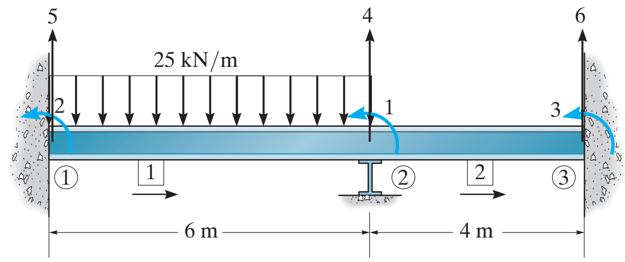
15–1. Determine the moments at ① and ③. Assume ② is a roller and ① and ③ are fixed. EI is constant.

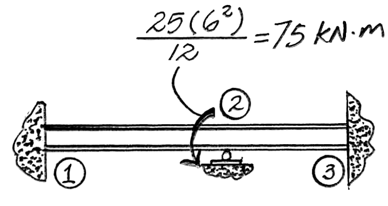


Member Stiffness Matrices. For member $\boxed{1}$,

$$\frac{12EI}{L^3} = \frac{12EI}{6^3} = 0.05556EI \qquad \frac{6EI}{L^2} = \frac{6EI}{6^2} = 0.16667EI$$

$$\frac{4EI}{L} = \frac{4EI}{6} = 0.66667EI \qquad \frac{2EI}{L} = \frac{2EI}{6} = 0.33333EI$$

$$\begin{array}{cc} & \begin{array}{cccc} 5 & 2 & 4 & 1 \end{array} \\ \mathbf{k}_1 = EI & \left[\begin{array}{cccc} 0.05556 & 0.16667 & -0.05556 & 0.16667 \\ 0.16667 & 0.66667 & -0.16667 & 0.33333 \\ -0.05556 & -0.16667 & 0.05556 & -0.16667 \\ 0.16667 & 0.33333 & -0.16667 & 0.66667 \end{array}\right] \begin{array}{c} 5 \\ 2 \\ 4 \\ 1 \end{array} \end{array}$$



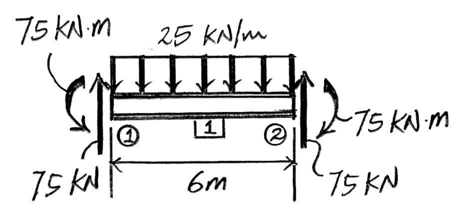
(a)

For member $\boxed{2}$,

$$\frac{12EI}{L^3} = \frac{12EI}{4^3} = 0.1875EI \qquad \frac{6EI}{L^2} = \frac{6EI}{4^2} = 0.375EI$$

$$\frac{4EI}{L} = \frac{4EI}{4} = EI \qquad \frac{2EI}{L} = \frac{2EI}{4} = 0.5EI$$

$$\begin{array}{cc} & \begin{array}{cccc} 4 & 1 & 6 & 3 \end{array} \\ \mathbf{k}_2 = EI & \left[\begin{array}{cccc} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1.00 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1.00 \end{array}\right] \begin{array}{c} 4 \\ 1 \\ 6 \\ 3 \end{array} \end{array}$$



(b)

Known Nodal Loads and Deflection. The nodal load acting on the unconstrained degree of freedom (Code number 1) is shown in Fig. *a*. Thus;

$$\mathbf{Q}_k = [75]\,1 \quad \text{and} \quad \mathbf{D}_k = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right] \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

Load-Displacement Relation. The structure stiffness matrix is a 6×6 matrix since the highest Code number is 6. Applying $\mathbf{Q} = \mathbf{KD}$

$$\left[\begin{array}{c} 75 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{array}\right] = EI \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{c|ccccc} 1.6667 & 0.33333 & 0.5 & 0.20833 & 0.16667 & -0.375 \\ \hline 0.33333 & 0.66667 & 0 & -0.16667 & 0.16667 & 0 \\ 0.5 & 0 & 1.00 & 0.375 & 0 & -0.375 \\ 0.20833 & -0.16667 & 0.375 & 0.24306 & -0.05556 & -0.1875 \\ 0.16667 & 0.16667 & 0 & -0.05556 & 0.05556 & 0 \\ -0.375 & 0 & -0.375 & -0.1875 & 0 & 0.1875 \end{array}\right] \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \left[\begin{array}{c} D_1 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right]$$

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

$$75 = 1.66667EID_1 + 0 \qquad D_1 = \frac{45}{EI}$$

15–1. Continued

Also, $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$,

$$Q_2 = 0.33333EI\left(\frac{45}{EI}\right) + 0 = 15 \text{ kN}\cdot\text{m}$$

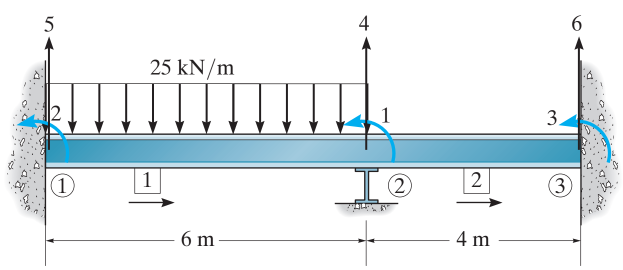
$$Q_3 = 0.5EI\left(\frac{45}{EI}\right) + 0 = 22.5 \text{ kN}\cdot\text{m}$$

Superposition of these results and the (FEM) in Fig. *b*,

$M_1 = 15 + 75 = 90 \text{ kN}\cdot\text{m}$ ⤸ **Ans.**

$M_3 = 22.5 + 0 = 22.5 \text{ kN}\cdot\text{m}$ ⤸ **Ans.**

15–2. Determine the moments at ① and ③ if the support ② moves upward 5 mm. Assume ② is a roller and ① and ③ are fixed. $EI = 60(10^6)\ \text{N}\cdot\text{m}^2$.

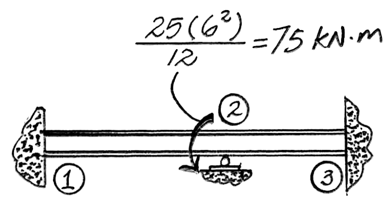


Member Stiffness Matrices. For member $\boxed{1}$,

$$\frac{12EI}{L^3} = \frac{12EI}{6^3} = 0.05556\,EI \qquad \frac{6EI}{L^2} = \frac{6EI}{6^2} = 0.16667\,EI$$

$$\frac{4EI}{L} = \frac{4EI}{6} = 0.66667EI \qquad \frac{2EI}{L} = \frac{2EI}{6} = 0.33333\,EI$$

$$\mathbf{k}_1 = EI\begin{array}{c} \begin{array}{cccc} 5 & 2 & 4 & 1 \end{array} \\ \begin{bmatrix} 0.05556 & 0.16667 & -0.05556 & 0.16667 \\ 0.16667 & 0.66667 & -0.16667 & 0.33333 \\ -0.05556 & -0.16667 & 0.05556 & -0.16667 \\ 0.16667 & 0.33333 & -0.16667 & 0.66667 \end{bmatrix} \end{array}\begin{array}{c} \\ 5 \\ 2 \\ 4 \\ 1 \end{array}$$

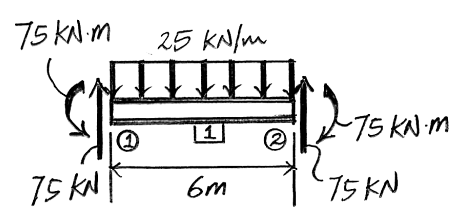


(a)

For member $\boxed{2}$,

$$\frac{12EI}{L^3} = \frac{12EI}{4^3} = 0.1875EI \qquad \frac{6EI}{L^2} = \frac{6EI}{4^2} = 0.375EI$$

$$\frac{4EI}{L} = \frac{4EI}{4} = EI \qquad \frac{2EI}{L} = \frac{2EI}{4} = 0.5EI$$



(b)

15–2. Continued

$$\mathbf{k}_2 = EI \begin{array}{c} \begin{array}{cccc} 4 & 1 & 6 & 3 \end{array} \\ \begin{bmatrix} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1.00 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1.00 \end{bmatrix} \end{array} \begin{array}{c} \\ 4 \\ 1 \\ 6 \\ 3 \end{array}$$

Known Nodal Loads and Deflection. The nodal load acting on the unconstrained degree of freedoom (code number 1) is shown in Fig. *a*. Thus,

$$Q_k = [75(10^3)]\ 1 \qquad \text{and} \qquad D_k = \begin{bmatrix} 0 \\ 0 \\ 0.005 \\ 0 \\ 0 \end{bmatrix} \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

Load-Displacement Relation. The structure stiffness matrix is a 6×6 matrix since the highest code number is 6. Applying $\mathbf{Q} = \mathbf{kD}$

$$\begin{bmatrix} 75(10^3) \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = EI \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1.66667 & 0.33333 & 0.5 & 0.20833 & 0.16667 & -0.375 \\ 0.33333 & 0.66667 & 0 & -0.16667 & 0.16667 & 0 \\ 0.5 & 0 & 1.00 & 0.375 & 0 & -0.375 \\ 0.20833 & -0.16667 & 0.375 & 0.24306 & -0.05556 & -0.1875 \\ 0.16667 & 0.16667 & 0 & -0.05556 & 0.05556 & 0 \\ -0.375 & 0 & -0.375 & -0.1875 & 0 & 0.1875 \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{bmatrix} D_1 \\ 0 \\ 0 \\ 0.005 \\ 0 \\ 0 \end{bmatrix}$$

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

$$75(10^3) = [1.6667D_1 + 0.20833(0.005)][60(10^6)]$$

$$D_1 = 0.125(10^{-3})\text{ rad}$$

Using this result and apply, $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$,

$$Q_2 = \{0.33333[0.125(10^{-3})] + (-0.16667)(0.005)\}[60(10^6)] = -47.5\text{ kN}\cdot\text{m}$$

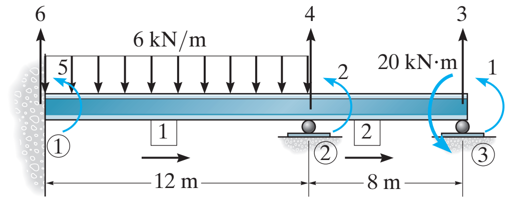
$$Q_3 = \{0.5[0.125(10^{-3})] + 0.375(0.005)\}[60(10^6)] = 116.25\text{ kN}\cdot\text{m}$$

Superposition these results to the (FEM) in Fig. *b*,

$M_1 = -47.5 + 75 = 27.5\text{ kN}\cdot\text{m}$ **Ans.**

$M_3 = 116.25 + 0 = 116.25\text{ kN.m} = 116\text{ kN}\cdot\text{m}$ **Ans.**

15–3. Determine the reactions at the supports. Assume the rollers can either push or pull on the beam. EI is constant.

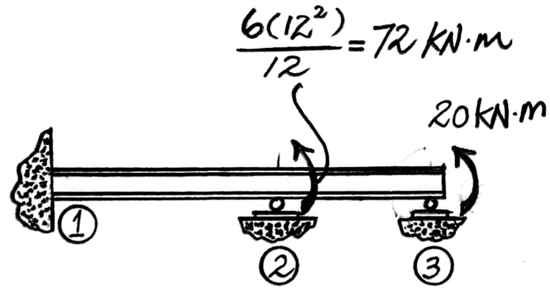


Member Stiffness Matrices. For member $\boxed{1}$,

$$\frac{12EI}{L^3} = \frac{12EI}{12^3} = 0.006944EI \qquad \frac{6EI}{L^2} = \frac{6EI}{12^2} = 0.041667EI$$

$$\frac{4EI}{L} = \frac{4EI}{12} = 0.333333EI \qquad \frac{2EI}{L} = \frac{2EI}{12} = 0.166667EI$$

$$\mathbf{k}_1 = EI \begin{array}{c} \begin{array}{cccc} 6 & 5 & 4 & 2 \end{array} \\ \begin{bmatrix} 0.006944 & 0.041667 & -0.006944 & 0.041667 \\ 0.041667 & 0.333333 & -0.041667 & 0.166667 \\ -0.006944 & -0.041667 & 0.006944 & -0.041667 \\ 0.041667 & 0.166667 & -0.041667 & 0.333333 \end{bmatrix} \end{array} \begin{array}{c} 6 \\ 5 \\ 4 \\ 2 \end{array}$$



(a)

For member $\boxed{2}$,

$$\frac{12EI}{L^3} = \frac{12EI}{8^3} = 0.0234375EI \qquad \frac{6EI}{L^2} = \frac{6EI}{8^2} = 0.09375EI$$

$$\frac{4EI}{L} = \frac{4EI}{8} = 0.5EI \qquad \frac{2EI}{L} = \frac{2EI}{8} = 0.25EI$$

$$\mathbf{k}_2 = EI \begin{array}{c} \begin{array}{cccc} 4 & 2 & 3 & 1 \end{array} \\ \begin{bmatrix} 0.0234375 & 0.09375 & -0.0234375 & 0.09375 \\ 0.09375 & 0.5 & -0.09375 & 0.25 \\ -0.0234375 & -0.09375 & 0.0234375 & -0.09375 \\ 0.09375 & 0.25 & -0.09375 & 0.5 \end{bmatrix} \end{array} \begin{array}{c} 4 \\ 2 \\ 3 \\ 1 \end{array} \begin{bmatrix} 0 \\ 87 \\ 0 \\ -3.76 \end{bmatrix}$$

Known Nodal Loads And Deflection. The nodal loads acting on the unconstrained degree of freedoom (code number 1 and 2) are shown in Fig. *a*. Thus,

$$\mathbf{Q}_k = \begin{bmatrix} 20 \\ 72 \end{bmatrix} \begin{array}{c} 1 \\ 2 \end{array} \quad \text{and} \quad \mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{array}{c} 3 \\ 4 \\ 5 \\ 6 \end{array}$$

Load-Displacement Relation. The structure stiffness matrix is a 6 × 6 matrix since the highest code number is 6. Applying $\mathbf{Q} = \mathbf{KD}$

$$\begin{bmatrix} 20 \\ 72 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = EI \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{cc|cccc} 0.5 & 0.25 & -0.09375 & 0.09375 & 0 & 0 \\ 0.25 & 0.833333 & -0.09375 & 0.052083 & 0.166667 & 0.041667 \\ \hline -0.09375 & -0.09375 & 0.0234375 & -0.0234375 & 0 & 0 \\ 0.09375 & 0.052083 & -0.0234375 & 0.0303815 & -0.041667 & -0.006944 \\ 0 & 0.166667 & 0 & -0.041667 & 0.333333 & 0.041667 \\ 0 & 0.041667 & 0 & -0.006944 & 0.041667 & 0.006944 \end{array}\right] \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{bmatrix} D_1 \\ D_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

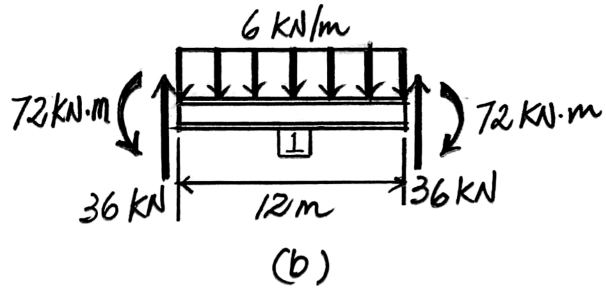
$$20 = EI[0.5D_1 + 0.25D_2] \qquad (1)$$

$$72 = EI[0.25D_1 + 0.833333D_2] \qquad (2)$$

15–3. Continued

Solving Eqs. (1) and (2),

$$D_1 = -\frac{3.7647}{EI} \qquad D_2 = \frac{87.5294}{EI}$$



Also, $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$

$$\begin{bmatrix} Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = EI \begin{bmatrix} -0.09375 & -0.09375 \\ 0.09375 & 0.052083 \\ 0 & 0.166667 \\ 0 & 0.041667 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} -3.7647 \\ 87.5294 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Q_3 = -0.09375(-3.7647) + (-0.09375)(87.5294) = -7.853 \text{ kN}$$

$$Q_4 = 0.09375(-3.7647) + 0.052083(87.5294) = 4.206 \text{ kN}$$

$$Q_5 = 0 + 0.166667(87.5294) = 14.59 \text{ kN}\cdot\text{m}$$

$$Q_6 = 0 + 0.041667(87.5294) = 3.647 \text{ kN}$$

Superposition these results with the (FEM) in Fig. *b*,

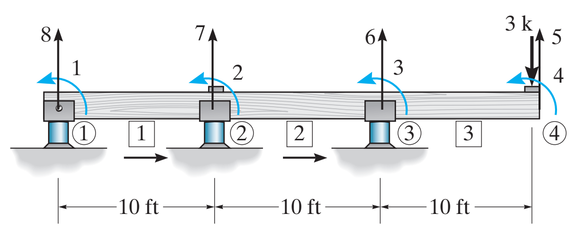
$R_3 = -7.853 + 0 = -7.853 \text{ kN} = 7.85 \text{ kN} \downarrow$ **Ans.**

$R_4 = 4.206 + 36 = 40.21 \text{ kN} = 40.2 \text{ kN} \uparrow$ **Ans.**

$M_5 = 14.59 + 72 = 86.59 \text{ kN}\cdot\text{m} = 86.6 \text{ kN}\cdot\text{m} \uparrow$ **Ans.**

$R_6 = 3.647 + 36 = 39.64 \text{ kN} = 39.6 \text{ kN} \uparrow$ **Ans.**

***15–4.** Determine the reactions at the supports. Assume ① is a pin and ② and ③ are rollers that can either push or pull on the beam. *EI* is constant.



Member Stiffness Matrices. For member $\boxed{1}$, $\boxed{2}$ and $\boxed{3}$,

$$\frac{12EI}{L^3} = \frac{12EI}{10^3} = 0.012 \qquad \frac{6EI}{L^2} = \frac{6EI}{10^2} = 0.06$$

$$\frac{4EI}{L} = \frac{4EI}{10} = 0.4 \qquad \frac{2EI}{L} = \frac{2EI}{10} = 0.2$$

$$\mathbf{k}_1 = EI \begin{array}{c} \begin{array}{cccc} 8 & 1 & 7 & 2 \end{array} \\ \begin{bmatrix} 0.012 & 0.06 & -0.012 & 0.06 \\ 0.06 & 0.4 & -0.06 & 0.2 \\ -0.012 & -0.06 & 0.012 & -0.06 \\ 0.06 & 0.2 & -0.06 & 0.4 \end{bmatrix} \end{array} \begin{array}{c} \\ 8 \\ 1 \\ 7 \\ 2 \end{array}$$

15–4. Continued

$$\mathbf{k}_2 = EI \begin{array}{c} \begin{matrix} \quad 7 & \quad 2 & \quad 6 & \quad 3 \end{matrix} \\ \begin{bmatrix} 0.012 & 0.06 & -0.012 & 0.06 \\ 0.06 & 0.4 & -0.06 & 0.2 \\ -0.012 & -0.06 & 0.012 & -0.06 \\ 0.06 & 0.2 & -0.06 & 0.4 \end{bmatrix} \begin{matrix} 7 \\ 2 \\ 6 \\ 3 \end{matrix} \end{array}$$

$$\mathbf{k}_3 = EI \begin{array}{c} \begin{matrix} \quad 6 & \quad 3 & \quad 5 & \quad 4 \end{matrix} \\ \begin{bmatrix} 0.012 & 0.06 & -0.012 & 0.06 \\ 0.06 & 0.4 & -0.06 & 0.2 \\ -0.012 & -0.06 & 0.012 & -0.06 \\ 0.06 & 0.2 & -0.06 & 0.4 \end{bmatrix} \begin{matrix} 6 \\ 3 \\ 5 \\ 4 \end{matrix} \end{array}$$

Known Nodal Load and Deflection. The nodal loads acting on the unconstrained degree of freedom (code number 1, 2, 3, 4 and 5) is

$$\mathbf{Q}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -3 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \quad \text{and} \quad \mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 6 \\ 7 \\ 8 \end{matrix}$$

Load-Displacement Relation. The structure stiffness matrix is a 8×8 matrix since the highest code number is 8. Applying $\mathbf{Q} = \mathbf{KD}$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -3 \\ \hline Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = EI \begin{array}{c} \begin{matrix} \quad 1 & \quad 2 & \quad 3 & \quad 4 & \quad 5 & \quad 6 & \quad 7 & \quad 8 \end{matrix} \\ \left[\begin{array}{ccccc|ccc} 0.4 & 0.2 & 0 & 0 & 0 & 0 & -0.06 & 0.06 \\ 0.2 & 0.8 & 0.2 & 0 & 0 & -0.06 & 0 & 0.06 \\ 0 & 0.2 & 0.8 & 0.2 & -0.06 & 0 & 0.06 & 0 \\ 0 & 0 & 0.2 & 0.4 & -0.06 & 0.06 & 0 & 0 \\ 0 & 0 & -0.06 & -0.06 & 0.012 & -0.012 & 0 & 0 \\ \hline 0 & -0.06 & 0 & 0.06 & -0.012 & 0.024 & -0.012 & 0 \\ -0.06 & 0 & 0.06 & 0 & 0 & -0.012 & 0.024 & -0.012 \\ 0.06 & 0.06 & 0 & 0 & 0 & 0 & -0.012 & 0.012 \end{array}\right] \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{matrix} \end{array} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix}$$

From the matrix partition, $\mathbf{Q}_k = \mathbf{k}_{11}\mathbf{D}_u + \mathbf{k}_{12}\mathbf{D}_k$,

$$0 = 04D_1 + 0.2D_2 \qquad (1)$$

$$0 = 0.2D_1 + 0.8D_2 + 0.2D_3 \qquad (2)$$

$$0 = 0.2D_2 + 0.8D_3 + 0.2D_4 - 0.06D_5 \qquad (3)$$

$$0 = 0.2D_3 + 0.4D_4 - 0.06D_5 \qquad (4)$$

$$-3 = -0.06D_3 - 0.06D_4 + 0.012D_5 \qquad (5)$$

Solving Eq. (1) to (5)

$$D_1 = -12.5 \quad D_2 = 25 \quad D_3 = -87.5 \quad D_4 = -237.5 \quad D_5 = -1875$$

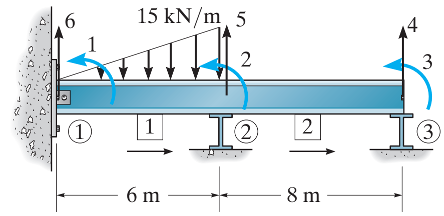
Using these results, $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{k}_{22}\mathbf{D}_k$

$Q_6 = 6.75$ kN **Ans.**

$Q_7 = -4.5$ kN **Ans.**

$Q_8 = -0.75$ kN **Ans.**

15–5. Determine the support reactions. Assume ② and ③ are rollers and ① is a pin. EI is constant.



Member Stiffness Matrices. For member $\boxed{1}$

$$\frac{12EI}{L^3} = \frac{12EI}{6^3} = 0.05556EI \qquad \frac{6EI}{L^2} = \frac{6EI}{6^2} = 0.16667EI$$

$$\frac{4EI}{L} = \frac{4EI}{6} = 0.066667EI \qquad \frac{2EI}{L} = \frac{2EI}{6} = 0.033333EI$$

$$\mathbf{k}_1 = EI \begin{array}{c} \begin{array}{cccc} 6 & 1 & 5 & 2 \end{array} \\ \begin{bmatrix} 0.05556 & 0.16667 & -0.05556 & 0.16667 \\ 0.16667 & 0.66667 & -0.16667 & 0.33333 \\ -0.05556 & -0.16667 & 0.05556 & -0.16667 \\ 0.16667 & 0.33333 & -0.16667 & 0.66667 \end{bmatrix} \end{array} \begin{array}{c} 6 \\ 1 \\ 5 \\ 2 \end{array}$$

For Member $\boxed{2}$,

$$\frac{12EI}{L^3} = \frac{12EI}{8^3} = 0.0234375EI \qquad \frac{6EI}{L^2} = \frac{6EI}{8^2} = 0.09375EI$$

$$\frac{4EI}{L} = \frac{4EI}{8} = 0.5EI \qquad \frac{2EI}{L} = \frac{2EI}{8} = 0.025EI$$

$$\mathbf{k}_2 = EI \begin{array}{c} \begin{array}{cccc} 5 & 2 & 4 & 3 \end{array} \\ \begin{bmatrix} 0.0234375 & 0.09375 & -0.0234375 & 0.09375 \\ 0.09375 & 0.5 & -0.09375 & 0.25 \\ -0.0234375 & -0.09375 & 0.0234375 & -0.09375 \\ 0.09375 & 0.25 & -0.09375 & 0.5 \end{bmatrix} \end{array} \begin{array}{c} 5 \\ 2 \\ 4 \\ 3 \end{array}$$

Known Nodal Load and Deflection. The nodal loads acting on the unconstrained degree of freedom (code number 1, 2, and 3) are shown in Fig. *a*

$$\mathbf{Q}_k = \begin{bmatrix} 0 \\ 36 \\ 0 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \quad \text{and} \quad \mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{array}{c} 4 \\ 5 \\ 6 \end{array}$$

Load-Displacement Relation. The structure stiffness matrix is a 6×6 matirx since the highest code number is 6. Applying $\mathbf{Q} = \mathbf{KD}$,

$$\begin{bmatrix} 0 \\ 36 \\ 0 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = EI \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{ccc|ccc} 0.66667 & 0.33333 & 0 & 0 & 0.16667 & 0.16667 \\ 0.33333 & 1.16667 & 0.25 & -0.09375 & -0.07292 & 0.16667 \\ 0 & 0.25 & 0.5 & -0.09375 & 0.09375 & 0 \\ \hline 0 & -0.09375 & -0.09375 & 0.0234375 & -0.0234375 & 0 \\ -0.16667 & -0.07292 & 0.09375 & -0.0234375 & 0.0789931 & -0.05556 \\ 0.16667 & 0.16667 & 0 & 0 & -0.05556 & 0.05556 \end{array}\right] \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

From the matrix partition, $\mathbf{Q}_k = \mathbf{k}_{11}\mathbf{D}_u + \mathbf{k}_{12}\mathbf{D}_k$

$$0 = 0.66667D_1 + 0.33333D_2 \qquad (1)$$

$$36 = 0.33333D_1 + 1.16667D_2 + 0.25D_3 \qquad (2)$$

$$0 = 0.25D_2 + 0.5D_3 \qquad (3)$$

15–5. Continued

Solving Eqs. (1) to (3),

$$D_1 = \frac{-20.5714}{EI}$$

$$D_2 = \frac{41.1429}{EI}$$

$$D_3 = \frac{-20.5714}{EI}$$

Using these results and apply $\mathbf{Q}_u = \mathbf{k}_{21}\,\mathbf{D}_u + {}_{+}\mathbf{k}_{22}\,\mathbf{D}_k$

$$Q_4 = 0 + (-0.09375EI)\left(\frac{41.1429}{EI}\right) + (-0.09375EI)\left(-\frac{20.5714}{EI}\right) = -1.929 \text{ kN}$$

$$Q_5 = -0.16667EI\left(-\frac{20.5714}{EI}\right) + (-0.07292EI)\left(\frac{41.1429}{EI}\right) + 0.09375EI\left(-\frac{20.5714}{EI}\right)$$

$$= -1.500 \text{ kN}$$

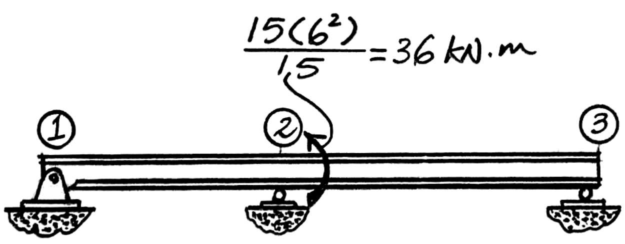
$$Q_6 = 0.16667EI\left(-\frac{20.5714}{EI}\right) + 0.16667EI\left(\frac{41.1429}{EI}\right) = 3.429 \text{ kN}$$

Superposition these results with the FEM show in Fig. *b*

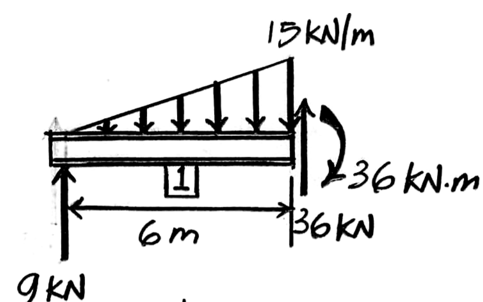
$R_4 = -1.929 + 0 = -1.929 \text{ kN} = 1.93 \text{ kN} \downarrow$ **Ans.**

$R_5 = -1.500 + 36 = 34.5 \text{ kN} \uparrow$

$R_6 = 3.429 + 9 = 12.43 \text{ kN} = 12.4 \text{ kN} \uparrow$ **Ans.**

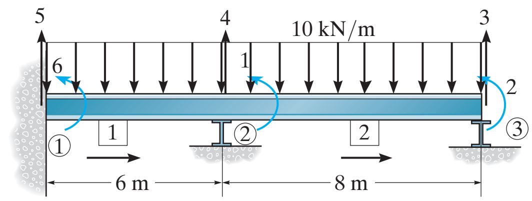


(a)



(b)

15–6. Determine the reactions at the supports. Assume ① is fixed ② and ③ are rollers. EI is constant.



Member Stiffness Matrices. For member $\boxed{1}$,

$$\frac{12EI}{L^3} = \frac{12EI}{6^3} = 0.05556EI \qquad \frac{6EI}{L^2} = \frac{6EI}{6^2} = 0.16667EI$$

$$\frac{4EI}{L} = \frac{4EI}{6} = 0.066667EI \qquad \frac{2EI}{L} = \frac{2EI}{6} = 0.33333EI$$

$$\mathbf{k}_1 = EI \begin{array}{c} \begin{array}{cccc} 5 & 6 & 4 & 1 \end{array} \\ \left[\begin{array}{cccc} 0.05556 & 0.16667 & -0.05556 & 0.16667 \\ 0.16667 & 0.66667 & -0.16667 & 0.33333 \\ -0.05556 & -0.16667 & 0.05556 & -0.16667 \\ 0.16667 & 0.33333 & -0.16667 & 0.66667 \end{array}\right] \end{array} \begin{array}{c} \\ 5 \\ 6 \\ 4 \\ 1 \end{array}$$

For Member $\boxed{2}$,

$$\frac{12EI}{L^3} = \frac{12EI}{8^3} = 0.0234375EI \qquad \frac{6EI}{L^2} = \frac{8EI}{8^2} = 0.09375EI$$

$$\frac{4EI}{L} = \frac{4EI}{8} = 0.5EI \qquad \frac{2EI}{L} = \frac{2EI}{8} = 0.025EI$$

$$\mathbf{k}_2 = EI \begin{array}{c} \begin{array}{cccc} 4 & 1 & 3 & 2 \end{array} \\ \left[\begin{array}{cccc} 0.0234375 & 0.09375 & -0.0234375 & 0.09375 \\ 0.09375 & 0.5 & -0.09375 & 0.25 \\ -0.0234375 & -0.09375 & 0.0234375 & -0.09375 \\ 0.09375 & 0.25 & -0.09375 & 0.5 \end{array}\right] \end{array} \begin{array}{c} \\ 4 \\ 1 \\ 3 \\ 2 \end{array}$$

Known Nodal Load and Deflections. The nodal loads acting on the unconstrained degree of freedom (code number 1 and 2) are shown in Fig. *a*

$$\mathbf{Q}_k = \left[\begin{array}{c} -50 \\ 0 \end{array}\right] \begin{array}{c} 1 \\ 2 \end{array} \quad \text{and} \quad \mathbf{D}_k = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array}\right] \begin{array}{c} 3 \\ 4 \\ 5 \\ 6 \end{array}$$

Load-Displacement Relation. The structure stiffness matrix is a 6 × 6 matirx since the highest code number is 6. Applying $\mathbf{Q} = \mathbf{KD}$,

$$\left[\begin{array}{c} -50 \\ 0 \\ \hline Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{array}\right] = EI \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{cc|cccc} 1.16667 & 0.25 & -0.09375 & -0.07292 & 0.16667 & 0.33333 \\ 0.25 & 0.5 & -0.09375 & 0.09375 & 0 & 0 \\ \hline -0.09375 & -0.09375 & 0.0234375 & -0.0234375 & 0 & 0 \\ -0.07292 & 0.09375 & -0.0234375 & 0.0789931 & -0.05556 & -0.16667 \\ 0.16667 & 0 & 0 & -0.05556 & 0.05556 & 0.16667 \\ 0.33333 & 0 & 0 & -0.16667 & 0.16667 & 0.66667 \end{array}\right] \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \left[\begin{array}{c} D_1 \\ D_2 \\ \hline 0 \\ 0 \\ 0 \\ 0 \end{array}\right]$$

15–6. Continued

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$

$$-50 = EI(1.16667D_1 + 0.25D_2)$$

$$0 = EI\,(0.25D_1 + 0.5D_2)$$

Solving Eqs. (1) and (2),

$$D_1 = \frac{48}{EI} \qquad D_2 = \frac{24}{EI}$$

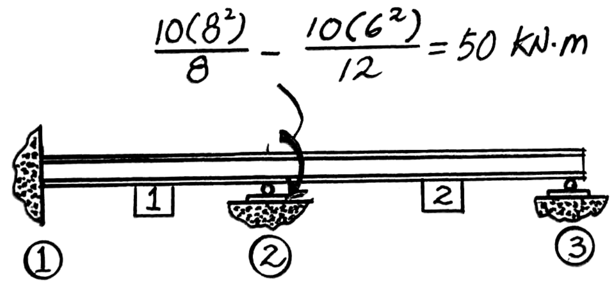
Using these results and apply in $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$

$$Q_3 = -0.09375EI\left(-\frac{48}{EI}\right) + (-0.09375EI)\left(\frac{24}{EI}\right) + 0 = 2.25\text{ kN}$$

$$Q_4 = -0.07292EI\left(-\frac{48}{EI}\right) + 0.09375EI\left(\frac{24}{EI}\right) + 0 = 5.75\text{ kN}$$

$$Q_5 = 0.16667EI\left(-\frac{48}{EI}\right) + 0 + 0 = -8.00\text{ kN}$$

$$Q_6 = (0.33333EI)\left(-\frac{48}{EI}\right) + 0 + 0 = -16.0\text{ kN}$$



(a)

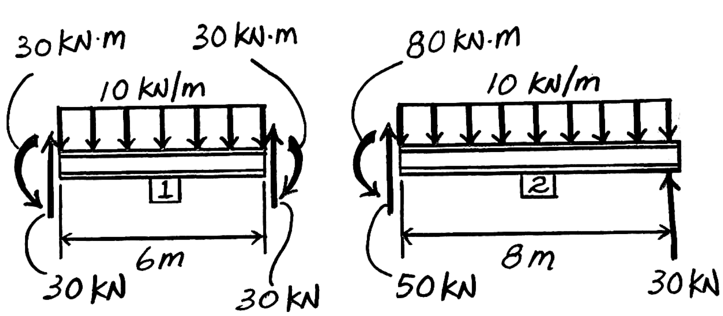
Superposition these results with the FEM show in Fig. *b*

$R_3 = 2.25 + 30 = 32.25\text{ kN}\uparrow$ **Ans.**

$R_4 = 5.75 + 30 + 50 = 85.75\text{ kN}\uparrow$ **Ans.**

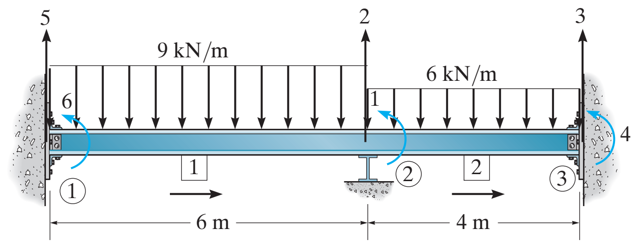
$R_5 = -8.00 + 30 = 22.0\text{ kN}\uparrow$ **Ans.**

$R_6 = -16.0 + 30 = 14.0\text{ kN}\cdot\text{m}\circlearrowleft$ **Ans.**



(b)

15–7. Determine the reactions at the supports. Assume ① and ③ are fixed and ② is a roller. EI is constant.



Member Stiffness Matrices. For member $\boxed{1}$,

$$\frac{12EI}{L^3} = \frac{12EI}{6^3} = 0.05556EI \qquad \frac{6EI}{L^2} = \frac{6EI}{6^2} = 0.16667EI$$

$$\frac{4EI}{L} = \frac{4EI}{6} = 0.66667EI \qquad \frac{2EI}{L} = \frac{2EI}{6} = 0.33333EI$$

$$\mathbf{k}_1 = EI \begin{array}{c} \begin{array}{cccc} 5 & 6 & 2 & 1 \end{array} \\ \begin{bmatrix} 0.05556 & 0.16667 & -0.05556 & 0.16667 \\ 0.16667 & 0.66667 & -0.16667 & 0.33333 \\ -0.05556 & -0.16667 & 0.05556 & -0.16667 \\ 0.16667 & 0.33333 & -0.16667 & 0.66667 \end{bmatrix} \end{array} \begin{array}{c} \\ 5 \\ 6 \\ 2 \\ 1 \end{array}$$

For member $\boxed{2}$,

$$\frac{12EI}{L^3} = \frac{12EI}{4^3} = 0.1875EI \qquad \frac{6EI}{L^2} = \frac{6EI}{4^2} = 0.375EI$$

$$\frac{4EI}{L} = \frac{4EI}{4} = EI \qquad \frac{2EI}{L} = \frac{2EI}{4} = 0.5EI$$

$$\mathbf{k}_2 = EI \begin{array}{c} \begin{array}{cccc} 2 & 1 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1.00 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1.00 \end{bmatrix} \end{array} \begin{array}{c} \\ 2 \\ 1 \\ 3 \\ 4 \end{array}$$

Known Nodal Loads and Deflections. The nodal load acting on the unconstrained degree of freedom (code number 1) are shown in Fig. *a*

$$\mathbf{Q}_k = [19]\ 1 \qquad \text{and} \qquad \mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

15–7. Continued

Load-Displacement Relation. The structure stiffness matrix is a 6×6 matirx since the highest code number is 6. Applying $\mathbf{Q} = \mathbf{KD}$,

$$\begin{bmatrix}19\\Q_2\\Q_3\\Q_4\\Q_5\\Q_6\end{bmatrix} = EI \begin{array}{c}\begin{array}{cccccc}\quad 1 & \qquad 2 & \qquad 3 & \qquad 4 & \qquad 5 & \qquad 6\end{array}\\ \left[\begin{array}{c|ccccc} 1.66667 & 0.20833 & -0.375 & 0.5 & 0.16667 & 0.33333\\ \hline 0.20833 & 0.24306 & -0.1875 & 0.375 & -0.05556 & -0.16667\\ -0.375 & -0.1875 & 0.1875 & -0.375 & 0 & 0\\ 0.5 & 0.375 & -0.375 & 1.00 & 0 & 0\\ 0.16667 & -0.05556 & 0 & 0 & 0.05556 & 0.16667\\ 0.33333 & -0.16667 & 0 & 0 & 0.16667 & 0.66667 \end{array}\right]\end{array} \begin{array}{c}\\1\\2\\3\\4\\5\\6\end{array} \begin{bmatrix}D_1\\0\\0\\0\\0\\0\end{bmatrix}$$

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$

$$19 = 1.66667EID_1 \qquad D_1 = \frac{11.4}{EI}$$

Using this result and applying $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$

$$Q_2 = 0.20833EI\left(\frac{11.4}{EI}\right) = 2.375 \text{ kN}$$

$$Q_3 = -0.375EI\left(\frac{11.4}{EI}\right) = -4.275 \text{ kN}$$

$$Q_4 = 0.5EI\left(\frac{11.4}{EI}\right) = 5.70 \text{ kN}\cdot\text{m}$$

$$Q_5 = 0.16667\left(\frac{11.4}{EI}\right) = 1.90 \text{ kN}$$

$$Q_6 = 0.33333\left(\frac{11.4}{EI}\right) = 3.80 \text{ kN}\cdot\text{m}$$

Superposition these results with the FEM shown in Fig. *b*,

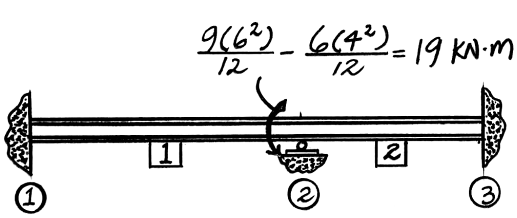
$R_2 = 2.375 + 27 + 12 = 41.375 \text{ kN} = 41.4 \text{ kN} \uparrow$ **Ans.**

$R_3 = -4.275 + 12 = 7.725 \text{ kN} \uparrow$ **Ans.**

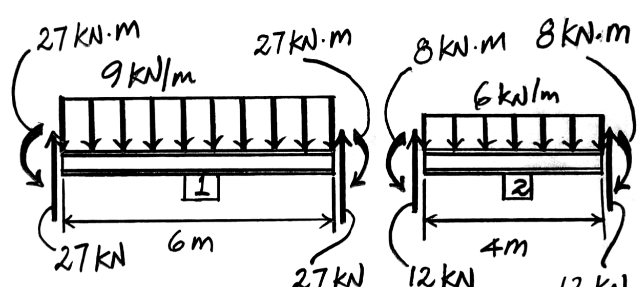
$R_4 = 5.70 - 8 = -2.30 \text{ kN}\cdot\text{m} = 2.30 \text{ kN.m} \circlearrowright$ **Ans.**

$R_5 = 1.90 + 27 = 28.9 \text{ kN} \uparrow$ **Ans.**

$R_6 = 3.80 + 27 = 30.8 \text{ kN.m} \uparrow$ **Ans.**

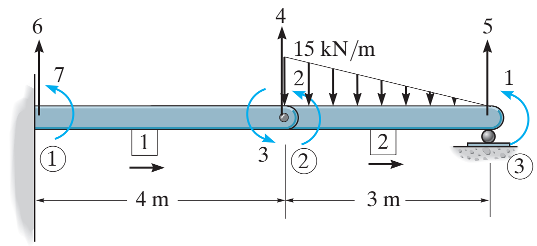


(a)



(b)

***15–8.** Determine the reactions at the supports. EI is constant.



Member Stiffness Matrices. For member $\boxed{1}$

$$\frac{12EI}{L^3} = \frac{12EI}{4^3} = 0.1875EI \qquad \frac{6EI}{L^2} = \frac{6EI}{4^2} = 0.375EI$$

$$\frac{4EI}{L} = \frac{4EI}{4} = EI \qquad \frac{2EI}{L} = \frac{2EI}{4} = 0.5EI$$

$$\mathbf{k}_1 = EI \begin{array}{c} \begin{array}{cccc} 6 & 7 & 4 & 3 \end{array} \\ \begin{bmatrix} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1.00 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1.00 \end{bmatrix} \end{array} \begin{array}{c} \\ 6 \\ 7 \\ 4 \\ 3 \end{array}$$

For member $\boxed{2}$,

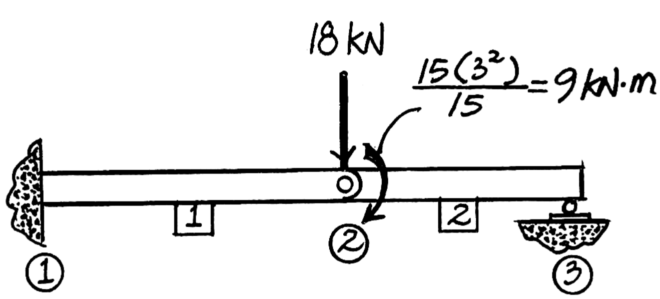
$$\frac{12EI}{L^3} = \frac{12EI}{3^3} = 0.44444EI \qquad \frac{6EI}{L^2} = \frac{6EI}{3^2} = 0.66667EI$$

$$\frac{4EI}{L} = \frac{4EI}{3} = 1.33333EI \qquad \frac{2EI}{L} = \frac{2EI}{3} = 0.66667EI$$

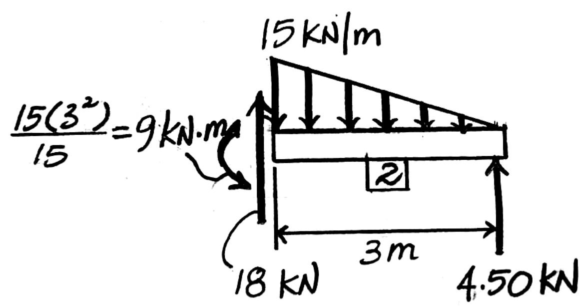
$$\mathbf{k}_2 = EI \begin{array}{c} \begin{array}{cccc} 4 & 2 & 5 & 1 \end{array} \\ \begin{bmatrix} 0.44444 & 0.66667 & -0.44444 & 0.66667 \\ 0.66667 & 1.33333 & -0.66667 & 0.66667 \\ -0.44444 & -0.66667 & 0.44444 & -0.66667 \\ 0.66667 & 0.66667 & -0.66667 & 1.33333 \end{bmatrix} \end{array} \begin{array}{c} \\ 4 \\ 2 \\ 5 \\ 1 \end{array}$$

Known Nodal Loads and Deflections. The nodal loads acting on the unconstrained degree of freedom (code number 1, 2, 3, and 4) are shown in Fig. *a* and *b*.

$$\mathbf{Q}_k = \begin{bmatrix} 0 \\ -9 \\ 0 \\ -18 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \quad \text{and} \quad \mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{array}{c} 5 \\ 6 \\ 7 \end{array}$$



(a)



(b)

15–8. Continued

Load-Displacement Relation. The structure stiffness matrix is a 7 $\times$ 7 matirx since the highest code number is 7. Applying $\mathbf{Q} = \mathbf{KD}$,

$$\begin{bmatrix} 0 \\ -9 \\ 0 \\ -18 \\ Q_5 \\ Q_6 \\ Q_7 \end{bmatrix} = EI \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 1.33333 & 0.66667 & 0 & 0.66667 & -0.66667 & 0 & 0 \\ 0.66667 & 1.33333 & 0 & 0.66667 & -0.66667 & 0 & 0 \\ 0 & 0 & 1.00 & -0.375 & 0 & 0.375 & 0.5 \\ 0.66667 & 0.66667 & -0.375 & 0.63194 & -0.44444 & -0.1875 & -0.375 \\ -0.66667 & -0.66667 & 0 & -0.44444 & 0.44444 & 0 & 0 \\ 0 & 0 & 0.375 & -0.1875 & 0 & 0.1875 & 0.375 \\ 0 & 0 & 0.5 & -0.375 & 0 & 0.375 & 1.00 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

$$0 = EI(1.33333D_1 + 0.66667D_2 + 0.66667D_4) \quad (1)$$

$$-9 = EI(0.66667D_1 + 1.33333D_2 + 0.66667D_4) \quad (2)$$

$$0 = EI(D_3 - 0.375D_4) \quad (3)$$

$$-18 = EI(0.66667D_1 + 0.66667D_2 - 0.375D_3 + 0.63194D_4) \quad (4)$$

Solving Eqs. (1) to (4),

$$D_1 = \frac{111.167}{EI} \qquad D_2 = \frac{97.667}{EI} \qquad D_3 = -\frac{120}{EI} \qquad D_4 = -\frac{320}{EI}$$

Using these result and applying $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$

$$Q_5 = -0.66667EI\left(\frac{111.167}{EI}\right) + \left(-0.66667EI\right)\left(\frac{97.667}{EI}\right) + (-0.44444EI)\left(\frac{-320}{EI}\right) + 0 = 3.00 \text{ kN}$$

$$Q_6 = 0.375EI\left(-\frac{120}{EI}\right) + (-0.1875EI)\left(-\frac{320}{EI}\right) + 0 = 15.00 \text{ kN}$$

$$Q_7 = 0.5EI\left(-\frac{120}{EI}\right) + (-0.375EI)\left(-\frac{320}{EI}\right) + 0 = 60.00 \text{ kN}\cdot\text{m}$$

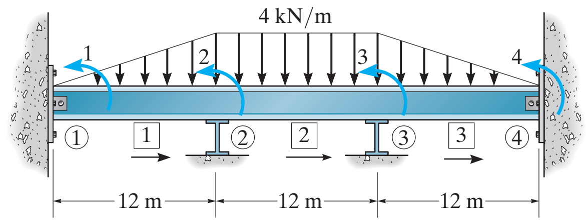
Superposition of these results with the (FEM),

$R_5 = 3.00 + 4.50 = 7.50$ kN $\uparrow$ **Ans.**

$R_6 = 15.00 + 0 = 15.0$ kN $\uparrow$ **Ans.**

$R_7 = 60.00 + 0 = 60.0$ kN $\cdot$ m $\circlearrowleft$ **Ans.**

15–9. Determine the moments at ② and ③ . EI is constant. Assume ① , ② , and ③ are rollers and ④ is pinned.



The FEMs are shown on the figure.

$$\mathbf{Q}_k = \begin{bmatrix} -19.2 \\ -19.2 \\ 19.2 \\ 19.2 \end{bmatrix} \qquad \mathbf{D}_k = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix}$$

$$\mathbf{k}_1 = EI\begin{bmatrix} 0.3333 & 0.16667 \\ 0.16667 & 0.3333 \end{bmatrix}$$

$$\mathbf{k}_2 = EI\begin{bmatrix} 0.3333 & 0.16667 \\ 0.16667 & 0.3333 \end{bmatrix}$$

$$\mathbf{k}_3 = EI\begin{bmatrix} 0.3333 & 0.16667 \\ 0.16667 & 0.3333 \end{bmatrix}$$

$$\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3$$

$$\mathbf{K} = EI\begin{bmatrix} 0.3333 & 0.16667 & 0 & 0 \\ 0.16667 & 0.6667 & 0.16667 & 0 \\ 0 & 0.16667 & 0.6667 & 0.16667 \\ 0 & 0 & 0.16667 & 0.3333 \end{bmatrix}$$

$$\mathbf{Q} = \mathbf{K}\mathbf{D}$$

$$\begin{bmatrix} -19.2 \\ -19.2 \\ 19.2 \\ 19.2 \end{bmatrix} = EI\begin{bmatrix} 0.3333 & 0.16667 & 0 & 0 \\ 0.16667 & 0.6667 & 0.16667 & 0 \\ 0 & 0.16667 & 0.6667 & 0.16667 \\ 0 & 0 & 0.16667 & 0.3333 \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix}$$

$$-19.2 = EI[0.3333D_1 + 0.16667D_2]$$

$$-19.2 = EI[0.16667D_1 + 0.6667D_2 + 0.16667D_3]$$

$$19.2 = EI[0.16667D_2 + 0.6667D_3 + 0.16667D_4]$$

$$19.2 = EI[0.16667D_3 + 0.16667D_4]$$

Solving,

$$D_1 = -46.08/EI$$

$$D_2 = -23.04/EI$$

$$D_3 = 23.04/EI$$

$$D_4 = 46.08/EI$$

$$\mathbf{q} = \mathbf{k}_1\mathbf{D}$$

15–9. Continued

$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = EI \begin{bmatrix} 0.3333 & 0.16667 \\ 0.16667 & 0.3333 \end{bmatrix} \begin{bmatrix} -46.08/EI \\ -23.04/EI \end{bmatrix}$$

$$q_1 = EI[0.3333(-46.08/EI) + 0.16667(-23.04/EI)]$$

$$q_1 = -19.2 \text{ kN} \cdot \text{m}$$

$$q_2 = EI[0.16667(-46.08/EI) + 0.3333(-23.04/EI)]$$

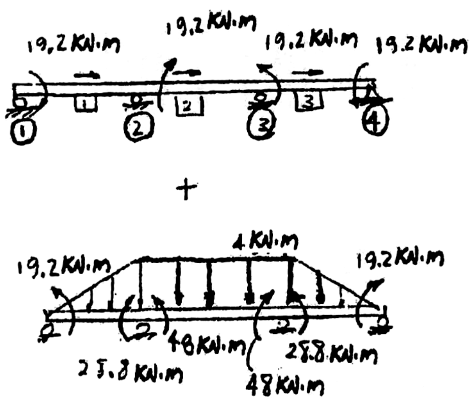
$$q_2 = -15.36 \text{ kN} \cdot \text{m}$$

Since the opposite FEM $= 19.2$ kN · m is at node 1, then

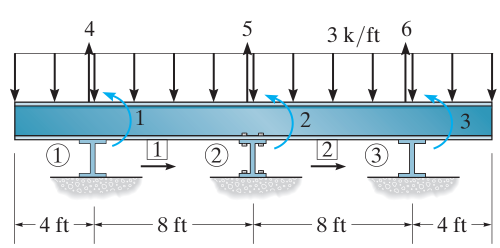
$$M_1 = M_4 = 19.2 - 19.2 = 0$$

Since the FEM $= -28.8$ kN · m is at node 2, then

$$M_2 = M_3 = -28.8 - 15.36 = 44.2 \text{ kN} \cdot \text{m}$$ **Ans.**



15–10. Determine the reactions at the supports. Assume ② is pinned and ① and ③ are rollers. EI is constant.



Member 1

$$\mathbf{k}_1 = \frac{EI}{8} \begin{bmatrix} 0.1875 & 0.75 & -0.1875 & 0.75 \\ 0.75 & 4 & -0.75 & 2 \\ -0.1875 & -0.75 & 0.1875 & -0.75 \\ 0.75 & 2 & -0.75 & 4 \end{bmatrix}$$

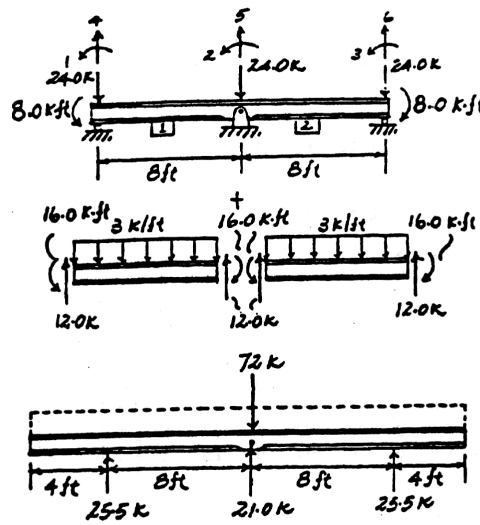
15–10. Continued

Member 2

$$\mathbf{k}_2 = \frac{EI}{8}\begin{bmatrix} 0.1875 & 0.75 & -0.1875 & 0.75 \\ 0.75 & 4 & -0.75 & 2 \\ -0.1875 & -0.75 & 0.1875 & -0.75 \\ 0.75 & 2 & -0.75 & 4 \end{bmatrix}$$

$\mathbf{Q} = \mathbf{KD}$

$$\begin{bmatrix} 8.0 \\ 0 \\ -8.0 \\ Q_4 - 24.0 \\ Q_5 - 24.0 \\ Q_6 - 24.0 \end{bmatrix} = \frac{EI}{8}\begin{bmatrix} 4 & 2 & 0 & 0.75 & -0.75 & 0 \\ 2 & 8 & 2 & 0.75 & 0 & -0.75 \\ 0 & 2 & 4 & 0 & 0.75 & -0.75 \\ 0.75 & 0.75 & 0 & 0.1875 & -0.1875 & 0 \\ -0.75 & 0 & 0.75 & -0.1875 & 0.375 & -0.1875 \\ 0 & -0.75 & -0.75 & 0 & -0.1875 & 0.1875 \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



$$8.0 = \frac{EI}{8}[4D_1 + 2D_2]$$

$$0 = \frac{EI}{8}[2D_1 + 8D_2 + 2D_3]$$

$$-8.0 = \frac{EI}{8}[2D_2 + 4D_3]$$

Solving:

$$D_1 = \frac{16.0}{EI}, \qquad D_2 = 0, \qquad D_3 = -\frac{16.0}{EI}$$

$$Q_4 - 24.0 = \frac{EI}{8}(0.75)\left(\frac{16.0}{EI}\right) + 0 + 0$$

$Q_4 = 25.5 \text{ k}$ **Ans.**

$$Q_5 - 24.0 = \frac{EI}{8}(-0.75)\left(\frac{16.0}{EI}\right) + 0 + \frac{EI}{8}(0.75)\left(-\frac{16.0}{EI}\right)$$

$Q_5 = 21.0 \text{ k}$ **Ans.**

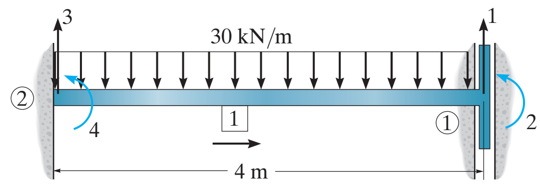
$$Q_6 - 24.0 = 0 + 0 + \frac{EI}{8}(-0.75)\left(\frac{-16.0}{EI}\right)$$

$Q_6 = 25.5 \text{ k}$ **Ans.**

$\circlearrowleft + \sum M_2 = 0; \quad 25.5(8) - 25.5(8) = 0 \quad$ (Check)

$+\uparrow \sum F = 0; \quad 25.5 + 21.0 + 25.5 - 72 = 0 \quad$ (Check)

15–11. Determine the reactions at the supports. There is a smooth slider at ①. EI is constant.

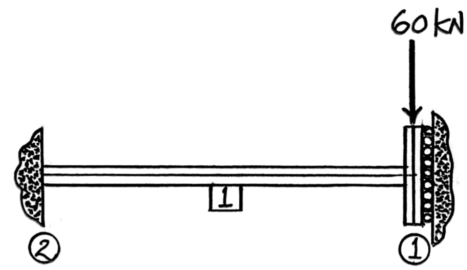


Member Stiffness Matrix. For member $\boxed{1}$,

$$\frac{12EI}{L^3} = \frac{12EI}{4^3} = 0.1875EI \qquad \frac{6EI}{L^2} = \frac{6EI}{4^2} = 0.375EI$$

$$\frac{4EI}{L} = \frac{4EI}{4} = EI \qquad \frac{2EI}{L} = \frac{2EI}{4} = 0.5EI$$

$$\mathbf{k}_1 = EI \begin{array}{c} \begin{array}{cccc} 3 & 4 & 1 & 2 \end{array} \\ \left[\begin{array}{cccc} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1.00 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1.00 \end{array}\right] \end{array} \begin{array}{c} \\ 3 \\ 4 \\ 1 \\ 2 \end{array}$$



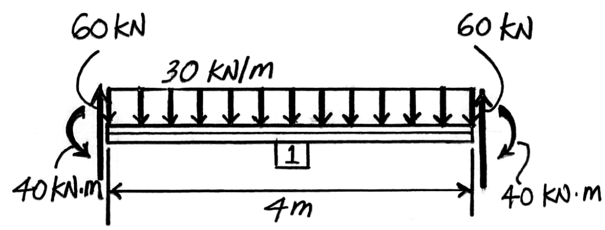
(a)

Known Nodal Loads And Deflections. The nodal load acting on the unconstrained degree of freedom (code number 1) is shown in Fig. *a*. Thus,

$$\mathbf{Q}_k = [-60]\ 1 \quad \text{and} \quad \mathbf{D}_k = \left[\begin{array}{c} 0 \\ 0 \\ 0 \end{array}\right] \begin{array}{c} 2 \\ 3 \\ 4 \end{array}$$

Load-Displacement Relation. The structure stiffness matrix is a 4 × 4 matirx since the highest code number is 4. Applying $\mathbf{Q} = \mathbf{KD}$,

$$\left[\begin{array}{c} -60 \\ \hline Q_2 \\ Q_3 \\ Q_4 \end{array}\right] = EI \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{c|ccc} 0.1875 & -0.375 & -0.1875 & -0.375 \\ \hline -0.375 & 1.00 & 0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & 0.375 \\ -0.375 & 0.5 & 0.375 & 1.00 \end{array}\right] \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \left[\begin{array}{c} D_1 \\ \hline 0 \\ 0 \\ 0 \end{array}\right]$$



(b)

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

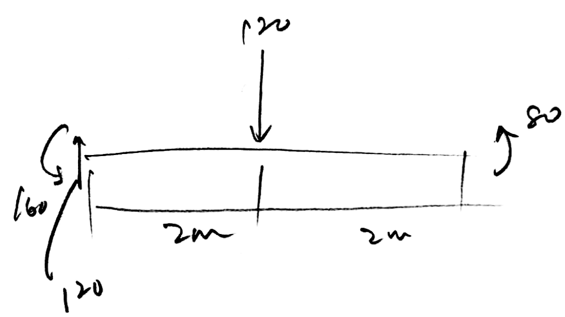
$$-60 = 0.1875EID_1 \qquad D_1 = -\frac{320}{EI}$$

Using this result, and applying $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$,

$$Q_2 = -0.375EI\left(-\frac{320}{EI}\right) + 0 = 120 \text{ kN}\cdot\text{m}$$

$$Q_3 = -0.1875EI\left(-\frac{320}{EI}\right) + 0 = 60 \text{ kN}$$

$$Q_4 = -0.375EI\left(-\frac{320}{EI}\right) + 0 = 120 \text{ kN}\cdot\text{m}$$



Superposition these results with the FEM shown in Fig. *b*,

$R_2 = 120 - 40 = 80 \text{ kN}\cdot\text{m}$ ↺ **Ans.**

$R_3 = 60 + 60 = 120 \text{ kN} \uparrow$ **Ans.**

$R_4 = 120 + 40 = 160 \text{ kN}\cdot\text{m}$ ↺ **Ans.**