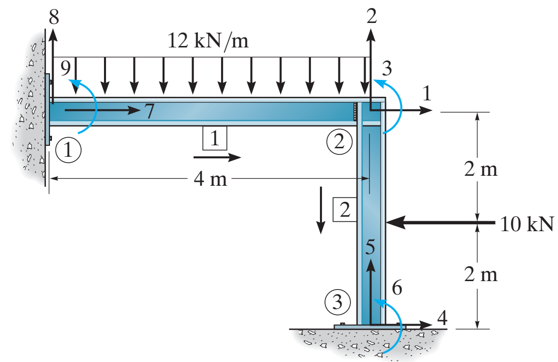
16–1. Determine the structure stiffness matrix **K** for the frame. Assume ① and ③ are fixed. Take $E = 200$ GPa, $I = 300(10^6)$ mm^4, $A = 10(10^3)$ mm^2 for each member.



Member Stiffness Matrices.

The orgin of the global coordinate system will be set at joint ①.

For member $\boxed{1}$ and $\boxed{2}$, $L = 4$m

$$\frac{AE}{L} = \frac{0.01[200(10^9)]}{4} = 500(10^6)\text{ N/m}$$

$$\frac{12EI}{L^3} = \frac{12[200(10^9)][300(10^{-6})]}{4^3} = 11.25(10^6)\text{ N/m}$$

$$\frac{6EI}{L^2} = \frac{6[200(10^9)][300(10^{-6})]}{4^2} = 22.5(10^6)\text{ N}$$

$$\frac{4EI}{L} = \frac{4[200(10^9)][300(10^{-6})]}{4} = 60(10^6)\text{ N}\cdot\text{m}$$

$$\frac{2EI}{L} = \frac{2[200(10^9)][300(10^{-6})]}{4} = 30(10^6)\text{ N}\cdot\text{m}$$

For member $\boxed{1}$, $\lambda_x = \dfrac{4-0}{4} = 1$ and $\lambda_y = \dfrac{0-0}{4} = 0$. Thus,

$$\mathbf{k}_1 = \begin{matrix} 7 & 8 & 9 & 1 & 2 & 3 & \\ \end{matrix}$$
$$\mathbf{k}_1 = \begin{bmatrix} 500 & 0 & 0 & -500 & 0 & 0 \\ 0 & 11.25 & 22.5 & 0 & -11.25 & 22.5 \\ 0 & 22.5 & 60 & 0 & -22.5 & 30 \\ -500 & 0 & 0 & 500 & 0 & 0 \\ 0 & -11.25 & -22.5 & 0 & 11.25 & -22.5 \\ 0 & 22.5 & 30 & 0 & -22.5 & 60 \end{bmatrix} \begin{matrix} 7 \\ 8 \\ 9 \\ 1 \\ 2 \\ 3 \end{matrix} (10^6)$$

For member $\boxed{2}$, $\lambda_x = \dfrac{4-4}{4} = 0$ and $\lambda_y = \dfrac{-4-0}{4} = -1$. Thus,

$$\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix}$$
$$\mathbf{k}_2 = \begin{bmatrix} 11.25 & 0 & 22.5 & -11.25 & 0 & 22.5 \\ 0 & 500 & 0 & 0 & -500 & 0 \\ 22.5 & 0 & 60 & -22.5 & 0 & 30 \\ -11.25 & 0 & -22.5 & 11.25 & 0 & -22.5 \\ 0 & -500 & 0 & 0 & 500 & 0 \\ 22.5 & 0 & 30 & -22.5 & 0 & 60 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} (10^6)$$

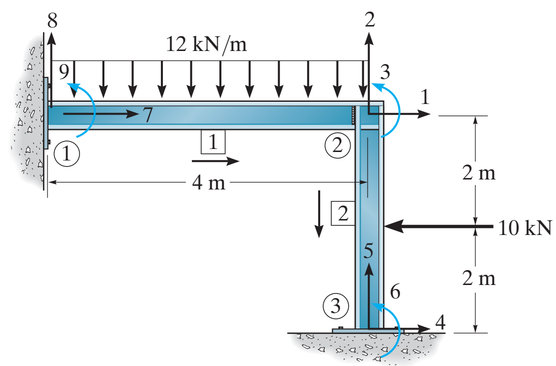
16–1. Continued

Structure Stiffness Matrix. It is a 9×9 matrix since the highest code number is 9. Thus,

$$\mathbf{K} = \begin{array}{c} \\ \end{array}\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array}$$

$$\mathbf{K} = \left[\begin{array}{ccccccccc} 511.25 & 0 & 22.5 & -11.25 & 0 & 22.5 & -500 & 0 & 0 \\ 0 & 511.25 & -22.5 & 0 & -500 & 0 & 0 & -11.25 & -22.25 \\ 22.5 & -22.5 & 120 & -22.5 & 0 & 30 & 0 & 22.5 & 30 \\ -11.25 & 0 & -22.5 & 11.25 & 0 & -22.5 & 0 & 0 & 0 \\ 0 & -500 & 0 & 0 & 500 & 0 & 0 & 0 & 0 \\ 22.5 & 0 & 30 & -22.5 & 0 & 60 & 0 & 0 & 0 \\ -500 & 0 & 0 & 0 & 0 & 0 & 500 & 0 & 0 \\ 0 & -11.25 & 22.5 & 0 & 0 & 0 & 0 & 11.25 & 22.5 \\ 0 & -22.5 & 30 & 0 & 0 & 0 & 0 & 25.5 & 60 \end{array}\right]\begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \\ 5\ (10^6) \\ 6 \\ 7 \\ 8 \\ 9 \end{array} \qquad \textbf{Ans.}$$

16–2. Determine the support reactions at the fixed supports ① and ③. Take $E = 200$ GPa, $I = 300(10^6)$ mm^4, $A = 10(10^3)$ mm^2 for each member.



Known Nodal Loads and Deflections. The nodal load acting on the unconstrained degree of freedom (code number 1, 2 and 3) are shown in Fig. *a* and *b*.

$$Q_k = \left[\begin{array}{c} -5(10^3) \\ -24(10^3) \\ 11(10^3) \end{array}\right]\begin{array}{l} 1 \\ 2 \\ 3 \end{array} \quad \text{and} \quad D_k = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right]\begin{array}{l} 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array}$$

Loads-Displacement Relation. Applying $\mathbf{Q} = \mathbf{KD}$,

$$\left[\begin{array}{c} -5(10^3) \\ -24(10^3) \\ 11(10^3) \\ \hline Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \end{array}\right] = \left[\begin{array}{ccc|cccccc} 511.25 & 0 & 22.5 & -11.25 & 0 & 22.5 & -500 & 0 & 0 \\ 0 & 511.25 & -22.5 & 0 & -500 & 0 & 0 & -11.25 & -22.5 \\ 22.5 & -22.5 & 120 & -22.5 & 0 & 30 & 0 & 22.5 & 30 \\ \hline -11.25 & 0 & -22.5 & 11.25 & 0 & -22.5 & 0 & 0 & 0 \\ 0 & -500 & 0 & 0 & 500 & 0 & 0 & 0 & 0 \\ 22.5 & 0 & 30 & -22.5 & 0 & 60 & 0 & 0 & 0 \\ -500 & 0 & 0 & 0 & 0 & 0 & 500 & 0 & 0 \\ 0 & -11.25 & 22.5 & 0 & 0 & 0 & 0 & 11.25 & 22.5 \\ 0 & -22.5 & 30 & 0 & 0 & 0 & 0 & 22.5 & 60 \end{array}\right](10^6)\left[\begin{array}{c} D_1 \\ D_2 \\ D_3 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right]$$

16–2. Continued

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

$$-5(10^3) = (511.25D_1 + 22.5D_3)(10^6) \qquad (1)$$

$$-24(10^3) = (511.25D_2 - 22.5D_3)(10^6) \qquad (2)$$

$$11(10^3) = (22.5D_1 - 22.5D_2 + 120D_3)(10^6) \qquad (3)$$

Solving Eqs. (1) to (3),

$$D_1 = -13.57(10^{-6})\text{ m} \qquad D_2 = -43.15(10^{-6})\text{m} \qquad D_3 = 86.12(10^{-6})\text{ rad}$$

Using these results and applying $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$,

$$Q_4 = -11.25(10^6)(-13.57)(10^{-6}) + (-22.5)(10^6)(86.12)(10^{-6}) = -1.785\text{ kN}$$

$$Q_5 = -500(10^6)(-43.15)(10^{-6}) = 21.58\text{ kN}$$

$$Q_6 = 22.5(10^6)(-13.57)(10^{-6}) + 30(10^6)(86.12)(10^{-6}) = 2.278\text{ kN}\cdot\text{m}$$

$$Q_7 = -500(10^6)(-13.57)(10^{-6}) = 6.785\text{ kN}$$

$$Q_8 = -11.25(10^6)(-43.15)(10^{-6}) + 22.5(10^6)(86.12)(10^{-6}) = 2.423\text{ kN}$$

$$Q_9 = -22.5(10^6)(-43.15)(10^{-6}) + 30(10^6)(86.12)(10^{-6}) = 3.555\text{ kN}\cdot\text{m}$$

Superposition these results to those of FEM shown in Fig. *a*,

$R_4 = -1.785 + 5 = 3.214\text{ kN} = 3.21\text{ kN} \quad \rightarrow$ **Ans.**

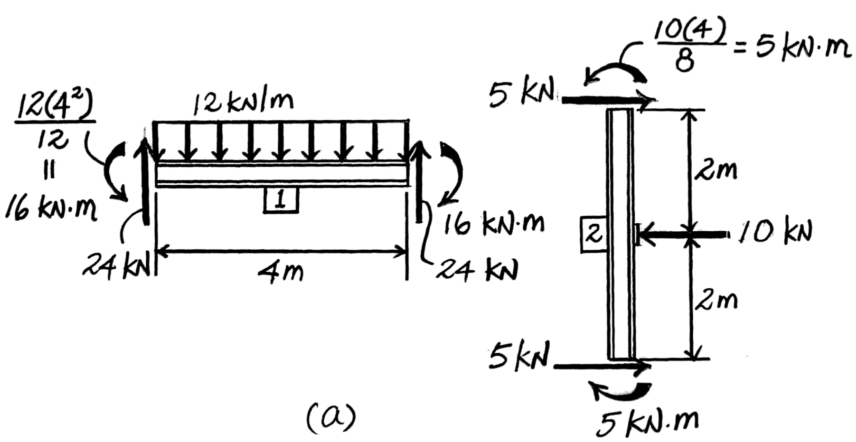
$R_5 = 21.58 + 0 = 21.58\text{ kN} = 21.6\text{ kN} \quad \uparrow$ **Ans.**

$R_6 = 2.278 - 5 = -2.722\text{ kN}\cdot\text{m} = 2.72\text{ kN}\cdot\text{m}$ ↻ **Ans.**

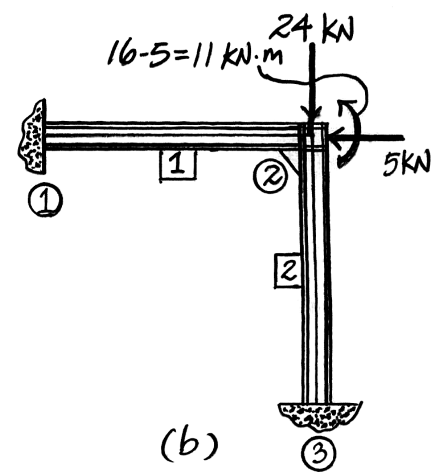
$R_7 = 6.785 + 0 = 6.785\text{ kN} = 6.79\text{ kN} \quad \rightarrow$ **Ans.**

$R_8 = 2.423 + 24 = 26.42\text{ kN} = 26.4\text{ kN} \quad \uparrow$ **Ans.**

$R_9 = 3.555 + 16 = 19.55\text{ kN}\cdot\text{m} = 19.6\text{ kN}\cdot\text{m}$ ↺ **Ans.**

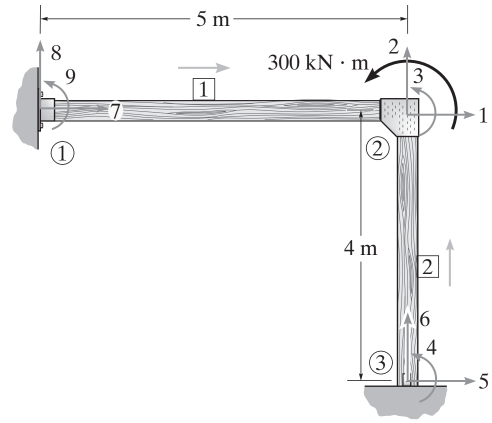


(a)



(b)

16–3. Determine the structure stiffness matrix **K** for the frame. Assume ③. is pinned and ①. is fixed. Take $E = 200$ MPa, $I = 300(10^6)$ mm^4, $A = 21(10^3)$ mm^2 for each member.



For member 1

$$\lambda_x = \frac{5-0}{5} = 1 \quad \lambda_y = 0$$

$$\frac{AE}{L} = \frac{(0.021)(200)(10^6)}{5} = 840000 \qquad \frac{12EI}{L^3} = \frac{(12)(200)(10^6)(300)(10^{-6})}{5^3} = 5760$$

$$\frac{6EI}{L^2} = \frac{6(200)(10^6)(300)(10^{-6})}{5^2} = 14400 \qquad \frac{2EI}{L} = \frac{2(200)(10^6)(300)(10^{-6})}{5} = 24000$$

$$\frac{4EI}{L} = \frac{4(200)(10^6)(300)(10^{-6})}{5} = 48000$$

$$\mathbf{k}_1 = \begin{bmatrix} 840000 & 0 & 0 & -840000 & 0 & 0 \\ 0 & 5760 & 14400 & 0 & -5760 & 14400 \\ 0 & 14400 & 48000 & 0 & -14400 & 24000 \\ -840000 & 0 & 0 & 840000 & 0 & 0 \\ 0 & -5760 & -14400 & 0 & 5760 & -14400 \\ 0 & 14400 & 24000 & 0 & -14400 & 48000 \end{bmatrix}$$

For member 2

$$\lambda_x = 0 \quad \lambda_y = \frac{0-(-4)}{4} = 1$$

$$\frac{AE}{L} = \frac{(0.021)(200)(10^6)}{5} = 1050000 \qquad \frac{12EI}{L^3} = \frac{(12)(200)(10^6)(300)(10^{-6})}{4^3} = 11250$$

$$\frac{6EI}{L^2} = \frac{6(200)(10^6)(300)(10^{-6})}{4^2} = 22500 \qquad \frac{2EI}{L} = \frac{2(200)(10^6)(300)(10^{-6})}{4} = 30000$$

$$\frac{4EI}{L} = \frac{4(200)(10^6)(300)(10^{-6})}{4} = 60000$$

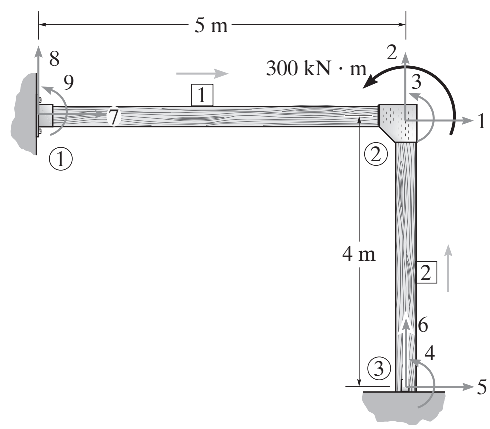
$$\mathbf{k}_2 = \begin{bmatrix} 11250 & 0 & -22500 & -11250 & 0 & -22500 \\ 0 & 1050000 & 0 & 0 & -1050000 & 0 \\ -22500 & 0 & 60000 & 22500 & 0 & 30000 \\ -11250 & 0 & 22500 & 11250 & 0 & 22500 \\ 0 & -1050000 & 0 & 0 & 1050000 & 0 \\ -22500 & 0 & 30000 & 22500 & 0 & 60000 \end{bmatrix}$$

16–3. Continued

Structure Stiffness Matrix.

$$\mathbf{K} = \begin{bmatrix} 851250 & 0 & 22500 & 22500 & -11250 & 0 & -840000 & 0 & 0 \\ 0 & 1055760 & -14400 & 0 & 0 & -1050000 & 0 & -5760 & -14400 \\ 22500 & -14400 & 108000 & 30000 & -22500 & 0 & 0 & 144000 & 24000 \\ 22500 & 0 & 30000 & 60000 & -22500 & 0 & 0 & 0 & 0 \\ -11250 & 0 & -22500 & -22500 & 11250 & 0 & 0 & 0 & 0 \\ 0 & -1050000 & 0 & 0 & 0 & 1050000 & 0 & 0 & 0 \\ -840000 & 0 & 0 & 0 & 0 & 0 & 840000 & 0 & 0 \\ 0 & -5760 & 14400 & 0 & 0 & 0 & 0 & 5760 & 14400 \\ 0 & -14400 & 24000 & 0 & 0 & 0 & 0 & 14400 & 48000 \end{bmatrix} \quad \textbf{Ans.}$$

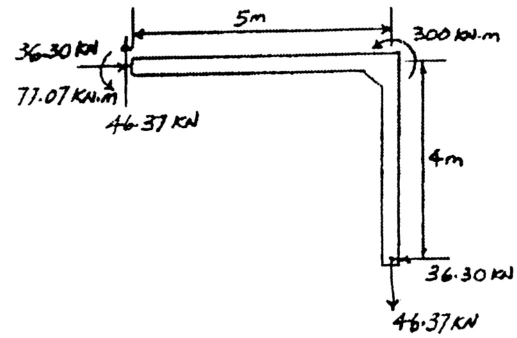
***16–4.** Determine the support reactions at ①. and ③. Take $E = 200$ MPa, $I = 300(10^6)$ mm^4, $A = 21(10^3)$ mm^2 for each member.



$$\mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{Q}_k = \begin{bmatrix} 0 \\ 0 \\ 300 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 300 \\ 0 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \end{bmatrix} = \begin{bmatrix} 851250 & 0 & 22500 & 22500 & -11250 & 0 & -840000 & 0 & 0 \\ 0 & 1055760 & -14400 & 0 & 0 & -1050000 & 0 & -5760 & -14400 \\ 22500 & -14400 & 108000 & 30000 & -22500 & 0 & 0 & 14400 & 24000 \\ 22500 & 0 & 30000 & 60000 & -22500 & 0 & 0 & 0 & 0 \\ -11250 & 0 & -22500 & -22500 & 11250 & 0 & 0 & 0 & 0 \\ 0 & -1050000 & 0 & 0 & 0 & 1050000 & 0 & 0 & 0 \\ -840000 & 0 & 0 & 0 & 0 & 0 & 840000 & 0 & 0 \\ 0 & -5760 & 14400 & 0 & 0 & 0 & 0 & 5760 & 14400 \\ 0 & -14400 & 24000 & 0 & 0 & 0 & 0 & 1440 & 48000 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

16–4. Continued



Partition matrix

$$\begin{bmatrix} 0 \\ 0 \\ 300 \\ 0 \end{bmatrix} = \begin{bmatrix} 851250 & 0 & 22500 & 22500 \\ 0 & 1055760 & -14400 & 0 \\ 22500 & -14400 & 108000 & 30000 \\ 22500 & 0 & 30000 & 60000 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$0 = 851250D_1 + 22500D_3 + 22500D_4$

$0 = 1055760D_2 - 14400D_3$

$300 = 22500D_1 - 14400D_2 + 108000D_3 + 30000D_4$

$0 = 22500D_1 + 30000D_3 + 60000D_4$

Solving.

$D_1 = -0.00004322$ m $\quad D_2 = 0.00004417$ m $\quad D_3 = 0.00323787$ rad
$D_4 = -0.00160273$ rad

$$\begin{bmatrix} Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \end{bmatrix} = \begin{bmatrix} -11250 & 0 & -22500 & -22500 \\ 0 & -1050000 & 0 & 0 \\ -840000 & 0 & 0 & 0 \\ 0 & -5760 & 14400 & 0 \\ 0 & -14400 & 24000 & 0 \end{bmatrix} \begin{bmatrix} -0.00004322 \\ 0.00004417 \\ 0.00323787 \\ -0.00160273 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$Q_5 = -36.3$ kN **Ans.**

$Q_6 = -46.4$ kN **Ans.**

$Q_7 = 36.3$ kN **Ans.**

$Q_8 = 46.4$ kN **Ans.**

$Q_9 = 77.1$ kN · m **Ans.**

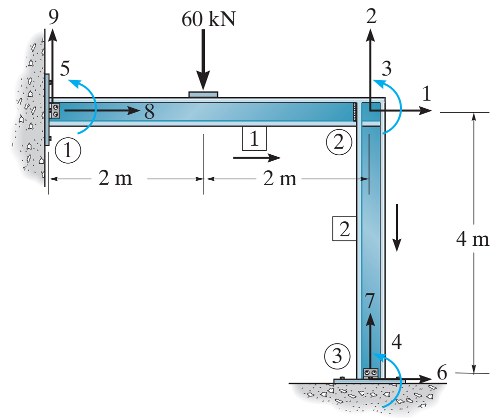
Check equilibrium

$\circlearrowleft \sum F_x = 0; \quad 36.30 - 36.30 = 0$ (Check)

$+\uparrow \sum F_y = 0; \quad 46.37 - 46.37 = 0$ (Check)

$\circlearrowleft + \sum M_1 = 0; \quad 300 + 77.07 - 36.30(4) - 46.37(5) = 0$ (Check)

16–5. Determine the structure stiffness matrix **K** for the frame. Take $E = 200$ GPa, $I = 350(10^6)\ \text{mm}^4$, $A = 15(10^3)\ \text{mm}^2$ for each member. Joints at ① and ③ are pins.



Member Stiffness Matrices. The origin of the global coordinate system will be set at joint ①. For member [1] and [2], L = 4m.

$$\frac{AE}{L} = \frac{0.015[200(10^9)]}{4} = 750(10^6)\ \text{N/m}$$

$$\frac{12EI}{L^3} = \frac{12[200(10^9)][350(10^{-6})]}{4^3} = 13.125(10^6)\ \text{N/m}$$

$$\frac{6EI}{L^2} = \frac{4[200(10^9)][350(10^{-6})]}{4^2} = 26.25(10^6)\ \text{N}$$

$$\frac{4EI}{L} = \frac{4[200(10^9)][350(10^{-6})]}{4} = 70(10^6)\ \text{N}\cdot\text{m}$$

$$\frac{2EI}{L} = \frac{2[200(10^9)][350(10^{-6})]}{4} = 35(10^6)\ \text{N}\cdot\text{m}$$

For member [1], $\lambda_x = \dfrac{4-0}{4} = 1$ and $\lambda_y = \dfrac{0-0}{4} = 0$. Thus,

$$\mathbf{k}_1 = \begin{array}{c} \begin{matrix} 8 & 9 & 5 & 1 & 2 & 3 \end{matrix} \\ \begin{bmatrix} 750 & 0 & 0 & -750 & 0 & 0 \\ 0 & 13.125 & 26.25 & 0 & -13.125 & 26.25 \\ 0 & 26.25 & 70 & 0 & -26.25 & 35 \\ -750 & 0 & 0 & 750 & 0 & 0 \\ 0 & -13.125 & -26.25 & 0 & 13.125 & -26.25 \\ 0 & 26.25 & 35 & 0 & -26.25 & 70 \end{bmatrix} \end{array} \begin{matrix} 8 \\ 9 \\ 5 \\ 1 \\ 2 \\ 3 \end{matrix} (10^6)$$

For member [2], $\lambda_x = \dfrac{4-4}{4} = 0$, and $\lambda_y = \dfrac{-4-0}{4} = -1$. Thus,

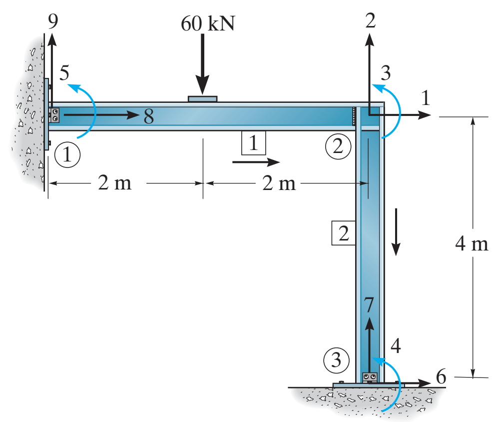
$$\mathbf{k}_2 = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 6 & 7 & 4 \end{matrix} \\ \begin{bmatrix} 13.125 & 0 & 26.25 & -13.125 & 0 & 26.25 \\ 0 & 750 & 0 & 0 & -750 & 0 \\ 26.25 & 0 & 70 & -26.25 & 0 & 35 \\ -13.125 & 0 & -26.25 & 13.125 & 0 & -26.25 \\ 0 & -750 & 0 & 0 & 750 & 0 \\ 26.25 & 0 & 35 & -26.25 & 0 & 70 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 6 \\ 7 \\ 4 \end{matrix} (10^6)$$

16–5. Continued

Structure Stiffness Matrix. It is a 9 × 9 matrix since the highest code number is 9. Thus

$$\mathbf{K} = \begin{array}{c} \\ \end{array}\begin{bmatrix} \overset{1}{763.125} & \overset{2}{0} & \overset{3}{26.25} & \overset{4}{26.25} & \overset{5}{0} & \overset{6}{-13.125} & \overset{7}{0} & \overset{8}{-750} & \overset{9}{0} \\ 0 & 763.125 & -26.25 & 0 & -26.25 & 0 & -750 & 0 & -13.125 \\ 26.25 & -26.25 & 140 & 35 & 35 & -26.25 & 0 & 0 & 26.25 \\ 26.25 & 0 & 35 & 70 & 0 & -26.25 & 0 & 0 & 0 \\ 0 & -26.25 & 35 & 0 & 70 & 0 & 0 & 0 & 26.25 \\ -13.125 & 0 & -26.25 & -26.25 & 0 & 13.125 & 0 & 0 & 0 \\ 0 & -750 & 0 & 0 & 0 & 0 & 750 & 0 & 0 \\ -750 & 0 & 0 & 0 & 0 & 0 & 0 & 750 & 0 \\ 0 & -13.125 & 26.25 & 0 & 26.25 & 0 & 0 & 0 & 13.125 \end{bmatrix}\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{matrix}(10^6) \qquad \textbf{Ans.}$$

16–6. Determine the support reactions at pins ① and ③. Take $E = 200$ GPa, $I = 350(10^6)$ mm^4, $A = 15(10^3)$ mm^2 for each member.



Known Nodal Loads and Deflections. The nodal load acting on the unconstrained degree of freedom (code numbers 1, 2, 3, 4, and 5) are shown in Fig. *a* and Fig. *b*.

$$\mathbf{Q}_k = \begin{bmatrix} 0 \\ -41.25(10^3) \\ 45(10^3) \\ 0 \\ 0 \end{bmatrix}\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \quad \text{and} \quad \mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{matrix} 6 \\ 7 \\ 8 \\ 9 \end{matrix}$$

Loads-Displacement Relation. Applying $\mathbf{Q} = \mathbf{KD}$,

$$\begin{bmatrix} 0 \\ -41.25(10^3) \\ 45(10^3) \\ 0 \\ 0 \\ \hline Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \end{bmatrix} = \left[\begin{array}{ccccc|cccc} 763.125 & 0 & 26.25 & 26.25 & 0 & -13.125 & 0 & -750 & 0 \\ 0 & 763.125 & -26.25 & 0 & -26.25 & 0 & -750 & 0 & -13.125 \\ 26.25 & -26.25 & 140 & 35 & 35 & -26.25 & 0 & 0 & 26.25 \\ 26.25 & 0 & 35 & 70 & 0 & -26.25 & 0 & 0 & 0 \\ 0 & -26.25 & 35 & 0 & 70 & 0 & 0 & 0 & 26.25 \\ \hline -13.125 & 0 & -26.25 & -26.25 & 0 & 13.125 & 0 & 0 & 0 \\ 0 & -750 & 0 & 0 & 0 & 0 & 750 & 0 & 0 \\ -750 & 0 & 0 & 0 & 0 & 0 & 0 & 750 & 0 \\ 0 & -13.125 & 26.25 & 0 & 26.25 & 0 & 0 & 0 & 13.125 \end{array}\right](10^6) \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ \hline 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

16–6. Continued

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

$$0 = (763.125D_1 + 26.25D_3 + 26.25D_4)(10^6) \quad (1)$$

$$-41.25(10^3) = (763.125D_2 - 26.25D_3 - 26.25D_5)(10^6) \quad (2)$$

$$45(10^3) = (26.25D_1 - 26.25D_2 + 140D_3 + 35D_4 + 35D_5)(10^6) \quad (3)$$

$$0 = (26.25D_1 + 35D_3 + 70D_4)(10^6) \quad (4)$$

$$0 = (-26.25D_2 + 35D_3 + 70D_5)(10^6) \quad (5)$$

Solving Eqs. (1) to (5)

$D_1 = -7.3802(10^{-6}) \quad D_2 = -47.3802(10^{-6}) \quad D_3 = 423.5714(10^{-6})$

$D_4 = -209.0181(10^{-6}) \quad D_5 = -229.5533(10^{-6})$

Using these results and applying $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$,

$Q_6 = (-13.125)(10^6) - 7.3802(10^{-6}) - 26.25(10^6)423.5714(10^{-6}) - 26.25(10^6) - 209.0181(10^{-6}) + 0 = -5.535 \text{ kN}$

$Q_7 = -750(10^6) - 47.3802(10^{-6}) + 0 = 35.535 \text{ kN}$

$Q_8 = -750(10^6) - 7.3802(10^{-6}) + 0 = 5.535 \text{ kN}$

$Q_9 = -13.125(10^6) - 47.3802(10^{-6}) + 26.25(10^6) + 423.5714(10^{-6}) + 26.25(10^6) - 229.5533(10^{-6}) + 0 = 5.715 \text{ kN}$

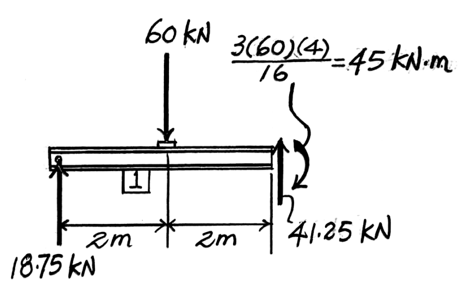
Superposition these results to those of FEM shown in Fig. *a*,

$R_6 = -5.535 \text{ kN} + 0 = 5.54 \text{ kN}$ **Ans.**

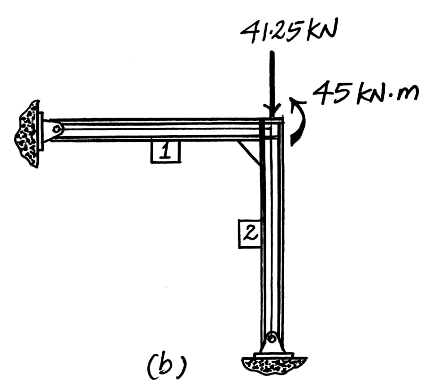
$R_7 = 35.535 + 0 = 35.5 \text{ kN}$ **Ans.**

$R_8 = 5.535 + 0 = 5.54 \text{ kN}$ **Ans.**

$R_9 = 5.715 + 18.75 = 24.5 \text{ kN}$ **Ans.**

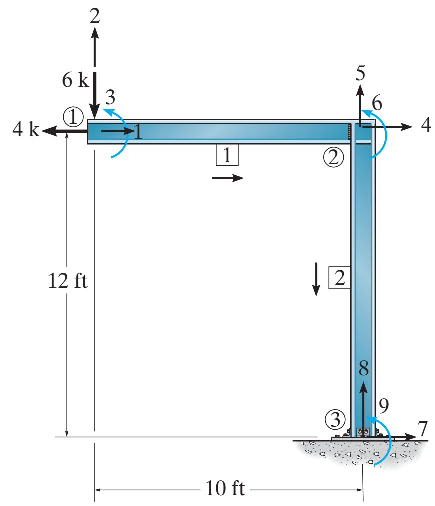


(a)



(b)

16–7. Determine the structure stiffness matrix **K** for the frame. Take $E = 29(10^3)$ ksi, $I = 650\ \text{in}^4$, $A = 20\ \text{in}^2$ for each member.



Member 1.

$$\lambda_x = \frac{10 - 0}{10} = 1 \qquad \lambda_y = 0$$

$$\frac{AE}{L} = \frac{20(29)(10^3)}{10(12)} = 4833.33 \qquad \frac{12EI}{L^3} = \frac{12(29)(10^3)(650)}{(10)^3(12)^3} = 130.90$$

$$\frac{6EI}{L^2} = \frac{6(29)(10^3)(650)}{(10)^2(12)^2} = 7854.17 \qquad \frac{4EI}{L} = \frac{4(29)(10^3)(650)}{(10)(12)} = 628333.33$$

$$\frac{2EI}{L} = \frac{2(29)(10^3)(650)}{(10)(12)} = 314166.67$$

$$\mathbf{k}_1 = \begin{bmatrix} 4833.33 & 0 & 0 & -4833.33 & 0 & 0 \\ 0 & 130.90 & 7854.17 & 0 & -130.90 & 7854.17 \\ 0 & 7854.17 & 628333.33 & 0 & -7854.17 & 314166.67 \\ -4833.33 & 0 & 0 & 4833.33 & 0 & 0 \\ 0 & -130.90 & -7854.17 & 0 & 130.90 & -7854.17 \\ 0 & 7854.17 & 314166.67 & 0 & -7854.17 & 628333.33 \end{bmatrix}$$

Member 2.

$$\lambda_x = 0 \qquad \lambda_y = \frac{-12 - 0}{12} = -1$$

$$\frac{AE}{L} = \frac{(20)(29)(10^3)}{(12)(12)} = 4027.78 \qquad \frac{12EI}{L^3} = \frac{12(29)(10^3)(650)}{(12)^3(12)^3} = 75.75$$

$$\frac{6EI}{L} = \frac{6(29)(10^3)(650)}{(12)^2(12)^2} = 5454.28 \qquad \frac{4EI}{L} = \frac{4(29)(10^3)(650)}{(12)(12)} = 523611.11$$

$$\frac{2EI}{L^2} = \frac{2(29)(10^3)(650)}{(12)(12)} = 261805.55$$

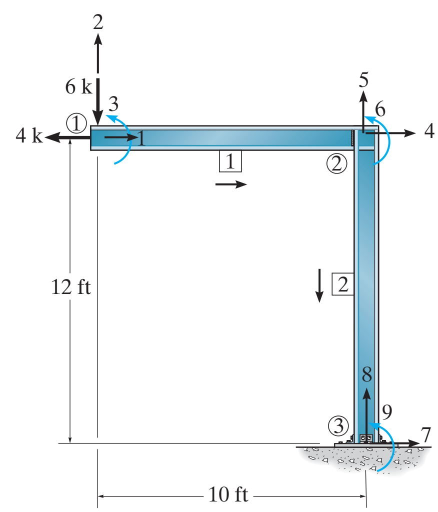
$$\mathbf{k}_2 = \begin{bmatrix} 75.75 & 0 & 5454.28 & -75.75 & 0 & 5454.28 \\ 0 & 4027.78 & 0 & 0 & -4027.78 & 0 \\ 5454.28 & 0 & 523611.11 & -5454.28 & 0 & 261805.55 \\ -75.75 & 0 & -5454.28 & 75.75 & 0 & -5454.28 \\ 0 & -4027.78 & 0 & 0 & 4027.78 & 0 \\ 5454.28 & 0 & 261805.55 & -5454.28 & 0 & 523611.11 \end{bmatrix}$$

16–7. Continued

Structure Stiffness Matrix.

$$\mathbf{K} = \begin{bmatrix} 4833.33 & 0 & 0 & -4833.33 & 0 & 0 & 0 & 0 & 0 \\ 0 & 130.90 & 7854.17 & 0 & -130.90 & 7854.17 & 0 & 0 & 0 \\ 0 & 7854.17 & 628333.33 & 0 & -7854.17 & 314166.67 & 0 & 0 & 0 \\ -4833.33 & 0 & 0 & 4909.08 & 0 & 5454.28 & -75.75 & 0 & 5454.28 \\ 0 & -130.90 & -7854.17 & 0 & 4158.68 & -7854.37 & 0 & -4027.78 & 0 \\ 0 & 7854.17 & 314166.67 & 5454.28 & -7854.17 & 1151944.44 & -5454.28 & 0 & 261805.55 \\ 0 & 0 & 0 & -75.75 & 0 & -5454.28 & 75.75 & 0 & -5454.28 \\ 0 & 0 & 0 & 0 & -4027.78 & 0 & 0 & 4027.78 & 0 \\ 0 & 0 & 0 & 5454.28 & 0 & 261805.55 & -5454.28 & 0 & 523611.11 \end{bmatrix} \quad \textbf{Ans.}$$

***16–8.** Determine the components of displacement at ①. Take $E = 29(10^3)$ ksi, $I = 650 \text{ in}^4$, $A = 20 \text{ in}^2$ for each member.



$$\mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{Q}_k = \begin{bmatrix} -4 \\ -6 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -4 \\ -6 \\ 0 \\ 0 \\ 0 \\ 0 \\ Q_7 \\ Q_8 \\ Q_9 \end{bmatrix} = \begin{bmatrix} 4833.33 & 0 & 0 & -4833.33 & 0 & 0 & 0 & 0 & 0 \\ 0 & 130.90 & 7854.17 & 0 & -130.90 & 7854.17 & 0 & 0 & 0 \\ 0 & 7854.17 & 628333.33 & 0 & -7854.17 & 314166.67 & 0 & 0 & 0 \\ -4833.33 & 0 & 0 & 4909.08 & 0 & 5454.28 & -75.75 & 0 & 5454.28 \\ 0 & -130.90 & -7854.17 & 0 & 4158.68 & -7854.17 & 0 & -4027.78 & 0 \\ 0 & 7854.17 & 314166.67 & 5454.28 & -7854.17 & 1151944.44 & -5454.28 & 0 & 261805.55 \\ 0 & 0 & 0 & -75.75 & 0 & -5454.28 & 75.75 & 0 & -5454.28 \\ 0 & 0 & 0 & 0 & -4027.78 & 0 & 0 & 4027.78 & 0 \\ 0 & 0 & 0 & 5454.28 & 0 & 261805.55 & -5454.28 & 0 & 523611.11 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

16–8. Continued

Partition Matrix.

$$\begin{bmatrix} -4 \\ -6 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 4833.33 & 0 & 0 & -4833.33 & 0 & 0 \\ 0 & 130.90 & 7854.17 & 0 & -130.90 & 7854.17 \\ 0 & 7854.17 & 628333.33 & 0 & -7854.17 & 314166.67 \\ -4833.33 & 0 & 0 & 4909.08 & 0 & 5454.28 \\ 0 & -130.90 & -7854.17 & 0 & 4158.68 & -7854.17 \\ 0 & 7854.17 & 314166.67 & 5454.28 & -7854.17 & 1151944.44 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$-4 = 4833.33D_1 - 4833.33D_4$

$-6 = 130.90D_2 + 7854.17D_3 - 130.90D_5 + 7854.17D_6$

$0 = 7854.17D_2 + 628333.33D_3 - 7854.17D_5 + 314166.67D_6$

$0 = -4833.33D_1 + 4909.08D_4 + 5454.28D_6$

$0 = -130.90D_2 - 7854.17D_3 + 4158.68D_5 - 7854.17D_6$

$0 = 7854.17D_2 + 314166.67D_3 + 5454.28D_4 - 7854.17D_5 + 1151944.44D_6$

Solving the above equations yields

$D_1 = -0.608$ in. **Ans.**

$D_2 = -1.12$ in. **Ans.**

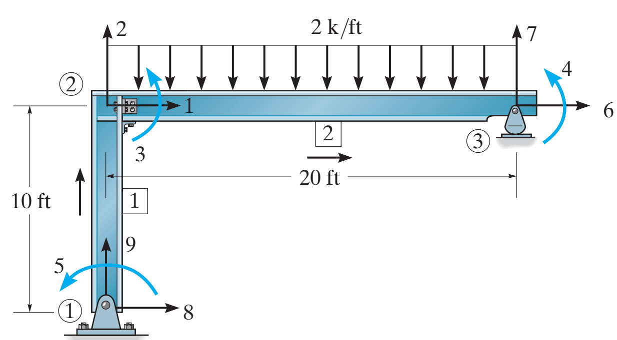
$D_3 = 0.0100$ rad **Ans.**

$D_4 = -0.6076$ in.

$D_5 = -0.001490$ in.

$D_6 = 0.007705$ rad

16–9. Determine the stiffness matrix **K** for the frame. Take $E = 29(10^3)$ ksi, $I = 300 \text{ in}^4$, $A = 10 \text{ in}^2$ for each member.



Member Stiffness Matrices. The origin of the global coordinate system will be set at joint ①. For member [1], $L = 10$ ft, $\lambda_x = \dfrac{0-0}{10} = 0$ and $\lambda_y = \dfrac{10-0}{10} = 1$

$$\frac{AE}{L} = \frac{10[29(10^3)]}{10(12)} = 2416.67 \text{ k/in} \qquad \frac{12EI}{L^3} = \frac{12[29(10^3)](300)}{[10(12)]^3} = 60.4167 \text{ k/in}$$

$$\frac{6EI}{L^2} = \frac{6[29(10^3)](300)}{[10(12)]^2} = 3625 \text{ k} \qquad \frac{4EI}{L} = \frac{4[29(10^3)](300)}{10(12)} = 290000 \text{ k}\cdot\text{in}$$

16–9. Continued

$$\frac{2EI}{L} = \frac{2[29(10^3)](300)}{10(12)} = 145000 \text{ k} \cdot \text{in}$$

$$\mathbf{k}_1 = \begin{array}{c} \\ \left[\begin{array}{cccccc} 8 & 9 & 5 & 1 & 2 & 3 \\ 60.4167 & 0 & -3625 & -60.4167 & 0 & -3625 \\ 0 & 2416.67 & 0 & 0 & -2416.67 & 0 \\ -3625 & 0 & 290000 & 3625 & 0 & 145000 \\ -60.4167 & 0 & 3625 & 60.4167 & 0 & 3625 \\ 0 & -2416.67 & 0 & 0 & 2416.67 & 0 \\ -3625 & 0 & 145000 & 3625 & 0 & 290000 \end{array}\right] \end{array} \begin{array}{c} \\ 8 \\ 9 \\ 5 \\ 1 \\ 2 \\ 3 \end{array}$$

For member $\boxed{2}$, $L = 20$ ft, $\lambda_x = \dfrac{20 - 0}{20} = 1$ and $\lambda_y = \dfrac{10 - 10}{20} = 0$.

$$\frac{AE}{L} = \frac{10[29(10^3)]}{20(12)} = 1208.33 \text{ k/in} \qquad \frac{12EI}{L^3} = \frac{12[29(10^3)](300)}{[20(12)]^3} = 7.5521 \text{ k/in}$$

$$\frac{6EI}{L^2} = \frac{6[29(10^3)](300)}{[20(12)]^2} = 906.25 \text{ k} \qquad \frac{4EI}{L} = \frac{4[29(10^3)](300)}{20(12)} = 145000 \text{ k} \cdot \text{in}$$

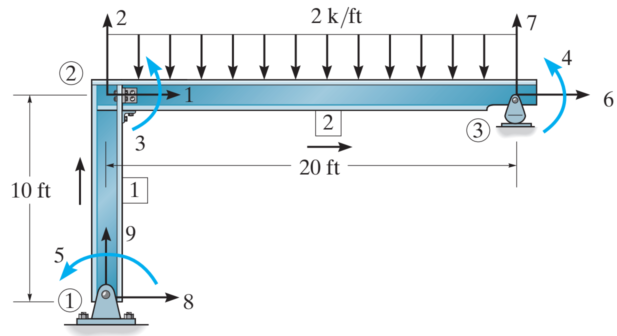
$$\frac{2EI}{L} = \frac{2[29(10^3)](300)}{20(12)} = 72500 \text{ k} \cdot \text{in}$$

$$\mathbf{k}_2 = \begin{array}{c} \\ \left[\begin{array}{cccccc} 1 & 2 & 3 & 6 & 7 & 4 \\ 1208.33 & 0 & 0 & -1208.33 & 0 & 0 \\ 0 & 7.5521 & 906.25 & 0 & -7.5521 & 906.25 \\ 0 & 906.25 & 145000 & 0 & -906.25 & 72500 \\ -1208.33 & 0 & 0 & 1208.33 & 0 & 0 \\ 0 & -7.5521 & -906.25 & 0 & 7.5521 & -906.25 \\ 0 & 906.25 & 72500 & 0 & -906.25 & 145000 \end{array}\right] \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 6 \\ 7 \\ 4 \end{array}$$

Structure Stiffness Matrix. It is a 9×9 matrix since the highest code number is 9. Thus,

$$\mathbf{K} = \begin{bmatrix} 1268.75 & 0 & 3625 & 0 & 3625 & -1208.33 & 0 & -60.4167 & 0 \\ 0 & 2424.22 & 906.25 & 906.25 & 0 & 0 & -7.5521 & 0 & -2416.67 \\ 3625 & 906.25 & 435000 & 72500 & 145000 & 0 & -906.25 & -3625 & 0 \\ 0 & 906.25 & 72500 & 145000 & 0 & 0 & -906.25 & 0 & 0 \\ 3625 & 0 & 145000 & 0 & 290000 & 0 & 0 & -3625 & 0 \\ -1208.33 & 0 & 0 & 0 & 0 & 1208.33 & 0 & 0 & 0 \\ 0 & -7.5521 & -906.25 & -906.25 & 0 & 0 & 7.5521 & 0 & 0 \\ -60.4167 & 0 & -3625 & 0 & -3625 & 0 & 0 & 60.4167 & 0 \\ 0 & -2416.67 & 0 & 0 & 0 & 0 & 0 & 0 & 2416.67 \end{bmatrix}$$

16–10. Determine the support reactions at ① and ③. Take $E = 29(10^3)$ ksi, $I = 300\text{ in}^4$, $A = 10\text{ in}^2$ for each member.



Known Nodal Loads and Deflections. The nodal loads acting on the unconstrained degree of freedom (code number 1, 2, 3, 4, 5, and 6) are shown in Fig. *a* and *b*.

$$\mathbf{Q}_k = \begin{bmatrix} 0 \\ -25 \\ -1200 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} \quad \text{and} \quad \mathbf{D}_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 7 \\ 8 \\ 9 \end{matrix}$$

Loads–Displacement Relation. Applying $\mathbf{Q} = \mathbf{KD}$.

$$\begin{bmatrix} 0 \\ -25 \\ -1200 \\ 0 \\ 0 \\ 0 \\ \hline Q_7 \\ Q_8 \\ Q_9 \end{bmatrix} = \left[\begin{array}{cccccc|ccc} 1268.75 & 0 & 3625 & 0 & 3625 & -1208.33 & 0 & -60.4167 & 0 \\ 0 & 2424.22 & 906.25 & 906.25 & 0 & 0 & -7.5521 & 0 & -2416.67 \\ 3625 & 906.25 & 435000 & 72500 & 145000 & 0 & -906.25 & -3625 & 0 \\ 0 & 906.25 & 72500 & 145000 & 0 & 0 & -906.25 & 0 & 0 \\ 3625 & 0 & 145000 & 0 & 290000 & 0 & 0 & -3625 & 0 \\ -1208.33 & 0 & 0 & 0 & 0 & 1208.33 & 0 & 0 & 0 \\ \hline 0 & -7.5521 & -906.25 & -906.25 & 0 & 0 & 7.5521 & 0 & 0 \\ -60.4167 & 0 & -3625 & 0 & -3625 & 0 & 0 & 60.4167 & 0 \\ 0 & -2416.67 & 0 & 0 & 0 & 0 & 0 & 0 & 2416.37 \end{array}\right] \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix}$$

$20 = -2416.67D_2$

$D_2 = -8.275862071(10^{-3})$

$5 = -7.5521(-8.2758)(10^{-3}) - 906.25D_3 - 906.25D_4$

$0 = 906.25(-8.2758)(10^{-3}) + 72500D_3 + 145000D_4$

$4.937497862 = -906.25D_3 - 906.25D_4$

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

$0 = 1268.75D_1 + 3625D_3 + 3625D_5 - 1208.33D_6$ (1)

$-25 = 2424.22D_2 + 906.25D_3 + 906.25D_4$ (2)

$-1200 = 3625D_1 + 906.25D_2 + 435000D_3 + 72500D_4 + 145000D_5$ (3)

$0 = 906.25D_2 + 72500D_3 + 145000D_4$ (4)

$0 = 3625D_1 + 145000D_3 + 290000D_5$ (5)

$0 = -1208.33D_1 + 1208.33D_6$ (6)

16–10. Continued

Solving Eqs. (1) to (6)

$D_1 = 1.32 \qquad D_2 = -0.008276 \qquad D_3 = -0.011 \qquad D_4 = 0.005552$

$D_5 = -0.011 \qquad D_6 = 1.32$

Using these results and applying $\mathbf{Q}_k = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$,

$Q_7 = -7.5521(-0.008276) - 906.25(-0.011) - 906.25(0.005552) = 5$

$Q_8 = 60.4167(1.32) - 3625(-0.011) - 3625(-0.011) = 0$

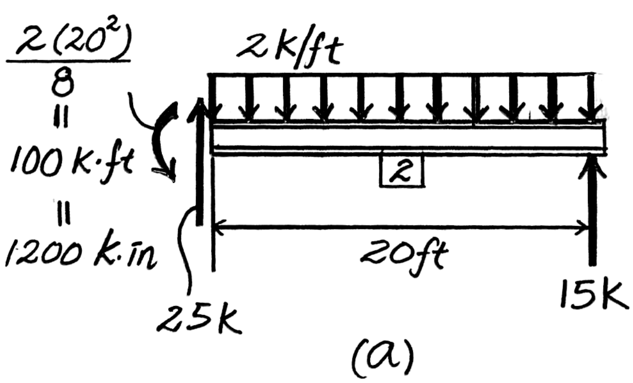
$Q_9 = -2416.67(-0.008276) = 20$

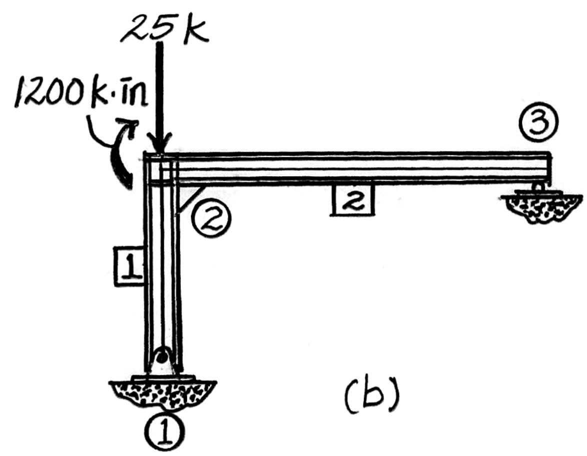
Superposition these results to those of FEM shown in Fig. *a*.

$R_7 = 5 + 15 = 20\text{ k}$ **Ans.**

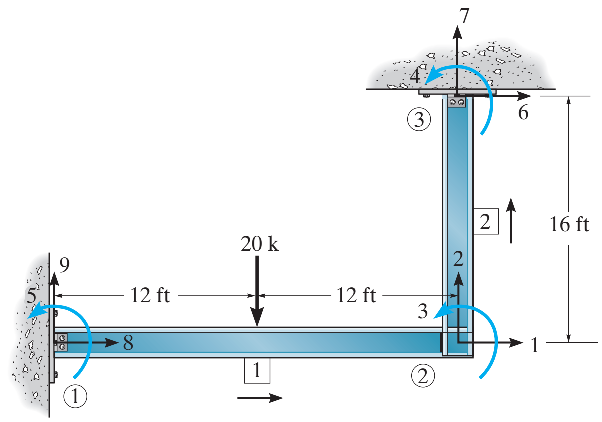
$R_8 = 0 + 0 = 0$ **Ans.**

$R_9 = 20 + 0 = 20\text{ k}$ **Ans.**





16–11. Determine the structure stiffness matrix **K** for the frame. Take $E = 29(10^3)$ ksi, $I = 700$ in^4, $A = 20$ in^2 for each member.



Member Stiffness Matrices. The origin of the global coordinate system will be set at joint ①. For member [1], $L = 24$ ft, $\lambda_x = \dfrac{24 - 0}{24} = 1$ and $\lambda_y = \dfrac{0 - 0}{24} = 0$

$$\frac{AE}{L} = \frac{20[29(10^3)]}{24(12)} = 2013.89 \text{ k/in} \qquad \frac{12EI}{L^3} = \frac{12[29(10^3)](700)}{[24(12)]^3} = 10.1976 \text{ k/in}$$

$$\frac{6EI}{L^2} = \frac{6[29(10^3)](700)}{[24(12)]^2} = 1468.46 \text{ k} \qquad \frac{4EI}{L} = \frac{4[29(10^3)](700)}{[24(12)]} = 281944 \text{ k}\cdot\text{in}$$

$$\frac{2EI}{L} = \frac{2[29(10^3)](700)}{[24(12)]} = 140972 \text{ k}\cdot\text{in}$$

$$\mathbf{k}_1 = \begin{array}{c} \begin{matrix} 8 & 9 & 5 & 1 & 2 & 3 \end{matrix} \\ \begin{bmatrix} 2013.89 & 0 & 0 & -2013.89 & 0 & 0 \\ 0 & 10.1976 & 1468.46 & 0 & -10.1976 & 1468.46 \\ 0 & 1468.46 & 281944 & 0 & -1468.46 & 140972 \\ -2013.89 & 0 & 0 & 2013.89 & 0 & 0 \\ 0 & -10.1976 & -1468.46 & 0 & 10.1976 & -1468.46 \\ 0 & 1468.46 & 140972 & 0 & -1468.46 & 281944 \end{bmatrix} \end{array} \begin{matrix} 8 \\ 9 \\ 5 \\ 1 \\ 2 \\ 3 \end{matrix}$$

For member [2], $L = 16$ ft, $\lambda_x = \dfrac{24 - 24}{16} = 0$ and $\lambda_y = \dfrac{16 - 0}{16} = 1.$

$$\frac{AE}{L} = \frac{20[29(10^3)]}{16(12)} = 3020.83 \text{ k/in.} \qquad \frac{12EI}{L^3} = \frac{12[29(10^3)](700)}{[16(12)]^3} = 34.4170 \text{ k/in}$$

$$\frac{6EI}{L^2} = \frac{6[29(10^3)](700)}{[16(12)]^2} = 3304.04 \text{ k} \qquad \frac{4EI}{L} = \frac{4[29(10^3)](700)}{[16(12)]} = 422917 \text{ k}\cdot\text{in}$$

$$\frac{2EI}{L} = \frac{2[29(10^3)](700)}{[16(12)]} = 211458 \text{ k}\cdot\text{in}$$

16–11. Continued

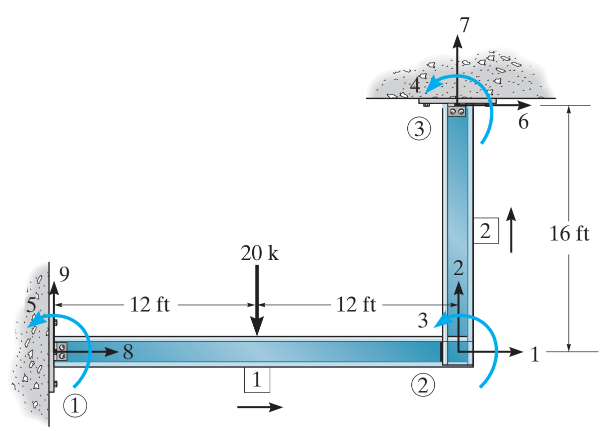
$$\mathbf{k}_2 = \begin{array}{c} \\ \end{array}\begin{array}{cccccc} 1 & 2 & 3 & 6 & 7 & 4 \end{array}$$

$$\mathbf{k}_2 = \left[\begin{array}{cccccc} 34.4170 & 0 & -3304.04 & -34.4170 & 0 & -3304.04 \\ 0 & 3020.83 & 0 & 0 & -3020.83 & 0 \\ -3304.04 & 0 & 422917 & 3304.04 & 0 & 211458 \\ -34.4170 & 0 & 3304.04 & 34.4170 & 0 & 3304.04 \\ 0 & -3020.83 & 0 & 0 & 3020.83 & 0 \\ -3304.04 & 0 & 211458 & 3304.04 & 0 & 422917 \end{array}\right]\begin{array}{c} 1 \\ 2 \\ 3 \\ 6 \\ 7 \\ 4 \end{array}$$

Structure Stiffness Matrix. It is a 9×9 matrix since the highest code number is 9. Thus,

$$\mathbf{K} = \left[\begin{array}{ccccccccc} 2048.31 & 0 & -3304.04 & -3304.04 & 0 & -34.4170 & 0 & -2013.89 & 0 \\ 0 & 3031.03 & -1468.46 & 0 & -1468.46 & 0 & -3020.83 & 0 & -10.1976 \\ -3304.04 & -1468.46 & 704861 & 211458 & 140972 & 3304.04 & 0 & 0 & 1468.46 \\ -3304.04 & 0 & 211458 & 422917 & 0 & 3304.04 & 0 & 0 & 0 \\ 0 & -1468.46 & 140972 & 0 & 281944 & 0 & 0 & 0 & 1468.46 \\ -34.4170 & 0 & 3304.04 & 3304.04 & 0 & 34.4170 & 0 & 0 & 0 \\ 0 & -3020.83 & 0 & 0 & 0 & 0 & 3020.83 & 0 & 0 \\ -2013.89 & 0 & 0 & 0 & 0 & 0 & 0 & 2013.89 & 0 \\ 0 & -10.1976 & 1468.46 & 0 & 1468.46 & 0 & 0 & 0 & 10.1976 \end{array}\right]$$

***16–12.** Determine the support reactions at the pins ① and ③. Take $E = 29(10^3)$ ksi, $I = 700\ \text{in}^4$, $A = 20\ \text{in}^2$ for each member.



Known Nodal Loads and Deflections. The nodal loads acting on the unconstrained degree of freedom (code number 1, 2, 3, 4, and 5) are shown in Fig. *a* and *b*.

$$\mathbf{Q}_k = \left[\begin{array}{c} 0 \\ -13.75 \\ 1080 \\ 0 \\ 0 \end{array}\right]\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \quad \text{and} \quad \mathbf{D}_k = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array}\right]\begin{array}{c} 6 \\ 7 \\ 8 \\ 9 \end{array}$$

16–12. Continued

Loads-Displacement Relation. Applying $\mathbf{Q} = \mathbf{KD}$,

$$
\begin{bmatrix} 0 \\ -13.75 \\ 90 \\ 0 \\ 0 \\ \hline Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \end{bmatrix}
=
\left[\begin{array}{ccccc|cccc}
2048.31 & 0 & -3304.04 & -3304.04 & 0 & -34.4170 & 0 & -2013.89 & 0 \\
0 & 3031.03 & -1468.46 & 0 & -1468.46 & 0 & -3020.83 & 0 & -10.1976 \\
-3304.04 & -1468.46 & 704861 & 211458 & 140972 & 3304.04 & 0 & 0 & 1468.46 \\
-3304.04 & 0 & 211458 & 422917 & 0 & 3304.04 & 0 & 0 & 0 \\
0 & -1468.46 & 140972 & 0 & 281944 & 0 & 0 & 0 & 1468.46 \\
\hline
-34.4170 & 0 & 3304.04 & 3304.04 & 0 & 34.4170 & 0 & 0 & 0 \\
0 & -3020.83 & 0 & 0 & 0 & 0 & 3020.83 & 0 & 0 \\
-2013.89 & 0 & 0 & 0 & 0 & 0 & 0 & 2013.89 & 0 \\
0 & -10.1976 & 1468.46 & 0 & 1468.46 & 0 & 0 & 0 & 10.1976
\end{array}\right]
\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ \hline 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

From the matrix partition, $\mathbf{Q}_k = \mathbf{K}_{11}\mathbf{D}_u + \mathbf{K}_{12}\mathbf{D}_k$,

$$0 = 2048.31D_1 - 3304.04D_3 - 3304.04D_4 \qquad (1)$$

$$-13.75 = 3031.03D_2 - 1468.46D_3 - 1468.46D_5 \qquad (2)$$

$$90 = -3304.04D_1 - 1468.46D_2 + 704861D_3 + 211458D_4 + 140972D_5 \qquad (3)$$

$$0 = -3304.04D_1 + 211458D_3 + 422917D_4 \qquad (4)$$

$$0 = -1468.46D_2 + 140972D_3 + 281944D_5 \qquad (5)$$

Solving Eqs. (1) to (5),

$D_1 = 0.001668 \quad D_2 = -0.004052 \quad D_3 = 0.002043 \quad D_4 = -0.001008 \quad D_5 = -0.001042$

Using these results and applying $\mathbf{Q}_u = \mathbf{K}_{21}\mathbf{D}_u + \mathbf{K}_{22}\mathbf{D}_k$,

$Q_6 = -34.4170(0.001668) + 3304.04(0.002043) + 3304.04(-0.001008) = 3.360$

$Q_7 = -3020.83(-0.004052) = 12.24$

$Q_8 = -2013.89(0.001668) = -3.360$

$Q_9 = -10.1976(-0.004052) + 1468.46(0.002043) + 1468.46(-0.001008) = 1.510$

Superposition these results to those of FEM shown in Fig. *a*.

$R_6 = 3.360 + 0 = 3.36\text{ k}$ **Ans.**

$R_7 = 12.24 + 0 = 12.2\text{ k}$ **Ans.**

$R_8 = -3.360 + 0 = -3.36\text{ k}$ **Ans.**

$R_9 = 1.510 + 6.25 = 7.76\text{ k}$ **Ans.**