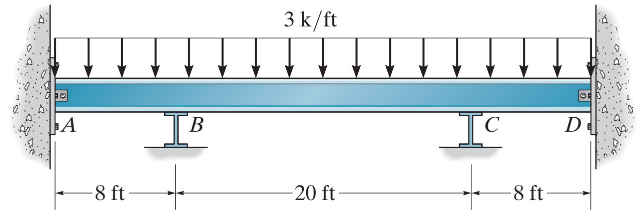
12–1. Determine the moments at B and C. EI is constant. Assume B and C are rollers and A and D are pinned.



$$\text{FEM}_{AB} = \text{FEM}_{CD} = -\frac{wL^2}{12} = -16, \qquad \text{FEM}_{BA} = \text{FEM}_{DC} = \frac{wL^2}{12} = 16$$

$$\text{FEM}_{BC} = -\frac{wL^2}{12} = -100 \qquad \text{FEM}_{CB} = \frac{wL^2}{12} = 100$$

$$K_{AB} = \frac{3EI}{8}, \qquad K_{BC} = \frac{4EI}{20}, \qquad K_{CD} = \frac{3EI}{8}$$

$$\text{DF}_{AB} = 1 = DF_{DC}$$

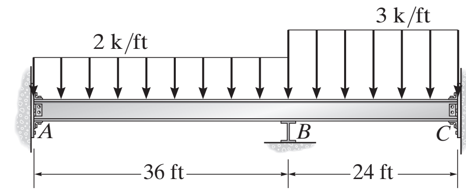
$$\text{DF}_{BA} = \text{DF}_{CD} = \frac{\frac{3EI}{8}}{\frac{3EI}{8} + \frac{4EI}{20}} = 0.652$$

$$\text{DF}_{BA} = \text{DF}_{CB} = 1 - 0.652 = 0.348$$

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	1	0.652	0.348	0.348	0.652	1
FEM	–16	16	–100	100	–16	16
	16	54.782	29.218	–29.218	–54.782	–16
		8	–14.609	14.609	–8	
		4.310	2.299	–2.299	–4.310	
			–1.149	1.149		
		0.750	0.400	–0.400	–0.750	
			–0.200	0.200		
		0.130	0.070	–0.070	–0.130	
			–0.035	0.035		
		0.023	0.012	–0.012	–0.023	
$\sum M$	0	84.0	–84.0	84.0	–84.0	0 k·ft

Ans.

12–2. Determine the moments at A, B, and C. Assume the support at B is a roller and A and C are fixed. EI is constant.



$(\text{DF})_{AB} = 0 \qquad (\text{DF})_{BA} = \dfrac{I/36}{I/36 + I/24} = 0.4$

$(\text{DF})_{BC} = 0.6 \qquad (\text{DF})_{CB} = 0$

$(\text{FEM})_{AB} = \dfrac{-2(36)^2}{12} = -216 \text{ k} \cdot \text{ft}$

$(\text{FEM})_{BA} = 216 \text{ k} \cdot \text{ft}$

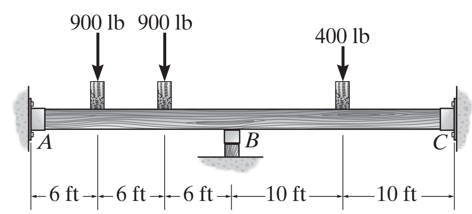
$(\text{FEM})_{BC} = \dfrac{-3(24)^2}{12} = -144 \text{ k} \cdot \text{ft}$

$(\text{FEM})_{CB} = 144 \text{ k} \cdot \text{ft}$

Joint	A	B		C
Mem.	AB	BA	BC	CB
DF	0	0.4	0.6	0
FEM	–216	216	–144	144
		–28.8	–43.2	
	–14.4			–21.6
$\sum M$	–230	187	–187	–122 k · ft

Ans.

12–3. Determine the moments at A, B, and C, then draw the moment diagram. Assume the support at B is a roller and A and C are fixed. EI is constant.



$(\text{DF})_{AB} = 0 \qquad (\text{DF})_{BA} = \dfrac{I/18}{I/18 + I/20} = 0.5263$

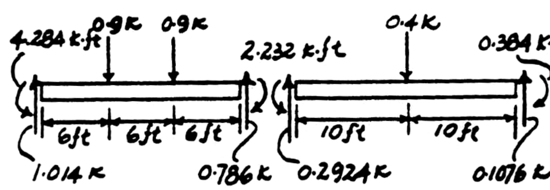
$(DF)_{CB} = 0 \qquad (DF)_{BC} = 0.4737$

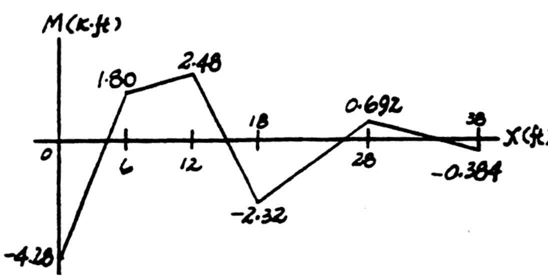
$(\text{FEM})_{AB} = \dfrac{-2(0.9)(18)}{9} = -3.60 \text{ k} \cdot \text{ft}$

$(\text{FEM})_{BA} = 3.60 \text{ k} \cdot \text{ft}$

$(\text{FEM})_{BC} = \dfrac{-0.4(20)}{8} = -1.00 \text{ k} \cdot \text{ft}$

$(\text{FEM})_{CB} = 1.00 \text{ k} \cdot \text{ft}$

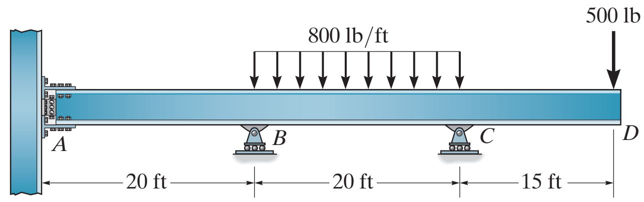




Joint	A	B		C
Mem.	AB	BA	BC	CB
DF	0	0.5263	0.4737	0
FEM	–3.60	3.60	–1.00	1.00
		–1.368	–1.232	
	–0.684			–0.616
$\sum M$	–4.28	2.23	–2.23	0.384 k · ft

Ans.

***12–4.** Determine the reactions at the supports and then draw the moment diagram. Assume A is fixed. EI is constant.



$$\text{FEM}_{BC} = -\frac{wL^2}{12} = -26.67, \quad \text{FEM}_{CB} = \frac{wL^2}{12} = 26.67$$

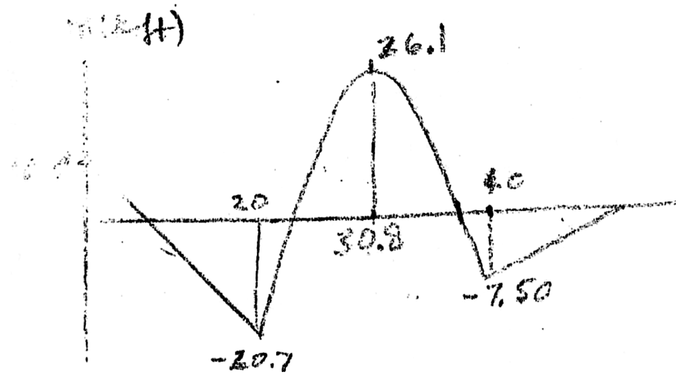
$$M_{CD} = 0.5(15) = 7.5 \text{ k} \cdot \text{ft}$$

$$K_{AB} = \frac{4EI}{20}, \quad K_{BC} = \frac{4EI}{20}$$

$$\text{DF}_{AB} = 0$$

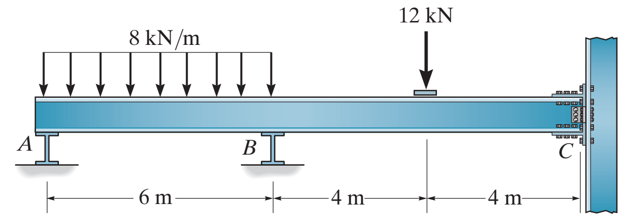
$$\text{DF}_{BA} = \text{DF}_{BC} = \frac{\frac{4EI}{20}}{\frac{4EI}{20} + \frac{4EI}{20}} = 0.5$$

$$\text{DF}_{CB} = 1$$



Joint	A	B		C	
Member	AB	BA	BC	CB	CD
DF	0	0.5	0.5	1	0
FEM			–26.67	26.67	–7.5
		13.33	13.33	–19.167	
	6.667		–9.583	6.667	
		4.7917	4.7917	–6.667	
	2.396		–3.333	2.396	
		1.667	1.667	–2.396	
	0.8333		–1.1979	0.8333	
		0.5990	0.5990	–0.8333	
	0.2994		–0.4167	0.2994	
		0.2083	0.2083	–0.2994	
	0.1042		–0.1497	0.1042	
		0.07485	0.07485	–0.1042	
	10.4	20.7	–20.7	7.5	–7.5 k · ft

12–5. Determine the moments at B and C, then draw the moment diagram for the beam. Assume C is a fixed support. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{BA} = \frac{3EI}{L_{BA}} = \frac{3EI}{6} = \frac{EI}{2} \qquad K_{BC} = \frac{4EI}{L_{BC}} = \frac{4EI}{8} = \frac{EI}{2}$$

$$(\mathrm{DF})_{AB} = 1 \qquad (\mathrm{DF})_{BA} = \frac{EI/2}{EI/2 + EI/2} = 0.5$$

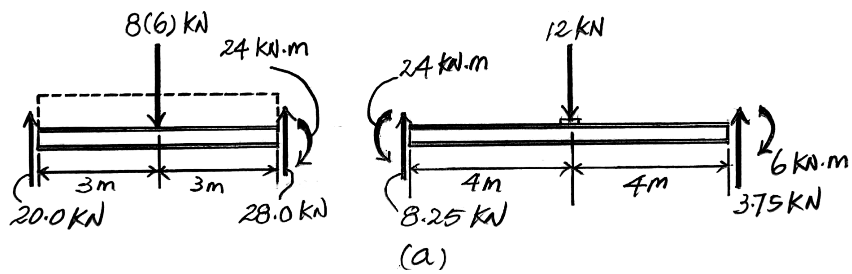
$$(\mathrm{DF})_{BC} = \frac{EI/2}{EI/2 + EI/2} = 0.5 \qquad (\mathrm{DF})_{CB} = 0$$

Fixed End Moments. Referring to the table on the inside back cover,

$$(\mathrm{FEM})_{BA} = \frac{wL^2}{8} = \frac{8(6^2)}{8} = 36\ \mathrm{kN \cdot m}$$

$$(\mathrm{FEM})_{BC} = -\frac{PL}{8} = -\frac{12(8)}{8} = -12\ \mathrm{kN \cdot m}$$

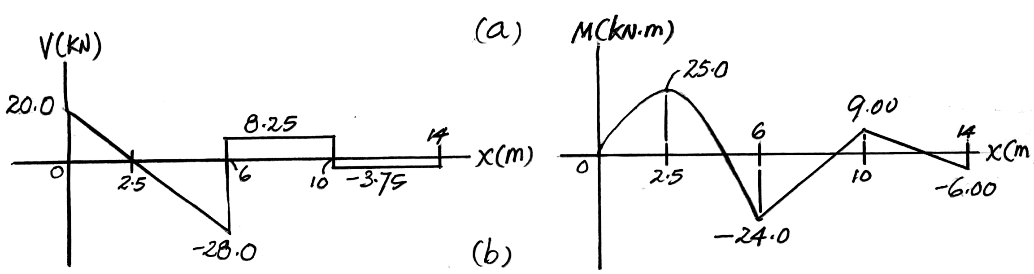
$$(\mathrm{FEM})_{CB} = \frac{PL}{8} = \frac{12(8)}{8} = 12\ \mathrm{kN \cdot m}$$



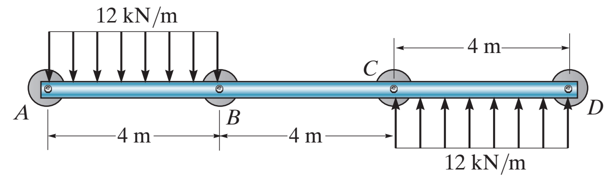
Moment Distribution. Tabulating the above data,

Joint	A	B		C
Member	AB	BA	BC	CB
DF	1	0.5	0.5	0
FEM	0	36	–12	12
Dist.		–12	–12	
				–6
$\sum M$	0	24	–24	6

Using these results, the shear and both ends of members AB and BC are computed and shown in Fig. a. Subsequently, the shear and moment diagram can be plotted, Fig. b.



12–6. Determine the moments at B and C, then draw the moment diagram for the beam. All connections are pins. Assume the horizontal reactions are zero. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{AB} = \frac{3EI}{L_{AB}} = \frac{3EI}{4} \qquad K_{BC} = \frac{6EI}{L_{BC}} = \frac{6EI}{4} = \frac{3EI}{2}$$

$$(\mathrm{DF})_{AB} = 1 \quad (\mathrm{DF})_{BA} = \frac{3EI/4}{3EI/4 + 3EI/2} = \frac{1}{3} \quad (\mathrm{DF})_{BC} = \frac{3EI/2}{3EI/4 + 3EI/2} = \frac{2}{3}$$

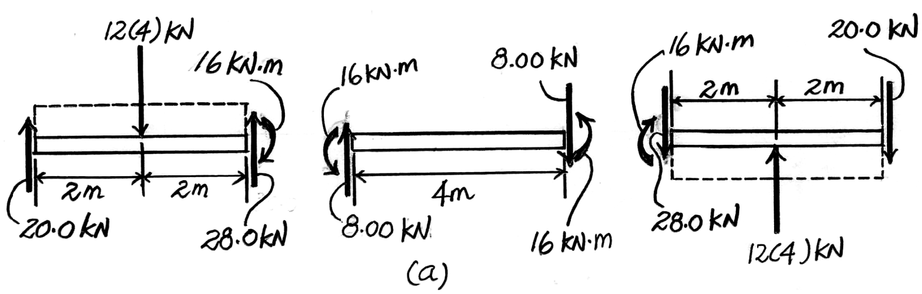
Fixed End Moments. Referring to the table on the inside back cover,

$$(\mathrm{FEM})_{BA} = \frac{wL^2}{8} = \frac{12(4^2)}{8} = 24 \text{ kN}\cdot\text{m} \qquad (\mathrm{FEM})_{BC} = 0$$

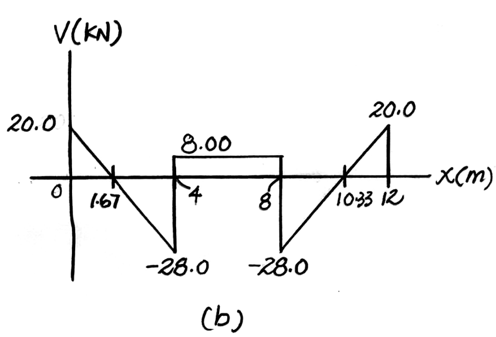
Moment Distribution. Tabulating the above data,

Joint	A	B	
Member	AB	BA	BC
DF	1	1/3	2/3
FEM	0	24	0
Dist.		–8	–16
$\sum M$	0	16	–16

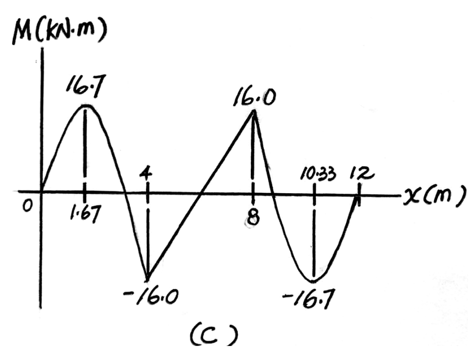
Using these results, the shear at both ends of members AB, BC, and CD are computed and shown in Fig. a. Subsequently the shear and moment diagram can be plotted, Fig. b and c, respectively.



(a)

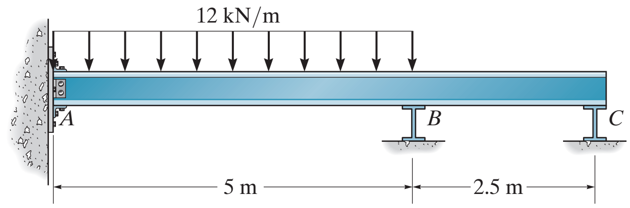


(b)



(c)

12–7. Determine the reactions at the supports. Assume A is fixed and B and C are rollers that can either push or pull on the beam. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{AB} = \frac{4EI}{L_{AB}} = \frac{4EI}{5} = 0.8EI \qquad K_{BC} = \frac{3EI}{L_{BC}} = \frac{3EI}{2.5} = 1.2EI$$

$$(\text{DF})_{AB} = 0 \qquad (\text{DF})_{BA} = \frac{0.8EI}{0.8EI + 1.2EI} = 0.4$$

$$(\text{DF})_{BC} = \frac{1.2.EI}{0.8EI + 1.2EI} = 0.6$$

$$(\text{DF})_{CB} = 1$$

Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{AB} = -\frac{wL^2}{12} = -\frac{12(5^2)}{12} = -25 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{BA} = \frac{wL^2}{12} = \frac{12(5^2)}{12} = 25 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{BC} = (\text{FEM})_{CB} = 0$$

Moment Distribution. Tabulating the above data,

Joint	A	B		C
Member	AB	BA	BC	CB
DF	0	0.4	0.6	1
FEM	–25	25	0	0
Dist.		–10	–15	
CO	–5			
$\sum M$	–30	15	–15	

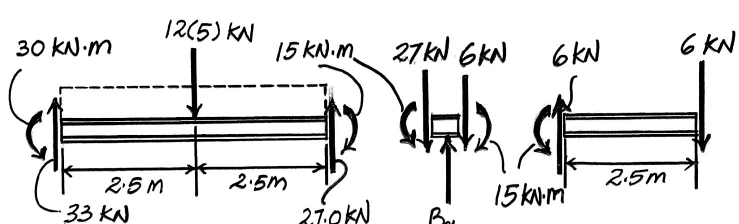
Ans.

Using these results, the shear at both ends of members AB and BC are computed and shown in Fig. *a*.

From this figure,

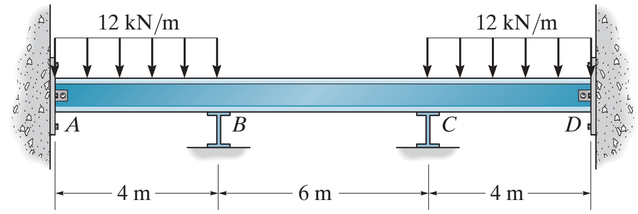
$A_x = 0 \quad A_y = 33 \text{ kN}\uparrow \quad B_y = 27 + 6 = 33 \text{ kN}\uparrow$ **Ans.**

$M_A = 30 \text{ kN}\cdot\text{m}$ ↺ $\quad C_y = 6 \text{ kN}\downarrow$ **Ans.**



(a)

***12–8.** Determine the moments at B and C, then draw the moment diagram for the beam. Assume the supports at B and C are rollers and A and D are pins. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{AB} = \frac{3EI}{L_{AB}} = \frac{3EI}{4} \qquad K_{BC} = \frac{2EI}{L_{BC}} = \frac{2EI}{6} = \frac{EI}{3}$$

$$(\text{DF})_{AB} = 1 \quad (\text{DF})_{BA} = \frac{3EI/4}{3EI/4 + 3EI/3} = \frac{9}{13} \quad (\text{DF})_{BC} = \frac{EI/3}{3EI/4 + EI/3} = \frac{4}{13}$$

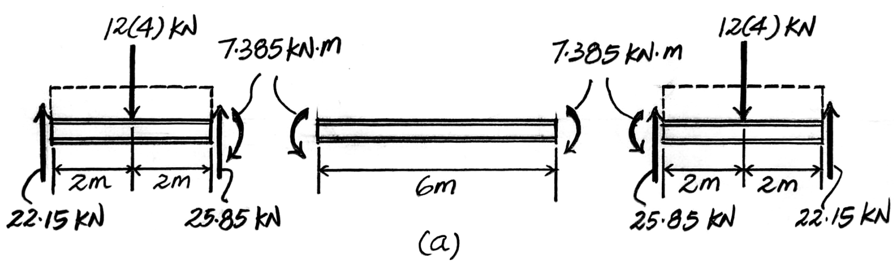
Fixed End Moments. Referring to the table on the inside back cover,

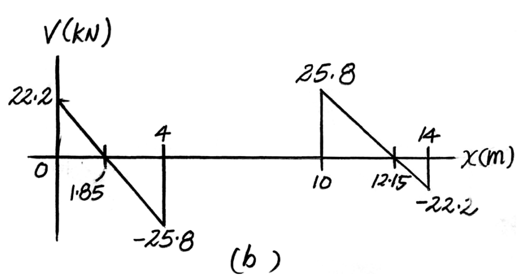
$$(\text{FEM})_{AB} = (\text{FEM})_{BC} = 0 \qquad (\text{FEM})_{BA} = \frac{wL^2}{8} = \frac{12(4^2)}{8} = 24 \text{ kN} \cdot \text{m}$$

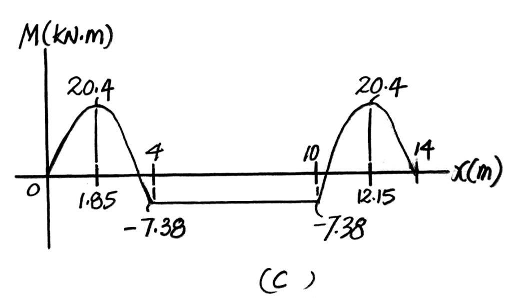
Moment Distribution. Tabulating the above data,

Joint	*A*	*B*	
Member	*AB*	*BA*	*BC*
DF	1	$\frac{9}{13}$	$\frac{4}{13}$
FEM	0	24	0
Dist.		–16.62	–7.385
$\sum M$	0	7.385	–7.385

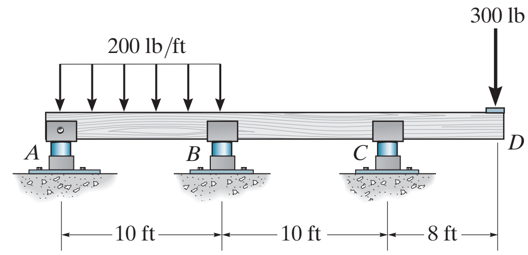
Using these results, the shear at both ends of members AB, BC, and CD are computed and shown in Fig. a. Subsequently, the shear and moment diagram can be plotted, Fig. b and c, respectively.







12–9. Determine the moments at B and C, then draw the moment diagram for the beam. Assume the supports at B and C are rollers and A is a pin. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{AB} = \frac{3EI}{L_{AB}} = \frac{3EI}{10} = 0.3EI \qquad K_{BC} = \frac{4EI}{L_{BC}} = \frac{4EI}{10} = 0.4EI.$$

$$(\text{DF})_{BA} = \frac{0.3EI}{0.3EI + 0.4EI} = \frac{3}{7} \qquad (\text{DF})_{BC} = \frac{0.4EI}{0.3EI + 0.4EI} = \frac{4}{7}$$

$$(\text{DF})_{CB} = 1 \qquad (\text{DF})_{CD} = 0$$

Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{CD} = -300(8) = 2400 \text{ lb} \cdot \text{ft} \qquad (\text{FEM})_{BC} = (\text{FEM})_{CB} = 0$$

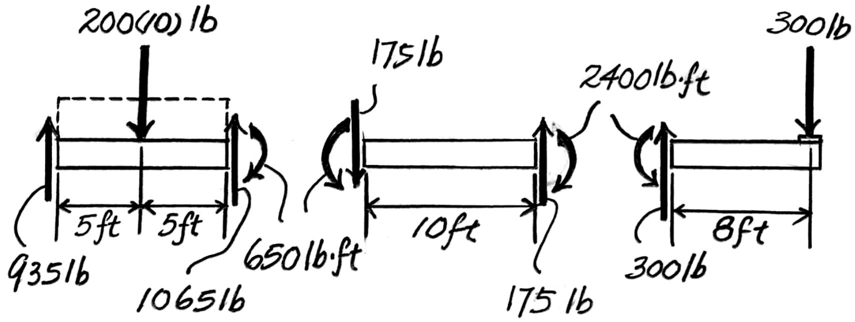
$$(\text{FEM})_{BA} = \frac{wL_{AB}^2}{8} = \frac{200(10^2)}{8} = 2500 \text{ lb} \cdot \text{ft}$$

Moment Distribution. Tabulating the above data,

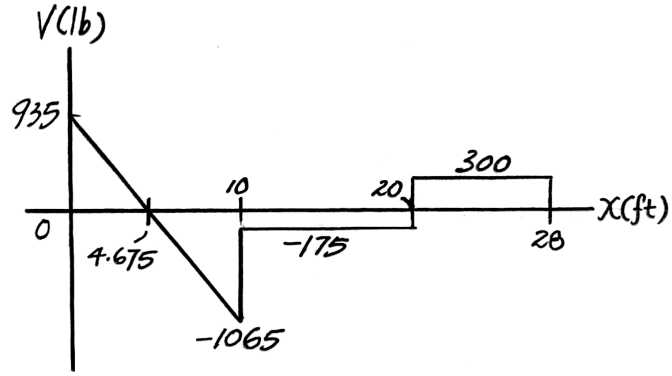
Joint	*A*	*B*		*C*	
Member	*AB*	*BA*	*BC*	*CB*	*CD*
DF	1	3/7	4/7	1	0
FEM	0	2500	0	0	–2400
Dist.		–1071.43	–1428.57	2400	
CO			1200	–714.29	
Dist.		–514.29	–685.71	714.29	
CO			357.15	–342.86	
Dist.		–153.06	–204.09	342.86	
CO			171.43	–102.05	
Dist.		–73.47	–97.96	102.05	
CO			51.03	–48.98	
Dist.		–21.87	–29.16	48.98	
CO			24.99	–14.58	
Dist.		–10.50	–13.99	14.58	
CO			7.29	–7.00	
Dist.		–3.12	–4.17	7.00	
CO			3.50	–2.08	
Dist.		–1.50	–2.00	2.08	
CO			1.04	–1.00	
Dist.		–0.45	–0.59	1.00	
CO			0.500	–0.30	
Dist.		–0.21	–0.29	0.30	
CO			0.15	–0.15	
Dist.		–0.06	–0.09	0.15	
CO			0.07	–0.04	
Dist.		–0.03	–0.04	0.04	
$\sum M$	0	650.01	–650.01	2400	–2400

Using these results, the shear at both ends of members AB, BC, and CD are computed and shown in Fig. *a*. Subsequently, the shear and moment diagrams can be plotted, Fig. *b* and *c*, respectively.

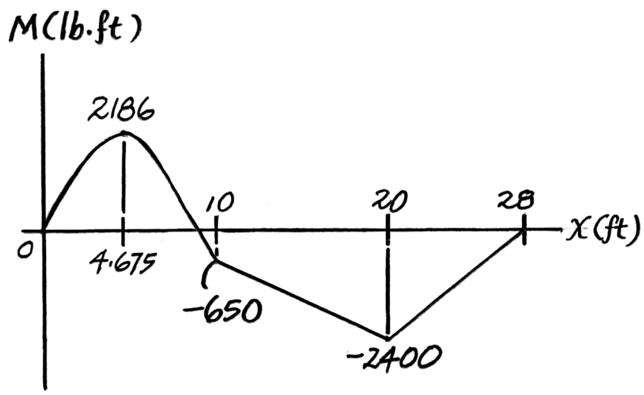
12–9. Continued



(a)

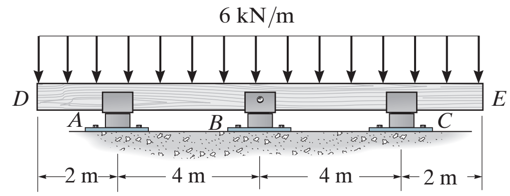


(b)



(c)

12–10. Determine the moment at B, then draw the moment diagram for the beam. Assume the supports at A and C are rollers and B is a pin. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{AB} = \frac{4EI}{L_{AB}} = \frac{4EI}{4} = EI \qquad K_{BC} = \frac{4EI}{L_{BC}} = \frac{4EI}{4} = EI$$

$$(\text{DF})_{AB} = 1 \quad (\text{DF})_{AD} = 0 \quad (\text{DF})_{BA} = (\text{DF})_{BC} = \frac{EI}{EI + EI} = 0.5$$

$$(\text{DF})_{CB} = 1 \qquad (\text{DF})_{CE} = 0$$

Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{AD} = 6(2)(1) = 12\text{ kN}\cdot\text{m} \qquad (\text{FEM})_{CE} = -6(2)(1) = -12\text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{AB} = \frac{-wL_{AB}^2}{12} = -\frac{6(4^2)}{12} = -8\text{ kN}\cdot\text{m}$$

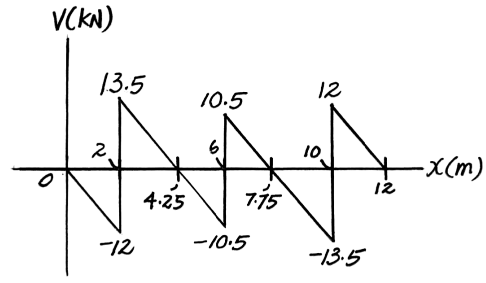
$$(\text{FEM})_{BA} = \frac{wL_{AB}^2}{12} = \frac{6(4^2)}{12} = 8\text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{BC} = \frac{-wL_{BC}^2}{12} = -\frac{6(4^2)}{12} = -8\text{ kN}\cdot\text{m}$$

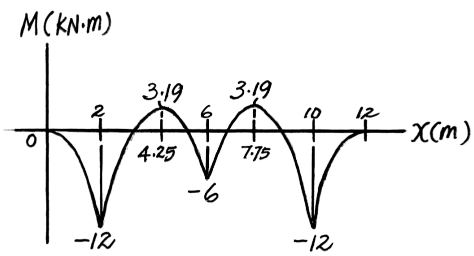
$$(\text{FEM})_{CB} = \frac{wL_{BC}^2}{12} = \frac{6(4^2)}{12} = 8\text{ kN}\cdot\text{m}$$

Moment Distribution. Tabulating the above data,

Joint	A		B		C	
Member	AD	AB	BA	BC	CB	CE
DF	0	1	0.5	0.5	1	0
FEM	12	–8	8	–8	8	–12
Dist.		–4	0	0	4	
CO			–2	2		
$\sum M$	12	–12	6	–6	12	–12

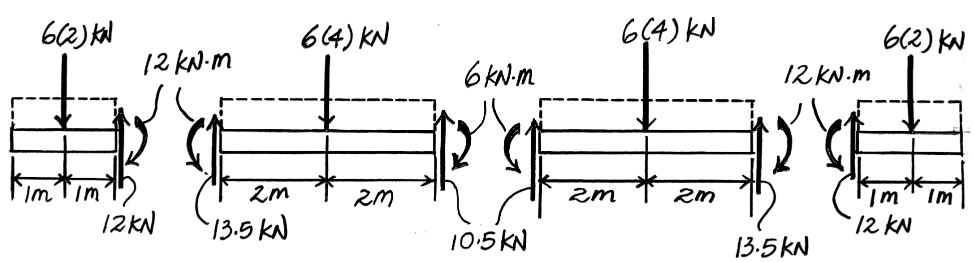


(b)



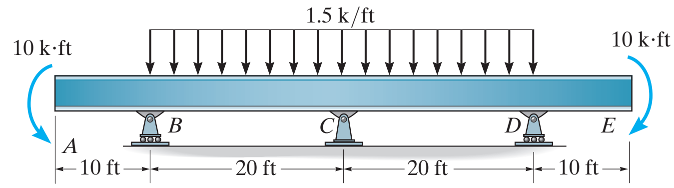
(c)

Using these results, the shear at both ends of members AD, AB, BC, and CE are computed and shown in Fig. a. Subsequently, the shear and moment diagram can be plotted, Fig. b and c, respectively.



(a)

12–11. Determine the moments at B, C, and D, then draw the moment diagram for the beam. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{BC} = \frac{4EI}{L_{BC}} = \frac{4EI}{20} = 0.2\,EI \qquad K_{CD} = \frac{4EI}{L_{CD}} = \frac{4EI}{20} = 0.2\,EI$$

$$(\text{DF})_{BA} = (\text{DF})_{DE} = 0 \qquad (\text{DF})_{BC} = (\text{DF})_{DC} = 1$$

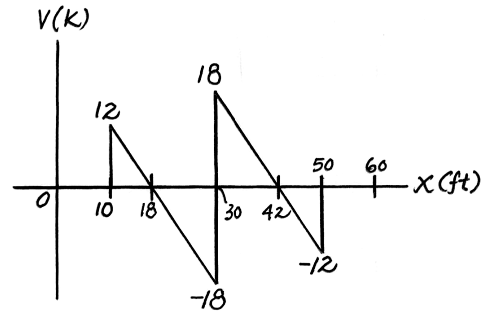
$$(\text{DF})_{CB} = (\text{DF})_{CD} = \frac{0.2EI}{0.2EI + 0.2EI} = 0.5$$

Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{BA} = 10\text{ k}\cdot\text{ft} \qquad (\text{FEM})_{DE} = -10\text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = (\text{FEM})_{CD} = -\frac{wL^2}{12} = -\frac{1.5(20^2)}{12} = -50\text{ k}\cdot\text{ft}$$

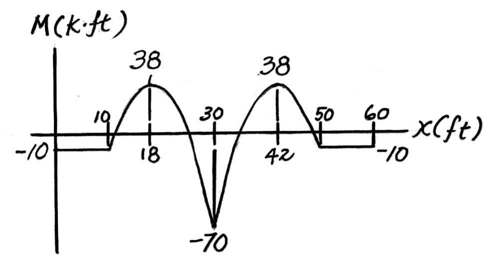
$$(\text{FEM})_{CB} = (\text{FEM})_{DC} = \frac{wL^2}{12} = -\frac{1.5(20^2)}{12} = 50\text{ k}\cdot\text{ft}$$



(b)

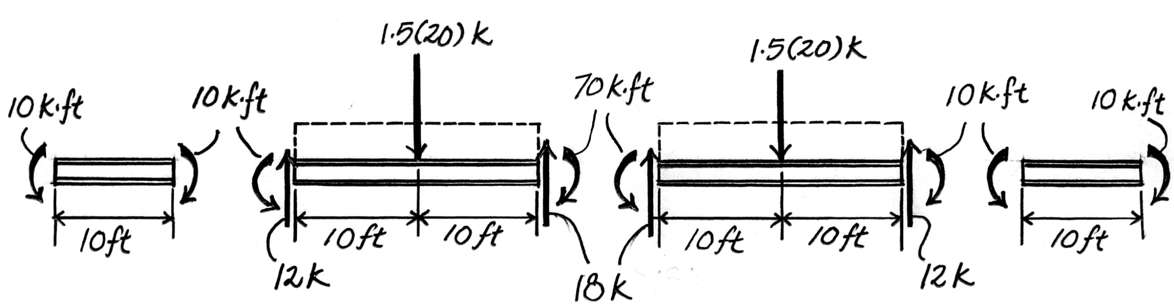
Moment Distribution. Tabulating the above data,

Joint	B		C		D	
Member	*BA*	*BC*	*CB*	*CD*	*DC*	*DE*
DF	0	1	0.5	0.5	1	0
FEM	10	–50	50	–50	50	–10
Dist.		40	0	0	–40	
CO			20	–20		
$\sum M$	10	–10	70	–70	10	–10



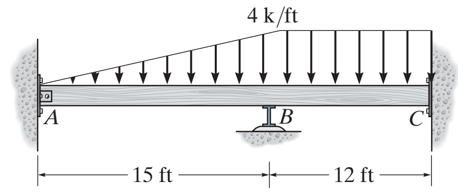
(c)

Using these results, the shear at both ends of members AB, BC, CD, and DE are computed and shown in Fig. a. Subsequently, the shear and moment diagram can be plotted, Fig. b and c, respectively.



(a)

***12–12.** Determine the moment at B, then draw the moment diagram for the beam. Assume the support at A is pinned, B is a roller and C is fixed. EI is constant.



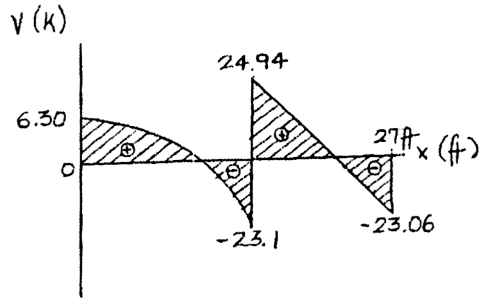
$$\text{FEM}_{AB} = \frac{wL^2}{30} = \frac{4(15^2)}{30} = 30 \text{ k}\cdot\text{ft}$$

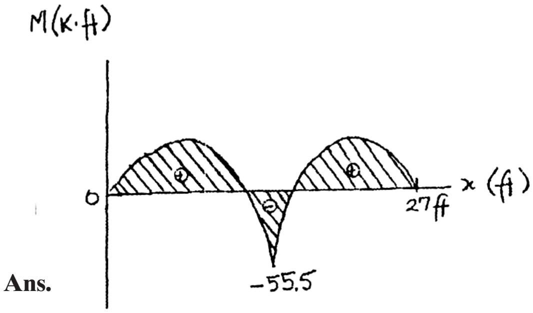
$$\text{FEM}_{BA} = \frac{wL^2}{20} = \frac{4(15^2)}{20} = 45 \text{ k}\cdot\text{ft}$$

$$\text{FEM}_{BC} = \frac{wL^2}{12} = \frac{(4)(12^2)}{12} = 48 \text{ k}\cdot\text{ft}$$

$$\text{FEM}_{CB} = 48 \text{ k}\cdot\text{ft}$$

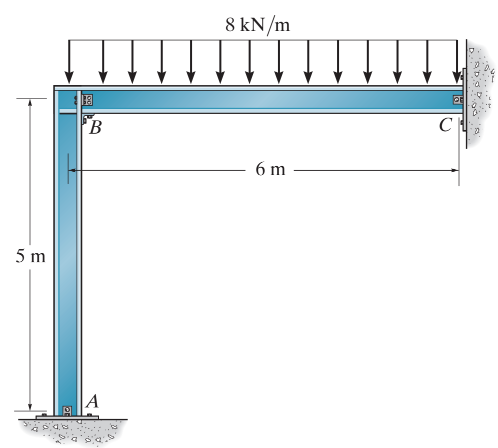
Joint	A	B		C
Member	AB	BA	BC	CB
DF	1	0.375	0.625	0
FEM	–30	45	–48	48
	30	1.125	1.875	
		15		0.9375
		–5.625	–9.375	
				–4.688
$\sum M$	0	55.5	–55.5	44.25





$M_B = -55.5 \text{ k}\cdot\text{ft}$ **Ans.**

12–13. Determine the moment at B, then draw the moment diagram for each member of the frame. Assume the supports at A and C are pins. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{BC} = \frac{3EI}{L_{BC}} = \frac{3EI}{6} = 0.5\,EI$$

$$K_{BA} = \frac{3EI}{L_{AB}} = \frac{3EI}{5} = 0.6\,EI$$

$$(\text{DF})_{AB} = (\text{DF})_{CB} = 1 \qquad (\text{DF})_{BC} = \frac{0.5EI}{0.5EI + 0.6EI} = \frac{5}{11}$$

$$(\text{DF})_{BA} = \frac{0.6EI}{0.5EI + 0.6EI} = \frac{6}{11}$$

Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{CB} = (\text{FEM})_{AB} = (\text{FEM})_{BA} = 0$$

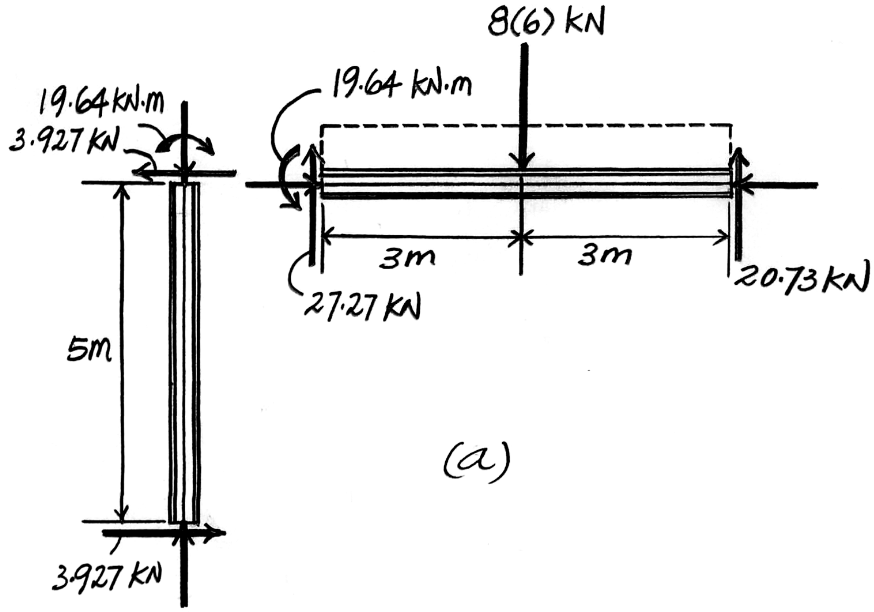
$$(\text{FEM})_{BC} = -\frac{wL_{BC}^2}{8} = -\frac{8(6^2)}{8} = -36 \text{ kN}\cdot\text{m}$$

12–13. Continued

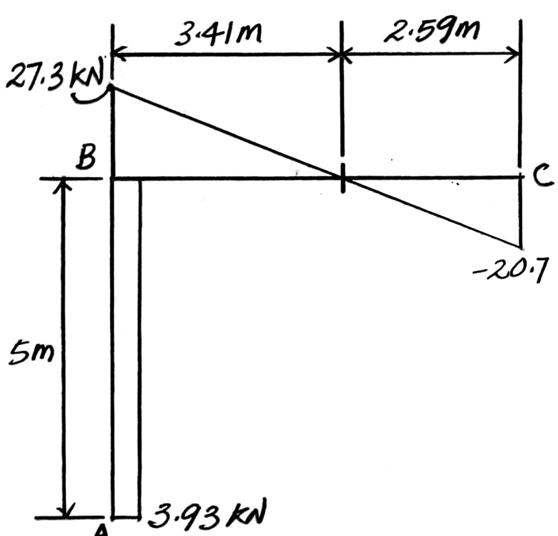
Moment Distribution. Tabulating the above data,

Joint	*A*	*B*		*C*
Member	*AB*	*BA*	*BC*	*CB*
DF	1	$\frac{6}{11}$	$\frac{5}{11}$	1
FEM	0	0	–36	0
Dist.		19.64	16.36	
$\sum M$	0	19.64	–19.64	0

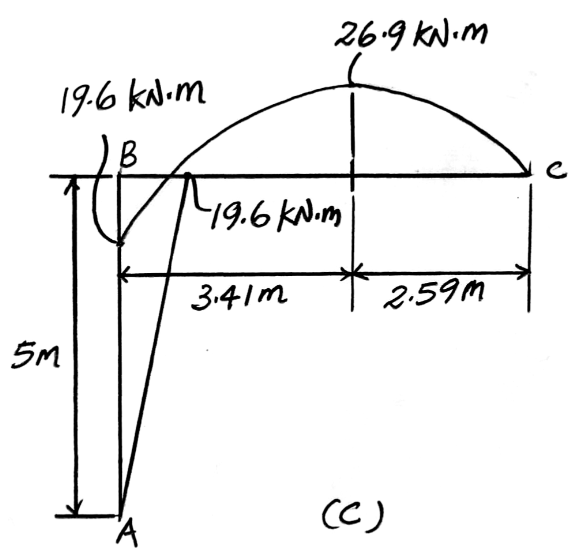
Using these results, the shear at both ends of member *AB* and *BC* are computed and shown in Fig. *a*. Subsequently, the shear and moment diagram can be plotted, Fig. *b* and *c*, respectively.



(a)

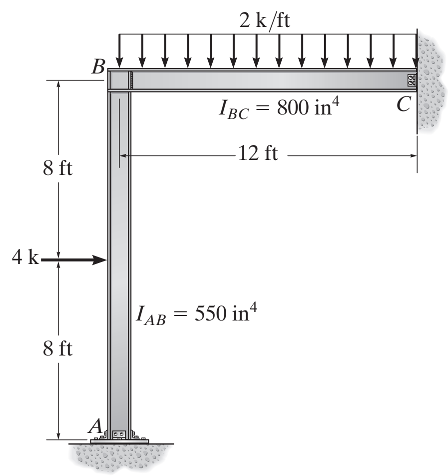


(b)



(c)

12–14. Determine the moments at the ends of each member of the frame. Assume the joint at B is fixed, C is pinned, and A is fixed. The moment of inertia of each member is listed in the figure. $E = 29(10^3)$ ksi.



$$(\text{DF})_{AB} = 0$$

$$(\text{DF})_{BA} = \frac{4(0.6875 I_{BC})/16}{4(0.6875 I_{BC})/16 + 3I_{BC}/12} = 0.4074$$

$$(\text{DF})_{BC} = 0.5926 \qquad (\text{DF})_{CB} = 1$$

$$(\text{FEM})_{AB} = \frac{-4(16)}{8} = -8 \text{ k} \cdot \text{ft}$$

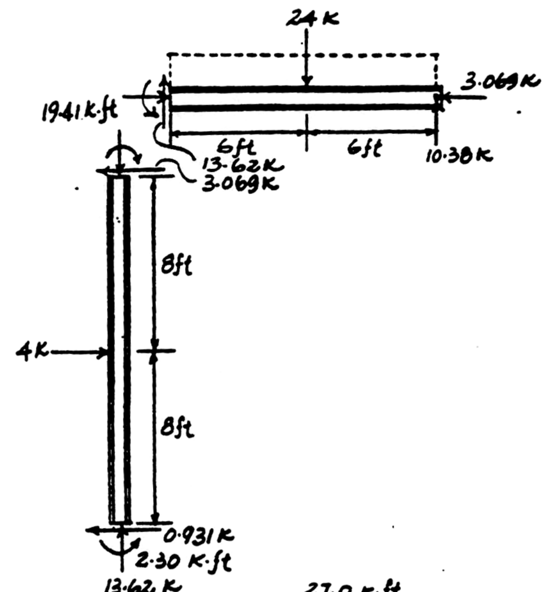
$$(\text{FEM})_{BA} = 8 \text{ k} \cdot \text{ft}$$

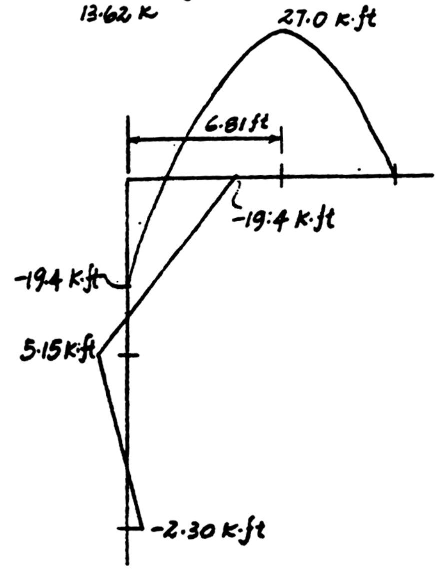
$$(\text{FEM})_{BC} = \frac{-2(12^2)}{12} = -24 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{CB} = 24 \text{ k} \cdot \text{ft}$$

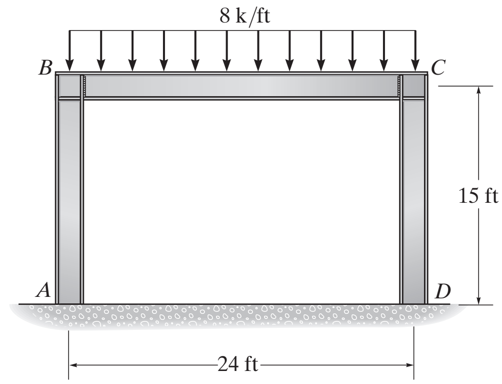
Joint	A	B		C
Mem.	AB	BA	BC	CB
DF	0	0.4047	0.5926	1
FEM	–8.0	8.0	–24.0	24.0
		6.518	9.482	–24.0
	3.259		–12.0	
		4.889	7.111	
	2.444			
$\sum M$	–2.30	19.4	–19.4	0

Ans.





12–15. Determine the reactions at A and D. Assume the supports at A and D are fixed and B and C are fixed connected. EI is constant.



$$(\text{DF})_{AB} = (\text{DF})_{DC} = 0$$

$$(\text{DF})_{BA} = (\text{DF})_{CD} = \frac{I/15}{I/15 + I/24} = 0.6154$$

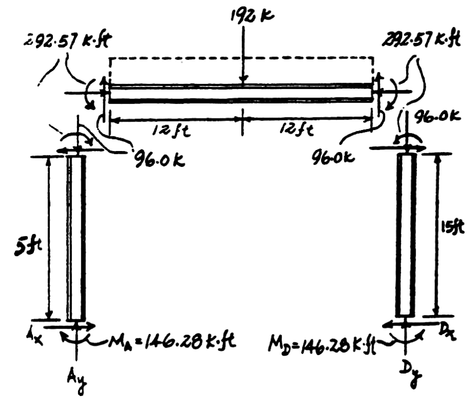
$$(\text{DF})_{BC} = (\text{DF})_{CB} = 0.3846$$

$$(\text{FEM})_{AB} = (\text{FEM})_{BA} = 0$$

$$(\text{FEM})_{BC} = \frac{-8(24)^2}{12} = -384 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{CB} = 384 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{CD} = (\text{FEM})_{DC} = 0$$



Joint	A	B		C		D
Mem.	AB	BA	BC	CB	CD	DC
DF	0	0.6154	0.3846	0.3846	0.6154	0
FEM			−384	384		
		236.31	147.69	−147.69	−236.31	
	118.16		−73.84	73.84		−118.16
		45.44	28.40	−28.40	−45.44	
	22.72		−14.20	14.20		−22.72
		8.74	5.46	−5.46	−8.74	
	4.37		−2.73	2.73		−4.37
		1.68	1.05	−1.05	−1.68	
	0.84		−0.53	0.53		−0.84
		0.32	0.20	−0.20	−0.33	
	0.16		−0.10	0.10		−0.17
		0.06	0.04	−0.04	−0.06	
	0.03		−0.02	0.02		−0.03
		0.01	0.01	−0.01	−0.01	
$\sum M$	146.28	292.57	−292.57	292.57	−292.57	−146.28

Thus from the free-body diagrams:

$A_x = 29.3$ k **Ans.**

$A_y = 96.0$ k **Ans.**

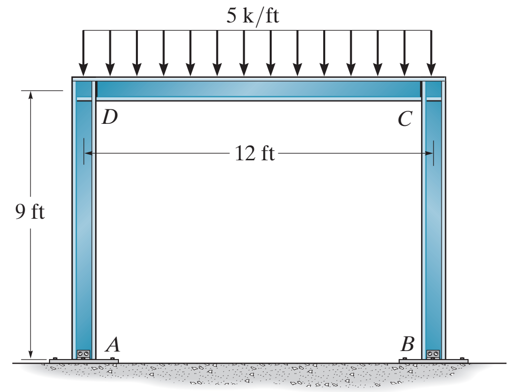
$M_A = 146$ k · ft **Ans.**

$D_x = 29.3$ k **Ans.**

$D_y = 96.0$ k **Ans.**

$M_D = 146$ k · ft **Ans.**

***12–16.** Determine the moments at D and C, then draw the moment diagram for each member of the frame. Assume the supports at A and B are pins and D and C are fixed joints. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{AD} = K_{BC} = \frac{3EI}{L} = \frac{3EI}{9} = \frac{EI}{3} \qquad K_{CD} = \frac{4EI}{L} = \frac{4EI}{12} = \frac{EI}{3}$$

$$(\text{DF})_{AD} = (\text{DF})_{BC} = 1 \qquad (\text{DF})_{DA} = (\text{DF})_{DC} = (\text{DF})_{CD} = \text{DF}_{CB} = \frac{EI/3}{EI/3 + EI/3} = \frac{1}{2}$$

Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{AD} = (\text{FEM})_{DA} = (\text{FEM})_{BC} = (\text{FEM})_{CB} = 0$$

$$(\text{FEM})_{DC} = -\frac{wL_{CD}^2}{12} = -\frac{5(12^2)}{12} = -60 \text{ k}\cdot\text{ft}$$

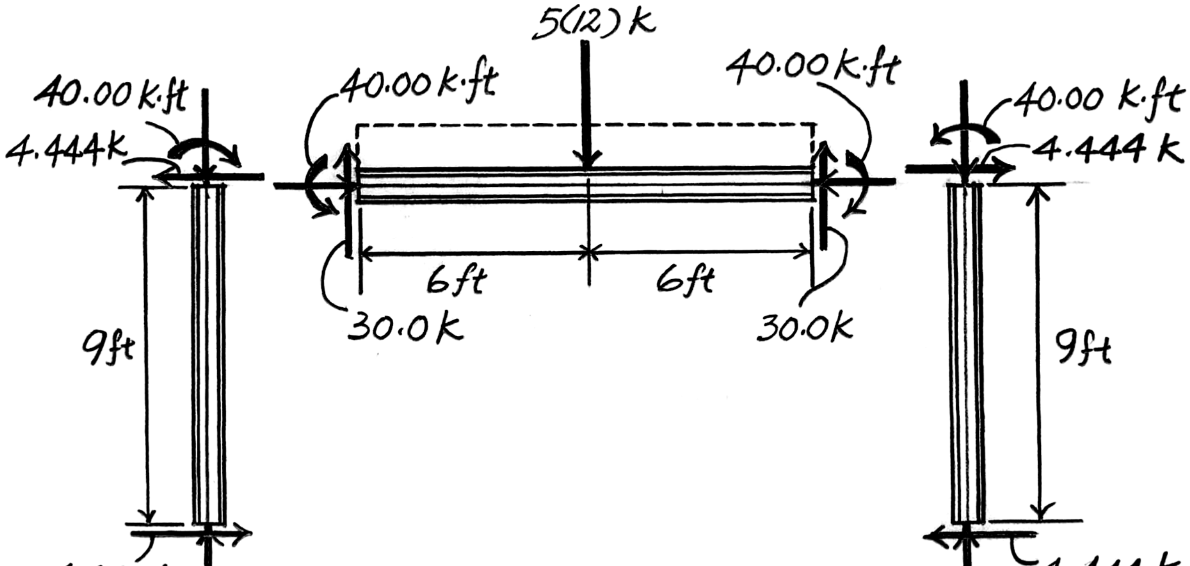
$$(\text{FEM})_{CD} = \frac{wL_{CD}^2}{12} = \frac{5(12^2)}{12} = 60 \text{ k}\cdot\text{ft}$$

Moments Distribution. Tabulating the above data,

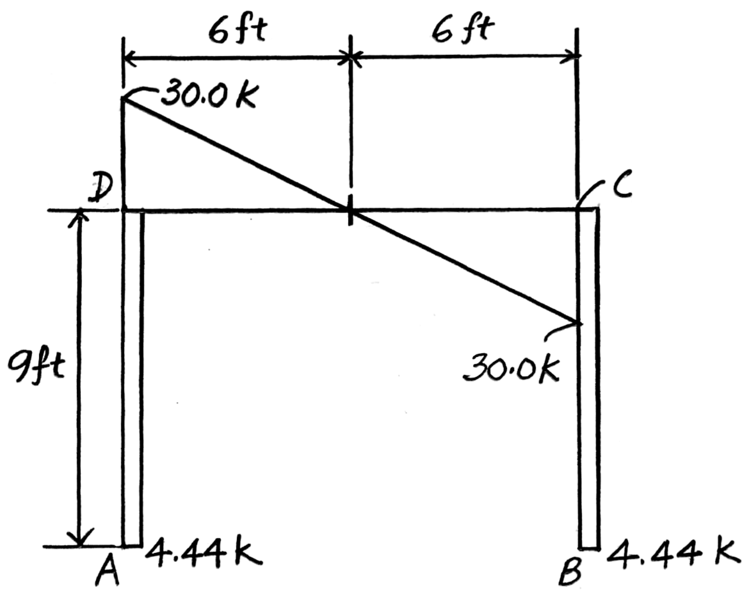
Joint	A	D		C		B
Member	AD	DA	DC	CD	CB	BC
DF	1	0.5	0.5	0.5	0.5	1
FEM	0	0	–60	60	0	0
Dist.		30	30	–30	–30	
CO			–15	15		
Dist.		7.50	7.50	–7.50	–7.50	
C0			–3.75	3.75		
Dist.		1.875	1.875	–1.875	–1.875	
C0			–0.9375	0.9375		
Dist.		0.4688	0.4688	–0.4688	–0.4688	
C0			–0.2344	0.2344		
Dist.		0.1172	0.1172	–0.1172	–0.1172	
C0			–0.0586	0.0586		
Dist.		0.0293	0.0293	–0.0293	–0.0293	
C0			–0.0146	0.0146		
Dist.		0.0073	0.0073	–0.0073	–0.0073	
$\sum M$	0	40.00	–40.00	40.00	–40.00	

Using these results, the shear at both ends of members AD, CD, and BC are computed and shown in Fig. a. Subsequently, the shear and moment diagram can be plotted.

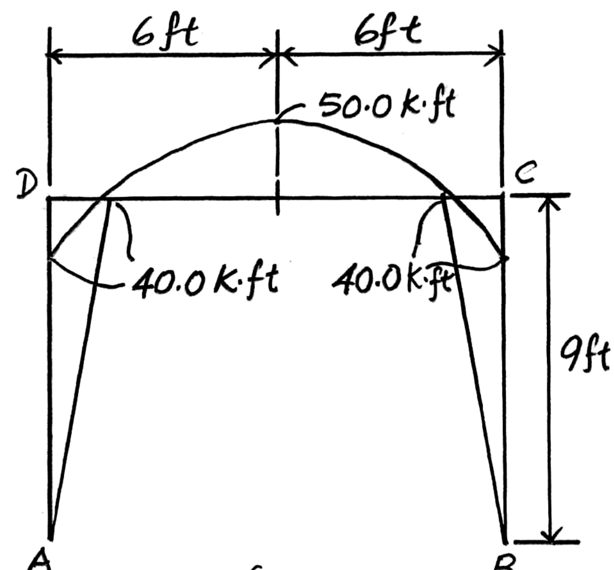
12–16. Continued



(a)

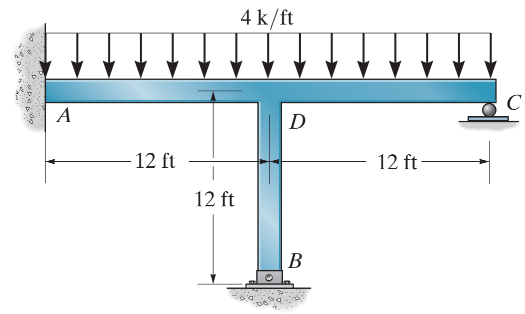


(b)



(c)

12–17. Determine the moments at the fixed support A and joint D and then draw the moment diagram for the frame. Assume B is pinned.



Member Stiffness Factor and Distribution Factor.

$$K_{AD} = \frac{4EI}{L_{AD}} = \frac{4EI}{12} = \frac{EI}{3} \qquad K_{DC} = K_{DB} = \frac{3EI}{L} = \frac{3EI}{12} = \frac{EI}{4}$$

$$(\text{DF})_{AD} = O \qquad (\text{DF})_{DA} = \frac{EI/3}{EI/3 + EI/4 + EI/4} = 0.4$$

$$(\text{DF})_{DC} = (\text{DF})_{DB} = \frac{EI/4}{EI/3 + EI/4 + EI/4} = 0.3$$

$$(\text{DF})_{CD} = (\text{DF})_{BD} = 1$$

Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{AD} = -\frac{wL_{AD}^2}{12} = -\frac{4(12^2)}{12} = -48\ \text{k} \cdot \text{ft}$$

$$(\text{FEM})_{DA} = \frac{wL_{AD}^2}{12} = \frac{4(12^2)}{12} = 48\ \text{k} \cdot \text{ft}$$

$$(\text{FEM})_{DC} = -\frac{wL_{CD}^2}{8} = -\frac{4(12^2)}{8} = -72\ \text{k} \cdot \text{ft}$$

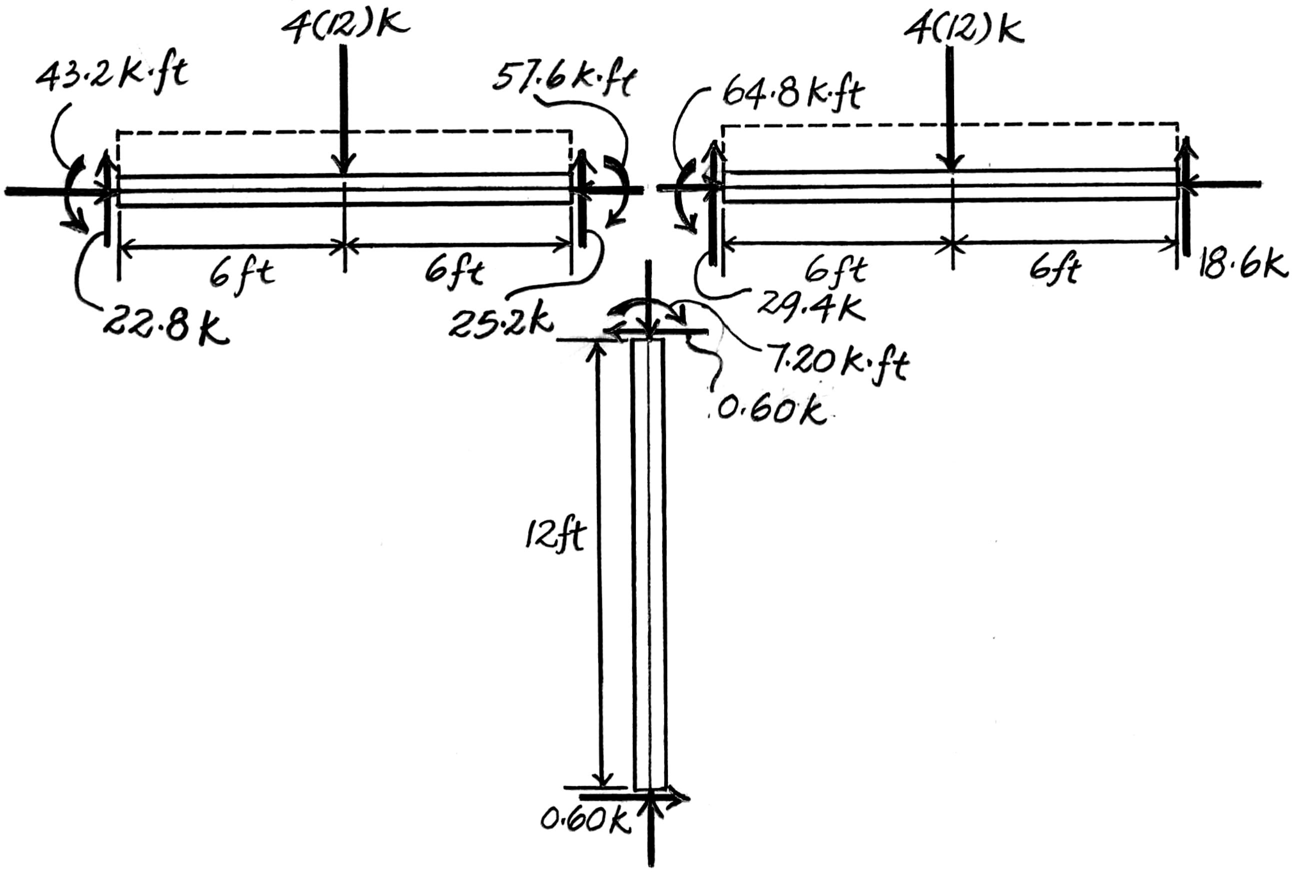
$$(\text{FEM})_{CD} = (\text{FEM})_{BD} = (\text{FEM})_{DB} = 0$$

Moments Distribution. Tabulating the above data,

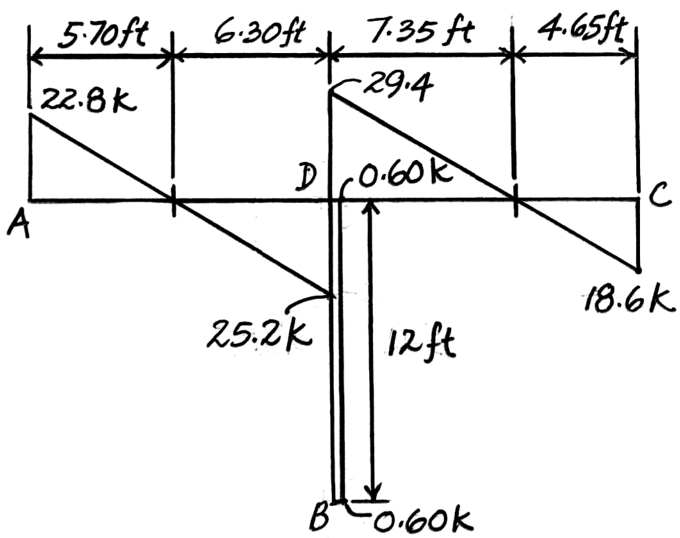
Joint	A	D		C		B
Member	AD	DA	DB	DC	CD	BD
DF	0	0.4	0.3	0.3	1	1
FEM	−48	48	0	−72	0	0
Dist.		9.60	7.20	7.20		
CO	4.80					
$\sum M$	−43.2	57.6	7.20	−64.8	0	0

Using these results, the shears at both ends of members AD, CD, and BD are computed and shown in Fig. *a*. Subsequently, the shear and moment diagram can be plotted, Fig. *b* and *c*, respectively.

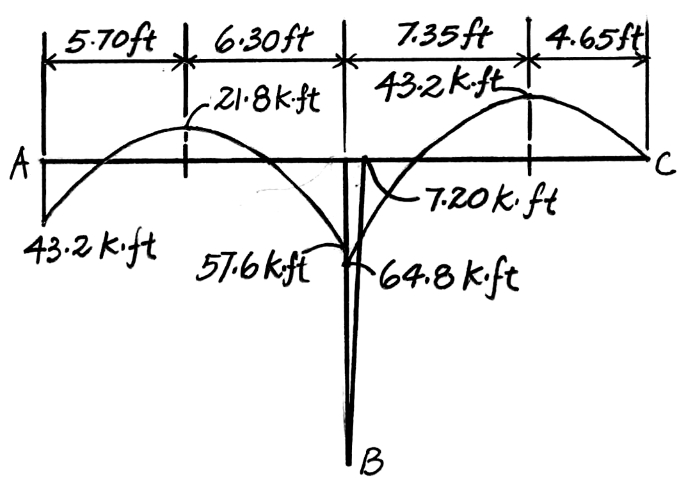
12–17. Continued



(a)

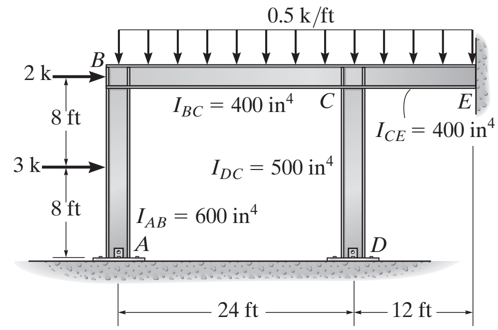


(b)



(c)

12–18. Determine the moments at each joint of the frame, then draw the moment diagram for member BCE. Assume B, C, and E are fixed connected and A and D are pins. $E = 29(10^3)$ ksi.



$(\text{DF})_{AB} = (\text{DF})_{DC} = 1 \quad (\text{DF})_{DC} = 0$

$$(\text{DF})_{BA} = \frac{3(A1.5I_{BC})/16}{3(1.5I_{BC})/16 + 4I_{BC}/24} = 0.6279$$

$(\text{DF})_{BC} = 0.3721$

$$(\text{DF})_{CB} = \frac{4I_{BC}/24}{4I_{BC}/24 + 3(1.25I_{BC})/16 + 4I_{BC}/12} = 0.2270$$

$(\text{DF})_{CD} = 0.3191$

$(\text{DF})_{CE} = 0.4539$

$$(\text{FEM})_{AB} = \frac{-3(16)}{8} = -6\ \text{k}\cdot\text{ft}$$

$(\text{FEM})_{BA} = 6\ \text{k}\cdot\text{ft}$

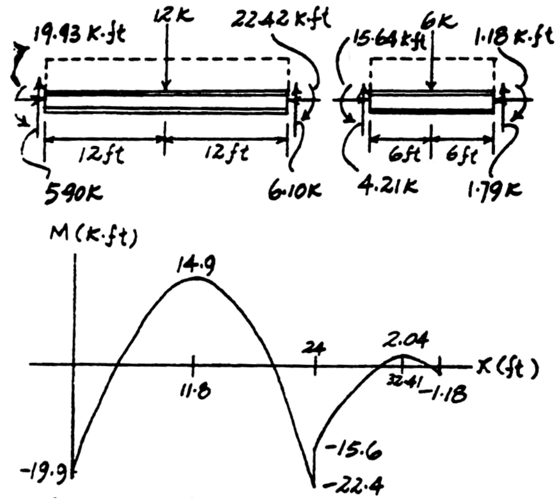
$$(\text{FEM})_{BC} = \frac{-(0.5)(24)^2}{12} = -24\ \text{k}\cdot\text{ft}$$

$(\text{FEM})_{CB} = 24\ \text{k}\cdot\text{ft}$

$$(\text{FEM})_{CE} = \frac{-(0.5)(12)^2}{12} = -6\ \text{k}\cdot\text{ft}$$

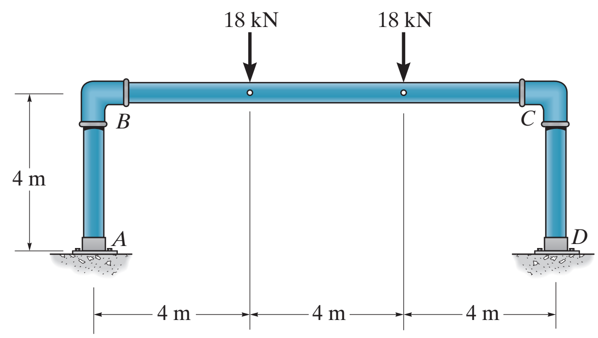
$(\text{FEM})_{EC} = 6\ \text{k}\cdot\text{ft}$

$(\text{FEM})_{CD} = (\text{FEM})_{DC} = 0$



Joint	A	B		C		E		D
Mem.	AB	BA	BC	CB	CD	CE	EC	DC
DF	1	0.6279	0.3721	0.2270	0.3191	0.4539	0	1
FEM	−6.0	6.0	−24.0	24.0		−6.0	6.0	
	6.0	11.30	6.70	−4.09	−5.74	−8.17		
		3.0	−2.04	3.35			−4.09	
		−0.60	−0.36	−0.76	−1.07	−1.52		
			−0.38	−0.18			−0.76	
		0.24	0.14	0.04	0.06	0.08		
			0.02	0.07			0.04	
		−0.01	−0.01	−0.02	−0.02	−0.03		
							−0.02	
$\sum M$	0	19.9	−19.9	22.4	−6.77	−15.6	1.18	0

12–19. The frame is made from pipe that is fixed connected. If it supports the loading shown, determine the moments developed at each of the joints. *EI* is constant.



$$\text{FEM}_{BC} = -\frac{2PL}{9} = -48, \quad \text{FEM}_{CB} = \frac{2PL}{9} = 48$$

$$K_{AB} = K_{CD} = \frac{4EI}{4}, \quad K_{BC} = \frac{4EI}{12}$$

$$\text{DF}_{AB} = \text{DF}_{DC} = 0$$

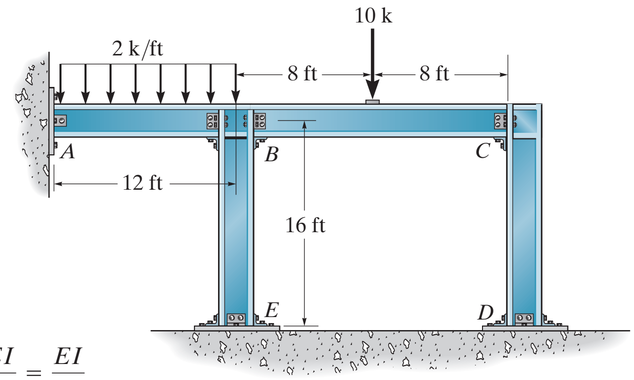
$$\text{DF}_{BA} = \text{DF}_{CD} = \frac{\frac{4EI}{5}}{\frac{4EI}{4} + \frac{4EI}{12}} = 0.75$$

$$\text{DF}_{BC} = \text{DF}_{CB} = 1 - 0.75 = 0.25$$

Joint	*A*	*B*		*C*		*D*
Member	*AB*	*BA*	*BC*	*CB*	*CD*	*DC*
DF	0	0.75	0.25	0.25	0.75	0
FEM			−48	48		
		36	12	−12	−36	
	18		−6	6		−18
		4.5	1.5	−1.5	−4.5	
	2.25		−0.75	0.75		−2.25
		0.5625	0.1875	−0.1875	−0.5625	
	0.281		−0.0938	0.0938		−0.281
		0.0704	0.0234	−0.0234	−0.0704	
	20.6	41.1	−41.1	41.1	−41.1	−20.6

Ans.

***12–20.** Determine the moments at B and C, then draw the moment diagram for each member of the frame. Assume the supports at A, E, and D are fixed. EI is constant.



Member Stiffness Factor and Distribution Factor.

$$K_{AB} = \frac{4EI}{L_{AB}} = \frac{4EI}{12} = \frac{EI}{3} \qquad K_{BC} = K_{BE} = K_{CD} = \frac{4EI}{L} = \frac{4EI}{16} = \frac{EI}{4}$$

$$(\text{DF})_{AB} = (\text{DF})_{EB} = (\text{DF})_{DC} = 0 \quad (\text{DF})_{BA} = \frac{EI/3}{EI/3 + EI/4 + EI/4} = 0.4$$

$$(\text{DF})_{BC} = (\text{DF})_{BE} = \frac{EI/4}{EI/3 + EI/4 + EI/4} = 0.3$$

$$(\text{DF})_{CB} = (\text{DF})_{CD} = \frac{EI/4}{EI/4 + EI/4} = 0.5$$

Fixed End Moments. Referring to the table on the inside back cover,

$$(\text{FEM})_{AB} = -\frac{wL_{AB}^2}{12} = -\frac{2(12^2)}{12} = -24\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{BA} = \frac{wL_{AB}^2}{12} = \frac{2(12^2)}{12} = 24\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = -\frac{PL_{BC}}{8} = -\frac{10(16)}{8} = -20\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{CB} = \frac{PL_{BC}}{8} = \frac{10(16)}{8} = 20\ \text{k}\cdot\text{ft}$$

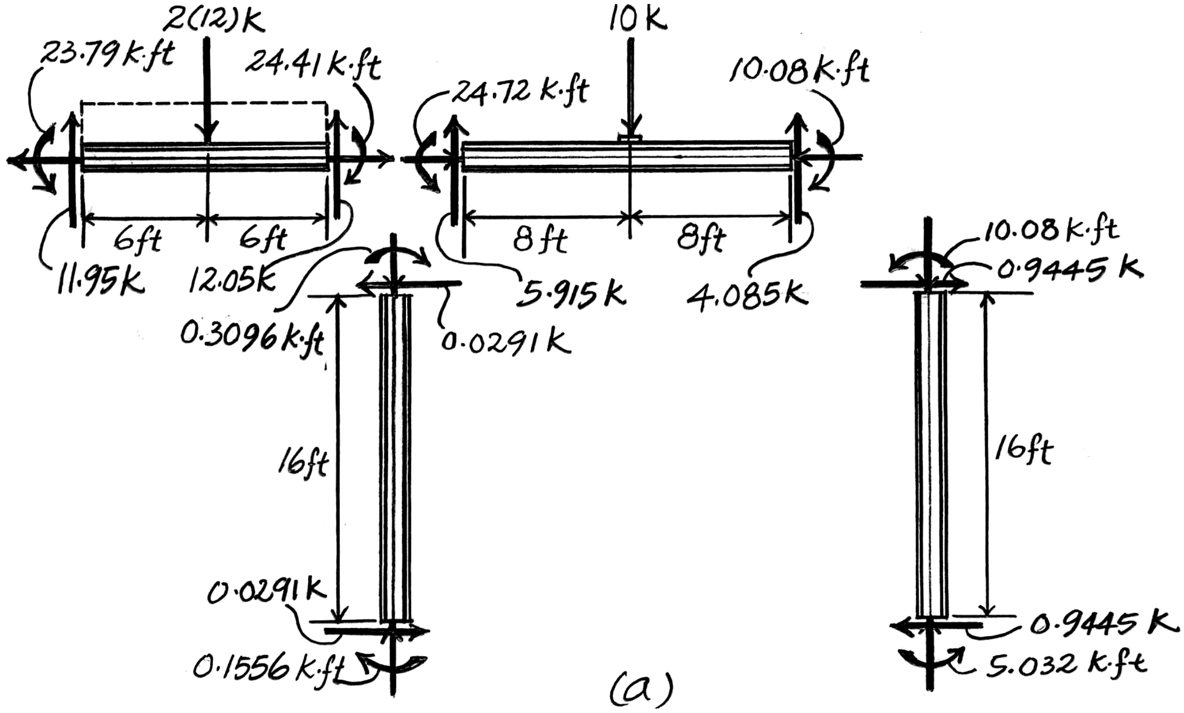
$$(\text{FEM})_{BE} = (\text{FEM})_{EB} = (\text{FEM})_{CD} = (\text{FEM})_{DC} = 0$$

Moment Distribution. Tabulating the above data,

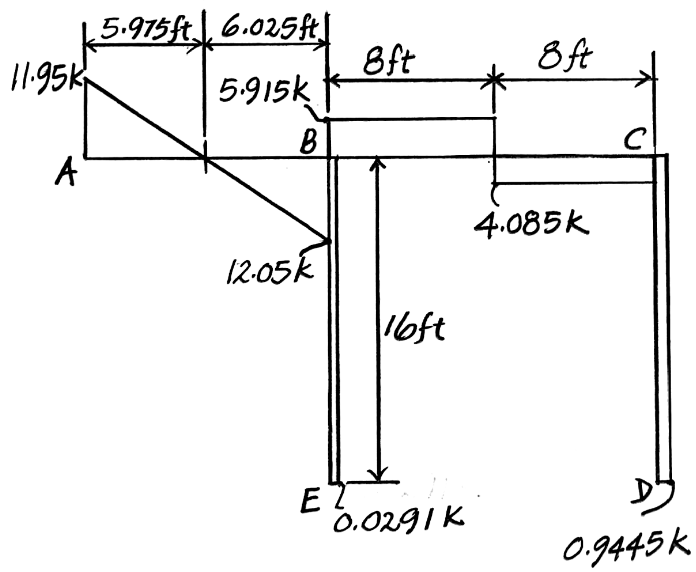
Joint	A	B			C		D	E
Member	AB	BA	BE	BC	CB	CD	DC	EB
DF	0	0.4	0.3	0.3	0.5	0.5	0	0
FEM	–24	24	0	–20	20	0	0	0
Dist.		–1.60	–1.20	–1.20	–10	–10		
CO	–0.80			–5	–0.60		–5	–0.6
Dist.		2.00	1.50	1.50	0.30	0.30		
CO	1.00			0.15	0.75		0.15	0.75
Dist.		–0.06	–0.045	–0.045	–0.375	–0.375		
CO	–0.03			–0.1875	–0.0225		–0.1875	–0.0225
Dist.		0.075	0.05625	0.05625	0.01125	0.01125		
CO	0.0375			0.005625	0.028125		0.005625	0.028125
Dist.		–0.00225	–0.0016875	–0.0016875	–0.01406	–0.01406		
$\sum M$	–23.79	24.41	0.3096	–24.72	10.08	–10.08	–5.031	0.1556

Using these results, the shear at both ends of members AB, BC, BE, and CD are computed and shown in Fig. a. Subsequently, the shear and moment diagram can be plotted.

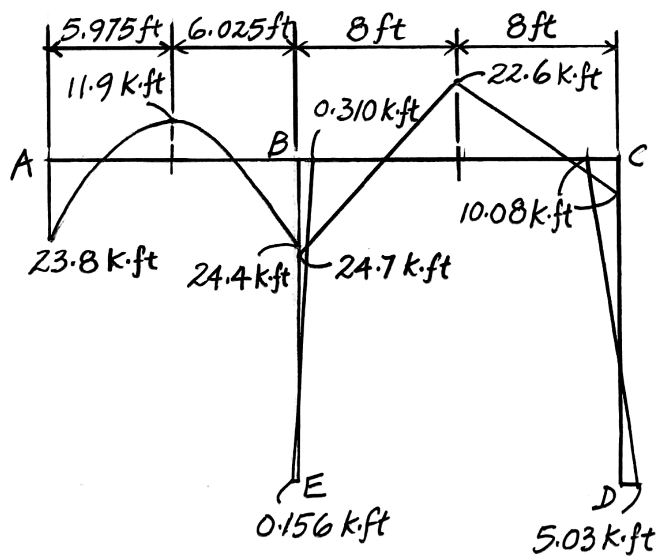
12–20. Continued



(a)

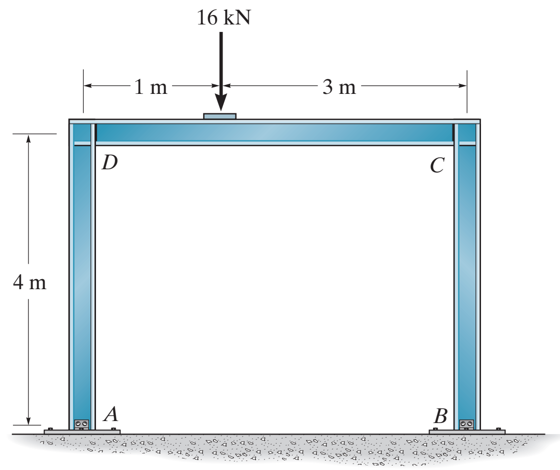


(b)



(c)

12–21. Determine the moments at D and C, then draw the moment diagram for each member of the frame. Assume the supports at A and B are pins. EI is constant.



Moment Distribution. No sidesway, Fig. *b*.

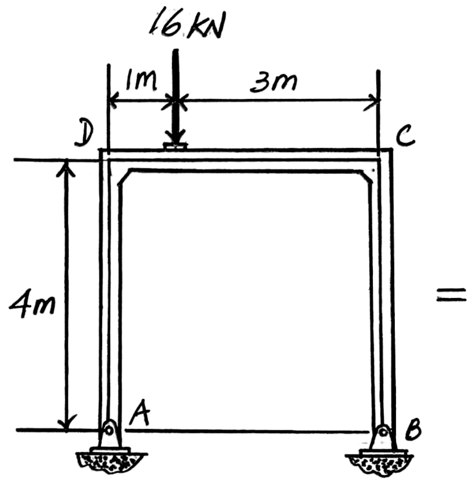
$$K_{DA} = K_{CB} = \frac{3EI}{L} = \frac{3EI}{4} \qquad K_{CD} = \frac{4EI}{L} = \frac{4EI}{4} = EI$$

$$(\text{DF})_{AD} = (\text{DF})_{BC} = 1 \quad (\text{DF})_{DA} = (\text{DF})_{CB} = \frac{3EI/4}{3EI/4 + EI} = \frac{3}{7}$$

$$(\text{DF})_{DC} = (\text{DF})_{CD} = \frac{EI}{3EI/4 + EI} = \frac{4}{7}$$

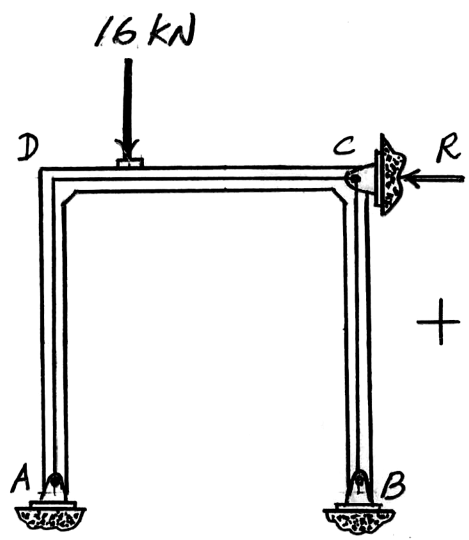
$$(\text{FEM})_{DC} = -\frac{Pb^2a}{L^2} = -\frac{16(3^2)(1)}{4^2} = -9 \text{ kN}\cdot\text{m}$$

$$(\text{FEM})_{CD} = -\frac{Pa^2b}{L^2} = -\frac{16(1^2)(3)}{4^2} = 3 \text{ kN}\cdot\text{m}$$



(a)

Joint	A	D		C		B
Member	AD	DA	DC	CD	CB	BC
DF	1	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{4}{7}$	$\frac{3}{7}$	1
FEM	0	0	–9	3	0	0
Dist.		3.857	5.143	–1.714	–1.286	
CO			–0.857	2.572		
Dist.		0.367	0.490	–1.470	–1.102	
CO			–0.735	0.245		
Dist.		0.315	0.420	–0.140	–0.105	
CO			–0.070	0.210		
Dist.		0.030	0.040	–0.120	–0.090	
CO			–0.060	0.020		
Dist.		0.026	0.034	–0.011	–0.009	
CO			–0.006	0.017		
Dist.		0.003	0.003	–0.010	–0.007	
$\sum M$	0	4.598	–4.598	2.599	–2.599	0



(b)

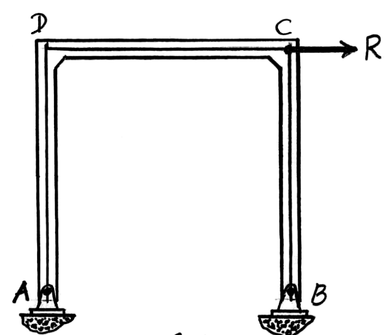
Using these results, the shears at A and B are computed and shown in Fig. *d*. Thus, for the entire frame

$$\xrightarrow{+} \Sigma F_x = 0; \quad 1.1495 - 0.6498 - R = 0 \quad R = 0.4997 \text{ kN}$$

12–21. Continued

For the frame in Fig. *e*,

Joint	*A*	*D*		*C*		*B*
Member	*AD*	*DA*	*DC*	*CD*	*CB*	*BC*
DF	1	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{4}{7}$	$\frac{3}{7}$	1
FEM	0	–10	0	0	–10	0
Dist.		4.286	5.714	5.714	4.286	
CO			2.857	2.857		
Dist.		–1.224	–1.633	–1.633	–1.224	
CO			–0.817	–0.817		
Dist.		0.350	0.467	0.467	0.350	
CO			0.234	0.234		
Dist.		–0.100	–0.134	–0.134	–0.100	
CO			–0.067	–0.067		
Dist.		0.029	0.038	0.038	0.029	
CO			0.019	0.019		
Dist.		–0.008	–0.011	–0.011	–0.008	
$\sum M$	0	–6.667	6.667	6.667	–6.667	0



(c)

Using these results, the shears at *A* and *B* caused by the application of *R′* are computed and shown in Fig. *f*. For the entire frame,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad R'1.667 - 1.667 = 0 \quad R' = 3.334 \text{ kN}$

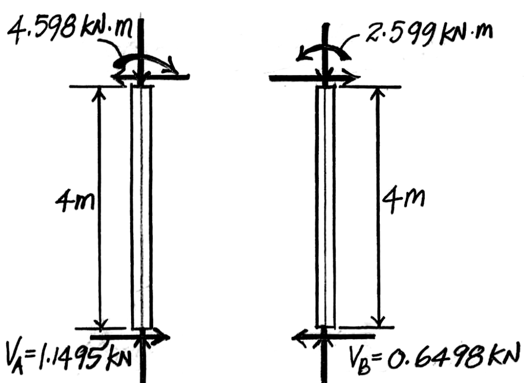
Thus,

$$M_{DA} = 4.598 + (-6.667)\left(\frac{0.4997}{3.334}\right) = 3.60 \text{ kN} \cdot \text{m}$$ **Ans.**

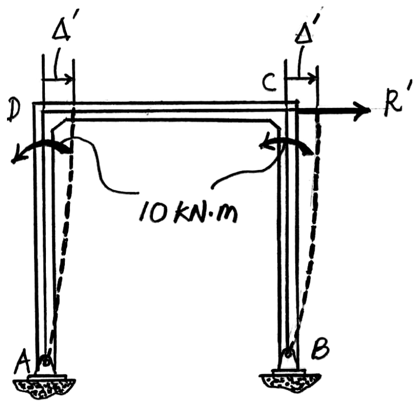
$$M_{DC} = -4.598 + (6.667)\left(\frac{0.4997}{3.334}\right) = -3.60 \text{ kN} \cdot \text{m}$$ **Ans.**

$$M_{CD} = 2.599 + (6.667)\left(\frac{0.4997}{3.334}\right) = -3.60 \text{ kN} \cdot \text{m}$$ **Ans.**

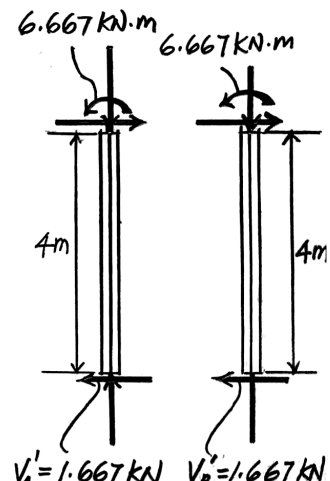
$$M_{CB} = 2.599 + (-6.667)\left(\frac{0.4997}{3.334}\right) = -3.60 \text{ kN} \cdot \text{m}$$ **Ans.**



(d)

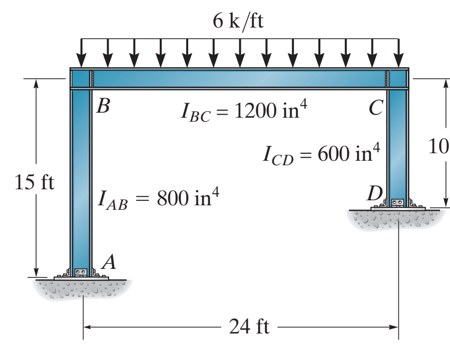


(e)



(f)

12–22. Determine the moments acting at the ends of each member. Assume the supports at A and D are fixed. The moment of inertia of each member is indicated in the figure. $E = 29(10^3)$ ksi.



Consider no sideway

$$(\text{DF})_{AB} = (\text{DF})_{DC} = 0$$

$$(\text{DF})_{BA} = \frac{(\frac{1}{12}I_{BC})/15}{(\frac{1}{12}I_{BC})/15 + I_{BC}/24} = 0.5161$$

$$(\text{DF})_{BC} = 0.4839$$

$$(\text{DF})_{CB} = \frac{I_{BC}/24}{0.5I_{BC}/10 + I_{BC}/24} = 0.4545$$

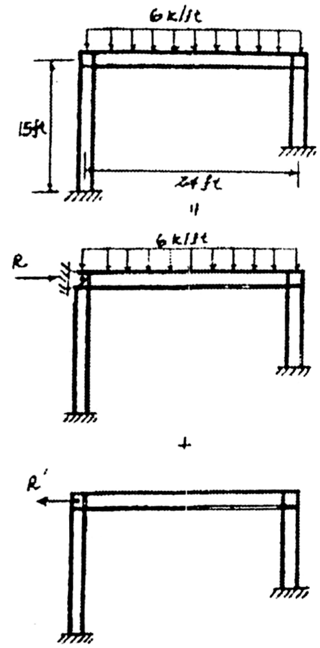
$$(\text{DF})_{CD} = 0.5455$$

$$(\text{FEM})_{AB} = (\text{FEM})_{BA} = 0$$

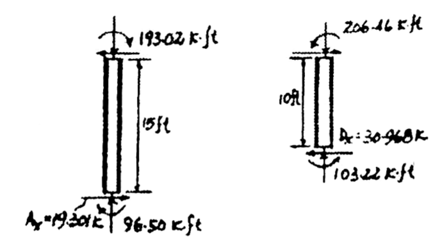
$$(\text{FEM})_{BC} = \frac{-6(24)^2}{12} = -288\ \text{k}\cdot\text{ft}$$

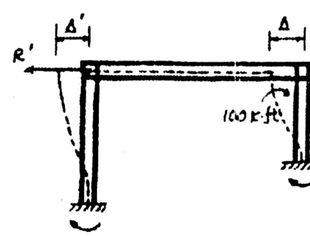
$$(\text{FEM})_{CB} = 288\ \text{k}\cdot\text{ft}$$

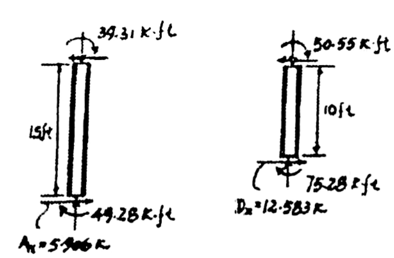
$$(\text{FEM})_{CD} = (\text{FEM})_{DC} = 0$$



Joint	A	B		C		D
Mem.	AB	BA	BC	CB	CD	DC
DF	0	0.5161	0.4839	0.4545	0.5455	0
FEM			−288	288		
		148.64	139.36	−130.90	−157.10	
	74.32		−65.45	69.68		−78.55
		33.78	31.67	−31.67	38.01	
	16.89		−15.84	15.84		−19.01
		8.18	7.66	−7.20	−8.64	
	4.09		−3.60	3.83		−4.32
		1.86	1.74	−1.74	−2.09	
	0.93		−0.87	0.87		−1.04
		0.45	0.42	−0.40	−0.47	
	0.22		0.20	0.21		−0.24
		0.10	0.10	−0.10	−0.11	
	0.05		−0.05	0.05		−0.06
		0.02	0.02	−0.02	−0.03	
$\sum M$	96.50	193.02	−193.02	206.46	−206.46	−103.22







12–22. Continued

$\xrightarrow{+} \sum F_x = 0$ (for the frame without sideway)

$R + 19.301 - 30.968 = 0$

$R = 11.666 \text{ k}$

$$(\text{FEM})_{CD} = (\text{FEM})_{DC} = 100 = \frac{6E(0.75I_{AB})\Delta'}{10^2}$$

$$\Delta' = \frac{100(10^2)}{6E(0.75I_{AB})}$$

$$(\text{FEM})_{AB} = (\text{FEM})_{BA} = \frac{6EI_{AB}\Delta'}{15^2} = \left(\frac{6EI_{AB}}{15^2}\right)\left(\frac{100(10^2)}{6E(0.75I_{AB})}\right) = 59.26 \text{ k} \cdot \text{ft}$$

Joint	A	B		C		D
Mem.	AB	BA	BC	CB	CD	DC
DF	0	0.5161	0.4839	0.4545	0.5455	0
FEM	59.26	59.26			100	100
		−30.58	−28.68	−45.45	−54.55	
	−15.29		−22.73	−14.34		−27.28
		11.73	11.00	6.52	7.82	
	5.87		3.26	5.50		3.91
		−1.68	−1.58	−2.50	−3.00	
	−0.84		−125	−0.79		−1.50
		0.65	0.60	0.36	0.43	
	0.32		0.18	0.30		0.22
		−0.09	−0.09	−0.14	−0.16	
	−0.05		−0.07	−0.04		−0.08
		0.04	0.03	0.02	0.02	
	0.02		0.01	0.02		0.01
$\sum M$	49.28	39.31	−39.31	−50.55	50.55	75.28

$R' = 5.906 + 12.585 = 18.489 \text{ k}$

$$M_{AB} = 96.50 + \left(\frac{11.666}{18.489}\right)(49.28) = 128 \text{ k} \cdot \text{ft}$$ **Ans.**

$$M_{BA} = 193.02 - \left(\frac{11.666}{18.489}\right)(39.31) = 218 \text{ k} \cdot \text{ft}$$ **Ans.**

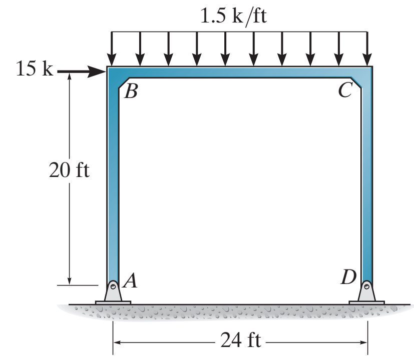
$$M_{BC} = -193.02 + \left(\frac{11.666}{18.489}\right)(-39.31) = 218 \text{ k} \cdot \text{ft}$$ **Ans.**

$$M_{CB} = 206.46 - \left(\frac{11.666}{18.489}\right)(-50.55) = 175 \text{ k} \cdot \text{ft}$$ **Ans.**

$$M_{CD} = -206.46 + \left(\frac{11.666}{18.489}\right)(50.55) = 175 \text{ k} \cdot \text{ft}$$ **Ans.**

$$M_{DC} = -103.21 + \left(\frac{11.666}{18.489}\right)(75.28) = -55.7 \text{ k} \cdot \text{ft}$$ **Ans.**

12–23. Determine the moments acting at the ends of each member of the frame. EI is the constant.



Consider no sideway

$(\text{DF})_{AB} = (\text{DF})_{DC} = 1$

$$(\text{DF})_{BA} = (\text{DF})_{CD} = \frac{3I/20}{3I/20 + 4I/24} = 0.4737$$

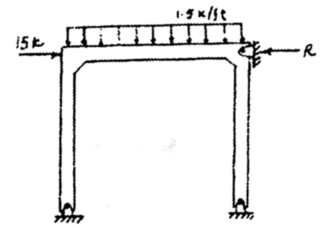
$(\text{DF})_{BC} = (\text{DF})_{CB} = 0.5263$

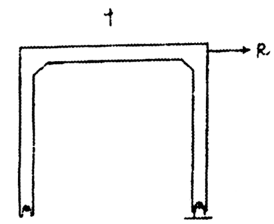
$(\text{FEM})_{AB} = (\text{FEM})_{BA} = 0$

$$(\text{FEM})_{BC} = \frac{-1.5(24)^2}{12} = -72\ \text{k}\cdot\text{ft}$$

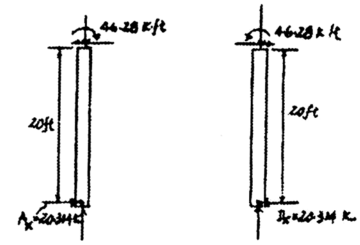
$(\text{FEM})_{CB} = 72\ \text{k}\cdot\text{ft}$

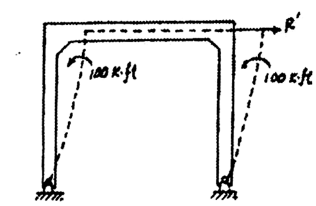
$(\text{FEM})_{CD} = (\text{FEM})_{DC} = 0$

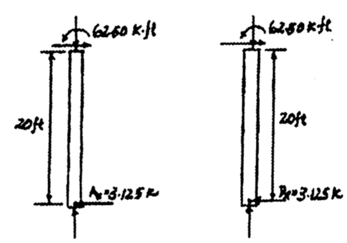




Joint	*A*	*B*		*C*		*D*
Member	*AB*	*BA*	*BC*	*CB*	*CD*	*DC*
DF	1	0.4737	0.5263	0.5263	0.4737	1
FEM			−72.0	72.0		
		34.41	37.89	−37.89	−34.11	
			−18.95	18.95		
		8.98	9.97	−9.97	−8.98	
			−4.98	4.98		
		2.36	2.62	−2.62	−2.36	
			−1.31	1.31		
		0.62	0.69	−0.69	−0.62	
			−0.35	0.35		
		0.16	0.18	−0.18	−0.16	
			−0.09	0.09		
		0.04	0.05	−0.05	−0.04	
			−0.02	0.02		
		0.01	0.01	−0.01	−0.01	
$\sum M$		46.28	−46.28	46.28	−46.28	







12–23. Continued

$\overset{+}{\leftarrow} \Sigma F_x = 0$ (for the frame without sidesway)

$R + 2.314 - 2.314 - 15 = 0$

$R = 15.0 \text{ k}$

Joint	A	B		C		D
Mem.	AB	BA	BC	CB	CD	DC
DF	1	0.4737	0.5263	0.5263	0.4737	1
FEM		–100			–100	
		47.37	52.63	52.63	47.37	
			26.32	26.32		
		–12.47	–13.85	–13.85	–12.47	
			–6.93	–6.93		
		3.28	3.64	3.64	3.28	
			1.82	1.82		
		–0.86	–0.96	–0.96	–0.86	
			–0.48	–0.48		
		0.23	0.25	0.25	0.23	
			0.13	0.13		
		–0.06	–0.07	–0.07	–0.06	
			–0.03	–0.03		
		0.02	0.02	0.02	0.02	
		–62.50	62.50	62.50	–62.50	

$R' = 3.125 + 3.125 = 6.25 \text{ k}$

$M_{BA} = 46.28 + \left(\frac{15}{6.25}\right)(-62.5) = -104 \text{ k} \cdot \text{ft}$ **Ans.**

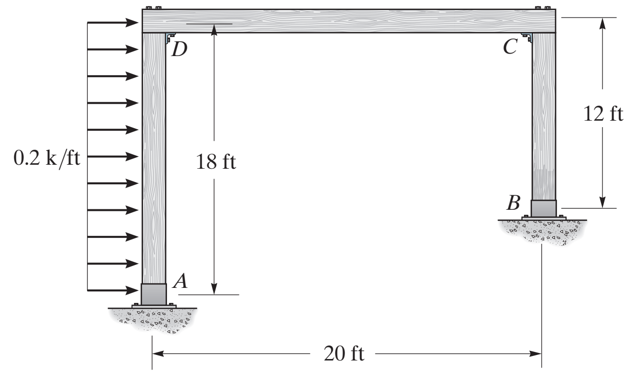
$M_{BC} = -46.28 + \left(\frac{15}{6.25}\right)(62.5) = 104 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CB} = 46.28 + \left(\frac{15}{6.25}\right)(62.5) = 196 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CD} = -46.28 + \left(\frac{15}{6.25}\right)(-62.5) = -196 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{AB} = M_{DC} = 0$ **Ans.**

***12–24.** Determine the moments acting at the ends of each member. Assume the joints are fixed connected and A and B are fixed supports. EI is constant.



Moment Distribution. No sidesway, Fig. b,

$$K_{AD} = \frac{4EI}{L_{AD}} = \frac{4EI}{18} = \frac{2EI}{9} \qquad K_{CD} = \frac{4EI}{L_{CD}} = \frac{4EI}{20} = \frac{EI}{5}$$

$$K_{BC} = \frac{4EI}{L_{BC}} = \frac{4EI}{12} = \frac{EI}{3}$$

$$(\text{DF})_{AD} = (\text{DF})_{BC} = 0 \qquad (\text{DF})_{DA} = \frac{2EI/59}{2EI/9 + EI/5} = \frac{10}{9}$$

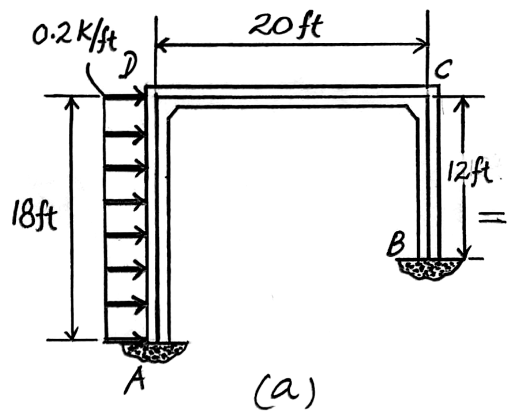
$$(\text{DF})_{DC} = \frac{EI/5}{2EI/9 + EI/5} = \frac{9}{19}$$

$$(\text{DF})_{CD} = \frac{EI/5}{EI/5 + EI/3} = \frac{3}{8} \qquad (\text{DF})_{CB} = \frac{EI/3}{EI/5 + EI/3} = \frac{5}{8}$$

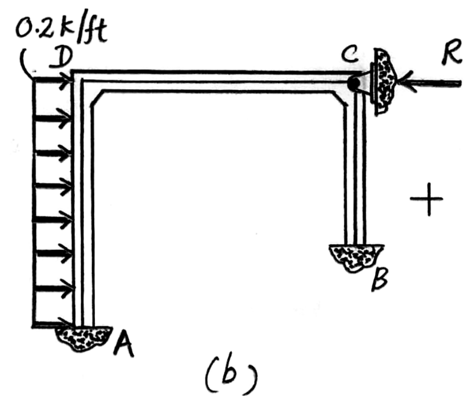
$$(\text{FEM})_{AD} = -\frac{wL_{AD}^2}{12} = -\frac{0.2(18^2)}{12} = -5.40\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{DA} = \frac{wL_{AD}^2}{12} = \frac{0.2(18^2)}{12} = 5.40\ \text{k}\cdot\text{ft}$$

$$(\text{FEM})_{DC} = (\text{FEM})_{CD} = (\text{FEM})_{CB} = (\text{FEM})_{BC} = 0$$



Joint	A	D		C		B
Member	AD	DA	DC	CD	CB	BC
DF	0	$\frac{10}{19}$	$\frac{9}{19}$	$\frac{3}{8}$	$\frac{5}{8}$	0
FEM	–5.40	5.40	0	0	0	0
Dist.		–2.842	–2.558			
CO	–1.421			–1.279		
Dist.				0.480	0.799	
CO			0.240			0.400
Dist.		–0.126	–0.114			
CO	–0.063			–0.057		
Dist.				0.021	0.036	
CO			0.010			0.018
Dist.		–0.005	–0.005			
$\sum M$	–6.884	2.427	–2.427	–0.835	0.835	0.418



12–24. Continued

Using these results, the shears at A and B are computed and shown in Fig. d. Thus, for the entire frame,

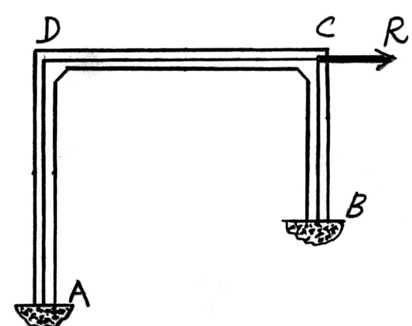
$\xrightarrow{+} \Sigma F_x = 0; \quad 0.2(18) + 0.104 - 2.048 - R = 0 \quad R = 1.656 \text{ k}$

For the frame in Fig. e,

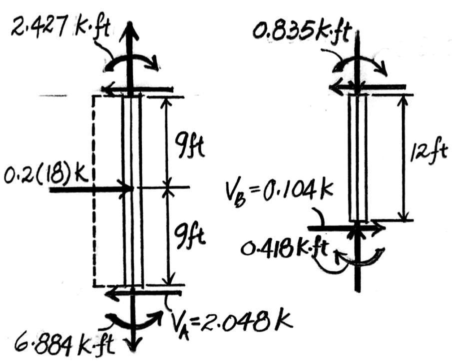
$(\text{FEM})_{BC} = (\text{FEM})_{CB} = -10 \text{ k} \cdot \text{ft}; \quad -\dfrac{6EI\Delta'}{L^2} = -10 \quad \Delta' = \dfrac{240}{EI}$

$(\text{FEM})_{AD} = (\text{FEM})_{DA} = -\dfrac{6EI\Delta'}{L^2} = -\dfrac{6EI(240/EI)}{18^2} = -4.444 \text{ k} \cdot \text{ft}$

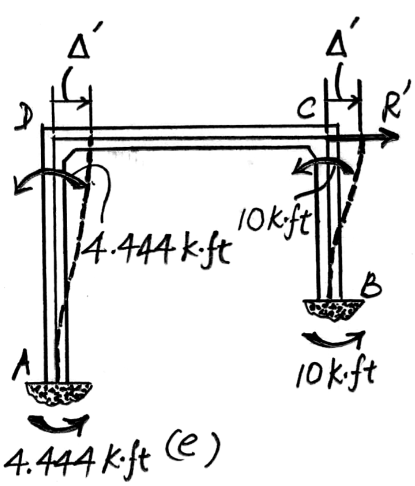
Joint	A	D		C		B
Member	AD	DA	DC	CD	CB	BC
DF	0	$\frac{10}{19}$	$\frac{9}{19}$	$\frac{3}{8}$	$\frac{5}{8}$	0
FEM	–4.444	–4.444			–10	–10
Dist.		2.339	2.105	3.75	6.25	
CO	1.170		1.875	1.053		3.125
Dist.		–0.987	–0.888	–0.395	–0.658	
CO	–0.494		–0.198	–0.444		–0.329
Dist.		0.104	0.094	0.767	0.277	
CO	0.052		0.084	0.047		0.139
Dist.		0.044	–0.040	–0.018	–0.029	
CO	–0.022		–0.009	–0.020		–0.015
Dist.		0.005	0.004	0.008	0.012	
CO	0.003		0.004	0.002		0.006
Dist.		–0.002	–0.002	–0.001	–0.001	
$\sum M$	–3.735	–3.029	3.029	4.149	–4.149	–7.074



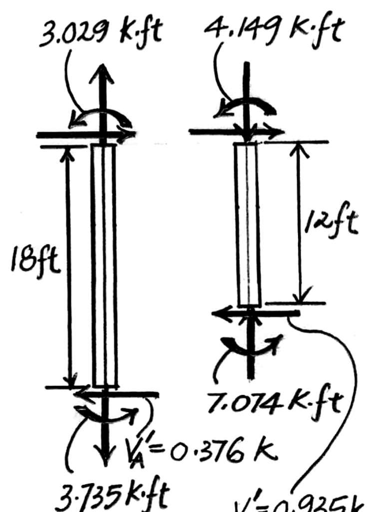
(c)



(d)



(e)



(f)

12–24. Continued

Using these results, the shears at both ends of members AD and BC are computed and shown in Fig. f. For the entire frame,

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad R' - 0.376 - 0.935 = 0 \quad R' = 1.311 \text{ k}$$

Thus,

$$M_{AD} = -6.884 + \left(\frac{1.656}{1.311}\right)(-3.735) = 11.6 \text{ k}\cdot\text{ft}$$ **Ans.**

$$M_{DA} = 2.427 + \left(\frac{1.656}{1.311}\right)(-3.029) = -1.40 \text{ k}\cdot\text{ft}$$ **Ans.**

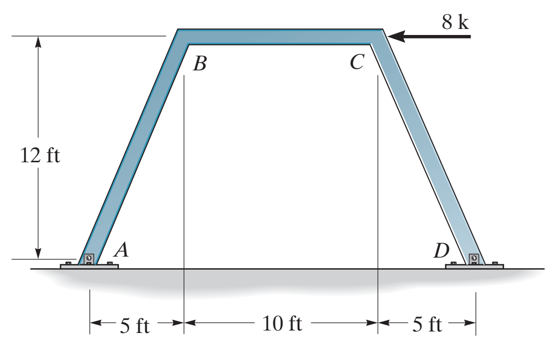
$$M_{DC} = -2.427 + \left(\frac{1.656}{1.311}\right)(3.029) = 1.40 \text{ k}\cdot\text{ft}$$ **Ans.**

$$M_{CD} = -0.835 + \left(\frac{1.656}{1.311}\right)(4.149) = 4.41 \text{ k}\cdot\text{ft}$$ **Ans.**

$$M_{CB} = 0.835 + \left(\frac{1.656}{1.311}\right)(-4.149) = -4.41 \text{ k}\cdot\text{ft}$$ **Ans.**

$$M_{CD} = 0.418 + \left(\frac{1.656}{1.311}\right)(-7.074) = -8.52 \text{ k}\cdot\text{ft}$$ **Ans.**

12–25. Determine the moments at joints B and C, then draw the moment diagram for each member of the frame. The supports at A and D are pinned. EI is constant.



Moment Distribution. For the frame with **P** acting at C, Fig. a,

$$K_{AB} = K_{CD} = \frac{3EI}{L} = \frac{3EI}{13} \quad K_{BC} = \frac{4EI}{10} = \frac{2EI}{5}$$

$$(\text{DF})_{AB} = (\text{DF})_{DC} = 1 \quad (\text{DF})_{BA} = (\text{DF})_{CD} = \frac{3EI/13}{3EI/13 + 2EI/5} = \frac{15}{41}$$

$$(\text{DF})_{BC} = (\text{DF})_{CB} = \frac{2EI/5}{3EI/13 + 2EI/5} = \frac{26}{41}$$

$$(\text{FEM})_{BA} = (\text{FEM})_{CD} = 100 \text{ k}\cdot\text{ft}; \quad \frac{3EI\Delta'}{L^2} = 100 \quad \Delta' = \frac{16900}{3EI}$$

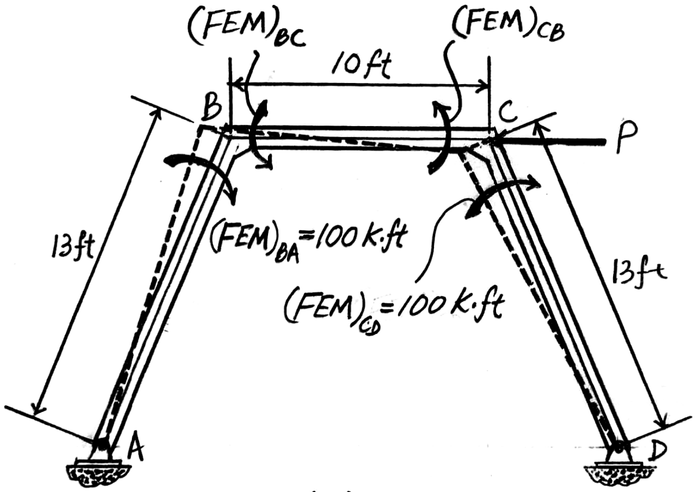
From the geometry shown in Fig. b,

$$\Delta'_{BC} = \frac{5}{13}\Delta' + \frac{5}{13}\Delta' = \frac{10}{13}\Delta'$$

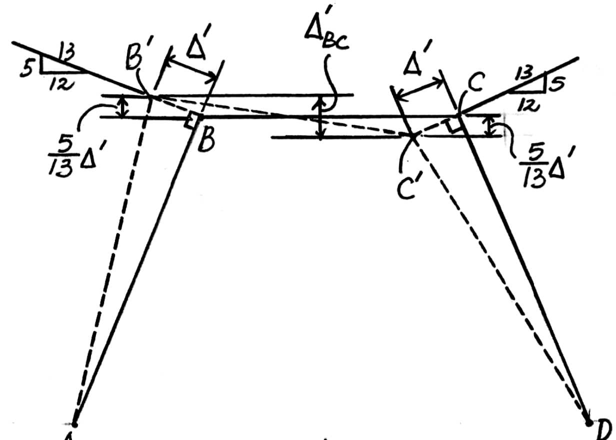
Thus

$$(\text{FEM})_{BC} = (\text{FEM})_{CB} = -\frac{6EI\Delta'_{BC}}{L_{BC}^2} = -\frac{6EI\left(\frac{10}{13}\right)\left(\frac{16900}{3EI}\right)}{10^2} = -260 \text{ k}\cdot\text{ft}$$

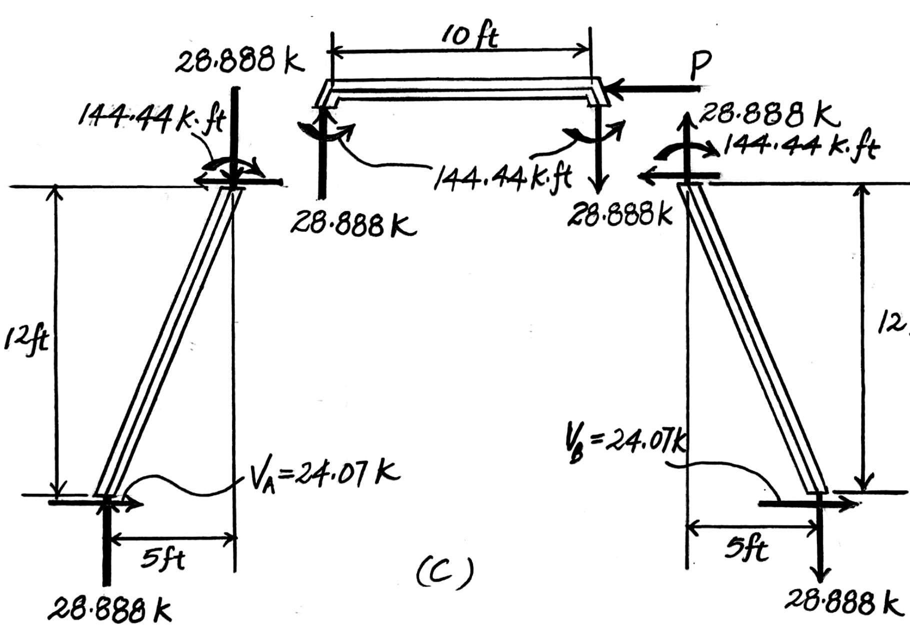
12–25. Continued



(a)



(b)



(c)

12–25. Continued

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	1	15/41	26/41	26/41	15/41	1
FEM	0	100	−260	−260	100	0
Dist.		58.54	101.46	101.46	58.54	
CO			50.73	50.73		
Dist.		18.56	−32.17	−32.17	−18.56	
CO			−16.09	−16.09		
Dist.		5.89	10.20	10.20	5.89	
CO			5.10	5.10		
Dist.		−1.87	−3.23	−3.23	−1.87	
CO			−1.62	−1.62		
Dist.		0.59	1.03	1.03	0.59	
CO			0.51	0.51		
Dist.		−0.19	−0.32	−0.32	−0.19	
CO			−0.16	−0.16		
Dist.		0.06	0.10	0.10	0.06	
CO			0.05	0.05		
Dist.		−0.02	−0.03	−0.03	−0.02	
$\sum M$	0	144.44	−144.44	−144.44	−144.44	0

Using these results, the shears at A and D are computed and shown in Fig. *c*. Thus for the entire frame,

$\xrightarrow{+} \sum F_x = 0; \quad 24.07 + 24.07 - P = 0 \quad P = 48.14\text{ k}$

Thus, for **P** = 8 k,

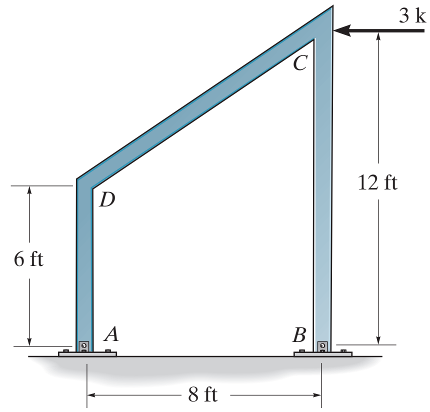
$M_{BA} = \left(\frac{8}{48.14}\right)(144.44) = 24.0\text{ k}\cdot\text{ft}$ **Ans.**

$M_{BC} = \left(\frac{8}{48.14}\right)(-144.44) = -24.0\text{ k}\cdot\text{ft}$ **Ans.**

$M_{CB} = \left(\frac{8}{48.14}\right)(-144.44) = -24.0\text{ k}\cdot\text{ft}$ **Ans.**

$M_{CD} = \left(\frac{8}{48.14}\right)(144.44) = 24.0\text{ k}\cdot\text{ft}$ **Ans.**

12–26. Determine the moments at C and D, then draw the moment diagram for each member of the frame. Assume the supports at A and B are pins. EI is constant.



Moment Distribution. For the frame with **P** acting at C, Fig. a,

$$K_{AD} = \frac{3EI}{L_{AD}} = \frac{3EI}{6} = \frac{EI}{2} \qquad K_{BC} = \frac{3EI}{L_{BC}} = \frac{3EI}{12} = \frac{EI}{4}$$

$$K_{CD} = \frac{4EI}{L_{CD}} = \frac{4EI}{10} = \frac{2EI}{5}$$

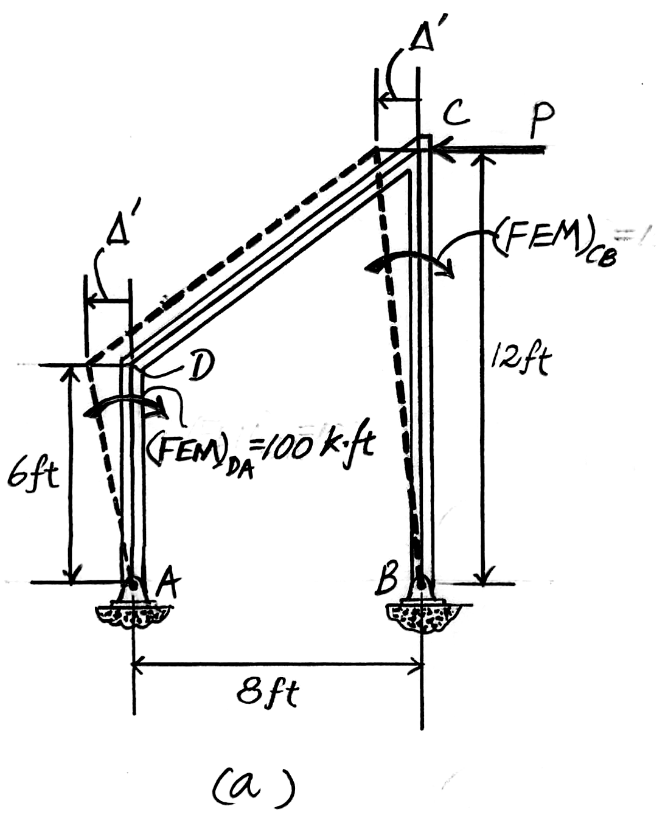
$$(\text{DF})_{AD} = (\text{DF})_{BC} = 1 \quad (\text{DF})_{DA} = \frac{EI/2}{EI/2 + 2EI/5} = \frac{5}{9}$$

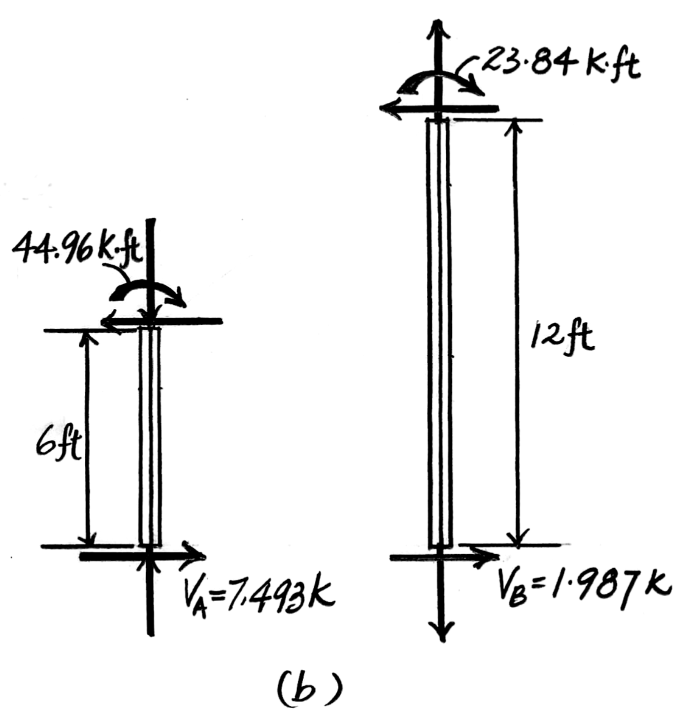
$$(\text{DF})_{DC} = \frac{2EI/5}{EI/2 + 2EI/5} = \frac{4}{9}$$

$$(\text{DF})_{CD} = \frac{2EI/5}{2EI/5 + EI/4} = \frac{8}{13} \quad (\text{DF})_{CB} = \frac{EI/4}{2EI/5 + EI/4} = \frac{5}{13}$$

$$(\text{FEM})_{DA} = 100\ \text{k}\cdot\text{ft}; \quad \frac{3EI\Delta'}{L_{DA}^2} = 100 \quad \Delta' = \frac{1200}{EI}$$

$$(\text{FEM})_{CB} = \frac{3EI\Delta'}{L_{CB}^2} = \frac{3EI(1200/EI)}{12^2} = 25\ \text{k}\cdot\text{ft}$$





12–26. Continued

Joint	A	D		C		B
Member	AD	DA	DC	CD	CB	BC
DF	1	$\frac{5}{9}$	$\frac{4}{9}$	$\frac{8}{13}$	$\frac{5}{13}$	1
FEM	0	100	0	0	25	0
Dist.		−55.56	−44.44	−15.38	−9.62	
CO			−7.69	−22.22		
Dist.		4.27	3.42	13.67	8.55	
CO			6.84	1.71		
Dist.		−3.80	−3.04	−1.05	−0.66	
CO			−0.53	−1.52		
Dist.		0.29	0.24	0.94	0.58	
CO			0.47	0.12		
Dist.		−0.26	−0.21	−0.07	−0.05	
CO			−0.04	−0.11		
Dist.		−0.02	−0.02	0.07	0.04	
$\sum M$	0	44.96	−44.96	−23.84	23.84	0

Using the results, the shears at A and B are computed and shown in Fig. c. Thus, for the entire frame,

$\overset{+}{\rightarrow}\sum F_X = 0; \quad 7.493 + 1.987 - P = 0 \quad P = 9.480\ k$

Thus, for $P = 3$ k,

$M_{DA} = \left(\frac{3}{9.480}\right)(44.96) = 14.2\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{DC} = \left(\frac{3}{9.480}\right)(-44.96) = -14.2\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{CD} = \left(\frac{3}{9.480}\right)(-23.84) = -7.54\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{CB} = \left(\frac{3}{9.480}\right)(23.84) = 7.54\ \text{k}\cdot\text{ft}$ **Ans.**