Structural Analysis II

→ What Do We mean by structural Analysis?

We are trying to identify:

(1) Forces

- ↳ Internal forces (Shear + moment)
- ↳ External moments (Reaction forces)

(2) Deformations

Applied loadings Are External Known loadings.

→ What have we learned in structure Analysis I?

Calculate deformations using:

↳ Elastic Beam theory

$$K = \frac{M}{EI} = \frac{d^2y}{dx^2}$$

[Direct integration method
Conjugate beam method
Moment Area method]

↳ Virtual work Method

- For beams and frames
- Trusses systems
- Include shear, torsion def.

$$\Delta = \sum n \frac{NL}{EA} + \sum n d_i$$

Elastic Beam theory → Flexural sys
Can't Be applied on trusses

→ In structure Analysis I We solved determinate systems

It means that No. of equilibrium eq. = No. of unkowns

→ Indeterminate systems:

Used:

- Equilebrium equations
- Comtability equations
- Force deformation equations

→ Force method

↳ Displacement method

→ Analysis to indeterminate systems using force method:

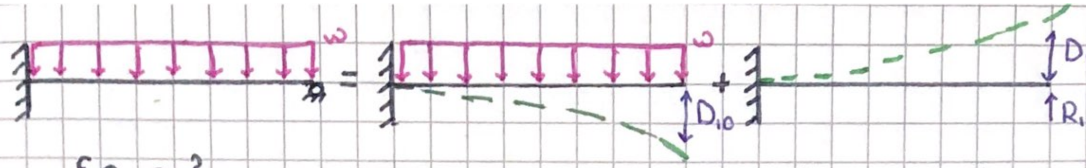
Statically indet. (1st degree)

Displacement at B = 0
↑
Known

Displacement at horizontal and moment is unkown

∴ 2nd DOF

Kinematic indet.
(Moment, Degree of freedom)
Dof



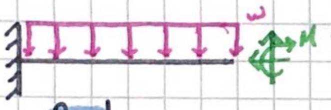
$[R_1 = B_y]$

primary system
+
Applied loading (stable loading)

Primary system
+
Redundant

Comtability equation = $0 = D_{10} + D_{11}$
(Vertical displacement at B)

Know we calculate D_{10}:



Real

$M = \frac{-wx^2}{2}$

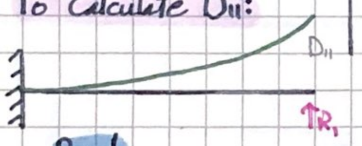
Virtual

$m = 1x$

$0<x<L$

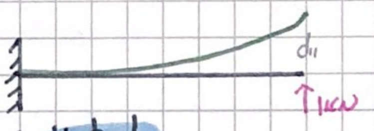
$$D_{10} = \int \frac{mM}{EI}dx = \int_0^L \frac{-wx^2}{2}(x)\left(\frac{dx}{IE}\right) = \frac{-wL^4}{8EI}$$

To Calculate D_{11}:



Real

$D_{11} = R_1 d_1$



Virtual

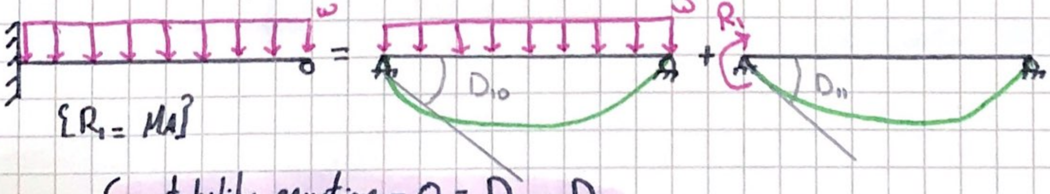
$d_{11} = \int_0^L \frac{m^2}{IE}dx = \frac{x^3}{3IE}\Big|_0^L = \frac{L^3}{3IE}$

$$D_{11} = \frac{R_1 L^3}{3IE}$$

From the first equation:

$$0 = \frac{-wL^{\cancel{4}}}{8E\cancel{I}} + \frac{R_1 \cancel{L^3}}{3\cancel{IE}} \longrightarrow \boxed{R_1 = \frac{3wL}{8} = B_y\uparrow}$$

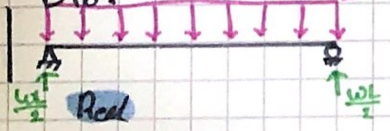
* In Another Way :



$[R_1 = M_A]$

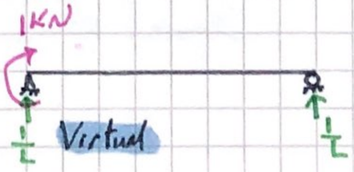
Comtability equation = $0 = D_{10} + D_{11}$
(Rotation at A)

D_{10}:



Real

$M = \frac{wLx}{2} - \frac{wx^2}{2}$



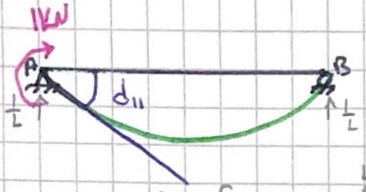
Virtual

$m = \frac{1}{L}(x)$

$$D_{10} = \int \frac{mM}{IE}dx = \int_0^L \frac{x}{L}\left(\frac{wLx}{2} - \frac{wx^2}{2}\right)\frac{dx}{IE} = \int_0^L \left(\frac{wx^2}{2} - \frac{wx^3}{2L}\right)\frac{dx}{IE}$$

$$= \frac{wx^3}{6EI}\Big|_0^L - \frac{wx^4}{8L}\Big|_0^L = \frac{wL^3}{6EI} - \frac{wL^3}{8EI} = \frac{wL^3}{24EI}$$

D_{11}:



$$d_{11} = \int \frac{m^2}{EI}dx = \int_0^L \left(\frac{x}{L}\right)^2 \frac{dx}{EI} = \frac{x^3}{3L^2EI}\Big|_0^L = \frac{L^3}{3EI}$$

$$D_{11} = d_{11}R_1 = R_1\frac{L}{3EI} \rightarrow R_1 = \ominus\frac{wL^2}{8}$$

↳ opposite ↻ = ↺

$$D_{11} + D_{10} = 0$$

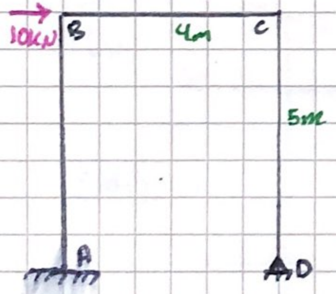
$$\frac{wL^3}{24IE} + \frac{R_1 L}{3EI} = 0 \rightarrow \boxed{R_1 = -\frac{wL^2}{8}}$$

Lecture(2)

Statically indeterminate to 1st degree

$R_1 = A_x$ → A_x is known
→ Can't work As primary

→ **Question:**



1- Static and Kinematic indet.?
2- Draw Bending moment diagram
3- Sketch deformed shape.
4- Find the drift of the frame (Horizontal dis. At B).

Design Value means:

The horizontal displacement

1- # Unkowns = 5   # equilebrium eq. = 3

5 > 3

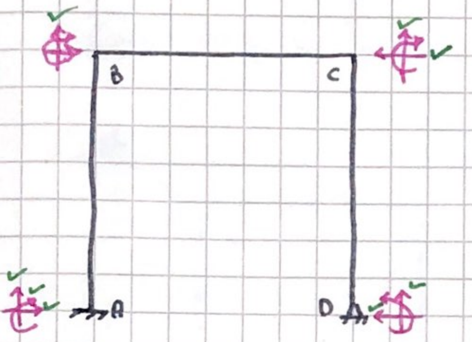
Statically indet. to 2nd degree

we have 4 nodes each have 3 DOF, Known 5, then

12-5 = 7

7th Degree of Freedom DOF (In general)

In Analysis I, we used to say Ignore axial deformation And shear deformation becouse they are small becouse the cross-section of the members are very small compared to its length



Ignoring Axial deformation

$L_{AB} = L'_{AB}$, $L'_{CD} = L_{CD}$

which means that that vertical displacement At B and C is Known

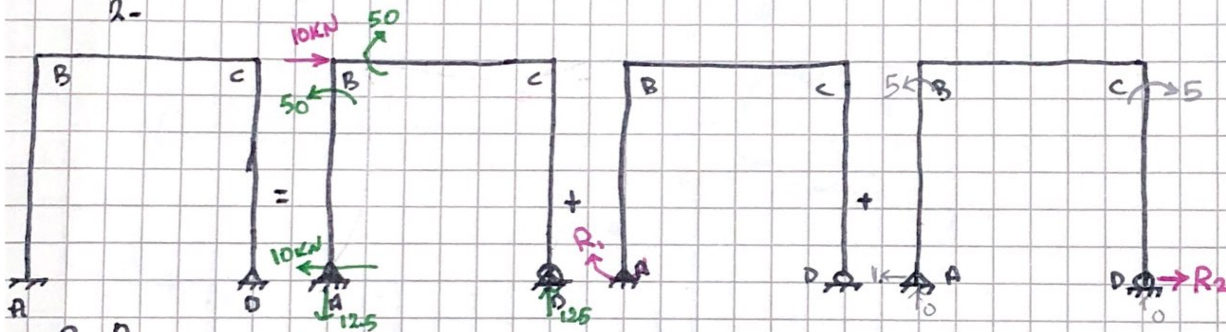
Δ_C (horizontal) = Δ_B (horizontal

which means that if one of them is Known the other will be Known

In this case it is Kinematically indet. to the 4th degree

[Δ_{Bh} , θ_B , θ_C , θ_D]

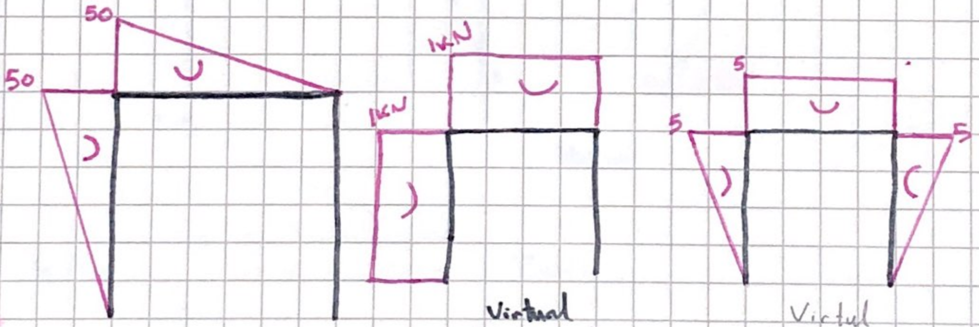
2-



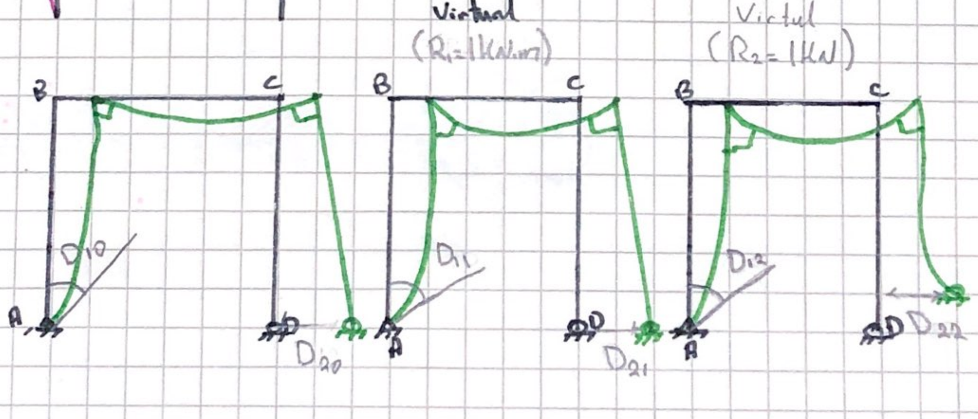
$R_1 = \theta_A$

$R_2 = D_x$

BMD



Deformed shape

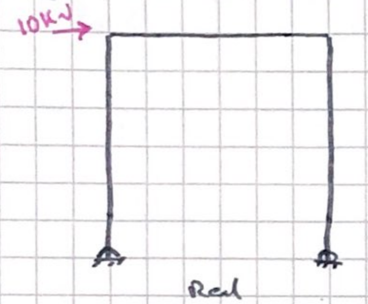


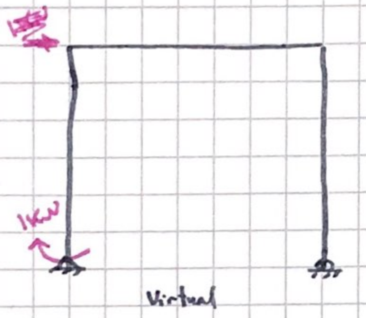
Comptability eq. (Rotation at A) → $0 = D_{10} + \underbrace{D_{11}}_{R_1 d_{11}} + \underbrace{D_{12}}_{R_2 d_{12}}$

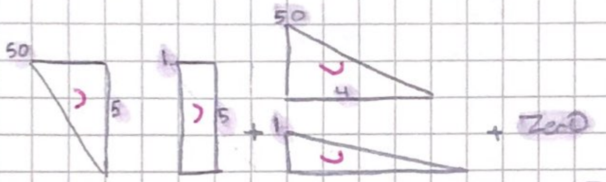
Comptability eq. (Horizontal dis. at D) → $0 = D_{20} + \underbrace{D_{21}}_{R_1 d_{21}} + \underbrace{D_{22}}_{R_2 d_{22}}$

→ solve for R_1 and R_2

→ D_{10}

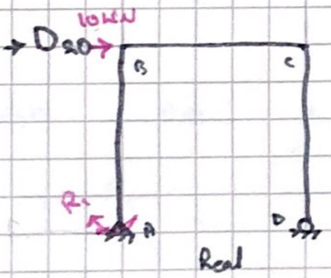


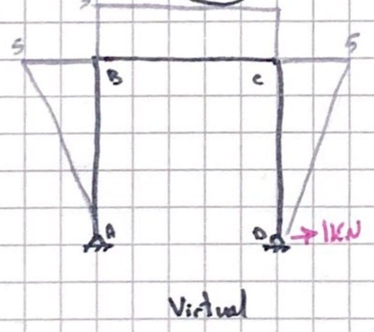


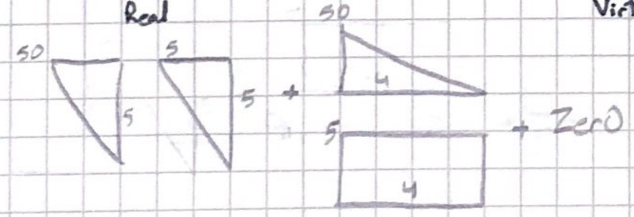


$$\left(\frac{1}{2}\right)\left(\frac{(50)(1)(5)}{EI}\right) + \left(\frac{1}{3}\right)\left(\frac{(50)(1)(4)}{EI}\right) = \boxed{\frac{191.67}{IE}}$$

→ D_{20}

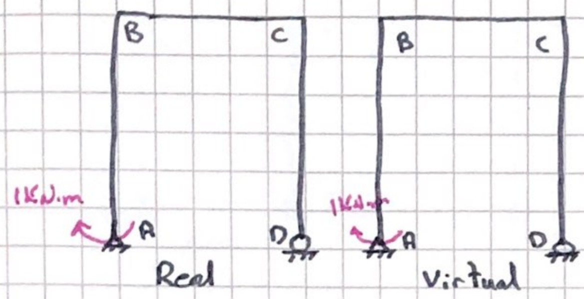


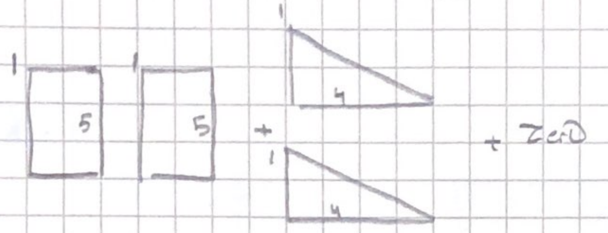




$$\left(\frac{1}{3}\right)\frac{(50)(5)(50)}{IE} + \frac{1}{2}\frac{(50)(5)(4)}{IE} = \boxed{\frac{918.67}{IE}}$$

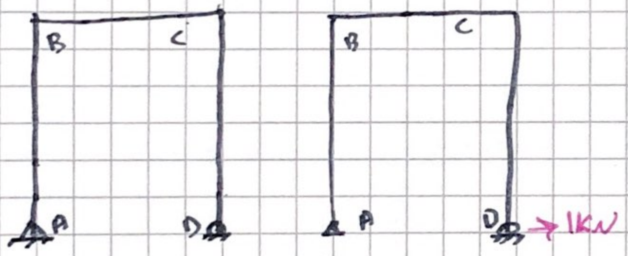
d_{11} :

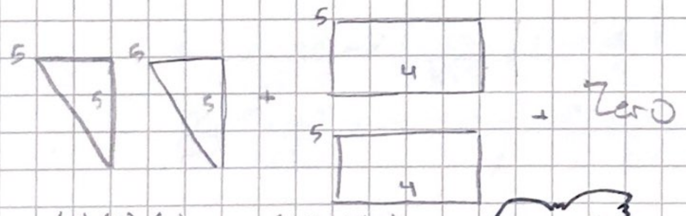




$$\frac{(1)(1)(5)}{IE} + \left(\frac{1}{3}\right)\frac{(1)(1)(4)}{IE} = \frac{6.33}{EI}$$

d_{22}:



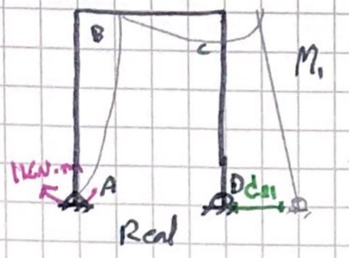


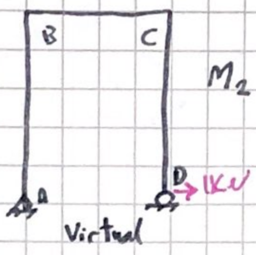
$$\frac{1}{3}\frac{(5)(5)(5)}{EI} + \frac{(5)(5)(4)}{EI} = \boxed{\frac{183.33}{EI}}$$

$$0 = \frac{191.67}{EI} + R_1\left(\frac{6.33}{IE}\right) + R_2\left(d_{12}\right)$$

$$0 = \frac{916.67}{EI} + R_1\, d_{21} + R_2\left(\frac{183.33}{EI}\right)$$

d_{21} :



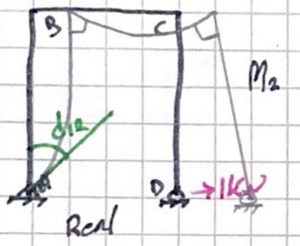


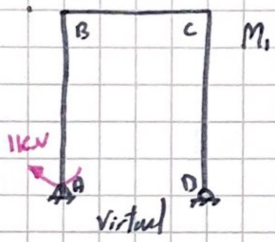
$$d_{21} = \int M_2 \frac{M_1}{EI} \, dx$$

This is An rule (always happens)

$$d_{ij} = d_{ji}$$

d_{12}:





And this Is the Maxwell's theorem of recipiol coeft.

$[d_{11}, d_{12}, d_{21}, d_{22}] \rightarrow$ Called flexibility coeffi. →

Deformations due to unit load.

$$d_{12} = \int M_2 \frac{M_1}{IE} \, dx$$

$$d_{12} = d_{21} = \left(\frac{1}{2}\right) \frac{(1)(5)(5)}{EI} + \left(\frac{1}{2}\right) \frac{(1)(5)(4)}{EI} = \frac{22.5}{EI}$$

Flexibility coeffit :

بتعطيني A good measure عشان أعرف كيف

بتعامل ال system الي عندي مع ال loads

$$0 = \frac{191.67}{EI} + R_1\left(\frac{6.33}{EI}\right) + R_2\left(\frac{22.5}{EI}\right)$$

$$0 = \frac{916.67}{EI} + R_1\left(\frac{22.5}{EI}\right) + R_2\left(\frac{183.33}{EI}\right)$$

$$-\frac{191.67}{EI} = R_1\left(\frac{6.33}{EI}\right) + R_2\left(\frac{22.5}{EI}\right)$$

$$-\frac{916.67}{EI} = R_1\left(\frac{22.5}{EI}\right) + R_2\left(\frac{183.33}{EI}\right)$$

$$\begin{bmatrix} -\frac{191.67}{EI} \\ -\frac{916.67}{EI} \end{bmatrix} = \begin{bmatrix} \frac{6.33}{IE} & \frac{22.5}{EI} \\ \frac{22.5}{EI} & \frac{183.33}{EI} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

- * Flexibility Matrix
- * It is symmetric
- * It's possitive definite
- * We can find the inverse
- * A square Matrix

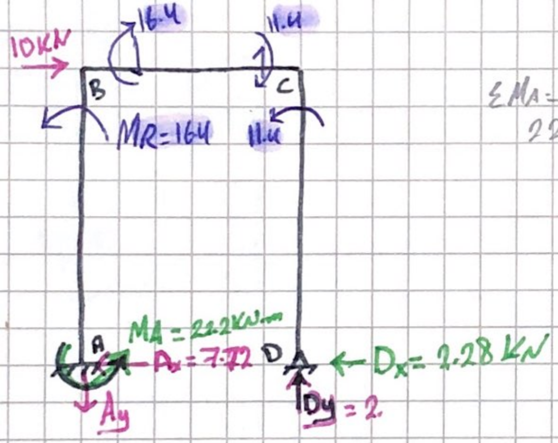
Lecture (3)

$$R_1 = \frac{\begin{vmatrix} -191.67 & 22.5 \\ -916.67 & 183.33 \end{vmatrix}}{\begin{vmatrix} 6.33 & 22.5 \\ 22.5 & 183.33 \end{vmatrix}} = \boxed{\ominus 22.2\ kN\cdot m} \quad \circlearrowleft R_1 = M_A$$

عكس الاتجاه المفروض

$$R_2 = \frac{\begin{vmatrix} 6.33 & -191.67 \\ 22.5 & -916.67 \end{vmatrix}}{\begin{vmatrix} 6.33 & 22.5 \\ 22.5 & 183.33 \end{vmatrix}} = \boxed{\ominus 2.28\ kN} \quad \leftarrow R_2 = D_x$$

عكس الاتجاه المفروض



$\Sigma M_A = 0$

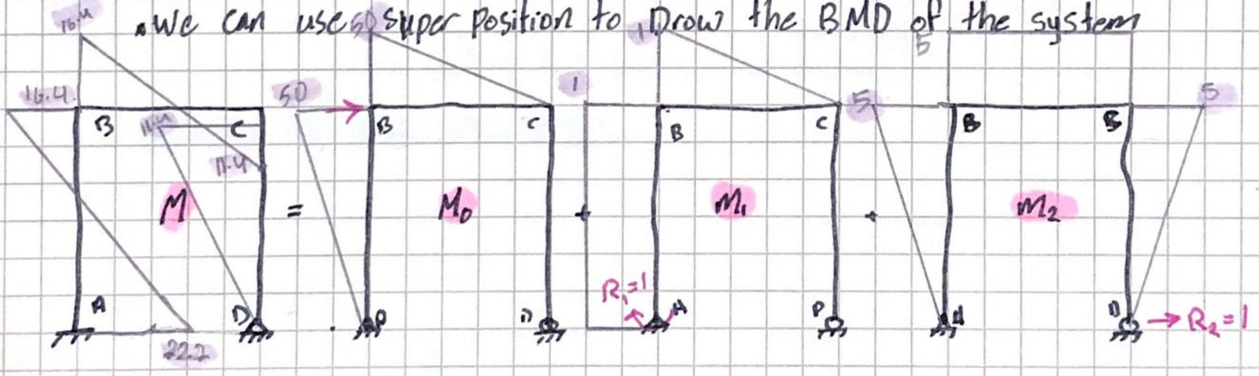
$22.2 - (10)(5) + 4D_y = 0$

$D_y = 6.95\ kN$, $A_y = 6.95\ kN$

$M_B = -22.2 + (7.72)(5)$

$= 16.4\ kN.m$

- We can use super position to Draw the BMD of the system



$M = M_0 + m_1 R_1 + m_2 R_2$

لما اني فرضت إلي الموجب، إلي طلع سالب

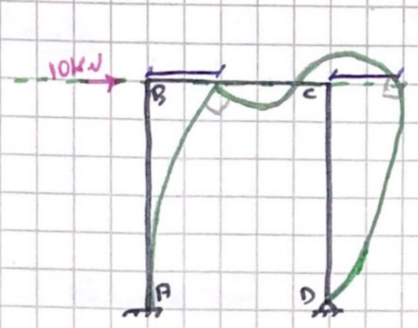
$M_A = 0 + (1)(-22.2) + 0 = \ominus 22.2\ kN.m$

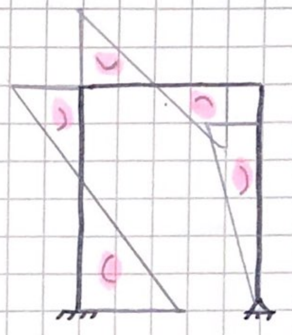
$M_B = 50 + 1(-22.2) + (5)(-2.28) = +16.4\ kN.m$

$M_C = 0 + 0 + (5)(-2.28) = -11.4\ kN.m$

$M_D = 0 + 1(-22.2) + 0 = -22.2\ kN.m$

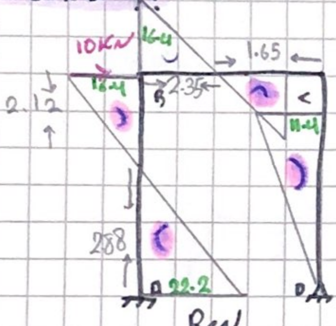
3- Deformed Shape



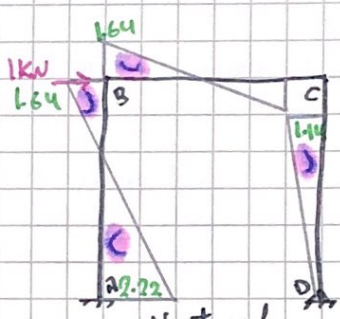


$\Delta_{B,h} = \Delta_{C,h} = \Delta$ (drift of the frame)

4- drift = ??



Real



Virtual

In general, we have to analyze the two systems to find M, m and find Δ, here we have a special case Real are Virtual beam has the same but load devided to 10, So the BMD of Real is devided by 10 to get Virtual Bending moment diagram

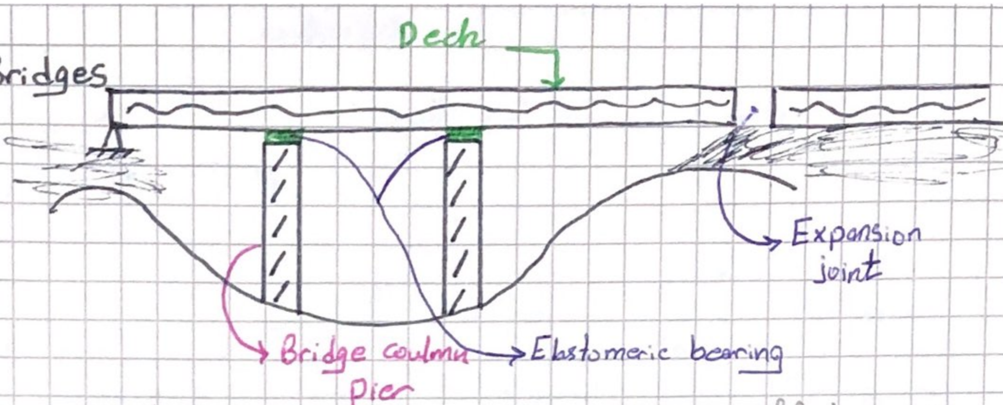
Using Virtual Integration factor:-

$$\Delta = \left(\frac{1}{3}\right)\frac{(22.2)(2.22)(2.88)}{IE} + \left(\frac{1}{3}\right)\frac{(16.4)(1.64)(2.12)}{EI} + \left(\frac{1}{3}\right)\frac{(16.4)(1.64)(2.35)}{EI}$$

$$+ \left(\frac{1}{3}\right)\frac{(11.4)(1.14)(1.65)}{EI} + \left(\frac{1}{3}\right)\frac{(11.4)(1.14)(5)}{EI}$$

→ **Beam Bridges**

Reality

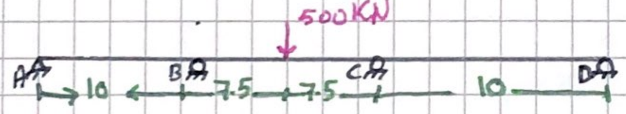


Expansion joint: In jeneral, there is temprature and other effectors make A horizontal displacement. And we have to give our bridge the permission to move (Roller support) even though it fixed in one point.

عدم عمل Permission لحركة الـ Bridge الـ horizontal بولد Internal stresses في الـ Beam الـ وهي شي انا ما بدي اياها بالـ structure بعمل Roller support

Elastomeric bearing: like a model of Roller support, Vertical displacement is not permitted, but horizontal is permitted, Elastic material.

بتعزز الـ Elastomeric bearing بـ steel plates
وبس بيصير تتحرك horizontal movement
زي ما موضح بالرسمة (زي طبقات الإسفنج)



Model

$\Delta_{s,a} = 27.5\,mm \downarrow$

$\Delta_{s,b} = 47.5\,mm \downarrow$

$\Delta_{s,c} = 22\,mm \downarrow$

$\Delta_{s,d} = 10\,mm \downarrow$

$E = 250\,MPa$

$I = 300 \times 10^{-6}\,m^4$

- Find Reaction Forces
- Draw BMD

Δ_s → displacement from soil can be downward and upward (نتيجة الـ Compression الـ soil!!)

* Static indet. to 2nd degree.

* choose two Redundands

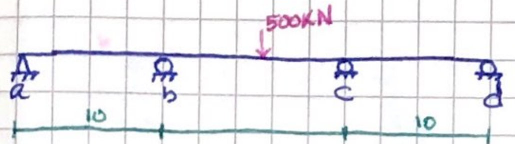
(a_x, d_y) ✗, (a_x, c_y) ✗, (b_y, c_y) ✓
(c_y, d_y) ✓, (b_x, c_y) ✗, (c_y, d_y) ✓
(b_y, d_y) ✓, ✓ (a_y, b_y)

→ I can't release a point which is already free.

Unstable
a_x is known and needed

Primary system

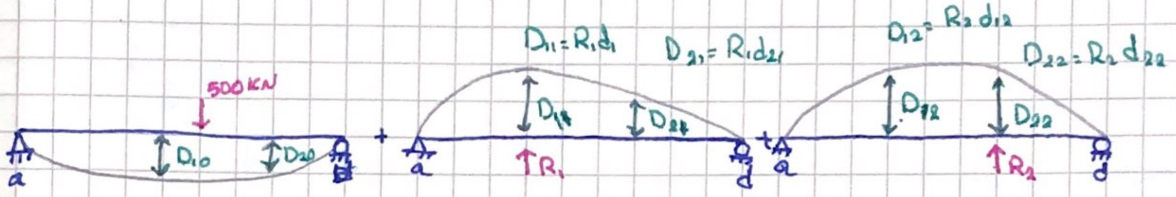
Lecture (4)



Statically indet. to second degree

$R_1 = B_y$

$R_2 = C_y$



We can't put 0, because there is a settlement

Comptability equation $= \cancel{0} = D_{10} + R_1 d_{11} + R_2 \bigcirc\!\!\!\!\!\;d_{12}$

(Vertical dis. at b)

$D_{10} = \frac{-341145}{IE}$

$\boxed{d_{12} = d_{21}}$

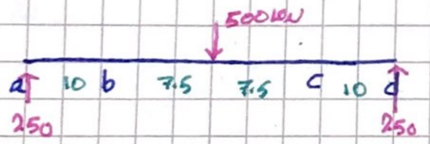
Comptability equation $= \quad = D_{20} + R_1 d_{21} + R_2 d_{22}$

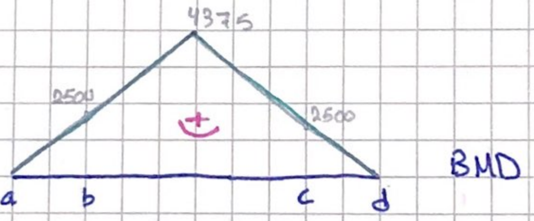
(Vertical dis. at c)

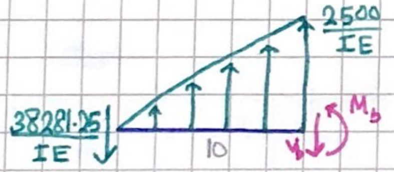
$D_{20} = \frac{-341145}{IE}$

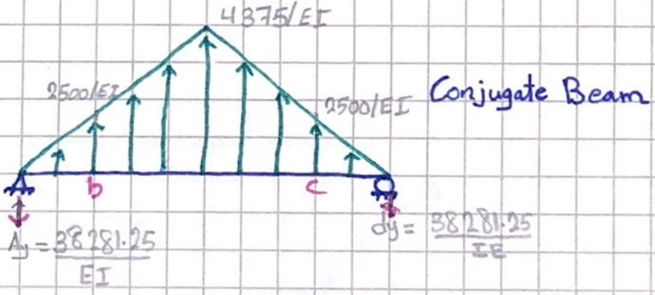
Using Conjugate Beam Method:

→ D_{10} , D_{20}







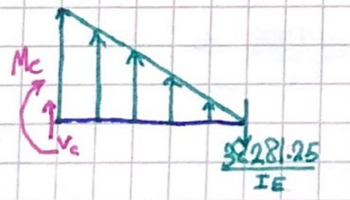


$\Sigma M_{cut} = 0$

$$M_b = -\left(\frac{38281.25}{IE}\right)(10) + \left(\frac{1}{2}\times 10\times \frac{2500}{IE}\right)\left(\frac{10}{3}\right)$$

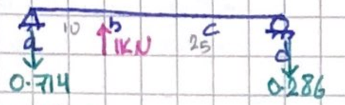
$$= -\frac{341145}{IE}$$

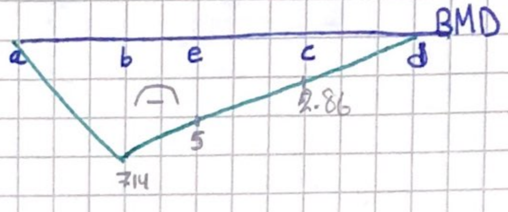
لما آجي أحسب ويطلع سالب بعوضها
بالـ comptability بوضو سالبة.

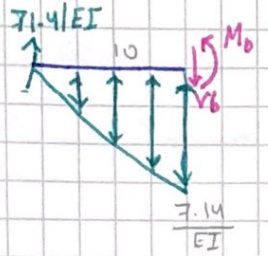


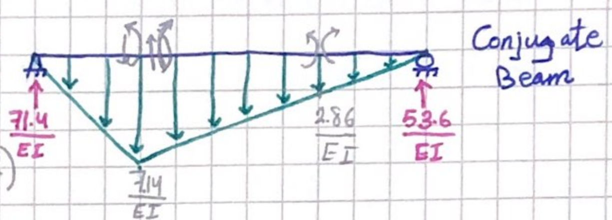
Mc, Mb will have the same value (Just in this case

$d_{11}\ d_{21}$:-



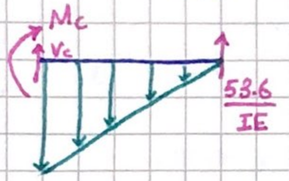






$$M_b = d_{11} = \left(\frac{71.41}{EI}\right)(10) - \left(\frac{1}{2}\times 10 \times \frac{7.14}{IE}\right)\left(\frac{10}{3}\right)$$

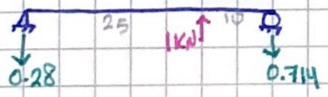
$$= \boxed{+\frac{595}{IE}}$$

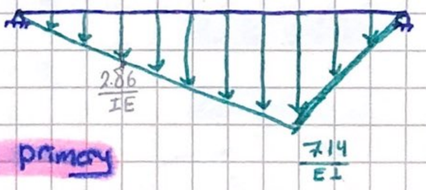


$$M_c = d_{21} = \left(\frac{53.6}{IE}\right)(10) - \left(\frac{1}{2}\times 10 \times \frac{2.86}{IE}\right)\left(\frac{2\times 10}{3}\right)$$

$$= \boxed{\frac{488.3}{IE}}$$

d_{22} :-





due to symm. In the primary

$$d_{22} = M_c = \frac{595}{IE} \text{ (Special case)}$$

$$= -\frac{341145}{EI} + R_1\frac{595}{IE} + R_2\frac{488.33}{IE}$$

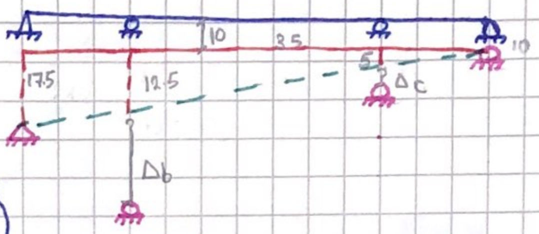
$$= -\frac{341145}{EI} + R_1\frac{488.33}{EI} + R_2\frac{595}{EI}$$

$\Delta_{s,a} = 27.5\ mm \downarrow$

$\Delta_{s,b} = 47.5\ mm \downarrow$

$\Delta_{s,c} = 22\ mm \downarrow$

$\Delta_{s,d} = 10\ mm \downarrow$



The Refrence line when we calculated deflections is ad

$\Delta_b = 47.5 - 10 - 12.5 = 25mm \rightarrow$ In eq. $= -25mm$

$\Delta_c = 22 - 10 - 5 = 7mm \rightarrow$ In eq. $= -7\ mm$

$$\left(-25\times10^{-3} = \frac{-341145}{EI} + R_1\frac{595}{IE} + R_2\frac{488.33}{IE}\right) —— ① \times EI$$

$$\left(-7\times10^{-3} = \frac{-341145}{EI} + R_1\frac{488.33}{EI} + R_2\frac{595}{EI}\right) —— ② \times EI$$

$E = \underline{200\ GPa}$

$I = 3000\times10^{-6}\ m^4$

$EI = 200\times10^{-6}\ \frac{KN}{m^2} \times 3000\times10^{-6}\ m^4$

$= 600\,000\ KN\cdot m^2$

$-1500 = -34145 + 595R_1 + 488.33R_2$

$-4200 = -34145 + 488.33R_1 + 595R_2$

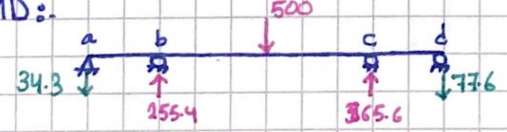
$$\begin{bmatrix} 326145 \\ 488.83 \end{bmatrix} = \begin{bmatrix} 595 & 488.33 \\ 488.33 & 595 \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

$R_1 = 255.4\ KN$

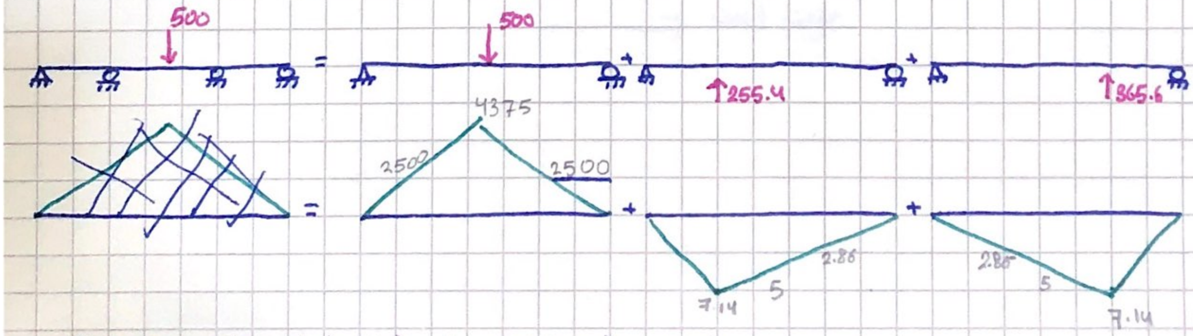
$R_2 = 365.6\ KN$

Lecture(6)

2-BMD:-



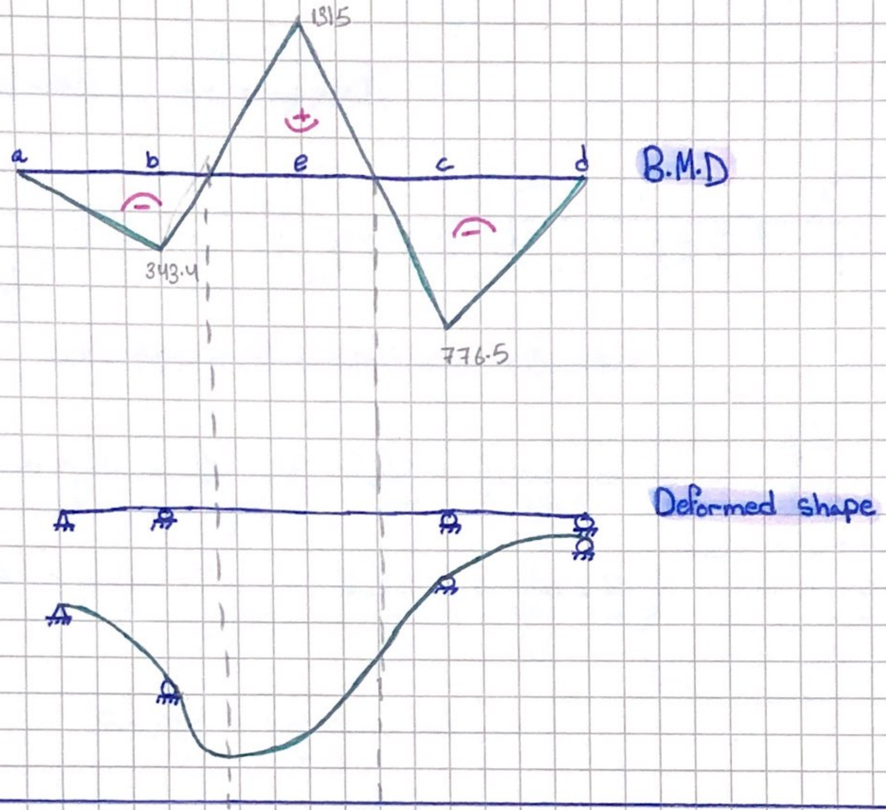
$R_1 \neq R_2$, Why?
Because there was nonsemmetric settlements



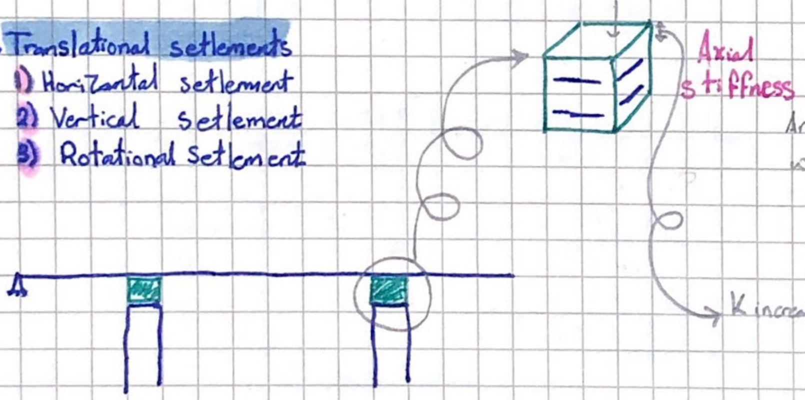
$M_b = 2500 + (255.4)(-7.14) + (365.6)(2.86) = 343.4$

$M_e = 4375 + (255.4)(^-5) + (365.6)(^-5) = 1315$

$M_c = 2500 + (255.4)(-2.86) + (3656)(-7.14) = -776.5$



* Translational setlements
1) Horizontal setlement
2) Vertical setlement
3) Rotational setlement



$K_{axial} = \infty$ means a roller sup.

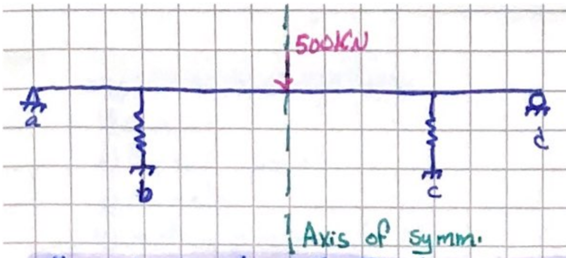
Arealy realy stiff support which means no move

we want to say

K_{axial} = Finite Value

K increase → displacement decrease

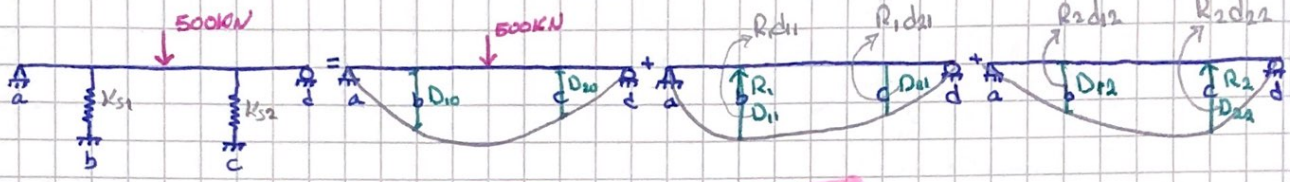
→ Next page



- In this case, its allways good to choose Redundance in the spring elements.

- **How to Deal with Elastic Supports?!**

$R_1 = b_y \qquad R_2 = C_y$



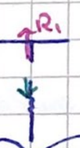
Roller support is a Vertical spring with $K = \infty$

Comptability equation $= D_{10} + R_1 d_{11} + R_2 d_{12} = \frac{R_1}{K_{s1}}$

(Vertical dis. at b) $= \frac{-341145}{EI} + R_1 \frac{595}{IE} + R_2 \frac{488.3}{EI}$

Comptability equation $= D_{20} + \cancel{R_2} d_{21} + R_2 d_{22} = \frac{R_2}{K_{s2}}$

(Vertical dis. at C) $= \frac{-341145}{EI} + R_1 \frac{488.3}{EI} + R_2 \frac{595}{EI}$



ال spring دائماً بعمل force عكس ال redundance لي عندي فأي اشارة ضيف للمعادلة بعكسها عالطرف الثاني

From physics, We Know that

$K\Delta = F \rightarrow \Delta = F/K$

$$0 = \frac{-34145}{EI} + \left(\frac{595}{IE} + \frac{1}{K_s}\right) R_1 + R_2 \frac{488.3}{EI}$$

$$0 = \frac{-34145}{EI} + \frac{595}{IE} R_1 + \left(\frac{488.3}{EI} + \frac{1}{K_s}\right) R_2$$

$$\begin{bmatrix} \frac{34145}{EI} \\ \frac{34145}{EI} \end{bmatrix} = \begin{bmatrix} \left(\frac{595}{EI} + \frac{1}{K_{s1}}\right) & \left(\frac{488.3}{IE}\right) \\ \left(\frac{488.33}{EI}\right) & \left(\frac{595}{EI} + \frac{1}{K_{s2}}\right) \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

↳ Flixibility matrix

$K_{s1} = K_{s2} = 0.006\ EI$

$$\frac{1}{\cancel{EI}} \begin{bmatrix} 34145 \\ 34145 \end{bmatrix} = \frac{1}{\cancel{IE}} \begin{bmatrix} (595 + 166.67) & (488.33) \\ (488.33) & (595 + 166.67) \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

- Symm in the system:-
  1) Support
  2) Applied loading
  3) Geometry
  4) Section properties (EI)

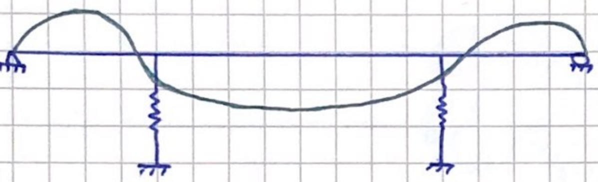
→ In the question:

I can say from symmetry that $R_1 = R_2$ and Instead of solving a two Variables eq. I solve a one Variable equation.

$$\frac{341145}{\cancel{EI}} = \left(\frac{595}{\cancel{EI}} + \frac{1}{0.006\cancel{EI}}\right)R_1 + \frac{488.3R_1}{\cancel{EI}}$$

$$34145 = (595 + 166.67)R_1 + 488.3R_1$$

$$\boxed{R_1 = R_2 = 272.9\ KN}$$



Deformed shape

BMD

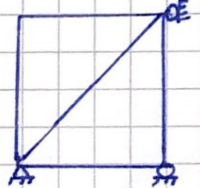
Lecture(7)

- Force Method for trusses:

members + # Reactions ☐ 2j

6 + 3   2(4)

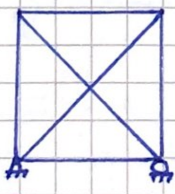
9 > 8



[Statically indet. to 1st degree]

[External indeterminacy]

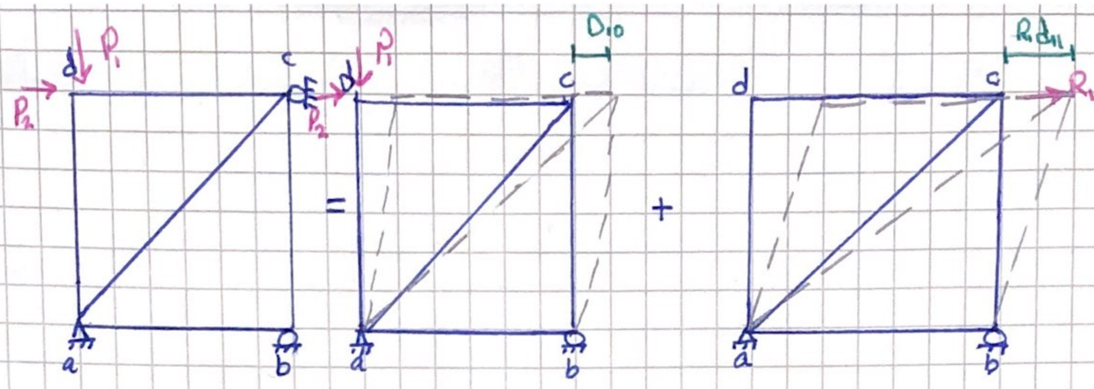
[Redundant : Reaction force (External)]



[Statically indet. to 1st degree]

[Internal indeterminacy]

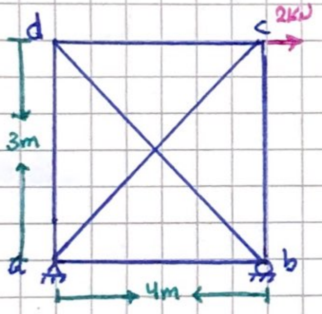
[Redundant: member force (Internal)]



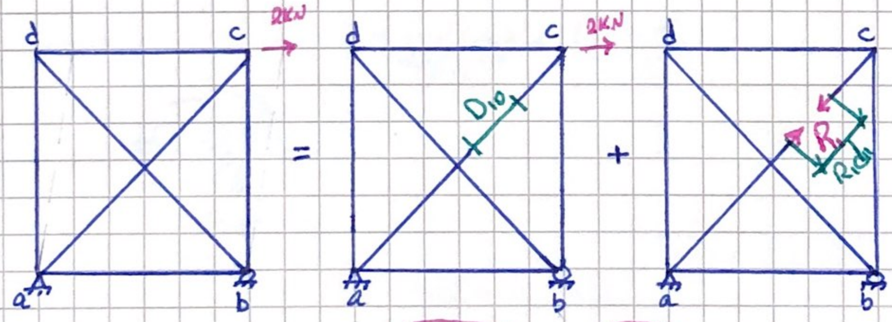
Comptability equation=0= $D_{10} + R_1 d_{11}$
(horizontal dis. at point c)

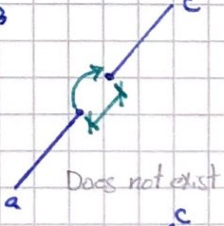
$$0 = \Sigma \frac{NnL}{EA} + R_1 \Sigma \frac{n^2L}{EA}$$

$$R_1 = \frac{-\Sigma NnL/EA}{\frac{\Sigma n^2L}{EA}}$$



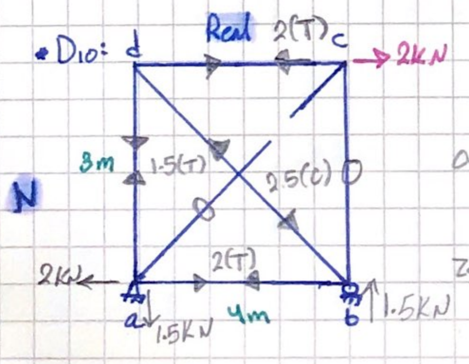
- Find member forces of the truss?
  member force is the redundant
  (we want to choose AC)

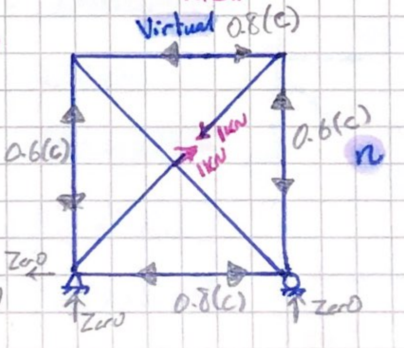


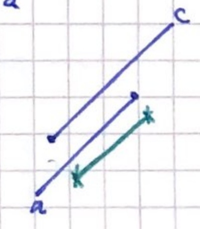


Comptability equation $= \boxed{\Sigma\, n_1 \frac{NL}{EA}} + \boxed{\frac{\Sigma n_1^2 L}{EA}}$
(The relative dis. between cut points of member AC)
D_{10} $R_1 d_{11}$

- D_{10}:







Member	L	N	n_1	EA	n_1NL/EA	n_1^2L/EA
ab						
ac						
ad						
bc						
bd						
cd						
da						

[There is No need for A table here].

$$D_{10} = \frac{(-0.6)(1.5)(3)}{EA} + \frac{(-0.8)(2)(4)}{EA} + 0 + \frac{(-0.8)(2)(4)}{EA} + \frac{(1)(-2.5)(5)}{EA} + 0$$

$$= \boxed{\frac{-28}{EA}}$$

*** d_{11}:**

$$d_{11} = \frac{(-0.6)(-0.6)(3)}{EA} + \frac{(-0.8)(-08)(4)}{EA} + \frac{(-0.6)(-0.6)(3)}{EA} + \frac{(-0.8)^2(4)}{EA} + \frac{(1)^2(5)}{EA} + \frac{(1)^2(5)}{EA}$$

$$= \boxed{\frac{17.28}{EA}}$$

$$0 = -28 + 17.28R_1$$

$$R_1 = \frac{28}{17.28} = 1.62\ KN(T)$$

Final Force = $N_1 + R_1 n_1$ (Using super-position method)

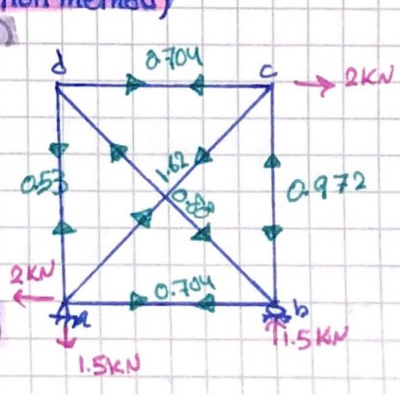
$F_{ad} = (1.5) + (1.62)(-0.6) = 0.53KN\ (T)$

$F_{ab} = (2) + (1.62)(-0.8) = 0.704(T)$

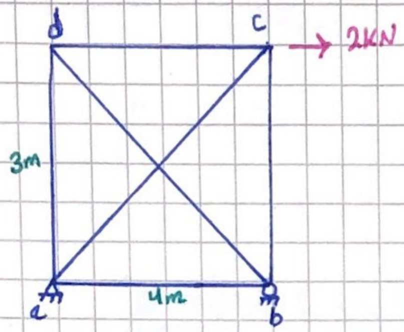
$F_{bd} = -(2.5) + (1.62)(1) = -0.88\ (c)$

$F_{bc} = (0) + (1.62)(-0.6) = -0.972(C)$

$F_{cd} = (2) + (1.62)(-0.8) = 0.704\ (T)$



Lectue(8)



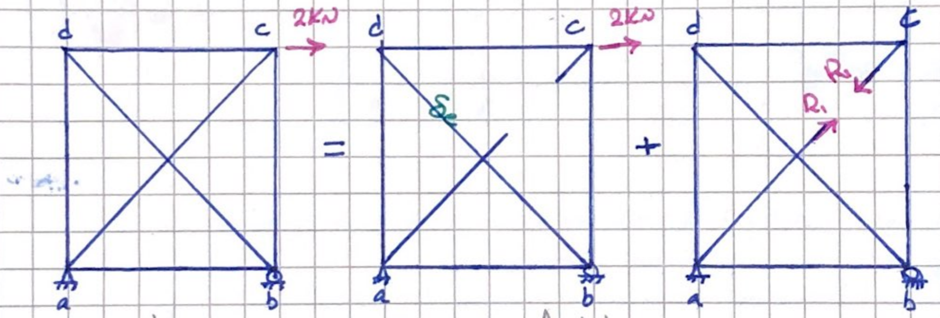
$\delta_{ac} = 2$ mm too short

$\delta_{bd} = 3$mm too short

* Revesion From Analysis I:

- Calculate deformation in Truss systems:

$$\Delta = \underbrace{\sum_{i=1}^{m} n \frac{NL}{EA}}_{\text{Applied loading}} + \underbrace{\sum n(\alpha \Delta T L)}_{\text{Thermal action}} + \underbrace{\sum_e n \, \delta e}_{\text{Error}}$$



Fabrication errors in members ac and bd

Primary + Applied loading + action + error Fabrication

Comptability equation = $D_{10} + R\, d_{11}$

(For the relative dis. of cut elements for member AC) .

$$= \quad + R_1\left(\sum \frac{n_1^2 L}{EA}\right) \rightarrow \text{will stay the same}$$

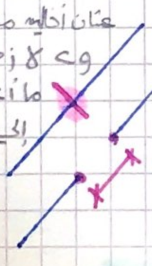
*** D_{10}:**

$$D_{10} = \sum n_1 \frac{N_0 L}{EA} + \sum n_i \, \delta e$$

$$= \frac{-28}{EA} + (1)\overset{\ominus \rightarrow \text{too short}}{}(3\times10^{-3})$$

$$+2\times10^{-3} = \left(\frac{-28}{EA} - (1)(-3\times10^{-3})\right) + R_1\left(\frac{17.28}{AE}\right)$$

ال member عنده too short
عشان أخليه متواصل بين a
و c لازم أشدّه لحد
ما أعبي ال 2mm
الي خفي.



E = 70MPa
A = 100mm^2

$$EA = \left(70\times10^{6}\frac{kN}{m^2}\right)\left(100\times10^{-6}\, m^2\right)$$

$$= 7000$$

→

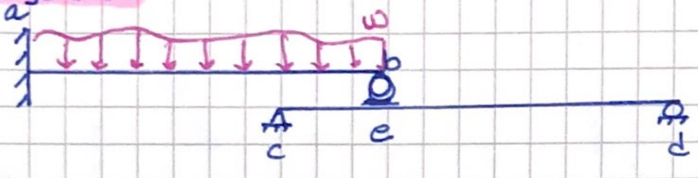
$$\left[+2\times10^{-3} = \left(\frac{-28}{EA} - 3\times10^{-3}\right) + R_1\left(\frac{17.28}{EA}\right)\right] \times 7000$$

$$14 = -28 - 21 + 17.28R_1$$

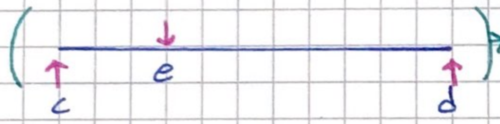
$$R_1 = \frac{14+28+21}{17.28} = \boxed{3.646\,KN} = F_{AC}$$

- These Imperfition forces can effect the system because system is sensitive to them, F_{AC} was dublicated.

Combined system:-

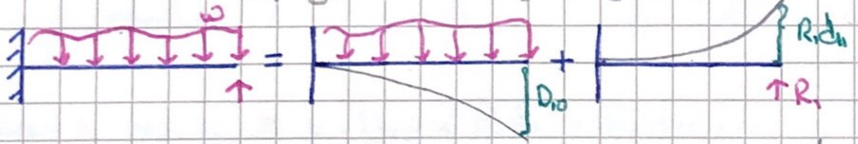


Indet. system, As soon as I solve it



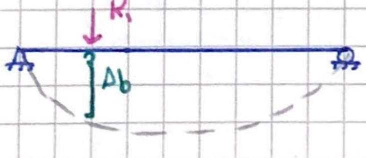
This will be determinate system.

* بقول ال systems التنين من نفس النوع بس Combined *



$$\boxed{\Delta b} = D_{10} + R_1 d_{11}$$

* To Find Δb:



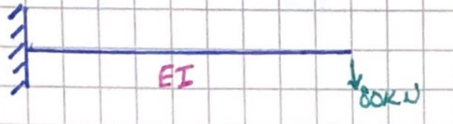
- The same Idea as I have an Spring, but here we have a system not spring.

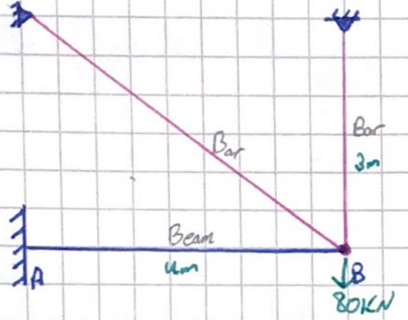
The Written above is The prosedure.

Lecture(9)

Force method (combined systems):

* عندي ال Beam التالي إلي بتأثر عليه load
مقدارها 80KN، لقيت انو ال displacement تبعها عالي
فكان لازم الاقي طريقة اقتصادية عشان اخففه غير
تغيير I عشانها حتزيد التكلفة فكان الحل كالتالي :-

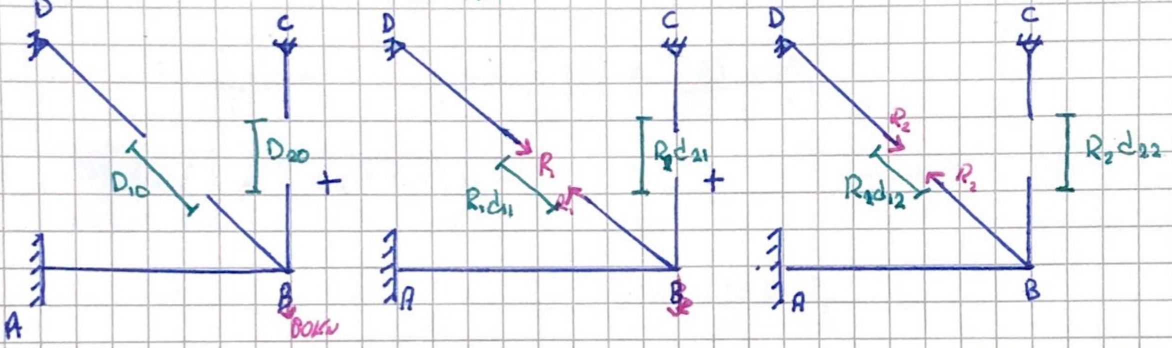




# unKowns		# Equations
5	>	3

Statically indet. to 2nd degree

$R_1 = F_{BD}$ $R_2 = F_{BC}$

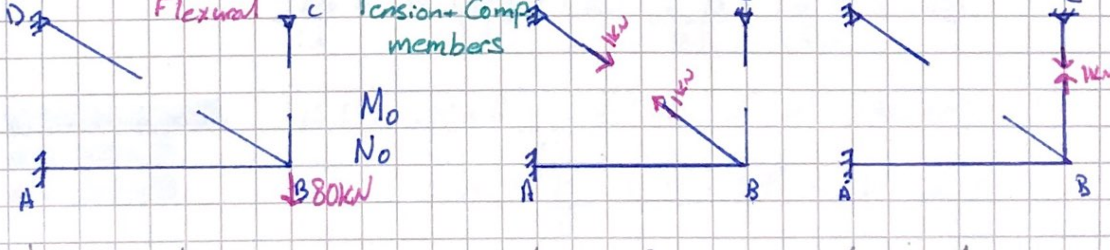


Comptability equation: $0 = D_{10} + R_1 d_{11} + R_2 d_{12}$
(Relative dis of cut in BD)

Comptability equation: $0 = D_{20} + R_1 d_{21} + R_2 d_{22}$
(Relative dis. of cut BC)

Equals zero Because there is no thermal effect or error given in the question.

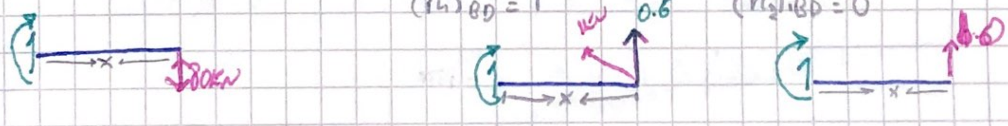
$$\Delta = \underbrace{\int \frac{mM}{EI} dx}_{\text{Flexural}} + \underbrace{\sum \frac{nNL}{EA}}_{\text{Tension+Comp members}}$$



$N_{0BC} = 0$, $N_{0BD} = 0$

$(n_1)_{BC} = 0$
$(n_1)_{BD} = 1$

$(n_2)_{BC} = 1$
$(n_2)_{BD} = 0$



$M_0 = -80x$ $m_1 = 0.6x$ $m_2 = x$

- $D_{10} = \int \frac{m_1 M_0}{EI} + \sum \frac{n_1 N_0 L}{EA}$

$$= \int_0^4 \frac{(0.6x)(-80x)}{EI}\,dx + 0 = -\frac{1024}{EI}$$

- $D_{20} = \int \frac{m_2 M_0}{EI} + \sum \frac{n_2 N_0 L}{EA}$

$$= \int_0^4 \frac{(x)(-80x)}{EI}\,dx + 0 = \frac{-1706.7}{EI}$$

- $d_{11} = \int \frac{m_1^2}{EI}\,dx + \sum \frac{n_1^2 L}{EA}$

$$= \int_0^4 \frac{(0.6x)^2\,dx}{EI} + \frac{(0)^2(3)}{EA} + \frac{(1)(5)}{EA} = \frac{7.68}{EI} + \frac{5}{EA}$$

- $d_{22} = \int \frac{m_2^2}{EI}\,dx + \sum \frac{n_2^2 L}{EA}$

$$= \int_0^4 \frac{x^2}{EI}\,dx + \frac{(1)^2(3)}{EA} + \frac{(0)^2(5)}{EA} = \frac{21.33}{EI} + \frac{3}{EA}$$

- $d_{21} = \int \frac{m_2 m_1}{EI}\,dx + \sum \frac{n_2 n_1 L}{EA}$

$$= \int_0^4 \frac{(x)(0.6x)}{EI}\,dx + 0 + 0 = \frac{12.8}{EI}$$

$$0 = \frac{-1024}{EI} + R_1\left(\frac{7.68}{EI} + \frac{5}{EA}\right) + R_2\left(\frac{12.8}{EI}\right) \text{———} ①$$

$$0 = \frac{-1706}{EI} + R_1\left(\frac{12.8}{EI}\right) + R_2\left(\frac{21.33}{EI} + \frac{3}{EA}\right) \text{———} ②$$

$I = 200\times 10^6\ mm^4$

$A = 100\ mm^2$

$E = 200\,GPa$

$$EI = 200\times\cancel{10^{6}}\,\frac{kN}{m^2} \times 200\times\cancel{10^{-6}}\,m^4 = 40.000$$

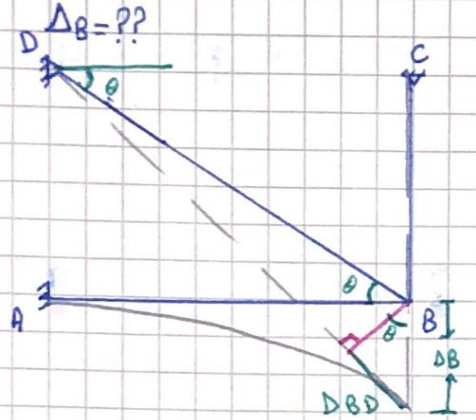
$$EA = 200\times\cancel{10^{6}}\,\frac{kN}{m^2} \times 100\times\cancel{10^{-6}} = 20\,000$$

$$25.6\times\cancel{10^{-3}} = R_1\left(0.442\times\cancel{10^{-3}}\right) + R_2\left(0.32\times\cancel{10^{-3}}\right)$$

$$42.7\times\cancel{10^{-3}} = R_1\left(0.32\times\cancel{10^{-3}}\right) + R_2\left(0.683\times\cancel{10^{-3}}\right)$$

$R_1 = F_{BD} = 19.2\,KN\,(T)$

$R_2 = F_{BC} = 53.4\,KN\,(T)$



→ $\Delta_B = \Delta_{BC}$

$= \left(\frac{F_{BC} L}{EA}\right)$

→ $\frac{\Delta_{BD}}{\Delta_B} = 0.6$ → Have to be the same

$\left(\Delta_B = \frac{\Delta_{BD}}{0.6}\right)$

Lecture(10)

→ **Influence lines And EnVelopes:**

we need them for design Values, we crate them becouse there is live loads with changing position, will give us the cretical design Values

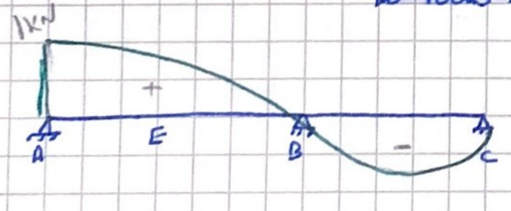
It's Drawn used:
1) Point - by - point
2) Equations
3) Qualitatine approch

We use I.L To decide load cases that produces critical design Values.

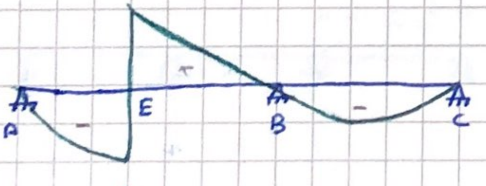
In Det. systems → Shape of influence lines are line segments

In this course, We are dealing with Indet. systems, So We are going to focus in Qualitative approach

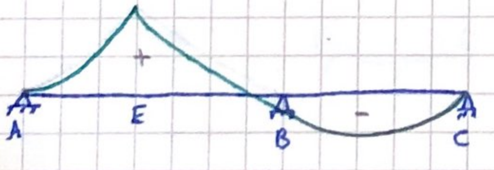
(Mueller principle)



Influence line for A_y

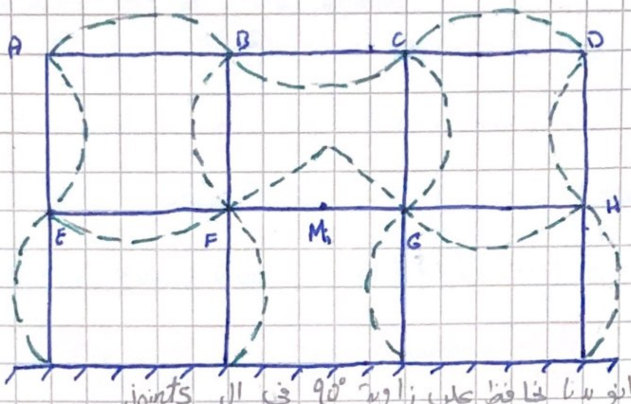


Influence line for shear V_E

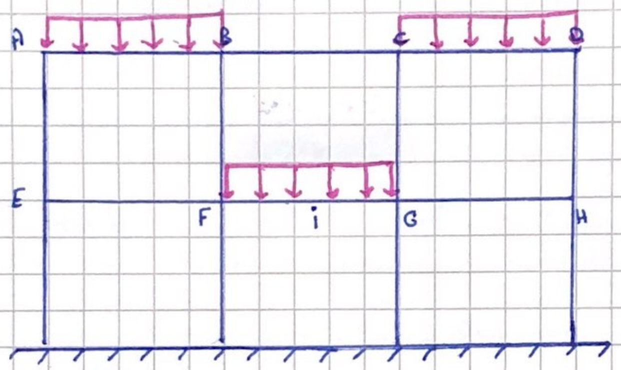


Influence line for B.M M_E

كلما بدي ال B M عند نقطة في البيم
بحط محلها Internal hinge و بشوف
ال Deformed shape

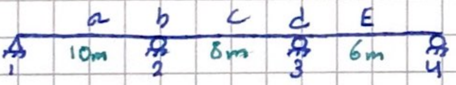


عا نرسم بنراعي انو بدنا نحافظ على زاوية 90° في ال joints



* The Shown load case is for Max. M_i

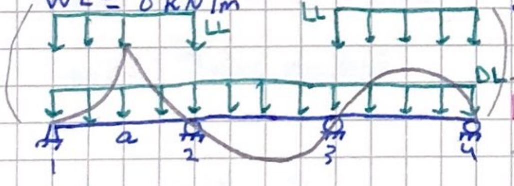
→ Example: Three span system



1. Draw the qualtitive influence lines for the Bending moment of the Beam, And develop the load cases
2. Draw qualtitive influence lines for the shear of the Beam
3. Draw the Invelope for Bending moment Values of the beam.

$W_D = 5 KN/m$

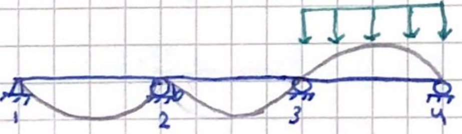
$W_L = 8 KN/m$



→ load case (1) For Maximum M_A

Influence line for M_A

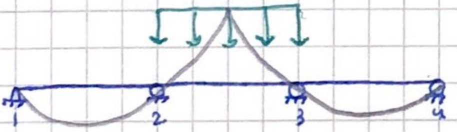
→ load case (2) For Min. M_A LL is at (2-3)



→ load case (3) for Max. M_B (LL is at 3-4)

Influence line for M_B

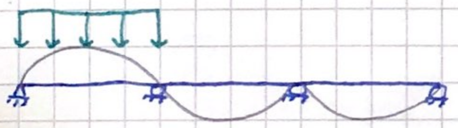
→ load case (4) For Min M_B (LL is at 1-2 & 3-4)



→ load case (5) For Max. M_C (LL at 2-3)

Influence line for M_C

→ load Case (6) For Min. M_C (LL at 1-2 & 2-3)

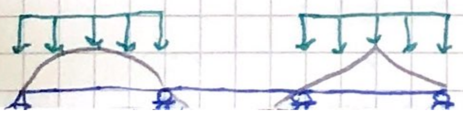


→ load case (7) for Max M_d (LL at 1-2)

Influence line for M_d

→ load Case (8) for Min. M_d (LL at 2-3 & 3-4)

→ Case (9) for Max M_e (LL at 1-2 and 3-4)



Influence line for M_e

→ Case (10) for Min M_e (LL at 2-3)

- We draw BMD's for all load cases

بعد هيك بروح باخد كل ال max Values و بوصل بيناتهم بخط و بعدها بروح
باخد كل ال min. Values و بوصل بيناتهم بكمان خط

→ Those Are Envelope lines

بفرق معي ال moment موجب ولا سالب في ال Design فمثلاً لو كان موجب بكون التسليح تحت بال Tension side أما لو كان فوق التسليح بكون ال moment سالب

Go Back to slide #12
For more Information

Lecture(11)

→ Displacement method		→ Force method
Kinematic determancy	⟷	Static indetermancy
Unkowns - deformations	⟷	Unkowns - Forces
Main equation - Equilibrium - Comtability	⟷	Main eq. - Comptability - Force deformation relation - Equilibrium
Stiffness coefficient → Force due to unit deformation	⟷	Flixibility coefficient → Deformation to unit load

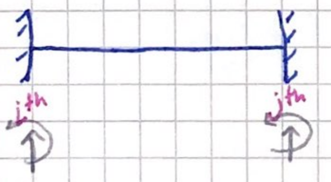
- Displacement method
  - Slope-diflection method
  - Moment distribution method

  } → Flexural systems

  - Direct stiffness method } → Finite element method in structural Analysis (FEM)

I Slope-diflection equations:-

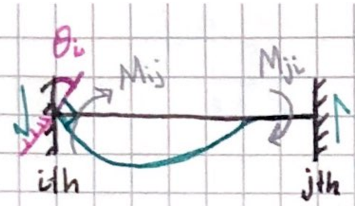
1) Darive stiffness coefficient → for a beam element



(Fixed ended Beam)
Kinematically determinate

$$M = \boxed{K}\ \theta_i$$

→ Stiffness coeff.



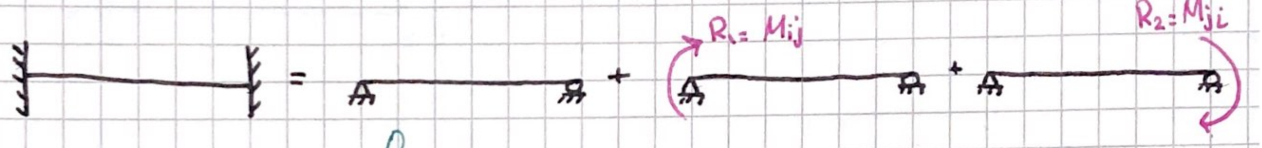
★ حيث Rotational settlement على ال ith ، بما معناه اني مسكت كل ال fixety إلي عندي ولفيتها بمقدار θ_i ★

$M_{ij} = \square\ \theta_i$

$M_{ji} = \square\ \theta_i$

M_{ij}: Moment in the element ij on the point i

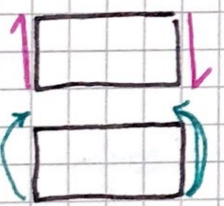
M_{ji}: Moment in the element ij on the point j



Comptability eq. = $\overset{0}{\cancel{D_{10}}} + R_1 d_{11} + R_2 d_{12} = 0$
(Rotation at ith)

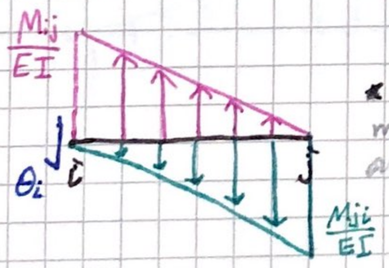
Comptability eq. = $\overset{0}{\cancel{D_{20}}} + R_1 d_{21} + R_2 d_{22} = \theta_i$
(Rotation at jth)

Use conjugate Beam to solve inet. system



Real	Conjugate
θ ↻	V(+)
Δ(↑)	M(+)

→ Sin Convension of the possitive conjugate Beam method



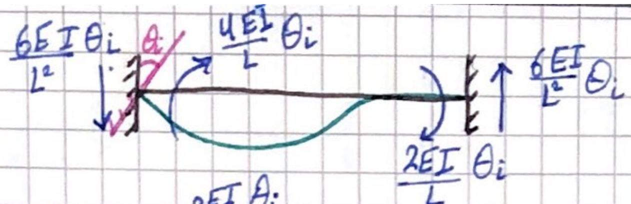
★ We used principle of super position, which means we saw the effect of each moment alone ★

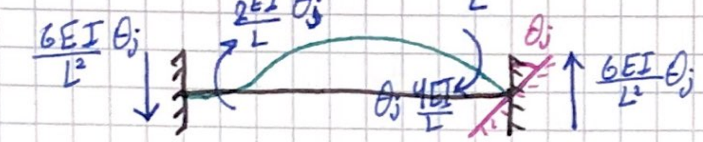
$$\Sigma M_i = 0 = \frac{1}{\cancel{2EI}} M_{ij} (\cancel{L}) \left(\frac{\cancel{L}}{3}\right) + -\frac{1}{\cancel{2EI}} M_{ji} (\cancel{L}) \left(\frac{2\cancel{L}}{3}\right)$$

$$\boxed{M_{ji} = \frac{1}{2} M_{ij}}$$

$$\Sigma M_j = 0 = \frac{1}{2} \frac{M_{ij}}{\cancel{EI}} (L) \left(\frac{2L}{3}\right) - \theta_i L - \frac{1}{2} \frac{M_{ij}}{\cancel{EI}} (L) \left(\frac{L}{3}\right)$$

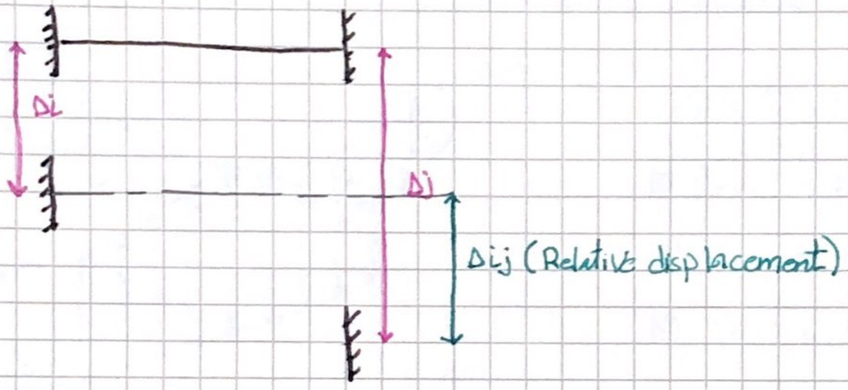
$$\boxed{M_{ij} = \frac{4EI\,\theta_i}{L}}$$



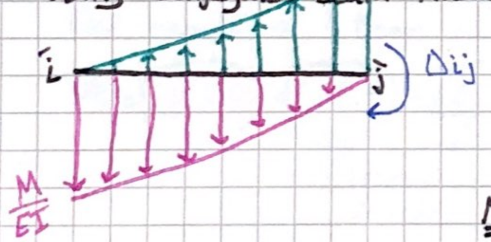


Lecture(12)

Displacement method : Slope-deflection equation



Using Conjugate Beam Analogy



Δ on Real ↓

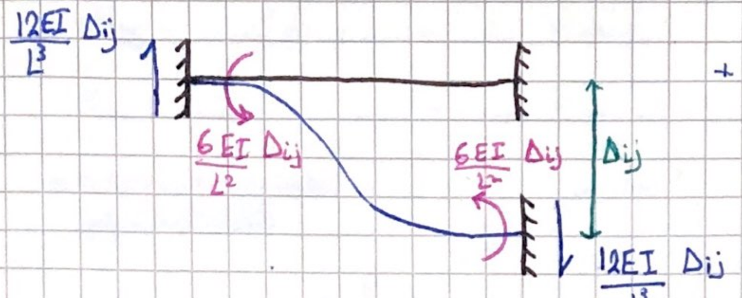
Moment on Conjogate

$$\Sigma M_j = 0$$

$$\left(\frac{1}{2}\right)\left(\frac{M}{EI}\right)(L)\left(\frac{L}{3}\right) + \Delta_{ij} = \frac{1}{2}\frac{M}{EI}(L)\left(\frac{2}{3}L\right)$$

$$M\left(\frac{L^2}{6EI}\right) = \Delta_{ij}$$

$$\boxed{M = \frac{6EI}{L^2}\Delta_{ij}}$$

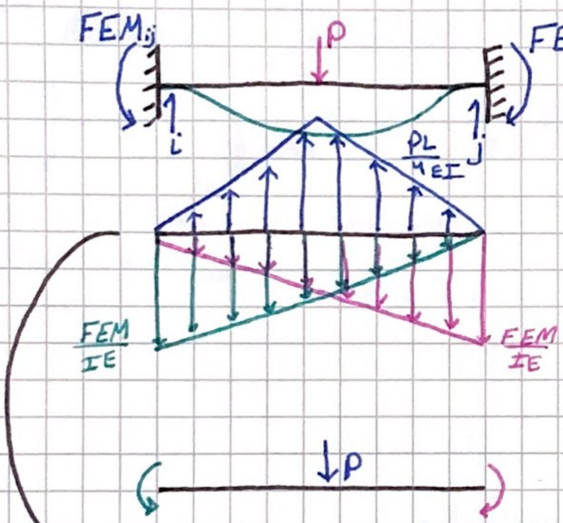


$+ FEM_{ij}, FEM_{ji}$

END FORCES $\xleftrightarrow[\text{مو}]{\text{مسما}}$ DEFORMATIONS

↑

APPLIED LOADING



(FEM: fixed end moment)

We can solve this using

1. force method
2. Conjugate Beam Analogy
3. Tables (for next lecture)

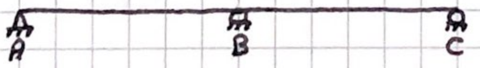
$\Sigma F_y = 0$ $\boxed{FEM = \frac{PL}{8}}$

End moment of A Beam element

Clock wise → ⊕

$$M_{ij} = \underbrace{\frac{4EI}{L}\theta_i}_{\text{Rotation on ith node}} + \underbrace{\frac{2EI}{L}\theta_j}_{\text{Rotation on jth node}} - \underbrace{\frac{6EI}{L^2}\Delta_{ij}}_{\text{Relative dis. in ij}} + \underbrace{FEM_{ij}}_{\text{Depends on the applied loading}}$$

$$= \left(\frac{2EI}{L}\left[2\theta_i + \theta_j - \frac{3\Delta_{ij}}{L}\right] + FEM_{ij}\right) \rightarrow \text{Slope-deflection equation}$$



→ How Many Beam elements in this system?

Two beam elements

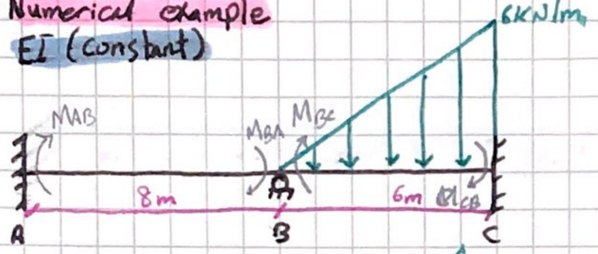
- ↳ Element AB
- ↳ Element BC

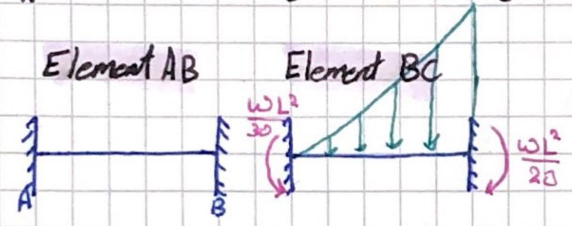
$M_{AB} =$ ____ + ____ - ____ + ____   * $\Delta_{ij} = \Delta_{ji}$

$M_{BC} =$ ____ + ____ - ____ + ____

- **Numerical example**

EI (constant)





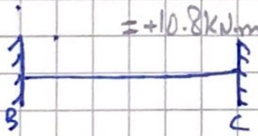
$0 = FEM_{AB} = FEM_{BA}$
There is No loading

From table

$FEM_{BC} = \frac{-6(6^2)}{30}$
$= -7.2$ KN.m

$FEM_{CB} = + \frac{(6)(6^2)}{20}$
$= +10.8$ KN.m

$\Delta_{AB} = \Delta_{BA} = 0$



$\Delta BC = \Delta CB = 0$

Fixity At A → $\theta_A = 0$

Fixity At C → $\theta_C = 0$

$M_{AB} = 1.54$ KN.m

$M_{BA} = 3.09$ KN.m

$M_{BC} = -3.09$ KN.m

$M_{CB} = 12.86$ KN.m

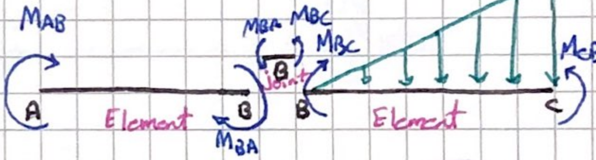
$$M_{AB} = \frac{2EI}{8}\left(\cancelto{0}{2\theta_A} + \theta_B - \cancelto{0}{\frac{3\Delta_{AB}}{8}}\right) + \cancelto{0}{FEM_{AB}}$$

$$M_{BA} = \frac{2EI}{8}\left(2\theta_B + \cancelto{0}{\theta_A} - \cancelto{0}{\frac{3\Delta_{AB}}{8}}\right) + \cancelto{0}{FEM_{BA}}$$

$$M_{BC} = \frac{2EI}{6}\left(2\theta_B + \cancelto{0}{\theta_C} - \cancelto{0}{\frac{3\Delta_{BC}}{6}}\right) + FEM_{BC}$$

$$M_{CB} = \frac{2EI}{6}\left(\cancelto{0}{2\theta_C} + \theta_B - \cancelto{0}{\frac{3\Delta_{CB}}{6}}\right) + FEM_{CB}$$

* Solve for θ_B (The only unkown)



ΣM at connecting joint beam B = 0
($M_{BA} + M_{BC} = 0$) → Equilebrium equation

$$\frac{2EI}{8}(2\theta B) + \left(\frac{2EI}{6}\right)(2\theta_B) - 7.2 = 0$$

$$\frac{4}{8} EI\,\theta_B + \frac{4}{6} EI\,\theta_B - 7.2 = 0$$

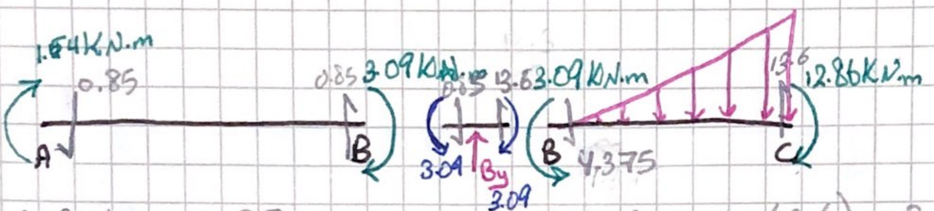
$$\theta_B = \frac{6.17}{EI}$$

System is Kinematicly indet. to 1st degree

Lecture(13)

Displacement method : Slope - deflection equations

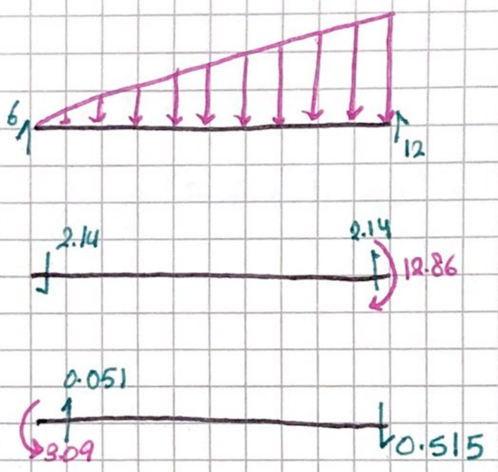
$$M_{ij} = \frac{2EI}{L}\left(2\theta_i + \theta_j + \frac{3\Delta_{ij}}{L}\right) + FEM_{ij}$$



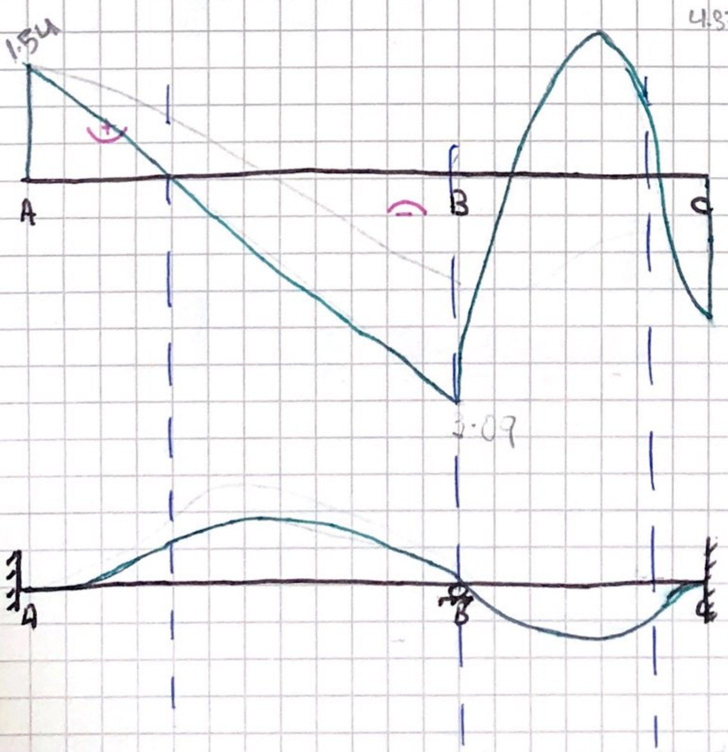
$$\frac{3.09 + 1.54}{8} = 0.85$$

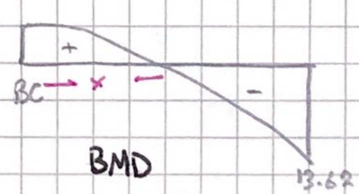
$$V_c = \frac{12.86 + (\frac{1}{2})(6)(6)(\frac{2}{3}6) - 3.09}{6} = 13.6$$

OR using super position

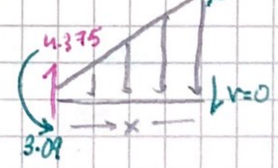


$$6 - 2.14 + 0.515 = 4.375$$





BMD



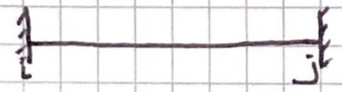
$$-\frac{x^2}{2} + 4.375 = 0$$

$$x^2 = 2(4.375) \rightarrow x = 2.96$$

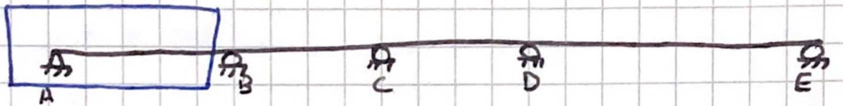
$$M = (4.375)(2.96) - 3.09 - \frac{1}{2}(1)(2.96)(\frac{2.96}{3})$$

$$= 5.5 \text{ KN.m}$$

• Elements when two end-moments are unkown



$$M_{ij} = \frac{2EI}{L}\left(2\theta_i + \theta_j - \frac{3\Delta_{ij}}{L}\right) + FEM_{ij}$$ → General equation



$M_{AB} = M_{ED} = 0$ → Known

• We want to derive A modified equation (One of end moments is Known)

$$\left(M_{ij} = \frac{2EI}{L}\left(2\theta_i + \theta_j - \frac{3\Delta_{ij}}{L}\right) + FEM_{ij}\right) \times 2$$

$$0 = M_{ji} = \frac{2EI}{L}\left(2\theta_j + \theta_i - \frac{3\Delta_{ij}}{L}\right) + \cancel{FEM_{ji}} \to 0$$ (Because its A pin, No Fixity)

$$2M_{ij} = \frac{2EI}{L}\left(4\theta_i + 2\theta_j - \frac{6\Delta_{ij}}{L}\right) + 2FEM_{ij}$$

$$-$$

$$0 = \frac{2EI}{L}\left(2\theta_j + \theta_i - \frac{3\Delta_{ij}}{L}\right)$$

$$\cancel{2}M_{ij} = \frac{\cancel{2}EI}{L}\left(3\theta_i - \frac{3\Delta_{ij}}{L}\right) + \cancel{2}FEM_{ij}$$

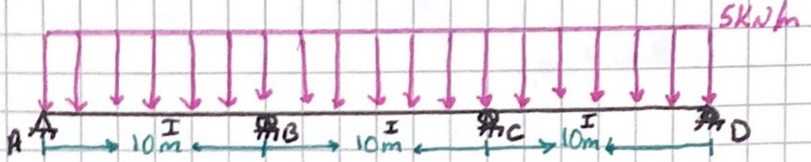
$$M_{ij} = \frac{EI}{L}\left(3\theta_i - \frac{3\Delta_{ij}}{L}\right) + FEM_{ij}$$

$$= \frac{3EI}{L}\left(\theta_i - \frac{\Delta_{ij}}{L}\right) + FEM_{ij}$$

when one of the end beam moments is Known

$$M_{ij} = \frac{3EI}{L}\left(\theta_i - \frac{\Delta_{ij}}{L}\right) + FEM_{ij}$$ → Modified equation

θ_j is Not equal to Zero, but we're not solving to it here so we decrease the number of unkowns



Support B 0.005m downward

Support C 0.01m downward

E = 200GPa

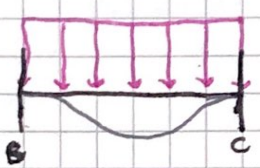
$I = 1.35 \times 10^{-3}\ m^4$

$\boxed{1}$ Reaction forces:

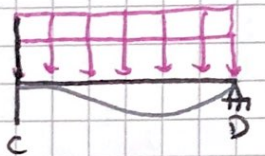
Modefied eq.

$M_{AB} = 0$

$M_{BA} = ??$



General eq.



Modified eq.

$M_{DC} = 0$

$M_{CD} = ??$

} Unkowns are θ_B, θ_C

If we used General equation for All parts then unkowns ar $\theta_A, \theta_B, \theta_C, \theta_D$

FEM_{BA}

$\frac{WL^2}{8} = 62.5$

FEM_{BC}

$\frac{WL^2}{12} = -41.67$

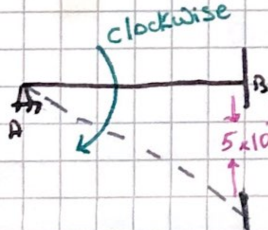
FEM_{CD}

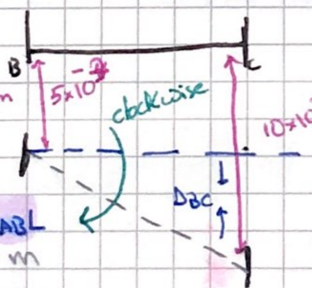
$\frac{WL^2}{8} = -62.5$

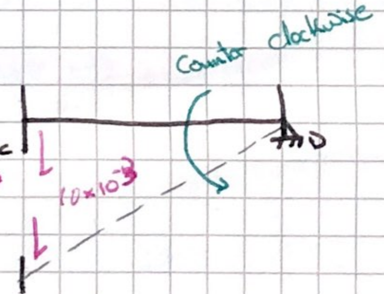
$\psi = \frac{\Delta}{L}$

FEM_{CB}

$+41.67$







$\Delta AB = \Delta BC = \psi_{AB} L$

$\Delta AB = \oplus 5 \times 10^{-3}\ m$

↳ clockwise

$\Delta_{CD} = \Delta_{DC} = \psi_{CD} L$

$\psi_{CD} = -(10 \times 10^{-3})/10$

$= -0.001$

$\Delta BC = \Delta_{CB} = \psi_{BC} L$

$\psi_{BC} = (10 \times 10^{-4} - 5 \times 10^{-4})/10$

$= +5 \times 10^{-4}\ m$

$$\left.\begin{aligned} \cdot M_{AB} &= 0 \\ \cdot M_{BA} &= \frac{3EI}{L}\left(\theta_b - \frac{\Delta_{AB}}{L}\right) + FEM_{BA} \\ &= \frac{3EI}{10}\left(\theta_b - 0.0005\right) + 62.5 \end{aligned}\right\} \rightarrow \text{Modefied}$$

$$\left.\begin{aligned} \cdot M_{BC} &= \frac{2EI}{L}\left(2\theta_B + \theta_c - 3\frac{\Delta_{BC}}{L}\right) + FEM_{BC} \\ &= \frac{2EI}{10}\left(2\theta_B + \theta_c - 3\times 0.0005\right) - 41.67 \\ \cdot M_{CB} &= \frac{2EI}{L}\left(2\theta_c + \theta_B - 3\times 0.0005\right) + 41.67 \end{aligned}\right\} \rightarrow \text{General}$$

Unkowns Are θ_B and θ_c

$$\left.\begin{aligned} \cdot M_{CD} &= \frac{3EI}{10}\left(\theta_c + 0.001\right) - 62.5 \\ \cdot M_{DC} &= 0 \end{aligned}\right\} \rightarrow \text{Modefied}$$

At joint b:

$\Sigma M_b = 0 \rightarrow M_{ba} + M_{bc} = 0$

$[0.3EI\theta_B - 40.5 + 62.5] + [0.4EI\theta_B + 0.2EI\theta_c - 81 - 41.67] = 0$

$0.7EI\theta_b + 0.2EI\theta_c - 100.67 = 0$ eq.1

At joint C:

$\Sigma M_c = 0 \rightarrow M_{cb} + M_{cd} = 0$

$[0.4EI\theta_c + 0.2EI\theta_B - 81 + 41.67] + [0.3EI\theta_c + 81 - 62.5] = 0$

$0.2EI\theta_b + 0.7EI\theta_c - 20.83 = 0$ eq.2

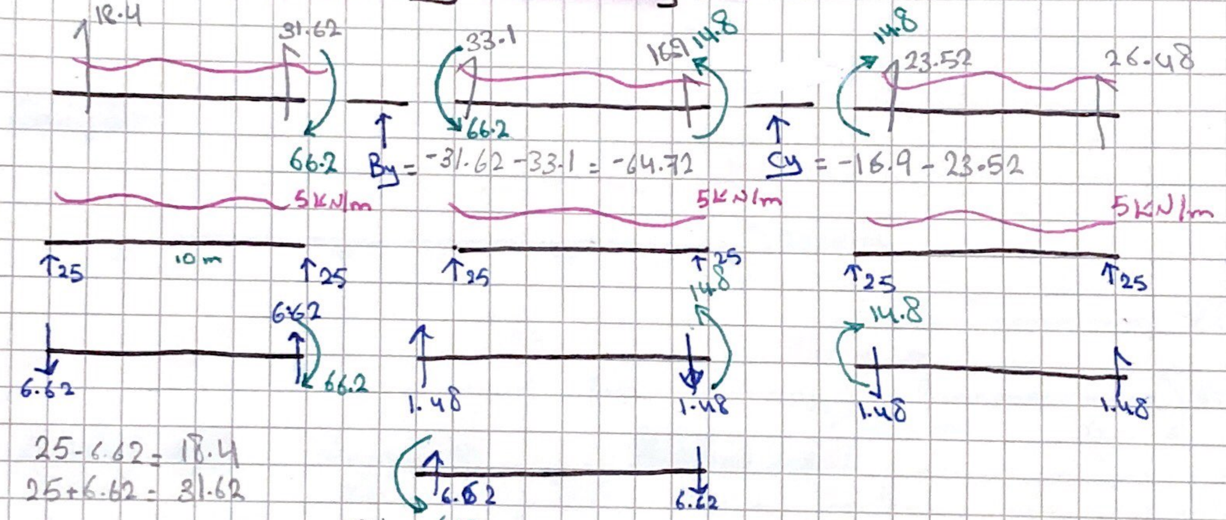
Stiffness Matrix

$$\begin{bmatrix} 0.7EI & 0.2EI \\ 0.2EI & 0.7EI \end{bmatrix}\begin{bmatrix} \theta_B \\ \theta_c \end{bmatrix} = \begin{bmatrix} 100.67 \\ 20.83 \end{bmatrix}$$

$\theta_B = 147.33/EI$ $\theta_C = -12.34/EI$

- $M_{AB} = 0$
- $M_{BA} = 66.2$
- $M_{BC} = -66.2$
- $M_{CB} = -14.8$
- $M_{CD} = 14.8$
- $M_{DC} = 0$

2 Draw shear And Bending moment diagrams

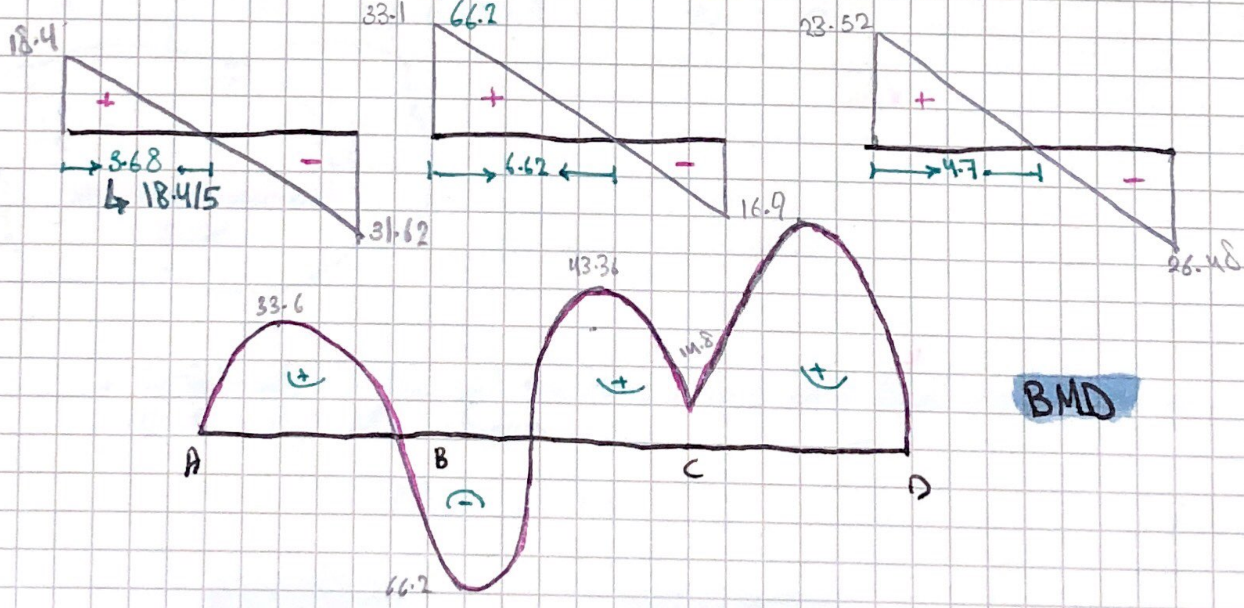


$B_y = -31.62 - 33.1 = -64.72$

$C_y = -16.9 - 23.52$

$25 - 6.62 = 18.4$

$25 + 6.62 = 31.62$



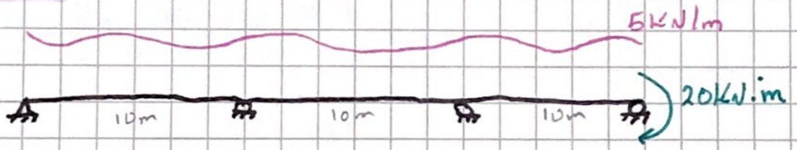
BMD

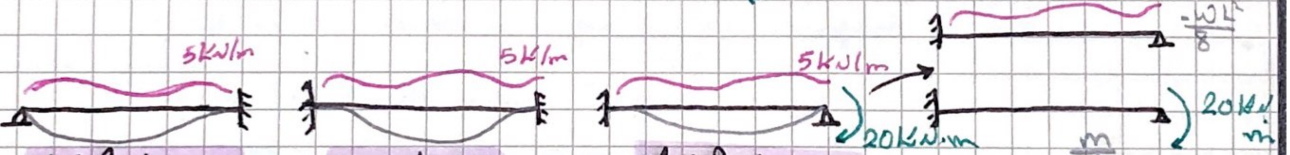
Deformed Shape

* Note:

معناها بقدر أقطع ثون وأوجد ال Shar وال Moment وبجيب نقلها
مباشرة عال support

Example:





Modefied eq.

$M_{AB} = 0$

$M_{BA} = \frac{WL^2}{8} = 62.5$

General eq.

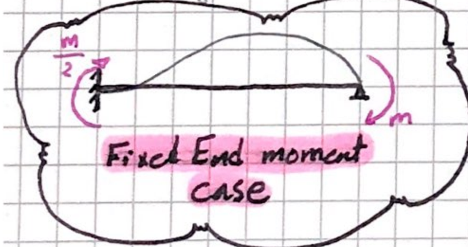
$M_{BC} = WL^2/12 = -41.67$

$M_{CB} = 41.67$

Modefied eq.

$M_{cd} = -52.5$

$M_{dc} = 0$



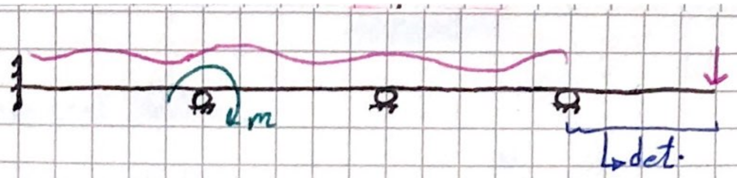
Fixed End moment case

* Settelments As the last question, And equation are the same But only M_{dc} change.

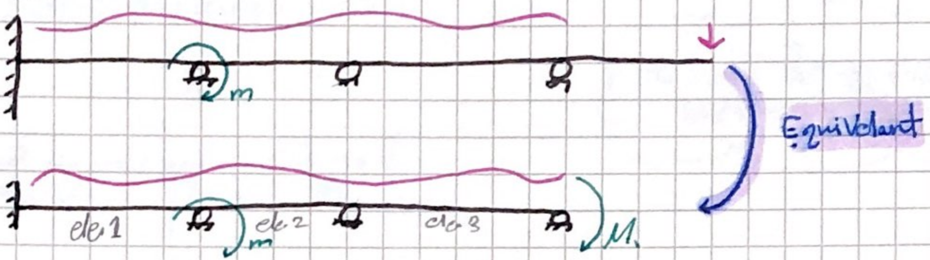
$M_{dc} = 20 \neq 0$

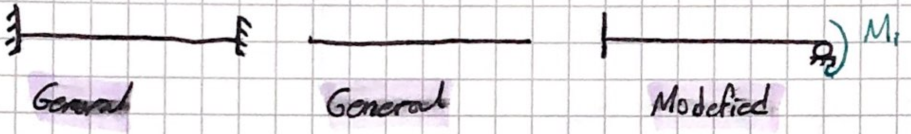
$\theta_B = 151.8/EI$ $\theta_c = -279/EI$

Assignment:



→ How Many Elements?





→ Where Do we use M_1?

FEM for An Element

→ Wher Do We use m?

it is used in the equilibrium equation of support B, where:

$$\Sigma M_B = 0 = M_{ba} + M_{bc} + -m_2$$

M_1: Is on exterior support

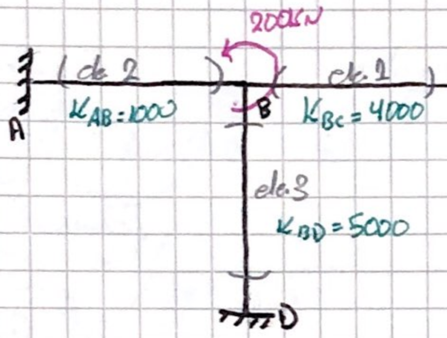
m: Is on Interior support

Lecture(15)

Displacement Metod

- → Slope-defletion equation ✓ $M_{ij} = \frac{2EI}{L}\left(2\theta_i + \theta_j - \frac{3\Delta}{L}\right) + FEM_{ij}$
- → Moment destribution method

Dist. of forces depends on the stiffness

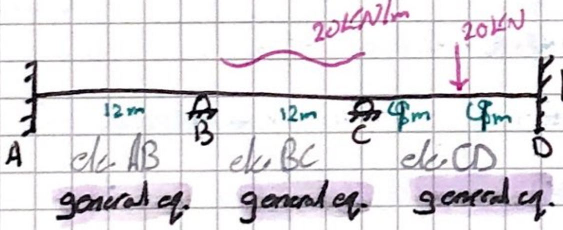


$M_{BA} = \frac{1000}{1000+5000+4000} \times 2000 = \boxed{0.1} \times 2000$
$= 200\ KN.m$

$M_{BC} = \frac{4000}{1000+5000+4000} \times 2000 = \boxed{0.4} \times 2000$
$= 800\ KN.m$

$M_{BD} = \frac{5000}{1000+5000+4000} \times 2000 = \boxed{0.5} \times 2000$
$= 1000\ KN.m$

Moment distrebution factors



EI Is constant

1 → Distribution factor

At A:
↳ DF_{AB} → The only Beam entering node A is Beam AB

At B:
↳ DF_{BA}
DF_{BC}

At C:
↳ DF_{CB}
DF_{CD}

2 → Rotational stiffness

$K_{AB} = K_{BA} = 4EI/12$

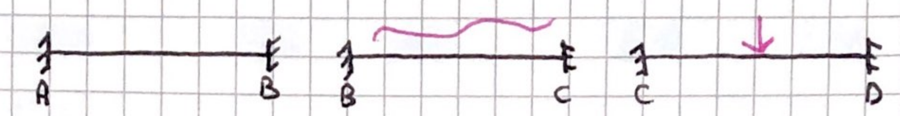
$K_{BC} = K_{CB} = 4EI/12$

$K_{CD} = K_{DC} = 4EI/8$

$DF_{AB} = \frac{K_{AB}}{K_{AB}+K_{pin}} = \frac{4EI/12}{4EI/12+0} = 1$

$DF_{BA} = \frac{K_{BA}}{K_{BA}+K_{An}} = \frac{4EI/12}{4EI/12+1} = 0.43$ $DF_{CB} = \frac{K_{CB}}{K_{CB}+K_{Pin}} = \frac{4EI/12}{}$

$DF_{BC} = \frac{K_{BC}}{K_{BC}+K_{Pin}} = \frac{4EI/12}{4EI/12+1} = 0.57$ $DF_{CD} = \frac{K_{CD}}{K_{CD}+K_{fixity}} = \frac{4EI/8}{4EI/8+\infty} =$



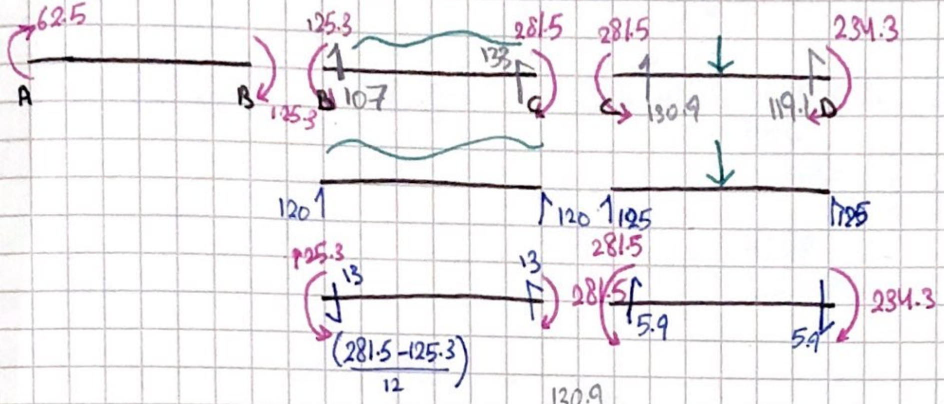
$FEM_{AB} = \cancel{A}FEM_{BA} = 0$ $FEM_{BC} = \frac{-20(12)}{12^2} = 240$ $FEM_{CD} = -250$

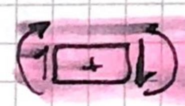
$FEM_{CB} = \frac{20(12)}{12^2} = 240$ $FEM_{DC} = 250$

		$\Sigma M_B = 0$		$\Sigma M_C = 0$		
Joint	A	B		C		D
END moments	AB	BA	BC	CB	CD	DC
DF	0	0.5	0.5	0.4	0.6	0
FEM	0	0	-240	240	-250	250
		0.5(240)	(0.5)(240)	(0.4)(10)	(0.6)(10)	
		120	120	4	6	
	(0.5)(120) = 60		(4)(0.5) = 2	120(1/2) = 60		(6)(0.5) = 3
		-2(0.5) = -1	(-2)(0.5)	-24	-36	
C.O Carry over	-0.5		-1			
			-12	-0.5		-18
		6	6	0.2	0.3	
	3		0.1	3		0.15
dist.		0.003	0.05	-1.2	-1.8	
	-0.025		-0.6	-0.025		-0.9
		0.3	0.3	0.01	0.015	
	62.475	125.25	-125.25	281.485	-281.485	234.25

Error = ∓0.3 KN·m

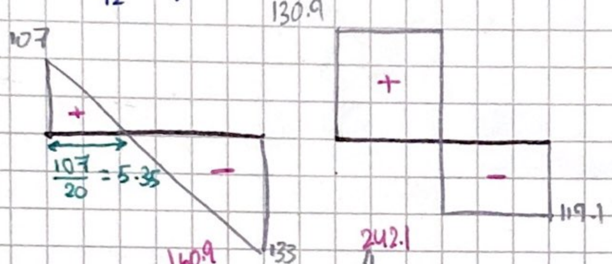
AB	BA	BC	CB	CD	DC
62.5	125.3	-125.3	281.5	-281.5	234.3



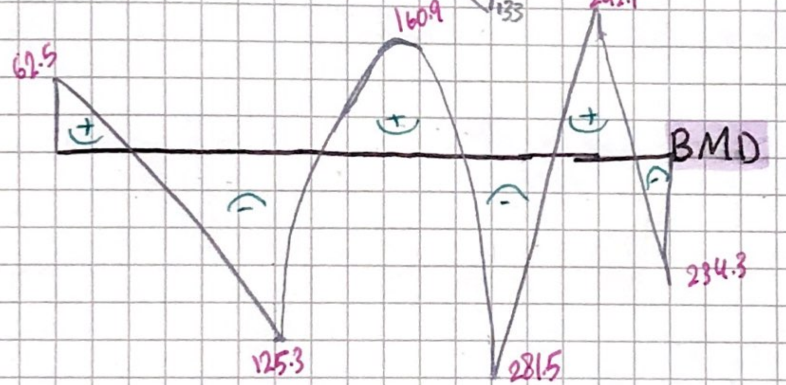


$V_C = 130$

$V_D = -119.1$

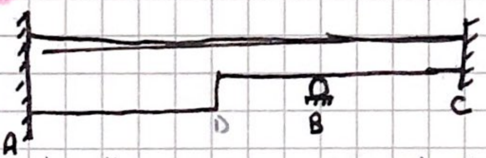


SFD



BMD

Note:



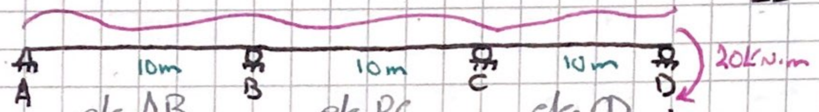
In this case we assume A Node D and solve for three elements instead of two.

We can Also derive New equations As before.

Deformed Shape

Know we will solve the slope deflection method problem using Moment-dist. Method, we must have same numbers.

EI Is constant



el. AB
Modified

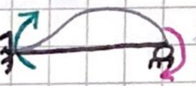
K_{AB}
$\overset{\shortparallel}{\frac{3EI}{L}}$

el. BC
General

K_{BC}
$\overset{\shortparallel}{\frac{4EI}{L}}$

el. CD
Modified

K_{CD}
$\overset{\shortparallel}{\frac{3EI}{L}}$



$FEM_{BA} = 62.5$

$FEM_{BC} = {}^{-}41.67$
$FEM_{CB} = 41.67$

$FEM_{CD} = {}^{-}62.5+10$
$= {}^{-}52.5$
$FEM_{DC} = 20KN.m$

→ Distrebution factors

At A:

$\hookrightarrow DF_{AB} = \frac{K_{AB}}{K_{AB} + \underset{0}{K_{pin}}} = 1$

At B:

$\hookrightarrow DF_{BA} = \frac{K_{AB}}{K_{AB} + K_{BC}} = \frac{3EI/10}{3EI/10 + \frac{4EI}{10}} = 0.43$

$\hookrightarrow DF_{BC} = \frac{K_{BC}}{K_{BC} + K_{CD}} = \frac{4EI/10}{4EI/10 + 3EI/10} = 0.57$

At C:

$\hookrightarrow DF_{CB} = \frac{K_{BC}}{K_{BC} + K_{CD}} = 0.57$

$\hookrightarrow DF_{CD} = \frac{K_{CD}}{\Sigma K} = 0.43$

At D:

$\hookrightarrow DF_{DC} = \frac{K_{CD}}{\Sigma K} = \frac{K_{CD}}{K_{CD} + 0} = 1$

What About settlments? How to deal with sittelments?

Slope-deflection equation

$$\frac{2EI}{L}\left(2\theta_i + \theta_j - \frac{3\boxed{\Delta}}{L}\right) + \underbrace{FEM}_{\text{loading}}$$

settlement

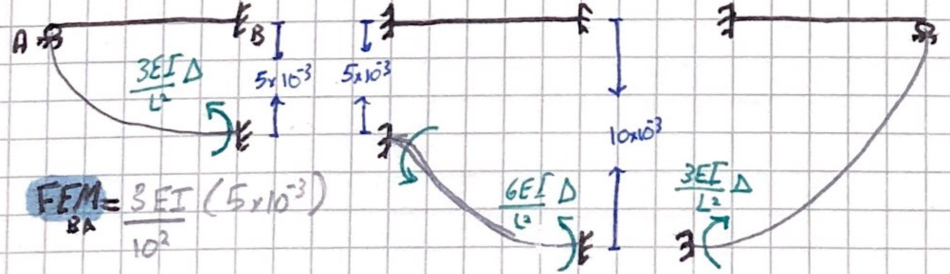
Moment dist. method

FEM

loading settlment

* Note:
In Moment dist. method, the settlment Shows its effect in FEM, which absolutly different than slope-deflection equation

FEM (II) settlment



$FEM_{BA} = \frac{3EI}{10^2}(5\times10^{-3})$

$= -40.5 KN.m$

$FEM_{BC} = {}^{-}81$

$FEM_{CB} = {}^{-}81$

$FEM_{CD} = \frac{3EI}{10^2}(10\times10^{-3})$

$= +81$

$FEM_{AB} = 62.5 - 40.5 = 22 KN.m$ $FEM_{CB} = 41.67 - 81 = -39.33 KN.m$

$FEM_{BC} = -41.67 - 81 = -122.67 KN.m$ $FEM_{CD} = -52.5 + 81 = 28.5 KN.m$

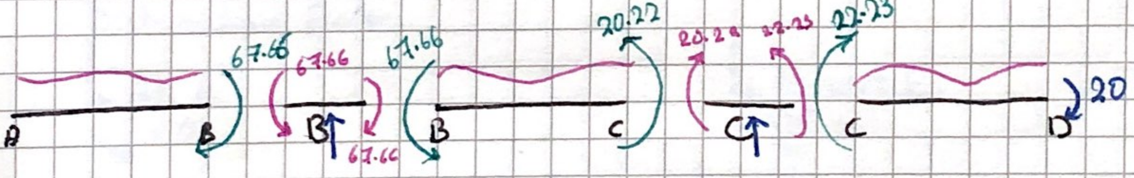
Joint	A	B		C		D
End moment	AB	BA	BC	CB	CD	DC
D.F.	1	0.43	0.57	0.57	0.43	1
FEM		22	-122.67	-39.33	28.5	
		43.28	57.38	6.17	4.65	
Dist			3.085	28.69		
C.O		-1.33	-1.76	-16.35	-12.34	
			-8.17	-0.88		
Dist		3.52	4.66	0.5	0.38	
C.O			0.25	2.33		
		-0.107	-0.143	-1.33	-1.281	
Dist						
C.O			-0.66	-0.07		
	0	0.284	0.376	0.04	0.03	20
		67.65	-67.66	-20.23	+20.22	

$M_{BA} + M_{BC} + M_{corr} = \boxed{0}$

If we have consantrated moment, 0 becomes this Moment

وهاد المومنت بس بيبين

في First dis. بعديها خلص



Now, we found Shear, BMD

Lecture (19)

Frames in Moment distrubution

Non-Sway frame

↓

Means that there is No horizontal drift.

يعني ما عندي حركة أفقية

↓

- When A frame is Non-sway:
  - [1] Symmetric system in
    - geometry
    - supports
    - Cross-section
    - loading
  - [2] propped from swaying movement

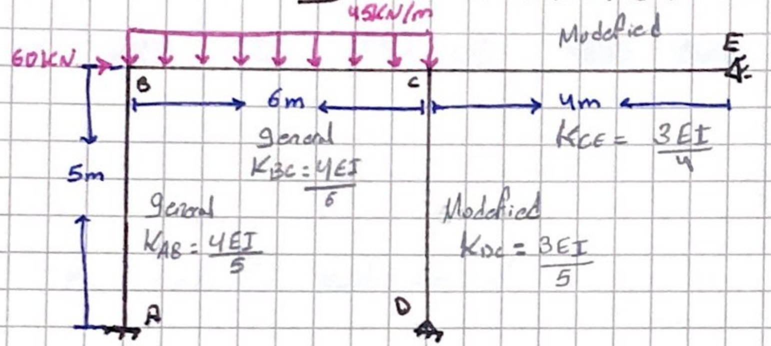
Sway Frame

↓

when [1] or [2] is not true, its a Sway frame.

---

*Ex: Solve Frame using Moment distr. method



① # Elements
② Rotational stiffness
③ Distribution factor
④ Fix -end. moment

→ Distribution Factor

↳ At A:

$DF_{AB} = 0$,

↳ At B:

$$DF_{BA} = \frac{4EI/5}{4EI/5 + 4EI/6} = 0.545$$

$$DF_{BC} = 1 - 0.545 = 0.455$$

↳ At C:

$$DF_{CB} = \frac{4EI/6}{4EI/6 + 3EI/5 + 3EI/4} = 0.330$$

$$DF_{CD} = \frac{4EI/5}{4EI/6 + 3EI/5 + 3EI/4} = 0.298$$

$$DF_{CE} = 1 - 0.298 - 0.330 = 0.372$$

↳ At D:

$DF_{DC} = 1$

↳ At joint E:

$DF_{EC} = 1$

IN our Question

45KN/m → member load
6 KN → Joint load

Member load show it effect in the FEM

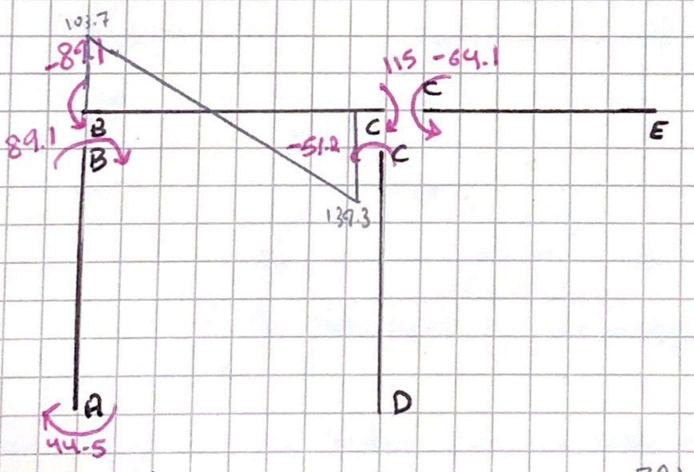
Joint load show it effect in equilibrium equation

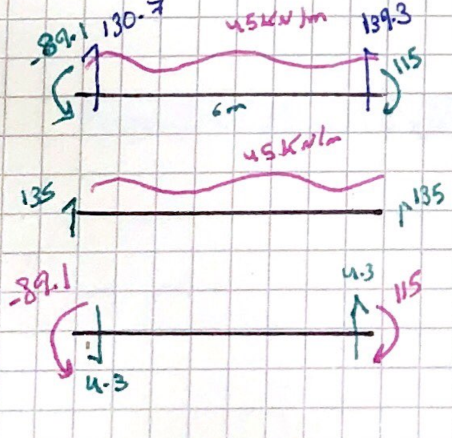
$$FEM_{BC} = \frac{wL^2}{12} = \frac{-45(6)^2}{12} = -135 \text{ KN.m}$$

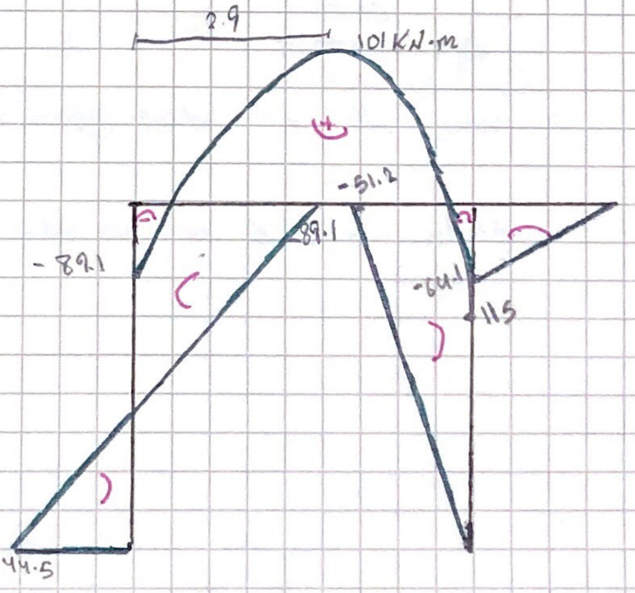
$$FEM_{CB} = \frac{wL^2}{12} = \frac{45(6)^2}{12} = 135 \text{ KN.m}$$

joint	A	B		C			D	E
Member	A B	B A	B C	C B	C D	C E	D C	E C
D F	0	0.545	0.455	0.330	0.298	0.372	1	1
FEM	0	0	-135	135	0	0		
Dist.		73.6	61.4	-44.6	-40.2	-50.2		
CO	36.8		-22.3	30.7				
Dits		12.2	10.1	-10.1	-9.1	-11.5		
CO	6.1		-5.1	5.1				
Dist.		2.8	2.3	-1.7	-1.5	-1.9		
CO	1.4		-0.8	1.2				
Dist.		0.4	0.4	-0.4	-0.4	-0.4		
C.O	0.2		-0.2	0.2				
Dist.		0.1	0.1	-0.1	0	-0.1		
Σ M	44.5	89.1	-89.1	115	-51.2	-64.1	0	0

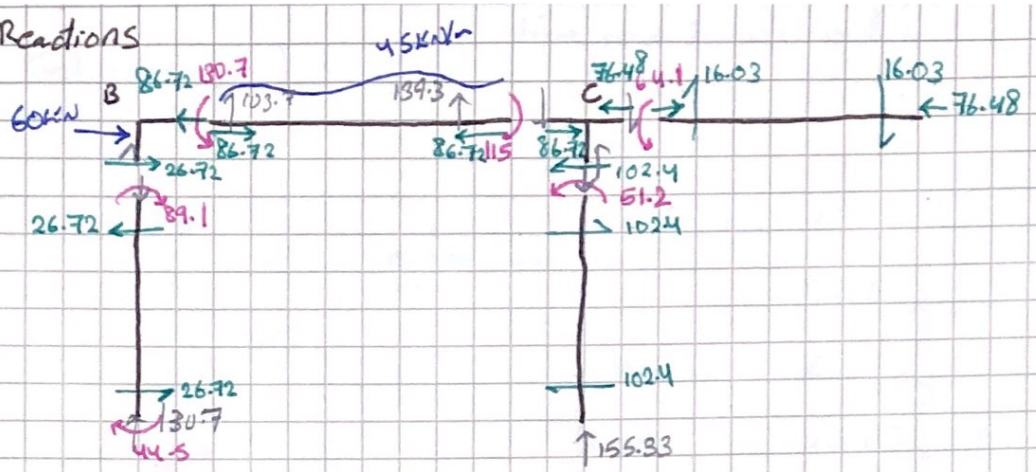
My joints in equilebrium ✓



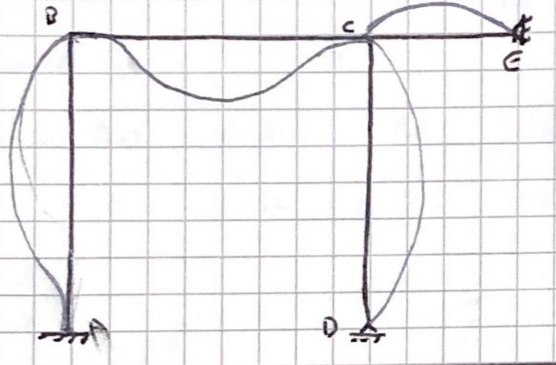




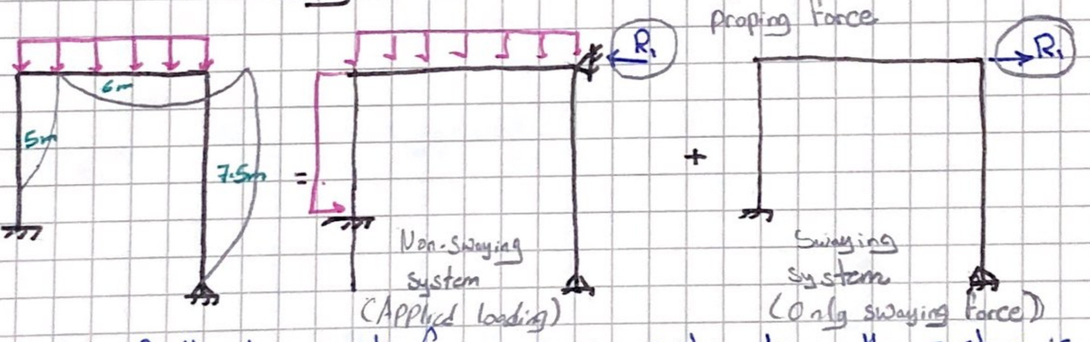
→ Reactions



→ Deformed shape:



Sway Frames (Short introduction)



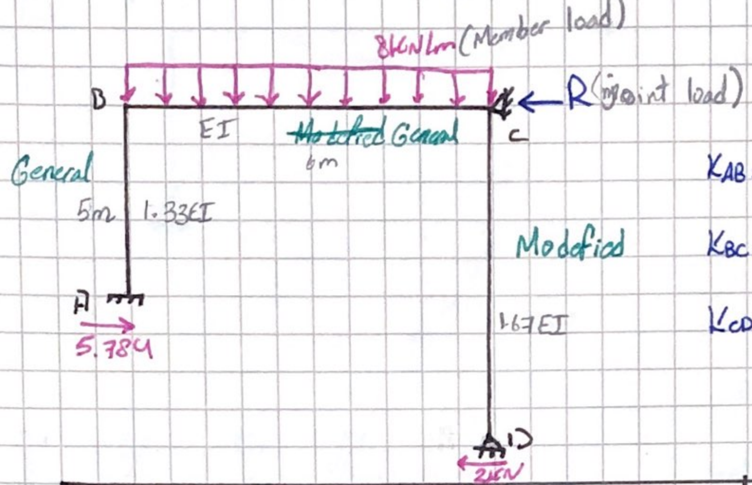
Since Non of the two points for non-swaying system true, the system is A swaying system

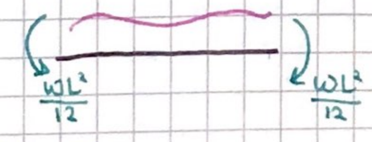
For solving swaying system, we use super position As shown above

$$\text{Moment} = M_I + (R/R')\, M_{II}$$

From phase I we want to know: M_I
R (Value & direction)

First: (Non-sway)





$K_{AB} = \frac{(4)(1.33EI)}{5}$

$K_{BC} = \frac{(4)(EI)}{6}$

$K_{CD} = \frac{3(1.67EI)}{7.5m}$

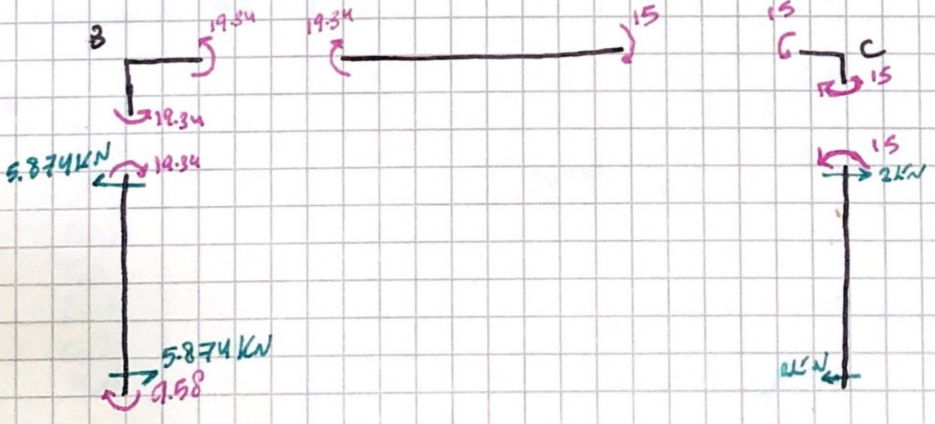
joint	A	B		C		D
Member	AB	BA	Bc	CB	CD	Dc
DF	0	0.615	0.385	0.5	0.5	1
FEM			-24	24		
Dist		14.76	9.24	-12	-12	
CO	7.38		-6	4.62		
Dist		3.69	2.31	-2.31	-2.31	
CO	1.84		-1.16	1.16		
Dist		0.713	0.447	-0.58	-0.58	
CO	0.357		-0.29	0.224		
Dist.		0.18	0.11	-0.11	-0.11	
ΣM	9.58	19.34	-19.34	15	-15	0

$DF_{BA} = K_{AB} / \Sigma K$

$DF_{BC} = K_{BC} / \Sigma K$

$DF_{CB} = K_{BC} / \Sigma K$

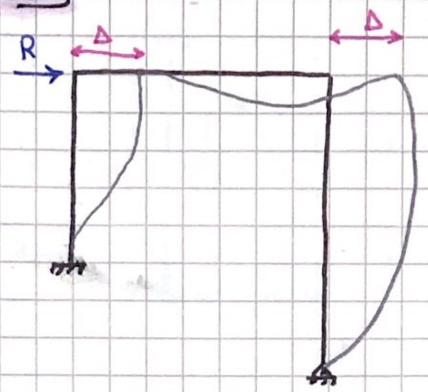
$DF_{CD} = K_{CD} / \Sigma K$



$R = 5.784 - 2 = 3.784$ KN ←

Case II : Sway

(Swaying force)

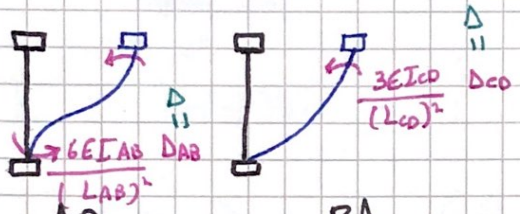


Is Δ ~~a~~ Known or unknown Value?
unkown, but can be calculated using Virtual work Method

$$\Delta = \int \frac{mM\,dx}{EI}$$

m, M, are unkowns or Known Values?
I can't know m, M, here becouse its an indeterminate system.

what to do?
we are going to assume a Value of Δ.

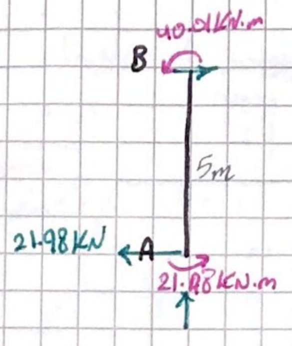


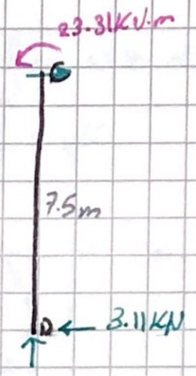
AB	BA	CD
$\frac{6(1.33\cancel{EI})}{5^2}\cancel{\Delta}$	$\frac{6(1.33\cancel{EI})\cancel{\Delta}}{5^2}$	$\frac{3\cancel{EI}}{(7.5)^2}\cancel{\Delta}$
0.3192	0.3192	0.0891

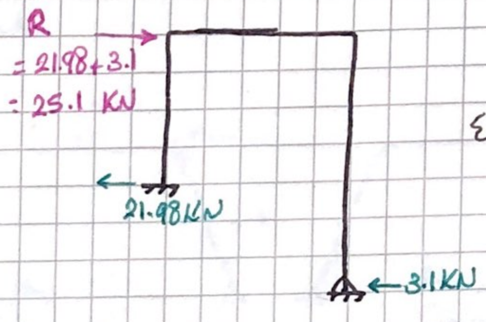
Assume $FEM_{AB} = 100$ kN.m

Ratio	$\frac{0.3192}{0.3192} = 1$	$\frac{0.3192}{0.3192} = 1$	$\frac{0.0891}{0.3192} = 0.2778$
FEM	100 (C.C.W)	100 (C.C.W)	27.28 (C.C.W)

joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	0	0.615	0.385	0.5	0.5	1
FEM	-100	-100	0	0	- 27.28	
Dist.		61.5	38.5	13.89	13.89	
CO	30.75		6.94	19.25		
Dist.		-4.27	-2.67	-9.625	-9.625	
CO	-2.14		-4.81	-1.34		
Dist.		2.96	1.85	0.67	0.67	
CO	1.48		0.33	0.92		
Dist.		-0.2	-0.13	-0.46	-0.46	
ΣM	-69.91	-40.01	40.01	23.31	-23.31	0







$\Sigma F_x = 0$

Phase I

M_I

$R = 3.784 KN$

Phase II

M_{II} (Assumed FEM)

$\grave{R} = 25.1$ KN

لما فرضت FEM طلع معي $\grave{R}=25.1KN$، بينما هي لازم تكون 3.784KN

← شو لازم أسوي بال FEM إلي فرضته؟ أصغره، أو أكبره؟

لازم نصغرها، عشان العلاقة بين المومنت و القوى طردية

← احنا على أي فرض بنشتغل؟

على فرض linear / elasticity

We can scale M_{II} In phase II in the ~~mom~~ R Reference $\left[\frac{R}{\grave{R}}\right] M_{II}$

$$M = M_I + \frac{R}{\grave{R}}(M_{II})$$

$$M_{AB} = (M_{AB})_I + \left(\frac{R}{\grave{R}}\right)(M_{AB})_{II}$$

$$M_{AB} = 9.58 + \left(\frac{3.78}{25.1}\right)(-69.91) = -0.948 KN.m$$

$$M_{BA} = 19.34 + \left(\frac{3.78}{25.1}\right)(-40.01) = 13.3 KN.m$$

$$M_{BC} = -19.34 + \left(\frac{3.78}{25.1}\right)(40.01) = -13.3 KN m$$

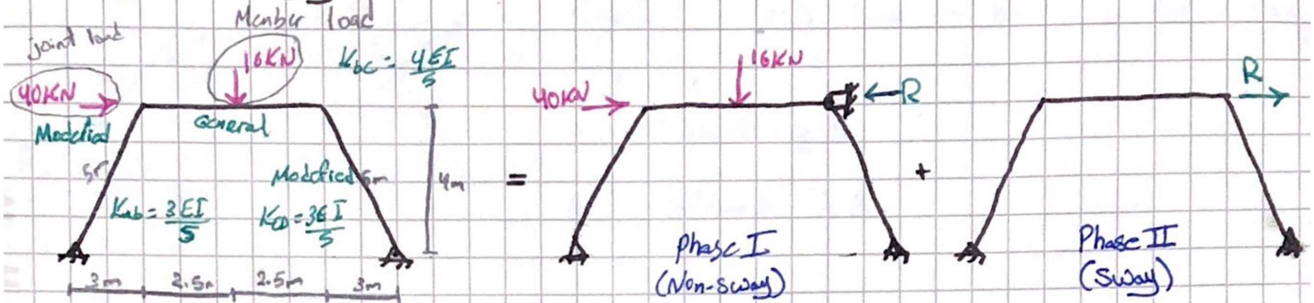
$$M_{CB} = 15 + \left(\frac{3.78}{25.1}\right)(23.31) = 18.5 KN.m$$

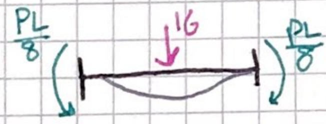
$$M_{CD} = -15 + \left(\frac{3.78}{25.1}\right)(-23.31) = -18.5 KN.m$$

→ Better to put in a table

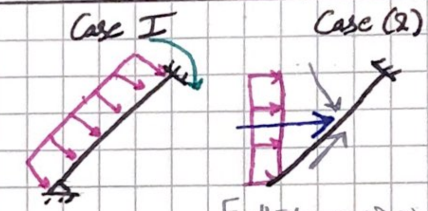
Lecture (21)

- Analysis of Swaying frame with inclined coulmns:



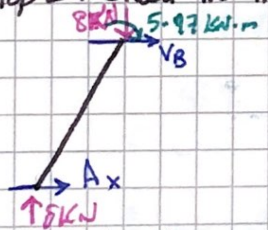


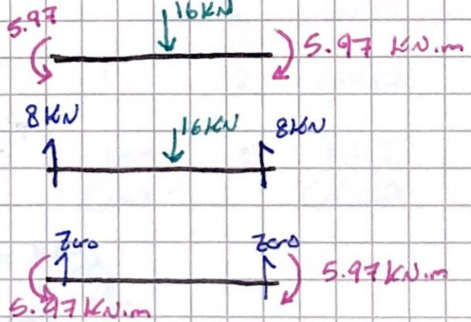
Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	1	0.429	0.571	0.571	0.429	1
FEM			-10	10		
Dist.		4.29	5.71	-5.71	-4.29	
CO			-2.86	2.86		
Dist.		1.23	1.63	-1.63	-1.23	
C.O			-0.82	0.82		
Dist.		0.35	0.47	-0.47	-0.35	
CO			-0.24	0.24		
Dist		0.1	0.13	-0.13	-0.1	
ΣM	0	5.97	-5.97	+5.97	-5.97	0



بوجد محصلة ال F
و بحللها بينتج اناتج
8 طول ال member

Step I: Shear in the beam:



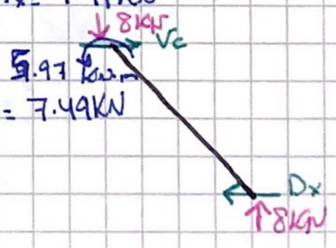


Step (II): Shear in the beam is Vertical to ading on coulmn

$\Sigma M_B = 0 \rightarrow (8)(3) - A_x(4) + 5.97 = 0 \rightarrow A_x = 7.49 KN$

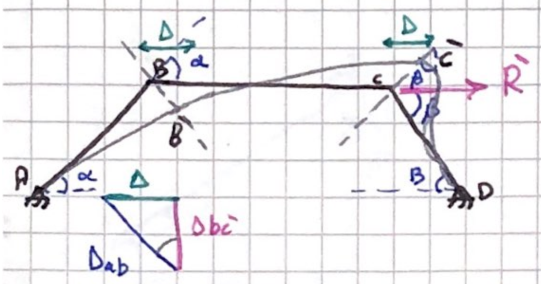
In the same way

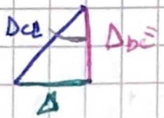
$\Sigma M_C = 0 \rightarrow 3C_y + -5.97 - 4D_x = 0 \rightarrow D_x = 7.49 KN$



$R = 40 + 7.49 - 7.49 = 40 KN$

Phase II (Swing frame)





*D: have to be ⊥ to the longitudinal axis

$\Delta_{cd} = \Delta / \sin\beta = \frac{5}{4}\Delta$

$\Delta_{bc''} = \Delta / \tan\beta = \frac{3}{4}\Delta$

$\Delta_{ab} = \Delta / \sin\alpha = \frac{5}{4}\Delta = 1.25\Delta$

$\Delta_{bc'} = \Delta / \tan\alpha = \frac{3}{4}\Delta$

$\boxed{\Delta_{bc} = \Delta_{bc'} + \Delta_{bc''} = 1.5\alpha}$

→ Set FEM

FEM-ba : FEM-bc : FEMcb : FEM-cd

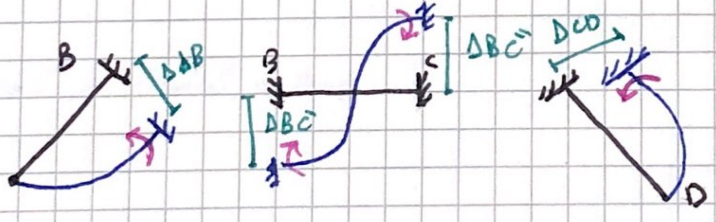
$3EI\,\Delta_{ab} / L_{ab}^2 : 6EI\,\Delta_{bc} / L_{cb}^2 : 6EI^{*}\,\Delta_{bc} / L_{cb}^2 : 3EI\,\Delta_{cd} (L_{cd})^2$

0.15 : 0.36 : 0.36 : 0.15

0.417 : 1 : 1 : 0.417

41.7 : 100 : 100 : 41.7

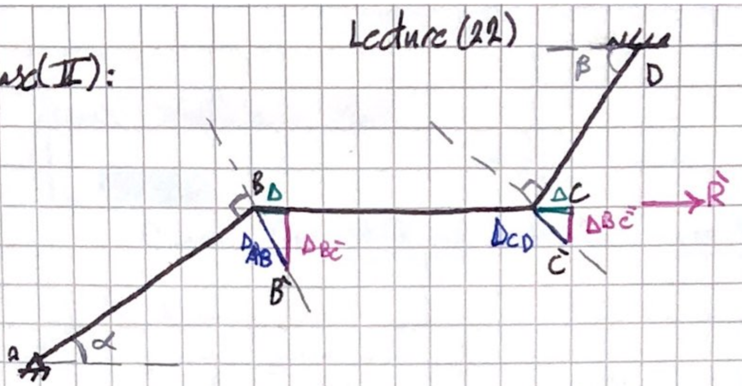
C.C.W C.W C.W C.C.W



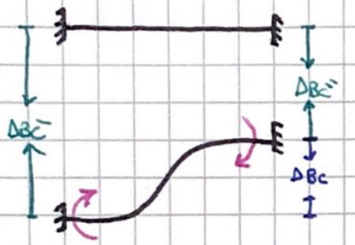
(Solution continued in the slides)

Lecture (22)

Phase(II):



To Find Δ_{BC}



$\Delta_{BC} = \Delta_{B\grave{C}} - \Delta_{BC''}$

→ Structure Analysis:

- Force method
- Displacement method
  - → Slope-deflection equations } → Flexural systems
  - → Moment distribution method } → Flexural systems
  - → Direct stiffness method
    - → Finite element method

Direct stiffness method:

↳ 1st step:

Derive the stiffness matrix of a structure element

عكس ال flixebility

Is A force due to unit deformation

$q = [K] d$

q → force factor; $[K]$ → Matrix; d → displacement factor

- Coulmn-beam element
- plate element (slab)
- Cable / Rod element
- Shell element
- Beam element
- Solid element (3D)

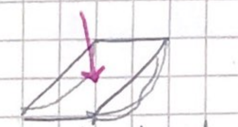
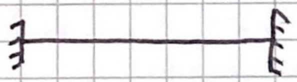


Plate element

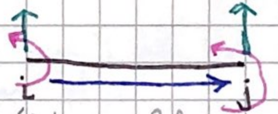
(out of plane and in plan)
Shell element

Solid element
(الثلاث أبعاد مجسمة)

→ Derive the stiffness matrix for A Beam element

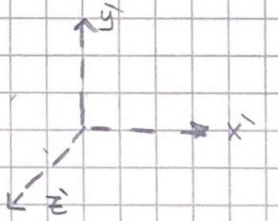


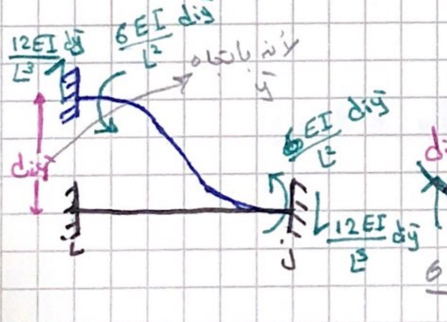
Kinematically det.

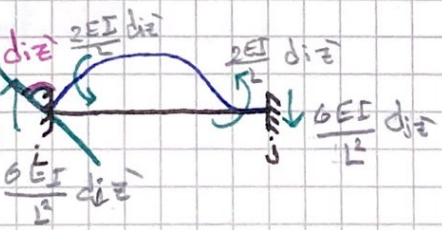


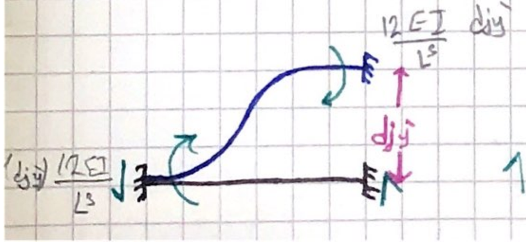
(2 degree of freedom for each node)

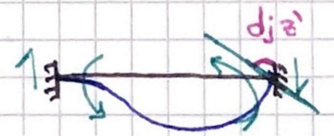
→ Sign convention for direct stiffness method.
(Vertical upward, C.C.W)











$$q_{iy'} = \frac{12EI}{L^3} d_{iy'} + \frac{6EI}{L^2} d_{iz'} - \frac{12EI}{L^3} d_{jy'} + \frac{6EI}{L^2} d_{jz'}$$

$$q_{iz'} = \frac{6EI}{L^2} d_{iy'} + \frac{4EI}{L} d_{iz'} - \frac{6EI}{L^2} d_{jy'} + \boxed{\frac{2EI}{L}} d_{jz'}$$

→ Moment at i coused by rotation at j

$$q_{jy'} = -\frac{12EI}{L^3} d_{iy'} - \frac{6EI}{L^2} d_{iz'} + \boxed{\frac{12EI}{L^3}} d_{jy'} - \frac{6EI}{L^2} d_{jz'}$$

→ shear at j coused by Δ at j

$$q_{jz'} = \frac{6EI}{L^2} d_{iy'} + \frac{2EI}{L} d_{iz'} - \frac{6EI}{L^2} d_{jy'} + \frac{4EI}{L} d_{jz'}$$

→ shear at i coused by rotation at j

$$\begin{bmatrix} q_{iy'} \\ q_{iz'} \\ q_{jy'} \\ q_{jz'} \end{bmatrix} = \begin{array}{cccc} & i & & j \\ d_{iy'} & d_{iz'} & d_{jy'} & d_{jz'} \\ \hline \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \boxed{\frac{6EI}{L^2}} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & \frac{2EI}{L} & \frac{6EI}{L^2} & \frac{4EI}{L} \\ \boxed{\frac{6EI}{L^2}} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{array} \begin{bmatrix} d_{iy'} \\ d_{iz'} \\ d_{iy'} \\ d_{iz'} \end{bmatrix} \begin{array}{c} i \\ \\ j \end{array}$$

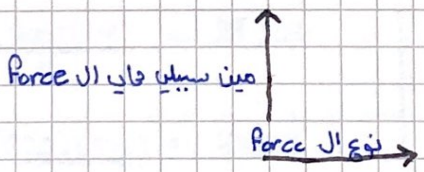
[q] [K] [d]

Force Vector Stiffness matrix disp. Vector

→ Moment at j coused by Δ at i

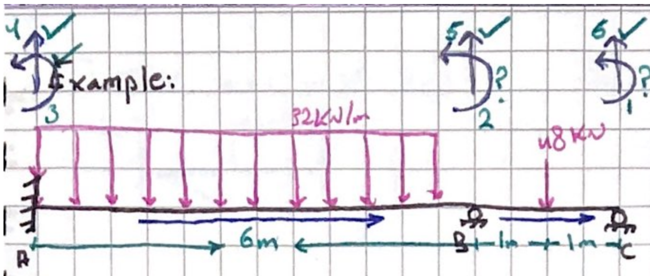
Pink block: These Values represent , Shears and moments (forces) at i th node coused by the different deformation at i

Green block: Forces at j coused by deformations at i



$$q = K d + \boxed{q_0} \rightarrow \text{Fix end forces}$$

Example:



∴ System is Kine-indet to 2^{nd} degree
θ_B , θ_C

1 Discritize the system to elements
we have 2 elements AB, BC
AB → 1
BC → 2

2 How Many nodes do we have?
elements+1 = 3nodes
A → ①
B → ②
C → ③

(No. of Degree of Freedom for Beam node = 2)

3 Decide the starting node and Ending node
A → B ① → ②
B → C ② → ③

4 Take element 1 and find stiffness matrix

$$\frac{12EI}{L^3} = \frac{(12)(200)(216)}{(6)^3} = 2400$$

$$\frac{6EI}{L^2} = \frac{6(200)(216)}{(6)^2} = 7200$$

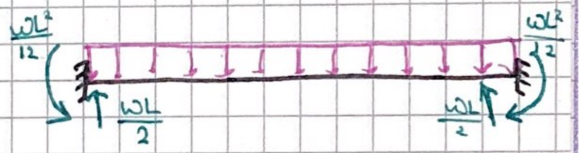
$$\frac{4EI}{L} = \frac{(4)(200)(216)}{6} = 28800$$

$$\frac{2EI}{L} = \frac{(2)(200)(216)}{6} = 14400$$

$$K = \begin{array}{c|cccc|c} & 4 & 3 & 5 & 2 & \\ \hline & 2400 & 7200 & -2400 & 7200 & 4 \\ & 7200 & 28800 & -7200 & 14400 & 3 \\ & -2400 & -7200 & 2400 & -7200 & 5 \\ & 7200 & 14400 & -7200 & 28800 & 2 \end{array}$$

لازم ننتبه عالاشارات

5 Find q_o for 1 :



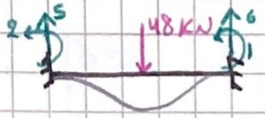
$$q_{o-4} = \frac{(32)(6)}{2} = 96$$

$$q_{o-3} = \frac{(32)(6^2)}{12} = 96$$

$$q_{o-5} = \frac{(32)(6)}{2} = 96$$

$$q_{o-2} = \frac{(32)(6)^2}{12} = -96$$

6 Take member **2** And find stiffness matrix



$$\frac{12EI}{L^3} = \frac{(12)(200)(216)}{2^3} = 64800$$

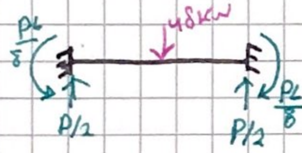
$$\frac{6EI}{L^2} = \frac{6(200)(216)}{2^2} = 64800$$

$$\frac{4EI}{L} = \frac{(4)(200)(216)}{2} = 86400$$

$$\frac{2EI}{L} = \frac{(2)(200)(216)}{2} = 43200$$

$$K_2 = \begin{array}{c|cccc|c} & 5 & 2 & 6 & 1 & \\ \hline & 64800 & 64800 & -64800 & 64800 & 5 \\ & 64800 & 86400 & -64800 & 43200 & 2 \\ & -64800 & -64800 & 64800 & -64800 & 6 \\ & 64800 & 43200 & -86400 & 86400 & 1 \end{array}$$

7 Find q_o for member **2**



$q_{o-5} = 24$

$q_{o-2} = 12$

$q_{o-6} = 24$

$q_{o-1} = -12$

Can be:
1) Reaction
2) Applied loading force

Nodal forces for system

$$Q = KD + Q_o$$

8 write system stiffness matrix: (size = #DOF × #DOF)

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix} = \begin{array}{cccccc|c} 1 & 2 & 3 & 4 & 5 & 6 & \\ \hline 86400 & 43200 & 0 & 0 & 64800 & -64800 & 1 \\ 43200 & 115200 & 14400 & 7200 & 57600 & -64800 & 2 \\ 0 & 14400 & 28800 & 7200 & -7200 & 0 & 3 \\ 0 & 7200 & 7200 & 2400 & -2400 & 0 & 4 \\ 64800 & 57600 & -7200 & -2400 & 67200 & -64800 & 5 \\ -64800 & -64800 & 0 & 0 & -64800 & 64800 & 6 \end{array} \begin{bmatrix} D_1 \\ D_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -12 \\ -84 \\ 96 \\ 96 \\ 120 \\ 24 \end{bmatrix}$$

$$K_{①}(2,5) + K_{②}(2,5) = K(2,5)$$

بعدما أخلص تعباية الكي بضل
فاضي بحطهم صفر

The effective matrix here is:

$$\begin{bmatrix} 86400 & 93200 \\ 43200 & 115200 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0+12 \\ 0+84 \end{bmatrix}$$

$$\left.\begin{aligned} 12 &= 86400 D_1 + 43\,200 D_2 \\ 84 &= 43200 D_1 + 115200 D_2 \end{aligned}\right] \rightarrow \begin{aligned} D_1 &= 0.2778 \times 10^{-3} \text{ m} \\ D_2 &= 0.8333 (10^{-3}) \text{ m} \end{aligned}$$

Reactions → Equations of Forces (on a system level بنحل)

Force members → I go back to the equation $q_1 = k_1 d + q_0$ and solve for it