

# ⇒ Displacement Method

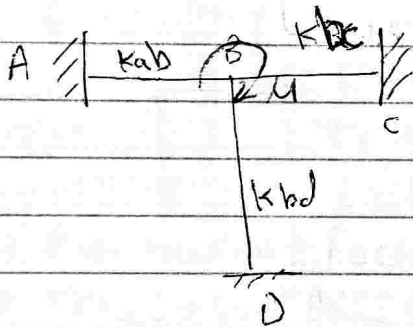
- slope-deflection equations & Flexural
- Moment distribution on Method system

↓  
 Tabulation of slope-deflection Eqn.  
 ↳ Direct stiffness method

## « Moment Distribution Method »

Force due to unit deformation

① Moment distribution coeff. factors



• concentrated moment at joint B  
 ↳ Boundary cond. fixed  
 EI  
 ↳ The element more stiffness

$$M_{ij} = \frac{2EI}{l} \left( 2\theta_i + \theta_j - \frac{3\Delta_{ij}}{l} \right) + FEM_{ij}$$

rotational stiffness

$$\left( \frac{4EI}{l} \right) + \frac{2EI}{l} \quad \frac{6EI}{l}$$

- rotational stiffness (general eq.) →  $K = 4EI/l$

- rotational stiffness (general eq.) →  $K = 3EI/l$

$$M_{BA} = \frac{k_{ab}}{\sum K} \mu, \quad M_{BD} = \frac{k_{bd}}{\sum K} \mu, \quad M_{BC} = \frac{k_{bc}}{\sum K} \mu$$

↳ distribution factor  $\frac{K}{\sum K}$

contains values for the example of last lecture :-

$$\theta_b = 151.8/EI$$

$$\theta_c = -27.9/EI$$

$$M_{ba} = 67.54 \text{ kN.m}$$

$$M_{bc} = -67.53 \text{ kN.m}$$

$$M_{cb} = -20.13 \text{ kN.m}$$

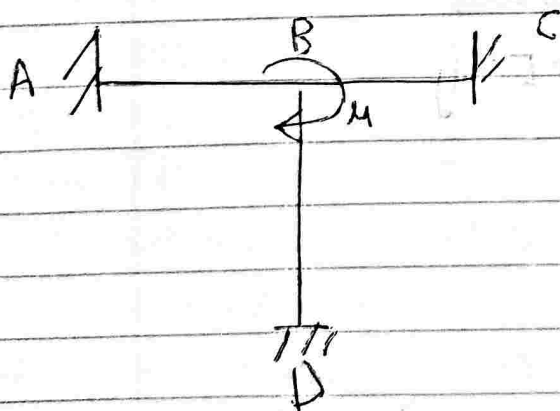
$$M_{cd} = 20.13 \text{ kN.m}$$

### « Displacement Method »

Moment distribution Method  
Tabulation of slope-deflection equations

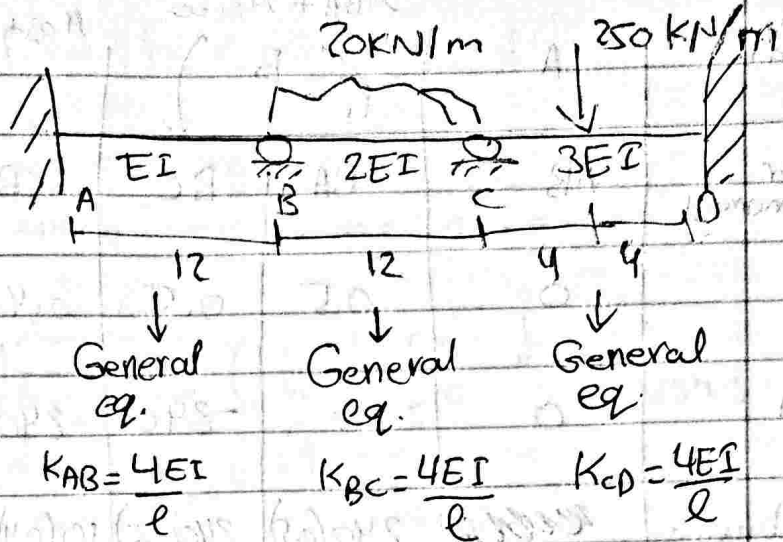
learn By example

### ⇒ Moment distribution factor

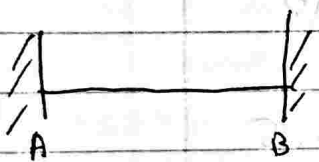


- general equation  
 $K = 4EI/L$
- Modified  
 $K = 3EI/L$

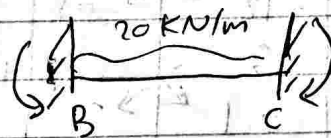
⇒ Example:  
 Draw B.M  
 Indicating Key  
 Values:



prismatic:  $EI$  is constant within the element]  $K_{AB} = K_{BA} = \frac{4EI}{l}$   
 Not prismatic  $K_{AB} \neq K_{BA}$ ] Derive



$FEM_{AB} = 0$   
 $FEM_{BA} = 0$



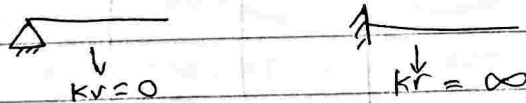
$FEM_{BC} = -\frac{20(12)^2}{12} = -240$   
 $FEM_{CB} = \frac{20(12)^2}{12} = 240$



$FEM_{CD} = -\frac{250(8)}{8} = -250$   
 $FEM_{DC} = +\frac{250(8)}{8} = 250$

• Moment distribution Factors

→ at A ⇒  $DF_{AB} = \frac{K_{AB}}{K_{AB} + K_{fixity}} = \frac{\frac{4EI}{l}}{\frac{4EI}{l} + \infty} = 0$



معاملات التوزيع، أي نسبة القوة من المرفق إلى المرفق  
 أي أن ال fixity  
 في كل المرفق

→ at D ⇒  $DF_{DC} = 0$

→ at B ⇒  $DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{4EI/12}{4EI/12 + 4EI/12} = 0.5$   
 $DF_{BC} = \frac{K_{BC}}{K_{BC} + K_{AB}} = 0.5$

→ at C ⇒  $DF_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD}} = \frac{4EI/12}{4EI/12 + 4EI/8} = 0.4$   
 $DF_{CD} = \frac{K_{CD}}{K_{CD} + K_{CB}} = \frac{4EI/8}{4EI/8 + 4EI/12} = 0.6$

	A	M <sub>BA</sub> + M <sub>BC</sub> = 0		M <sub>CB</sub> + M <sub>CD</sub> = 0		D
Joint		B	B	C	C	
member end moment	AB	BA	BC	CB	CD	Dc
DF	0	0.5	0.5	0.4	0.6	0
FEM	0	0	-240	-240	-250	250
Distribution	<del>240(0.5)</del>	240(0.5)	240(0.5)	10(0.4)	10(0.6)	
		120	120	4	6	
CO	60		2	60		3
Dist.		-1	-1	-24	-36	
CO	-0.5		-12	-0.5		-18
Dis		6	6	+0.24	+0.36	
CO	3		0.1	3		0.15
Dis		0.05	-0.05	-1.2	-1.8	
CO	-0.025		-0.6	0.025		-0.9
Dis		0.3	0.3	0.01	0.015	
Moment (KN.m)	62.5	125.3	-125.3	281.5	-218.5	234.3

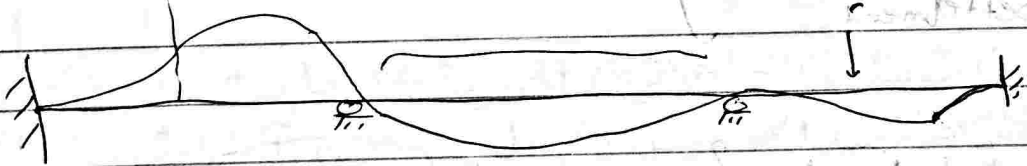
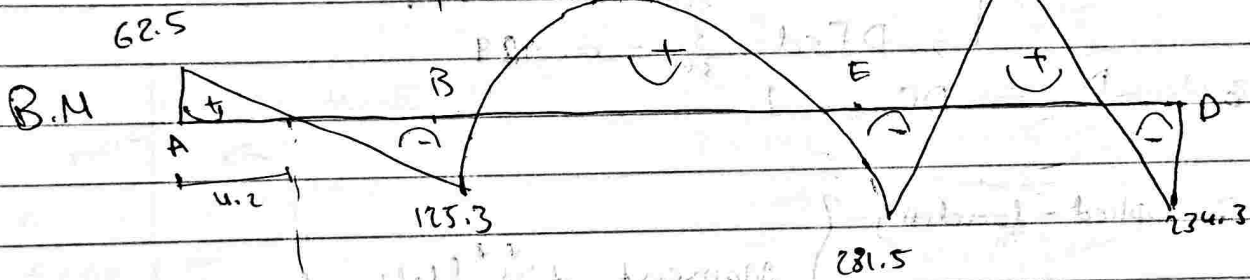
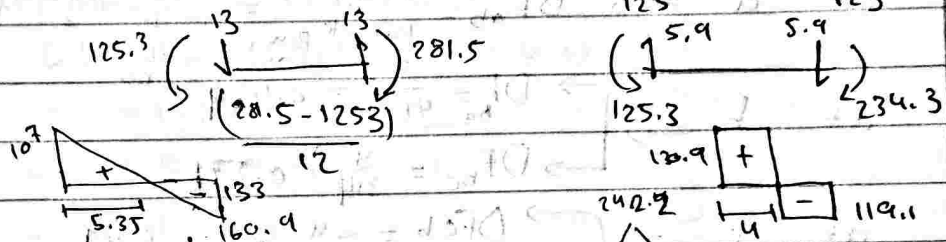
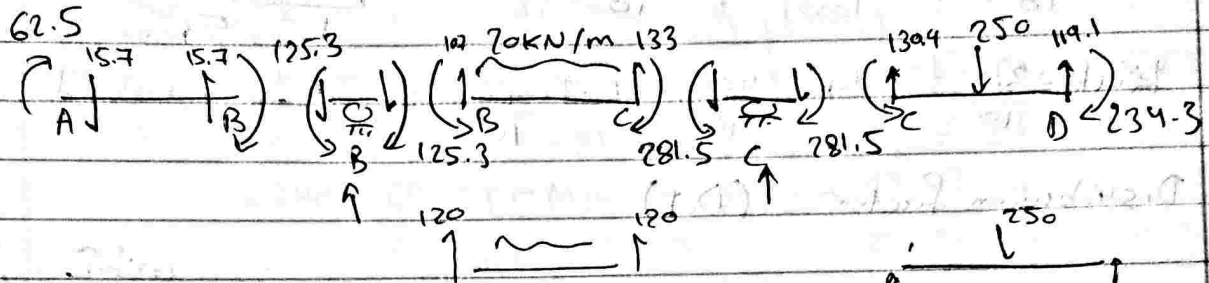
مومنت  
 با صفر  
 القوت  
 اقل  
 (مقارن)

قوت القوت (التي يشار إليها)

مومنت  
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# Moment Distribution Method

AB	BA	BC	CB	CD	DC
62.5	125.3	-125.3	281.5	-281.5	234.3

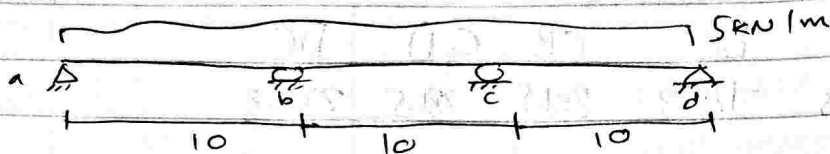


Handwritten notes and calculations at the bottom of the page, including the formula  $M = \frac{wL^2}{8}$  and other numerical values.

# Moment Distribution Method

$E = 200 \text{ GPa}$ ,  $I = 1.35 \times 10^{-3} \text{ m}^4$

$EI$  is constant



Settle

b.  $\rightarrow$  5 mm  $\downarrow$

c.  $\rightarrow$  10 mm  $\downarrow$

$\Rightarrow k_{ab} = \frac{3EI}{10}$      $k_{bc} = \frac{4EI}{10}$      $k_{cd} = \frac{3EI}{10}$

$\Rightarrow$  Distribution factor (D.F)

\* at a  $\Rightarrow DF_{ab} = \frac{k_{ab}}{k_{ab} + k_{pin}} = \frac{3EI/10}{3EI/10 + 0} = 1$

وإذا كان  
 $\rightarrow$   $\frac{3EI}{10}$

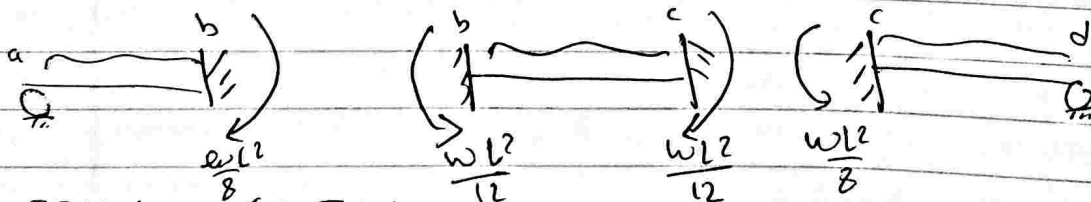
\* at b  $\Rightarrow$   $DF_{ba} = \frac{3}{4+3} = 0.429$   
 $DF_{bc} = \frac{4}{3+4} = 0.571$

\* at c  $\Rightarrow$   $DF_{cb} = \frac{4}{3+4} = 0.571$   
 $DF_{cd} = \frac{3}{3+4} = 0.429$

\* at D  $\Rightarrow DF_{dc} = 1$

FEM  $\rightarrow$  applied loading } Moment dist. Method  
+ FEM  $\rightarrow$  Settlement

FEM  $\Rightarrow$  applied loading } slope def. equation  
- Settlement }

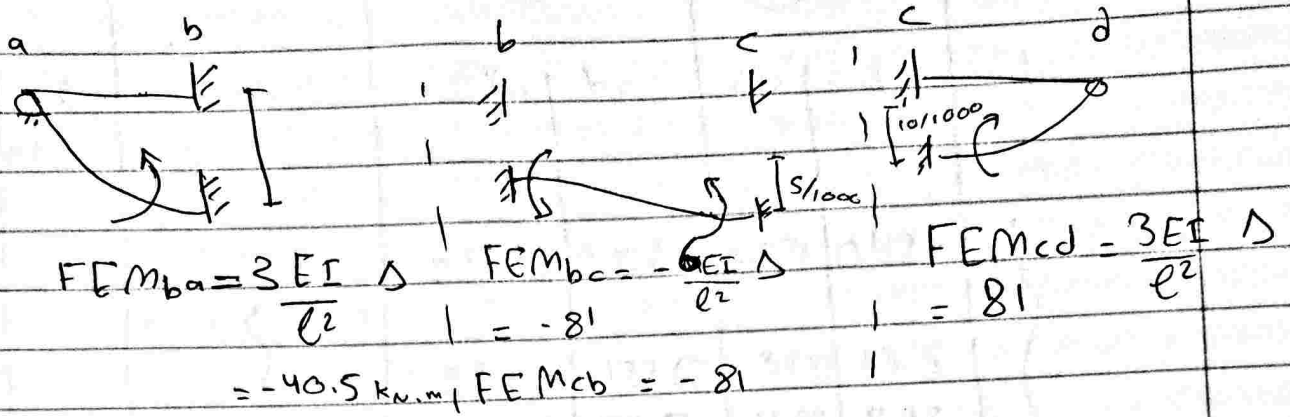


FEM  $b_a = 62.5 \text{ kN.m}$   
FEM  $b_c = -44.67 \text{ kN.m}$   
FEM  $c_b = 44.67 \text{ kN.m}$   
FEM  $c_d = -62.5 \text{ kN.m}$

} applied loading

~~Handwritten scribbles~~

Settlement :-

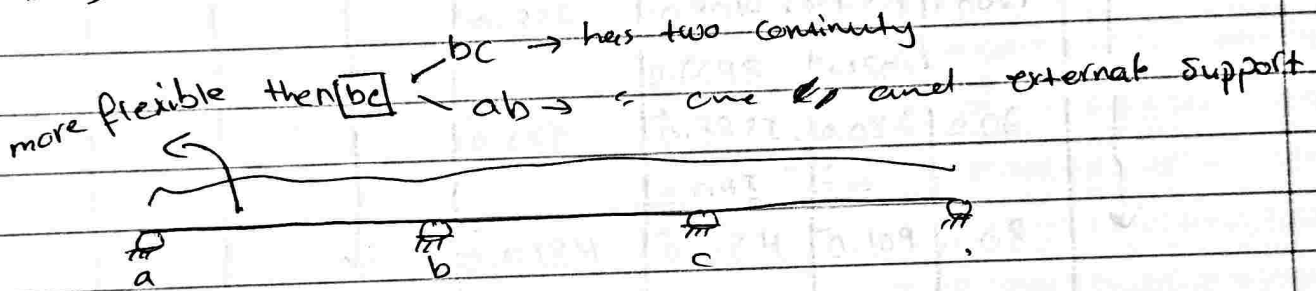


$\Rightarrow FEM_{ba} = (FEM_{ba})^{\text{applied}} + (FEM_{ba})^{\text{settle}}$   
 $= 62.5 + -40.5 = 22 \text{ kN.m}$

$\Rightarrow FEM_{bc} = -122.67 \text{ kN.m}$

$\Rightarrow FEM_{cd} = 18.5 \text{ kN.m}$

$\Rightarrow FEM_{cb} = -39.33 \text{ kN.m}$



لو في موقف خارجي (توتر) الجونية الخارجية في حال (pin) سبكر خانات الجونية بالجود وينحطم آخراش.

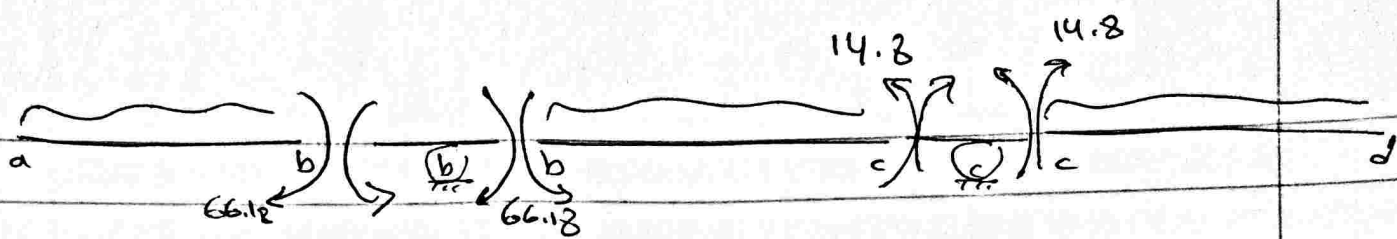
b, c  $\Rightarrow$  Joint  $\Rightarrow$  ترتيب بين الؤهنت  
 عندهم يك بنقل معادلات ال  
 Eqn:  $\sum M_b = 0$   $\sum M_c = 0$

Joint	a	b	c	d		
member end moment	ab	ba   bc	cb   cd	dc		
DF	1	0.429	0.571	0.571	0.429	1
FEM		22	-122.67	-39.33	18.5	
Dist.		43.1	57.5	11.89	8.93	
CO.			5.95	28.76		
Dist.		-2.551	-3.40	16.44	-12.33	
CO.		+3.526	-8.22	-1.85		
			+4.693	1.056	0.793	
			0.528	2.346		
		-0.226	-0.3014	-1.339	1.0061	
			0.6695	0.1507		
		+0.287	+0.3822	+0.086	0.06	
			0.043	0.19		
		0.0184	0.024	0.109	0.08	
		66.186	-66.18	-14.89	14.89	0
	0					

آخر اشئ بنقله مو Dis كى يكون محقق للتوازن

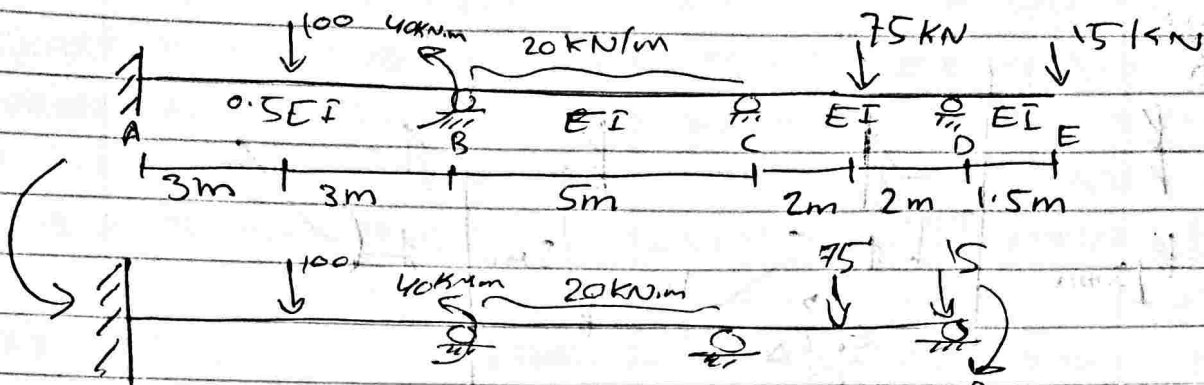
$\Rightarrow$  Max. Dist  $\pm 0.1$  kN.m  $\Rightarrow$  represented error





Shear + B.M diagram  $\rightarrow$  deforme (correctly)  
 (on the settlers support)

# Moment Distribution Method:-



①  $K_{AB} = \frac{4(0.5EI)}{6}$        $K_{BC} = \frac{4(EI)}{5}$        $K_{CD} = \frac{3EI}{4}$        $K_{DE} = \frac{3EI}{4}$        $K_{EF} = \frac{3EI}{4}$

② DF at Joints:-

at Joint A       $DF_{AB} = 0$

at Joint D       $DF_{DC} = 1$

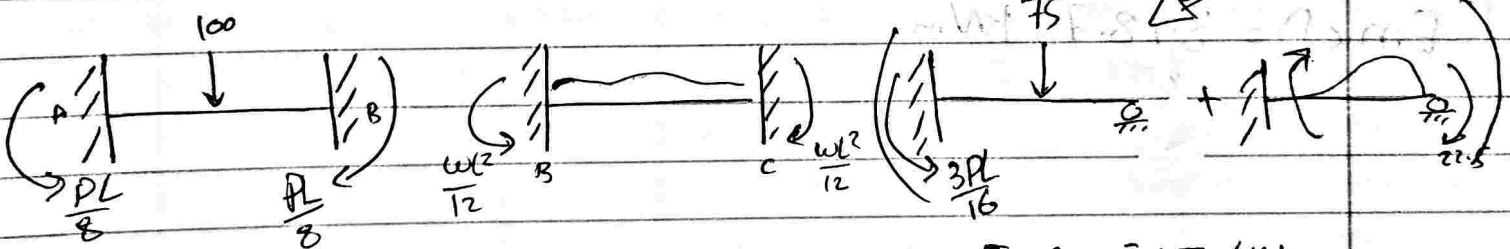
at Joint B       $DF_{BA} = 0.294$

$DF_{BC} = 0.706$

at Joint C       $DF_{CB} = 0.516$

$DF_{CD} = 0.484$

Fixed end moment



$FEM_{AB} = -75 \text{ kN.m}$

$FEM_{BC} = -41.6 \text{ kN.m}$

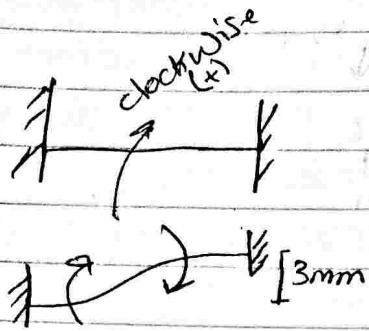
$FEM_{CD} = 45 \text{ kN.m}$

$FEM_{BA} = 75 \text{ kN.m}$

$FEM_{CB} = 41.6 \text{ kN.m}$

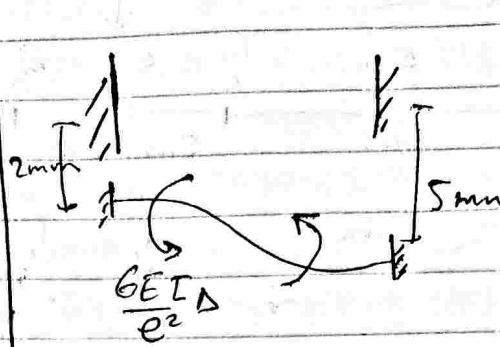
تأثير الحمل  
في السورن  
مباشرة ما باشع قوة الكمرات

→ Settlement:-



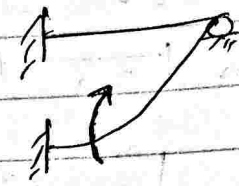
$$FEM_{AB} = \frac{6EI\Delta}{l^2} = 75$$

$$FEM_{BA} = \frac{6EI\Delta}{l^2} = 75$$



$$FEM_{BC} = -360$$

$$FEM_{CB} = -360$$



$$FEM_{CD} = 393.75$$

$$FEM_{AB} = 0$$

$$FEM_{BA} = 150 \text{ kN}\cdot\text{m}$$

$$FEM_{BC} = -401.67 \text{ kN}\cdot\text{m}$$

$$FEM_{CB} = -318.33 \text{ kN}\cdot\text{m}$$

$$FEM_{CD} = 348.75 \text{ kN}\cdot\text{m}$$

$$M_{BA} + M_{BC} = -40 \quad M_{CB} + M_{CD} = 0$$

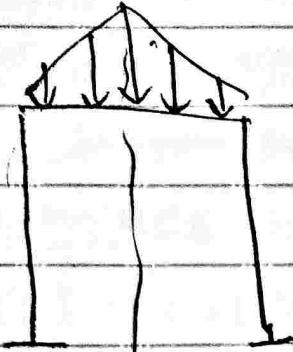
Joint	A	B	C	D		
member	AB	BA	BC	CB	CD	DC
End moment						
DF	0	0.294	0.706	0.516	0.484	1
FEM	0	150	-401.67	-318.33	348.75	
Dist.		62.23	149.44	-15.7	-14.72	
C.O.	31.11		-7.85	74.72		
Dist		2.3	5.54	-38.55	-36.16	
C.O.	1.15		19.275	2.77		
Dis	35.5	221.2	-266.5	-292.65	292.65	22.5

$$\sum m_B = 0$$

$$m_{BA} + m_{BC} = -40$$

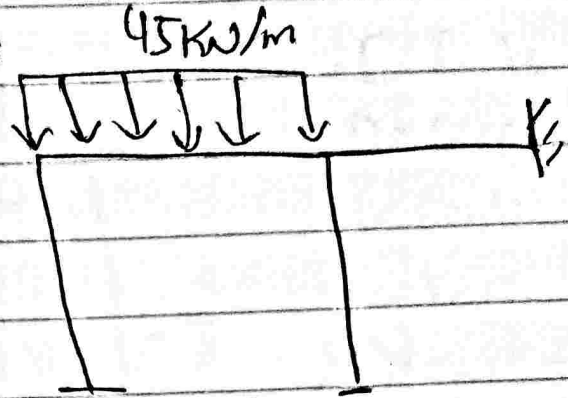
$$\text{Dist } \Rightarrow 150 - 401.67 + M = -40 \Rightarrow M = 211.67$$

« Non Sway Fram »



axis of symmetry

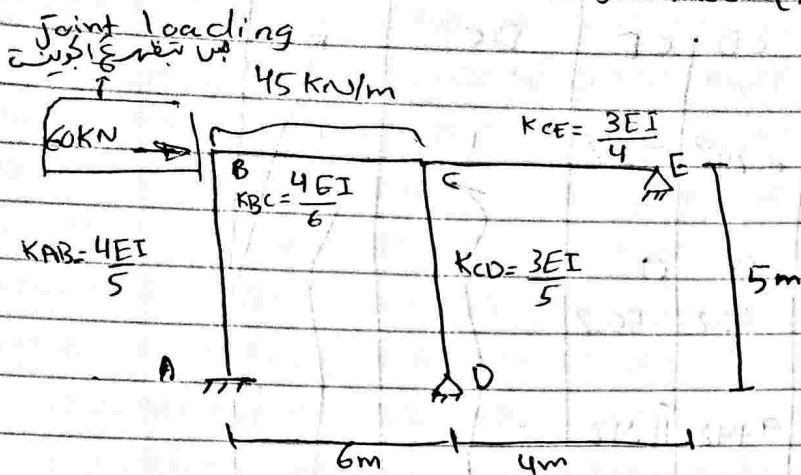
عائى سواى لائىءى السز  
أءء سبءرءى



↓

عائى سواى لائىء  
فءءر سبءرءى عءء

# Moment distribution Method (Non-sway Frames)



- EI is constant
- Determine end moments of the members.
- Draw the B.M.d indicating Key Values.
- Draw the deformed shape.

• DF at Joints:-

at A  $\Rightarrow DF_{AB} = 0$ , at D  $\Rightarrow DF_{DC} = 1$

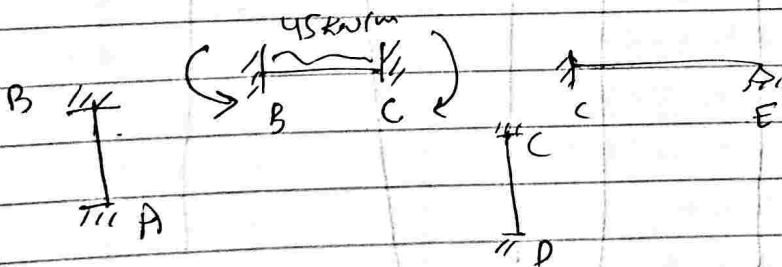
at E  $\Rightarrow DF_{EC} = 1$

at B  $\Rightarrow DF_{BA} = \frac{K_{AB}}{K_{AB} + K_{BC}} = 0.545$ ,  $DF_{BC} = \frac{K_{BC}}{K_{AB} + K_{BC}} = 0.455$

at C  $\Rightarrow DF_{CB} = \frac{K_{BC}}{K_{BC} + K_{CE} + K_{CD}} = 0.332$

$DF_{CD} = 0.298$ ,  $DF_{CE} = 0.372$

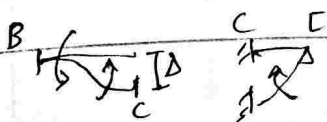
[FEM] for the members



$$FEM_{BC} = -\frac{45(6)^2}{12} = -135 \text{ kN}\cdot\text{m}$$

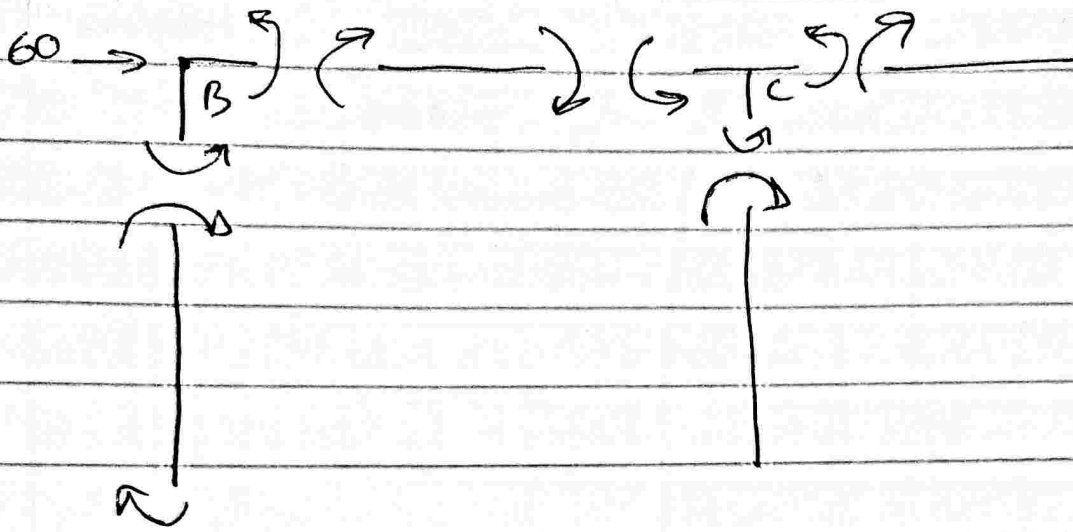
$$FEM_{CB} = \frac{45(6)^2}{12} = 135 \text{ kN}\cdot\text{m}$$

CE/BC إذا كان في كذا، The D is settled



Dis max  $\pm 0.1 \text{ kN/m}$

Joint	A	B		C			D	E
Member	AB	BA	BC	CB	CD	CE	DC	EC
End Moment								
D.F	0	0.545	0.455	0.33	0.298	0.372	1	1
FEM	0	0	-135	135	0	0		
Dist.		73.6	61.4	-44.6	-40.2	-50.2		
C.O	36.8		-22.3	30.7				
Dist.		12.15	10.146	-10.131	9.148	11.42		
C.O	6.075		-5.06	5.073				
Dis		2.75	2.31	-1.67	1.51	-1.88		
CO	1.378		-0.837	1.155				
Dis		0.456	0.381	-0.381	-0.344	0.429		
CO	0.22		-0.19	+0.19				
Dis		0.103	0.08	0.06	0.056	0.07	0	0
M	44.4	89.05	-89.07	114.12	-51.25	-63.7		

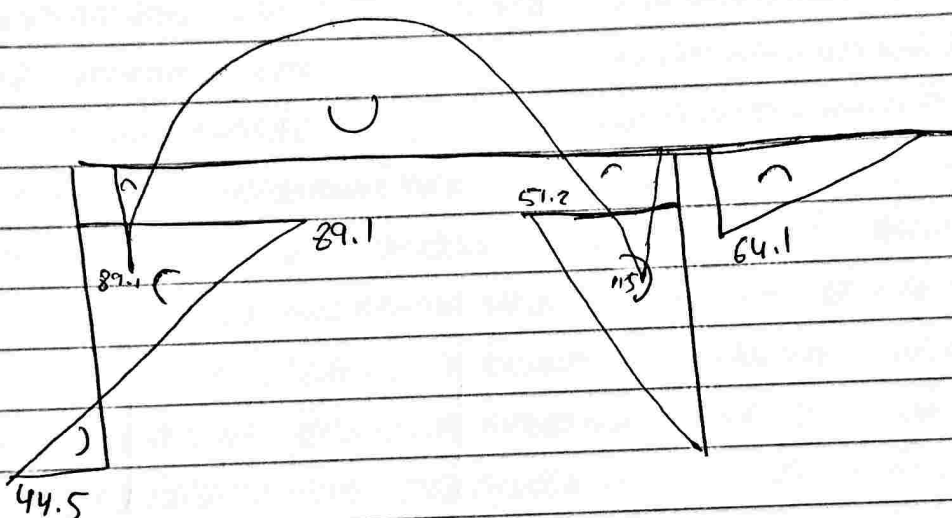
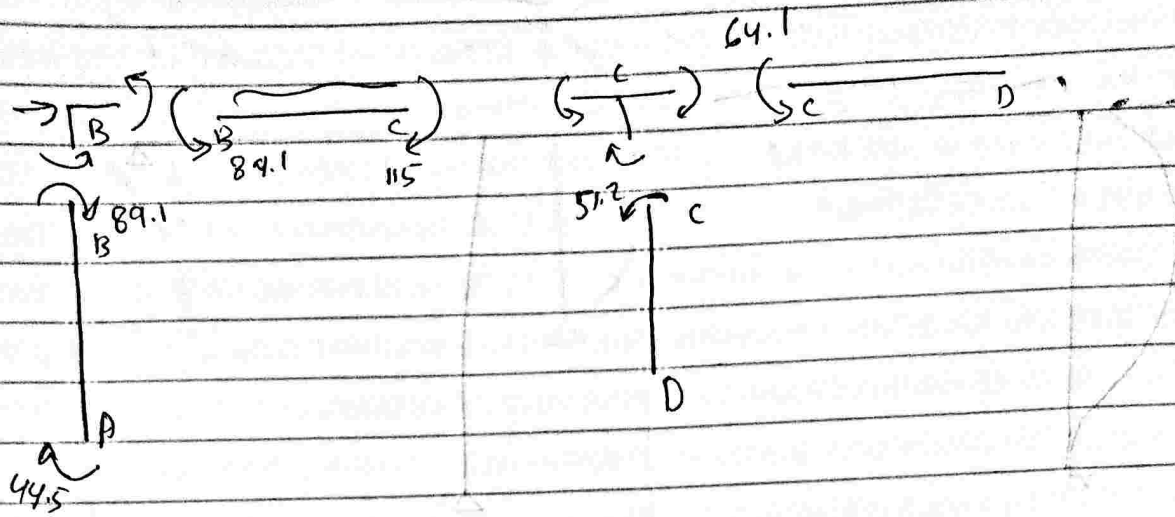


•  $\sum M_B = 0 \Rightarrow M_{BA} + M_{BC} = 0$

•  $\sum M_C = 0 \Rightarrow M_{CB} + M_{CD} + M_{CE} = 0$

الآن نكتب ما تبقى إلّا بالمدالات التي نحتاجها، لربما كنت





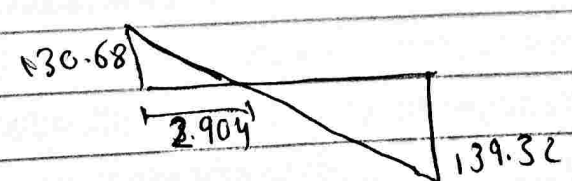
← (BC) لا نعلم توزع القم للبيج

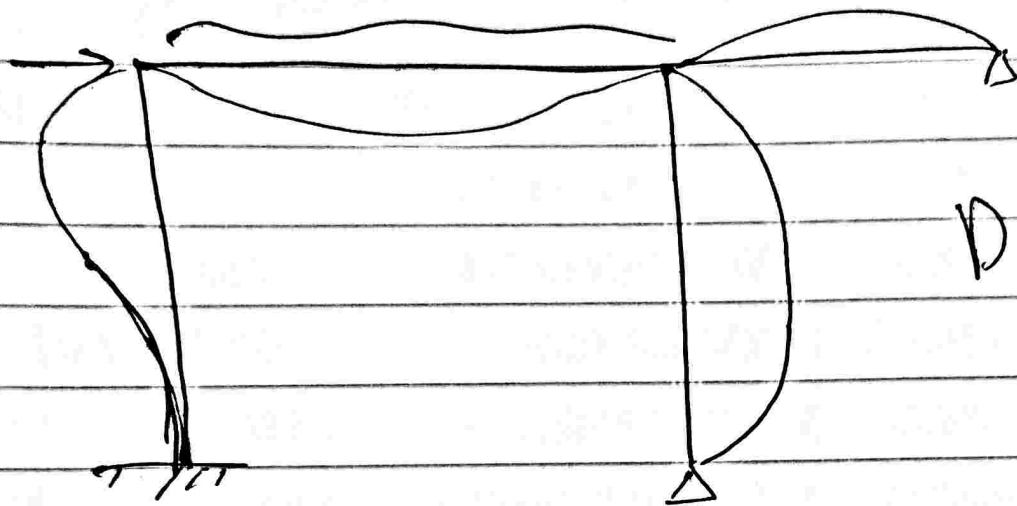
$$135 \quad \left[ \text{---} \right] \quad 135$$

$$89.1 \quad \left[ \text{---} \right] \quad 115$$

$$\frac{(115 - 89.1)}{6} = 4.32$$

$$130.68 \quad \left[ \text{---} \right] \quad 139.32$$





Deformed  
Shape