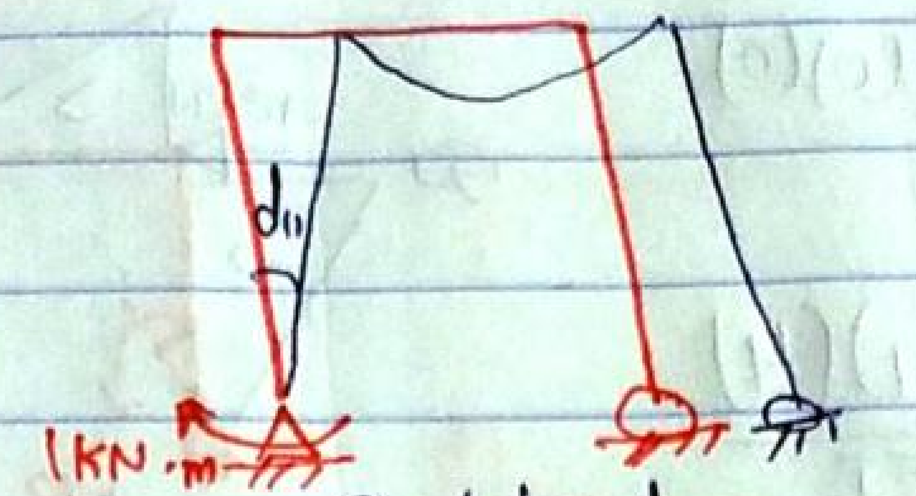
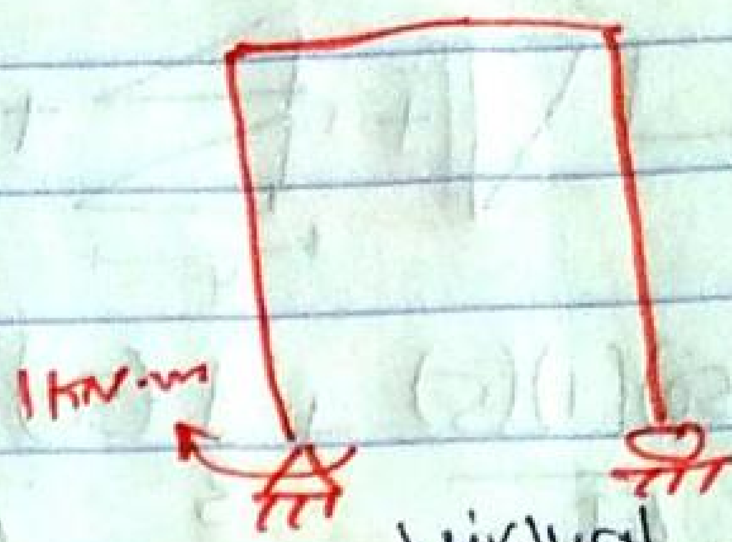
→ Find d_{11}



Real load system
M_1



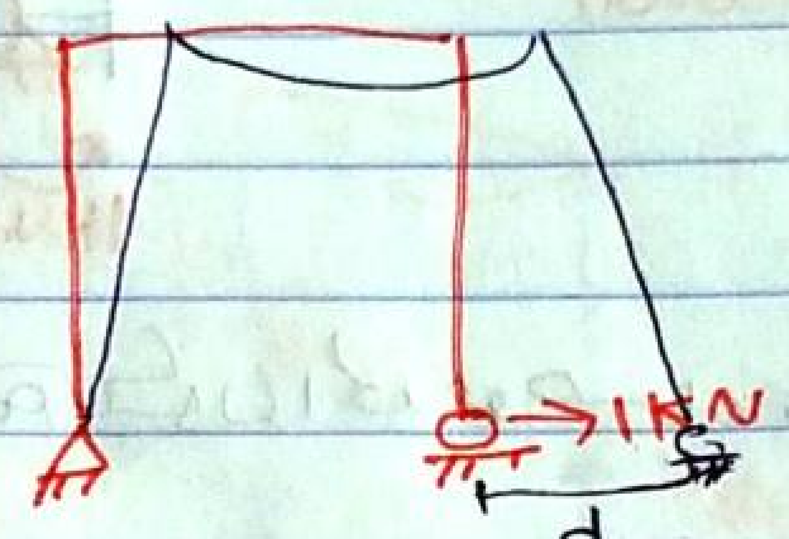
virtual load system
M_1

$$d_{11} = \int \frac{M_1^2}{EI}\,dx$$

$$= \frac{(1)(1)(5)}{EI} + \frac{1}{3}\frac{(1)(1)(4)}{EI}$$

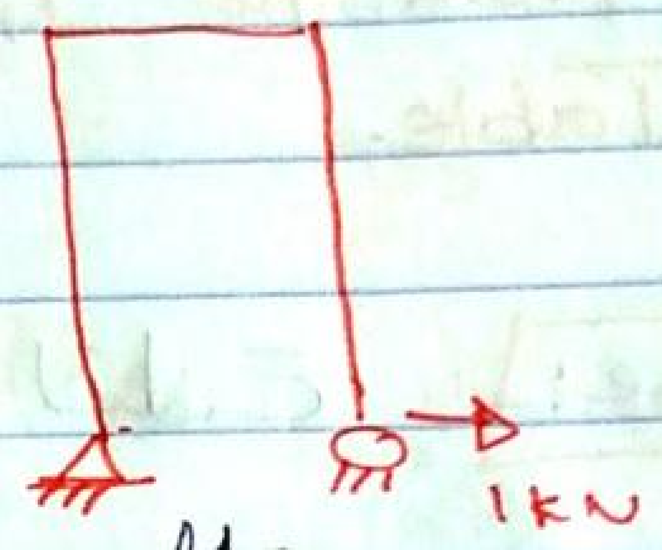
$$= \frac{6.33}{EI}$$

→ Find d_{22}



M_2

Real load system



M_2

virtual load system

$$d_{22} = \int \frac{M_2^2}{EI}\,dx$$

$$= 2\left[\frac{1}{3}\frac{(5)(5)(5)}{EI}\right] + \frac{(5)(5)(4)}{EI}$$

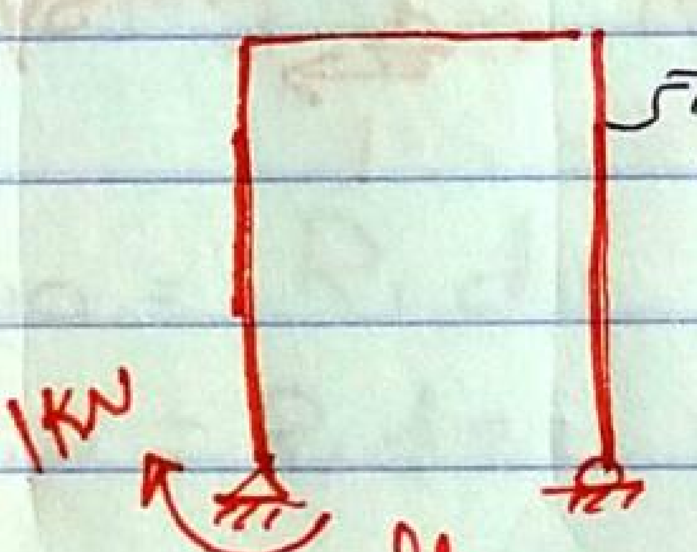
$$=$$

→ Find d_{12} = rotation at A caused by horizontal force at D.

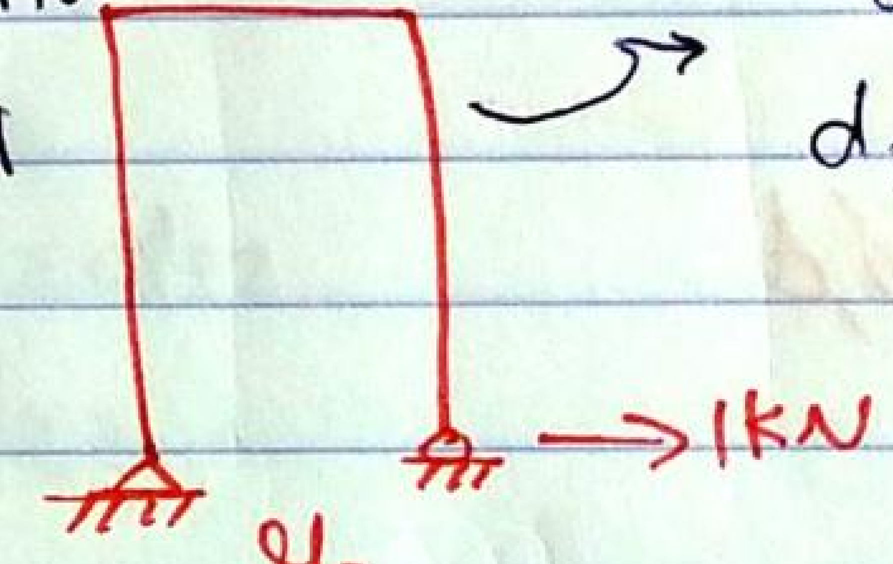
d_{21} = horizontal displa. at D caused by Moment at A.

$d_{12} \rightarrow$ virtual
$d_{21} \rightarrow$ real

$d_{12} \rightarrow$ Real
$d_{21} \rightarrow$ virtual



M_1



M_2

$$\therefore d_{12} = \int \frac{M_1 M_2}{EI}\,dx \quad , \quad d_{21} = \int \frac{M_2 M_1}{EI}\,dx$$

$$\therefore d_{12} = d_{21}$$

$$d_{12} = d_{21} = \frac{1}{2}\frac{(1)(5)(5)}{EI} + \frac{1}{2}\frac{(1)(5)(4)}{EI}$$

$$= \frac{22.5}{EI}$$

Maxwell's theorem of reciprocal displacements
* In general $d_{ij} = d_{ji}$

→ Equations solutions:-

$$-D_{10} = R_1 d_{11} + R_2 d_{12}$$
$$-D_{20} = R_1 d_{21} + R_2 d_{22}$$

$$\begin{Bmatrix} \frac{-191.67}{EI} \\ \frac{-916.67}{EI} \end{Bmatrix} = \begin{bmatrix} 6.33/EI & 22.5/EI \\ 22.5/EI & 183.33/EI \end{bmatrix} \begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix}$$

↳ flexibility matrix
- symmetric matrix ⇒ $d_{ij} = d_{ji}$
- square matrix

- Fllowing cramer's rule

$$R_1 = \frac{\begin{vmatrix} -191.67 & 22.5 \\ -916.67 & 183.33 \end{vmatrix}}{\begin{vmatrix} 6.33 & 22.5 \\ 22.5 & 183.33 \end{vmatrix}} = -22.2 \text{ KN.m}$$

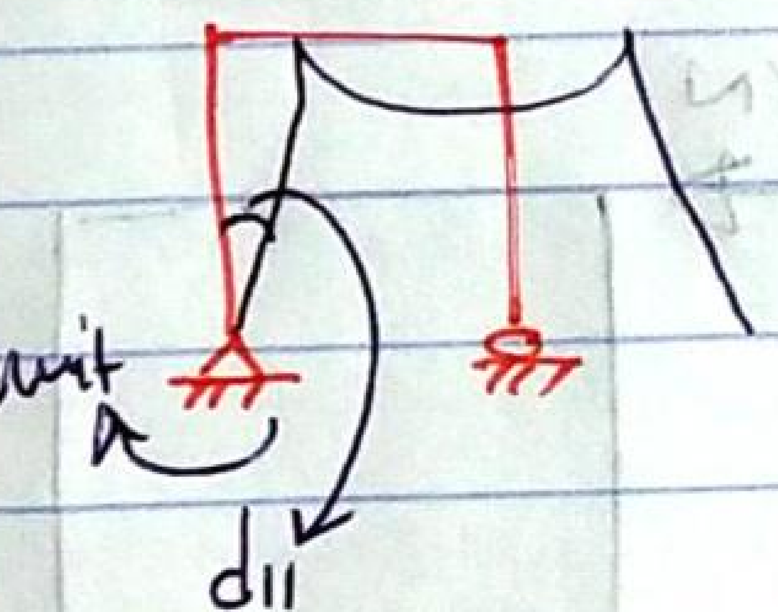
↳ عكس عقارب الساعة

$$\Rightarrow M_A = R_1 = 22.2 \text{ KN.m}$$

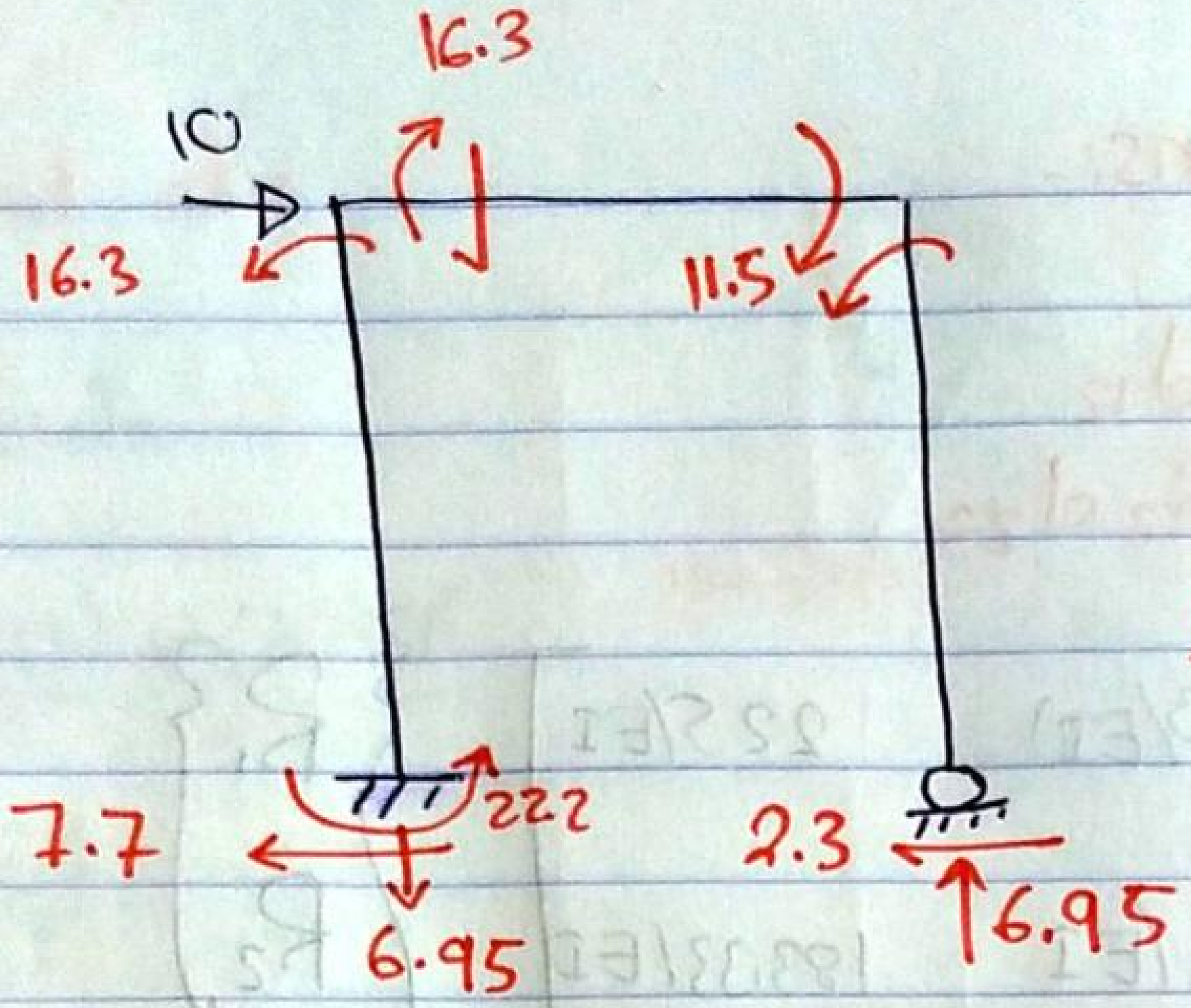
$$R_2 = \frac{\begin{vmatrix} 6.33 & -191.67 \\ 22.5 & -916.67 \end{vmatrix}}{\begin{vmatrix} 6.33 & 22.5 \\ 22.5 & 183.33 \end{vmatrix}} = -2.3 \text{ KN}$$

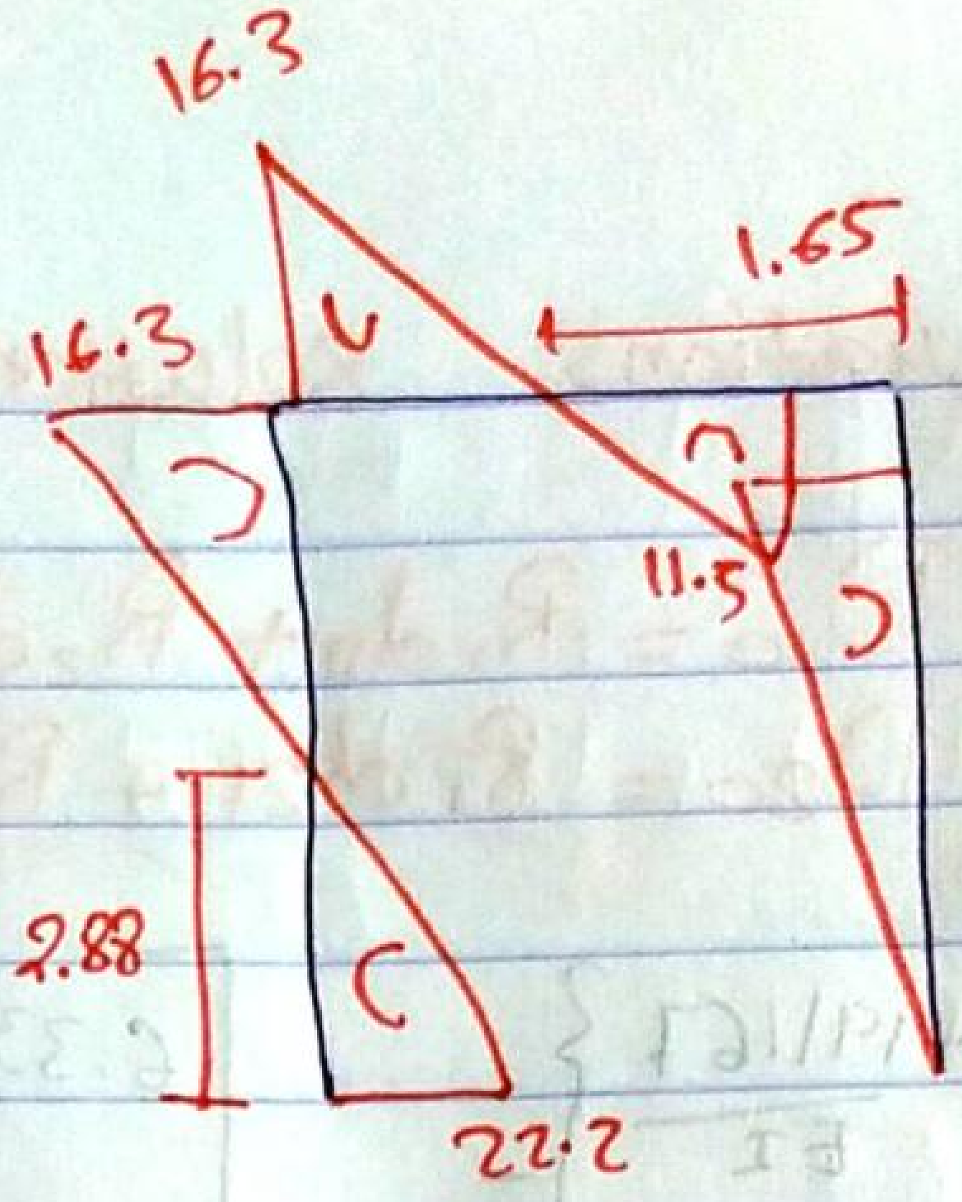
$$\Rightarrow D_x = R_2 = 2.3 \text{ KN} \leftarrow$$

⇒ Notes : flexibility cofficent.



- d_{11} flexibility Coff. deformation due to one unit force.
- Important to guss If the system is flexible or stuff
- depend on ① Geometry ② EI



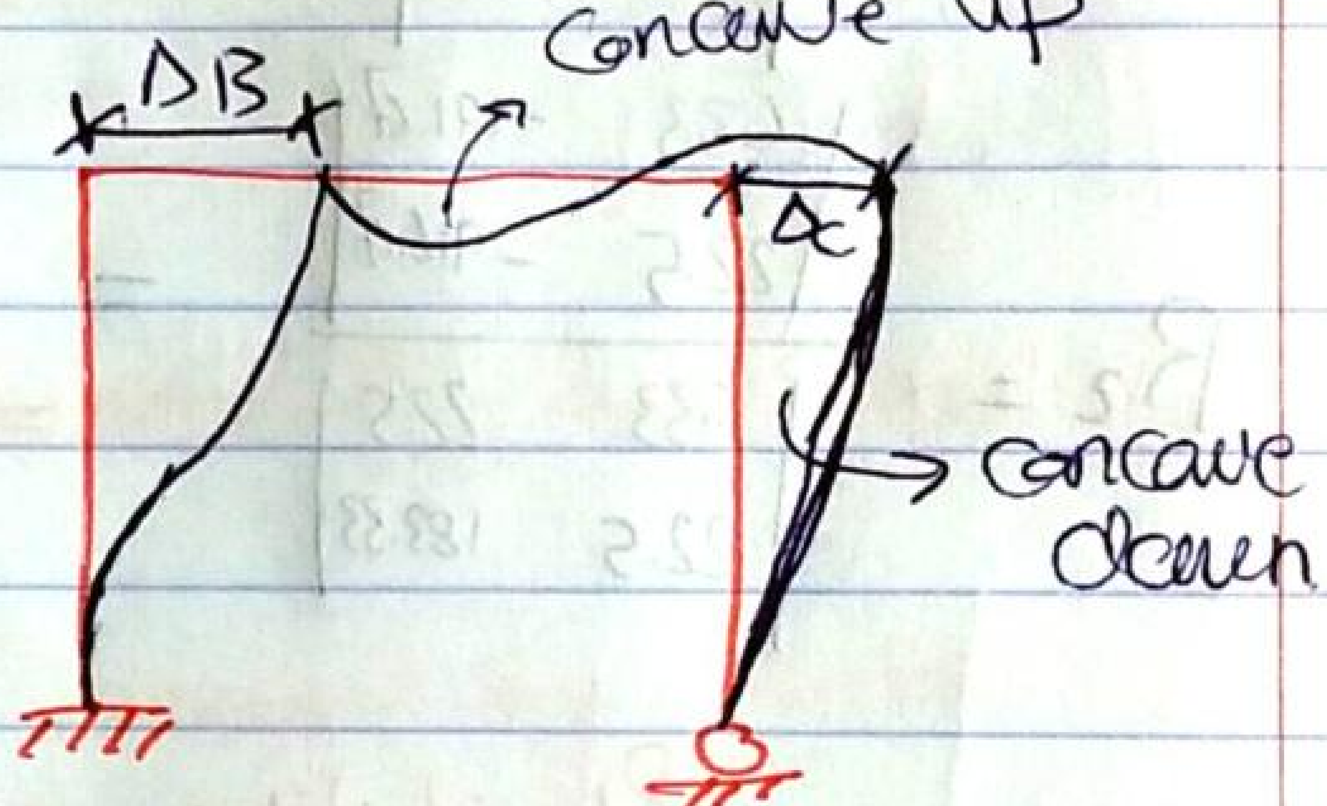


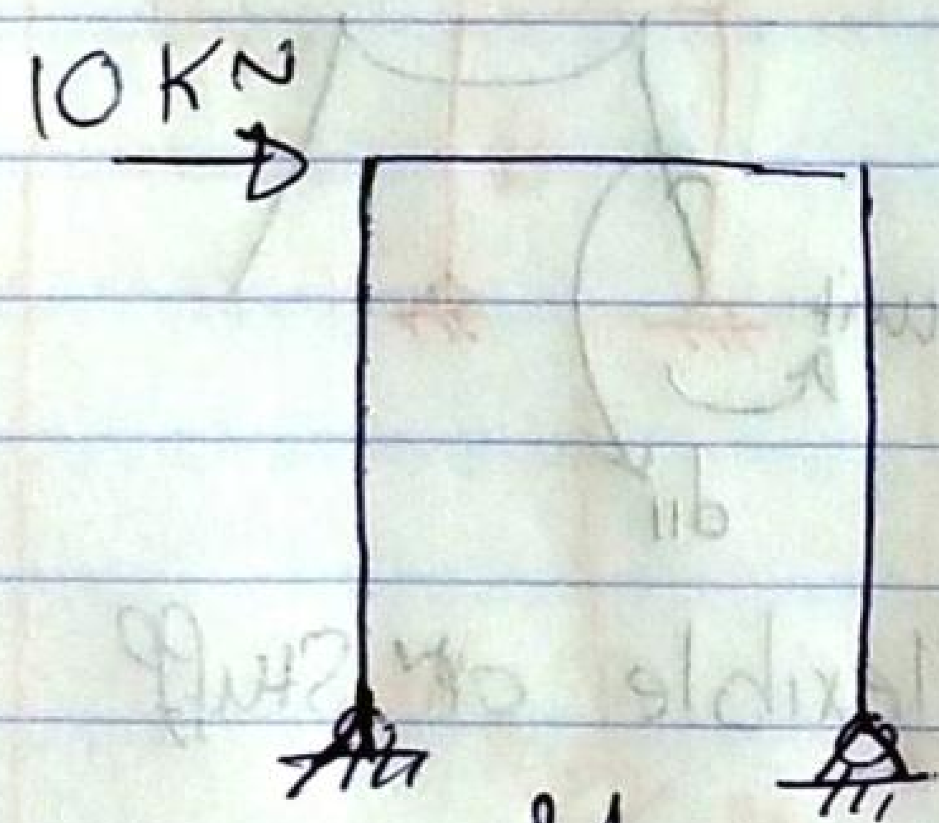
Using Eq. Equation.

- Steps:- ① write Compatabilit Equations.
  ② Use Force - deformation relationshipes
  ③ use Eqer Equations.

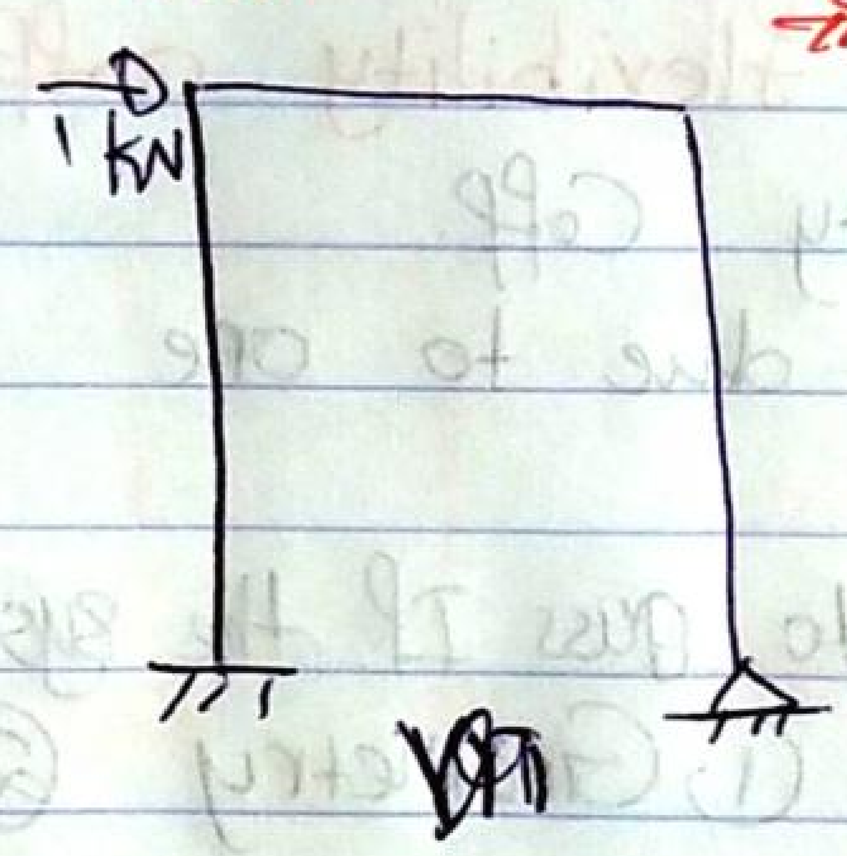
- $\Delta_B = \Delta_C = \Delta$

Ignore axial load deformation In frame elements.





Real load system



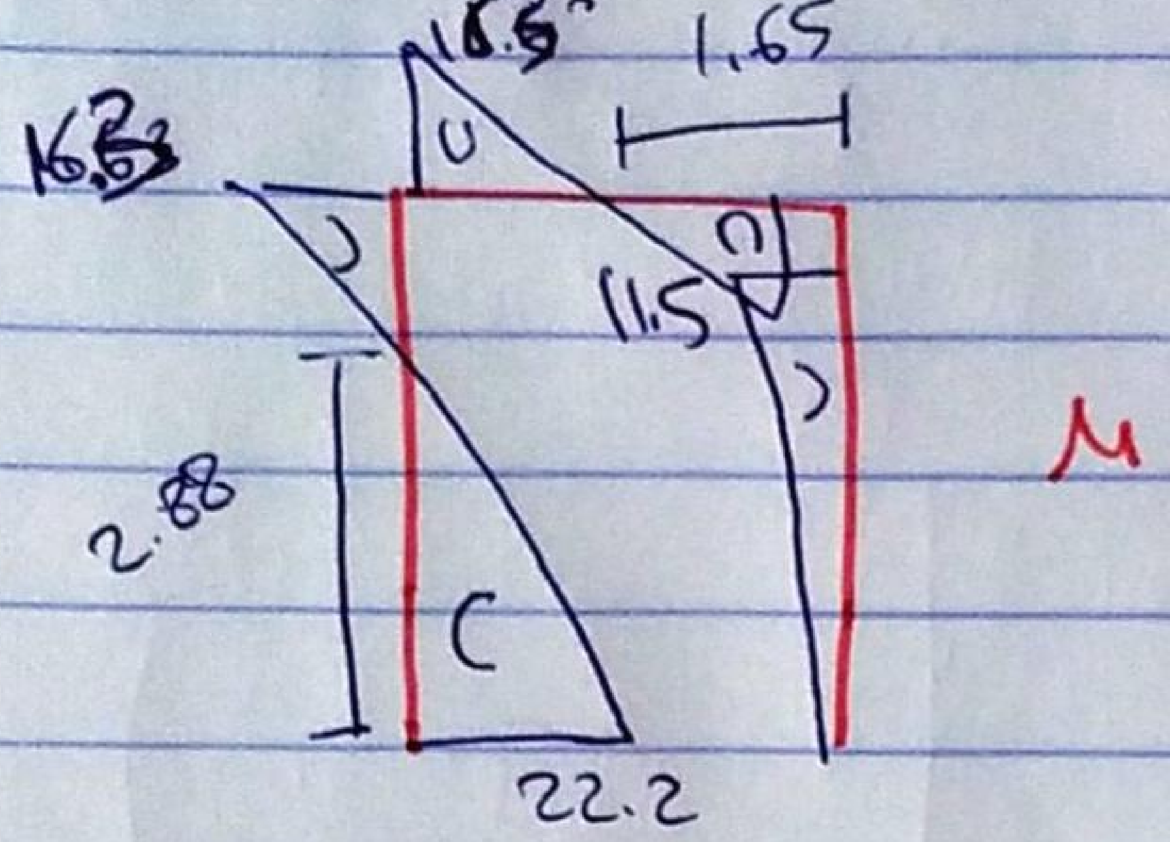
Virtual load System

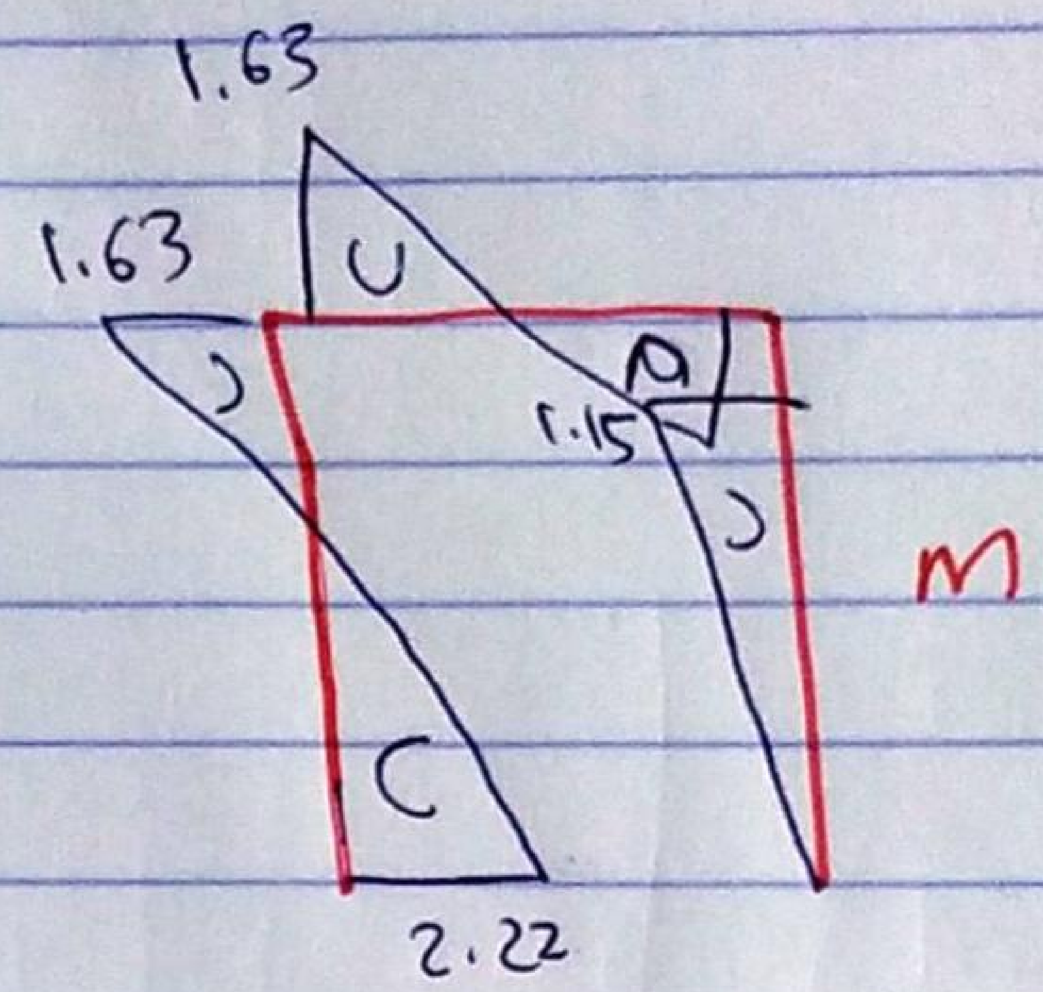
$$\Delta = \int \frac{m M}{EI} dx$$

Statically Indet.

- Find the Value of Δ?

* Speehial Case:-





$$\Delta = \int \frac{mM}{EI}\,dx$$

$$= \Big((22.2)\times(2.88)\times(2.22) + (2.12)(16.3)(1.63) + (1.63)(16.3)(2.35) + (1.15)(11.5)(1.65) + (5)(1.15)(11.5)\Big) \Big/ 3EI$$

$$= \frac{348.639}{3EI} = \frac{116.213}{EI}$$

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