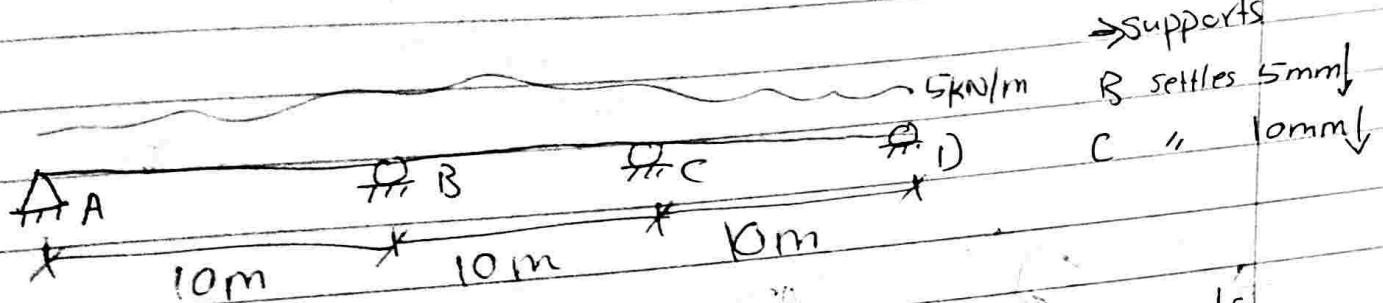
• Slope-deflection Equations:

$$\rightarrow M_{ij} = \frac{2EI}{L}\left(2\theta_i + \theta_j - \frac{3\Delta}{L}\right) + FE\,M_{ij}$$

→ EI constant , $E = 200\ GPa$

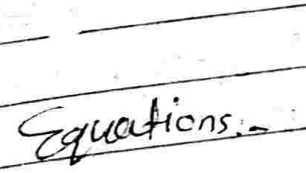
$I = 1.35 \times 10^{-3}\ m^4$



→ Supports
B settles 5mm ↓
C " 10mm ↓

- Find the end moments of the beam elements.
- Draw B.M diagram indicating key values.

→ Modified Equations:



$M_{ji} = 0$

$$2 \times M_{ij} = \frac{2EI}{L}\left(2\theta_i + \theta_j - \frac{3\Delta}{L}\right) + FE\,M_{ij}$$

$$0 = \frac{2EI}{L}\left(2\theta_j + \theta_i - \frac{3\Delta}{L}\right) + 0$$

$$\Rightarrow 2M_{ij} = \frac{3EI}{L}\theta_i + 0 - \frac{3EI\,\Delta}{L^2} + FE\,M_{ij}$$

→ Modified Equ $\Rightarrow M_{ij} = \frac{3EI}{L}\left(\theta_i - \frac{\Delta}{L}\right) + FE\,M_{ij}$

General Equ:
بستخدمها لما يكون العزم
مجهولات M_{ij}
fixed

Modified Equ:
بستخدمها لما يكون
أحد الأطراف مفصلة
فيه معروف

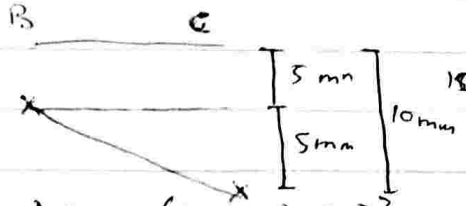
• There are 3 elements: Ele. AB, Ele BC, Ele CD
(Modified Equ., General Equ., Modified Equ.)

• Ele AB:

CCW = +

$\Delta_{AB} = 0.005m$ (C.W)

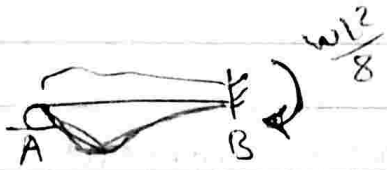
• Ele BC:

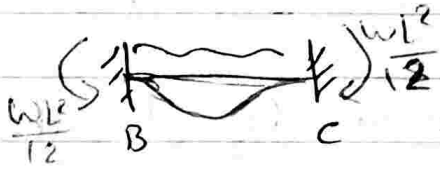


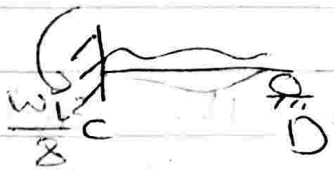
$\Delta_{BC} = (10-5)\times10^{-3}$ (C.W)

• Ele CD:

$\Delta_{CD} = -10\times10^{-3}$ (C.C.W)







$FEM_{BA} = \overset{+}{\frac{5(10)^2}{8}}$

$FEM_{BC} = -41.67$

$FEM_{CB} = \overset{+}{41.67}$

$FEM_{CD} = -62.5$

• $M_{AB} = 0$ (external pin support) + Modified Eq.

• $M_{BA} = \frac{3EI}{10}\left(\theta_B - \frac{0.005}{10}\right) + 62.5$

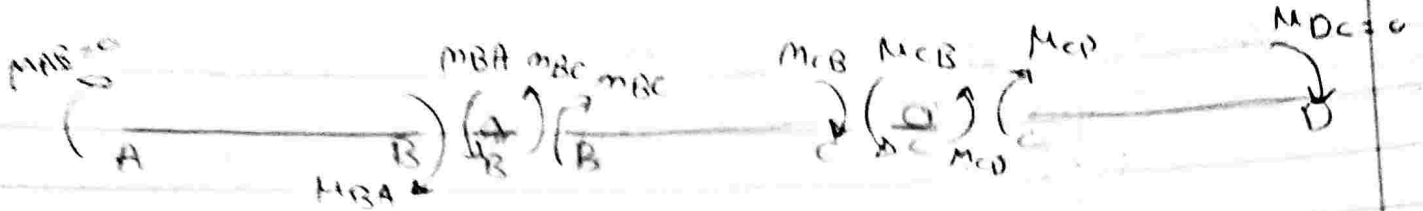
• $M_{BC} = \frac{2EI}{10}\left(2\theta_B + \theta_C - \frac{3(0.005)}{10}\right) - 41.67$

• $M_{CB} = \frac{2EI}{10}\left(2\theta_C + \theta_B - \frac{3(0.005)}{10}\right) + 41.67$

• $M_{CD} = \frac{3EI}{10}\left(\theta_C - -\frac{0.01}{10}\right) - 62.5$

• $M_{DC} = 0$ (external roller support) + Modified Eq.

Solve for θ_B, θ_C



⇒ Connected joint :

$$\Sigma M_{B=0} = M_{BA} + M_{BC} \quad (1)$$
$$\Sigma M_{C=0} = M_{CB} + M_{CD} \quad (2)$$

- $0.3EI\,\theta_B - 40.5 + 62.5 + 0.4EI\,\theta_b + 0.2EI\,\theta_c - 81 - 41.67 = 0$
  $0.7EI\,\theta_b + 0.2EI\,\theta_c - 100.67 = 0$ ——— (1)

→ $EI = 200 \times 10^6\ KN.m^2$ (...) m^4
لأنه الوزن بالكيلو نيوتن.

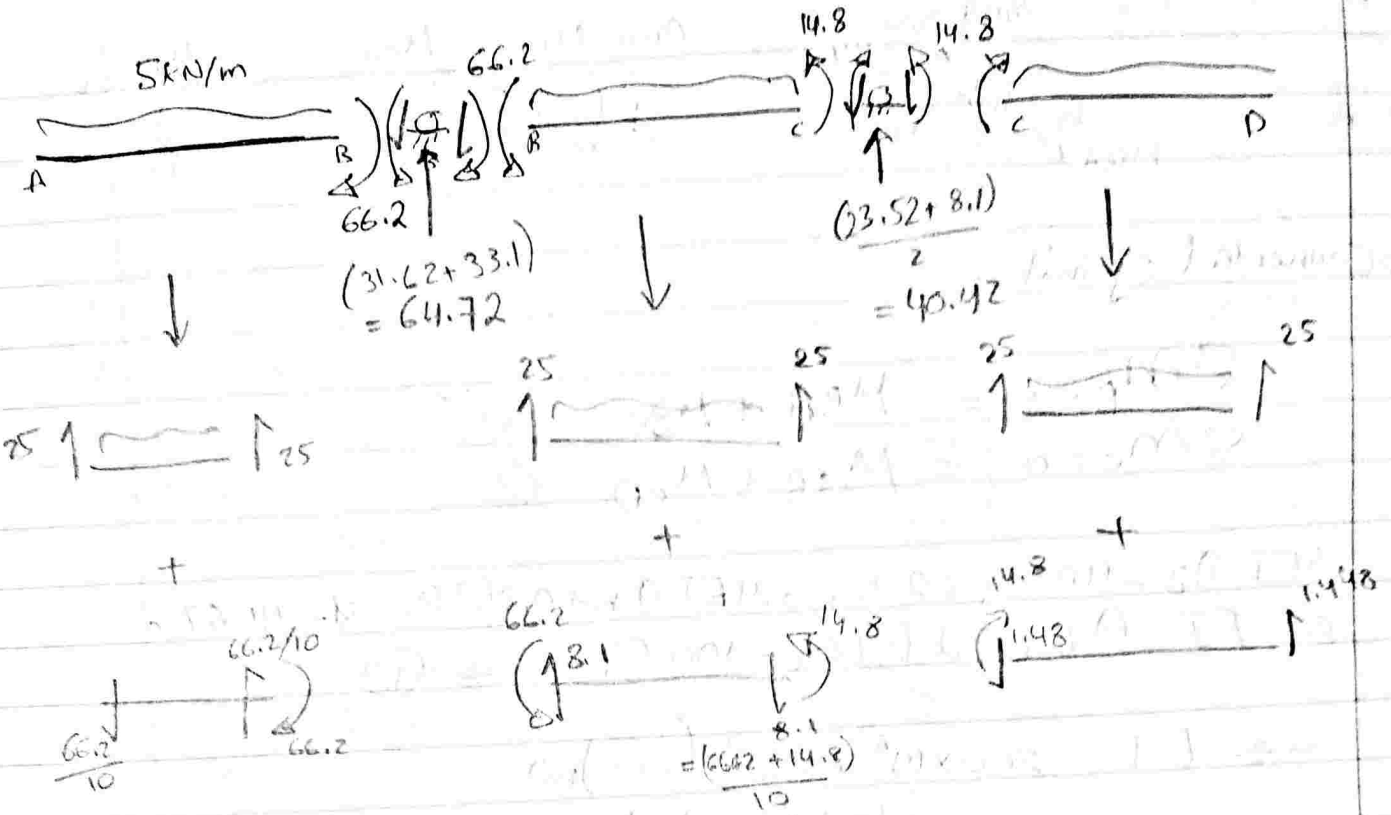
- $0.4EI\,\theta_c + 0.2EI\,\theta_b - 81 + 41.67 + 0.3EI\,\theta_c + 81 - 62.5 = 0$
  $0.2EI\,\theta_b + 0.7EI\,\theta_c - 20.83 = 0$ ——— (2)

$$EI \begin{bmatrix} 0.7 & 0.2 \\ 0.2 & 0.7 \end{bmatrix} \begin{Bmatrix} \theta_b \\ \theta_c \end{Bmatrix} = \begin{Bmatrix} 100.67 \\ 20.83 \end{Bmatrix}$$

$\theta_b = 147.33/EI$, $\theta_c = -12.34/EI$

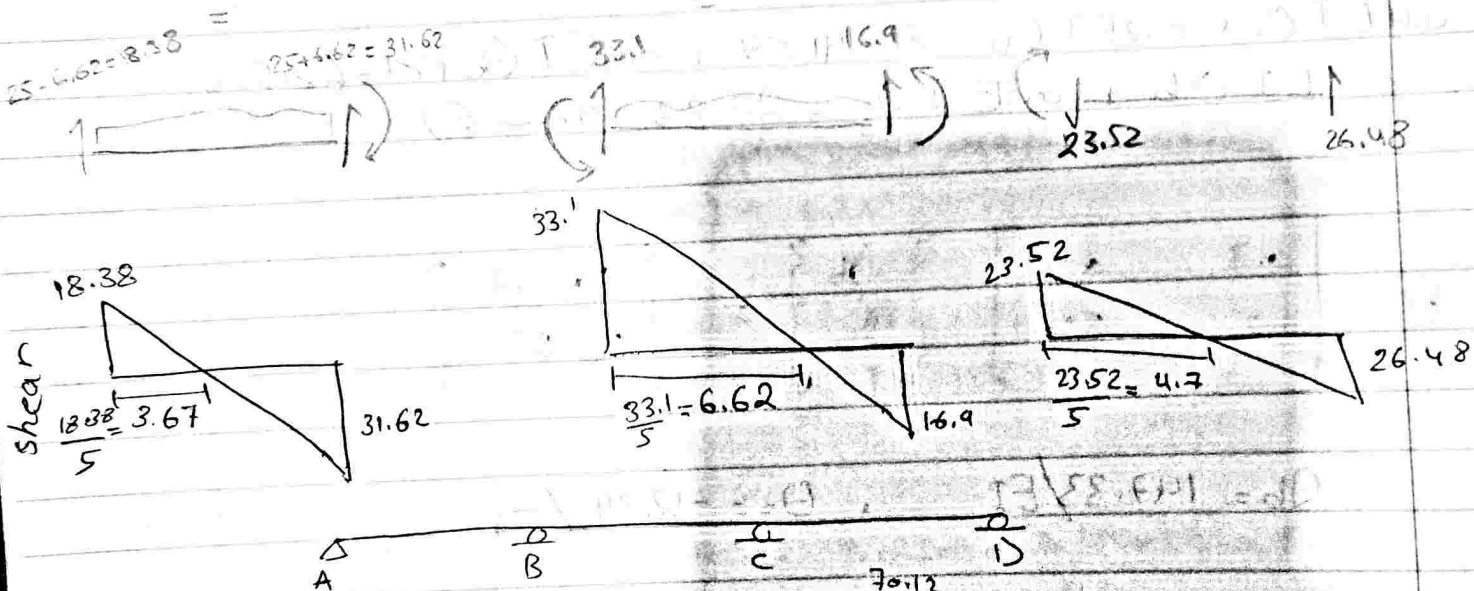
- $M_{BA} = 66.199\ KN.m$
- $M_{BC} = -66.2\ KN.m$
- $M_{CB} = -14.8\ KN.m$
- $M_{CD} = 14.8\ KN.m$

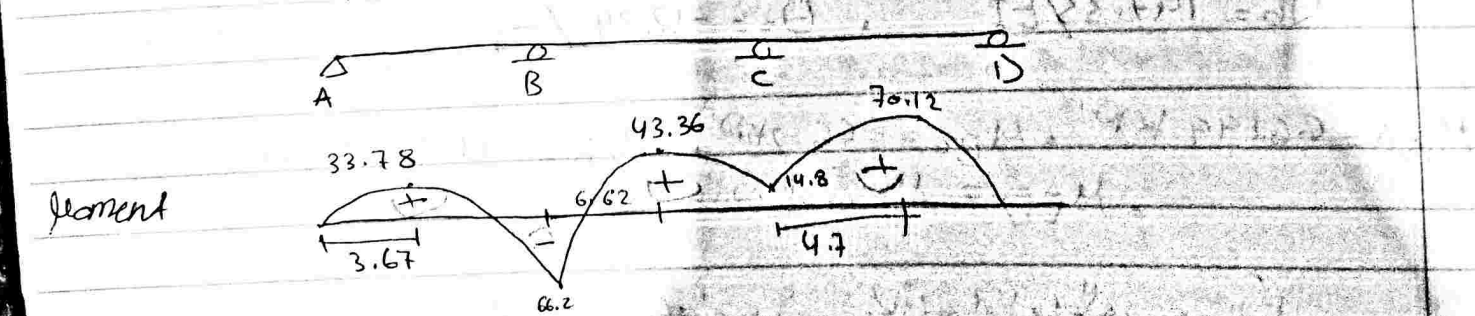
يكونوا نفس المقدار بعكس الاتجاه إذا صار حمل متماثل.
(أو ما يسمى Concentrated load)

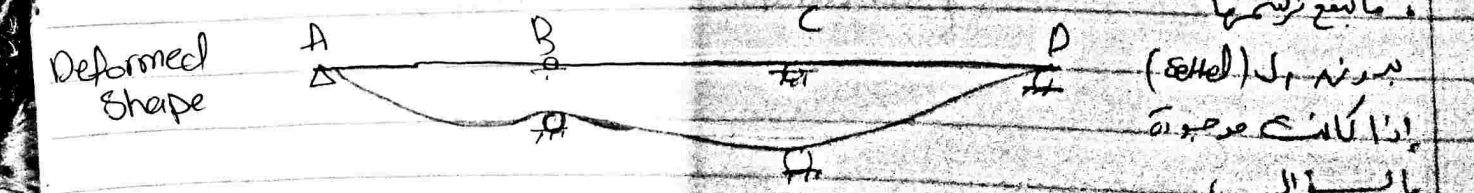


=

=





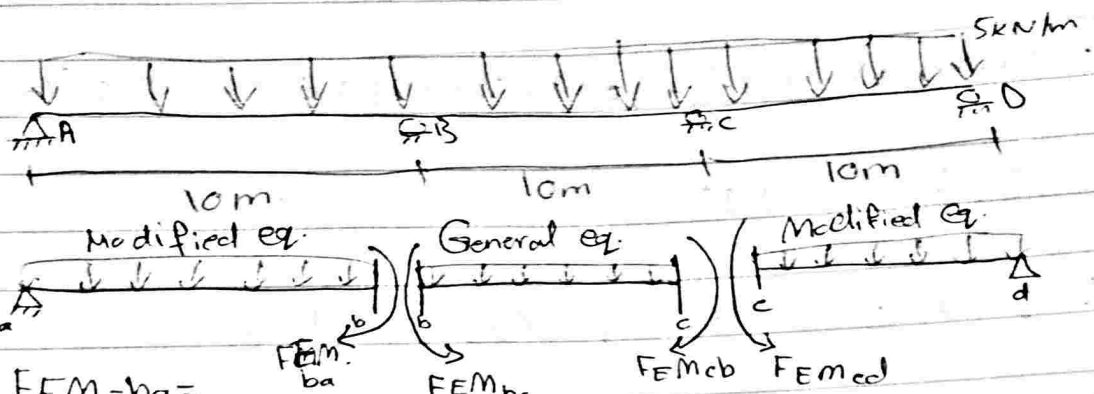


• ما ينفع نرسمها
بدونه ل (Shear)
إذا كانت موجودة
بالسؤال

- Example: uniformly distributed load of 5kN/m, as compute Reaction, draw shear force and Bending moment diagram.

settelment { Support B → 0.005 vertically downwards
Support C → 0.01 vertically downwards

$E = 200\ GPa$, $I = 1.35 \times 10^{-3}\ m^3$



$FEM_{ba} = WL^2/8 = 62.5$

$FEM_{bc} = -WL^2/12 = -41.67$

$FEM_{cd} = -WL^2/8 = -62.5$

$FEM_{cb} = 41.67$

$\psi_{ab} = +0.005/10 = 0.0005$

$\psi_{bc} = 0.01/10 - 0.005/10 = 0.005/10 = 0.0005$

$\psi_{cd} = -0.01/10 = -0.001$

(+) ⇒ CCW.

- Eq. of Modified: $M_{ab} = 0$

$M_{ba} = 3EI/L[\theta b - \psi ab] + 62.5 = 3EI/10[\theta b - 0.0005] + 62.5$

$M_{bc} = 2EI/L[2\theta b + \theta c - 3\psi bc] - 41.67 = 2EI/10[2\theta b + \theta c - 3*0.0005] - 41.67$

Now { $M_{cb} = 2EI/10[2\theta c + \theta b - 3*0.0005] + 41.67$

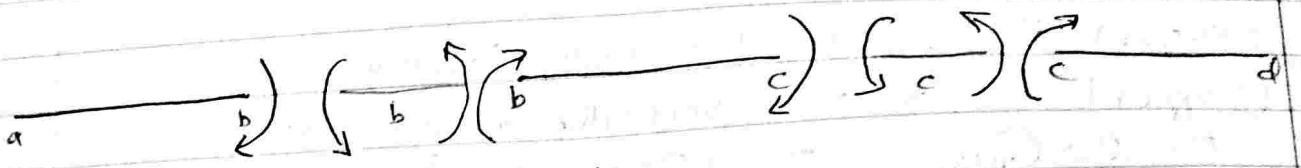
Modified { $M_{cd} = 3EI/L[\theta c - \psi cd] - 62.5 = 3EI/10[\theta c + 0.001] - 62.5$

$M_{dc} = 0$

Solve for $\theta b, \theta c$.

$EI = 200\times10^{6}\,kN/m^2 * 1.33\times10^{-3}\,m^4 =$ [ ] correct unit

- Equilibrium equation applied at Connecting Joints.



- At Joint b $\Sigma M = 0$, $M_{bc} + M_{ba} = 0$

$[0.3EI\theta_b - 40.5 + 62.5] + [0.4EI\theta_b + 0.2EI\theta_c - 81 - 41.67] = 0$

$0.7EI\theta_b + 0.2EI\theta_c - 100.67 = 0$ — Eq (1)

- At Joint c $\Sigma M = 0$, $M_{cb} + M_{cd} = 0$

$[0.4EI\theta_c + 0.2EI\theta_b - 81 + 41.67] + [0.3EI\theta_c + 81 - 62.5] = 0$

$0.2EI\theta_b + 0.7EI\theta_c - 20.83 = 0$ — Eq (2)

$$\begin{bmatrix} 0.7EI & 0.2EI \\ 0.2EI & 0.7EI \end{bmatrix} \begin{bmatrix} \theta_b \\ \theta_a \end{bmatrix} = \begin{bmatrix} 100.67 \\ 20.83 \end{bmatrix}$$

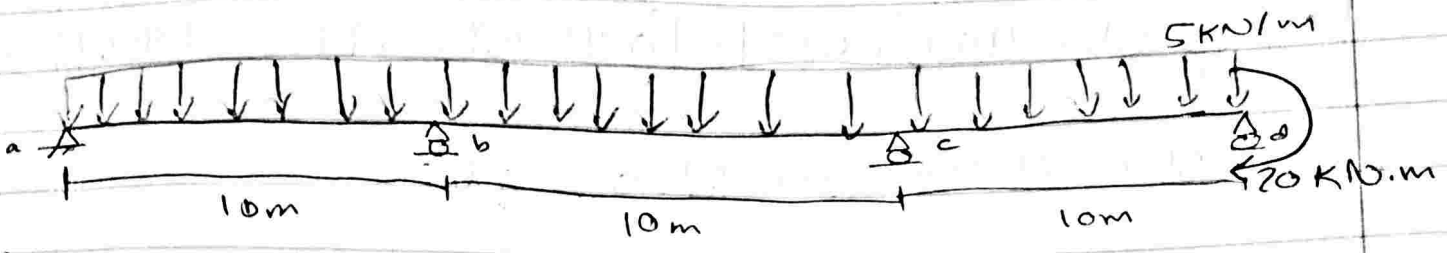
$\Rightarrow$ $\theta_b = 147.83/EI$, $\theta_c = -12.34/EI$

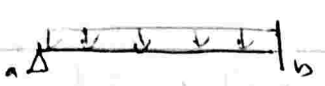
- $M_{ab} = 0$
- $M_{ba} = 66.2$
- $M_{bc} = -66.2$
- $M_{cb} = -14.8$
- $M_{cd} = 14.8$
- $M_{dc} = 0$

- Concentrated moment at internal pin or roller [support], this moment appears in the equilibrium equation.
  - → connecting joint.

- Concentrated moment at external pin support

• Example: same of the previous sample + external clockwise moment at Joint d.





+

FEM_ba = $wl^2/8$ = 62.5

FEM_bc = $-wl^2/12$ = -41.67

FEM_cb = 41.67

FEM_cd = $-wl^2/8 + m/2$ = -52.5

• The same settelments of the previous sample.

Member end-moments

• $M_{ab} = 0$

• $M_{ba} = 3EI/L\,[\theta b - \psi_{ab}] + 62.5 = 3EI\,[\theta b - 0.0005] + 62.5$

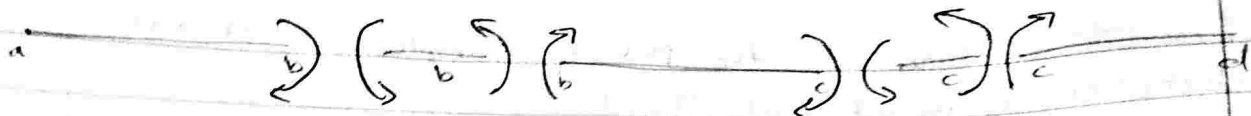
• $M_{bc} = 2EI/L\,[2\theta b + \theta c - 3\psi_{bc}] - 41.67 = 2EI/10\,[2\theta b + \theta c - 3*0.0005] - 41.67$

• $M_{cb} = 2EI/L\,[2\theta c + \theta b - 3\psi_{bc}] + 41.67 = 2EI/10\,[2\theta c + \theta b - 3*0.0005] + 41.67$

• $M_{cd} = 3EI/L\,[\theta c - \psi_{cd}] - 52.5 = 3EI/10\,[\theta c + 0.001] - 52.5$

• $M_{dc} = 20$

unknowns θb, θc.



- At joint b $\Sigma M = 0$, $M_{ba} + M_{bc} = 0$

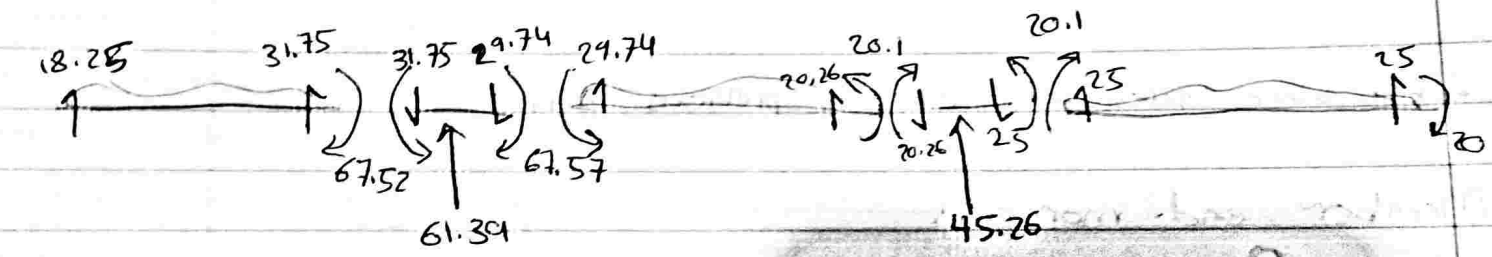
$[0.3EI\theta_b - 40.5 + 62.5] + [0.4EI\theta_b + 0.2EI\theta_c - 81 - 41.67] = 0$

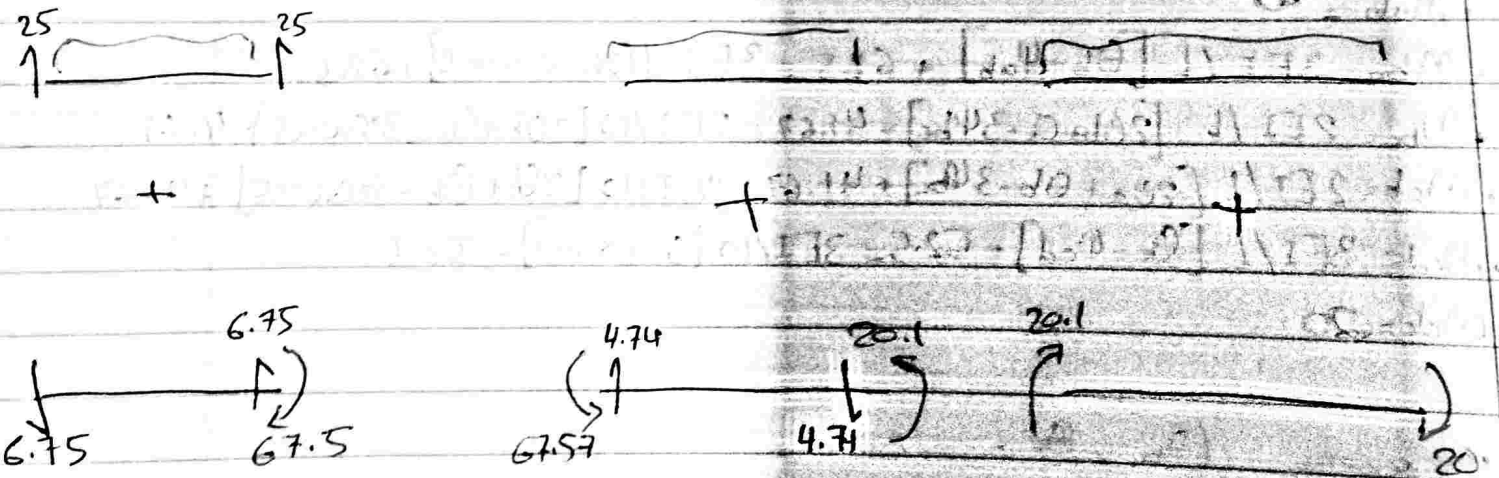
- $0.7EI\theta_b + 0.2EI\theta_c - 100.67 = 0 \rightarrow eq(1)$

- At Joint c $\Sigma M = 0$, $M_{cb} + M_{cd} = 0$

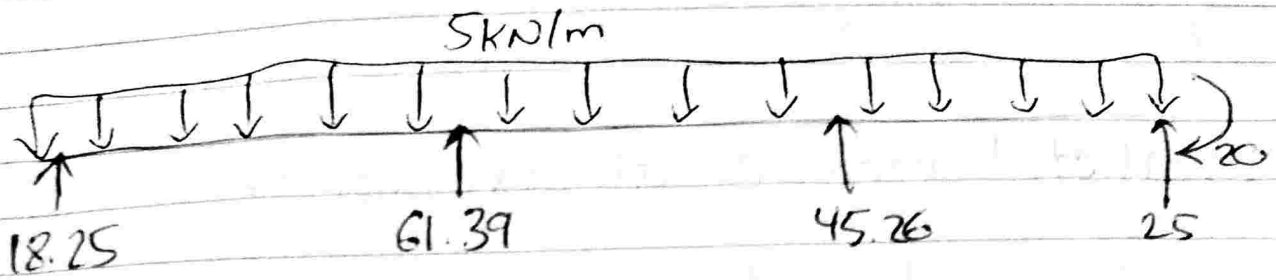
$[0.4EI\theta_c + 0.2EI\theta_b - 81 + 41.67] + [0.3EI\theta_c + 81 - 52.5] = 0$

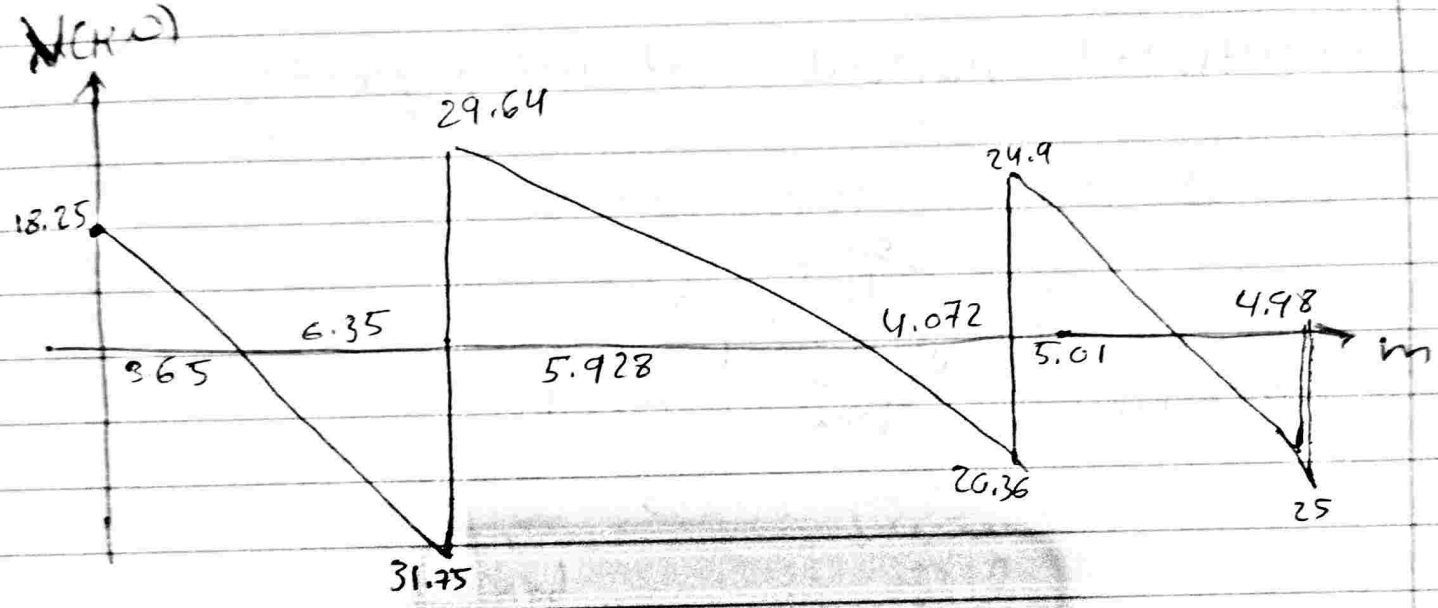
- $0.2EI\theta_b + 0.7EI\theta_c - 10.83 = 0 \rightarrow eq(2)$

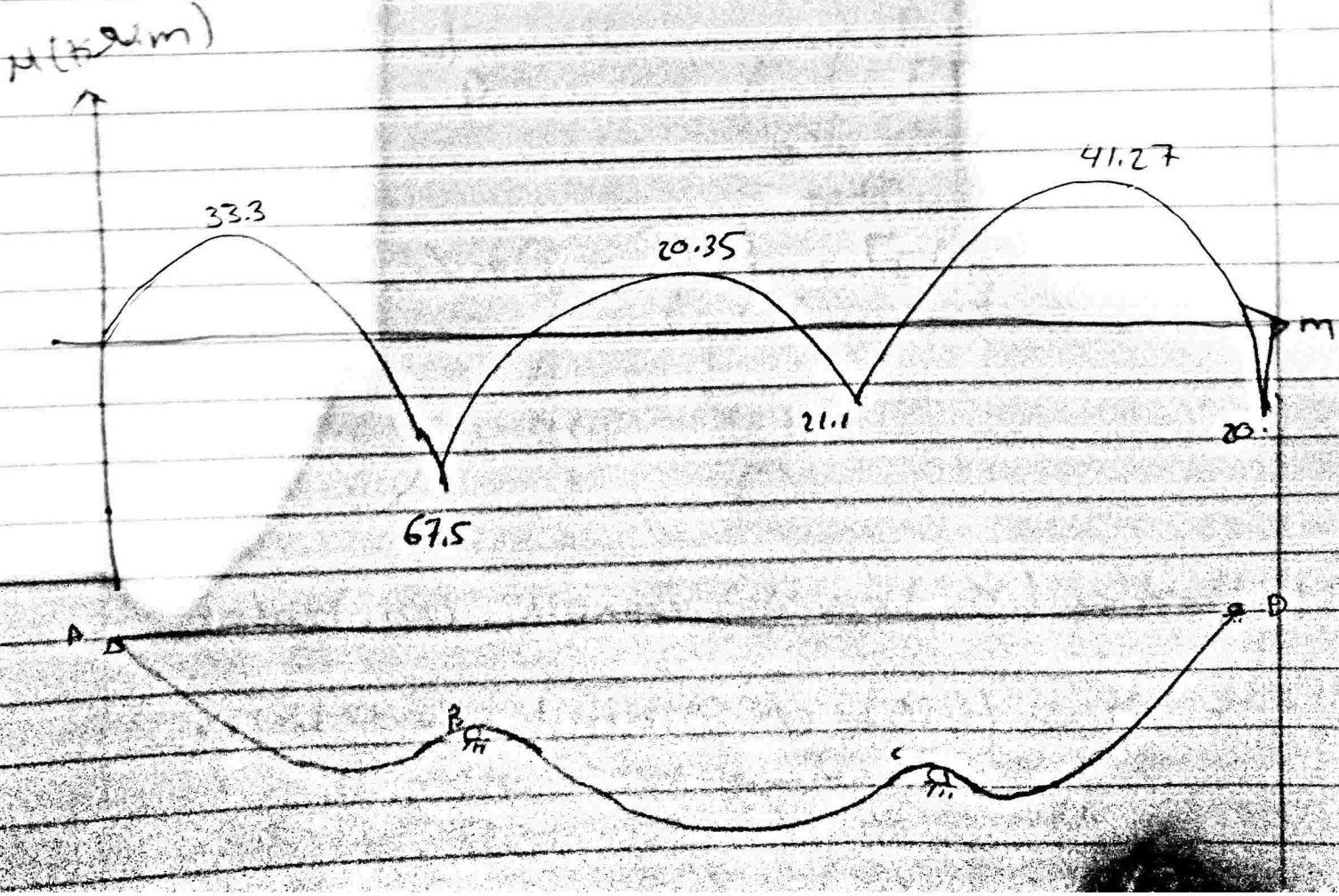


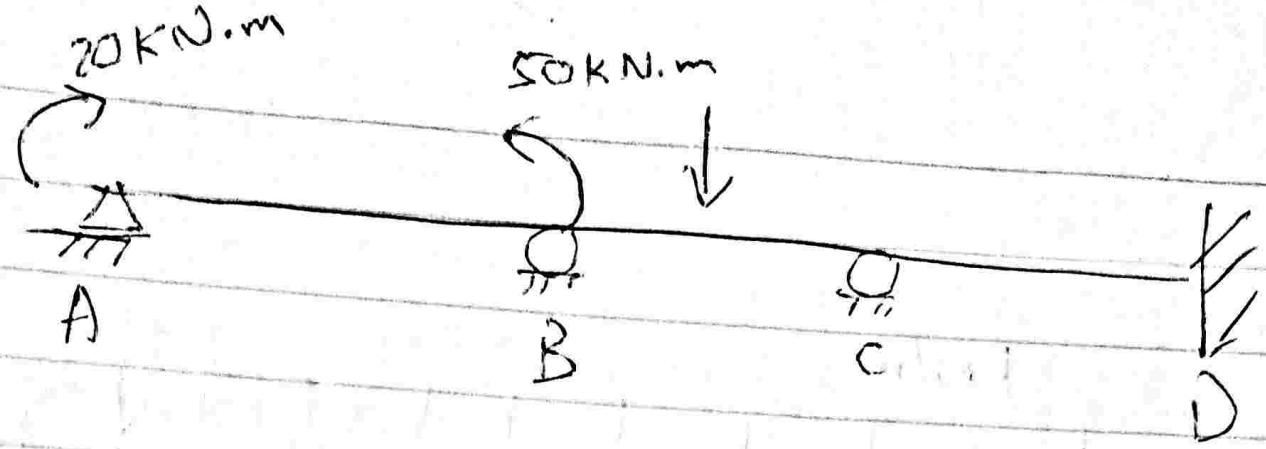


- $M_{ab} = 0$
- $M_{ba} = 67.52$
- $M_{bc} = -67.52$
- $M_{cb} = -20.16$
- $M_{cd} = 20.16$
- $M_{dc} = 20$


5KN/m
18.25
61.39
45.26
25


N(KN)
18.25
29.64
6.35
3.65
5.928
24.9
4.072
5.01
4.98
m
20.36
25
31.75


M(KN.m)
33.3
20.35
41.27
m
21.1
67.5
A
B
C
D



Concentrated moment at ext. Support

↳ fixed end moment

Concentrated moment at int. Support.

↳ equilibrium eqn.