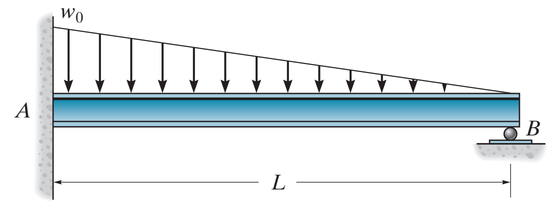
10–1. Determine the reactions at the supports A and B. EI is constant.



Support Reactions: FBD(a).

$$\xrightarrow{+} \sum F_x = 0; \qquad A_x = 0$$ **Ans.**

$$+\uparrow \sum F_y = 0; \qquad A_y + B_y - \frac{w_o L}{2} = 0 \qquad [1]$$

$$\circlearrowleft + \sum M_A = 0; \qquad B_y L + M_A - \frac{w_o L}{2}\left(\frac{L}{3}\right) = 0 \qquad [2]$$

Method of Superposition: Using the method of superposition as discussed in Chapter 4, the required displacements are

$$v_B{}' = \frac{w_o L^4}{30EI} \downarrow \qquad v_B{}'' = \frac{B_y L^3}{3EI} \uparrow$$

The compatibility condition requires

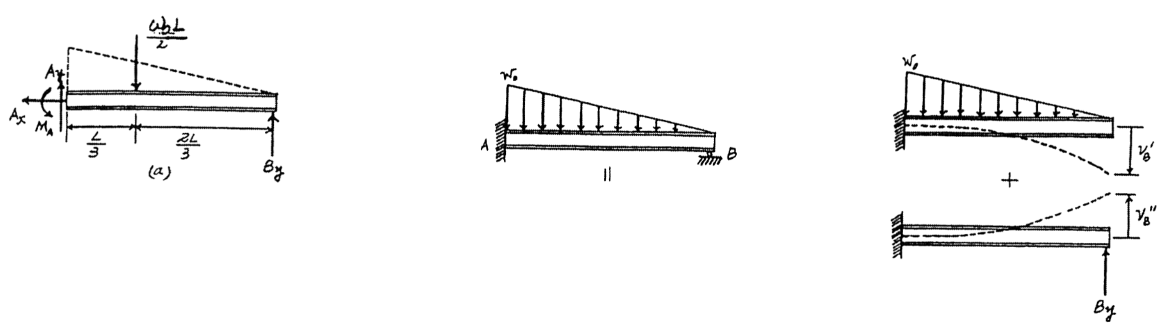
$$(+\downarrow) \qquad 0 = v_B{}' + v_B{}''$$

$$0 = \frac{w_o L^4}{30EI} + \left(-\frac{B_y L^3}{3EI}\right)$$

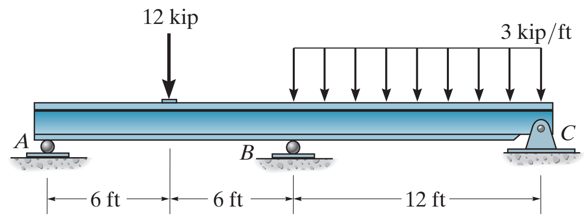
$$B_y = \frac{w_o L}{10}$$ **Ans.**

Substituting B_y into Eqs. [1] and [2] yields.

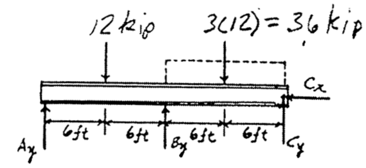
$$A_y = \frac{2w_o L}{5} \qquad M_A = \frac{w_o L^2}{15}$$ **Ans.**



10–2. Determine the reactions at the supports A, B, and C, then draw the shear and moment diagrams. EI is constant.



Support Reactions: FBD(a).



$\overset{+}{\rightarrow} \sum F_x = 0; \qquad C_x = 0$ **Ans.**

$+\uparrow \sum F_y = 0; \qquad A_y + B_y + C_y - 12 - 36.0 = 0$ [1]

$\circlearrowleft + \sum M_A = 0; \quad B_y(12) + C_y(24) - 12(6) - 36.0(18) = 0$ [2]

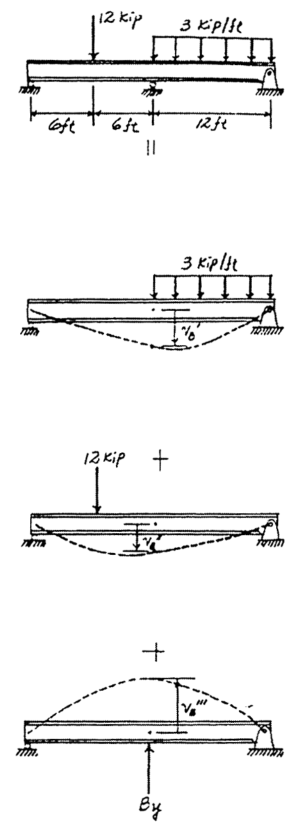
Method of Superposition: Using the method of superposition as discussed in Chapter 4, the required displacements are

$$v_B' = \frac{5wL^4}{768EI} = \frac{5(3)(24^4)}{768EI} = \frac{6480 \text{ kip} \cdot \text{ft}^3}{EI} \downarrow$$

$$v_B'' = \frac{Pbx}{6EIL}(L^2 - b^2 - x^2)$$

$$= \frac{12(6)(12)}{6EI(24)}(24^2 - 6^2 - 12^2) = \frac{2376 \text{ kip} \cdot \text{ft}^3}{EI} \downarrow$$

$$v_B''' = \frac{PL^3}{48EI} = \frac{B_y(24^3)}{48EI} = \frac{288B_y \text{ ft}^3}{EI} \uparrow$$



The compatibility condition requires

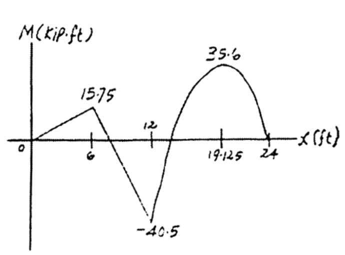
$$(+\downarrow) \qquad 0 = v_B' + v_B'' + v_B'''$$

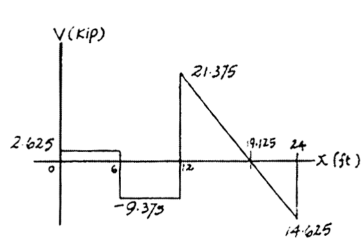
$$0 = \frac{6480}{EI} + \frac{2376}{EI} + \left(-\frac{288B_y}{EI}\right)$$

$$B_y = 30.75 \text{ kip}$$ **Ans.**

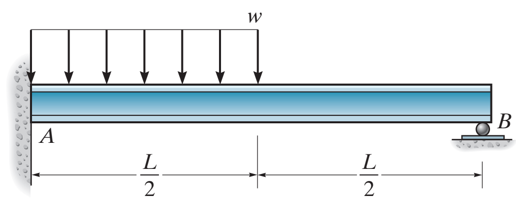
Substituting B_y into Eqs. [1] and [2] yields,

$$A_y = 2.625 \text{ kip} \qquad C_y = 14.625 \text{ kip}$$ **Ans.**

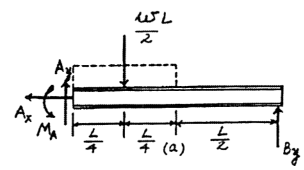




10–3. Determine the reactions at the supports A and B. EI is constant.



Support Reactions: FBD(a).



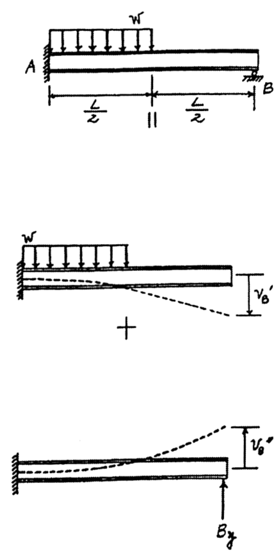
$\overset{+}{\rightarrow} \quad \sum F_x = 0; \quad A_x = 0$ **Ans.**

$+\uparrow \sum F_y = 0; \qquad A_y + B_y - \frac{wL}{2} = 0$ [1]

$\circlearrowleft + \sum M_A = 0; \qquad B_y(L) + M_A - \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) = 0$ [2]

Method of Superposition: Using the method of superposition as discussed in Chapter 4, the required displacements are

$$v_B{}' = \frac{7wL^4}{384EI}\downarrow \qquad v_B{}'' = \frac{PL^3}{3EI} = \frac{B_yL^3}{3EI}\uparrow$$



The compatibility condition requires

$$(+\downarrow) \qquad 0 = v_B{}' + v_B{}''$$

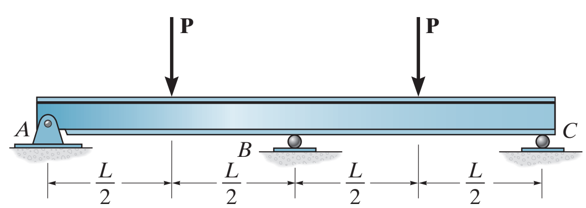
$$0 = \frac{7wL^4}{384EI} + \left(-\frac{B_yL^3}{3EI}\right)$$

$$B_y = \frac{7wL}{128}$$ **Ans.**

Substituting B_y into Eqs. [1] and [2] yields,

$$A_y = \frac{57wL}{128} \qquad M_A = \frac{9wL^2}{128}$$ **Ans.**

10–4. Determine the reactions at the supports A, B, and C; then draw the shear and moment diagrams. EI is constant.

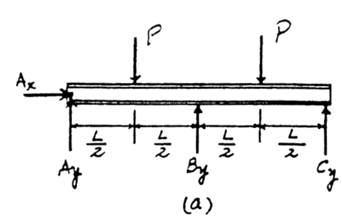


Support Reactions: FBD(a).

$\overset{+}{\rightarrow} \quad \sum F_x = 0; \qquad A_x = 0$ **Ans.**

$+\uparrow \sum F_y = 0; \qquad A_y + B_y + C_y - 2p = 0$ [1]

$\circlearrowleft + \sum M_A = 0; \qquad B_yL + C_y(2L) - P\left(\frac{L}{2}\right) - P\left(\frac{3L}{2}\right) = 0$ [2]



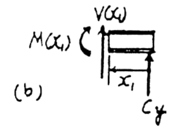
Moment Functions: FBD(b) and (c).

$$M(x_1) = C_yx_1$$

$$M(x_2) = C_yx_2 - Px_2 + \frac{PL}{2}$$

***10–4. Continued**

Slope and Elastic Curve:



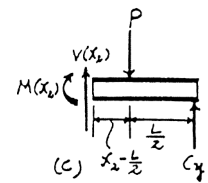
$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = C_y x_1$,

$$EI\frac{d^2v_1}{dx_1^2} = C_y x_1$$

$$EI\frac{dv_1}{dx_1} = \frac{C_y}{2}x_1^2 + C_1 \qquad [3]$$

$$EI\, v_1 = \frac{C_y}{6}x_1^3 + C_1 x_1 + C_2 \qquad [4]$$

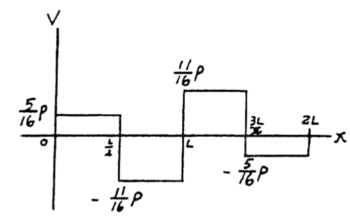


For $M(x_2) = C_y x_2 - P x_2 + \dfrac{PL}{2}$,

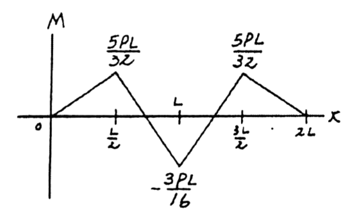
$$EI\frac{d^2v_2}{dx_2^2} = C_y x_2 - P x_2 + \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{C_y}{2}x_2^2 - \frac{P}{2}x_2^2 + \frac{PL}{2}x_2 + C_3 \qquad [5]$$

$$EIv_2 = \frac{C_y}{6}x_2^3 - \frac{P}{6}x_2^4 + \frac{PL}{4}x_2^2 + C_3 x_2 + C_4 \qquad [6]$$



Boundary Conditions:



$v_1 = 0$ *at* $x_1 = 0$. From Eq. [4] $C_2 = 0$

Due to symmetry, $\dfrac{dv_2}{dx_2} = 0$ *at* $x_2 = L$. From Eq. [5],

$$0 = \frac{C_y L^2}{2} - \frac{PL^2}{2} + \frac{PL^2}{2} + C_3 \qquad C_3 = \frac{C_y L^2}{2}$$

$v_2 = 0$ *at* $x_2 = L$. From Eq. [6],

$$0 = \frac{C_y L^3}{6} - \frac{PL^3}{6} + \frac{PL^3}{4} + \left(-\frac{C_y L^2}{2}\right)L + C_4$$

$$C_4 = \frac{C_y L^3}{3} - \frac{PL^3}{12}$$

Continuity Conditions:

At $x_1 = x_2 = \dfrac{L}{2}$, $\dfrac{dv_1}{dx_1} = \dfrac{dv_2}{dx_2}$. From Eqs. [3] and [5],

$$\frac{C_y}{2}\left(\frac{L}{2}\right)^2 + C_1 = \frac{C_y}{2}\left(\frac{L}{2}\right)^2 - \frac{P}{2}\left(\frac{L}{2}\right)^2 + \frac{PL}{2}\left(\frac{L}{2}\right) - \frac{C_y L^2}{2}$$

$$C_1 = \frac{PL^2}{8} - \frac{C_y L^2}{2}$$

At $x_1 = x_2 = \dfrac{L}{2}$, $v_1 = v_2$. From Eqs. [4] and [6].

$$\frac{C_y}{6}\left(\frac{L}{2}\right)^3 + \left(\frac{PL^2}{8} - \frac{C_y L^2}{2}\right)\left(\frac{L}{2}\right)$$

***10–4. Continued**

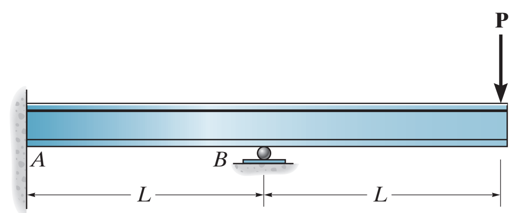
$$= \frac{C_y}{6}\left(\frac{L}{2}\right)^3 - \frac{P}{6}\left(\frac{L}{2}\right)^3 + \frac{PL}{4}\left(\frac{L}{2}\right)^2 + \left(-\frac{C_yL^2}{2}\right)\left(\frac{L}{2}\right) + \frac{C_yL^3}{3} - \frac{PL^3}{12}$$

$$C_y = \frac{5}{16}P \qquad \textbf{Ans.}$$

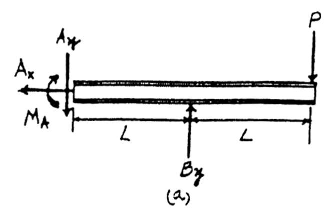
Substituting C_y into Eqs. [1] and [2],

$$B_y = \frac{11}{8}P \quad A_y = \frac{5}{16}P \qquad \textbf{Ans.}$$

10–5. Determine the reactions at the supports, then draw the shear and moment diagram. *EI* is constant.



Support Reactions: FBD(a) .

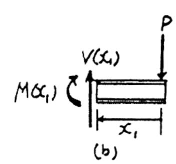


$\xrightarrow{+} \sum F_x = 0; \qquad A_x = 0 \qquad$ **Ans.**

$+\uparrow \sum F_y = 0; \qquad B_y - A_y - P = 0 \qquad [1]$

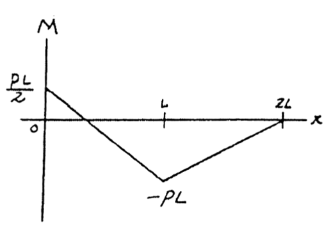
$\circlearrowleft + \sum M_B = 0; \qquad A_yL - M_A - PL = 0 \qquad [2]$

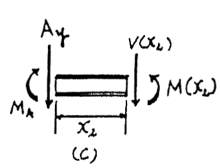
Moment Functions: FBD(b) and (c).



$$M(x_1) = -Px_1$$

$$M(x_2) = M_A - A_yx_2$$





10–5. Continued

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = -Px_1$.

$$EI\frac{d^2v_1}{dx_1^2} = -Px_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{P}{2}x_1^2 + C_1 \qquad [3]$$

$$EI\,v_1 = -\frac{P}{6}x_1^3 + C_1x_1 + C_2 \qquad [4]$$

For $M(x_2) = M_A - A_yx_2$

$$EI\frac{d^2v_2}{dx_2^2} = M_A - A_yx_2$$

$$EI\frac{dv_2}{dx_2} = M_Ax_2 - \frac{A_y}{2}x_2^2 + C_3 \qquad [5]$$

$$EI\,v_2 = \frac{M_A}{2}x_2^2 - \frac{A_y}{6}x_2^3 + C_3x_2 + C_4 \qquad [6]$$

Boundary Conditions:

$v_2 = 0$ *at* $x_2 = 0$. From Eq. [6], $C_4 = 0$

$\frac{dv_2}{dx_2} = 0$ at $x_2 = 0$. From Eq. [5], $C_3 = 0$

$v_2 = 0$ at $x_2 = L$. From Eq. [6].

$$0 = \frac{M_AL^2}{2} - \frac{A_yL^3}{6} \qquad [7]$$

Solving Eqs. [2] and [7] yields.

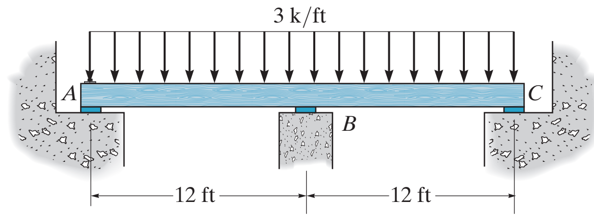
$$M_A = \frac{PL}{2} \qquad A_y = \frac{3P}{2}$$ **Ans.**

Substituting the value of A_y into Eq. [1],

$$B_y = \frac{5P}{2}$$ **Ans.**

Note: The other boundary and continuity conditions can be used to determine the constants C_1 and C_2 which are not needed here.

10–6. Determine the reactions at the supports, then draw the moment diagram. Assume B and C are rollers and A is pinned. The support at B settles downward 0.25 ft. Take $E = 29(10^3)$ ksi, $I = 500\ \text{in}^4$.



Compatibility Equation. Referring to Fig. a,

$$\Delta'_B = \frac{5wL_{AC}^4}{384EI} = \frac{5(3)(24^4)}{384EI} = \frac{12960\ \text{k}\cdot\text{ft}^3}{EI}$$

$$= \frac{12960(12^3)\ \text{k}\cdot\text{in}^3}{[29(10^3)\ \text{k/in}^2](500\ \text{in}^4)}$$

$$= 1.544\ \text{in} \downarrow$$

$$f_{BB} = \frac{L_{AC}^3}{48EI} = \frac{24^3}{48EI} = \frac{288\ \text{ft}^3}{EI}$$

$$= \frac{288(12^3)\ \text{in}^3}{[29(10^3)\ \text{k/in}^2](500\ \text{in}^4)}$$

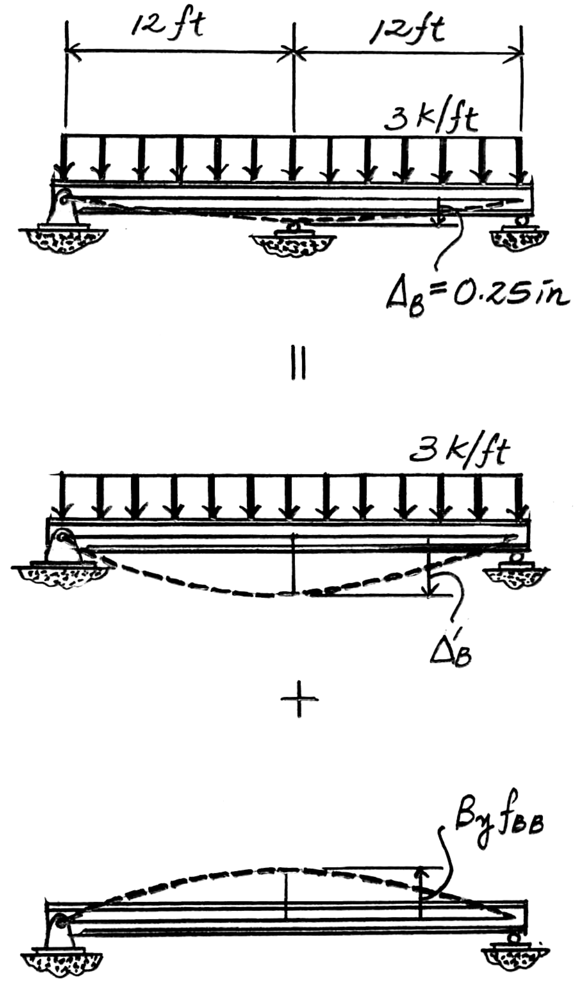
$$= 0.03432\ \frac{\text{in}}{\text{k}} \uparrow$$

Using the principle of superposition,

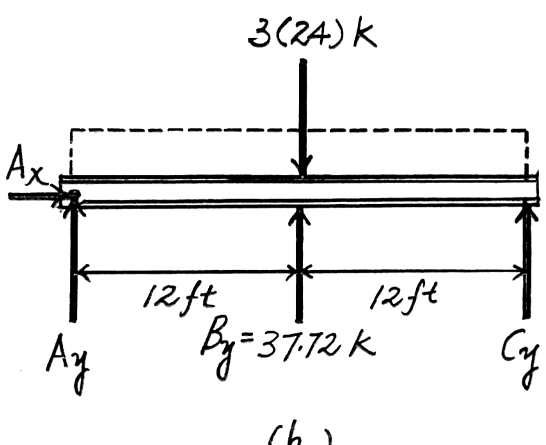
$$\Delta_B = \Delta'_B + B_y f_{BB}$$

$$(+\downarrow)\ 0.25\ \text{in} = 1.544\ \text{in} + B_y\left(-0.03432\ \frac{\text{in}}{\text{k}}\right)$$

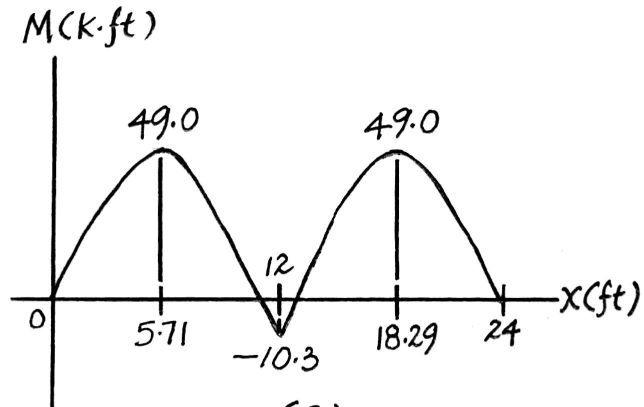
$$B_y = 37.72\,\text{k} = 37.7\,\text{k}$$ **Ans.**



(a)



(b)



(c)

10–6. Continued

Equilibrium. Referring to the FBD in Fig. *b*

$\xrightarrow{+} \sum F_x = 0; \quad A_x = 0$ **Ans.**

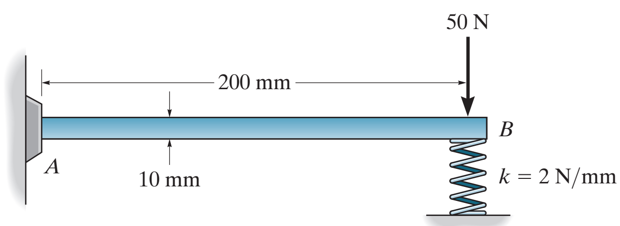
$\zeta + \sum M_A = 0; \quad C_y(24) + 37.72(12) - 3(24)(12) = 0$

$C_y = 17.14 \text{ k} = 17.1 \text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad A_y + 37.72 + 17.14 - 3(24) = 0$

$A_y = 17.14 \text{ k} = 17.1 \text{ k}$ **Ans.**

10–7. Determine the deflection at the end *B* of the clamped A-36 steel strip. The spring has a stiffness of $k = 2$ N/mm. The strip is 5 mm wide and 10 mm high. Also, draw the shear and moment diagrams for the strip.



$$I = \frac{1}{12}(0.005)(0.01)^3 = 0.4166(10^{-9}) \text{ m}^4$$

$$(\Delta_B)_1 = \frac{PL^3}{3EI} = \frac{50(0.2^3)}{3(200)(10^9)(0.4166)(10^{-9})} = 0.0016 \text{ m}$$

$$(\Delta_B)_2 = \frac{PL^3}{3EI} = \frac{2000\Delta_B(0.2^3)}{3(200)(10^9)(0.4166)(10^{-9})} = 0.064\ \Delta_B$$

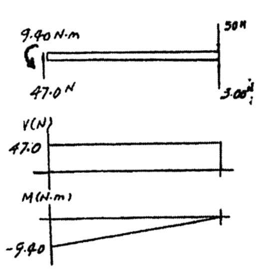
Compatibility Condition:

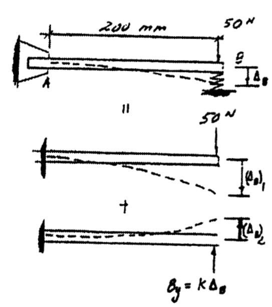
$+\downarrow \Delta_B = (\Delta_B)_1 - (\Delta_B)_2$

$\Delta_B = 0.0016 - 0.064\Delta_B$

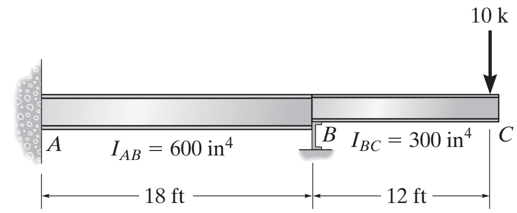
$\Delta_B = 0.001503 \text{ m} = 1.50 \text{ mm}$ **Ans.**

$B_y = k\Delta_B = 2(1.5) = 3.00 \text{ N}$

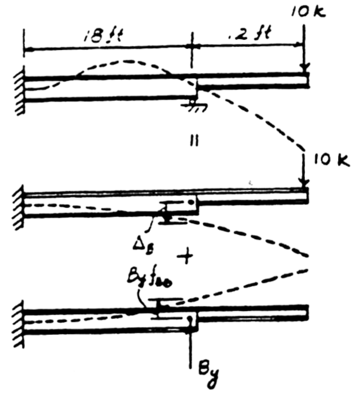




***10–8.** Determine the reactions at the supports. The moment of inertia for each segment is shown in the figure. Assume the support at B is a roller. Take $E = 29(10^3)$ ksi.



Compatibility Equation:



$(+\downarrow) \qquad \Delta_B - B_y f_{BB} = 0 \qquad (1)$

Use conjugate beam method:

$\circlearrowleft + \sum M_B' = 0; \qquad M_B' + \frac{2160}{EI_{AB}}(9) + \frac{1620}{EI_{AB}}(12) = 0$

$$\Delta_B = M_B' = -\frac{38\,880}{EI_{AB}}$$

$\circlearrowleft + \sum M_B' = 0; \qquad M_B' - \frac{162}{EI_{AB}}(12) = 0$

$$f_{BB} = M_B' = \frac{1944}{EI_{AB}}$$

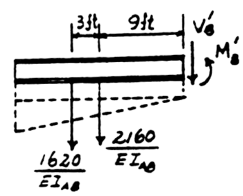
From Eq. 1 $\qquad \frac{38\,880}{EI_{AB}} - \frac{1944}{EI_{AB}} B_y = 0$

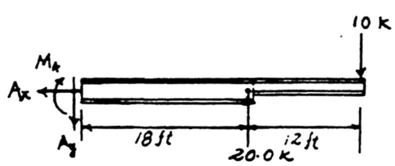
$B_y = 20 \text{ k}$ **Ans.**

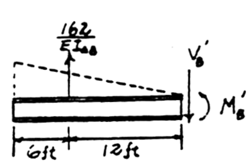
$A_y = 10 \text{ k}$ **Ans.**

$M_A = 60 \text{ k} \cdot \text{ft}$ **Ans.**

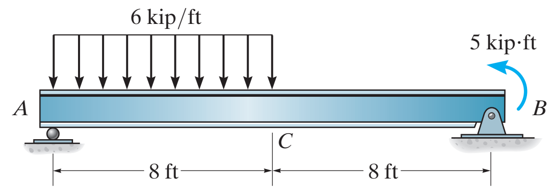
$A_x = 0$ **Ans.**







10–9. The simply supported beam is subjected to the loading shown. Determine the deflection at its center *C*. *EI* is constant.



Elastic Curves: The elastic curves for the uniform distributed load and couple moment are drawn separately as shown.

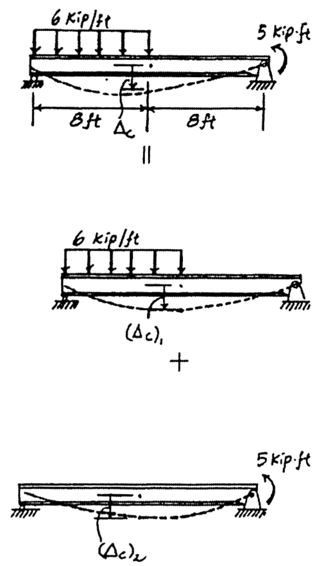
Method of Superposition: Using the method of superposition as discussed in Chapter 4, the required displacements are

$$(\Delta_C)_1 = \frac{-5wL^4}{768EI} = \frac{-5(6)(16^4)}{768EI} = \frac{2560 \text{ kip}\cdot\text{ft}^3}{EI}\downarrow$$

$$(\Delta_C)_2 = \frac{M_o x}{6EIL}(x^2 - 3Lx + 2L^2)$$

$$= -\frac{5(8)}{6EI(16)}[8^2 - 3(16)(8) + 2(16^2)]$$

$$= \frac{80 \text{ kip}\cdot\text{ft}^3}{EI}\downarrow$$



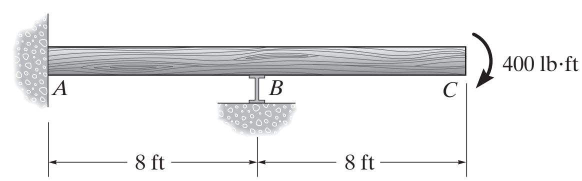
The displacement at C is

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2$$

$$= \frac{2560}{EI} + \frac{80}{EI}$$

$$= \frac{2640 \text{ kip}\cdot\text{ft}^3}{EI}$$ **Ans.**

10–10. Determine the reactions at the supports, then draw the moment diagram. Assume the support at *B* is a roller. *EI* is constant.



Compatibility Equation:

$$(+\downarrow) \qquad \Delta_B - 2_B - B_y f_{BB} = 0 \qquad (1)$$

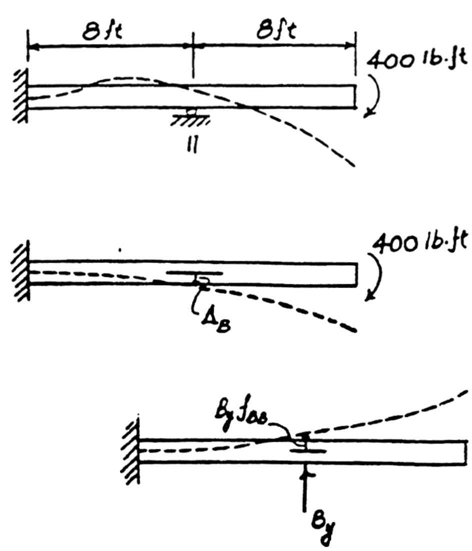
Use conjugate beam method:

$$\circlearrowleft + \sum M_{B'} = 0; \qquad M_{B'} + \frac{3200}{EI}(4) = 0$$

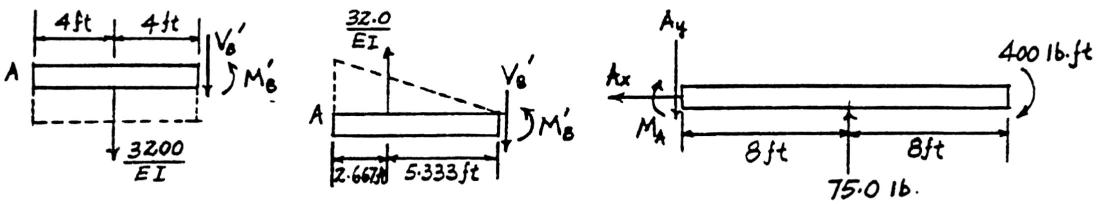
$$\Delta_B = M_{B'} = -\frac{12\,800}{EI}$$

$$\circlearrowleft + \sum M_{B'} = 0; \qquad M_{B'} - \frac{32}{EI}(5.333) = 0$$

$$f_{BB} = M_{B'} = \frac{170.67}{EI}$$



10–10. Continued



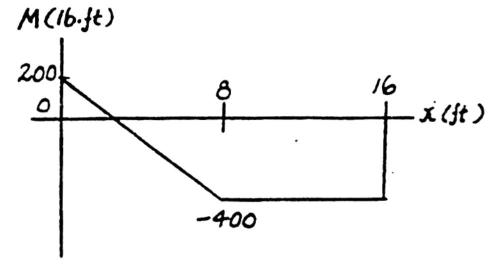
From Eq. 1 $\frac{12\,800}{EI} - B_y\left(\frac{170.67}{EI}\right) = 0$

$B_y = 75 \text{ lb}$ **Ans.**

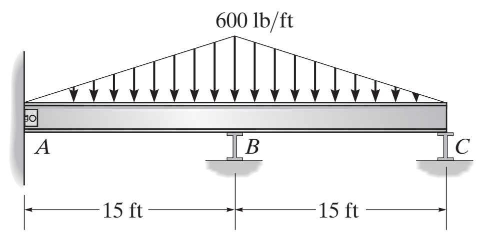
$A_x = 0$ **Ans.**

$A_y = 75 \text{ lb}$ **Ans.**

$M_A = 200 \text{ lb} \cdot \text{ft}$ **Ans.**



10–11. Determine the reactions at the supports, then draw the moment diagram. Assume *A* is a pin and *B* and *C* are rollers. *EI* is constant.



Compatibility Equation:

$(+\downarrow) \qquad \Delta_B - B_y f_{BB} = 0 \qquad (1)$

Use virtual work method:

$$\Delta_B = \int_0^L \frac{mM}{EI}dx = 2\int_0^{15} \frac{(4.5x - 0.00667x^3)(-0.5x)}{EI}dx = -\frac{4050}{EI}$$

$$f_{BB} = \int_0^L \frac{mm}{EI}dx = 2\int_0^{15} \frac{(-0.5x)^2}{EI}dx = \frac{562.5}{EI}$$

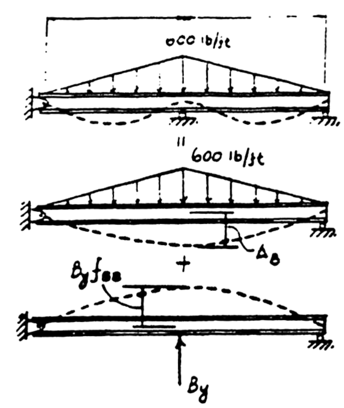
From Eq. 1 $\frac{4050}{EI} - B_y \frac{562.5}{EI} = 0$

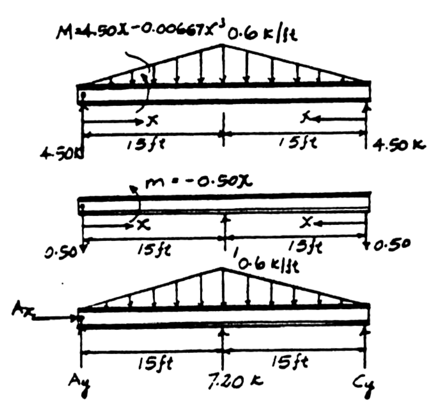
$B_y = 7.20 \text{k}$ **Ans.**

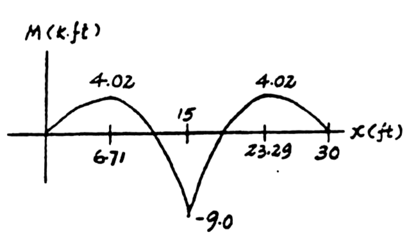
$A_y = 0.900 \text{k}$ **Ans.**

$A_x = 0$ **Ans.**

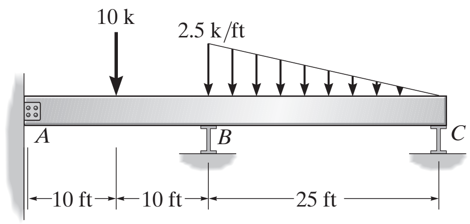
$C_y = 0.900 \text{k}$ **Ans.**

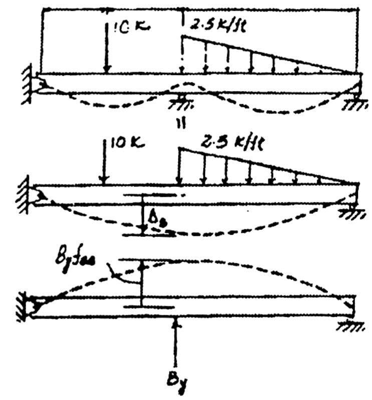






***10–12.** Determine the reactions at the supports, then draw the moment diagram. Assume the support at *A* is a pin and *B* and *C* are rollers. *EI* is constant.





Compatibility Equation:

$$(+\downarrow) \qquad \Delta_B - B_y f_{BB} = 0 \qquad (1)$$

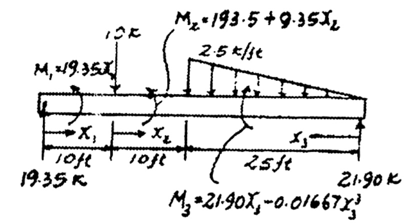
Use virtual work method:

$$\Delta_B = \int_0^L \frac{mM}{EI}dx = \int_0^{10} \frac{(-0.5556x_1)(19.35x_1)}{EI}dx_1$$

$$+ \int_0^{10} \frac{(-5.556 - 0.5556x_2)(193.5 + 9.35x_2)}{EI}dx_2$$

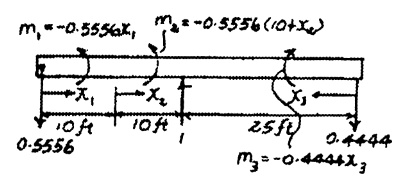
$$+ \int_0^{25} \frac{(-0.4444x_3)(21.9x_3 - 0.01667x_3^3)}{EI}dx_3$$

$$= -\frac{60\,263.53}{EI}$$



$$f_{BB} = \int_0^{10} \frac{(-0.5556x_1)^2}{EI}dx_1 + \int_0^{25} \frac{(-0.4444x_3)^2}{EI}dx_3 + \int_0^{10} \frac{(-5.556 - 0.5556x_2)^2}{EI}dx_2$$

$$= \frac{1851.85}{EI}$$



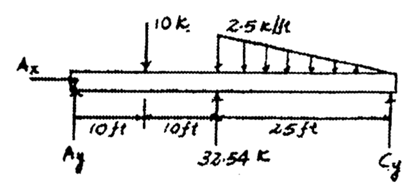
From Eq. 1 $\qquad \dfrac{60\,262.53}{EI} - B_y\dfrac{1851.85}{EI} = 0$

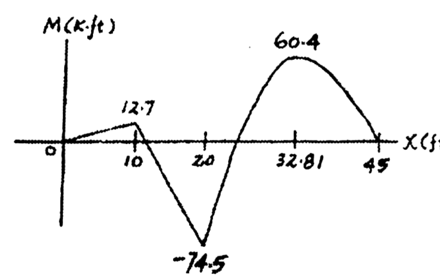
$B_y = 32.5\,\text{k}$ **Ans.**

$A_x = 0$ **Ans.**

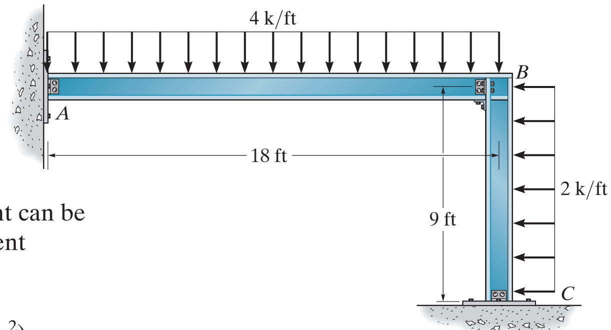
$A_y = 1.27\,\text{k}$ **Ans.**

$C_y = 7.44\,\text{k}$ **Ans.**





10–13. Determine the reactions at the supports. Assume A and C are pins and the joint at B is fixed connected. EI is constant.



Compatibility Equation: Referring to Fig *a*, the necessary displacement can be determined using virtual work method, using the real and virtual moment functions shown in Fig. *b* and *c*,

$$\Delta'_{C_n} = \int_0^L \frac{mM}{EI}\,dx = \int_0^{18\text{ ft}} \frac{(0.5x_1)(31.5x_1 - 2x_1^2)}{EI}\,dx_1 + \int_0^{9\text{ ft}} \frac{(x_2)(-x_2^2)}{EI}\,dx_2$$

$$= \frac{2733.75}{EI} \quad \rightarrow$$

$$f_{CC} = \int_0^L \frac{mm}{EI}\,dx = \int_0^{18\text{ ft}} \frac{(0.5x_1)(0.5x_1)}{EI}\,dx_1 + \int_0^{9\text{ ft}} \frac{(x_2)(x_2)}{EI}\,dx_2$$

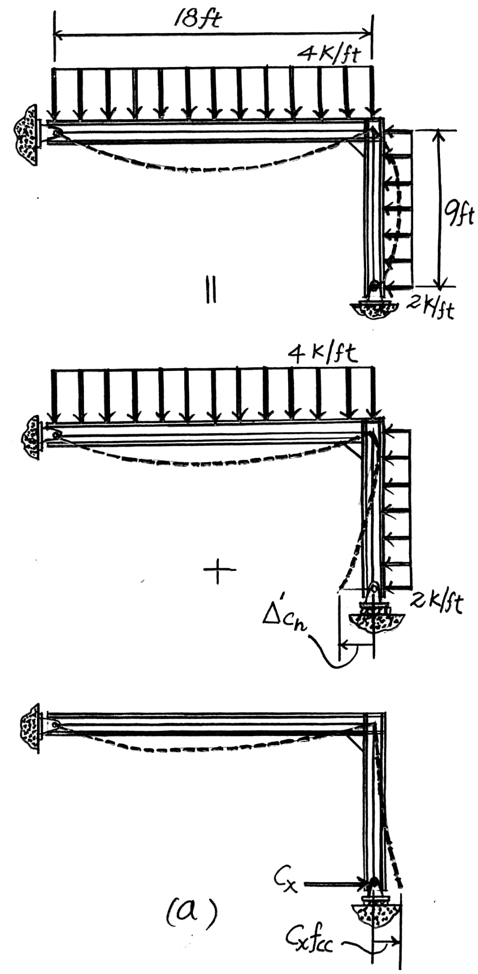
$$= \frac{729}{EI} \quad \rightarrow$$

Using the principle of superposition,

$$\Delta_{C_n} = \Delta'_{Cn} + C_x f_{CC}$$

$$O = \frac{2733.75}{EI} + C_x\left(\frac{729}{EI}\right)$$

$$C_x = -3.75\text{k} = 3.75\text{k} \quad \leftarrow$$ **Ans.**



Equilibrium: Referring to the FBD of the frame in Fig. *d*,

$\xrightarrow{+} \sum F_x = 0;\quad A_x - 2(9) - 3.75 = 0$

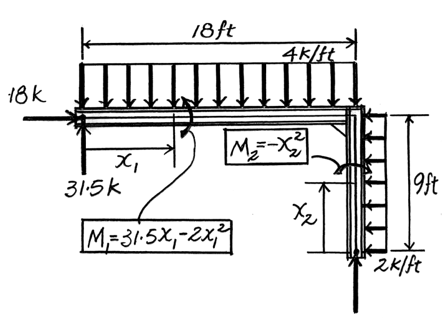
$A_x = 21.75\text{k}$ **Ans.**

$\circlearrowleft + \sum M_A = 0;\quad C_y(18) - 4(18)(9) - 2(9)(4.5) - 3.75(9) = 0$

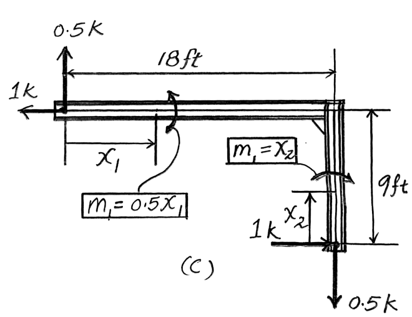
$C_y = 42.375\text{k} = 42.4\text{k}$ **Ans.**

$+\uparrow \sum F_y = 0;\quad A_y + 42.375 - 4(18) = 0$

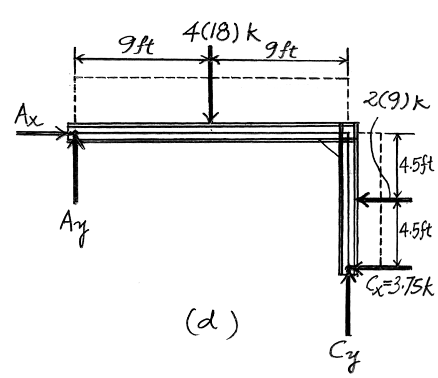
$A_y = 29.625\text{k} = 29.6\text{k}$ **Ans.**



(b)

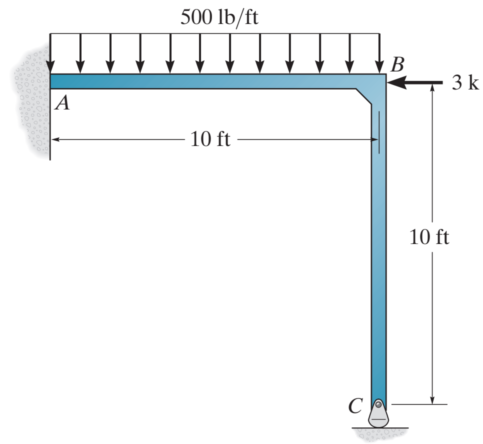


(c)



(d)

10–14. Determine the reactions at the supports. EI is constant.



Compatibility Equation:

$$(+\downarrow) \qquad 0 = \Delta_C - C_y f_{CC} \qquad (1)$$

Use virtual work method

$$\Delta_C = \int_0^L \frac{mM}{EI}dx = \int_0^{10} \frac{(x_1)(-0.25x_1^2)}{EI}dx_1 = \frac{-625}{EI}$$

$$f_{CC} = \int_0^L \frac{mm}{EI}dx = \int_0^{10} \frac{(x_1)^2}{EI}dx_1 = \frac{333.33}{EI}$$

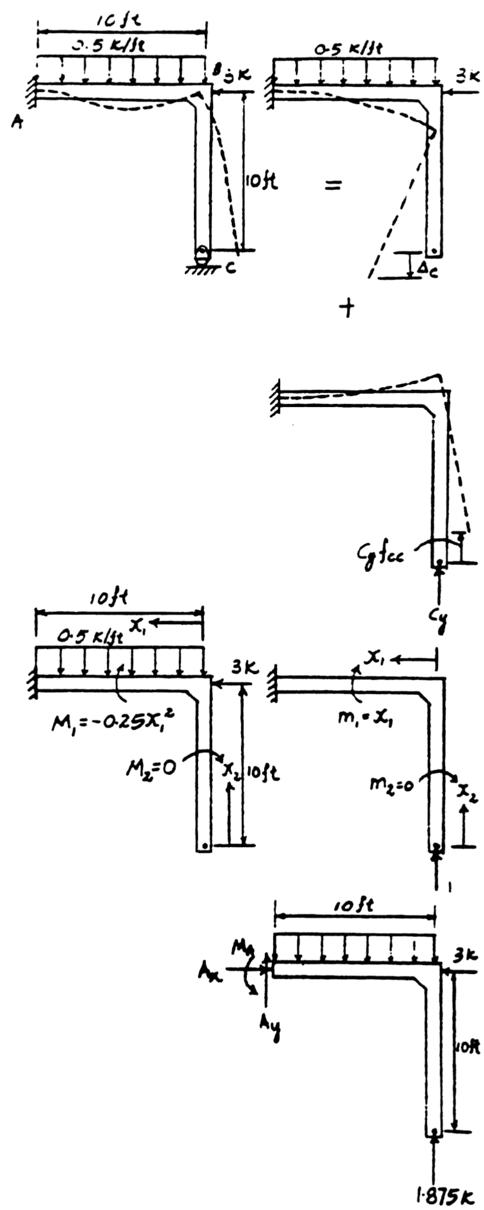
From Eq. 1 $\qquad 0 = \frac{625}{EI} - \frac{333.33}{EI}C_y$

$C_y = 1.875\,\text{k}$ **Ans.**

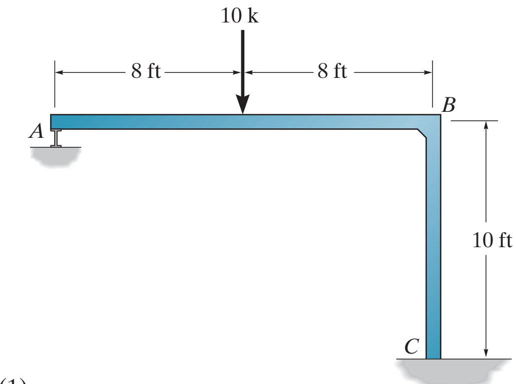
$A_x = 3.00\,\text{k}$ **Ans.**

$A_y = 3.125\,\text{k}$ **Ans.**

$M_A = 6.25\,\text{k}\cdot\text{ft}$ **Ans.**



10–15. Determine the reactions at the supports, then draw the moment diagram for each member. *EI* is constant.



Compatibility Equation:

$(+\downarrow) \qquad 0 = \Delta_A - A_y f_{AA} \qquad (1)$

Use virtual work method

$$\Delta_A = \int_0^L \frac{mM}{EI}dx = \int_0^8 \frac{(8 + x_2)(-10x_2)}{EI}dx_2 + \int_0^{10} \frac{(16)(-80)}{EI}dx_3 = \frac{-17066.67}{EI}$$

$$f_{AA} = \int_0^L \frac{mm}{EI}dx = \int_0^8 \frac{(x_1)^2}{EI}dx_1 + \int_0^8 \frac{(8 + x_2)^2}{EI}dx_2 + \int_0^{10} \frac{(16)^2}{EI}dx_3 = \frac{3925.33}{EI}$$

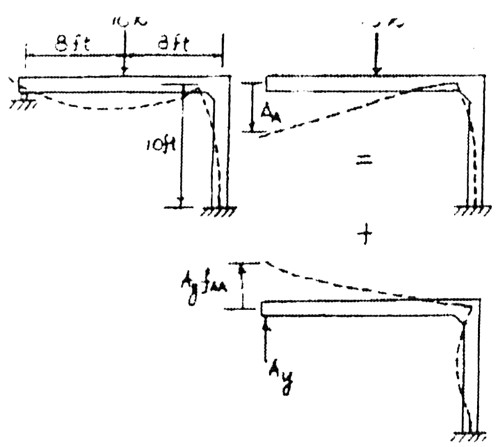
From Eq. 1 $\qquad 0 = \frac{17066.67}{EI} - \frac{3925.33}{EI}A_y$

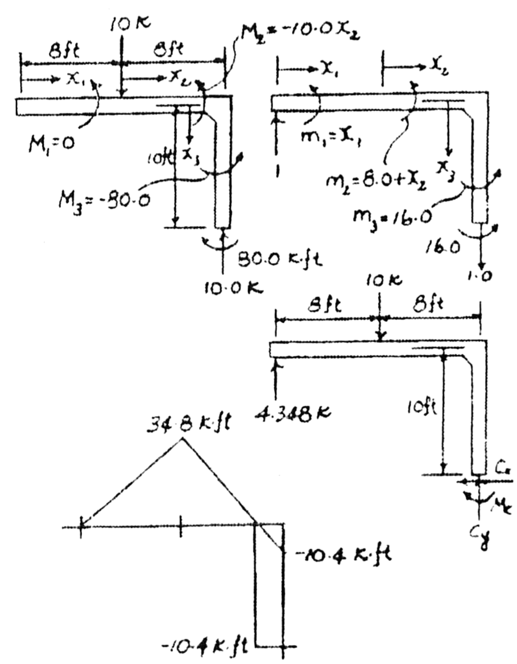
$A_y = 4.348\,\text{k} = 4.35\,\text{k}$ **Ans.**

$C_x = 0\,\text{k}$ **Ans.**

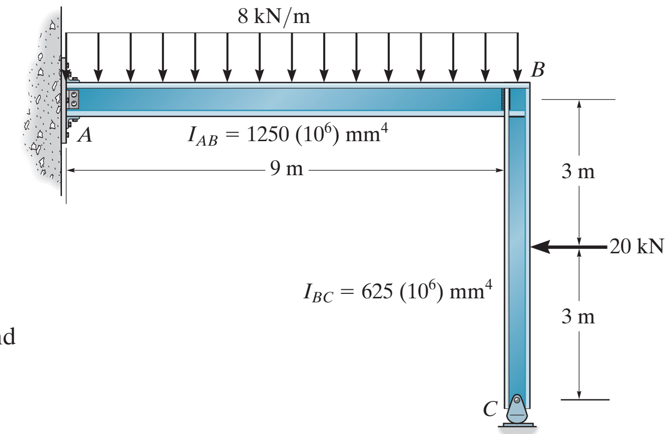
$C_y = 5.65\,\text{k}$ **Ans.**

$M_C = 10.4\,\text{k} \cdot \text{ft}$ **Ans.**





***10–16.** Determine the reactions at the supports. Assume A is fixed connected. E is constant.



Compatibility Equation. Referring to Fig. *a*, and using the real and virtual moment function shown in Fig. *b* and *c*, respectively,

$$\Delta'_{C_v} = \int_0^L \frac{mM}{EI}dx = \int_0^{9\,m} \frac{(-x_3)[-(4x_3^2 + 60)]}{EI_{AB}}dx_3 = \frac{8991}{EI_{AB}} \downarrow$$

$$f_{CC} = \int_0^L \frac{mm}{EI}dx = \int_0^{9\,m} \frac{(-x_3)(-x_3)}{EI_{AB}}dx_3 = \frac{243}{EI_{AB}} \downarrow$$

Using the principle of superposition,

$$\Delta_{C_v} = \Delta'_{C_v} + C_y f_{CC}$$

$$(+\downarrow) \quad 0 = \frac{8991}{EI_{AB}} + C_y\left(\frac{243}{EI_{AB}}\right)$$

$$C_y = -37.0\text{ kN} = 37.0\text{ kN} \uparrow \qquad \textbf{Ans.}$$

Equilibrium. Referring to the FBD of the frame in Fig. *d*,

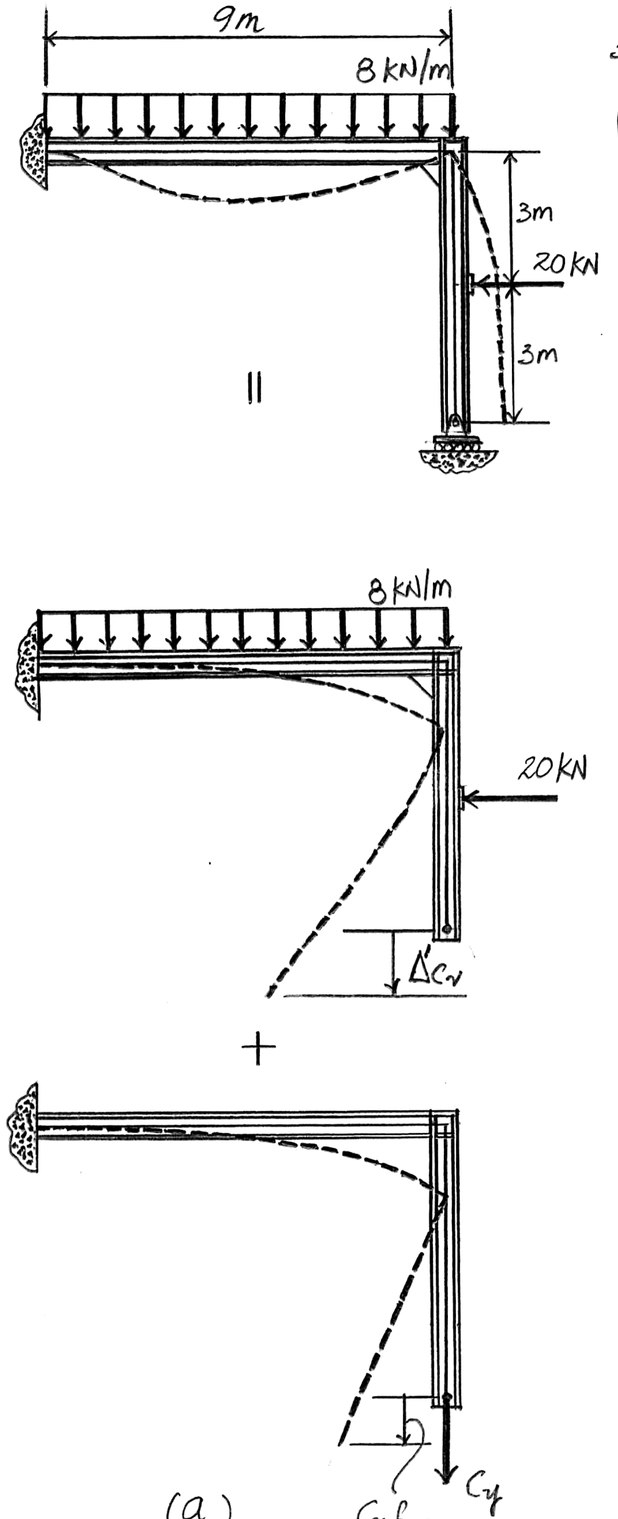
$\overset{+}{\rightarrow} \sum F_x = 0; \quad A_x - 20 = 0 \quad A_x = 20\text{ kN}$ **Ans.**

$\circlearrowleft + \sum M_A = 0; \quad M_A + 37.0(9) - 8(9)(4.5) - 20(3) = 0$

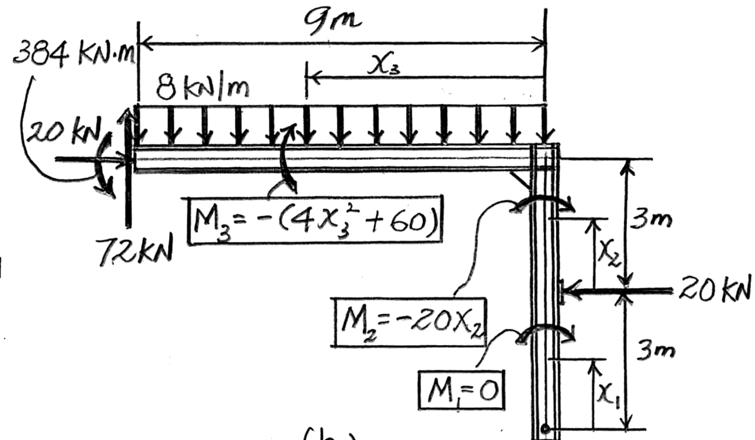
$M_A = 51.0\text{ kN}\cdot\text{m}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad A_y + 37.0 - 8(9) = 0 \quad A_y = 35.0\text{ kN}$ **Ans.**

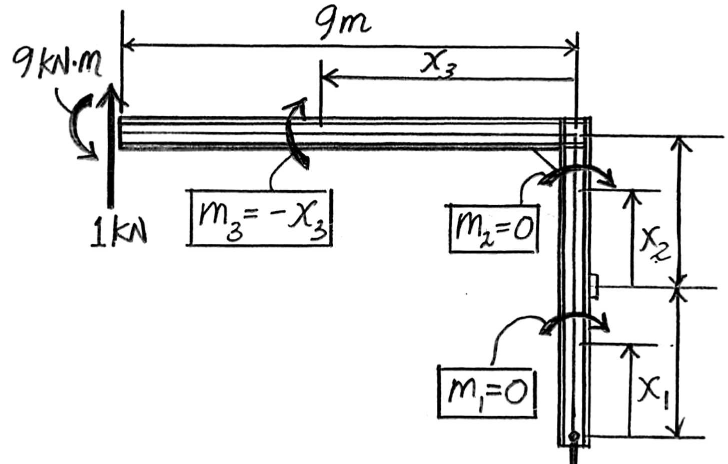
10–16. Continued



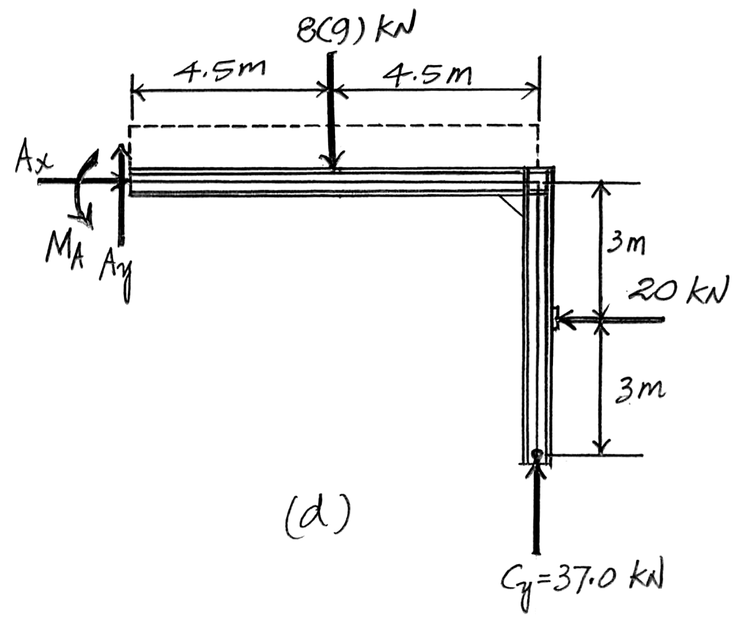
(a)



(b)

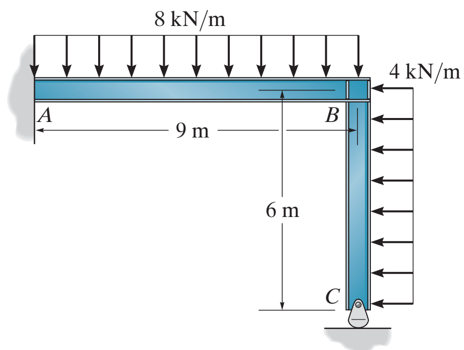


(c)



(d)

10–17. Determine the reactions at the supports. EI is constant.



***Compatibility Equation*:**

$$(+\downarrow) \qquad 0 = \Delta_C - C_y f_{CC} \qquad (1)$$

Use virtual work method:

$$\Delta_C = \int_0^L \frac{mM}{EI}dx = \int_0^9 \frac{(-x_1 + 9)(72x_1 - 4x_1^2 - 396)}{EI}dx_1 = \frac{-9477}{EI}$$

$$f_{CC} = \int_0^L \frac{mm}{EI}dx = \int_o^9 \frac{(-x_1 + 9)^2}{EI}dx_1 = \frac{243.0}{EI}$$

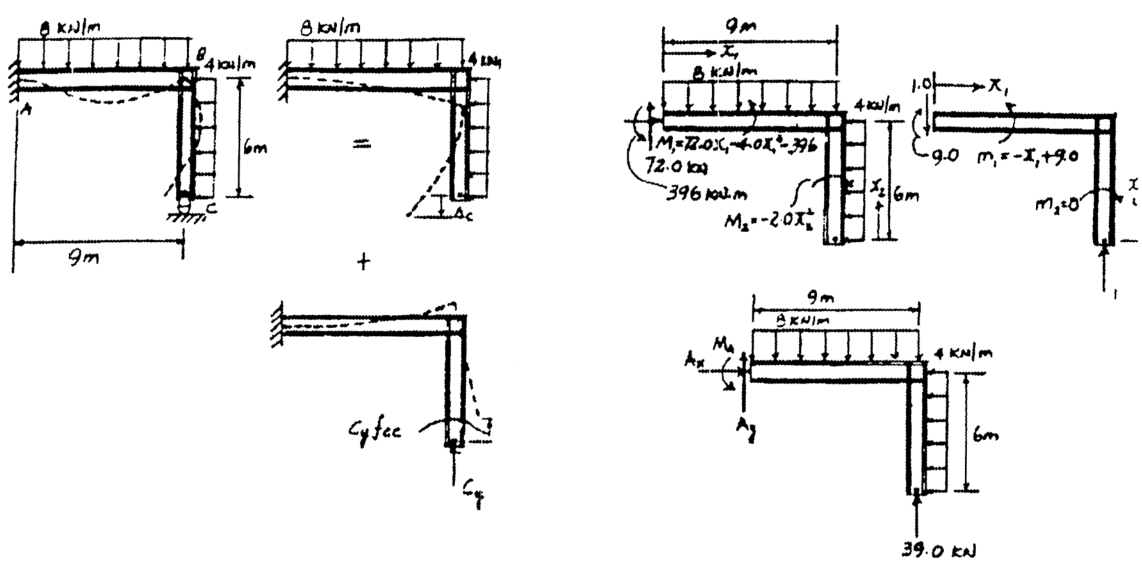
From Eq. 1 $\qquad 0 = \dfrac{9477}{EI} - \dfrac{243.0}{EI}C_y$

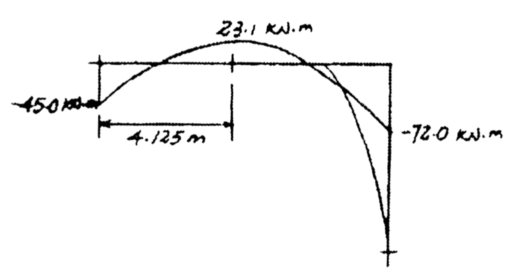
$$C_y = 39.0 \text{ kN}$$

$$A_y = 33.0 \text{ kN}$$

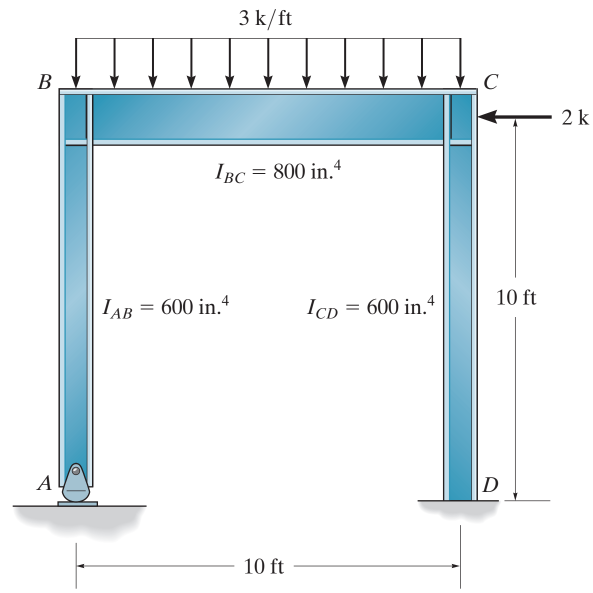
$$A_x = 24.0 \text{ kN}$$

$$M_A = 45.0 \text{ kN} \cdot \text{m}$$





10–18. Determine the reactions at the supports A and D. The moment of inertia of each segment of the frame is listed in the figure. Take $E = 29(10^3)$ ksi.



$$\Delta_A = \int_0^L \frac{mM}{EI}dx = 0 + \int_0^{10} \frac{(lx)(\frac{3}{2}x^2)}{EI_{BC}}dx + \int_0^{10} \frac{(10)(170\text{–}2x)}{EI_{CD}}dx$$

$$= \frac{18.8125}{EI_{CD}}$$

$$f_{AA} = \int_0^L \frac{m^2}{EI}dx = 0 + \int_0^{10} \frac{x^2}{EI_{BC}}dx + \int_0^{10} \frac{10^2}{EI_{CD}}dx = \frac{1250}{EI_{CD}}$$

$$+\downarrow \Delta_A + A_y f_{AA} = 0$$

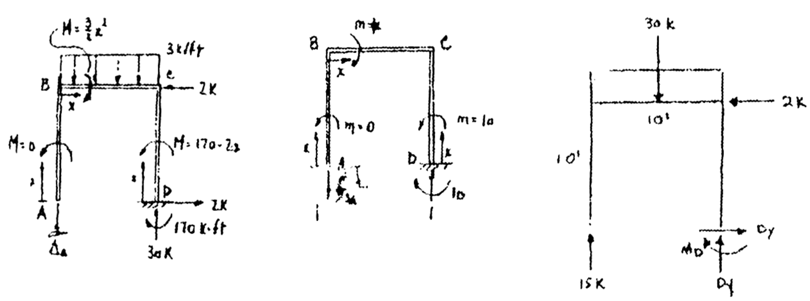
$$\frac{18{,}812.5}{EI_{CD}} + A_y\left(\frac{1250}{EI_{CD}}\right) = 0$$

$A_y = -15.0$ k **Ans.**

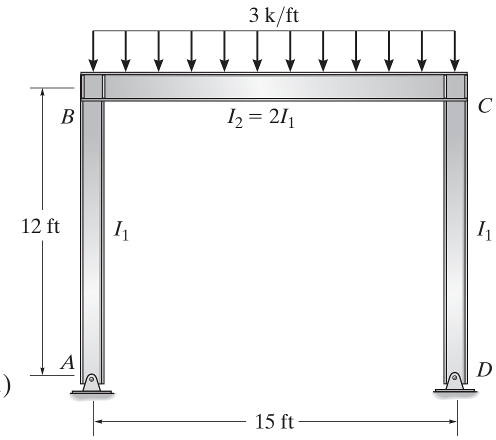
$+\uparrow \sum F_y = 0;\quad -30 + 15 + D_y = 0;\quad D_y = 15.0$ k **Ans.**

$\xrightarrow{+} \sum F_x = 0;\quad D_x = 2$ k **Ans.**

$\circlearrowleft + \sum M_D = 0;\quad 15.0(10) - 2(10) - 30(5) + M_D = 0;\quad M_D = 19.5$ k·ft **Ans.**



10–19. The steel frame supports the loading shown. Determine the horizontal and vertical components of reaction at the supports A and D. Draw the moment diagram for the frame members. E is constant.



Compatibility Equation:

$$\Delta_D + D_x f_{DD} = 0 \qquad (1)$$

Use virtual work method:

$$\Delta_D = \int_0^L \frac{mM}{EI} dx = 0 + \int_0^{15} \frac{12(22.5x - 1.5x^2)}{E(2I_1)} dx + 0 = \frac{5062.5}{EI_1}$$

$$f_{DD} = \int_0^L \frac{mm}{EI} dx = 2\int_o^{12} \frac{(1x)^2}{EI_1} dx + \int_0^{15} \frac{(12)^2}{E(2I_1)} dx = \frac{2232}{EI_1}$$

From Eq. 1

$$\frac{5062.5}{EI_1} + D_x \frac{2232}{EI_1} = 0$$

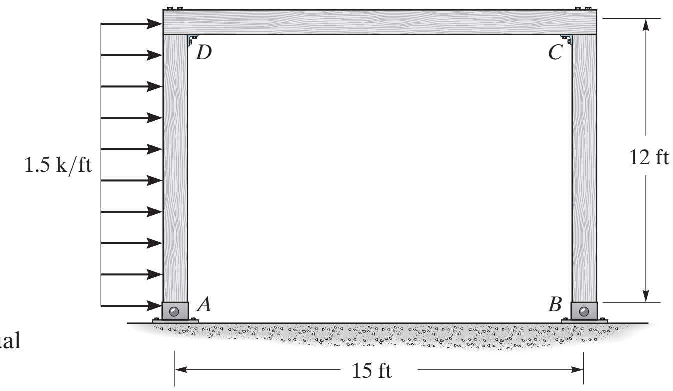
$D_x = -2.268 \text{ k} = -2.27 \text{ k}$ **Ans.**

$\circlearrowleft + \sum M_A = 0; \quad -45(7.5) + D_y(15) = 0 \quad D_y = 22.5 \text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad 22.5 - 45 + A_y = 0; \quad A_y = 22.5 \text{ k}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0; \quad A_x - 2.268 = 0; \quad A_x = 2.27 \text{ k}$ **Ans.**

***10–20.** Determine the reactions at the supports. Assume A and B are pins and the joints at C and D are fixed connections. EI is constant.



Compatibility Condition: Referring to Fig. *a*, the real and virtual moment functions shown in Fig. *b* and *c*, respectively,

$$\Delta'_{B_h} = \int_0^L \frac{mM}{EI}dx = \int_0^{12\text{ ft}} \frac{x_1(18x_1 - 0.75x_1^2)}{EI}dx_1 + \int_0^{15\text{ ft}} \frac{12(7.20x_2)}{EI}dx_2 + 0$$

$$= \frac{16200}{EI} \quad \rightarrow$$

$$f_{BB} = \int_0^L \frac{mm}{EI}dx = \int_0^{12\text{ ft}} \frac{x_1(x_1)}{EI}dx_1 + \int_0^{15\text{ ft}} \frac{12(12)}{EI}dx_2 + \int_0^{12\text{ ft}} \frac{x_3(x_3)}{EI}dx_3$$

Using the principle of superposition, Fig. *a*,

$$\Delta_{B_h} = \Delta'_{Bh} + B_x f_{BB}$$

$$(\overset{+}{\rightarrow}) \qquad 0 = \frac{16200}{EI} + B_x\left(\frac{3312}{EI}\right)$$

$B_x = -4.891 \text{ k} = 4.89 \text{ k} \leftarrow$ **Ans.**

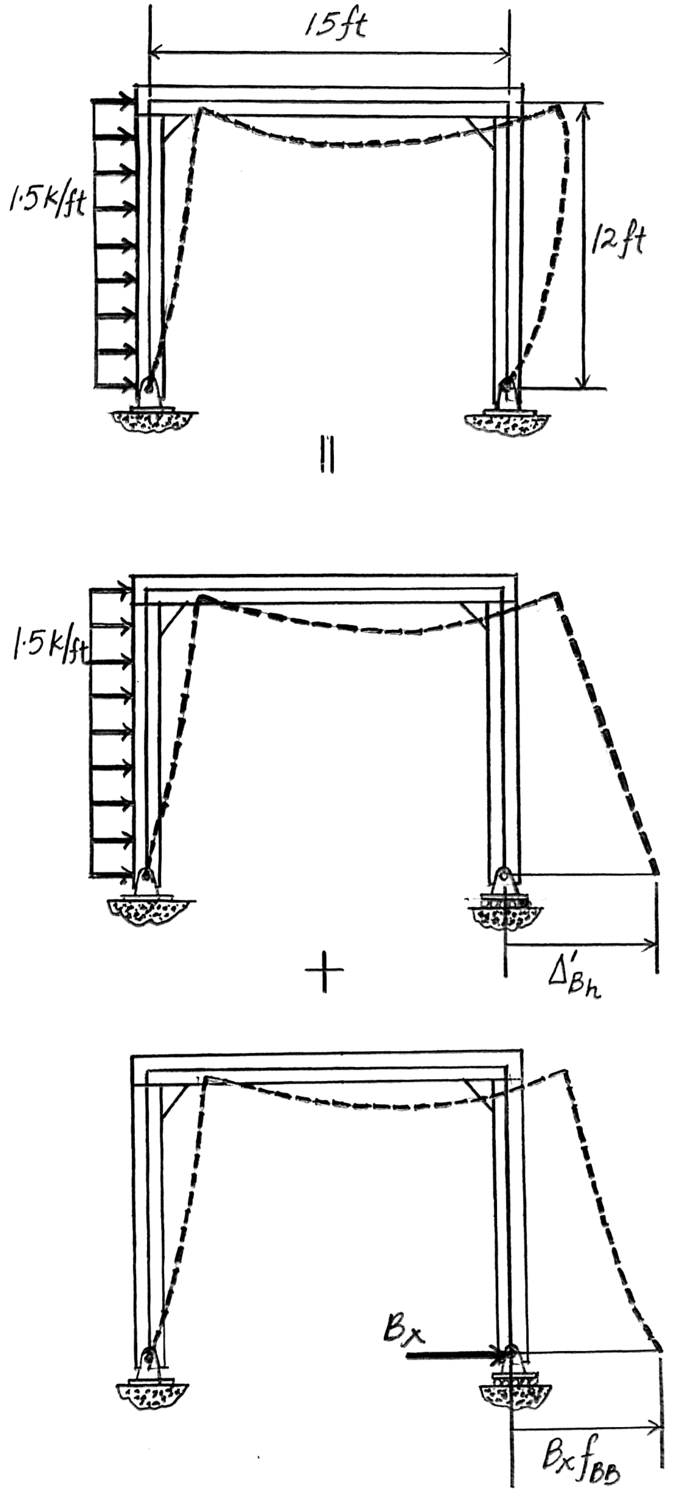
Equilibrium: Referring to the FBD of the frame in Fig. *d*,

$\overset{+}{\rightarrow} \sum F_x = 0; \quad 15(12) - 4.891 - A_x = 0 \quad A_x = 13.11 \text{ k} = 13.1 \text{ k}$ **Ans.**

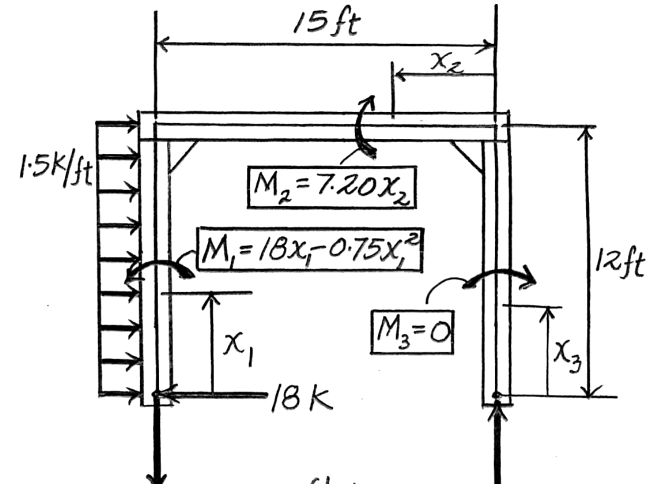
$\circlearrowleft + \sum M_A = 0; \quad B_y(15) - 1.5(12)(6) = 0 \quad B_y = 7.20 \text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad 7.20 - A_y = 0 \quad A_y = 7.20 \text{ k}$ **Ans.**

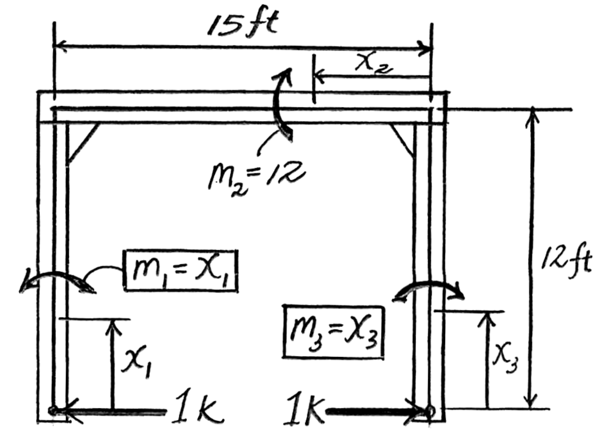
10–20. Continued



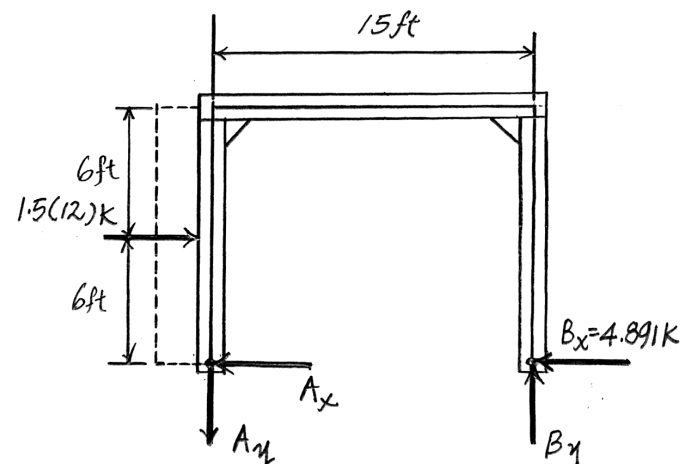
(a)



(b)

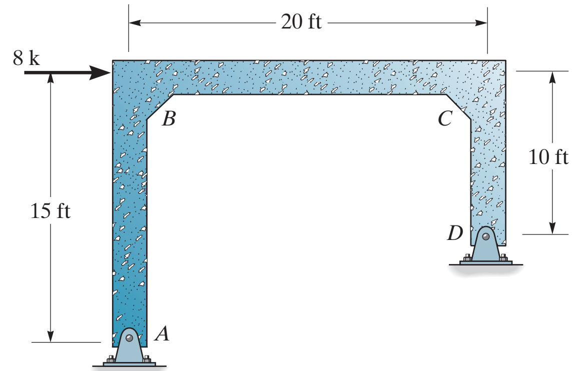


(c)



(d)

10–21. Determine the reactions at the supports. Assume *A* and *D* are pins. *EI* is constant.



Compatibility Equation: Referring to Fig. *a*, and the real and virtual moment functions shown in Fig. *b* and *c*, respectively.

$$\Delta' D_h = \int_0^L \frac{mM}{EI}dx = \int_0^{15\text{ ft}} \frac{(x_1)(8x_1)}{EI}dx_1 + \int_0^{20\text{ ft}} \frac{(0.25x_2 + 10)(6x_2)}{EI}dx_2 + 0$$

$$= \frac{25000}{EI} \quad \rightarrow$$

$$f_{DD} = \int_0^L \frac{mm}{EI}dx = \int_0^{15\text{ ft}} \frac{(x_1)(x_1)}{EI}dx_1 + \int_0^{20\text{ ft}} \frac{(0.25x_2 + 10)(0.25x_2 + 10)}{EI}dx_2 + \int_0^{10\text{ ft}} \frac{(x_3)(x_3)}{EI}dx_3$$

$$= \frac{4625}{EI} \quad \rightarrow$$

Using the principle of superposition, Fig. *a*,

$$\Delta_{D_h} = \Delta'_{Dh} + D_x f_{DD}$$

$$(\overset{+}{\rightarrow}) \qquad 0 = \frac{25000}{EI} + D_x\left(\frac{4625}{EI}\right)$$

$$D_x = -5.405 \text{ k} = 5.41\ k \quad \leftarrow$$ **Ans.**

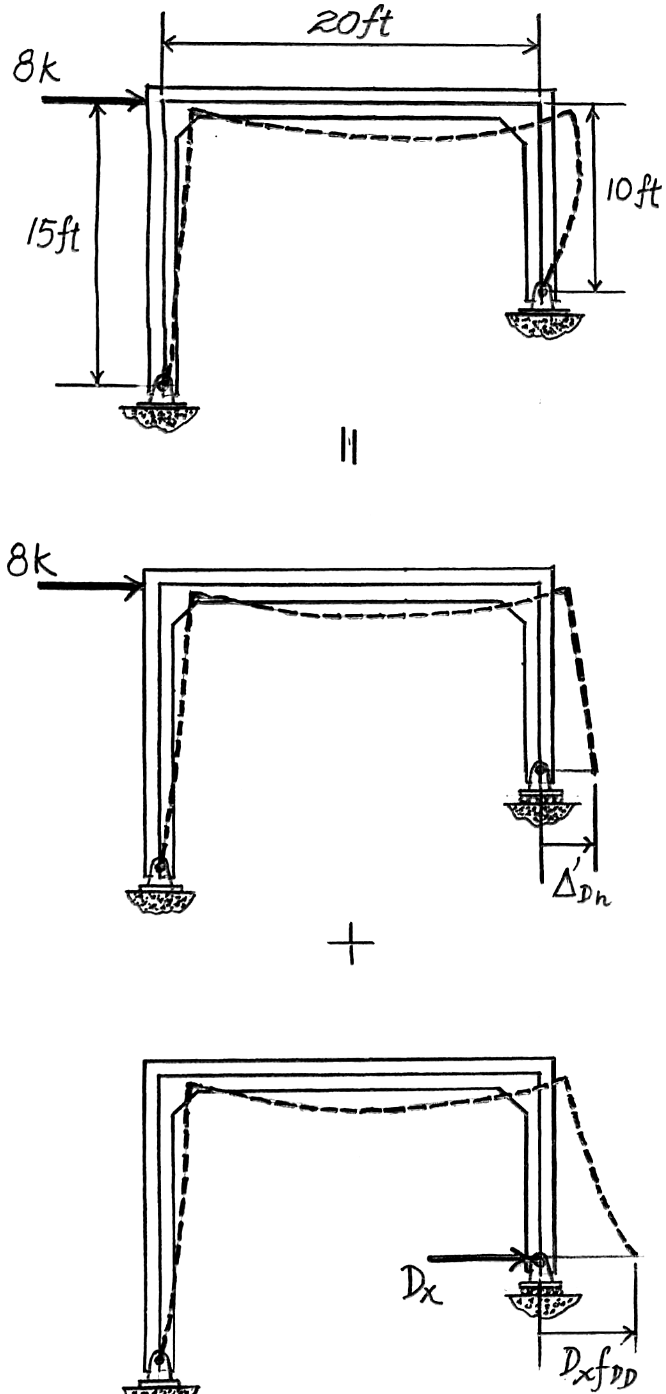
Equilibrium:

$\overset{+}{\rightarrow} \sum F_x = 0; \quad 8 - 5.405 - A_x = 0 \qquad A_x = 2.5946 \text{ k} = 2.59 \text{ k}$ **Ans.**

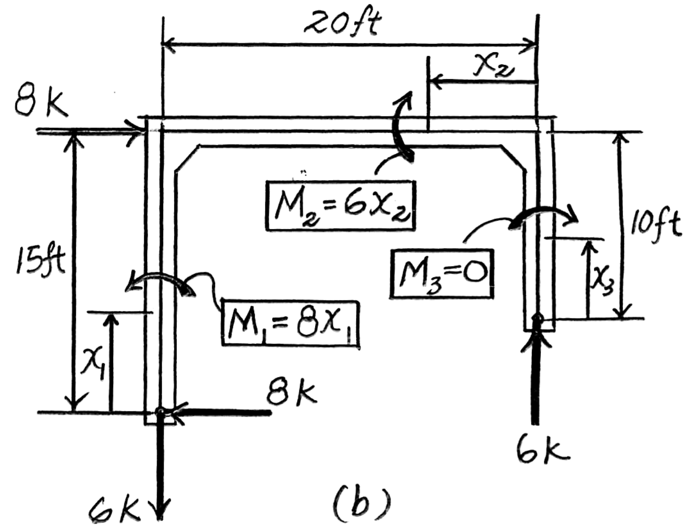
$\circlearrowleft + \sum M_A = 0; \quad D_y(20) + 5.405(5) - 8(15) = 0 \qquad D_y = 4.649 \text{ k} = 4.65 \text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0; \quad 4.649 - A_y = 0 \qquad A_y = 4.649 \text{ k} = 4.65 \text{ k}$ **Ans.**

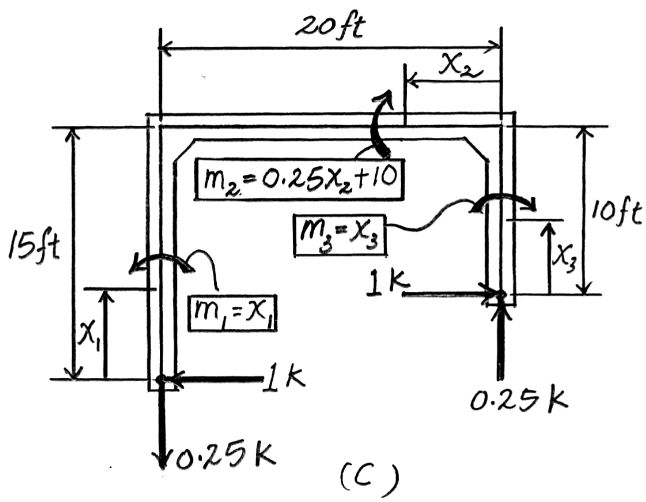
10–21. Continued



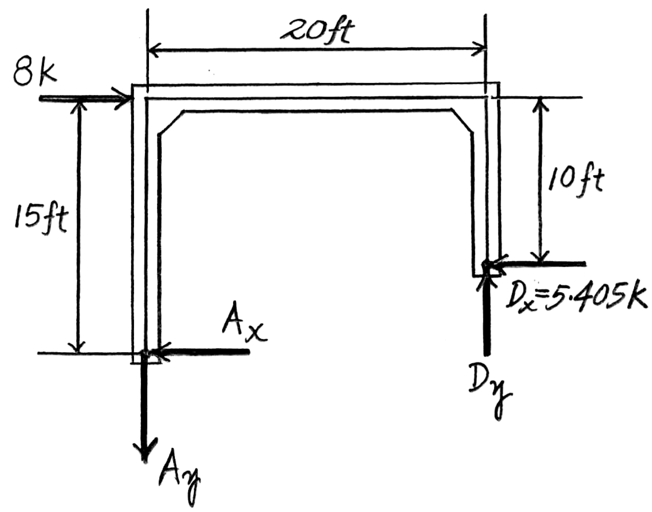
(a)



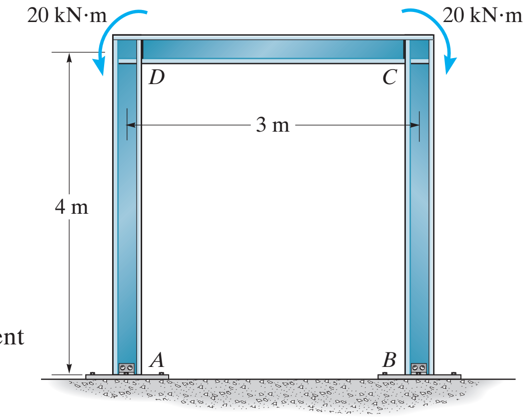
(b)



(c)



10–22. Determine the reactions at the supports. Assume A and B are pins. EI is constant.



Compatibility Condition: Referring to Fig. *a*, and the real and virtual moment functions shown in Fig. *b* and *c*, respectively,

$$\Delta'_{B_h} = \int_0^L \frac{mM}{EI}dx = 0 + \int_0^{3\text{m}} \frac{(-4)(-20)}{EI}dx_2 + 0 = \frac{240}{EI} \quad \leftarrow$$

$$f_{BB} = \int_0^L \frac{mm}{EI}dx = \int_0^{4\text{ m}} \frac{(-x_1)(-x_1)}{EI}dx_1 + \int_0^{3\text{ m}} \frac{(-4)(-4)}{EI}dx_2 + \int_0^{4\text{ m}} \frac{(-x_3)(-x_3)}{EI}dx_3$$

$$= \frac{90.67}{EI} \quad \leftarrow$$

Applying the principle of superposition, Fig. *a*,

$$\Delta_{B_h} = \Delta'_{B_h} + B_x f_{BB}$$

$$(\overset{+}{\leftarrow}) \qquad 0 = \frac{240}{EI} + B_x\left(\frac{90.67}{EI}\right)$$

$$B_x = -2.647 \text{ kN} = 2.65 \text{ kN} \quad \rightarrow$$ **Ans.**

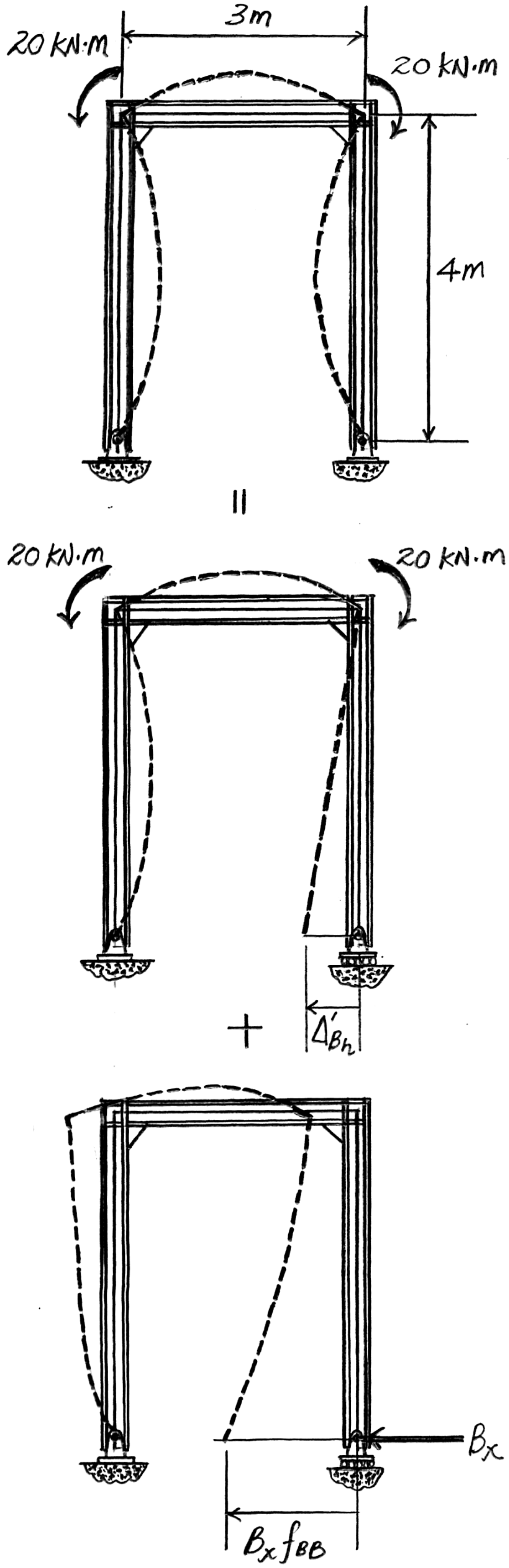
Equilibrium: Referring to the FBD of the frame shown in Fig. *d*,

$\overset{+}{\leftarrow} \sum F_x = 0; \qquad A_x - 2.647 = 0 \qquad A_x = 2.647 \text{ kN} = 2.65 \text{ kN}$ **Ans.**

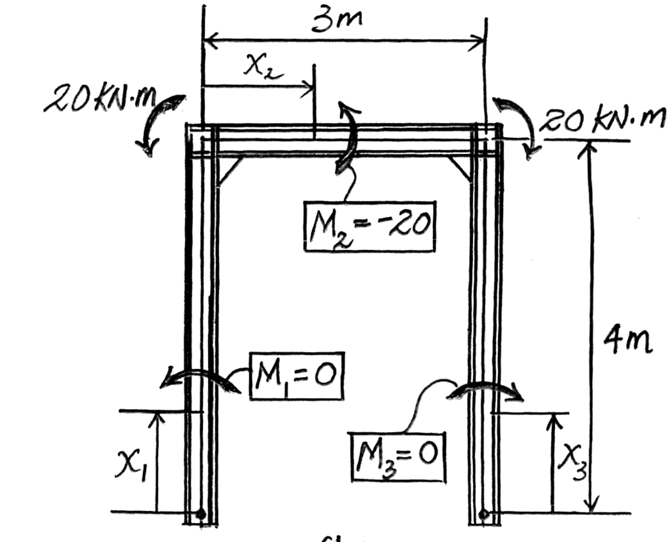
$\circlearrowleft + \sum M_A = 0; \qquad B_y(3) + 20 - 20 = 0 \qquad B_y = 0$ **Ans.**

$+\uparrow \sum F_y = 0; \qquad A_y = 0$ **Ans.**

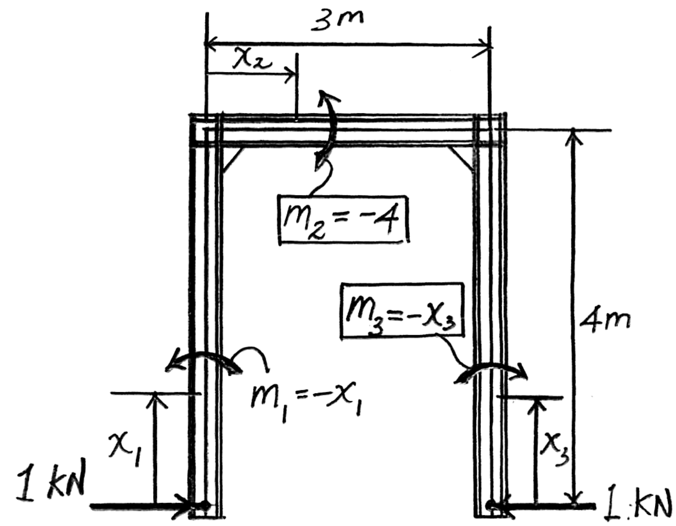
10–22. Continued



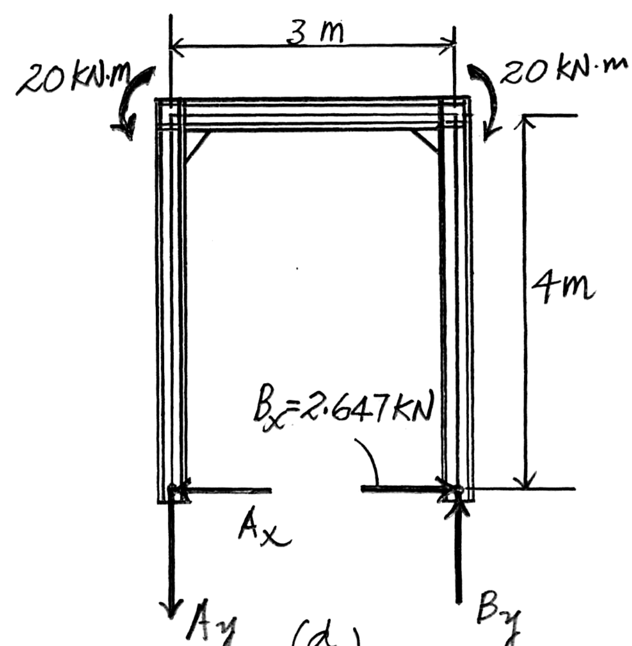
(a)



(b)

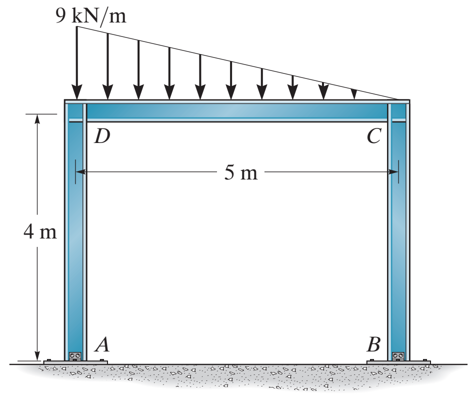


(c)



(d)

10–23. Determine the reactions at the supports. Assume A and B are pins. EI is constant.



Compatibility Equation: Referring to Fig. *a*, and the real and virtual moment functions in Fig. *b* and *c*, respectively,

$$\Delta' B_h = \int_0^L \frac{mM}{EI} dx = 0 + \int_0^{5\text{ m}} \frac{4(7.50x_2 - 0.3x_2^3)}{EI} dx_2 + 0 = \frac{187.5}{EI} \quad \rightarrow$$

$$f_{BB} = \int_0^L \frac{mm}{EI} dx = \int_0^{4\text{ m}} \frac{(x_1)(x_1)}{EI} dx_1 + \int_0^{5\text{ m}} \frac{4(4)}{EI} dx_2 + \int_0^{4\text{ m}} \frac{(x_3)(x_3)}{EI} dx_3$$

$$= \frac{122.07}{EI} \quad \rightarrow$$

Applying to the principle of superposition, Fig. *a*,

$$\Delta_{B_h} = \Delta'_{B_h} + B_x f_{BB}$$

$$(\overset{+}{\rightarrow}) \qquad 0 = \frac{187.5}{EI} + B_x\left(\frac{122.07}{EI}\right)$$

$$B_x = -1.529 \text{ kN} = 1.53 \text{ kN} \quad \leftarrow \qquad \textbf{Ans.}$$

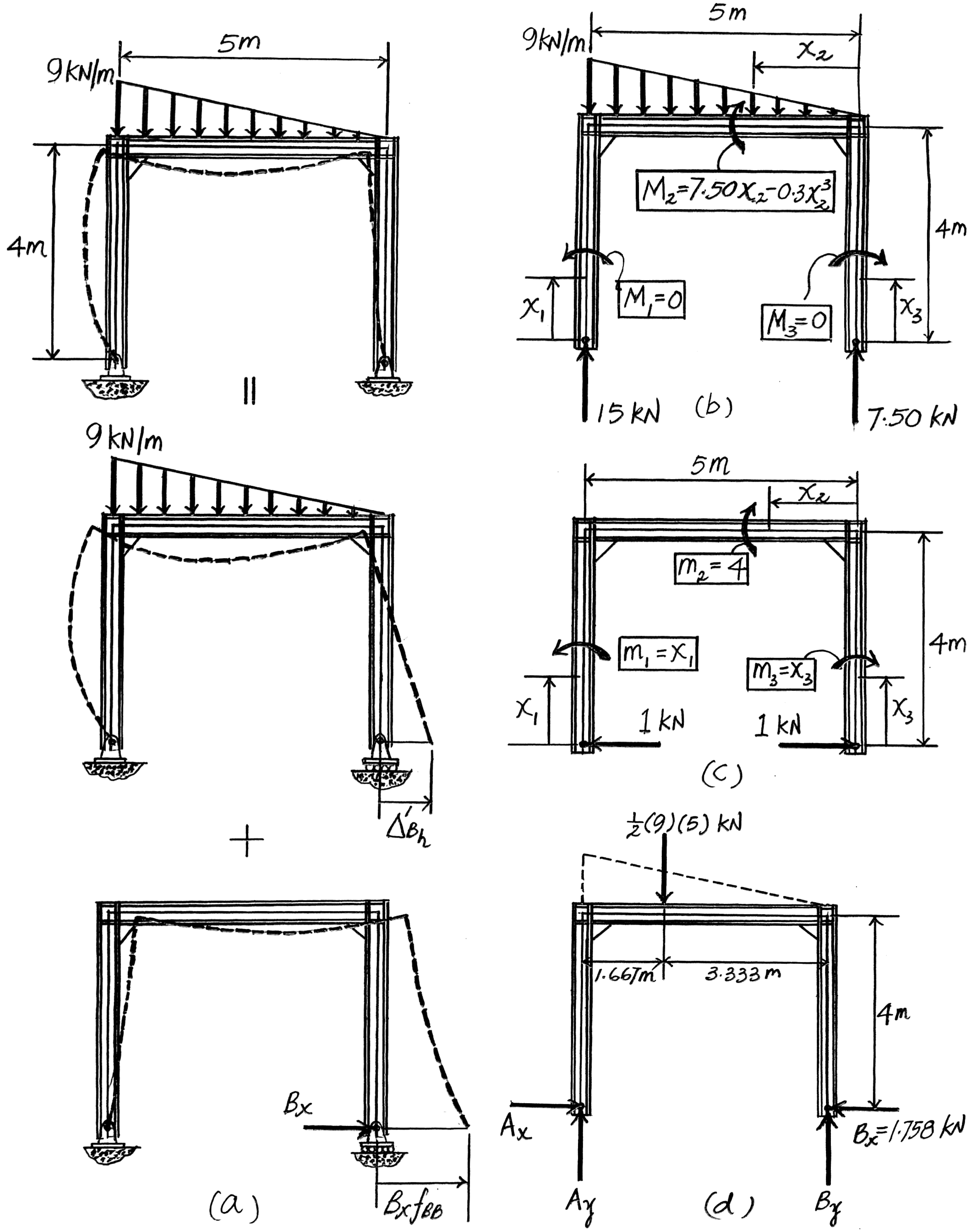
Equilibrium: Referring to the FBD of the frame in Fig. *d*,

$$\overset{+}{\rightarrow} \sum F_x = 0; \qquad A_x - 1.529 = 0 \qquad A_x = 1.529 \text{ kN} = 1.53 \text{ kN} \qquad \textbf{Ans.}$$

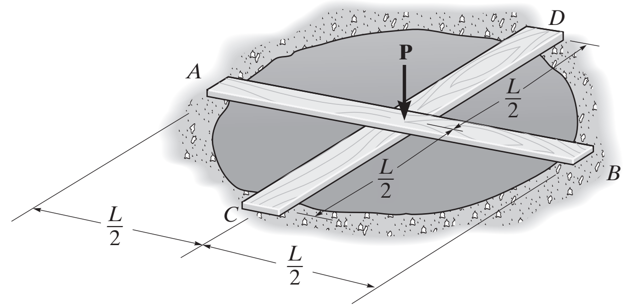
$$\circlearrowleft + \sum M_A = 0; \quad B_y(5) - \frac{1}{2}(9)(5)(1.667) = 0 \quad B_y = 7.50 \text{ kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \sum M_B = 0; \quad \frac{1}{2}(9)(5)(3.333) - A_y(5) = 0 \quad A_y = 15.0 \text{ kN} \qquad \textbf{Ans.}$$

10–23. Continued



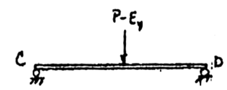
***10–24.** Two boards each having the same EI and length L are crossed perpendicular to each other as shown. Determine the vertical reactions at the supports. Assume the boards just touch each other before the load **P** is applied.

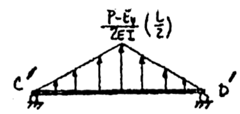


$$\Delta_E' = \Delta_E'$$

$$\Delta_E' = M_{E'} = -\frac{(P - E_y)L^2}{16EI}\left(\frac{L}{2}\right) + \frac{(P - E_y)L^2}{16EI}\left(\frac{L}{6}\right)$$

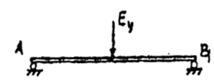
$$= -\frac{(P - E_y)L^3}{48EI}$$

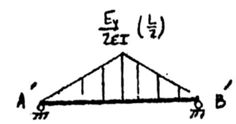




$$\Delta_E'' = M_{E''} = \frac{E_yL^2}{16EI}\left(\frac{L}{6}\right) - \frac{E_yL^2}{16EI}\left(\frac{L}{2}\right)$$

$$= -\frac{E_yL^3}{48EI}$$



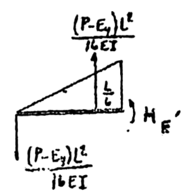


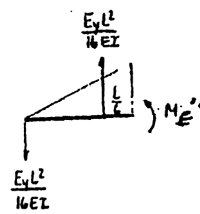
$$\Delta_E' = \Delta_E''$$

$$-\frac{(P - E_y)L^3}{48EI} = -\frac{E_yL^3}{48EI}$$

$$-(P-E_y) = -E_y$$

$$E_y = \frac{P}{2} \qquad \textbf{Ans.}$$

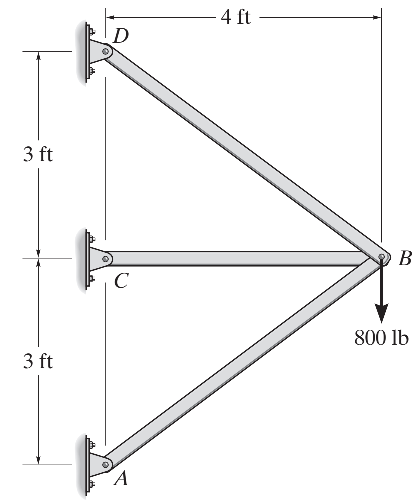




For equilibrium:

$$A_y = B_y = C_y = D_y = \frac{P}{4} \qquad \textbf{Ans.}$$

10–25. Determine the force in each member of the truss. AE is constant.



Compatibility Equation:

$$0 = \Delta_{AB} + F_{AB} f_{ABAB} \qquad (1)$$

Use virtual work method:

$$\Delta_{AB} = \sum \frac{nNL}{AE} = \frac{(1.0)(1.333)(5)}{AE} + \frac{(-1.6)(-1.067)(4)}{AE} = \frac{13.493}{AE}$$

$$f_{ABAB} = \sum \frac{nnL}{AE} = \frac{2(1)^2(5)}{AE} + \frac{(-1.6)^2(4)}{AE} = \frac{20.24}{AE}$$

From Eq. 1 $\qquad 0 = \dfrac{13.493}{AE} + \dfrac{20.24}{AE} F_{AB}$

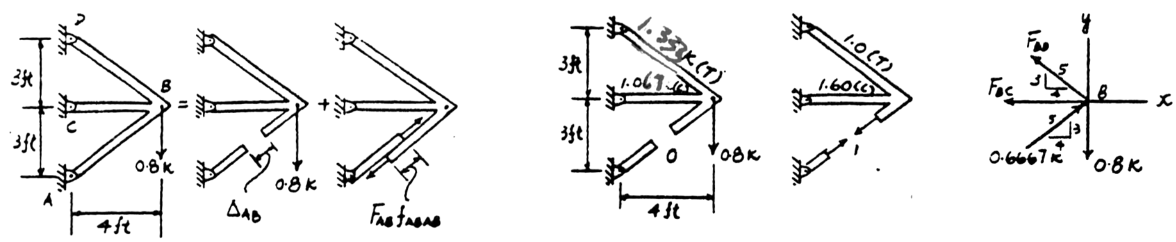
$F_{AB} = -0.667 \text{ k} = 0.667 \text{ k (C)}$ **Ans.**

Joint B:

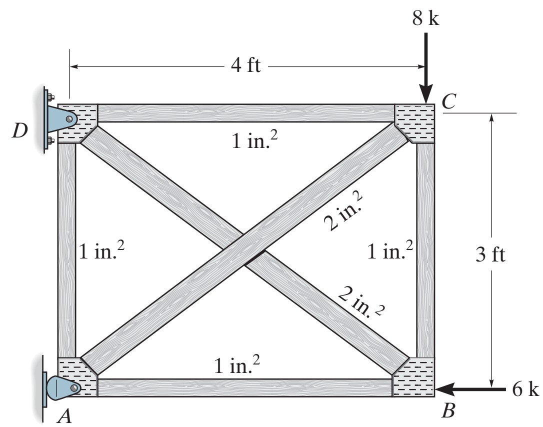
$+\uparrow \sum F_y = 0; \qquad \frac{3}{5}F_{BD} + \left(\frac{3}{5}\right)0.6666 - 0.8 = 0$

$F_{BD} = 0.667 \text{ k (T)}$ **Ans.**

$\overset{+}{\leftarrow} \sum F_x = 0; \qquad F_{BC} = 0$ **Ans.**

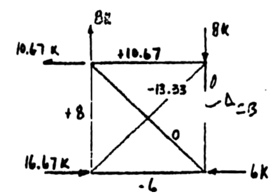


10–26. Determine the force in each member of the truss. The cross-sectional area of each member is indicated in the figure. $E = 29(10^3)$ ksi. Assume the members are pin connected at their ends.



$$\Delta_{CB} = \sum \frac{nNL}{AE} = \frac{1}{E}\left[\frac{(1.33)(10.67)(4)}{1} + \frac{(1.33)(-6)(4)}{1} + \frac{(1)(8)(3)}{1}\right] + \left[\frac{(-1.667)(-13.33)(5)}{2}\right]$$

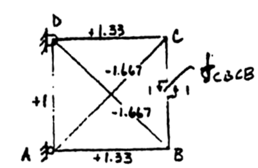
$$= \frac{104.4}{E}$$



$$f_{CBCB} = \sum \frac{n^2L}{AE} = \frac{1}{E}\left[\frac{2(1.33)^2(4)}{1} + \frac{2(1)^2(3)}{1} + \frac{2(-1.667)^2(5)}{2}\right]$$

$$= \frac{34.1}{E}$$

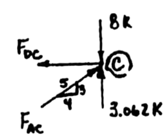
$$\Delta_{CB} + F_{CB} f_{CBCB} = 0$$



$$\frac{104.4}{E} + F_{CB}\left(\frac{34.1}{E}\right) = 0$$

$$F_{CB} = -3.062 \text{ k} = 3.06 \text{ k (C)}$$ **Ans.**

Joint C:



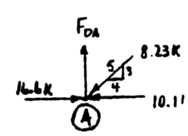
$+\uparrow \sum F_y = 0; \qquad \frac{3}{5}F_{AC} - 8 + 3.062 = 0;$

$F_{AC} = 823 \text{ k (C)}$ **Ans.**

$\xrightarrow{+} \sum F_x = 0; \qquad \frac{4}{5}(8.23) - F_{DC} = 0;$

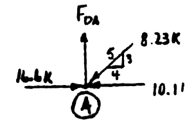
$F_{DC} = 6.58 \text{ k (T)}$ **Ans.**

Joint B:



$+\uparrow \sum F_y = 0; \qquad -3.062 + \left(\frac{3}{5}\right)(F_{DB}) = 0;$

$F_{DB} = 5.103 \text{ k} = 5.10 \text{ k (T)}$ **Ans.**



$\xrightarrow{+} \sum F_x = 0; \qquad F_{AB} - 6 - 5.103\left(\frac{4}{5}\right) = 0;$

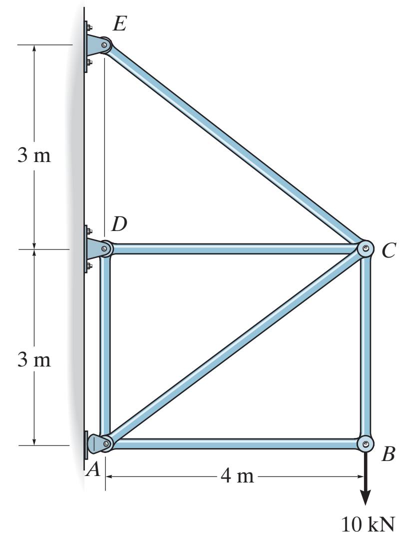
$F_{AB} = 10.1 \text{ k (C)}$ **Ans.**

Joint A:

$+\uparrow \sum F_y = 0; \qquad -8.23 + \left(\frac{3}{5}\right)F_{DA} = 0;$

$F_{DA} = 4.94 \text{ k (T)}$ **Ans.**

10–27. Determine the force in member *AC* of the truss. *AE* is constant.



Compatibility Equation: Referring to Fig. *a*, and using the real force and virtual force in each member shown in Fig. *b* and *c*, respectively,

$$\Delta'_{AC} = \sum\frac{nNL}{AE} = \frac{1(16.67)(5)}{AE} + \frac{(-1.60)(-13.33)(4)}{AE} = \frac{168.67}{AE}$$

$$f_{ACAC} = \sum\frac{n^2L}{AE} = 2\left[\frac{(1^2)(5)}{AE}\right] + \frac{[(-1.60)^2](4)}{AE} + \frac{[(-0.6)^2](3)}{AE}$$

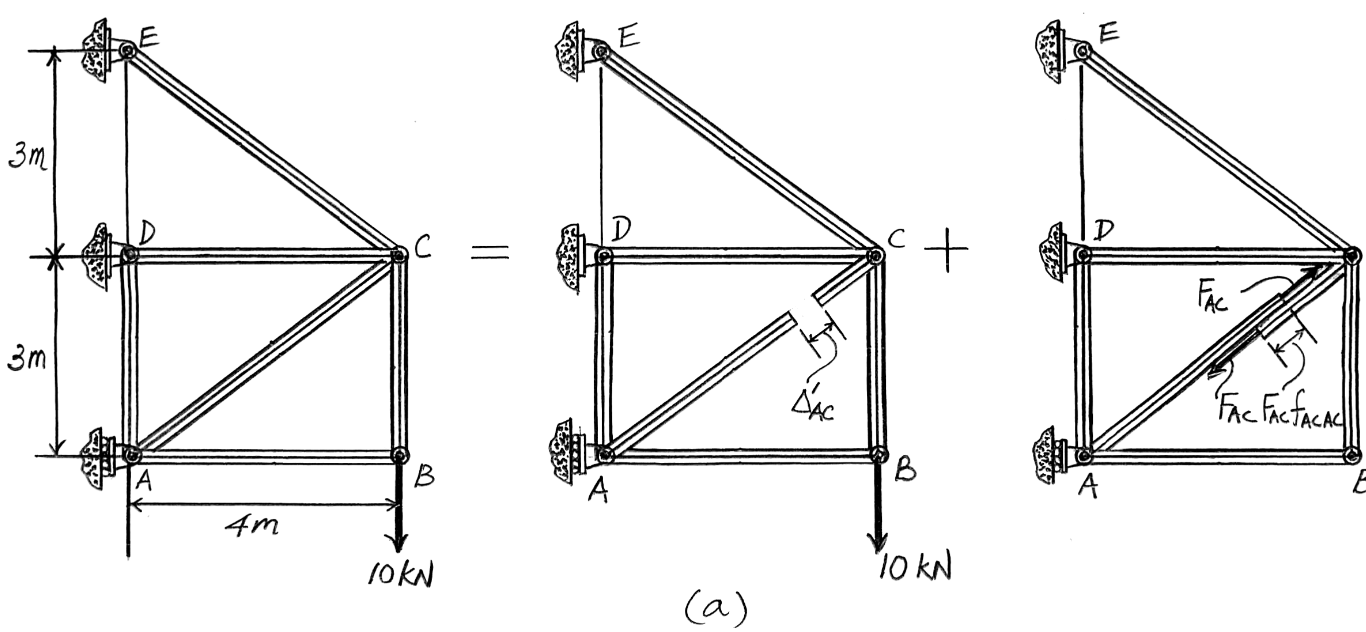
$$= \frac{21.32}{AE}$$

Applying the principle of superposition, Fig. *a*

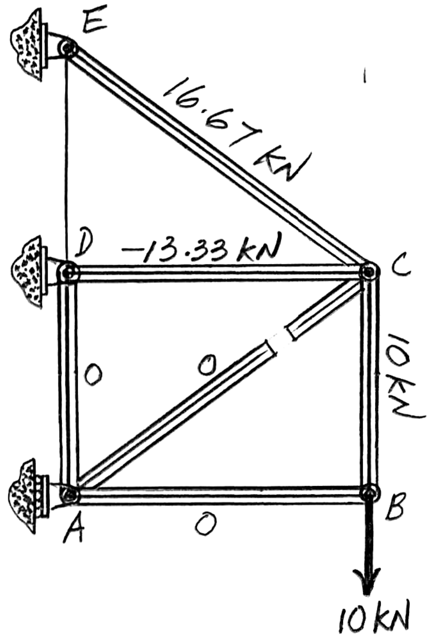
$$\Delta_{AC} = \Delta'_{AC} + F_{AC}f_{ACAC}$$

$$0 = \frac{168.67}{AE} + F_{AC}\left(\frac{21.32}{AE}\right)$$

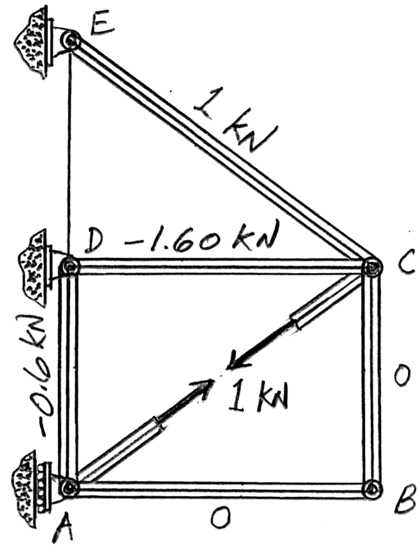
$$F_{AC} = -7.911 \text{ kN} = 7.91 \text{ kN (C)}$$ **Ans.**



10–27. Continued

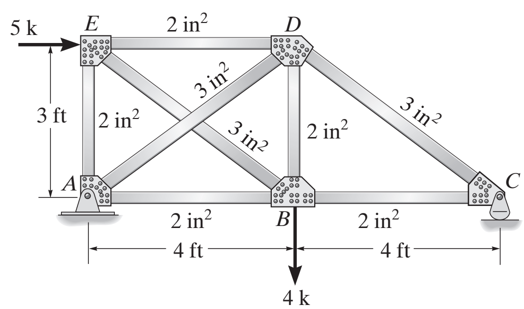


(b)



(C)

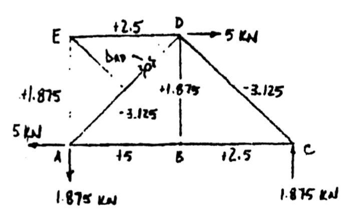
***10–28.** Determine the force in member AD of the truss. The cross-sectional area of each member is shown in the figure. Assume the members are pin connected at their ends. Take $E = 29(10^3)$ ksi.



$$\Delta_{AD} = \sum\frac{nNL}{AE} = \frac{1}{E}\left[\frac{1}{2}(-0.8)(2.5)(4) + (2)\left(\frac{1}{2}\right)(-0.6)(1.875)(3)\right.$$

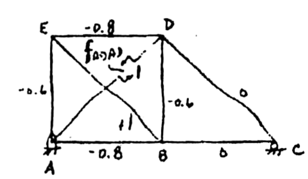
$$\left. + \frac{1}{2}(-0.8)(5)(4) + \frac{1}{3}(1)(-3.125)(5)\right]$$

$$= -\frac{20.583}{E}$$



$$f_{ADAD} = \sum\frac{n^2L}{AE} = \frac{1}{E}\left[2\left(\frac{1}{2}\right)(-0.8)^2(4) + 2\left(\frac{1}{2}\right)(-0.6)^2(3) + 2\left(\frac{1}{3}\right)(1)^2(5)\right]$$

$$= \frac{6.973}{E}$$

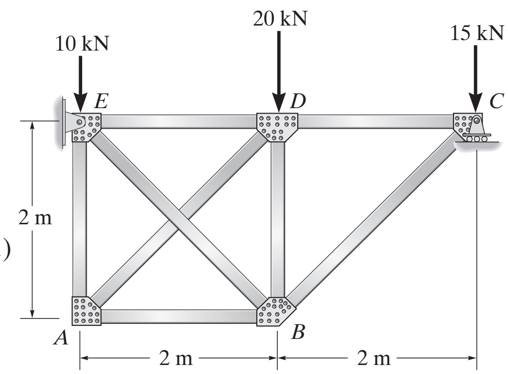


$$\Delta_{AD} + F_{AD}f_{ADAD} = 0$$

$$-\frac{20.583}{E} + F_{AD}\left(\frac{6.973}{E}\right) = 0$$

$$F_{AD} = 2.95 \text{ kN (T)} \qquad \textbf{Ans.}$$

10–29. Determine the force in each member of the truss. Assume the members are pin connected at their ends. AE is constant.



Compatibility Equation:

$$0 = \Delta_{AD} + F_{AD} f_{ADAD} \qquad (1)$$

Use virtual work method

$$\Delta_{AD} = \sum \frac{nNL}{AE} = \frac{(-0.7071)(-10)(2)}{AE} + \frac{(-0.7071)(-20)(2)}{AE} + \frac{(1)(14.142)(2.828)}{AE}$$

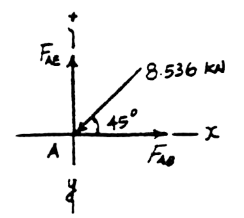
$$= \frac{82.43}{AE}$$

$$f_{ADAD} = \sum \frac{nnL}{AE} = \frac{4(-0.7071)^2(2)}{AE} + \frac{2(1)^2(2.828)}{AE} = \frac{9.657}{AE}$$

From Eq. 1

$$0 = \frac{82.43}{AE} + \frac{9.657}{AE} F_{AD}$$

$F_{AD} = -8.536\text{ kN} = 8.54\text{ kN (C)}$ **Ans.**



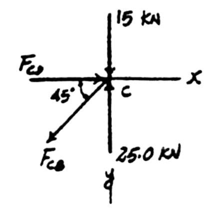
Joint *A*:

$+\uparrow \sum F_y = 0;$ $\quad F_{AE} - 8.536 \sin 45° = 0$

$F_{AE} = 6.04\text{ kN (T)}$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad F_{AB} - 8.536 \cos 45° = 0$

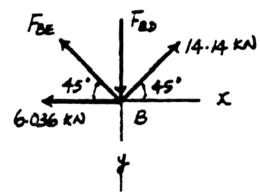
$F_{AB} = 6.036\text{ kN} = 6.04\text{ kN (T)}$ **Ans.**



Joint *C*:

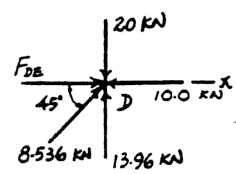
$+\uparrow \sum F_y = 0;$ $\quad -F_{CB} \sin 45° - 15 + 25 = 0$

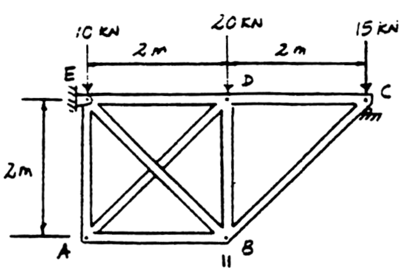
$F_{CB} = 14.14\text{ kN} = 14.1\text{ kN (T)}$ **Ans.**

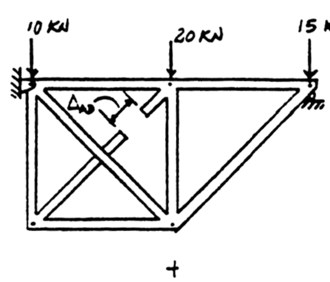


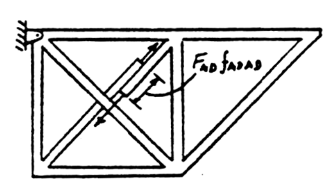
$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad F_{CD} - 14.14 \cos 45° = 0$

$F_{CD} = 10.0\text{ kN (C)}$ **Ans.**









10–29. Continued

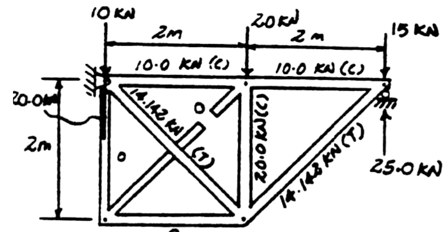
Joint *B*:

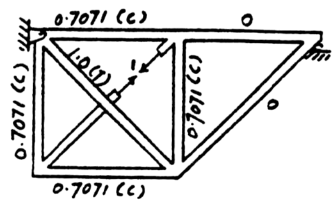
$\overset{+}{\leftarrow} \sum F_x = 0;$ $F_{BE} \cos 45° + 6.036 - 14.14 \cos 45° = 0$

$F_{BE} = 5.606 \text{ kN} = 5.61 \text{ kN (T)}$ **Ans.**

$+\uparrow \sum F_y = 0;$ $-F_{BD} + 5.606 \sin 45° + 14.14 \sin 45° = 0$

$F_{BD} = 13.96 \text{ kN} = 14.0 \text{ kN (C)}$ **Ans.**





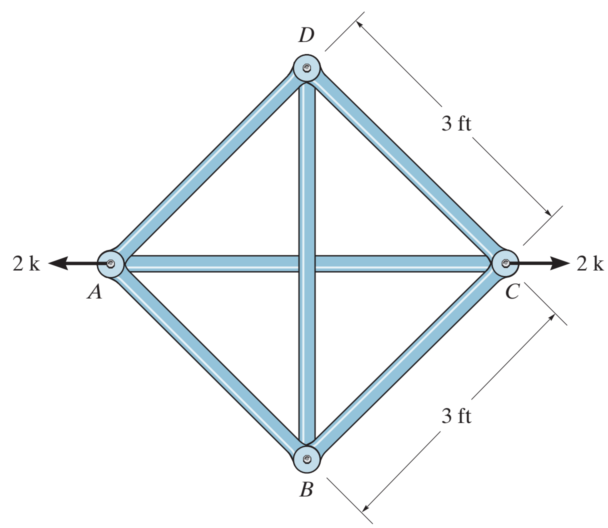
Joint *D*:

$\overset{+}{\rightarrow} \sum F_x = 0;$ $F_{DE} + 8.536 \cos 45° - 10 = 0$

$F_{DE} = 3.96 \text{ kN (C)}$ **Ans.**

$+\uparrow \sum F_y = 0;$ $8.536 \sin 45° + 13.96 - 20 = 0$ (Check) **Ans.**

10–30. Determine the force in each member of the pin-connected truss. AE is constant.



$$\Delta_{AC} = \sum \frac{nNL}{AE} = \frac{1}{AE}[(-0.707)(1.414)(3)(4) + (1)(-2)\sqrt{18}]$$

$$= -\frac{20.485}{AE}$$

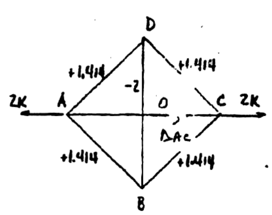
$$f_{ACAC} = \sum \frac{n^2 L}{AE} = \frac{1}{AE}[4(-0.707)^2(3) + 2(1)^2\sqrt{18}]$$

$$= \frac{14.485}{AE}$$

$$\Delta_{AC} + F_{AC} f_{ACAC} = 0$$

$$-\frac{20.485}{AE} + F_{AC}\left(\frac{14.485}{AE}\right) = 0$$

$F_{AC} = 1.414 \text{ k} = 1.41 \text{ k (T)}$ **Ans.**

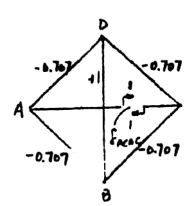


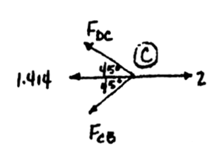
Joint *C*:

$+\uparrow \sum F_y = 0;$ $F_{DC} = F_{CB} = F$ **Ans.**

$\overset{+}{\rightarrow} \sum F_x = 0;$ $2 - 1.414 - 2F(\cos 45°) = 0;$

$F_{DC} = F_{CB} = 0.414 \text{ k (T)}$ **Ans.**





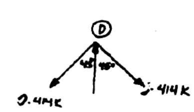
Due to symmetry:

$F_{AD} = F_{AB} = 0.414 \text{ k (T)}$ **Ans.**

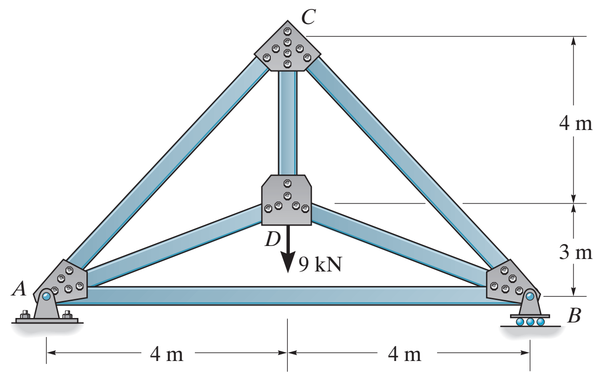
Joint *D*:

$+\uparrow \sum F_y = 0;$ $F_{DB} - 2(0.414)(\cos 45°) = 0;$

$F_{DB} = 0.586 \text{ k (C)}$ **Ans.**



10–31. Determine the force in member *CD* of the truss. *AE* is constant.



Compatibility Equation: Referring to Fig. *a* and using the real and virtual force in each member shown in Fig. *b* and *c*, respectively,

$$\Delta'_{CD} = \sum \frac{nNL}{AE} = 2\left[\frac{0.8333(-7.50)(5)}{AE}\right] + \frac{(-0.3810)(6.00)(8)}{AE} = -\frac{80.786}{AE}$$

$$f_{CDCD} = \sum \frac{n^2L}{AE} = 2\left[\frac{(-0.5759)^2(\sqrt{65})}{AE}\right] + 2\left[\frac{0.8333^2(5)}{AE}\right]$$

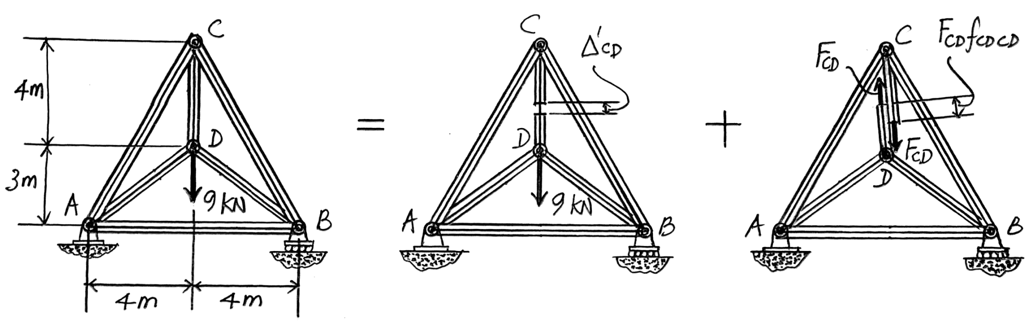
$$+ \frac{(-0.3810)^2(8)}{AE} + \frac{1^2(4)}{AE} = \frac{17.453}{AE}$$

Applying the principle of superposition, Fig. *a*,

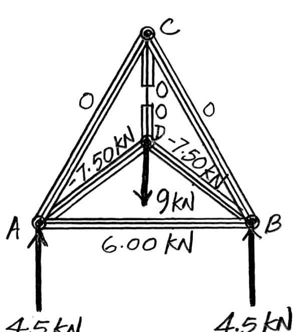
$$\Delta_{CD} = \Delta'_{CD} + F_{CD}f_{CDCD}$$

$$0 = -\frac{80.786}{AE} + F_{CD}\left(\frac{17.453}{AE}\right)$$

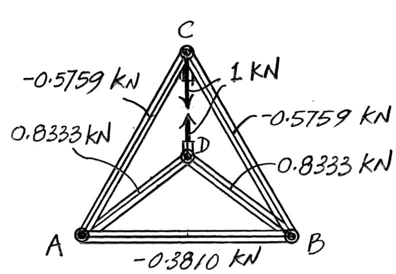
$$F_{CD} = 4.63 \text{ kN (T)}$$ **Ans.**



(a)

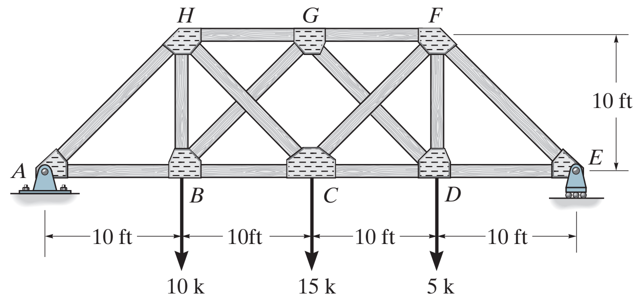


(b)



(c)

***10–32.** Determine the force in member GB of the truss. AE is constant.



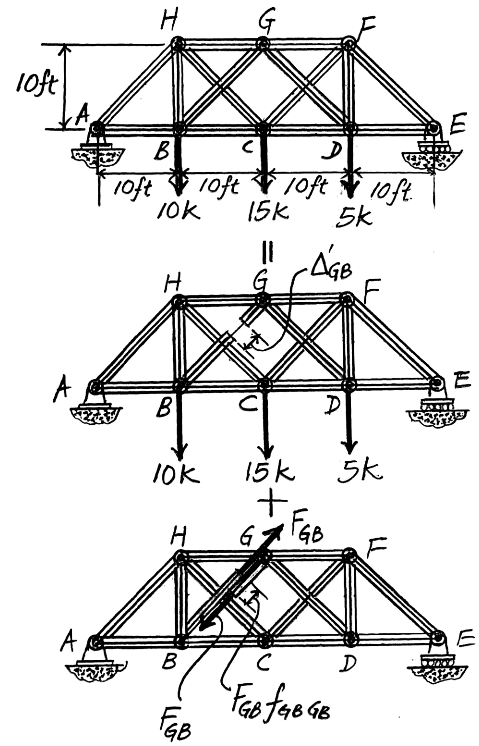
Compatibility Equation: Referring to Fig. *a*, and using the real and virtual force in each member shown in Fig. *b* and *c*, respectively,

$$\Delta'_{GB} = \sum \frac{nNL}{AE} = \frac{1}{AE}\Big[(-0.7071)(10)(10) + (-0.7071)(16.25)(10) + 0.7071(13.75)(10) + 0.7071(5)(10) + 0.7071(-22.5)(10) + (-0.7071)(-22.5)(10) + 1(8.839)(14.14) + (-1)(12.37)(14.14)\Big]$$

$$= -\frac{103.03}{AE}$$

$$f_{GBGB} = \sum \frac{n^2 L}{AE} = 3\left[\frac{0.7071^2(10)}{AE}\right] + 3\left[\frac{(-0.7071)^2(10)}{AE}\right] + 2\left[\frac{(-1)^2(14.14)}{AE}\right] + 2\left[\frac{(1^2)(14.14)}{AE}\right]$$

$$= \frac{86.57}{AE}$$



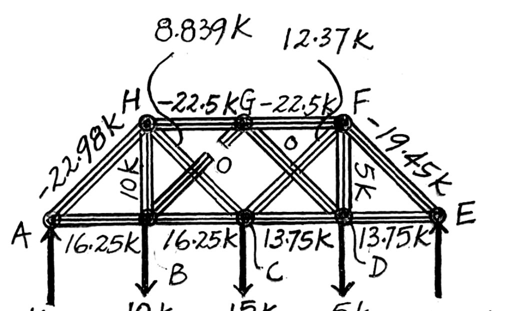
(a)

Applying the principle of superposition, Fig. *a*

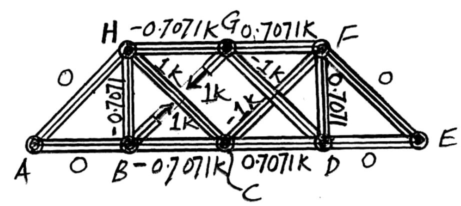
$$\Delta_{GB} = \Delta_{GB} + F_{GB} f_{GBGB}$$

$$0 = \frac{-103.03}{AE} + F_{GB}\left(\frac{86.57}{AE}\right)$$

$$F_{GB} = 1.190 \text{ k} = 1.19 \text{ k}(T)$$ **Ans.**

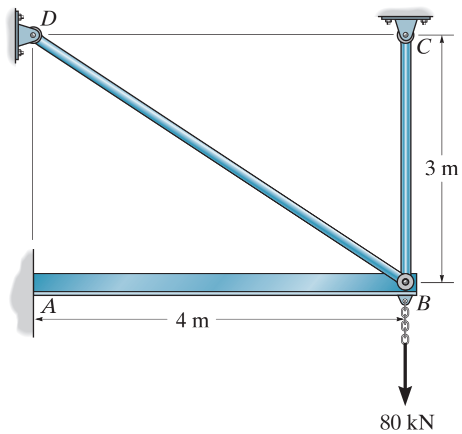


(b)



(C)

10–33. The cantilevered beam AB is additionally supported using two tie rods. Determine the force in each of these rods. Neglect axial compression and shear in the beam. For the beam, $I_b = 200(10^6)\ \text{mm}^4$, and for each tie rod, $A = 100\ \text{mm}^2$. Take $E = 200$ GPa.

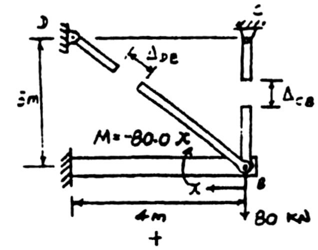


Compatibility Equations:

$$\Delta_{DB} + F_{DB}f_{DBDB} + F_{CB}f_{DBDB} = 0 \quad (1)$$

$$\Delta_{CB} + F_{DB}f_{CBDB} + F_{CB}f_{CBCB} = 0 \quad (2)$$

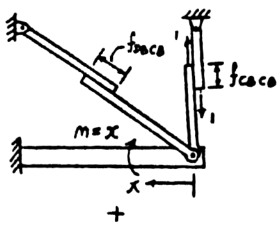
Use virtual work method



$$\Delta_{DB} = \int_0^L \frac{mM}{EI}dx = \int_0^4 \frac{(0.6x)(-80x)}{EI}dx = -\frac{1024}{EI}$$

$$\Delta_{CB} = \int_0^L \frac{mM}{EI}dx = \int_0^4 \frac{(1x)(-80x)}{EI}dx = -\frac{1706.67}{EI}$$

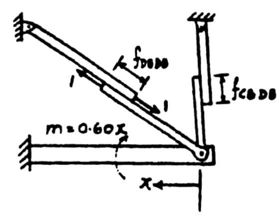
$$f_{CBCB} = \int_0^L \frac{mm}{EI}dx + \sum\frac{nnL}{AE} = \int_0^4 \frac{(1x)^2}{EI}dx + \frac{(1)^2(3)}{AE} = \frac{21.33}{EI} + \frac{3}{AE}$$



$$f_{DBDB} = \int_0^L \frac{mm}{EI}dx + \sum\frac{nnL}{AE} = \int_0^4 \frac{(0.6x)^2}{EI}dx + \frac{(1)^2(5)}{AE} = \frac{7.68}{EI} + \frac{5}{AE}$$

$$f_{DBCB} = \int_0^4 \frac{(0.6x)(1x)}{EI} = \frac{12.8}{AE}.$$

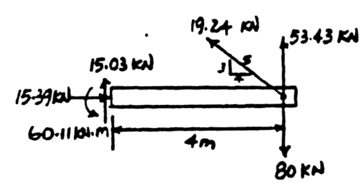
From Eq. 1



$$\frac{-1024}{E(200)(10^{-4})} + F_{DB}\left[\frac{7.68}{E(200)(10^{-4})} + \frac{5}{E(100)(10^{-4})}\right] + F_{CB}\left[\frac{12.8}{E(200)(10^{-4})}\right] = 0$$

$$0.0884F_{DB} + 0.064F_{CB} = 5.12$$

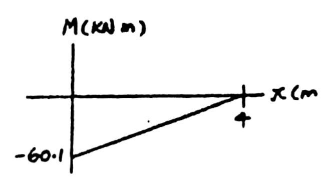
From Eq. 2



$$-\frac{1706.67}{E(200)(10^{-6})} + F_{DB}\frac{12.8}{E(200)(10^{-6})} + F_{CB}\left[\frac{21.33}{E(200)(10^{-6})} + \frac{3}{E(200)(10^{-6})}\right] = 0$$

$$0.064F_{DB} + 0.13667F_{CB} = 8.533$$

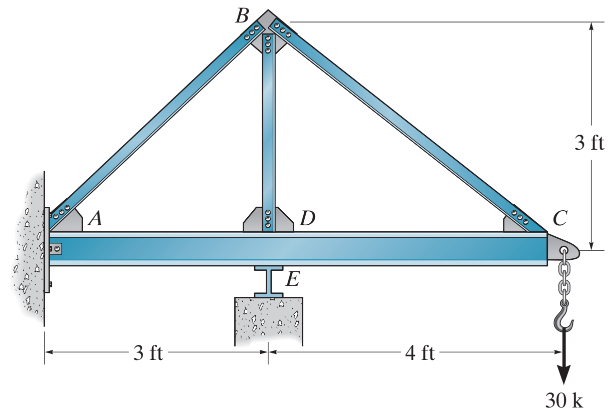
Solving



$F_{DB} = 19.24\ \text{kN} = 19.2\ \text{kN}$ **Ans.**

$F_{CB} = 53.43\ \text{kN} = 53.4\ \text{kN}$ **Ans.**

10–34. Determine the force in members AB, BC and BD which is used in conjunction with the beam to carry the 30-k load. The beam has a moment of inertia of $I = 600\,\text{in}^4$, the members AB and BC each have a cross-sectional area of 2 in^2, and BD has a cross-sectional area of 4 in^2. Take $E = 29(10^3)$ ksi. Neglect the thickness of the beam and its axial compression, and assume all members are pin-connected. Also assume the support at F is a pin and E is a roller.



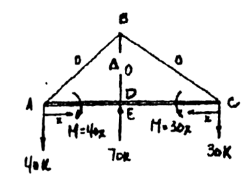
$$\Delta = \int_0^L \frac{mM}{EI} = \sum \frac{nNL}{AE} = \int_0^3 \frac{(0.57143x)(40x)}{EI}dx + \int_0^4 \frac{(0.42857x)(30x)}{EI}dx + 0$$

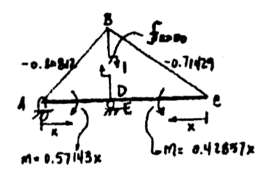
$$= \frac{480}{EI}$$

$$f_{BDBD} = \int_0^L \frac{m^2}{EI}dx + \sum \frac{n^2L}{AE} = \int_0^3 \frac{(0.57143x)^2dx}{EI} + \int_0^4 \frac{(0.42857x)^2dx}{EI}$$

$$+ \frac{(1)^2(3)}{4E} + \frac{(0.80812)^2\sqrt{18}}{2E} + \frac{(0.71429)^2(5)}{2E}$$

$$= \frac{6.8571}{EI} + \frac{3.4109}{E}$$

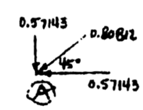




$$\Delta + F_{BD}f_{BDBD} = 0$$

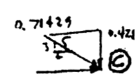
$$\frac{480(12^3)}{E(600)} + F_{BD}\left(\frac{6.8571(12^3)}{E(600)} + \frac{3.4109(12)}{E}\right) = 0$$

$$F_{BD} = -22.78 \text{ k} = 22.8 \text{ k } (C)$$ **Ans.**



Joint *B*:

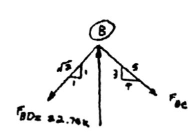
$$\xrightarrow{+} \sum F_x = 0; \quad -F_{AB}\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{4}{5}\right)F_{BC} = 0;$$



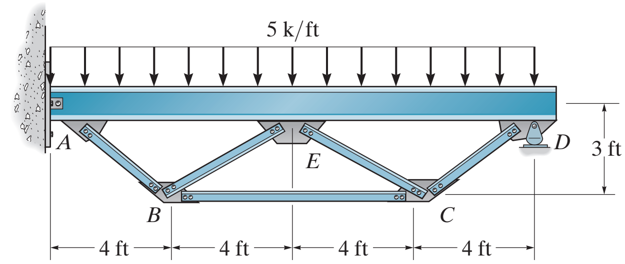
$$+\uparrow \sum F_y = 0; \quad 22.78 - \left(\frac{3}{5}\right)F_{BC} - F_{AB}\left(\frac{1}{\sqrt{2}}\right) = 0;$$

$$F_{AB} = 18.4 \text{ k (T)}$$ **Ans.**

$$F_{BC} = 16.3 \text{ k (T)}$$ **Ans.**



10–35. The trussed beam supports the uniform distributed loading. If all the truss members have a cross-sectional area of 1.25 in^2, determine the force in member BC. Neglect both the depth and axial compression in the beam. Take $E = 29(10^3)$ ksi for all members. Also, for the beam $I_{AD} = 750 \text{ in}^4$. Assume A is a pin and D is a rocker.



Compatibility Equation: Referring to Fig. a, and using the real and virtual loadings in each member shown in Fig. b and c, respectively,

$$\Delta'_{BC} = \int_0^L \frac{mM}{EI}dx + \sum \frac{nNL}{AE} = 2\int_0^{8\text{ ft}} \frac{(-0.375x)(40x-25x^2)}{EI}dx + 0$$

$$= -\frac{3200 \text{ k}\cdot\text{ft}^3}{EI} = -\frac{3200(12^2)\text{ k}\cdot\text{in}^3}{[29(10^3)\text{ k/in}^2](750 \text{ in}^2)} = -0.254$$

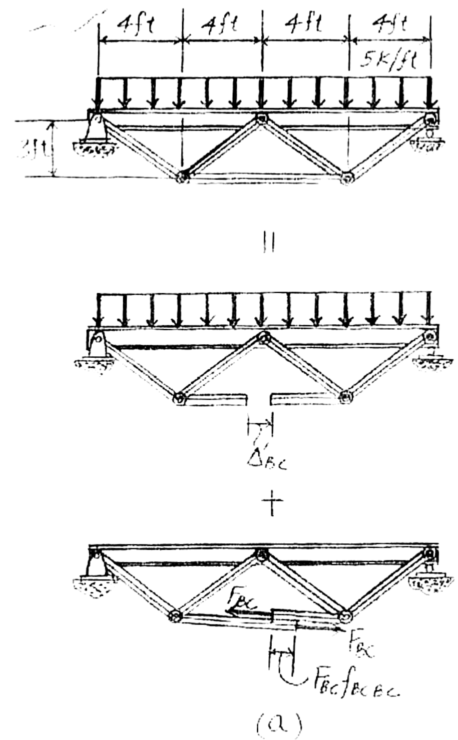
$$f_{BCBC} = \int_0^L \frac{m^2}{EI}dx + \sum \frac{n^2L}{AE} = 2\int_0^{8\text{ ft}} \frac{(-0.375x)^2}{EI}dx$$

$$+ \frac{1}{AE}[1^2(8) + 2(0.625^2)(5) + 2(-0.625)^2]$$

$$= \frac{48 \text{ ft}^3}{EI} + \frac{15.8125 \text{ ft}}{AE}$$

$$= \frac{48(12^2)\text{ in}^3}{[29(10)^3 \text{ k/in}^3](750 \text{ in}^4)} + \frac{15.8125(12)\text{ in}}{(1.25 \text{ in}^2)[29(10^3)\text{ k/in}^2]}$$

$$= 0.009048 \text{ in/k}$$

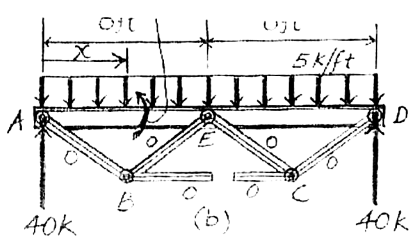


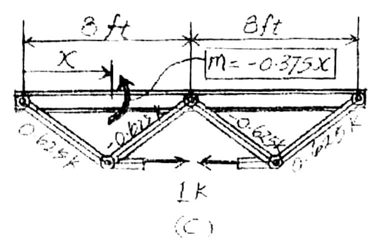
Applying principle of superposition, Fig. a

$$\Delta_{BC} = \Delta'_{BC} + F_{BC}f_{BCBC}$$

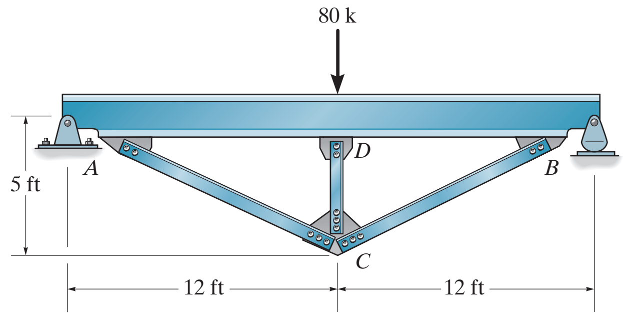
$$0 = -0.2542 \text{ in} + F_{BC}(0.009048 \text{ in/k})$$

$$F_{BC} = 28.098 \text{ k (T)} = 28.1 \text{ k (T)}$$ **Ans.**





***10–36.** The trussed beam supports a concentrated force of 80 k at its center. Determine the force in each of the three struts and draw the bending-moment diagram for the beam. The struts each have a cross-sectional area of 2 in^2. Assume they are pin connected at their end points. Neglect both the depth of the beam and the effect of axial compression in the beam. Take $E = 29(10^3)$ ksi for both the beam and struts. Also, for the beam $I = 400\,\text{in}^4$.



$$\Delta_{CD} = \int_0^L \frac{mM}{EI}dx + \sum \frac{nNL}{AE} = 2\int_0^{12} \frac{(0.5x)(40x)}{EI}dx = \frac{23040}{EI}$$

$$f_{CDCD} = \int_0^L \frac{m^2}{EI}dx + \sum \frac{n^2L}{AE} = 2\int_0^{12} \frac{(0.5x)^2}{EI}dx + \frac{(1)^2(5)}{AE} + \frac{2(1.3)^2(13)}{AE}$$

$$= \frac{288}{EI} + \frac{48.94}{AE}$$

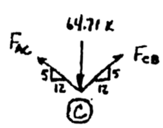
$$\Delta_{CD} + F_{CD} f_{CDCD} = 0$$

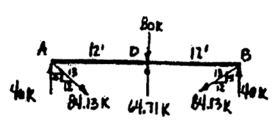
$$= \frac{23{,}040}{\frac{400}{12^4}} + F_{CD}\left(\frac{288}{\frac{400}{12^4}} + \frac{48.94}{\frac{2}{14^4}}\right) = 0$$

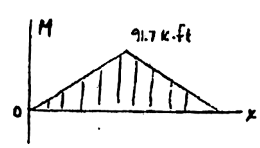
$$F_{CD} = -64.71 = 64.7\,\text{k (C)}$$ **Ans.**

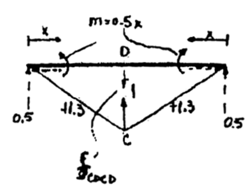
Equilibrium of joint C:

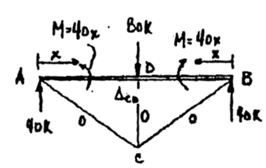
$$F_{CD} = F_{AC} = 84.1\,\text{k (T)}$$ **Ans.**



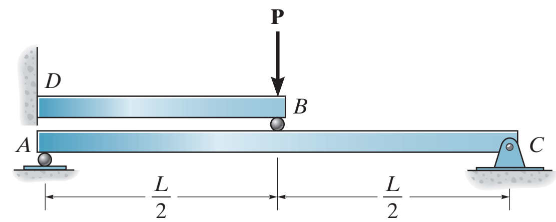








10–37. Determine the reactions at support C. EI is constant for both beams.



Support Reactions: FBD(a).

$$\overset{+}{\rightarrow} \sum F_x = 0; \qquad C_x = 0$$ **Ans.**

$$\circlearrowleft + \sum M_A = 0; \qquad C_y(L) - B_y\left(\frac{L}{2}\right) = 0 \qquad [1]$$

Method of Superposition: Using the method of superposition as discussed in Chapter 4, the required displacements are

$$v_B = \frac{PL^3}{48EI} = \frac{B_y L^3}{48EI} \downarrow$$

$$v_B' = \frac{PL_{3D}^3}{3EI} = \frac{P(\frac{L}{2})^3}{3EI} = \frac{PL^3}{24EI} \downarrow$$

$$v_B'' = \frac{PL_{3\,D}^3}{3EI} = \frac{B_y L^3}{24EI} \uparrow$$

The compatibility condition requires

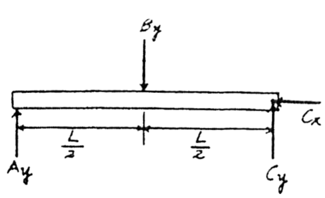
$$(+\downarrow) \qquad v_B = v_B' + v_B''$$

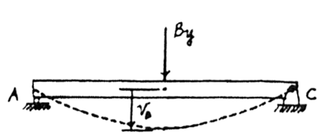
$$\frac{B_y L^3}{48EI} = \frac{PL^3}{24EI} + \left(-\frac{B_y L^3}{24EI}\right)$$

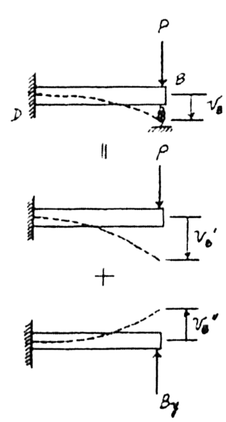
$$B_y = \frac{2P}{3}$$

Substituting B_y into Eq. [1] yields,

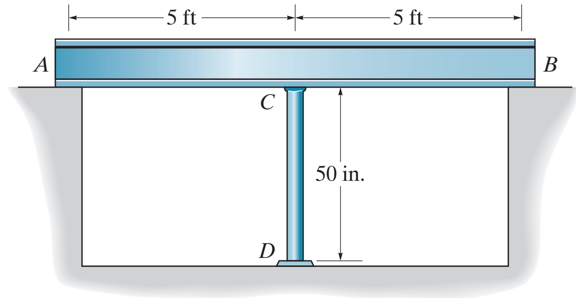
$$C_y = \frac{P}{3}$$ **Ans.**





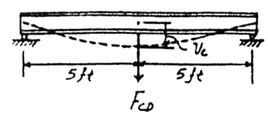


10–38. The beam AB has a moment of inertia $I = 475\,\text{in}^4$ and rests on the smooth supports at its ends. A 0.75-in.-diameter rod CD is welded to the center of the beam and to the fixed support at D. If the temperature of the rod is decreased by 150°F, determine the force developed in the rod. The beam and rod are both made of steel for which $E = 200$ GPa and $\alpha = 6.5(10^{-6})/\text{F}°$.



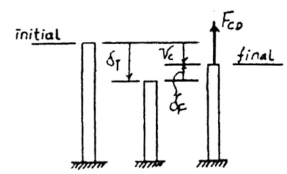
Method of Superposition: Using the method of superposition as discussed in Chapter 4, the required displacements are

$$v_C = \frac{PL^3}{48EI} = \frac{F_{CD}(120^3)}{48(29)(10^3)(475)} = 0.002613F_{CD} \;\downarrow$$



Using the axial force formula,

$$\delta_F = \frac{PL}{AE} = \frac{F_{CD}(50)}{\frac{6}{4}(0.75^2)(29)(10^3)} = 0.003903F_{CD} \;\uparrow$$



The thermal contraction is,

$$\delta_T = \alpha\Delta TL = 6.5(10^{-6})(150)(50) = 0.04875 \text{ in.} \;\downarrow$$

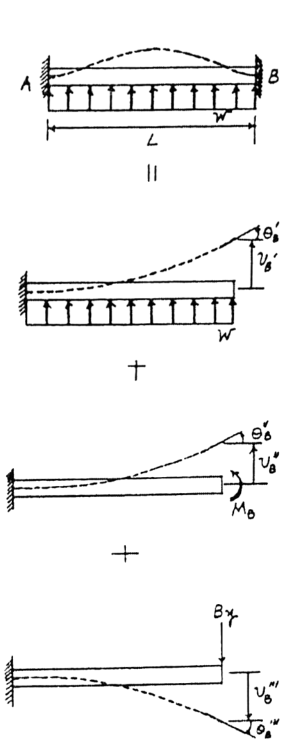
The compatibility condition requires

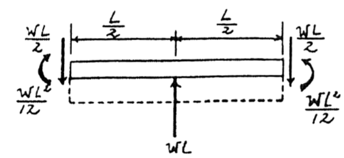
$$(+\downarrow) \qquad v_C = \delta_T + \delta_F$$

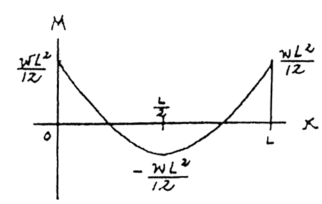
$$0.002613F_{CD} = 0.04875 + (-0.003903F_{CD})$$

$$F_{CD} = 7.48 \text{ kip}$$

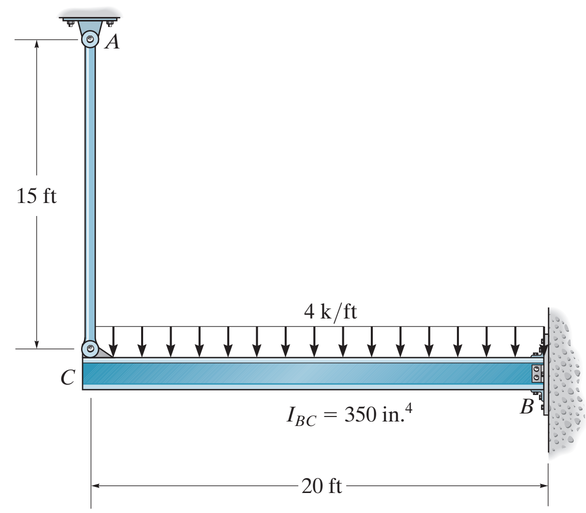
Ans.







10–39. The cantilevered beam is supported at one end by a $\frac{1}{2}$-in.-diameter suspender rod AC and fixed at the other end B. Determine the force in the rod due to a uniform loading of 4 k/ft. $E = 29(10^3)$ ksi for both the beam and rod.



$$\Delta_{AC} = \int_0^L \frac{mM}{EI}dx + \sum \frac{nNL}{AE} = \int_0^2 \frac{(1x)(-2x^2)}{EI}dx + 0 = -\frac{80.000}{EI}$$

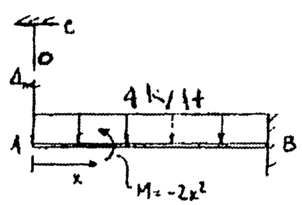
$$f_{ACAC} = \int_0^L \frac{m^2}{EI}dx + \sum \frac{n^2L}{AE} = \int_0^{20} \frac{x^2}{EI}dx + \frac{(1)^2(15)}{AE} = \frac{2666.67}{EI} + \frac{15}{AE}$$

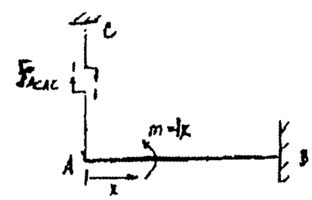
$+\downarrow \qquad \Delta_{AC} + F_{AC} f_{ACAC} = 0$

$$-\frac{80.000}{EI} + F_{AC}\left(\frac{2666.67}{EI} + \frac{15}{AE}\right) = 0$$

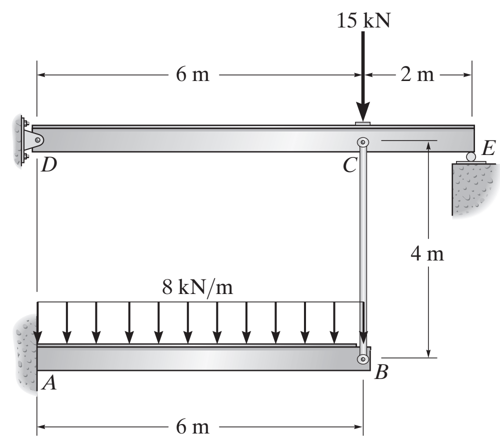
$$-\frac{80.000}{\frac{330}{12^*}} + F_{AC}\left(\frac{2666.67}{\frac{350}{17^*}} + \frac{15}{\pi(\frac{0.23}{12})^2}\right) = 0$$

$F_{AC} = 28.0$ k **Ans.**





***10–40.** The structural assembly supports the loading shown. Draw the moment diagrams for each of the beams. Take $I = 100(10^6)\ \text{mm}^4$ for the beams and $A = 200\ \text{mm}^2$ for the tie rod. All members are made of steel for which $E = 200$ GPa.



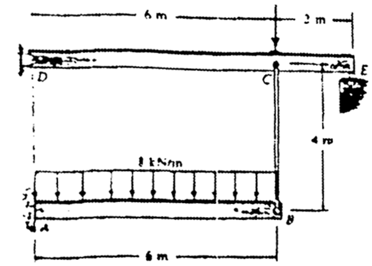
Compatibility Equation

$$0 = \Delta_{CB} + F_{CB} f_{CBCB} \quad (1)$$

Use virtual work method

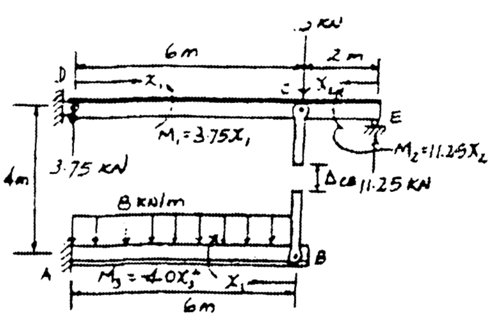
$$\Delta_{CB} = \int_0^L \frac{mM}{EI}dx = \int_0^6 \frac{(0.25x_1)(3.75x_1)}{EI}dx_1 + \int_0^2 \frac{(0.75x_2)(11.25x_2)}{EI}dx_2 + \int_0^6 \frac{(1x_3)(-4x_3^2)}{EI}dx_3$$

$$= \frac{-1206}{EI}$$



$$f_{CBCB} = \int_0^L \frac{mm}{EI}dx + \sum \frac{nnL}{AE} = \int_0^6 \frac{(0.25x_1)^2}{EI}dx_1 + \int_0^2 \frac{(0.75x_2)^2}{EI}dx_2 + \int_0^6 \frac{(1x_3)^2}{EI}dx_3 + \frac{(1)^2(4)}{AE}$$

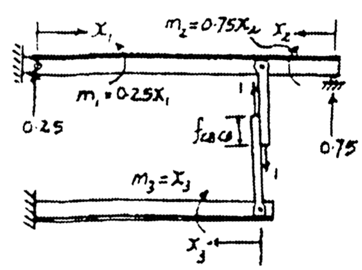
$$= \frac{78.0}{EI} + \frac{4.00}{AE}$$

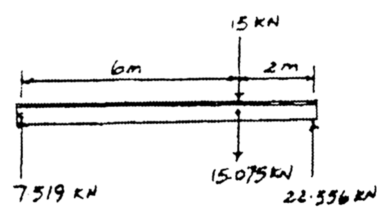


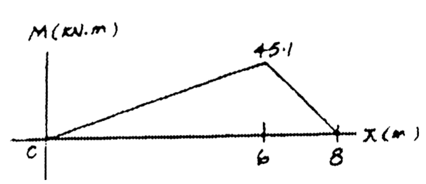
From Eq.1

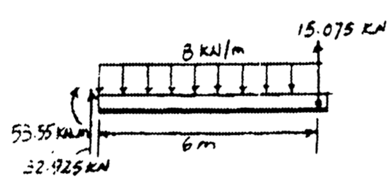
$$-\frac{1206}{E100(10^{-6})} + F_{CB}\left[\frac{78.0}{E(100)(10^{-6})} + \frac{4.00}{200(10^{-6})E}\right] = 0$$

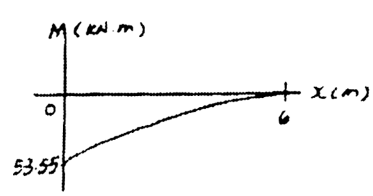
$F_{CB} = 15.075$ kN (T) $= 15.1$ kN (T)



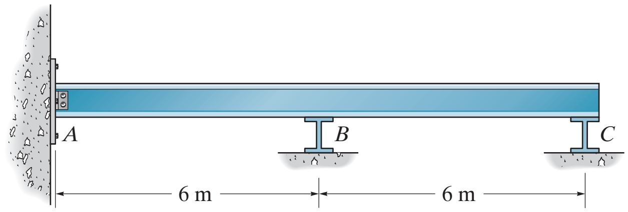








10–41. Draw the influence line for the reaction at C. Plot numerical values at the peaks. Assume A is a pin and B and C are rollers. EI is constant.



The primary real beam and qualitative influence line are shown in Fig. *a* and its conjugate beam is shown in Fig. *b*. Referring to Fig. *c*,

$$f_{AC} = M'_A = 0, \qquad f_{BC} = M'_B = 0 \qquad f_{CC} = M'_C = \frac{144}{EI}$$

The maximum displacement between A and B can be determined by referring to Fig *d*.

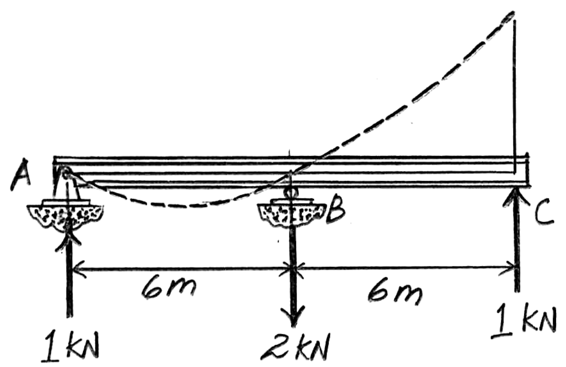
$$+\uparrow \sum F_y = 0; \qquad \frac{1}{2}\left(\frac{x}{EI}\right)x - \frac{6}{EI} = 0 \quad x = \sqrt{12}\ \text{m}$$

$$\circlearrowleft + \sum M = 0; \qquad M'_{\max} + \frac{6}{EI}\left(\sqrt{12}\right) - \frac{1}{2}\left(\frac{\sqrt{12}}{EI}\right)\left(\sqrt{12}\right)\left(\frac{\sqrt{12}}{3}\right) = 0$$

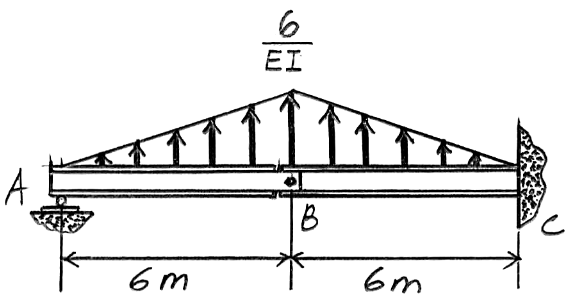
$$f_{\max} = -\frac{13.86}{EI}$$

Dividing f's by f_{CC}, we obtain

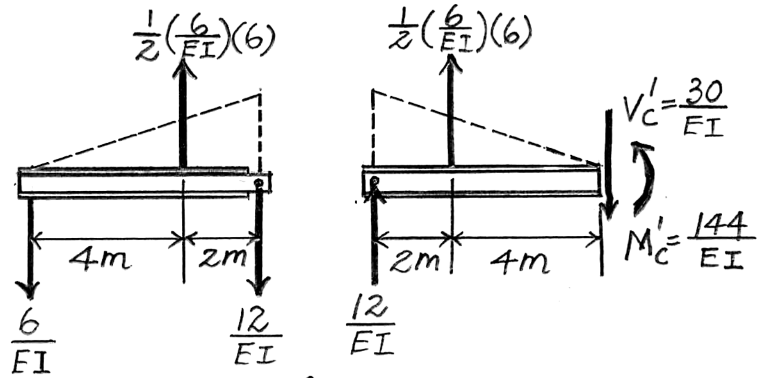
x (m)	0	$\sqrt{12}$	6	12
C_y (kN)	0	–0.0962	0	1



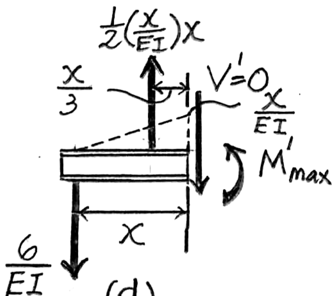
(a)

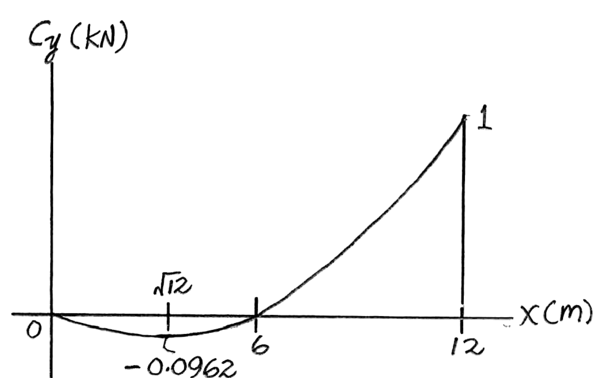


(b)



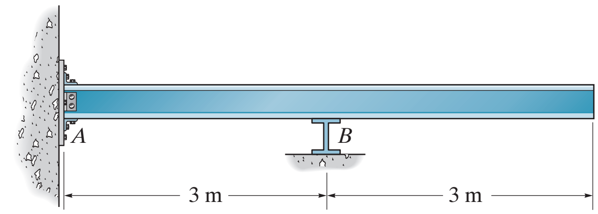
(c)





(e)

10–42. Draw the influence line for the moment at A. Plot numerical values at the peaks. Assume A is fixed and the support at B is a roller. EI is constant.



The primary real beam and qualitative influence line are shown in Fig. *a* and its conjugate beam is shown in Fig. *b*. Referring to Fig. *c*,

$$\alpha_{AA} = \frac{1}{EI}, \quad f_{AA} = M'_A = 0, \quad f_{BA} = M'_B = 0, \quad f_{CA} = M'_C = \frac{3}{2EI}$$

The maximum displacement between A and B can be determined by referring to Fig. *d*,

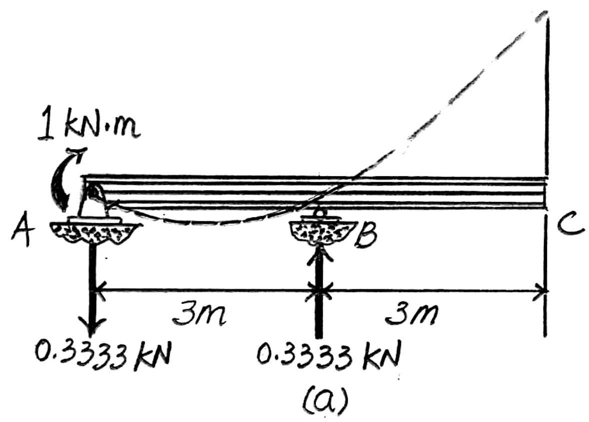
$$+\uparrow \sum F_y = 0; \qquad \frac{1}{2}\left(\frac{x}{3EI}\right)x - \frac{1}{2EI} = 0 \qquad x = \sqrt{3}\text{ m}$$

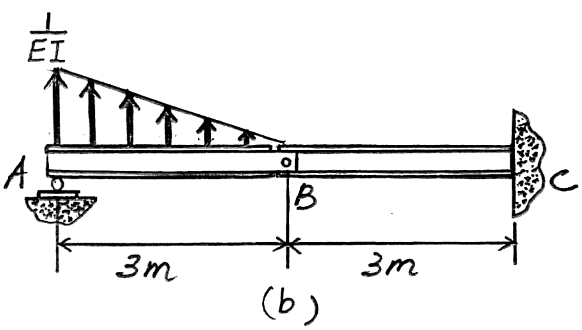
$$\circlearrowleft + \sum M = 0; \qquad \frac{1}{2}\left(\frac{\sqrt{3}}{3EI}\right)\left(\sqrt{3}\right)\left(\frac{\sqrt{3}}{3}\right) - \frac{1}{2EI}\left(\sqrt{3}\right) - M'_{max} = 0$$

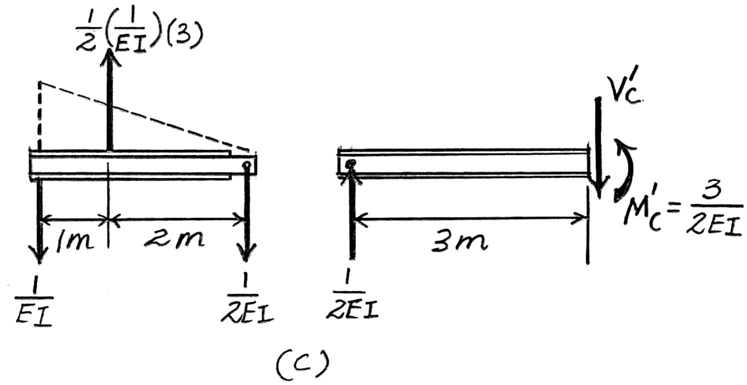
$$f_{max} = M'_{max} = -\frac{0.5774}{EI}$$

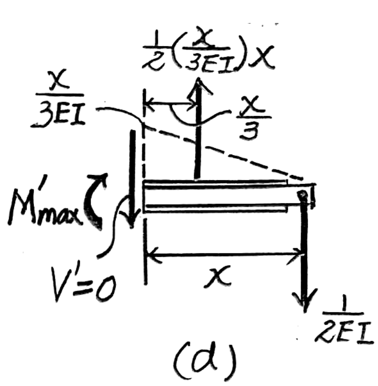
Dividing f's by α_{AA}, we obtain

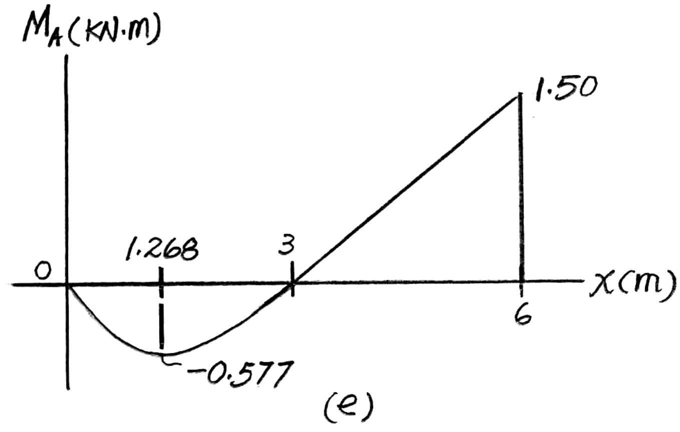
x (m)	0	1.268	3	6
M_A (kN · m)	0	–0.577	0	1.50



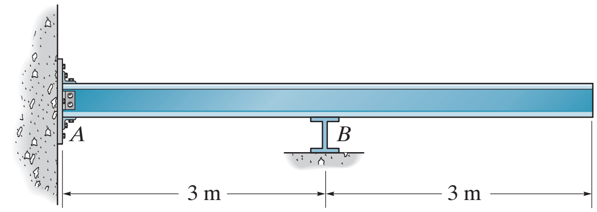








10–43. Draw the influence line for the vertical reaction at B. Plot numerical values at the peaks. Assume A is fixed and the support at B is a roller. EI is constant.



The primary real bean and qualitative influence line are shown in Fig. *a* and its conjugate beam is shown in Fig. *b*. Referring to Fig. *c*,

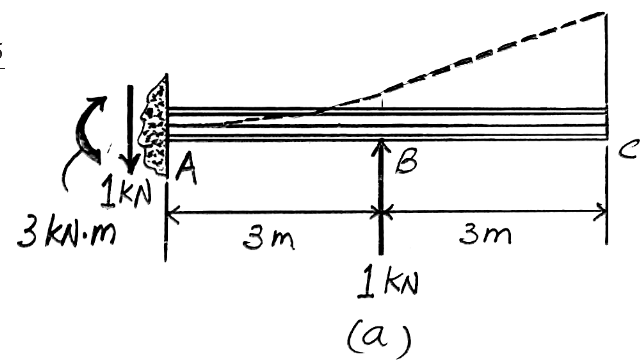
$$\zeta + \sum M_B = 0; \qquad M'_B - \frac{1}{2}\left(\frac{3}{EI}\right)(3)(2) = 0 \quad f_{BB} = M'_B = \frac{9}{EI}$$

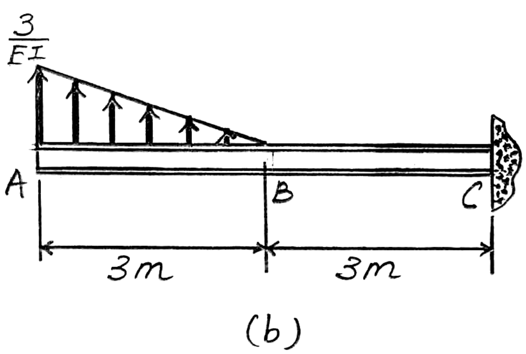
Referring to Fig. *d*,

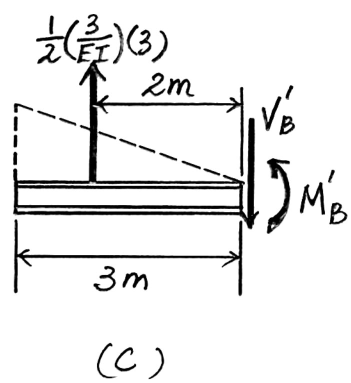
$$\zeta + \sum M_C = 0; \qquad M'_C - \frac{1}{2}\left(\frac{3}{EI}\right)(3)(5) = 0 \qquad f_{CB} = M'_C = \frac{22.5}{EI}$$

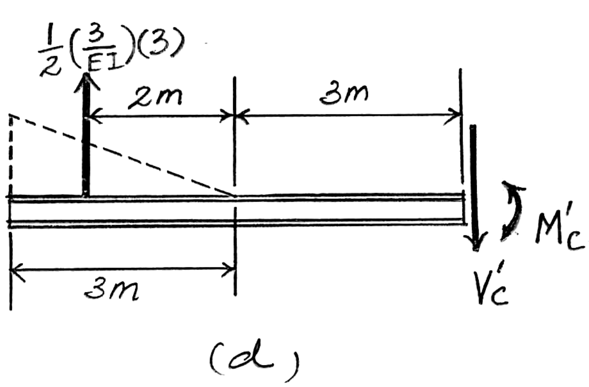
Also, $f_{AB} = 0$. Dividing f's by f_{BB}, we obtain

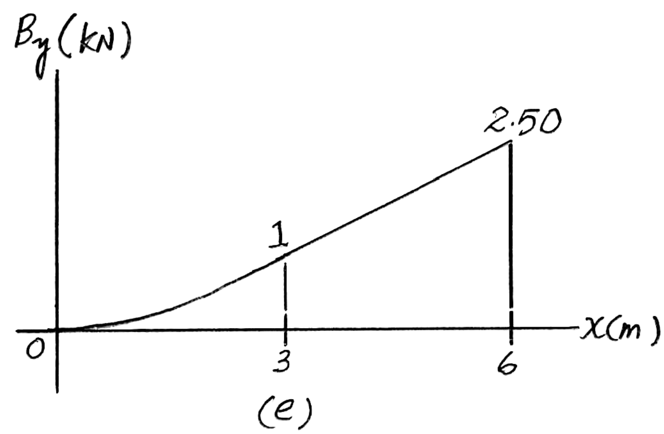
x (m)	0	3	6
B_y (kN)	0	1	2.5



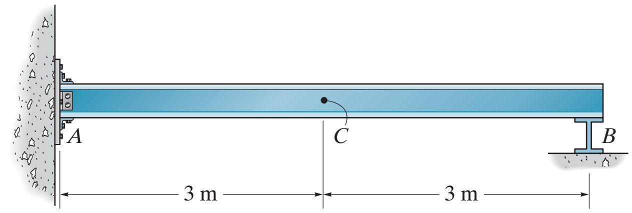








***10–44.** Draw the influence line for the shear at C. Plot numerical values every 1.5 m. Assume A is fixed and the support at B is a roller. EI is constant.



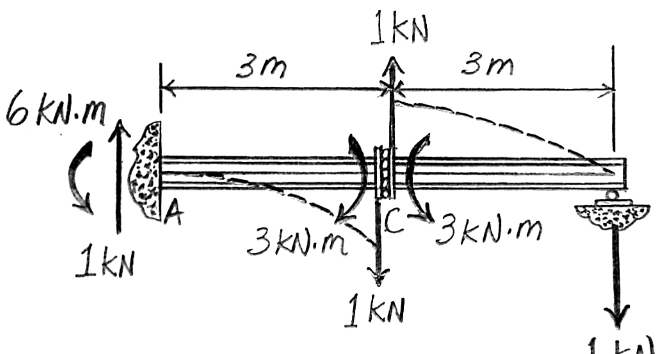
The primary real beam and qualitative influence line are shown in Fig. *a*, and its conjugate beam is shown in Fig. *b*. Referring to Figs. *c*, *d*, *e* and *f*,

$$f_{OC} = M'_0 = 0 \quad f_{1.5\,C} = M'_{1.5} = -\frac{6.1875}{EI} \quad f_{3\bar{C}} = M'_{3^-} = --\frac{22.5}{EI}$$

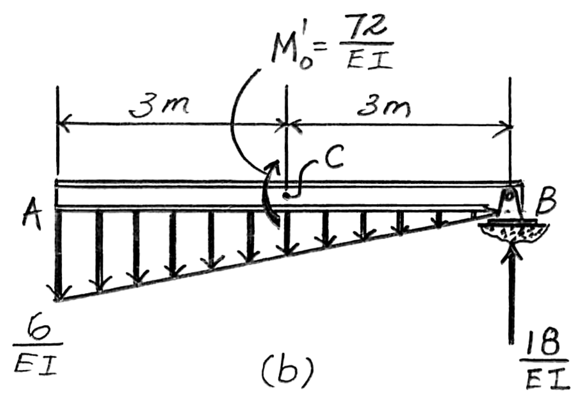
$$f_{3C}^{+} = M'_{3+} = \frac{49.5}{EI} \quad f_{4.5\,C} = M'_{4.5} = \frac{26.4375}{EI} \quad f_{6\,C} = M'_6 = 0$$

Dividing f's by $M'_0 = \dfrac{72}{EI}$, we obtain

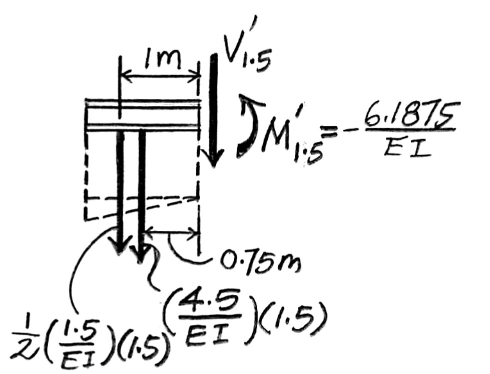
x (m)	0	1.5	3^-	3^+	4.5	6
V_C (kN)	0	–0.0859	–0.3125	0.6875	0.367	0



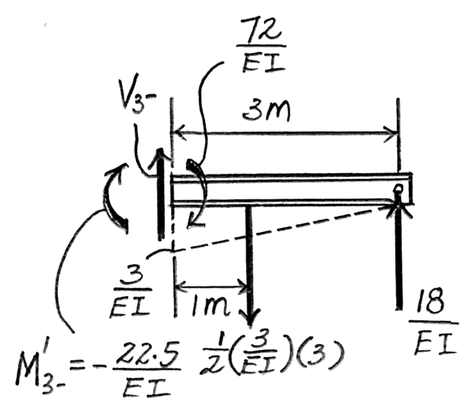
(a)



(b)

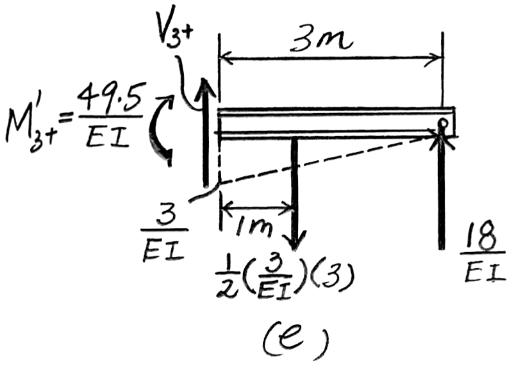


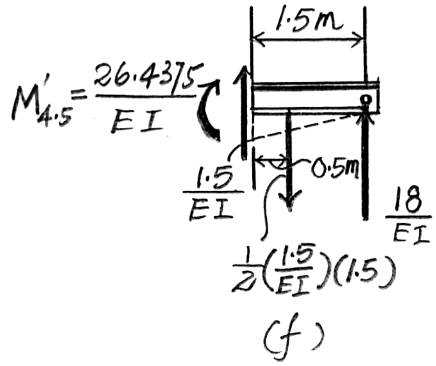
(c)

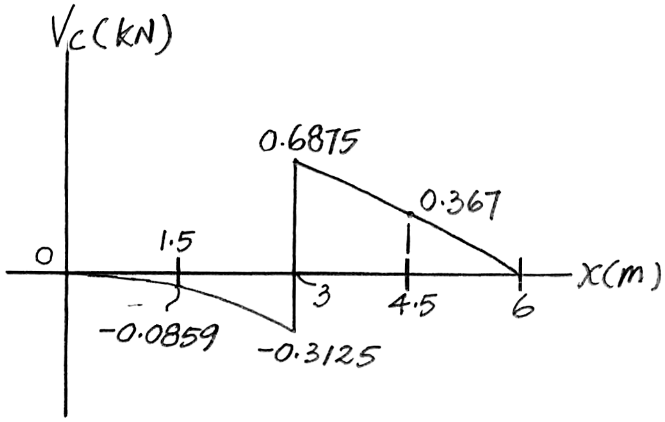


(d)

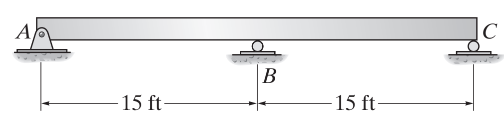
10–44. Continued







10–45. Draw the influence line for the reaction at C. Plot the numerical values every 5 ft. EI is constant.



$x = 0$ ft

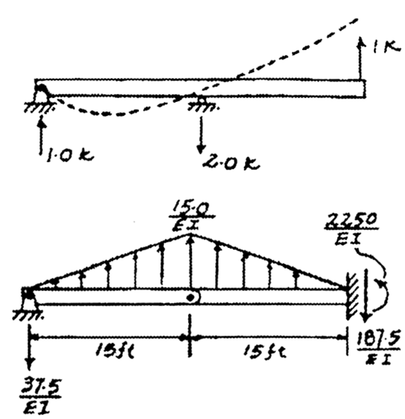
$$\Delta_0 = M_0' = 0$$

$x = 5$ ft

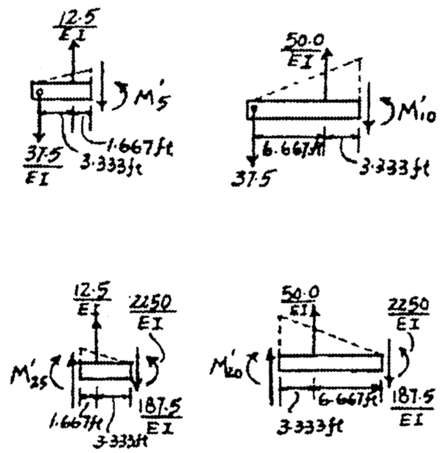
$$\Delta_5 = M_5' = \frac{12.5}{EI}1.667 - \frac{37.5}{EI}(5) = -\frac{166.67}{EI}$$

$x = 10$ ft

$$\Delta_{10} = M_{10}' = \frac{50}{EI}3.333 - \frac{37.5}{EI}(10) = -\frac{208.33}{EI}$$



10–45. Continued



$x = 15$ ft

$$\Delta_{15} = M_{15}' = 0$$

$x = 20$ ft

$$\Delta_{20} = M_{20}' = \frac{2250}{EI} + \frac{50}{EI}3.333 - \frac{187.5}{EI}(10) = \frac{541.67}{EI}$$

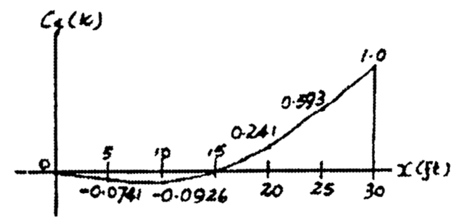
$x = 25$ ft

$$\Delta_{25} = M_{25}' = \frac{2250}{EI} + \frac{12.5}{EI}1.667 - \frac{187.5}{EI}(5) = \frac{1333.33}{EI}$$

$x = 30$ ft

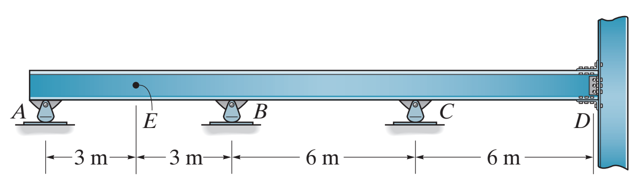
$$\Delta_{30} = M_{30}' = \frac{2250}{EI}$$

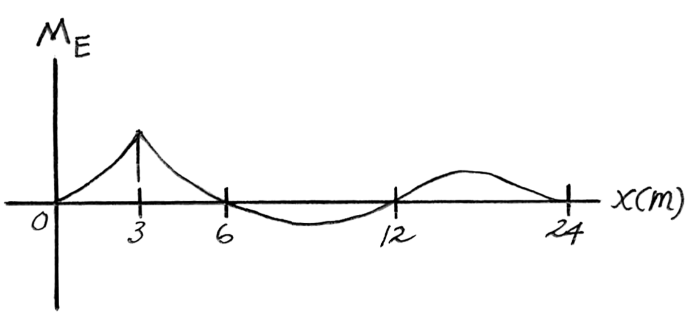
x	Δ_i/Δ_{30}
0	0
5	–0.0741
10	–0.0926
15	0
20	0.241
25	0.593
30	1.0

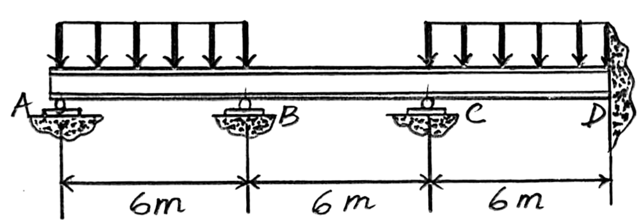


At 20 ft: $C_y = 0.241$ k **Ans.**

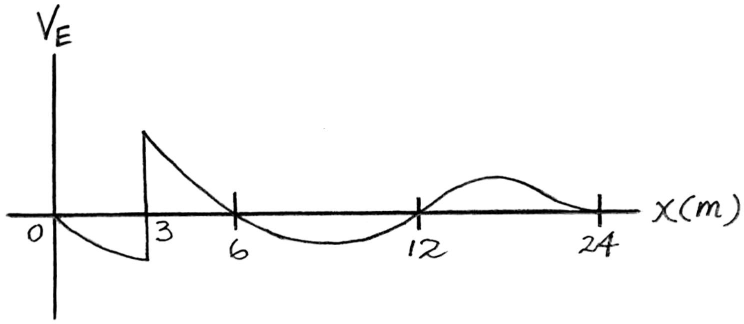
10–46. Sketch the influence line for (a) the moment at E, (b) the reaction at C, and (c) the shear at E. In each case, indicate on a sketch of the beam where a uniform distributed live load should be placed so as to cause a maximum positive value of these functions. Assume the beam is fixed at D.

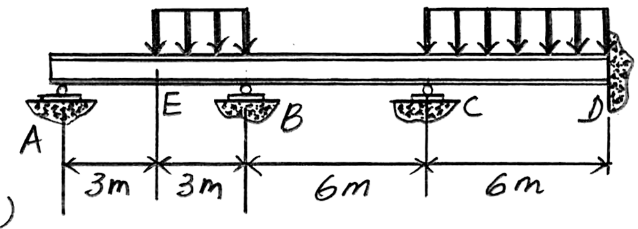


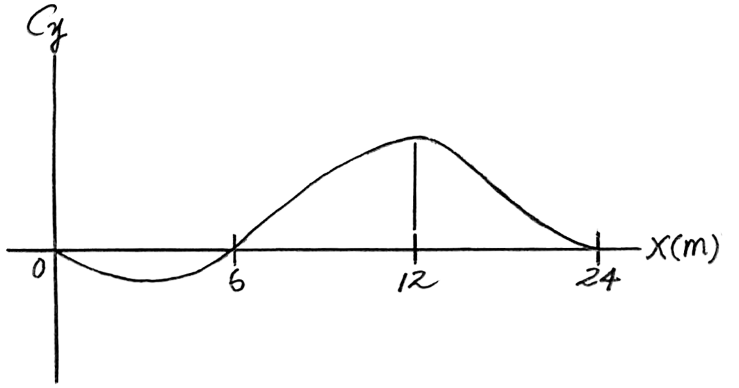


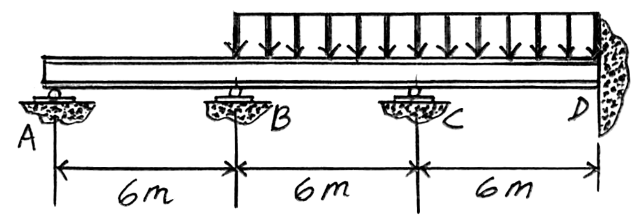


10–46. Continued

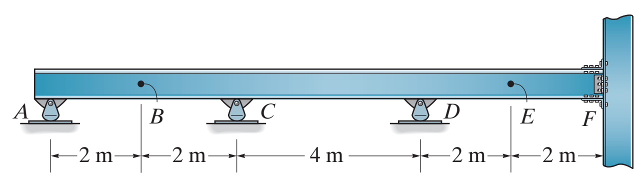


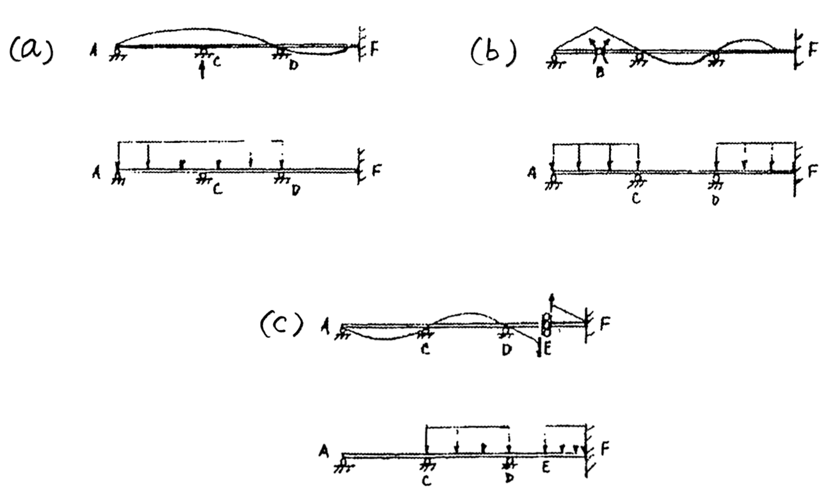




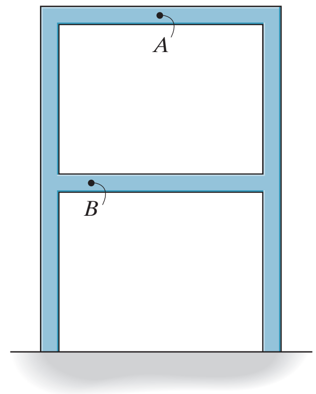


10–47. Sketch the influence line for (a) the vertical reaction at C, (b) the moment at B, and (c) the shear at E. In each case, indicate on a sketch of the beam where a uniform distributed live load should be placed so as to cause a maximum positive value of these functions. Assume the beam is fixed at F.

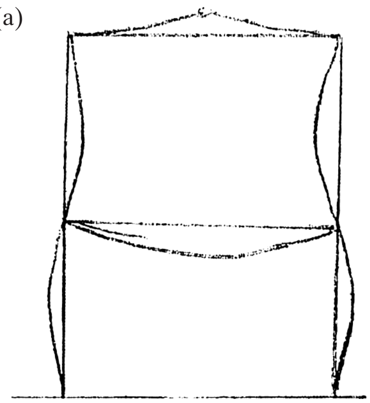




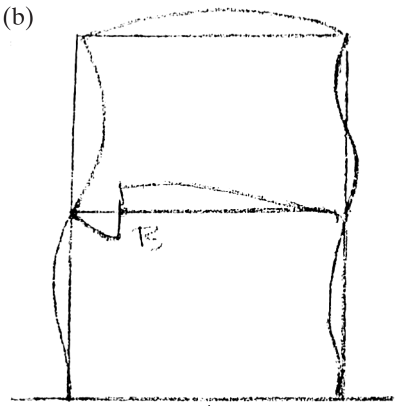
***10–48.** Use the Müller-Breslau principle to sketch the general shape of the influence line for (a) the moment at *A* and (b) the shear at *B*.



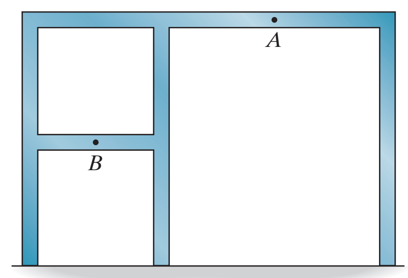
(a)



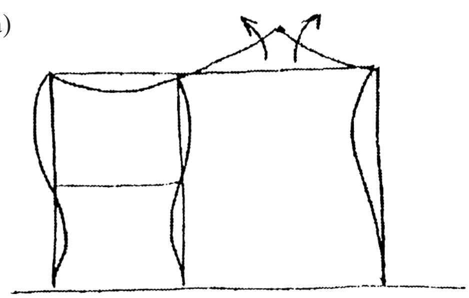
(b)



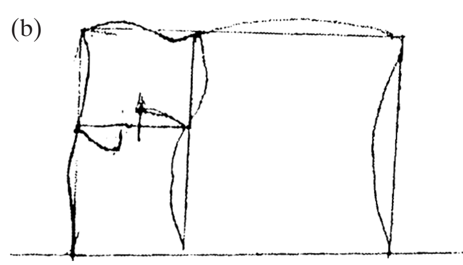
10–49. Use the Müller-Breslau principle to sketch the general shape of the influence line for (a) the moment at *A* and (b) the shear at *B*.



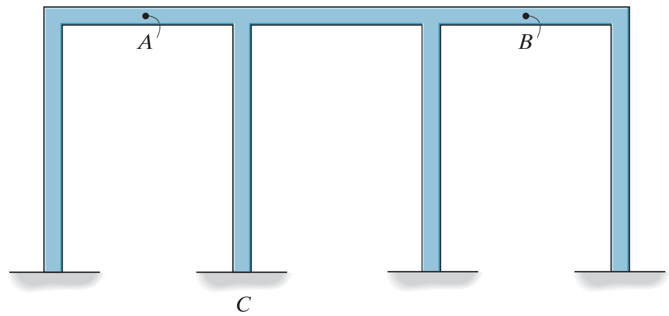
(a)

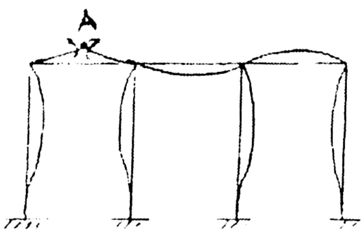


(b)

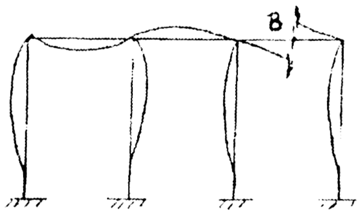


10–50. Use the Müller-Breslau principle to sketch the general shape of the influence line for (a) the moment at *A* and (b) the shear at *B*.



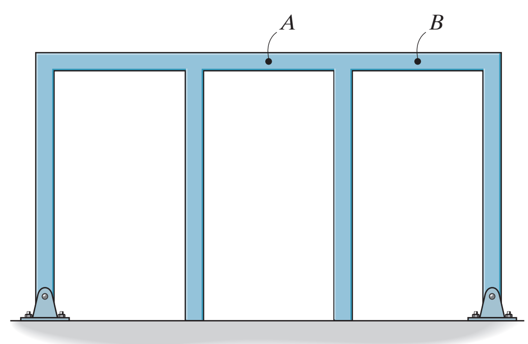


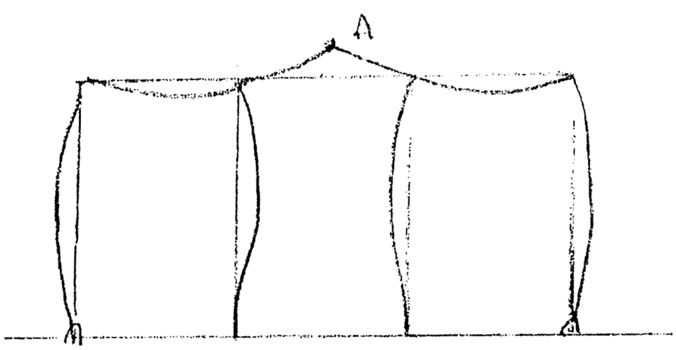
(a)



(b)

10–51. Use the Müller-Breslau principle to sketch the general shape of the influence line for (a) the moment at *A* and (b) the shear at *B*.





(a)

(b)