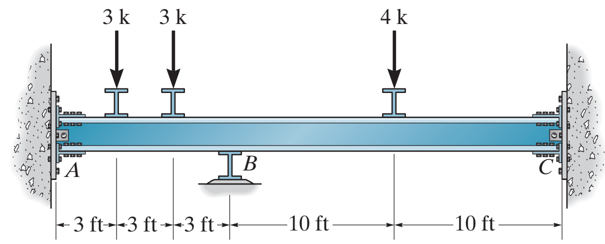
11–1. Determine the moments at A, B, and C and then draw the moment diagram. EI is constant. Assume the support at B is a roller and A and C are fixed.



Fixed End Moments. Referring to the table on the inside back cover

$$(\text{FEM})_{AB} = -\frac{2PL}{9} = -\frac{2(3)(9)}{9} = -6\text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BA} = \frac{2PL}{9} = \frac{2(3)(9)}{9} = 6\text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = -\frac{PL}{8} = -\frac{4(20)}{8} = -10\text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = \frac{PL}{8} = \frac{4(20)}{8} = 10\text{ k}\cdot\text{ft}$$

Slope-Deflection Equations. Applying Eq. 11–8,

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

For span AB,

$$M_{AB} = 2E\left(\frac{I}{9}\right)[2(0) + \theta_B - 3(0)] + (-6) = \left(\frac{2EI}{9}\right)\theta_B - 6 \qquad (1)$$

$$M_{BA} = 2E\left(\frac{I}{9}\right)[2\theta_B + 0 - 3(0)] + 6 = \left(\frac{4EI}{9}\right)\theta_B + 6 \qquad (2)$$

For span BC,

$$M_{BC} = 2E\left(\frac{I}{20}\right)[2\theta_B + 0 - 3(0)] + (-10) = \left(\frac{EI}{5}\right)\theta_B - 10 \qquad (3)$$

$$M_{CB} = 2E\left(\frac{I}{20}\right)[2(0) + \theta_B - 3(0)] + (10) = \left(\frac{EI}{10}\right)\theta_B + 10 \qquad (4)$$

Equilibrium. At Support B,

$$M_{BA} + M_{BC} = 0 \qquad (5)$$

Substitute Eq. 2 and 3 into (5),

$$\left(\frac{4EI}{9}\right)\theta_B + 6 + \left(\frac{EI}{5}\right)\theta_B - 10 = 0 \qquad \theta_B = \frac{180}{29EI}$$

Substitute this result into Eqs. 1 to 4,

$M_{AB} = -4.621\text{ k}\cdot\text{ft} = -4.62\text{ k}\cdot\text{ft}$ **Ans.**

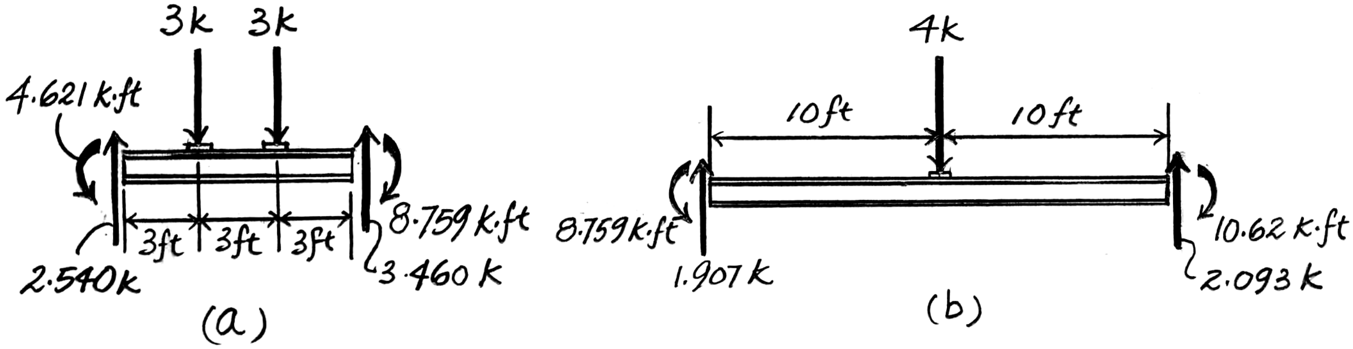
$M_{BA} = 8.759\text{ k}\cdot\text{ft} = 8.76\text{ k}\cdot\text{ft}$ **Ans.**

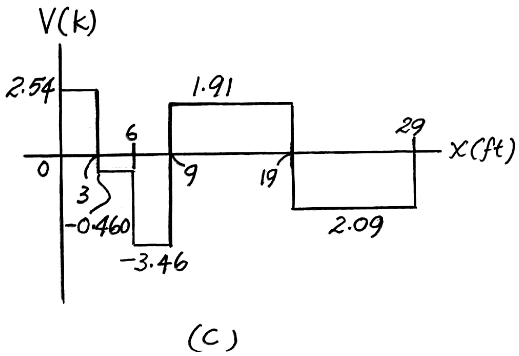
$M_{BC} = -8.759\text{ k}\cdot\text{ft} = -8.76\text{ k}\cdot\text{ft}$ **Ans.**

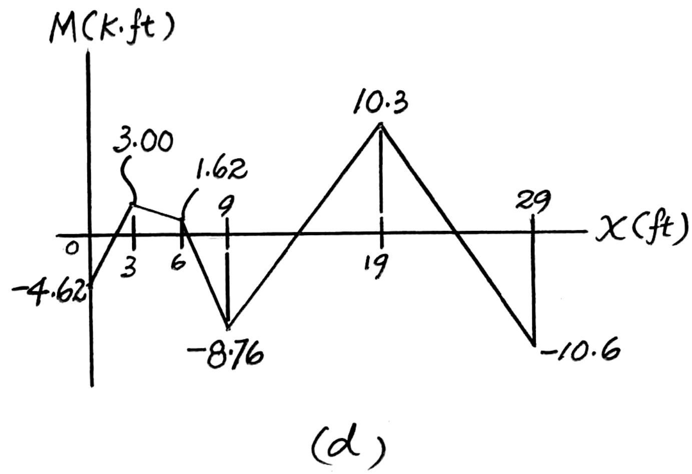
$M_{CB} = 10.62\text{ k}\cdot\text{ft} = 10.6\text{ k}\cdot\text{ft}$ **Ans.**

The Negative Signs indicate that $\mathbf{M}_{AB}$ and $\mathbf{M}_{BC}$ have the counterclockwise rotational sense. Using these results, the shear at both ends of span AB and BC are computed and shown in Fig. a and b, respectively. Subsequently, the shear and moment diagram can be plotted, Fig. c and d respectively.

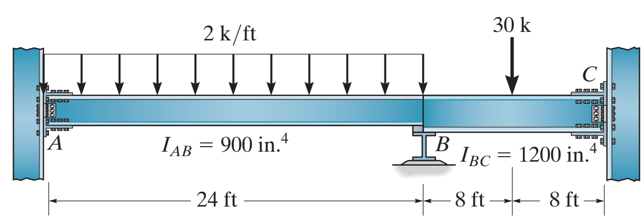
11–1. Continued







11–2. Determine the moments at A, B, and C, then draw the moment diagram for the beam. The moment of inertia of each span is indicated in the figure. Assume the support at B is a roller and A and C are fixed. $E = 29(10^3)$ ksi.



Fixed End Moments. Referring to the table on the inside back cover,

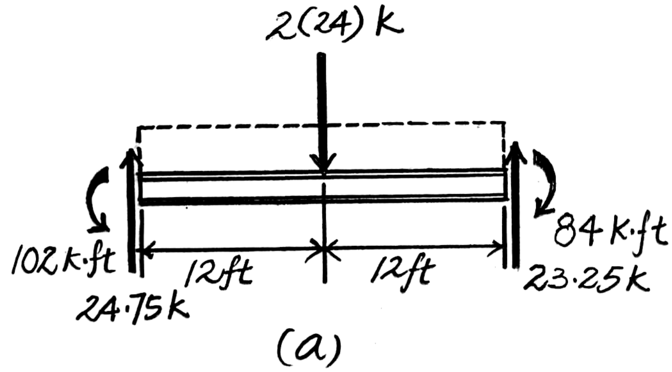
$$(\text{FEM})_{AB} = -\frac{wL^2}{12} = -\frac{2(24^2)}{12} = -96 \text{ k}\cdot\text{ft}$$

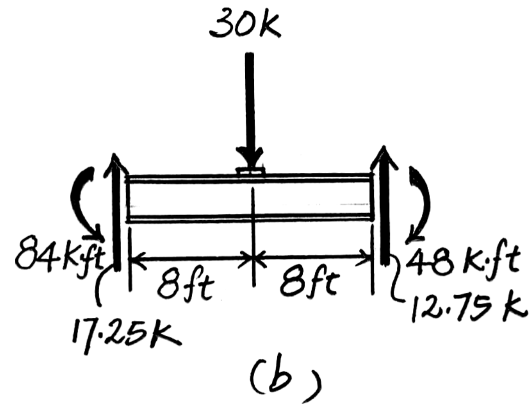
$$(\text{FEM})_{BA} = \frac{wL^2}{12} = \frac{2(24^2)}{12} = 96 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = -\frac{PL}{8} = -\frac{30(16)}{8} = -60 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{CB} = \frac{PL}{8} = \frac{30(16)}{8} = 60 \text{ k}\cdot\text{ft}$$

11–2. Continued





Slope-Deflection Equations. Applying Eq. 11–8,

$$M_N = 2Ek\,(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

For span AB,

$$M_{AB} = 2E\left[\frac{900\text{ in}^4}{24(12)\text{ in}}\right][2(0) + \theta_B - 3(0)] + [-96(12)\text{ k}\cdot\text{in}]$$

$$M_{AB} = 6.25E\theta_B - 1152 \qquad (1)$$

$$M_{BA} = 2E\left[\frac{900\text{ in}^4}{24(12)\text{ in}}\right][2\theta_B + 0 - 3(0)] + 96(12)\text{ k}\cdot\text{in}$$

$$M_{BA} = 12.5E\theta_B + 1152 \qquad (2)$$

For span BC,

$$M_{BC} = 2E\left[\frac{1200\text{ in}^4}{16(12)\text{ in}}\right][2\theta_B + 0 - 3(0)] + [-60(12)\text{ k}\cdot\text{in}]$$

$$M_{BC} = 25E\theta_B - 720 \qquad (3)$$

$$M_{CB} = 2E\left[\frac{1200\text{ in}^4}{16(12)\text{ in}}\right][2(0) + \theta_B - 3(0)] + 60(12)\text{ k}\cdot\text{in}$$

$$M_{CB} = 12.5E\theta_B + 720 \qquad (4)$$

Equilibrium. At Support B,

$$M_{BA} + M_{BC} = 0 \qquad (5)$$

Substitute Eqs. 3(2) and (3) into (5),

$$12.5E\theta_B + 1152 + 25E\theta_B - 720 = 0$$

$$\theta_B = -\frac{11.52}{E}$$

Substitute this result into Eqs. (1) to (4),

$M_{AB} = -1224\text{ k}\cdot\text{in} = -102\text{ k}\cdot\text{ft}$ **Ans.**

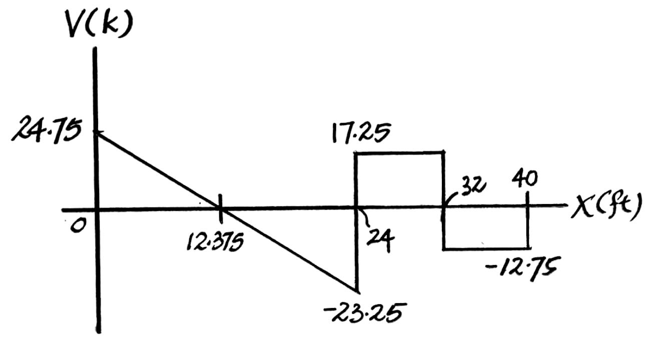
$M_{BA} = 1008\text{ k}\cdot\text{in} = 84\text{ k}\cdot\text{ft}$ **Ans.**

$M_{BC} = -1008\text{ k}\cdot\text{in} = -84\text{ k}\cdot\text{ft}$ **Ans.**

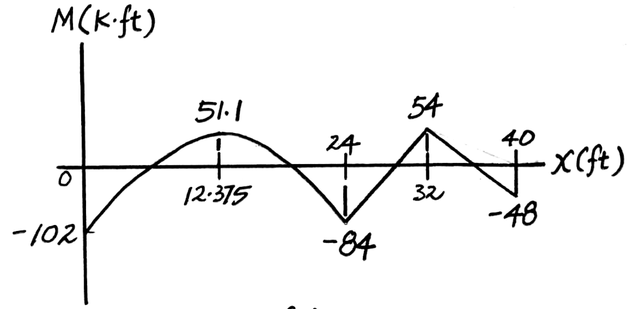
$M_{CB} = 576\text{ k}\cdot\text{in} = 48\text{ k}\cdot\text{ft}$ **Ans.**

The negative signs indicate that $\mathbf{M}_{AB}$ and $\mathbf{M}_{BC}$ have counterclockwise rotational senses. Using these results, the shear at both ends of spans AB and BC are computed and shown in Fig. *a* and *b*, respectively. Subsequently, the shear and moment diagram can be plotted, Fig. *c* and *d* respectively.

11–2. Continued

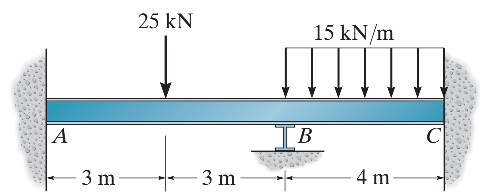


(c)



(d)

11–3. Determine the moments at the supports A and C, then draw the moment diagram. Assume joint B is a roller. EI is constant.



$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = \frac{2EI}{6}(0 + \theta_B) - \frac{(25)(6)}{8}$$

$$M_{BA} = \frac{2EI}{6}(2\theta_B) + \frac{(25)(6)}{8}$$

$$M_{BC} = \frac{2EI}{4}(2\theta_B) - \frac{(15)(4)^2}{12}$$

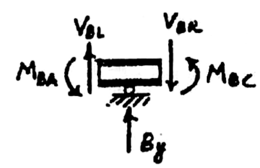
$$M_{CB} = \frac{2EI}{4}(\theta_B) + \frac{(15)(4)^2}{12}$$

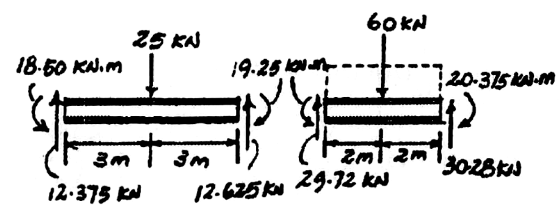
Equilibrium.

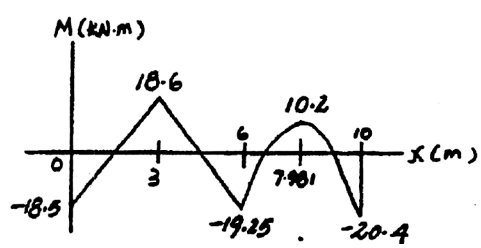
$$M_{BA} + M_{BC} = 0$$

$$\frac{2EI}{6}(2\theta_B) + \frac{25(6)}{8} + \frac{2EI}{4}(2\theta_B) - \frac{15(4)^2}{12} = 0$$

$$\theta_B = \frac{0.75}{EI}$$







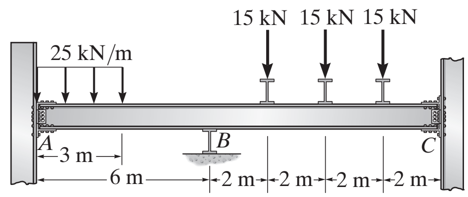
$M_{AB} = -18.5 \text{ kN}\cdot\text{m}$ **Ans.**

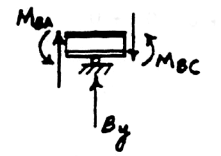
$M_{CB} = 20.375 \text{ kN}\cdot\text{m} = 20.4 \text{ kN}\cdot\text{m}$ **Ans.**

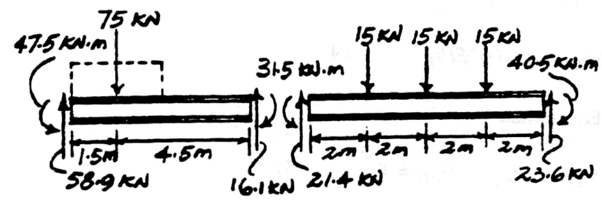
$M_{BA} = 19.25 \text{ kN}\cdot\text{m}$ **Ans.**

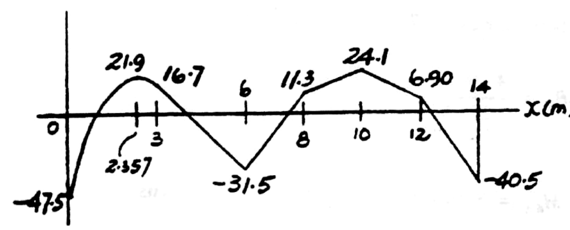
$M_{BC} = -19.25 \text{ kN}\cdot\text{m}$ **Ans.**

***11–4.** Determine the moments at the supports, then draw the moment diagram. Assume B is a roller and A and C are fixed. EI is constant.









$$(\text{FEM})_{AB} = -\frac{11(25)(6)^2}{192} = -51.5625 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{BA} = \frac{5(25)(6)^2}{192} = 23.4375 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{BC} = \frac{-5(15)(8)}{16} = -37.5 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{CB} = 37.5 \text{ kN}\cdot\text{m}$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2E\left(\frac{I}{6}\right)(2(0) + \theta_B - 0) - 51.5625$$

$$M_{AB} = \frac{EI\theta_B}{3} - 51.5625 \qquad (1)$$

$$M_{BA} = 2E\left(\frac{I}{6}\right)(2\theta_B + 0 - 0) + 23.4375$$

$$M_{BA} = \frac{2EI\theta_B}{3} + 23.4375 \qquad (2)$$

$$M_{BC} = 2E\left(\frac{I}{8}\right)(2\theta_B + 0 - 0) - 37.5$$

$$M_{BC} = \frac{EI\theta_B}{2} - 37.5 \qquad (3)$$

$$M_{CB} = 2E\left(\frac{I}{8}\right)(2(0) + \theta_B - 0) + 37.5$$

$$M_{CB} = \frac{EI\theta_B}{4} + 37.5 \qquad (4)$$

Equilibrium.

$$M_{BA} + M_{BC} = 0 \qquad (5)$$

Solving:

$$\theta_B = \frac{12.054}{EI}$$

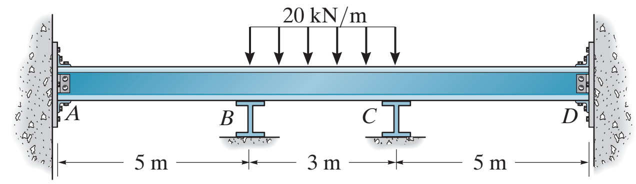
$M_{AB} = -47.5 \text{ kN}\cdot\text{m}$ **Ans.**

$M_{BA} = 31.5 \text{ kN}\cdot\text{m}$ **Ans.**

$M_{BC} = -31.5 \text{ kN}\cdot\text{m}$ **Ans.**

$M_{CB} = 40.5 \text{ kN}\cdot\text{m}$ **Ans.**

11–5. Determine the moment at A, B, C and D, then draw the moment diagram for the beam. Assume the supports at A and D are fixed and B and C are rollers. EI is constant.

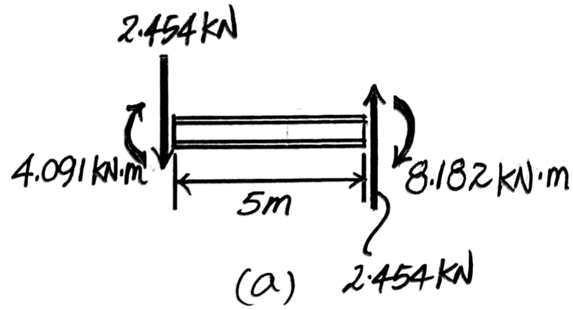


Fixed End Moments. Referring to the table on the inside back cover,

$(\text{FEM})_{AB} = 0 \qquad (\text{FEM})_{BA} = 0 \qquad (\text{FEM})_{CD} = 0 \qquad (\text{FEM})_{DC} = 0$

$$(\text{FEM})_{BC} = -\frac{wL^2}{12} = -\frac{20(3^2)}{12} = -15 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{CB} = \frac{wL^2}{12} = \frac{20(3^2)}{12} = 15 \text{ kN}\cdot\text{m}$$



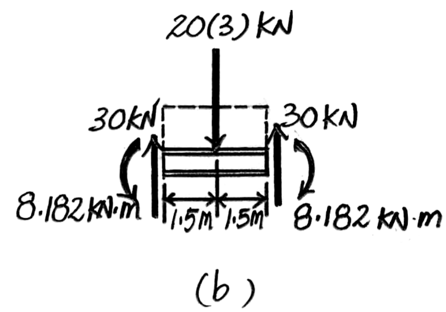
Slope-Deflection Equation. Applying Eq. 11–8,

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

For span AB,

$$M_{AB} = 2E\left(\frac{I}{5}\right)[2(0) + \theta_B - 3(0)] + 0 = \left(\frac{2EI}{5}\right)\theta_B \quad (1)$$

$$M_{BA} = 2E\left(\frac{I}{5}\right)[2\theta_B + 0 - 3(0)] + 0 = \left(\frac{4EI}{5}\right)\theta_B \quad (2)$$



For span BC,

$$M_{BC} = 2E\left(\frac{I}{3}\right)[2\theta_B + \theta_C - 3(0)] + (-15) = \left(\frac{4EI}{3}\right)\theta_B + \left(\frac{2EI}{3}\right)\theta_C - 15 \quad (3)$$

$$M_{CB} = 2E\left(\frac{I}{3}\right)[2\theta_C + \theta_B - 3(0)] + 15 = \left(\frac{4EI}{3}\right)\theta_C + \left(\frac{2EI}{3}\right)\theta_B + 15 \quad (4)$$

For span CD,

$$M_{CD} = 2E\left(\frac{I}{5}\right)[2\theta_C + 0 - 3(0)] + 0 = \left(\frac{4EI}{5}\right)\theta_C \quad (5)$$

$$M_{DC} = 2E\left(\frac{I}{5}\right)[2(0) + \theta_C - 3(0)] + 0 = \left(\frac{2EI}{5}\right)\theta_C \quad (6)$$

Equilibrium. At Support B,

$$M_{BA} + M_{BC} = 0$$

$$\left(\frac{4EI}{5}\right)\theta_B + \left(\frac{4EI}{3}\right)\theta_B + \left(\frac{2EI}{3}\right)\theta_C - 15 = 0$$

$$\left(\frac{32EI}{15}\right)\theta_B + \left(\frac{2EI}{3}\right)\theta_C = 15 \quad (7)$$

At Support C,

$$M_{CB} + M_{CD} = 0$$

$$\left(\frac{4EI}{3}\right)\theta_C + \left(\frac{2EI}{3}\right)\theta_B + 15 + \left(\frac{4EI}{5}\right)\theta_C = 0$$

11–5. Continued

$$\left(\frac{2EI}{3}\right)\theta_B + \left(\frac{32EI}{15}\right)\theta_C = -15 \qquad (8)$$

Solving Eqs. (7) and (8)

$$\theta_B = \frac{225}{22EI} \qquad \theta_C = -\frac{225}{22EI}$$

Substitute these results into Eqs. (1) to (6),

$M_{AB} = 4.091\ \text{kN} \cdot \text{m} = 4.09\ \text{kN} \cdot \text{m}$ **Ans.**

$M_{BA} = 8.182\ \text{kN} \cdot \text{m} = 8.18\ \text{kN} \cdot \text{m}$ **Ans.**

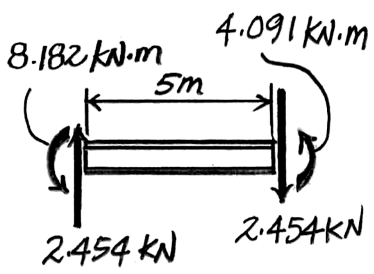
$M_{BC} = -8.182\ \text{kN} \cdot \text{m} = -8.18\ \text{kN} \cdot \text{m}$ **Ans.**

$M_{CB} = 8.182\ \text{kN} \cdot \text{m} = 8.18\ \text{kN} \cdot \text{m}$ **Ans.**

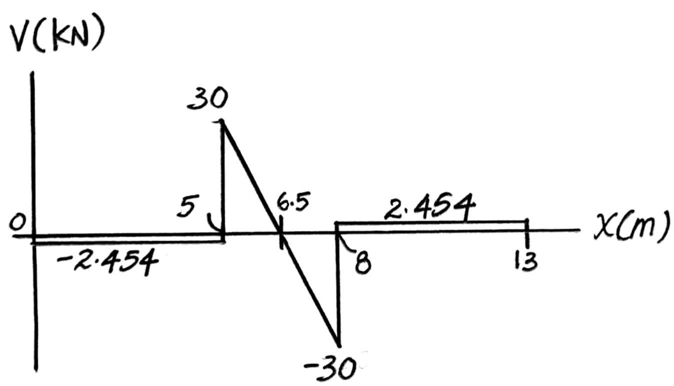
$M_{CD} = -8.182\ \text{kN} \cdot \text{m} = -8.18\ \text{kN} \cdot \text{m}$ **Ans.**

$M_{DC} = -4.091\ \text{kN} \cdot \text{m} = -4.09\ \text{kN} \cdot \text{m}$ **Ans.**

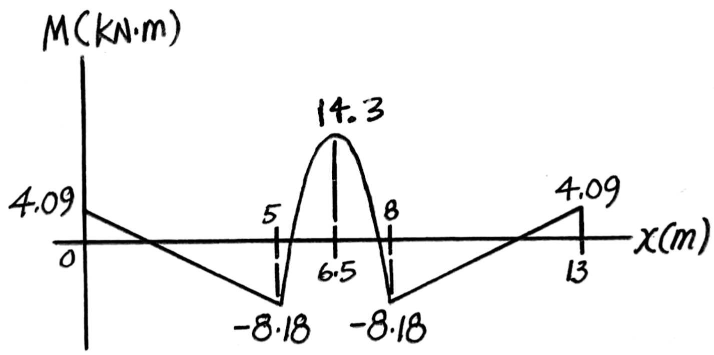
The negative sign indicates that $\mathbf{M}_{BC}$, $\mathbf{M}_{CD}$ and $\mathbf{M}_{DC}$ have counterclockwise rotational sense. Using these results, the shear at both ends of spans AB, BC, and CD are computed and shown in Fig. *a*, *b*, and *c* respectively. Subsequently, the shear and moment diagram can be plotted, Fig. *d*, and *e* respectively.



(c)

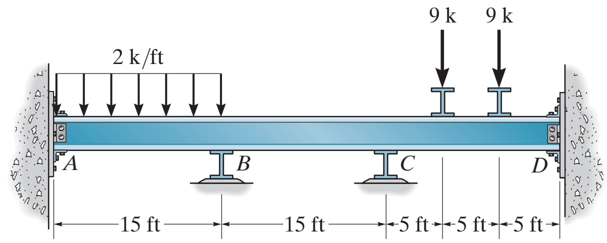


(d)

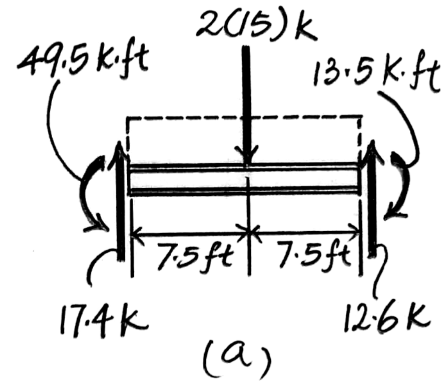


(e)

11–6. Determine the moments at A, B, C and D, then draw the moment diagram for the beam. Assume the supports at A and D are fixed and B and C are rollers. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,



$$(\text{FEM})_{AB} = -\frac{wL^2}{12} = -\frac{2(15)^2}{12} = -37.5\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{BA} = \frac{wL^2}{12} = \frac{2(15^2)}{12} = 37.5\ \text{k}\cdot\text{ft}$$

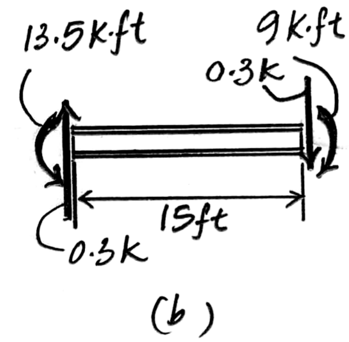
$$(\text{FEM})_{BC} = (\text{FEM})_{CB} = 0$$

$$(\text{FEM})_{CD} = \frac{-2PL}{9} = -\frac{2(9)(15)}{9} = -30\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{DC} = \frac{2PL}{9} = \frac{2(9)(15)}{9} = 30\ \text{k}\cdot\text{ft}$$

Slope-Deflection Equation. Applying Eq. 11–8,

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$



For span AB,

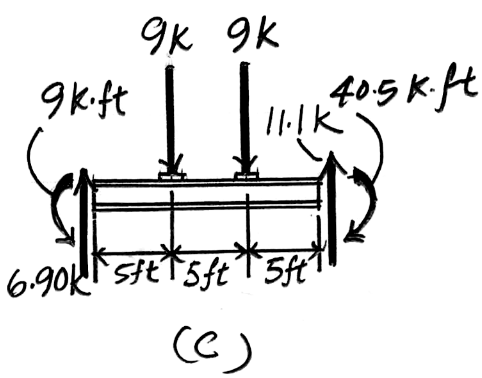
$$M_{AB} = 2E\left(\frac{I}{15}\right)[2(0) + \theta_B - 3(0)] + (-37.5) = \left(\frac{2EI}{15}\right)\theta_B - 37.5 \quad (1)$$

$$M_{BA} = 2E\left(\frac{I}{15}\right)[2\theta_B + 0 - 3(0)] + 37.5 = \left(\frac{4EI}{15}\right)\theta_B + 37.5 \quad (2)$$

For span BC,

$$M_{BC} = 2E\left(\frac{I}{15}\right)[2\theta_B + \theta_C - 3(0)] + 0 = \left(\frac{4EI}{15}\right)\theta_B + \left(\frac{2EI}{15}\right)\theta_C \quad (3)$$

$$M_{CB} = 2E\left(\frac{I}{15}\right)[2\theta_C + \theta_B - 3(0)] + 0 = \left(\frac{4EI}{15}\right)\theta_C + \left(\frac{2EI}{15}\right)\theta_B \quad (4)$$



11–6. Continued

For span CD,

$$M_{CD} = 2E\left(\frac{I}{15}\right)[2\theta_C + 0 - 3(0)] + (-30) = \left(\frac{4EI}{15}\right)\theta_C - 30 \quad (5)$$

$$M_{DC} = 2E\left(\frac{I}{15}\right)[2(0) + \theta_C - 3(0)] + 30 = \left(\frac{2EI}{15}\right)\theta_C + 30 \quad (6)$$

Equilibrium. At Support B,

$$M_{BA} + M_{BC} = 0$$

$$\left(\frac{4EI}{15}\right)\theta_B + 37.5 + \left(\frac{4EI}{15}\right)\theta_B + \left(\frac{2EI}{15}\right)\theta_C = 0$$

$$\left(\frac{8EI}{15}\right)\theta_B + \left(\frac{2EI}{15}\right)\theta_C = -37.5 \quad (7)$$

At Support C,

$$M_{CB} + M_{CD} = 0$$

$$\left(\frac{4EI}{15}\right)\theta_C + \left(\frac{2EI}{15}\right)\theta_B + \left(\frac{4EI}{15}\right)\theta_C - 30 = 0$$

$$\left(\frac{8EI}{15}\right)\theta_C + \left(\frac{2EI}{15}\right)\theta_B = 30 \quad (8)$$

Solving Eqs. (7) and (8),

$$\theta_C = \frac{78.75}{EI} \qquad \theta_B = -\frac{90}{EI}$$

Substitute these results into Eqs. (1) to (6),

$M_{AB} = -49.5 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{BA} = 13.5 \text{ k} \cdot \text{ft}$ **Ans.**

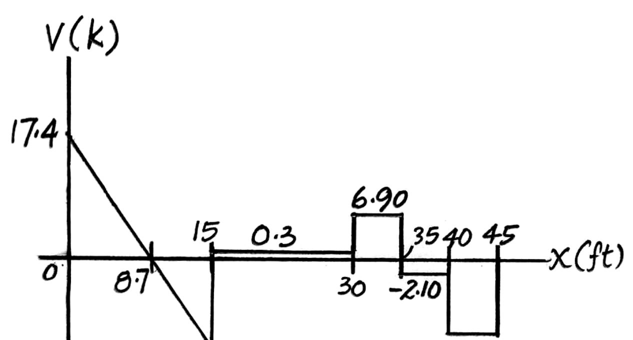
$M_{BC} = -13.5 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CB} = 9 \text{ k} \cdot \text{ft}$ **Ans.**

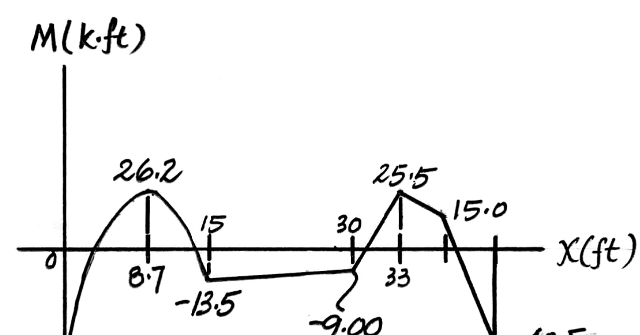
$M_{CD} = -9 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{DC} = 40.5 \text{ k} \cdot \text{ft}$ **Ans.**

The negative signs indicate that $\mathbf{M}_{AB}$, $\mathbf{M}_{BC}$ and $\mathbf{M}_{CD}$ have counterclockwise rotational sense. Using these results, the shear at both ends of spans AB, BC, and CD are computed and shown in Fig. *a*, *b*, and *c* respectively. Subsequently, the shear and moment diagram can be plotted, Fig. *d*, and *e* respectively.

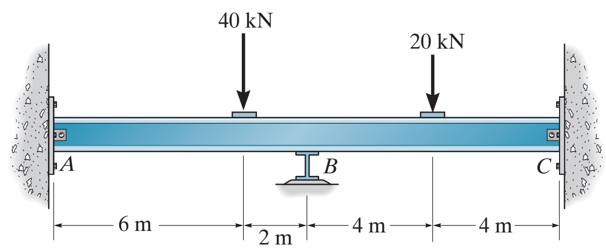


(d)



(e)

11–7. Determine the moment at B, then draw the moment diagram for the beam. Assume the supports at A and C are pins and B is a roller. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{BA} = \left(\frac{P}{L^2}\right)\left(b^2a + \frac{a^2b}{2}\right) = \left(\frac{40}{8^2}\right)\left[6^2(2) + \frac{2^2(6)}{2}\right] = 52.5\ \text{kN}\cdot\text{m}$$

$$(\text{FEM})_{BC} = -\frac{3PL}{16} = -\frac{3(20)(8)}{16} = -30\ \text{kN}\cdot\text{m}$$

Slope-Deflection Equations. Applying Eq. 11–10 Since one of the end's support for spans AB and BC is a pin.

$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

For span AB,

$$M_{BA} = 3E\left(\frac{I}{8}\right)(\theta_B - 0) + 52.5 = \left(\frac{3EI}{8}\right)\theta_B + 52.5 \qquad (1)$$

For span BC,

$$M_{BC} = 3E\left(\frac{I}{8}\right)(\theta_B - 0) + (-30) = \left(\frac{3EI}{8}\right)\theta_B - 30 \qquad (2)$$

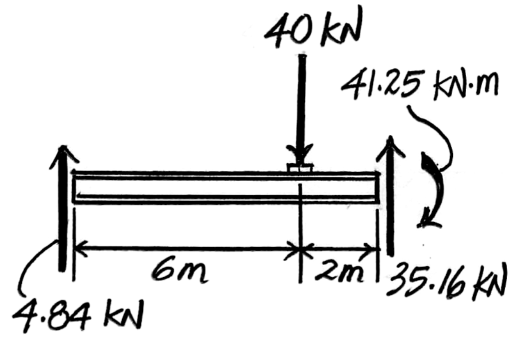
Equilibrium. At support B,

$$M_{BA} + M_{BC} = 0$$

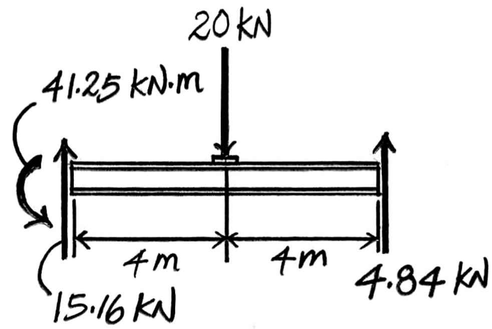
$$\left(\frac{3EI}{8}\right)\theta_B + 52.5 + \left(\frac{3EI}{8}\right)\theta_B - 30 = 0$$

$$\left(\frac{3EI}{4}\right)\theta_B = -22.5$$

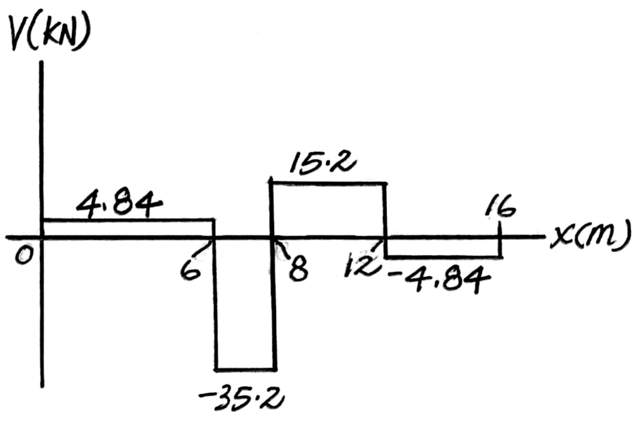
$$\theta_B = -\frac{30}{EI}$$



(a)

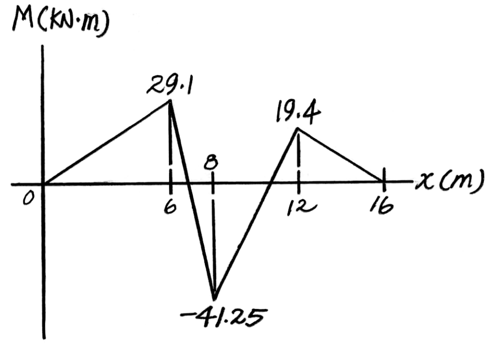


(b)



(c)

11–7. Continued



(d)

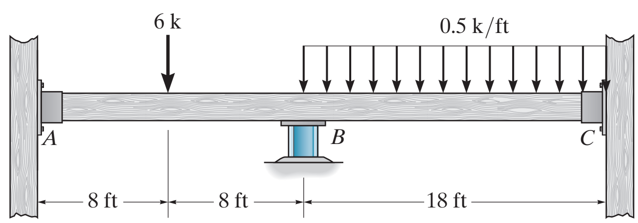
Substitute this result into Eqs. (1) and (2)

$M_{BA} = 41.25\ \text{kN}\cdot\text{m}$ **Ans.**

$M_{BC} = -41.25\ \text{kN}\cdot\text{m}$ **Ans.**

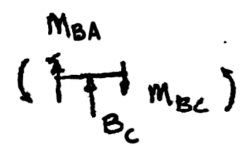
The negative sign indicates that $\mathbf{M}_{BC}$ has counterclockwise rotational sense. Using this result, the shear at both ends of spans *AB* and *BC* are computed and shown in Fig. *a* and *b* respectively. Subsequently, the shear and Moment diagram can be plotted, Fig. *c* and *d* respectively.

***11–8.** Determine the moments at *A*, *B*, and *C*, then draw the moment diagram. *EI* is constant. Assume the support at *B* is a roller and *A* and *C* are fixed.



$$(\text{FEM})_{AB} = -\frac{PL}{8} = -12, \qquad (\text{FEM})_{BC} = -\frac{wL^2}{12} = -13.5$$

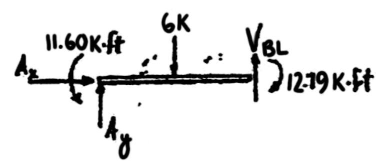
$$(\text{FEM})_{BA} = \frac{PL}{8} = 12, \qquad (\text{FEM})_{CB} = \frac{wL^2}{12} = 13.5$$



$$\theta_A = \theta_C = \psi_{AB} = \psi_{BC} = 0$$

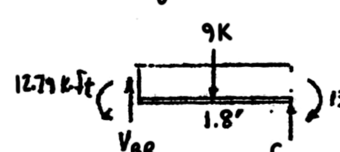
$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = \frac{2EI}{16}(\theta_B) - 12$$



$$M_{BA} = \frac{2EI}{16}(2\theta_B) + 12$$

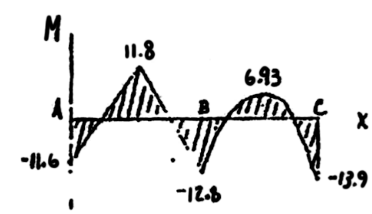
$$M_{BC} = \frac{2EI}{18}(2\theta_B) - 13.5$$



$$M_{CB} = \frac{2EI}{18}(\theta_B) + 13.5$$

Moment equilibrium at *B*:

$$M_{BA} + M_{BC} = 0$$



$$\frac{2EI}{16}(2\theta_B) + 12 + \frac{2EI}{18}(2\theta_B) - 13.5 = 0$$

$$\theta_B = \frac{3.1765}{EI}$$

11–8. Continued

Thus

$M_{AB} = -11.60 = -11.6\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{BA} = 12.79 = 12.8\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{BC} = -12.79 = -12.8\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{CB} = 13.853 = 13.9\ \text{k}\cdot\text{ft}$ **Ans.**

Left Segment

$\circlearrowleft + \sum M_A = 0;\quad -11.60 + 6(8) + 12.79 - V_{BL}(16) = 0$

$V_{BL} = 3.0744\ \text{k}$

$+\uparrow \sum F_y = 0;\quad A_y = 2.9256\ \text{k}$

Right Segment

$\circlearrowleft + \sum M_B = 0;\quad -12.79 + 9(9) - C_y(18) + 13.85 = 0$

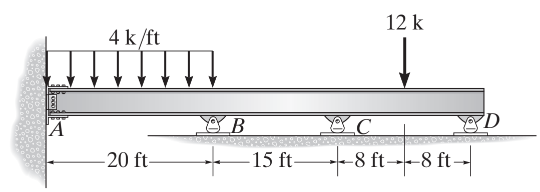
$C_y = 4.5588\ \text{k}$

$+\uparrow \sum F_y = 0;\quad V_{BK} = 4.412\ \text{k}$

At B

$B_y = 3.0744 + 4.4412 = 7.52\ \text{k}$

11–9. Determine the moments at each support, then draw the moment diagram. Assume A is fixed. EI is constant.



$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = \frac{2EI}{20}(2(0) + \theta_B - 0) - \frac{4(20)^2}{12}$$

$$M_{BA} = \frac{2EI}{20}(2\theta_B + 0 - 0) + \frac{4(20)^2}{12}$$

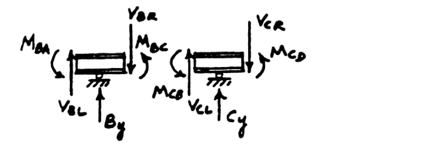
$$M_{BC} = \frac{2EI}{15}(2\theta_B + \theta_C - 0) + 0$$

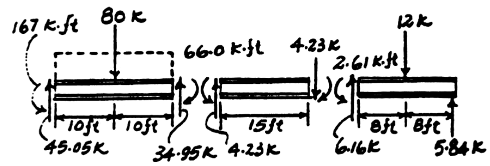
$$M_{CB} = \frac{2EI}{15}(2\theta_C + \theta_B - 0) + 0$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{CD} = \frac{3EI}{16}(\theta_C - 0) - \frac{3(12)16}{16}$$

11–9. Continued





Equilibrium.

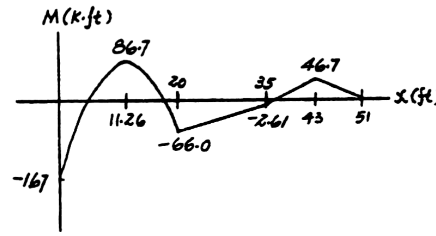
$M_{BA} + M_{BC} = 0$

$M_{CB} + M_{CD} = 0$

Solving

$$\theta_C = \frac{178.08}{EI}$$

$$\theta_B = -\frac{336.60}{EI}$$



$M_{AB} = -167\ \text{k}\cdot\text{ft}$ **Ans.**

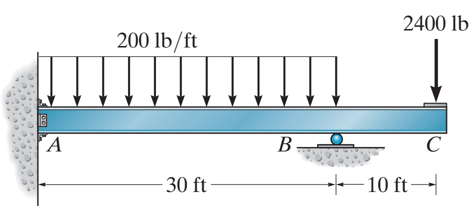
$M_{BA} = 66.0\ \text{k}\cdot\text{ft}$ **Ans.**

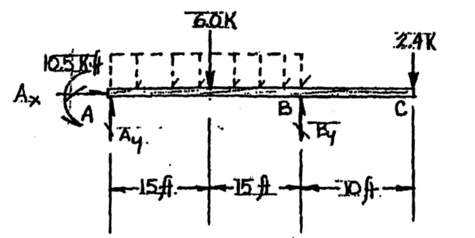
$M_{BC} = -66.0\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{CB} = 2.61\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{CD} = -2.61\ \text{k}\cdot\text{ft}$ **Ans.**

11–10. Determine the moments at A and B, then draw the moment diagram for the beam. EI is constant.

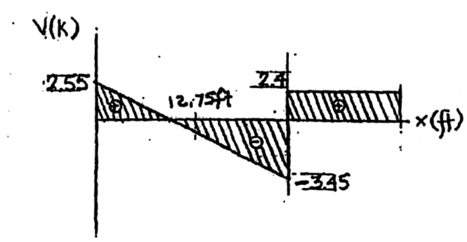




$$(\text{FEM})_{AB} = -\frac{1}{12}(w)(L^2) = -\frac{1}{12}(200)(30^2) = -15\ \text{k}\cdot\text{ft}$$

$$M_{AB} = \frac{2EI}{30}(0 + \theta_B - 0) - 15$$

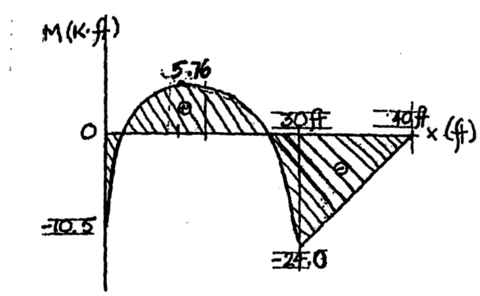
$$M_{BA} = \frac{2EI}{30}(2\theta_B + 0 - 0) + 15$$



$\sum M_B = 0; \qquad M_{BA} = 2.4(10)$

Solving,

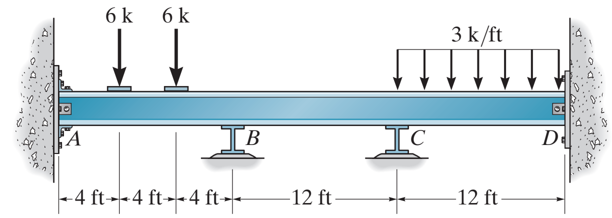
$$\theta_B = \frac{67.5}{EI}$$



$M_{AB} = -10.5\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{BA} = 24\ \text{k}\cdot\text{ft}$ **Ans.**

11–11. Determine the moments at A, B, and C, then draw the moment diagram for the beam. Assume the support at A is fixed, B and C are rollers, and D is a pin. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{AB} = -\frac{2PL}{9} = -\frac{2(6)(12)}{9} = -16\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{BA} = \frac{2PL}{9} = \frac{2(6)(12)}{9} = 16\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = (\text{FEM})_{CB} = 0 \quad (\text{FEM})_{CD} = -\frac{wL^2}{8} = -\frac{3(12^2)}{8} = -54\ \text{k}\cdot\text{ft}$$

Slope-Deflection Equations. Applying Eq. 11–8, for spans AB and BC.

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

For span AB,

$$M_{AB} = 2E\left(\frac{I}{12}\right)[2(0) + \theta_B - 3(0)] + (-16) = \left(\frac{EI}{6}\right)\theta_B - 16 \quad (1)$$

$$M_{BA} = 2E\left(\frac{I}{12}\right)[2\theta_B + 0 - 3(0)] + 16 = \left(\frac{EI}{3}\right)\theta_B + 16 \quad (2)$$

For span BC,

$$M_{BC} = 2E\left(\frac{I}{12}\right)[2\theta_B + \theta_C - 3(0)] + 0 = \left(\frac{EI}{3}\right)\theta_B + \left(\frac{EI}{6}\right)\theta_C \quad (3)$$

$$M_{CB} = 2E\left(\frac{I}{12}\right)[2\theta_C + \theta_B - 3(0)] + 0 = \left(\frac{EI}{3}\right)\theta_C + \left(\frac{EI}{6}\right)\theta_B \quad (4)$$

Applying Eq. 11–10 for span CD,

$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{CD} = 3E\left(\frac{I}{12}\right)(\theta_C - 0) + (-54) = \left(\frac{EI}{4}\right)\theta_C - 54 \quad (5)$$

Equilibrium. At support B,

$$M_{BA} + M_{BC} = 0$$

$$\left(\frac{EI}{3}\right)\theta_B + 16 + \left(\frac{EI}{3}\right)\theta_B + \left(\frac{EI}{6}\right)\theta_C = 0$$

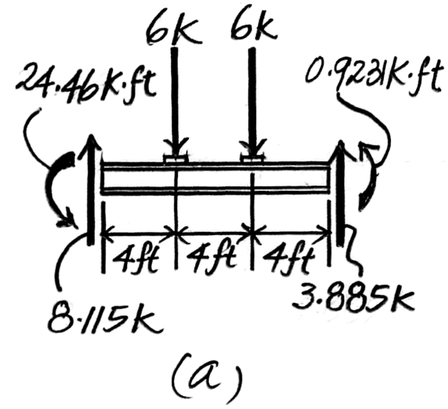
$$\left(\frac{2EI}{3}\right)\theta_B + \left(\frac{EI}{6}\right)\theta_C = -16 \quad (6)$$

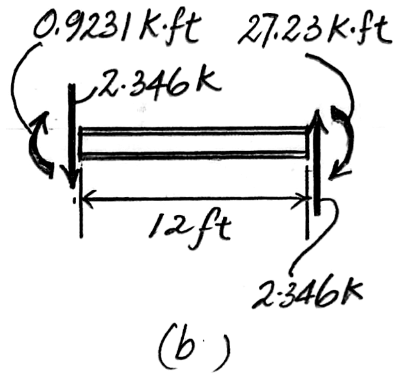
At support C,

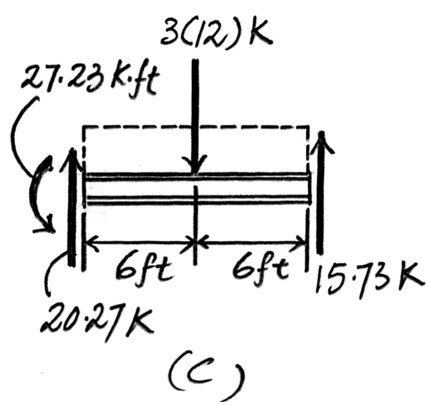
$$M_{CB} + M_{CD} = 0$$

$$\left(\frac{EI}{3}\right)\theta_C + \left(\frac{EI}{6}\right)\theta_B + \left(\frac{EI}{4}\right)\theta_C - 54 = 0$$

$$\left(\frac{7EI}{12}\right)\theta_C + \left(\frac{EI}{6}\right)\theta_B = 54 \quad (7)$$







11–11. Continued

Solving Eqs. (6) and (7)

$$\theta_C = \frac{1392}{13EI} \qquad \theta_B = -\frac{660}{13EI}$$

Substitute these results into Eq. (1) to (5)

$M_{AB} = -24.46 \text{ k} \cdot \text{ft} = -24.5 \text{ k} \cdot \text{ft}$ **Ans.**

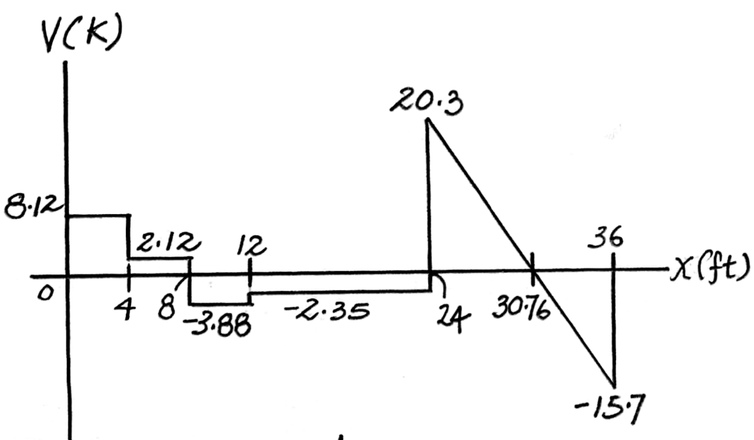
$M_{BA} = -0.9231 \text{ k} \cdot \text{ft} = -0.923 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{BC} = 0.9231 \text{ k} \cdot \text{ft} = 0.923 \text{ k} \cdot \text{ft}$ **Ans.**

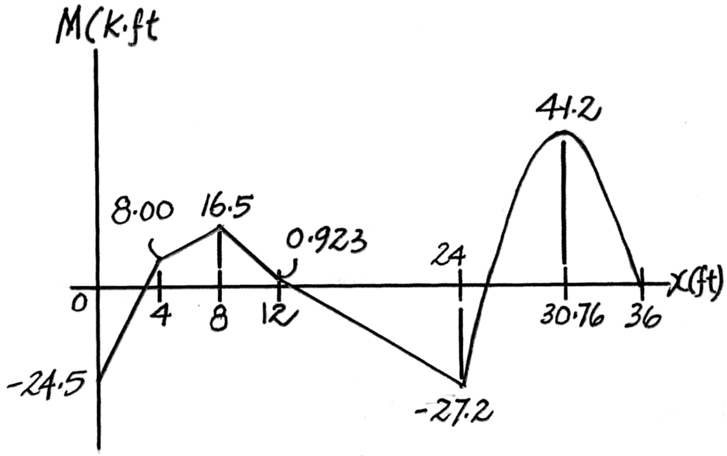
$M_{CB} = 27.23 \text{ k} \cdot \text{ft} = 27.2 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CD} = -27.23 \text{ k} \cdot \text{ft} = -27.2 \text{ k} \cdot \text{ft}$ **Ans.**

The negative signs indicates that $\mathbf{M}_{AB}$, $\mathbf{M}_{BA}$, and $\mathbf{M}_{CD}$ have counterclockwise rotational sense. Using these results, the shear at both ends of spans AB, BC, and CD are computed and shown in Fig. *a*, *b*, and *c* respectively. Subsequently, the shear and moment diagram can be plotted, Fig. *d* and *e* respectively.

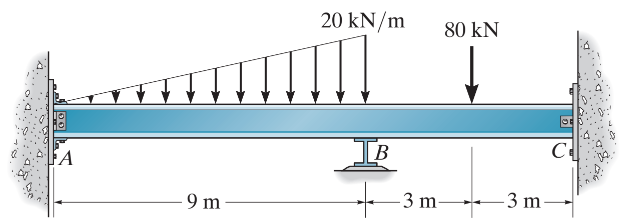


(d)

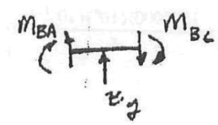


(e)

***11–12.** Determine the moments acting at A and B. Assume A is fixed supported, B is a roller, and C is a pin. EI is constant.



$$(FEM)_{AB} = \frac{wL^2}{30} = -54, \quad (FEM)_{BC} = \frac{3PL}{16} = -90$$



$$(FEM)_{BA} = \frac{wL^2}{20} = 81$$

Applying Eqs. 11–8 and 11–10,

$$M_{AB} = \frac{2EI}{9}(\theta_B) - 54$$

$$M_{BA} = \frac{2EI}{9}(2\theta_B) + 81$$

$$M_{BC} = \frac{3EI}{6}(\theta_B) - 90$$

Moment equilibrium at B:

$$M_{BA} + M_{BC} = 0$$

$$\frac{4EI}{9}(\theta_B) + 81 + \frac{EI}{2}\theta_B - 90 = 0$$

$$\theta_B = \frac{9.529}{EI}$$

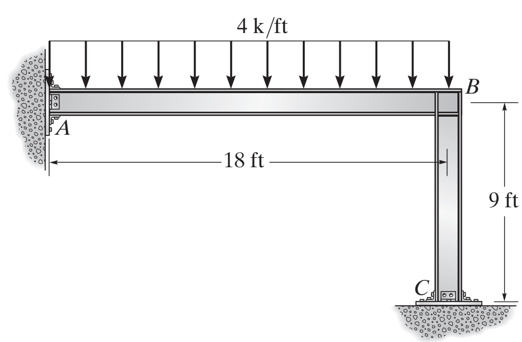
Thus,

$M_{AB} = -51.9 \text{ kN} \cdot \text{m}$ **Ans.**

$M_{BA} = 85.2 \text{ kN} \cdot \text{m}$ **Ans.**

$M_{BC} = -85.2 \text{ kN} \cdot \text{m}$ **Ans.**

11–13. Determine the moments at A, B, and C, then draw the moment diagram for each member. Assume all joints are fixed connected. EI is constant.



$$(FEM)_{AB} = \frac{-4(18)^2}{12} = -108 \text{ k} \cdot \text{ft}$$

$$(FEM)_{BA} = 108 \text{ k} \cdot \text{ft}$$

$$(FEM)_{BC} = (FEM)_{CB} = 0$$

11–13. Continued

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2E\left(\frac{I}{18}\right)(2(0) + \theta_B - 0) - 108$$

$$M_{AB} = 0.1111EI\theta_B - 108 \qquad (1)$$

$$M_{BA} = 2E\left(\frac{I}{18}\right)(2\theta_B + 0 - 0) + 108$$

$$M_{BA} = 0.2222EI\theta_B + 108 \qquad (2)$$

$$M_{BC} = 2E\left(\frac{I}{9}\right)(2\theta_B + 0 - 0) + 0$$

$$M_{BC} = 0.4444EI\theta_B \qquad (3)$$

$$M_{CB} = 2E\left(\frac{I}{9}\right)(2(0) + \theta_B - 0) + 0$$

$$M_{CB} = 0.2222EI\theta_B \qquad (4)$$

Equilibrium

$$M_{BA} + M_{BC} = 0 \qquad (5)$$

Solving Eqs. 1–5:

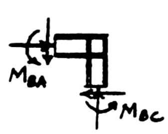
$$\theta_B = \frac{-162.0}{EI}$$

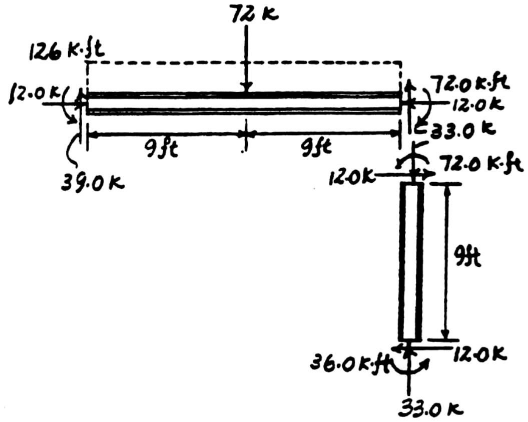
$M_{AB} = -126 \text{ k} \cdot \text{ft}$ **Ans.**

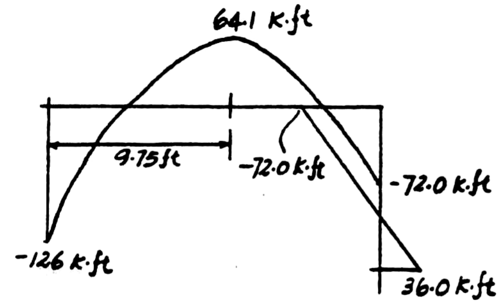
$M_{BA} = 72 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{BC} = -72 \text{ k} \cdot \text{ft}$ **Ans.**

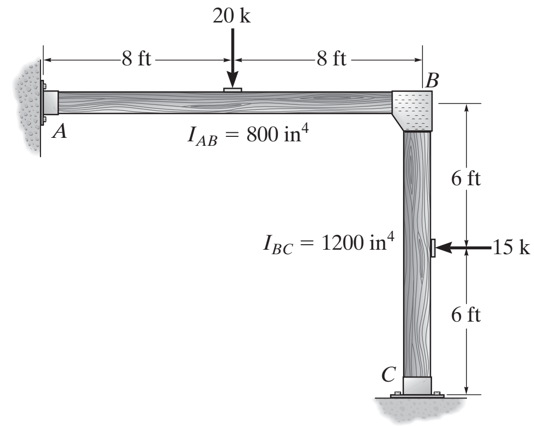
$M_{CB} = -36 \text{ k} \cdot \text{ft}$ **Ans.**







11–14. Determine the moments at the supports, then draw the moment diagram. The members are fixed connected at the supports and at joint B. The moment of inertia of each member is given in the figure. Take $E = 29(10^3)$ ksi.

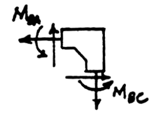


$$(\text{FEM})_{AB} = \frac{-20(16)}{8} = -40 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BA} = 40 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = \frac{-15(12)}{8} = -22.5 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{CB} = 22.5 \text{ k}\cdot\text{ft}$$



$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = \frac{2(29)(10^3)(800)}{16(144)}(2(0) + \theta_B - 0) - 40$$

$$M_{AB} = 20{,}138.89\theta_B - 40 \qquad (1)$$

$$M_{BA} = \frac{2(29)(10^3)(800)}{16(144)}(2\theta_B + 0 - 0) + 40$$

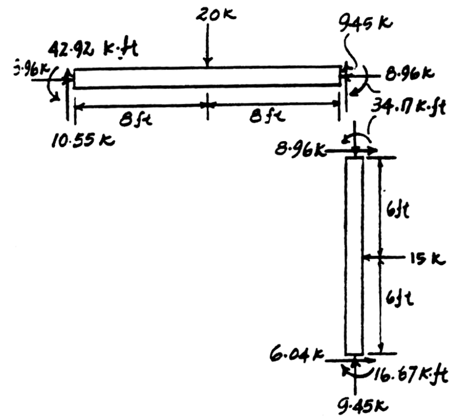
$$M_{BA} = 40{,}277.78\theta_B + 40 \qquad (2)$$

$$M_{BC} = \frac{2(29)(10^3)(1200)}{12(144)}(2\theta_B + 0 - 0) - 22.5$$

$$M_{BC} = 80{,}555.55\theta_B - 22.5 \qquad (3)$$

$$M_{CB} = \frac{2(29)(10^3)(1200)}{12(144)}(2(0) + \theta_B - 0) + 22.5$$

$$M_{CB} = 40{,}277.77\theta_B + 22.5 \qquad (4)$$



Equilibrium.

$$M_{BA} + M_{BC} = 0 \qquad (5)$$

Solving Eqs. 1–5:

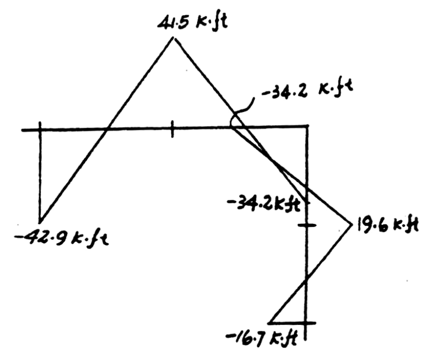
$$\theta_B = -0.00014483$$

$M_{AB} = -42.9$ k · ft **Ans.**

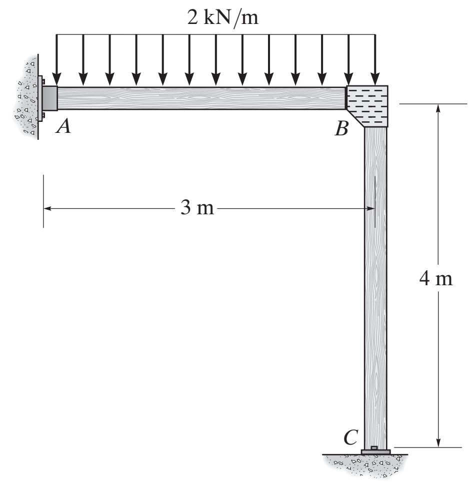
$M_{BA} = 34.2$ k · ft

$M_{BC} = -34.2$ k · ft

$M_{CB} = 16.7$ k · ft **Ans.**



11–15. Determine the moment at B, then draw the moment diagram for each member of the frame. Assume the support at A is fixed and C is pinned. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{AB} = -\frac{wL^2}{12} = -\frac{2(3^2)}{12} = -1.50\ \text{kN}\cdot\text{m}$$

$$(\text{FEM})_{BA} = \frac{wL^2}{12} = \frac{2(3^2)}{12} = 1.50\ \text{kN}\cdot\text{m}$$

$$(\text{FEM})_{BC} = 0$$

Slope-Deflection Equations. Applying Eq. 11–8 for member AB,

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2E\left(\frac{I}{3}\right)[2(0) + \theta_B - 3(0)] + (-1.50) = \left(\frac{2EI}{3}\right)\theta_B - 1.50 \qquad (1)$$

$$M_{BA} = 2E\left(\frac{I}{3}\right)[2\theta_B + 0 - 3(0)] + 1.50 = \left(\frac{4EI}{3}\right)\theta_B + 1.50 \qquad (2)$$

Applying Eq. 11–10 for member BC,

$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{BC} = 3E\left(\frac{I}{4}\right)(\theta_B - 0) + 0 = \left(\frac{3EI}{4}\right)\theta_B \qquad (3)$$

Equilibrium. At Joint B,

$$M_{BA} + M_{BC} = 0$$

$$\left(\frac{4EI}{3}\right)\theta_B + 1.50 + \left(\frac{3EI}{4}\right)\theta_B = 0$$

$$\theta_B = -\frac{0.72}{EI}$$

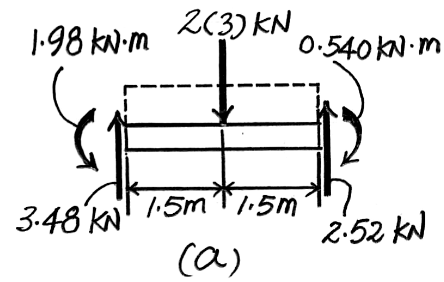
Substitute this result into Eqs. (1) to (3)

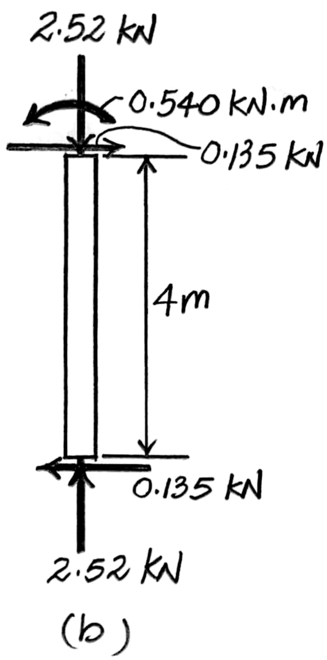
$M_{AB} = -1.98\ \text{kN}\cdot\text{m}$ **Ans.**

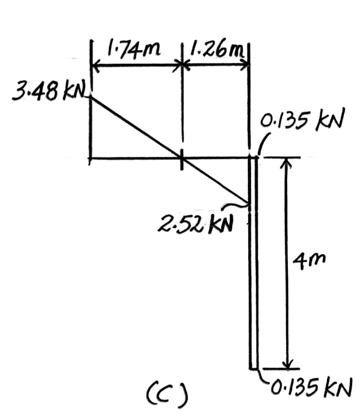
$M_{BA} = 0.540\ \text{kN}\cdot\text{m}$ **Ans.**

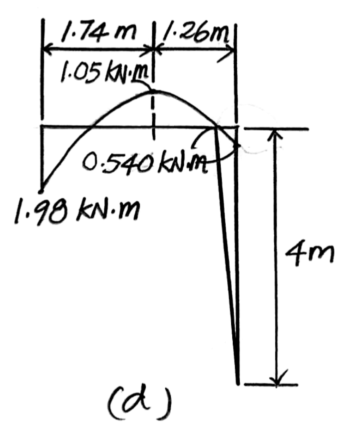
$M_{BC} = -0.540\ \text{kN}\cdot\text{m}$ **Ans.**

The negative signs indicate that $\mathbf{M}_{AB}$ and $\mathbf{M}_{BC}$ have counterclockwise rotational sense. Using these results, the shear at both ends of member AB and BC are computed and shown in Fig. a and b respectively. Subsequently, the shear and moment diagram can be plotted, Fig. c and d respectively.

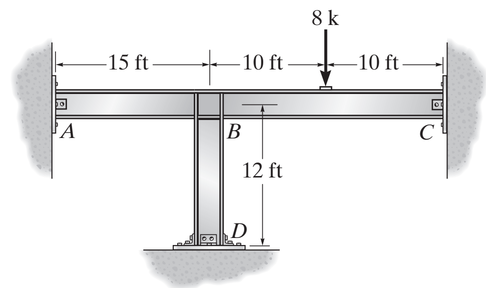


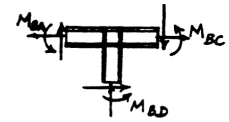






***11–16.** Determine the moments at B and D, then draw the moment diagram. Assume A and C are pinned and B and D are fixed connected. EI is constant.





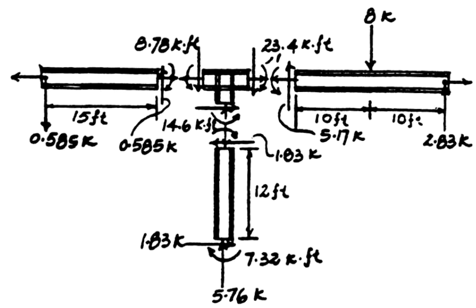
$$(\text{FEM})_{BA} = 0$$

$$(\text{FEM})_{BC} = \frac{-3(8)(20)}{16} = -30 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{BD} = (\text{FEM})_{DB} = 0$$

$$M_N = 3E\left(\frac{I}{L}\right)(\theta_N - \psi) + (\text{FEM})_N$$

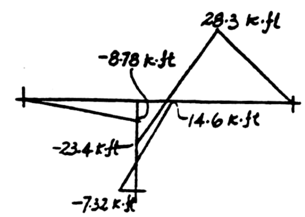
$$M_{BA} = 3E\left(\frac{I}{15}\right)(\theta_B - 0) + 0$$



$$M_{BA} = 0.2EI\theta_B \tag{1}$$

$$M_{BC} = 3E\left(\frac{I}{20}\right)(\theta_B - 0) - 30$$

$$M_{BC} = 0.15EI\theta_B - 30 \tag{2}$$



$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{BD} = 2E\left(\frac{I}{12}\right)(2\theta_B + 0 - 0) + 0$$

$$M_{BD} = 0.3333EI\theta_B \tag{3}$$

$$M_{DB} = 2E\left(\frac{I}{12}\right)(2(0) + \theta_B - 0) + 0$$

$$M_{DB} = 0.1667EI\theta_B \tag{4}$$

Equilibrium.

$$M_{BA} + M_{BC} + M_{BD} = 0 \tag{5}$$

Solving Eqs. 1–5:

$$\theta_B = \frac{43.90}{EI}$$

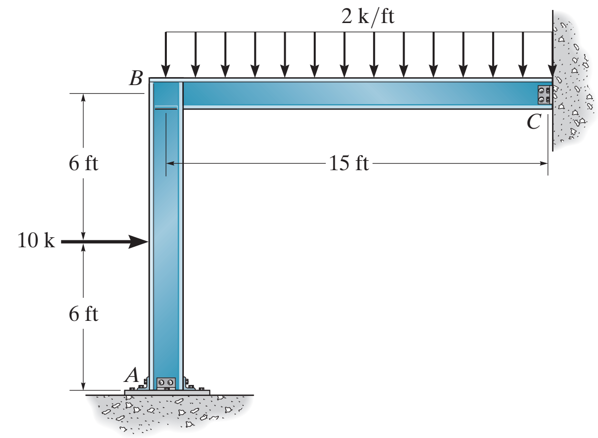
$M_{BA} = 8.78 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{BC} = -23.41 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{BD} = 14.63 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{DB} = 7.32 \text{ k} \cdot \text{ft}$ **Ans.**

11–17. Determine the moment that each member exerts on the joint at B, then draw the moment diagram for each member of the frame. Assume the support at A is fixed and C is a pin. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{AB} = -\frac{PL}{8} = -\frac{10(12)}{8} = -15 \text{ k} \cdot \text{ft} \qquad (\text{FEM})_{BA} = \frac{PL}{8} = \frac{10(12)}{8} = 15 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{BC} = -\frac{wL^2}{8} = -\frac{2(15^2)}{8} = -56.25 \text{ k} \cdot \text{ft}$$

Slope Reflection Equations. Applying Eq. 11–8 for member AB,

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2E\left(\frac{I}{12}\right)[2(0) + \theta_B - 3(0)] + (-15) = \left(\frac{EI}{6}\right)\theta_B - 15 \qquad (1)$$

$$M_{BA} = 2E\left(\frac{I}{12}\right)[2\theta_B + 0 - 3(0)] + 15 = \left(\frac{EI}{3}\right)\theta_B + 15 \qquad (2)$$

For member BC, applying Eq. 11–10

$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{BC} = 3E\left(\frac{I}{15}\right)(\theta_B - 0) + (-56.25) = \left(\frac{EI}{5}\right)\theta_B - 56.25 \qquad (3)$$

Equilibrium. At joint B,

$$M_{BA} + M_{BC} = 0$$

$$\left(\frac{EI}{3}\right)\theta_B + 15 + \left(\frac{EI}{5}\right)\theta_B - 56.25 = 0$$

$$\theta_B = \frac{77.34375}{EI}$$

Substitute this result into Eqs. (1) to (3)

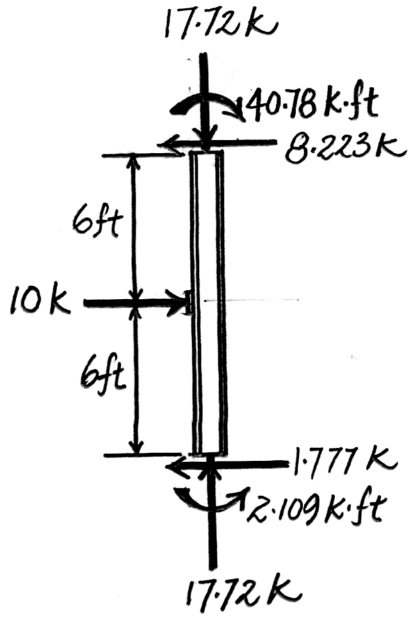
$M_{AB} = -2.109 \text{ k} \cdot \text{ft} = -2.11 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{BA} = 40.78 \text{ k} \cdot \text{ft} = 40.8 \text{ k} \cdot \text{ft}$ **Ans.**

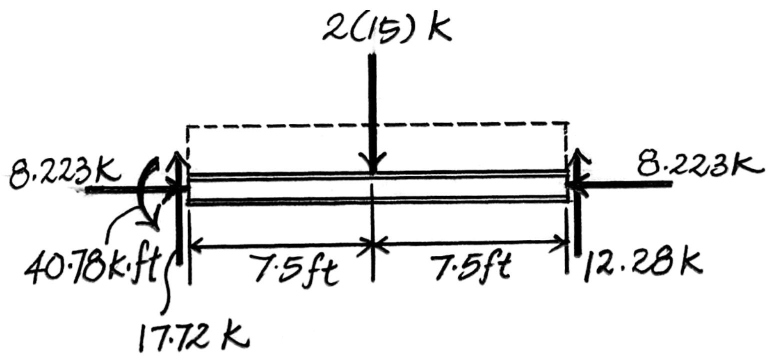
$M_{BC} = -40.78 \text{ k} \cdot \text{ft} = -40.8 \text{ k} \cdot \text{ft}$ **Ans.**

The negative signs indicate that $\mathbf{M}_{AB}$ and $\mathbf{M}_{BC}$ have counterclockwise rotational sense. Using these results, the shear at both ends of member AB and BC are computed and shown in Fig. *a* and *b* respectively. Subsequently, the shear and Moment diagram can be plotted, Fig. *c* and *d* respectively.

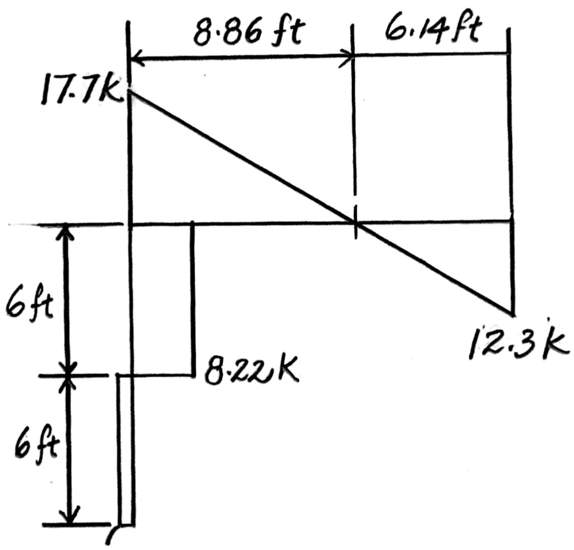
11–17. Continued



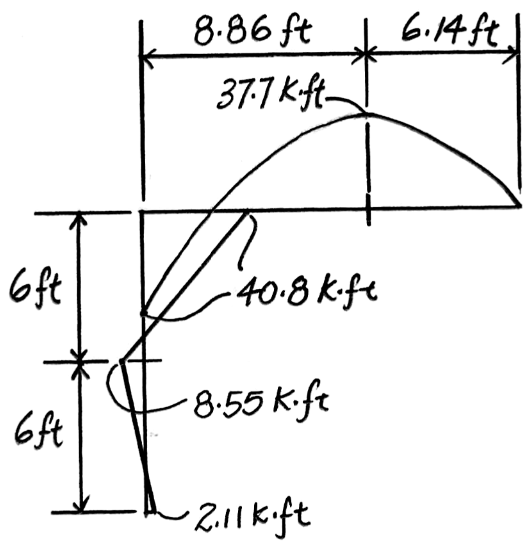
(a)



(b)

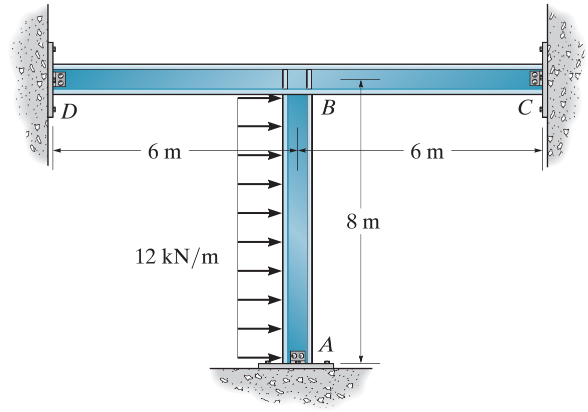


(c)



(d)

11–18. Determine the moment that each member exerts on the joint at B, then draw the moment diagram for each member of the frame. Assume the supports at A, C, and D are pins. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{BA} = \frac{wL^2}{8} = \frac{12(8^2)}{8} = 96\text{ kN}\cdot\text{m} \qquad (\text{FEM})_{BC} = (\text{FEM})_{BD} = 0$$

Slope-Reflection Equation. Since the far end of each members are pinned, Eq. 11–10 can be applied

$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

For member AB,

$$M_{BA} = 3E\left(\frac{I}{8}\right)(\theta_B - 0) + 96 = \left(\frac{3EI}{8}\right)\theta_B + 96 \qquad (1)$$

For member BC,

$$M_{BC} = 3E\left(\frac{I}{6}\right)(\theta_B - 0) + 0 = \left(\frac{EI}{2}\right)\theta_B \qquad (2)$$

For member BD,

$$M_{BD} = 3E\left(\frac{I}{6}\right)(\theta_B - 0) + 0 = \frac{EI}{2}\theta_B \qquad (3)$$

Equilibrium. At joint B,

$$M_{BA} + M_{BC} + M_{BD} = 0$$

$$\left(\frac{3EI}{8}\right)\theta_B + 96 + \left(\frac{EI}{2}\right)\theta_B + \frac{EI}{2}\theta_B = 0$$

$$\theta_B = -\frac{768}{11EI}$$

Substitute this result into Eqs. (1) to (3)

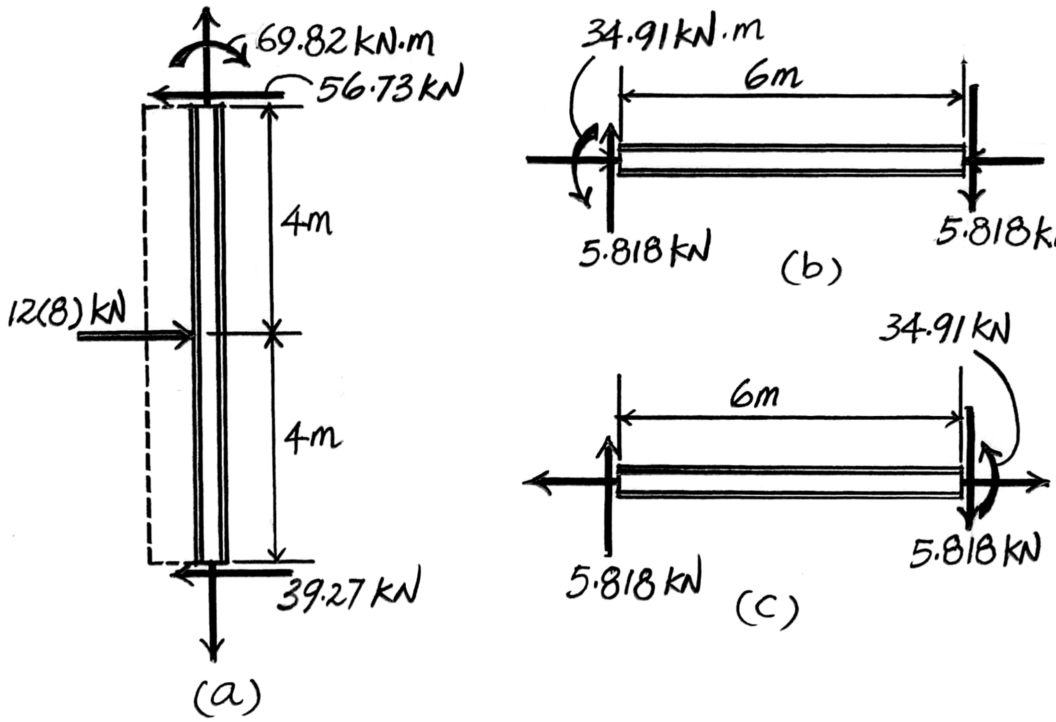
$$M_{BA} = 69.82\text{ kN}\cdot\text{m} = 69.8\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

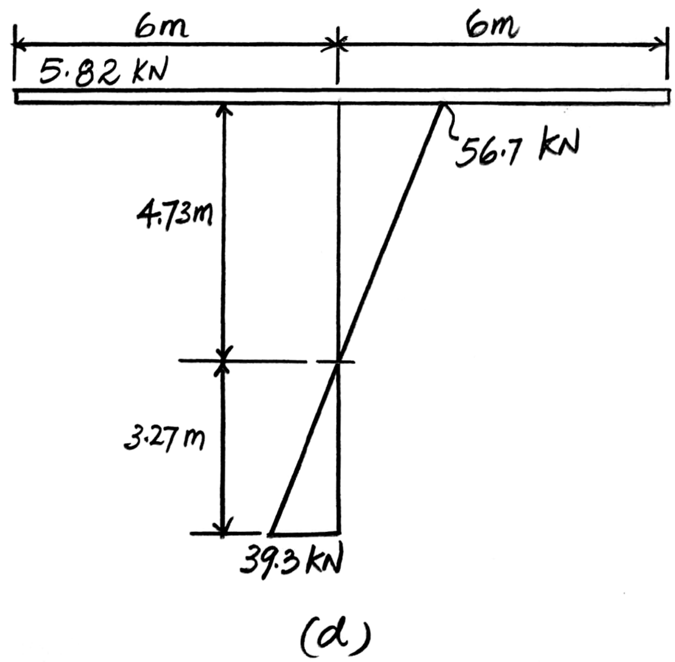
$$M_{BC} = -34.91\text{ kN}\cdot\text{m} = -34.9\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

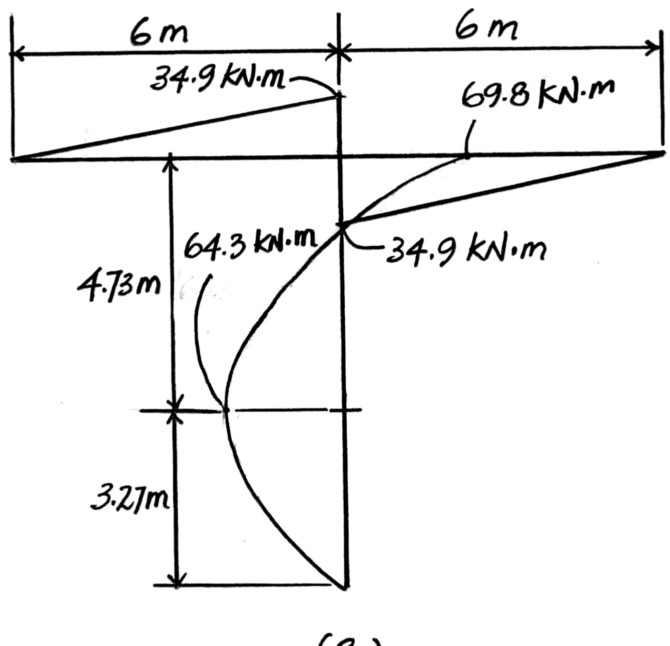
$$M_{BD} = -34.91\text{ kN}\cdot\text{m} = -34.9\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

The negative signs indicate that $\mathbf{M}_{BC}$ and $\mathbf{M}_{BD}$ have counterclockwise rotational sense. Using these results, the shear at both ends of members AB, BC, and BD are computed and shown in Fig. *a*, *b* and *c* respectively. Subsequently, the shear and moment diagrams can be plotted, Fig. *d* and *e* respectively.

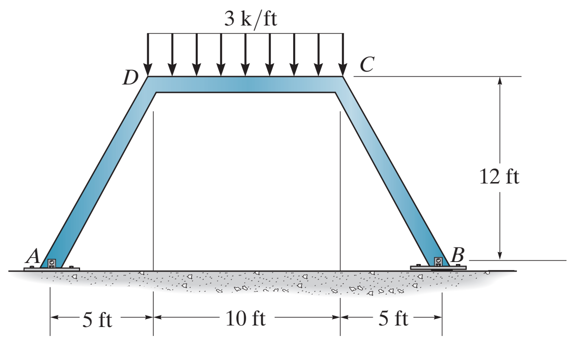
11–18. Continued







11–19. Determine the moment at joints D and C, then draw the moment diagram for each member of the frame. Assume the supports at A and B are pins. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{DC} = -\frac{wL^2}{12} = -\frac{3(10^2)}{12} = -25\ \text{k}\cdot\text{ft} \qquad (\text{FEM})_{CD} = \frac{wL^2}{12} = \frac{3(10^2)}{12} = 25\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{DA} = (\text{FEM})_{CB} = 0$$

Slope-Deflection Equations. For member CD, applying Eq. 11–8

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{DC} = 2E\left(\frac{I}{10}\right)[2\theta_D + \theta_C - 3(0)] + (-25) = \left(\frac{2EI}{5}\right)\theta_D + \left(\frac{EI}{5}\right)\theta_C - 25 \qquad (1)$$

$$M_{CD} = 2E\left(\frac{I}{10}\right)[2\theta_C + \theta_D - 3(0)] + 25 = \left(\frac{2EI}{5}\right)\theta_C + \left(\frac{EI}{5}\right)\theta_D + 25 \qquad (2)$$

For members AD and BC, applying Eq. 11–10

$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{DA} = 3E\left(\frac{I}{13}\right)(\theta_D - 0) + 0 = \left(\frac{3EI}{13}\right)\theta_D \qquad (3)$$

$$M_{CB} = 3E\left(\frac{I}{13}\right)(\theta_C - 0) + 0 = \left(\frac{3EI}{13}\right)\theta_C \qquad (4)$$

Equilibrium. At joint D,

$$M_{DC} + M_{DA} = 0$$

$$\left(\frac{2EI}{5}\right)\theta_D + \left(\frac{EI}{5}\right)\theta_C - 25 + \left(\frac{3EI}{13}\right)\theta_D = 0$$

$$\left(\frac{41EI}{65}\right)\theta_D + \left(\frac{EI}{5}\right)\theta_C = 25 \qquad (5)$$

At joint C,

$$M_{CD} + M_{CB} = 0$$

$$\left(\frac{2EI}{5}\right)\theta_C + \left(\frac{EI}{5}\right)\theta_D + 25 + \left(\frac{3EI}{13}\right)\theta_C = 0$$

$$\left(\frac{41EI}{65}\right)\theta_C + \left(\frac{EI}{5}\right)\theta_D = -25 \qquad (6)$$

Solving Eqs. (5) and (6)

$$\theta_D = \frac{1625}{28EI} \qquad \theta_C = -\frac{1625}{28EI}$$

Substitute these results into Eq. (1) to (4)

$M_{DC} = -13.39\ \text{k}\cdot\text{ft} = -13.4\ \text{k}\cdot\text{ft}$ **Ans.**

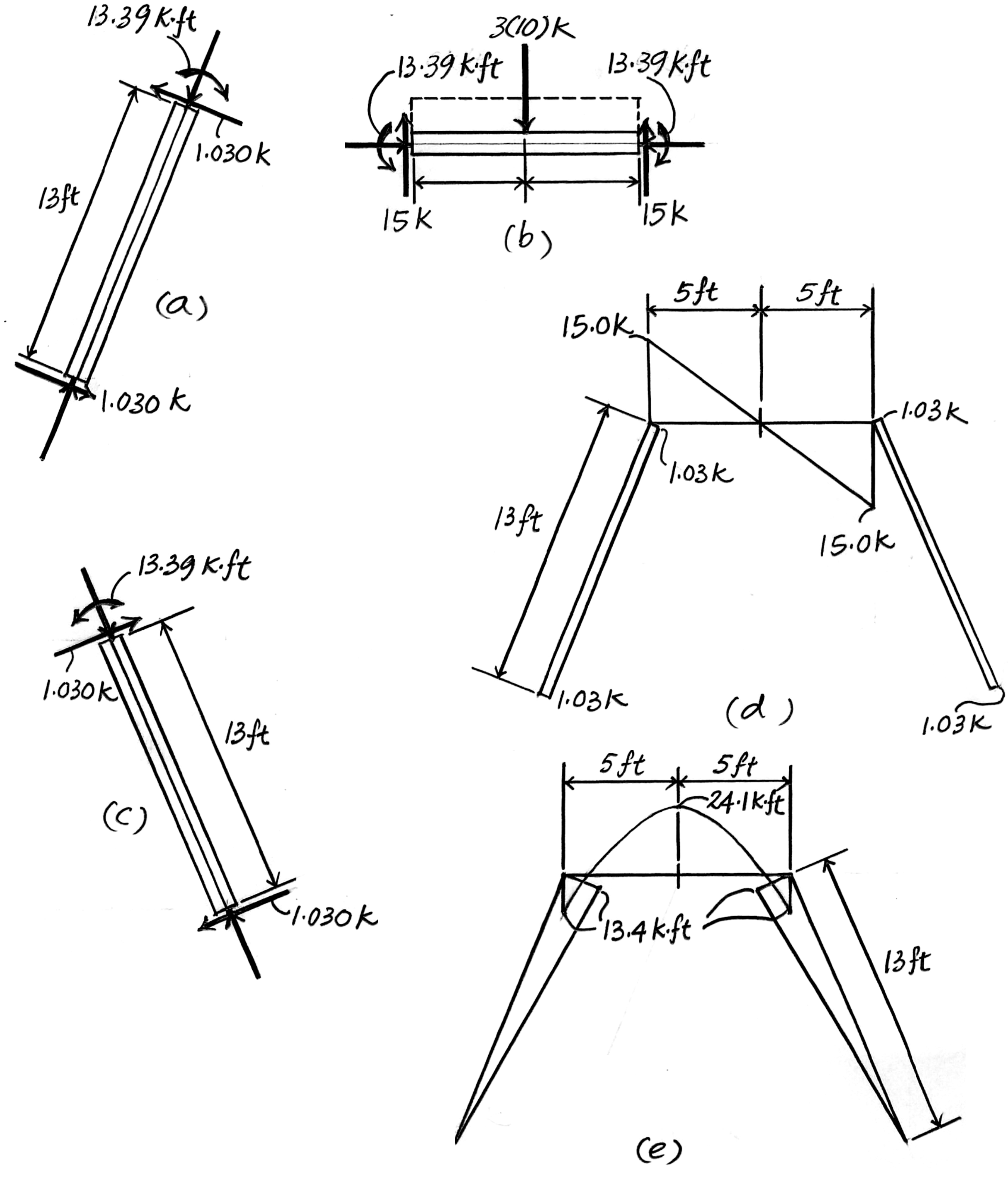
$M_{CD} = 13.39\ \text{k}\cdot\text{ft} = 13.4\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{DA} = 13.39\ \text{k}\cdot\text{ft} = 13.4\ \text{k}\cdot\text{ft}$ **Ans.**

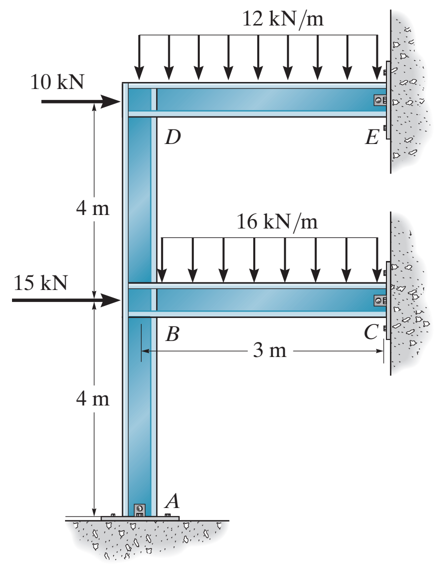
$M_{CB} = -13.39\ \text{k}\cdot\text{ft} = -13.4\ \text{k}\cdot\text{ft}$ **Ans.**

11–19. Continued

The negative signs indicate that $\mathbf{M}_{DC}$ and $\mathbf{M}_{CB}$ have counterclockwise rotational sense. Using these results, the shear at both ends of members *AD*, *CD*, and *BC* are computed and shown in Fig. *a*, *b*, and *c* respectively. Subsequently, the shear and moment diagrams can be plotted, Fig. *d* and *e* respectively.



***11–20.** Determine the moment that each member exerts on the joints at B and D, then draw the moment diagram for each member of the frame. Assume the supports at A, C, and E are pins. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{BA} = (\text{FEM})_{BD} = (\text{FEM})_{DB} = 0$$

$$(\text{FEM})_{BC} = -\frac{wL^2}{8} = -\frac{16(3^2)}{8} = -18 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{DE} = -\frac{wL^2}{8} = -\frac{12(3^2)}{8} = -13.5 \text{ kN}\cdot\text{m}$$

Slope-Deflection Equations. For member AB, BC, and ED, applying Eq. 11–10.

$$M_N = 3Ek(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{BA} = 3E\left(\frac{I}{4}\right)(\theta_B - 0) + 0 = \left(\frac{3EI}{4}\right)\theta_B \qquad (1)$$

$$M_{BC} = 3E\left(\frac{I}{3}\right)(\theta_B - 0) + (-18) = EI\theta_B - 18 \qquad (2)$$

$$M_{DE} = 3E\left(\frac{I}{3}\right)(\theta_D - 0) + (-13.5) = EI\theta_D - 13.5 \qquad (3)$$

For member BD, applying Eq. 11–8

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{BD} = 2E\left(\frac{I}{4}\right)[2\theta_B + \theta_D - 3(0)] + 0 = EI\theta_B + \left(\frac{EI}{2}\right)\theta_D \qquad (4)$$

$$M_{DB} = 2E\left(\frac{I}{4}\right)[2\theta_D + \theta_B - 3(0)] + 0 = EI\theta_D + \left(\frac{EI}{2}\right)\theta_B \qquad (5)$$

Equilibrium. At Joint B,

$$M_{BA} + M_{BC} + M_{BD} = 0$$

$$\left(\frac{3EI}{4}\right)\theta_B + EI\theta_B - 18 + EI\theta_B + \left(\frac{EI}{2}\right)\theta_D = 0$$

$$\left(\frac{11EI}{4}\right)\theta_B + \left(\frac{EI}{2}\right)\theta_D = 18 \qquad (6)$$

At joint D,

$$M_{DB} + M_{DE} = 0$$

$$EI\theta_D + \left(\frac{EI}{2}\right)\theta_B + EI\theta_D - 13.5 = 0$$

$$2EI\theta_D + \left(\frac{EI}{2}\right)\theta_B = 13.5 \qquad (7)$$

Solving Eqs. (6) and (7)

$$\theta_B = \frac{39}{7EI} \qquad \theta_D = \frac{75}{14EI}$$

11–20. Continued

Substitute these results into Eqs. (1) to (5),

$M_{BA} = 4.179\ \text{kN}\cdot\text{m} = 4.18\ \text{kN}\cdot\text{m}$ **Ans.**

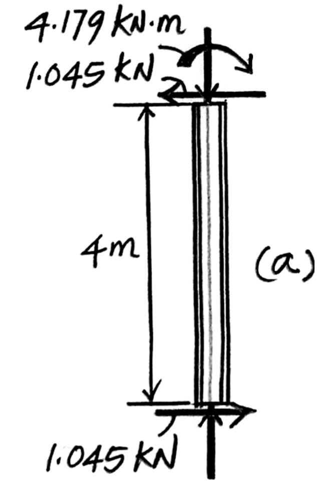
$M_{BC} = -12.43\ \text{kN}\cdot\text{m} = -12.4\ \text{kN}\cdot\text{m}$ **Ans.**

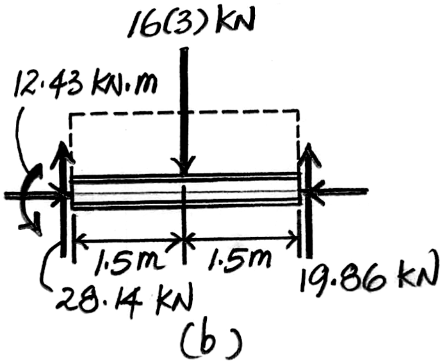
$M_{DE} = -8.143\ \text{kN}\cdot\text{m} = -8.14\ \text{kN}\cdot\text{m}$ **Ans.**

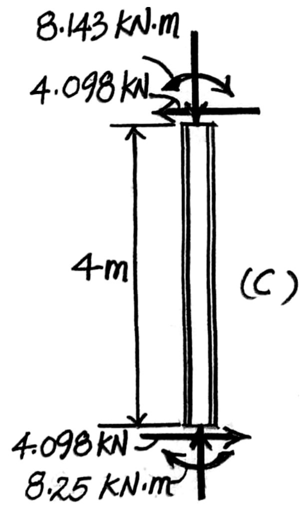
$M_{BD} = 8.25\ \text{kN}\cdot\text{m}$ **Ans.**

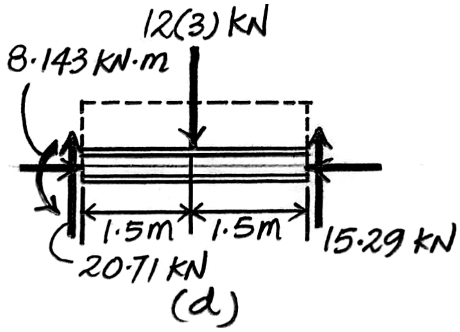
$M_{DB} = 8.143\ \text{kN}\cdot\text{m} = 8.14\ \text{kN}\cdot\text{m}$ **Ans.**

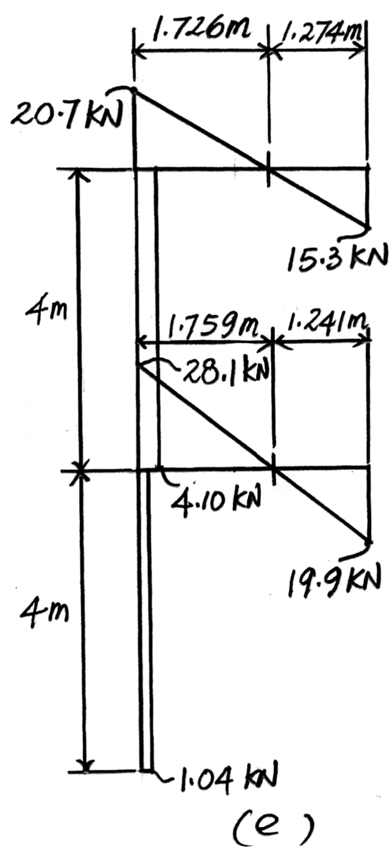
The negative signs indicate that $\mathbf{M}_{BC}$ and $\mathbf{M}_{DE}$ have counterclockwise rotational sense. Using these results, the shear at both ends of members AB, BC, BD and DE are computed and shown on Fig. a, b, c and d respectively. Subsequently, the shear and moment diagram can be plotted, Fig. e and f.

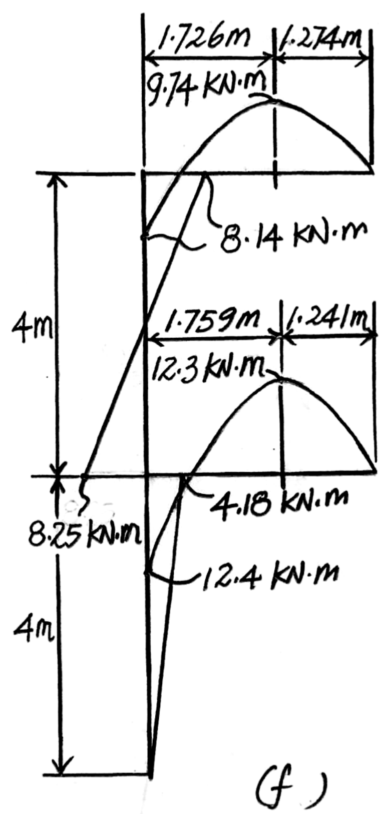




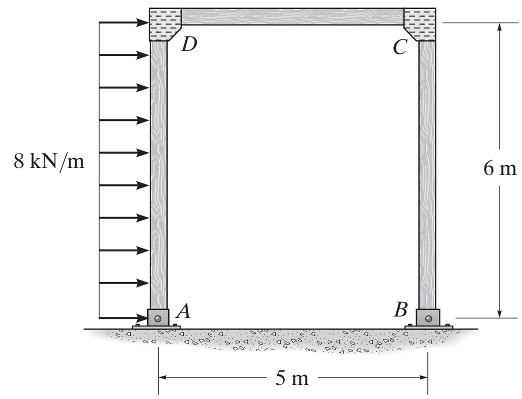








11–21. Determine the moment at joints C and D, then draw the moment diagram for each member of the frame. Assume the supports at A and B are pins. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{DA} = \frac{wL^2}{8} = \frac{8(6^2)}{8} = 36\ \text{kN}\cdot\text{m}$$

$$(\text{FEM})_{DC} = (\text{FEM})_{CD} = (\text{FEM})_{CB} = 0$$

Slope-Deflection Equations. Here, $\psi_{DA} = \psi_{CB} = \psi$ and $\psi_{DC} = \psi_{CD} = 0$

For member CD, applying Eq. 11–8,

$$M_N = 2Ek\,(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{DC} = 2E\left(\frac{I}{5}\right)[2\theta_D + \theta_C - 3(0)] + 0 = \left(\frac{4EI}{5}\right)\theta_D + \left(\frac{2EI}{5}\right)\theta_C \qquad (1)$$

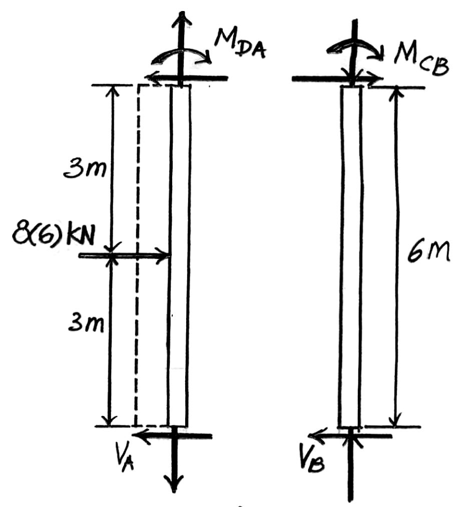
$$M_{CD} = 2E\left(\frac{I}{5}\right)[2\theta_C + \theta_D - 3(0)] + 0 = \left(\frac{4EI}{5}\right)\theta_C + \left(\frac{2EI}{5}\right)\theta_D \qquad (2)$$

For member AD and BC, applying Eq. 11–10

$$M_N = 3Ek\,(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{DA} = 3E\left(\frac{I}{6}\right)(\theta_D - \psi) + 36 = \left(\frac{EI}{2}\right)\theta_D - \left(\frac{EI}{2}\right)\psi + 36 \qquad (3)$$

$$M_{CB} = 3E\left(\frac{I}{6}\right)(\theta_C - \psi) + 0 = \left(\frac{EI}{2}\right)\theta_C - \left(\frac{EI}{2}\right)\psi \qquad (4)$$



(a)

Equilibrium. At joint D,

$$M_{DA} + M_{DC} = 0$$

$$\left(\frac{EI}{2}\right)\theta_D - \left(\frac{EI}{2}\right)\psi + 36 + \left(\frac{4EI}{5}\right)\theta_D + \left(\frac{2EI}{5}\right)\theta_C = 0$$

$$1.3EI\theta_D + 0.4EI\theta_C - 0.5EI\psi = -36 \qquad (5)$$

At joint C,

$$M_{CD} + M_{CB} = 0$$

$$\left(\frac{4EI}{5}\right)\theta_C + \left(\frac{2EI}{5}\right)\theta_D + \left(\frac{EI}{2}\right)\theta_C - \left(\frac{EI}{2}\right)\psi = 0$$

$$0.4EI\theta_D + 1.3EI\theta_C - 0.5EI\psi = 0 \qquad (6)$$

Consider the horizontal force equilibrium for the entire frame

$$\overset{+}{\rightarrow}\sum F_x = 0;\quad 8(6) - V_A - V_B = 0$$

Referring to the FBD of member AD and BC in Fig. a,

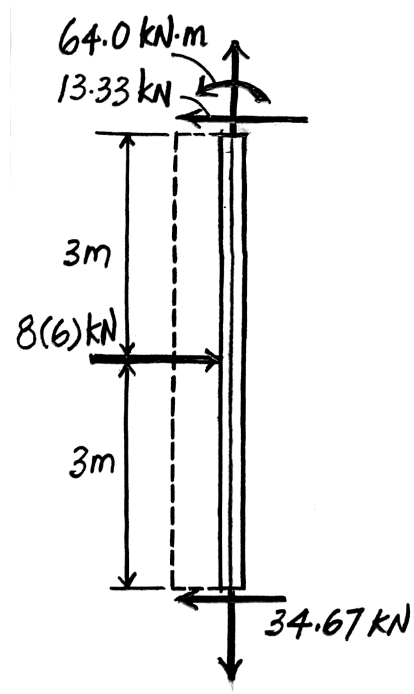
$$\circlearrowleft + \sum M_D = 0;\quad 8(6)(3) - M_{DA} - V_A(6) = 0$$

$$V_A = 24 - \frac{M_{DA}}{6}$$

and

$$\circlearrowleft + \sum M_C = 0;\quad -M_{CB} - V_B(6) = 0$$

$$V_B = -\frac{M_{CB}}{6} = 0$$



(b)

11–21. Continued

Thus,

$$8(6) - \left(24 - \frac{M_{DA}}{6}\right) - \left(-\frac{M_{CB}}{6}\right) = 0$$

$$M_{DA} + M_{CB} = -144$$

$$\left(\frac{EI}{2}\right)\theta_D - \left(\frac{EI}{2}\right)\psi + 36 + \left(\frac{EI}{2}\right)\theta_C - \left(\frac{EI}{2}\right)\psi = -144$$

$$0.5EI\theta_D + 0.5EI\theta_C - EI\psi = -180 \qquad (7)$$

Solving of Eqs. (5), (6) and (7)

$$\theta_C = \frac{80}{EI} \quad \theta_D = \frac{40}{EI} \quad \psi = \frac{240}{EI}$$

Substitute these results into Eqs. (1) to (4),

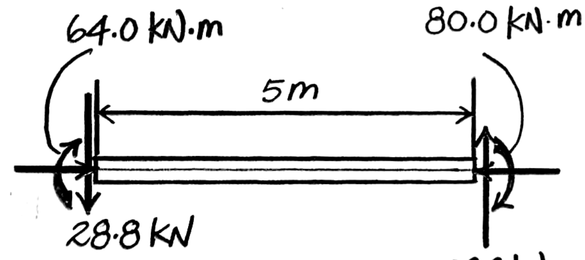
$M_{DC} = 64.0 \text{ kN} \cdot \text{m}$ **Ans.**

$M_{CD} = 80.0 \text{ kN} \cdot \text{m}$ **Ans.**

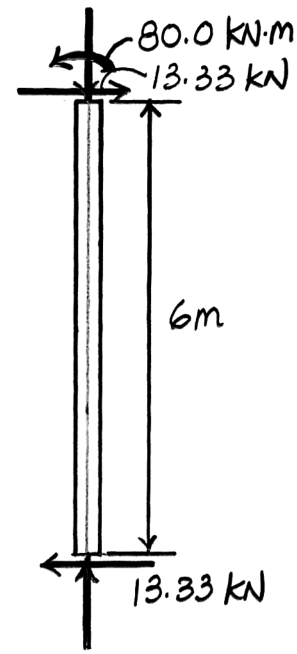
$M_{DA} = -64.0 \text{ kN} \cdot \text{m}$ **Ans.**

$M_{CB} = -80.0 \text{ kN} \cdot \text{m}$ **Ans.**

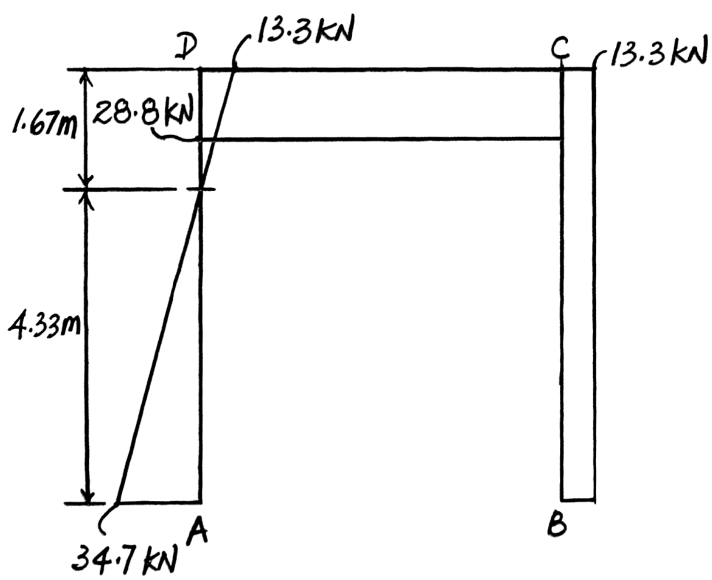
The negative signs indicate that $\mathbf{M}_{DA}$ and $\mathbf{M}_{CB}$ have counterclockwise rotational sense. Using these results, the shear at both ends of members AD, CD, and BC are computed and shown in Fig. b, c, and d, respectively. Subsequently, the shear and moment diagram can be plotted, Fig. e and f respectively.



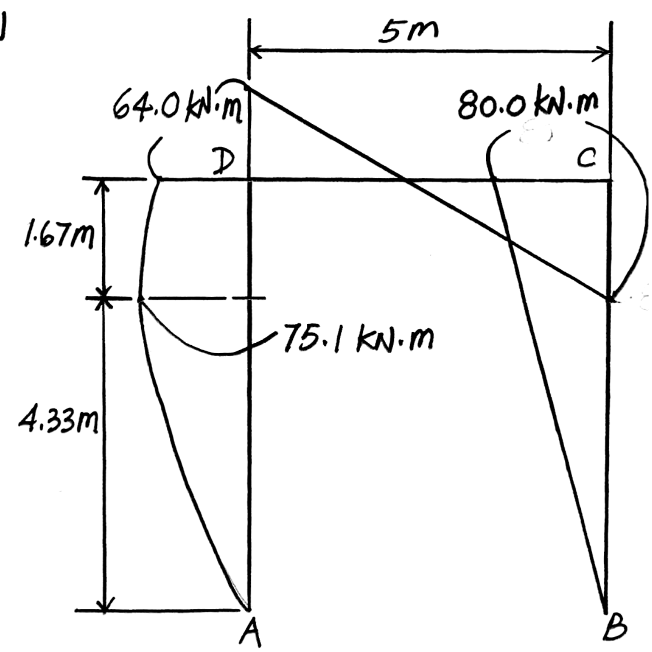
(c)



(d)

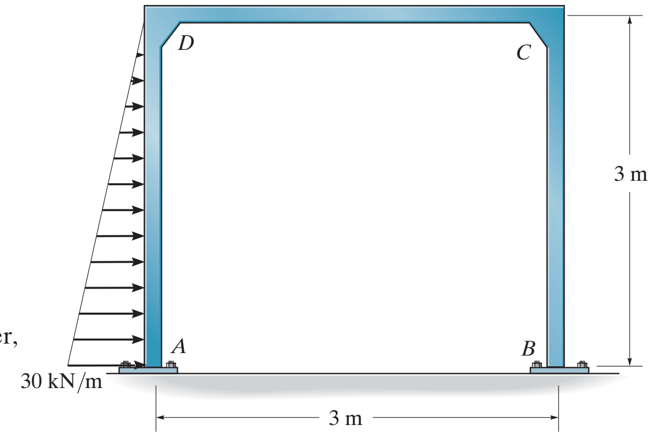


(e)



(f)

11–22. Determine the moment at joints A, B, C, and D, then draw the moment diagram for each member of the frame. Assume the supports at A and B are fixed. EI is constant.



Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{AD} = -\frac{wL^2}{20} = -\frac{30(3^2)}{20} = 13.5 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{DA} = \frac{wL^2}{30} = \frac{30(3^2)}{30} = 9 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{DC} = (\text{FEM})_{CD} = (\text{FEM})_{CB} = (\text{FEM})_{BC} = 0$$

Slope-Deflection Equations. Here, $\psi_{AD} = \psi_{DA} = \psi_{BC} = \psi_{CB} = \psi$ and $\psi_{CD} = \psi_{DC} = 0$

Applying Eq. 11–8,

$$M_N = 2Ek(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

For member AD,

$$M_{AD} = 2E\left(\frac{I}{3}\right)[2(0) + \theta_D - 3\psi] + (-13.5) = \left(\frac{2EI}{3}\right)\theta_D - 2EI\psi - 13.5 \quad (1)$$

$$M_{DA} = 2E\left(\frac{I}{3}\right)(2\theta_D + 0 - 3\psi) + 9 = \left(\frac{4EI}{3}\right)\theta_D - 2EI\psi + 9 \quad (2)$$

For member CD,

$$M_{DC} = 2E\left(\frac{I}{3}\right)[2\theta_D + \theta_C - 3(0)] + 0 = \left(\frac{4EI}{3}\right)\theta_D + \left(\frac{2EI}{3}\right)\theta_C \quad (3)$$

$$M_{CD} = 2E\left(\frac{I}{3}\right)[2\theta_C + \theta_D - 3(0)] + 0 = \left(\frac{4EI}{3}\right)\theta_C + \left(\frac{2EI}{3}\right)\theta_D \quad (4)$$

For member BC,

$$M_{BC} = 2E\left(\frac{I}{3}\right)[2(0) + \theta_C - 3\psi] + 0 = \left(\frac{2EI}{3}\right)\theta_C - 2EI\psi \quad (5)$$

$$M_{CB} = 2E\left(\frac{I}{3}\right)[2\theta_C + 0 - 3\psi] + 0 = \left(\frac{4EI}{3}\right)\theta_C - 2EI\psi \quad (6)$$

Equilibrium. At Joint D,

$$M_{DA} + M_{DC} = 0$$

$$\left(\frac{4EI}{3}\right)\theta_D - 2EI\psi + 9 + \left(\frac{4EI}{3}\right)\theta_D + \left(\frac{2EI}{3}\right)\theta_C = 0$$

$$\left(\frac{8EI}{3}\right)\theta_D + \left(\frac{2EI}{3}\right)\theta_C - 2EI\psi = -9 \quad (7)$$

At joint C,

$$M_{CD} + M_{CB} = 0$$

$$\left(\frac{4EI}{3}\right)\theta_C + \left(\frac{2EI}{3}\right)\theta_D + \left(\frac{4EI}{3}\right)\theta_C - 2EI\psi = 0$$

11–22. Continued

$$\left(\frac{2EI}{3}\right)\theta_D + \left(\frac{8EI}{3}\right)\theta_C - 2EI\psi = 0 \qquad (8)$$

Consider the horizontal force equilibrium for the entire frame,

$$\overset{+}{\rightarrow}\sum F_x = 0; \quad \frac{1}{2}(30)(3) - V_A - V_B = 0$$

Referring to the FBD of members AD and BC in Fig. a

$$\zeta + \sum M_D = 0; \quad \frac{1}{2}(30)(3)(2) - M_{DA} - M_{AD} - V_A(3) = 0$$

$$V_A = 30 - \frac{M_{DA}}{3} - \frac{M_{AD}}{3}$$

and

$$\zeta + \sum M_C = 0; \quad -M_{CB} - M_{BC} - V_B(3) = 0$$

$$V_B = -\frac{M_{CB}}{3} - \frac{M_{BC}}{3}$$

Thus,

$$\frac{1}{2}(30)(3) - \left(30 - \frac{M_{DA}}{3} - \frac{M_{AD}}{3}\right) - \left(-\frac{M_{CB}}{3} - \frac{M_{BC}}{3}\right) = 0$$

$$M_{DA} + M_{AD} + M_{CB} + M_{BC} = -45$$

$$\left(\frac{4EI}{3}\right)\theta_D - 2EI\psi + 9 + \left(\frac{2EI}{3}\right)\theta_D - 2EI\psi - 13.5 + \left(\frac{4EI}{3}\right)\theta_C - 2EI\psi$$

$$+ \left(\frac{2EI}{3}\right)\theta_C - 2EI\psi = -45$$

$$2EI\theta_D + 2EI\theta_C - 8EI\psi = -40.5 \qquad (9)$$

Solving of Eqs. (7), (8) and (9)

$$\theta_C = \frac{261}{56EI} \quad \theta_D = \frac{9}{56EI} \quad \psi = \frac{351}{56EI}$$

Substitute these results into Eq. (1) to (6),

$M_{AD} = -25.93 \text{ kN}\cdot\text{m} = -25.9 \text{ kN}\cdot\text{m}$ **Ans.**

$M_{DA} = -3.321 \text{ kN}\cdot\text{m} = -3.32 \text{ kN}\cdot\text{m}$ **Ans.**

$M_{DC} = 3.321 \text{ kN}\cdot\text{m} = 3.32 \text{ kN}\cdot\text{m}$ **Ans.**

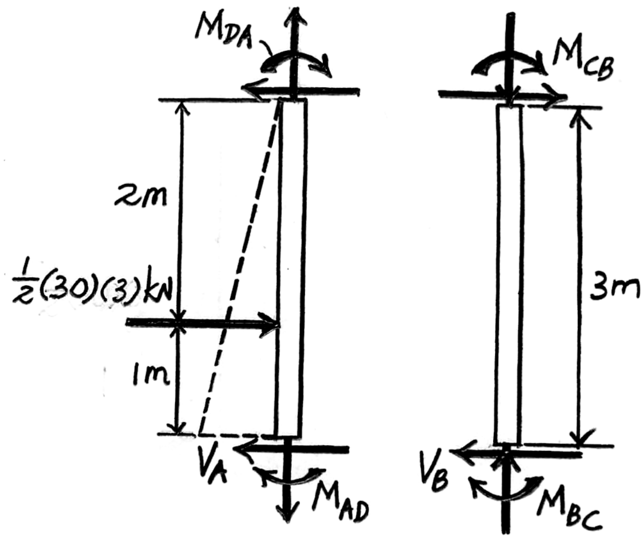
$M_{CD} = 6.321 \text{ kN}\cdot\text{m} = 6.32 \text{ kN}\cdot\text{m}$ **Ans.**

$M_{BC} = -9.429 \text{ kN}\cdot\text{m} = -9.43 \text{ kN}\cdot\text{m}$ **Ans.**

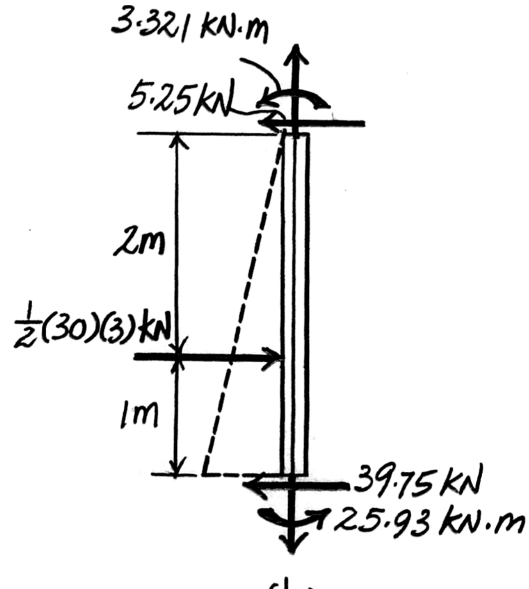
$M_{CB} = -6.321 \text{ kN}\cdot\text{m} = -6.32 \text{ kN}\cdot\text{m}$ **Ans.**

The negative signs indicate that $\mathbf{M}_{AD}$, $\mathbf{M}_{DA}$, $\mathbf{M}_{BC}$ and $\mathbf{M}_{CB}$ have counterclockwise rotational sense.Using these results, the shear at both ends of members AD, CD and BC are computed and shown on Fig. b, c and d, respectively. Subsequently, the shear and moment diagram can be plotted, Fig. e and d respectively.

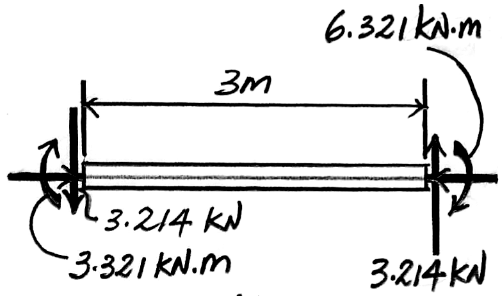
11–22. Continued



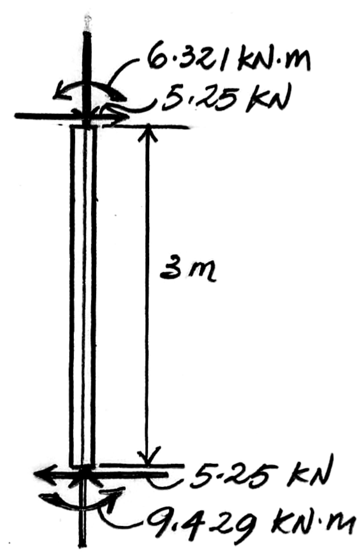
(a)



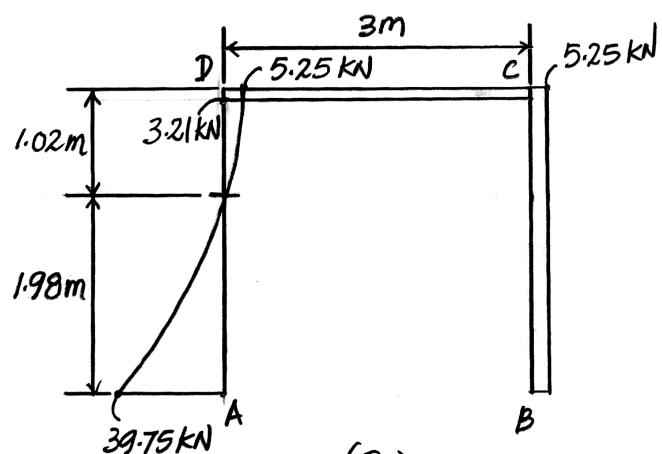
(b)



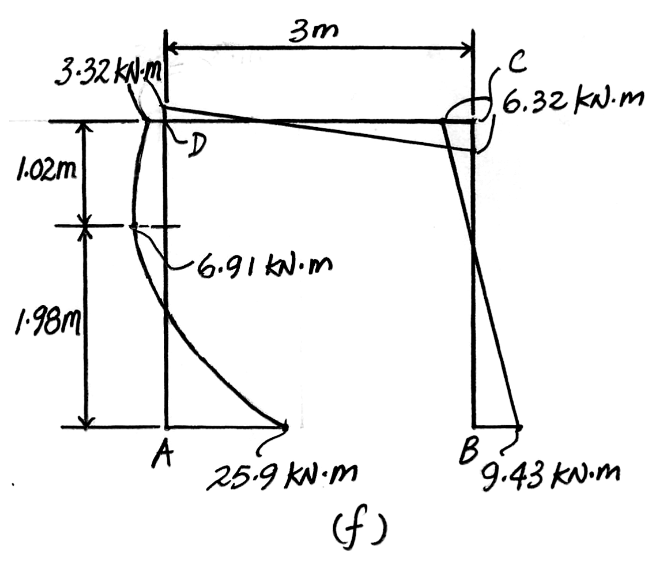
(c)



(d)

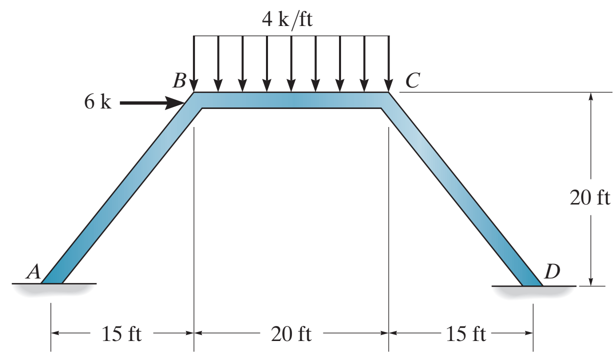


(e)



(f)

11–23. Determine the moments acting at the supports A and D of the battered-column frame. Take $E = 29(10^3)$ ksi, $I = 600\ \text{in}^4$.



$$(\text{FEM})_{BC} = -\frac{wL^2}{12} = -1600\ \text{k}\cdot\text{in.} \qquad (\text{FEM})_{CB} = \frac{wL^2}{12} = 1600\ \text{k}\cdot\text{in.}$$

$$\theta_A = \theta_D = 0$$

$$\psi_{AB} = \psi_{CD} = \frac{\Delta}{25}$$

$$\psi_{BC} = -\frac{1.2\Delta}{20}$$

$$\psi_{BC} = -1.5\psi_{CD} = -1.5\psi_{AB}$$

$$\psi = -1.5\psi \qquad (\text{where } \psi = \psi_{BC}, \psi = \psi_{AB} = \psi_{CD})$$

$$M_N = 2E\left(\frac{I}{L}\right)(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AB} = 2E\left(\frac{600}{25(12)}\right)(0 + \theta_B - 3\psi) + 0 = 116{,}000\theta_B - 348{,}000\psi$$

$$M_{BA} = 2E\left(\frac{600}{25(12)}\right)(2\theta_B + 0 - 3\psi) + 0 = 232{,}000\theta_B - 348{,}000\psi$$

$$M_{BC} = 2E\left(\frac{600}{20(12)}\right)(2\theta_B + \theta_C - 3(-1.5\psi)) - 1600$$

$$= 290{,}000\theta_B + 145{,}000\theta_C + 652{,}500\psi - 1600$$

$$M_{CB} = 2E\left(\frac{600}{20(12)}\right)(2\theta_C + \theta_B - 3(-1.5\psi)) + 1600$$

$$= 290{,}000\theta_C + 145{,}000\theta_B + 652{,}500\psi - 1600$$

$$M_{CD} = 2E\left(\frac{600}{20(12)}\right)(2\theta_C + 0 - 3\psi) + 0$$

$$= 232{,}000\theta_C - 348{,}000\psi$$

$$M_{DC} = 2E\left(\frac{600}{25(12)}\right)(0 + \theta_C - 3\psi) + 0$$

$$= 116{,}000\theta_C - 348{,}000\psi$$

Moment equilibrium at B and C:

$$M_{BA} + M_{BC} = 0$$

$$522{,}000\theta_B + 145{,}000\theta_C + 304{,}500\psi = 1600 \qquad (1)$$

$$M_{CB} + M_{CD} = 0$$

$$145{,}000\theta_B + 522{,}000\theta_C + 304{,}500\psi = -1600 \qquad (2)$$

11–23. Continued

using the FBD of the frame,

$\circlearrowleft + \sum M_0 = 0;$

$$M_{AB} + M_{DC} - \left(\frac{M_{BA} + M_{AB}}{25(12)}\right)(41.667)(12)$$

$$-\left(\frac{M_{DC} + M_{CD}}{25(12)}\right)(41.667)(12) - 6(13.333)(12) = 0$$

$$-0.667M_{AB} - 0.667M_{DC} - 1.667M_{BA} - 1.667M_{CD} - 960 = 0$$

$$464{,}000\theta_B + 464{,}000\theta_C - 1{,}624{,}000\psi = -960$$

Solving Eqs. (1), (2) and (3),

$\theta_B = 0.004030$ rad

$\theta_C = -0.004458$ rad

$\psi = 0.0004687$ in.

$M_{AB} = 25.4$ k · ft **Ans.**

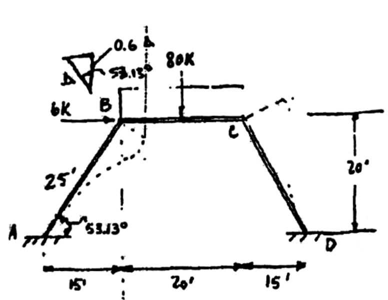
$M_{BA} = 64.3$ k · ft

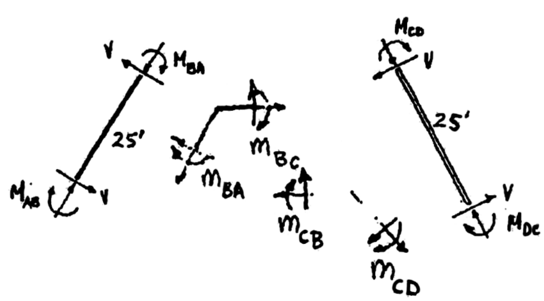
$M_{BC} = -64.3$ k · ft

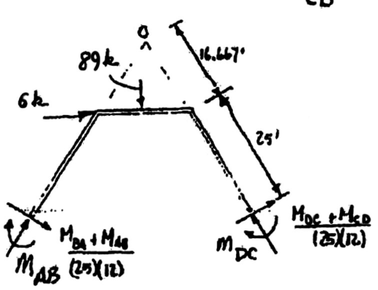
$M_{CB} = 99.8$ k · ft

$M_{CD} = -99.8$ k · ft

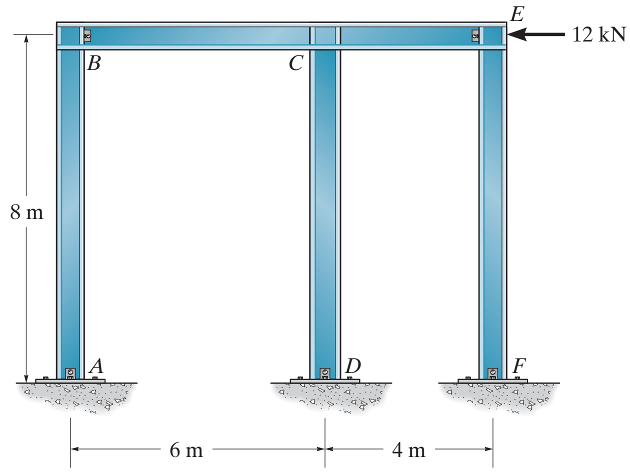
$M_{DC} = -56.7$ k · ft **Ans.**







***11–24.** Wind loads are transmitted to the frame at joint E. If A, B, E, D, and F are all pin connected and C is fixed connected, determine the moments at joint C and draw the bending moment diagrams for the girder BCE. EI is constant.



$\psi_{BC} = \psi_{CE} = 0$

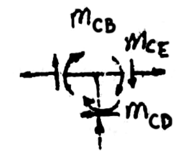
$\psi_{AB} = \psi_{CD} = \psi_{CF} = \psi$

Applying Eq. 11–10,

$$M_{CB} = \frac{3EI}{6}(\theta_C - 0) + 0$$

$$M_{CE} = \frac{3EI}{4}(\theta_C - 0) + 0$$

$$M_{CD} = \frac{3EI}{8}(\theta_C - \psi) + 0 \qquad (1)$$

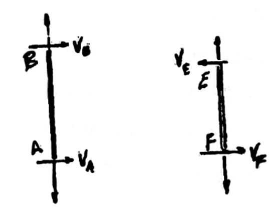


Moment equilibrium at C:

$$M_{CB} + M_{CE} + M_{CD} = 0$$

$$\frac{3EI}{6}(\theta_C) + \frac{3EI}{4}(\theta_C) + \frac{3EI}{8}(\theta_C - \psi) = 0$$

$$\psi = 4.333\theta_C \qquad (2)$$

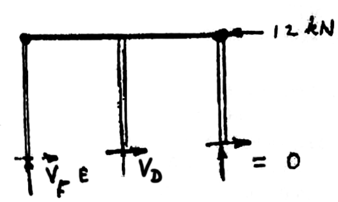


From FBDs of members AB and EF:

$\circlearrowleft + \sum M_B = 0; \quad V_A = 0$

$\circlearrowleft + \sum M_E = 0; \quad V_F = 0$

Since AB and FE are two-force members, then for the entire frame:

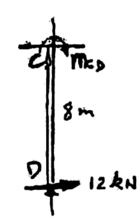


$\overset{+}{\rightarrow} \sum F_E = 0; \quad V_D - 12 = 0; \quad V_D = 12\text{ kN}$

From FBD of member CD:

$\circlearrowleft + \sum M_C = 0; \quad M_{CD} - 12(8) = 0$

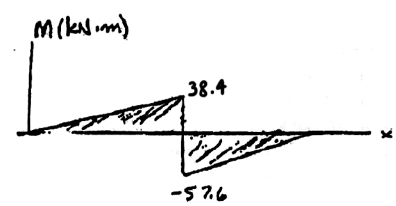
$$M_{CD} = 96\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$



From Eq. (1),

$$96 = \frac{3}{8}EI(\theta_C - 4.333\theta_C)$$

$$\theta_C = \frac{-76.8}{EI}$$



From Eq. (2),

$$\psi = \frac{-332.8}{EI}$$

Thus,

$$M_{CB} = -38.4\text{ kN}\cdot\text{in} \qquad \textbf{Ans.}$$

$$M_{CE} = -57.6\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$