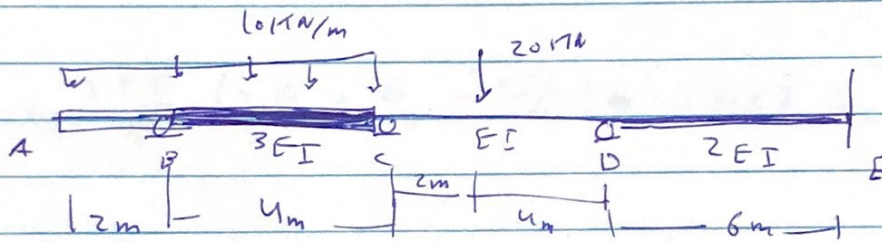
Support C \downarrow 6mm
 " D \downarrow 2mm

$E = 30 \text{ GPa}$

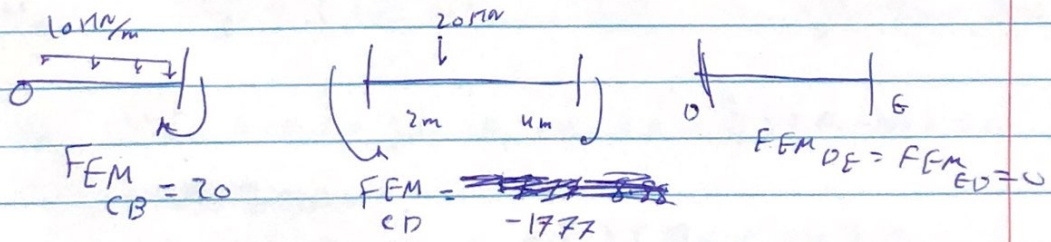
$I = 12 (10^8) \text{ mm}^4$

$S_1 = 4 \text{ m}$, $S_2 = 6 \text{ m}$

$w = 10 \text{ kN/m}$, $P = 20 \text{ kN}$

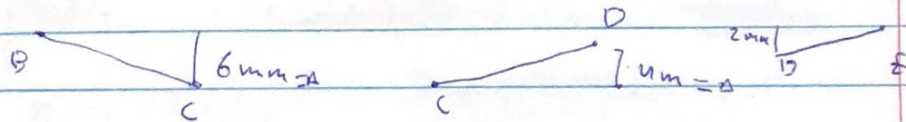


FEM



$FEM_{DC} = 8.88$

Satellito



$$EI \rightarrow 76000$$

END Moment equations

$$\left\{ \begin{array}{l} M_{BC} = -20 \end{array} \right.$$

$$M_{CB} = \frac{3(3EI)}{4} \left(\theta_c - \frac{0.006}{4} \right) + 20 = \frac{9}{4} EI \theta_c - 101.5$$

$$M_{CD} = \frac{2EI}{6} \left(2\theta_c + \theta_D - \frac{0.012}{6} \right) - 17.77 = \frac{2}{6} EI (2\theta_c + \theta_D) - 17.77$$

$$M_{DC} = \frac{2EI}{6} \left(2\theta_D + \theta_c - \frac{0.012}{6} \right) + 8.88 = \frac{2}{6} EI (2\theta_D + \theta_c) - 15.12$$

$$\boxed{\theta_E = 0} \quad M_{DE} = \frac{2(2EI)}{6} \left(2\theta_D + \theta_E - \frac{0.006}{6} \right) + 0 = \frac{4EI}{6} (2\theta_D) - 24$$

$$M_{ED} = \frac{2(2EI)}{6} \left(2\theta_E + \theta_D - \frac{0.006}{6} \right) = \frac{4EI}{6} \theta_D - 24$$

$$\sum M_C = 0 \Rightarrow \frac{3}{6} EI (2\theta_c) + \frac{3}{6} EI \theta_D - 41.77 + \frac{9}{4} EI \theta_c - 101.5 = 0$$

~~2666 EI~~

$$2.9166 \theta_c EI + 0.3 EI \theta_D = 143.3$$

$$\sum M_D = 0 \Rightarrow 39.12 = 2 EI \theta_D + 0.3 \theta_c EI$$

$$\begin{bmatrix} 143.3 \\ 39.12 \end{bmatrix} = \begin{bmatrix} 2.9166 & 0.3 \\ 0.3 & 2 \end{bmatrix} \begin{bmatrix} \theta_c \\ \theta_D \end{bmatrix} EI \quad \begin{matrix} (17.166) \\ (-10.47566) \end{matrix} EI \theta_D = \frac{199}{138.83}$$

$$\theta_D = 11.6/EI$$

$$\theta_c = 47.8/EI$$

~~$$\theta_D = 1325/EI$$~~
~~$$\theta_c = 102/EI$$~~

$$M_{BC} = -20$$

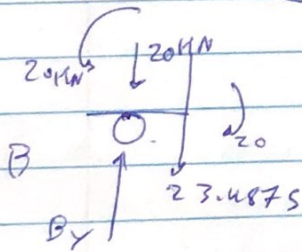
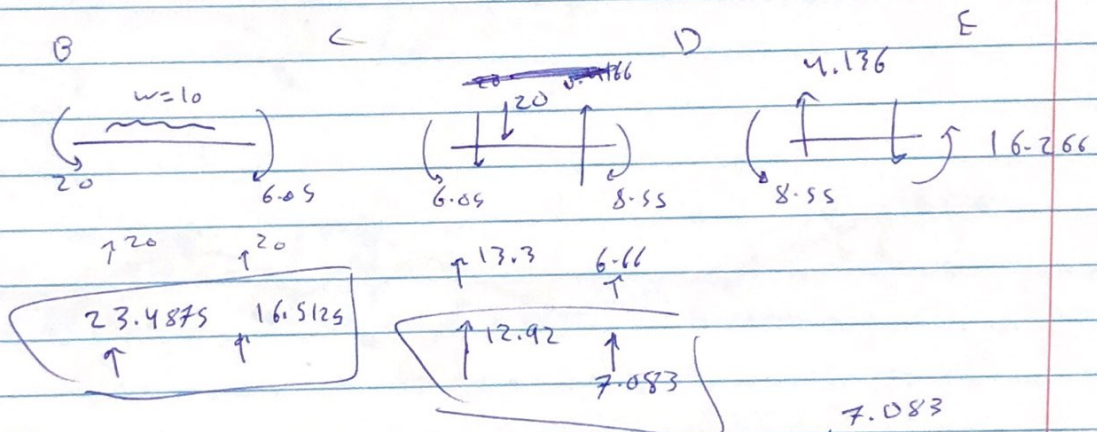
$$M_{CD} = 6.05 \text{ kN.m}$$

$$M_{CD} = \cancel{6.05} \cdot 6.05$$

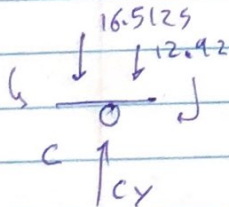
$$M_{DC} = 8.55$$

$$M_{DE} = -8.55$$

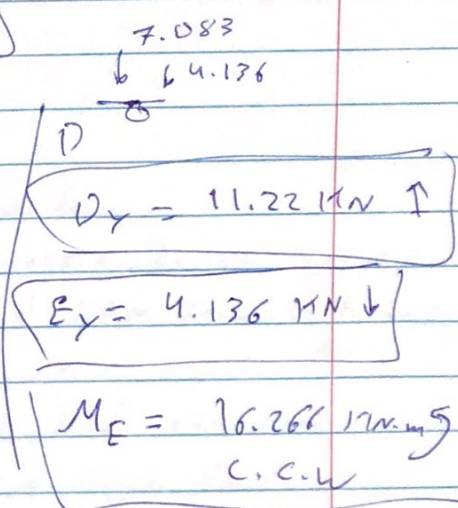
$$M_{ED} = -16.266$$

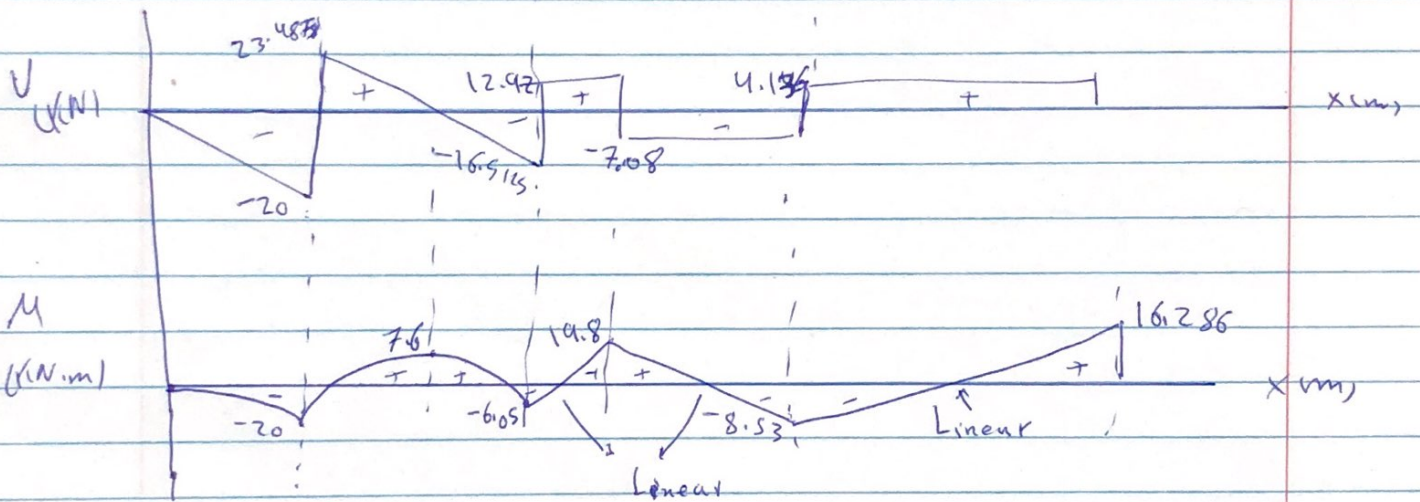
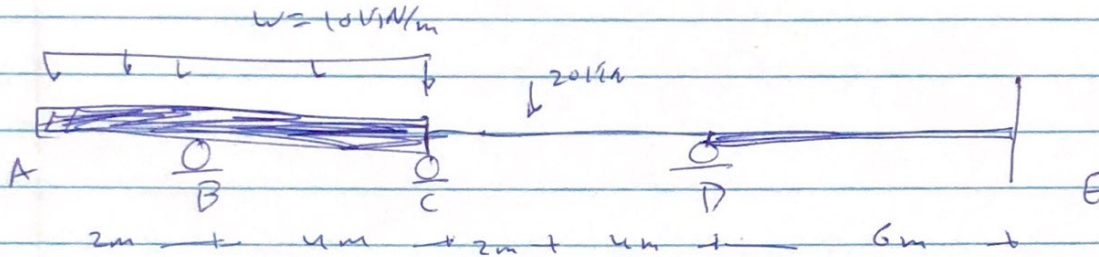


$$\Rightarrow B_y = 43.4875 \text{ kN } \uparrow$$



$$C_y = 29.4325 \text{ kN } \uparrow$$





- e. divide elements
determine FEM due to loading.
depend on distribute moment resistents at joints
in moment dis. method (we find FEM from settlement) but not in slope def. equation
we find END moment and find reaction
In both methods the positive sign convention is \oplus for C.W.