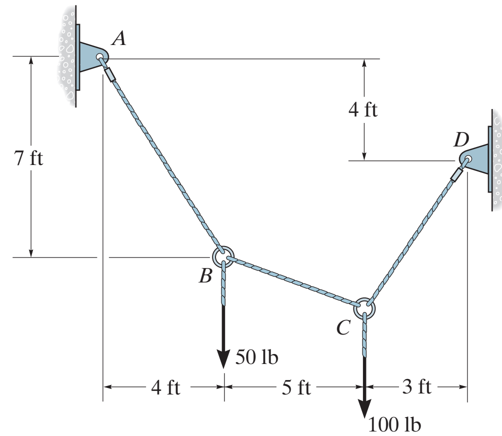
5–1. Determine the tension in each segment of the cable and the cable's total length.

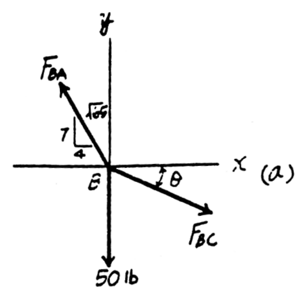


Equations of Equilibrium: Applying method of joints, we have

Joint *B*:

$$\overset{+}{\rightarrow}\sum F_x = 0; \qquad F_{BC}\cos\theta - F_{BA}\left(\frac{4}{\sqrt{65}}\right) = 0 \qquad [1]$$

$$+\uparrow \sum F_y = 0; \qquad F_{BA}\left(\frac{7}{\sqrt{65}}\right) - F_{BC}\sin\theta - 50 = 0 \qquad [2]$$



Joint *C*:

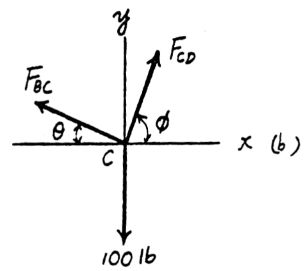
$$\overset{+}{\rightarrow}\sum F_x = 0; \qquad F_{CD}\cos\phi - F_{BC}\cos\theta = 0 \qquad [3]$$

$$+\uparrow \sum F_y = 0; \qquad F_{BC}\sin\theta + F_{CD}\sin\phi - 100 = 0 \qquad [4]$$

Geometry:

$$\sin\theta = \frac{y}{\sqrt{y^2 + 25}} \qquad \cos\theta = \frac{5}{\sqrt{y^2 + 25}}$$

$$\sin\phi = \frac{3 + y}{\sqrt{y^2 + 6y + 18}} \qquad \cos\phi = \frac{3}{\sqrt{y^2 + 6y + 18}}$$



Substitute the above results into Eqs. [1], [2], [3] and [4] and solve. We have

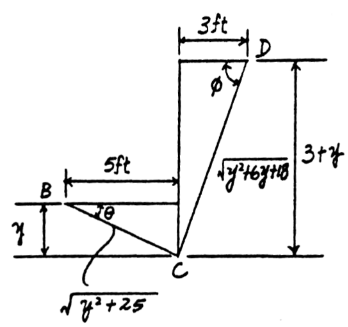
$F_{BC} = 46.7$ lb $\qquad F_{BA} = 83.0$ lb $\qquad F_{CD} = 88.1$ lb **Ans.**

$y = 2.679$ ft

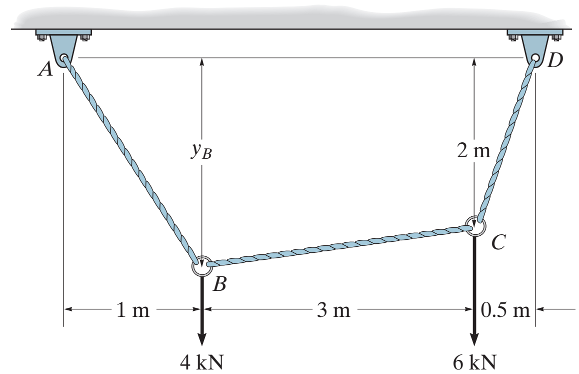
The total length of the cable is

$$l = \sqrt{7^2 + 4^2} + \sqrt{5^2 + 2.679^2} + \sqrt{3^2 + (2.679 + 3)^2}$$

$$= 20.2 \text{ ft}$$ **Ans.**



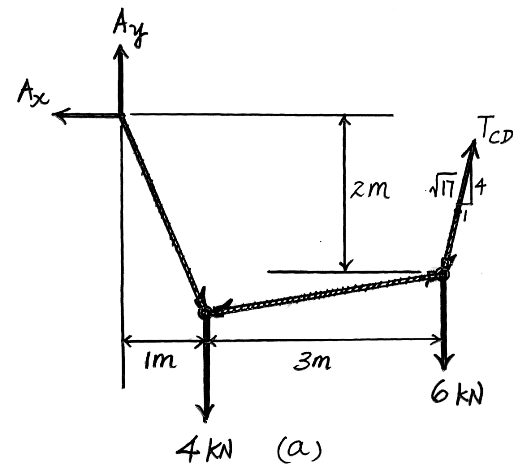
5–2. Cable *ABCD* supports the loading shown. Determine the maximum tension in the cable and the sag of point *B*.



Referring to the FBD in Fig. *a*,

$$\circlearrowleft + \sum M_A = 0; \quad T_{CD}\left(\frac{4}{\sqrt{17}}\right)(4) + T_{CD}\left(\frac{1}{\sqrt{17}}\right)(2) - 6(4) - 4(1) = 0$$

$$T_{CD} = 6.414 \text{ kN} = 6.41 \text{ kN(Max)}$$ **Ans.**



Joint *C*: Referring to the FBD in Fig. *b*,

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad 6.414\left(\frac{1}{\sqrt{17}}\right) - T_{BC}\cos\theta = 0$$

$$+\uparrow \sum F_y = 0; \quad 6.414\left(\frac{4}{\sqrt{17}}\right) - 6 - T_{BC}\sin\theta = 0$$

Solving,

$$T_{BC} = 1.571 \text{ kN} = 1.57 \text{ kN} \qquad (< T_{CD})$$

$$\theta = 8.130^\circ$$

Joint *B*: Referring to the FBD in Fig. *c*,

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad 1.571 \cos 8.130^\circ - T_{AB}\cos\phi = 0$$

$$+\uparrow \sum F_y = 0; \quad T_{AB}\sin\phi + 1.571 \sin 8.130^\circ - 4 = 0$$

Solving,

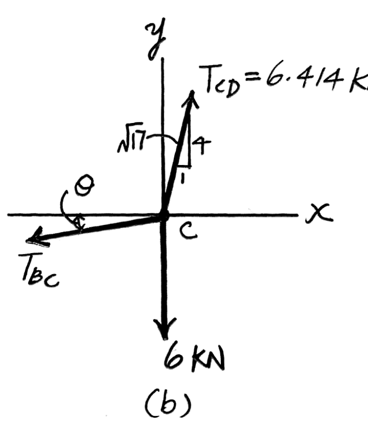
$$T_{AB} = 4.086 \text{ kN} = 4.09 \text{ kN} \qquad (< T_{CD})$$

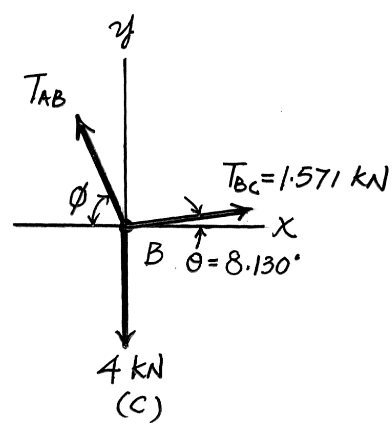
$$\phi = 67.62^\circ$$

Then, from the geometry,

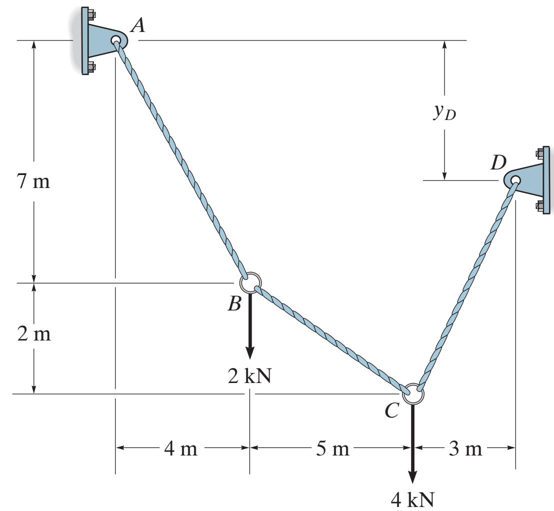
$$\frac{y_B}{1} = \tan\phi; \qquad y_B = 1 \tan 67.62^\circ$$

$$= 2.429 \text{ m} = 2.43 \text{ m}$$ **Ans.**





5–3. Determine the tension in each cable segment and the distance y_D.



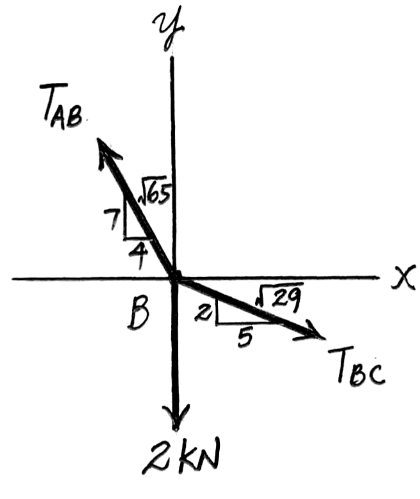
Joint *B*: Referring to the FBD in Fig. *a*,

$\xrightarrow{+} \sum F_x = 0;$ $$T_{BC}\left(\frac{5}{\sqrt{29}}\right) - T_{AB}\left(\frac{4}{\sqrt{65}}\right) = 0$$

$+\uparrow \sum F_y = 0;$ $$T_{AB}\left(\frac{7}{\sqrt{65}}\right) - T_{BC}\left(\frac{2}{\sqrt{29}}\right) - 2 = 0$$

Solving,

$T_{AB} = 2.986 \text{ kN} = 2.99 \text{ kN}$ $\quad T_{BC} = 1.596 \text{ kN} = 1.60 \text{ kN}$ **Ans.**



(a)

Joint *C*: Referring to the FBD in Fig. *b*,

$\xrightarrow{+} \sum F_x = 0;$ $$T_{CD}\cos\theta - 1.596\left(\frac{5}{\sqrt{29}}\right) = 0$$

$+\uparrow \sum F_y = 0;$ $$T_{CD}\sin\theta + 1.596\left(\frac{2}{\sqrt{29}}\right) - 4 = 0$$

Solving,

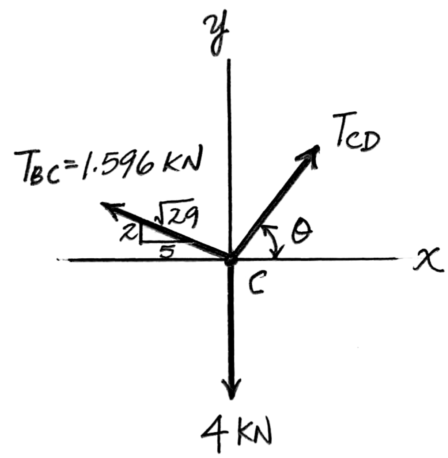
$T_{CD} = 3.716 \text{ kN} = 3.72 \text{ kN}$ **Ans.**

$\theta = 66.50°$

From the geometry,

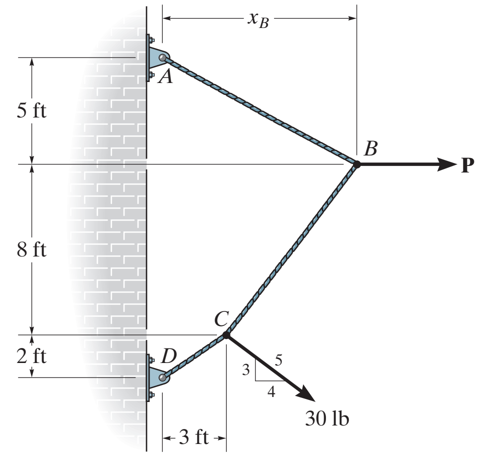
$$y_D + 3\tan\theta = 9$$

$$y_D = 9 - 3\tan 66.50° = 2.10 \text{ m}$$ **Ans.**



(b)

***5–4.** The cable supports the loading shown. Determine the distance x_B the force at point B acts from A. Set $P = 40$ lb.

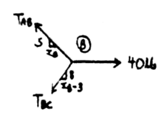


At B

$$\xrightarrow{+} \sum F_x = 0; \qquad 40 - \frac{x_B}{\sqrt{x_B^2 + 25}} T_{AB} - \frac{x_B - 3}{\sqrt{(x_B - 3)^2 + 64}} T_{BC} = 0$$

$$+\uparrow \sum F_y = 0; \qquad \frac{5}{\sqrt{x_B^2 + 25}} T_{AB} - \frac{8}{\sqrt{(x_B - 3)^2 + 64}} T_{BC} = 0$$

$$\frac{13x_B - 15}{\sqrt{(x_B - 3)^2 + 64}} T_{BC} = 200 \qquad (1)$$

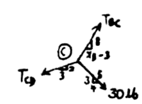


At C

$$\xrightarrow{+} \sum F_x = 0; \qquad \frac{4}{5}(30) + \frac{x_B - 3}{\sqrt{(x_B - 3)^2 + 64}} T_{BC} - \frac{3}{\sqrt{13}} T_{CD} = 0$$

$$+\uparrow \sum F_y = 0; \qquad \frac{8}{\sqrt{(x_B - 3)^2 + 64}} T_{BC} + \frac{2}{\sqrt{13}} T_{CD} - \frac{3}{5}(30) = 0$$

$$\frac{30 - 2x_B}{\sqrt{(x_B - 3)^2 + 64}} T_{BC} = 102 \qquad (2)$$

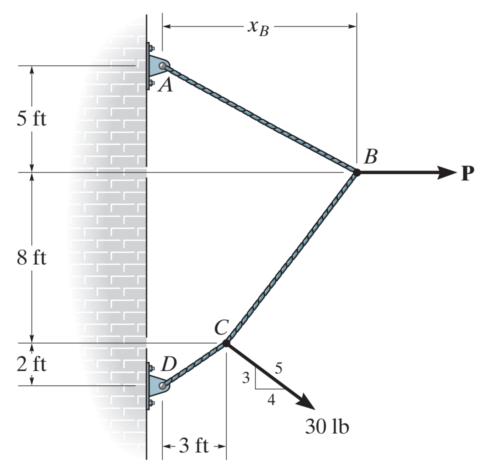


Solving Eqs. (1) & (2)

$$\frac{13x_B - 15}{30 - 2x_B} = \frac{200}{102}$$

$$x_B = 4.36 \text{ ft}$$ **Ans.**

5–5. The cable supports the loading shown. Determine the magnitude of the horizontal force **P** so that $x_B = 6$ ft.



At B

$$\xrightarrow{+} \sum F_x = 0; \qquad P - \frac{6}{\sqrt{61}} T_{AB} - \frac{3}{\sqrt{73}} T_{BC} = 0$$

$$+\uparrow \sum F_y = 0; \qquad \frac{5}{\sqrt{61}} T_{AB} - \frac{8}{\sqrt{73}} T_{BC} = 0$$

$$5P - \frac{63}{\sqrt{73}} T_{BC} = 0 \qquad (1)$$

At C

$$\xrightarrow{+} \sum F_x = 0; \qquad \frac{4}{5}(30) + \frac{3}{\sqrt{73}} T_{BC} - \frac{3}{\sqrt{13}} T_{CD} = 0$$

$$+\uparrow \sum F_y = 0; \qquad \frac{8}{\sqrt{73}} T_{BC} - \frac{2}{\sqrt{13}} T_{CD} - \frac{3}{5}(30) = 0$$

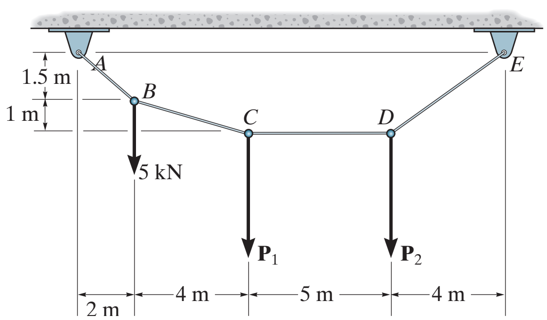
$$\frac{18}{\sqrt{73}} T_{BC} = 102 \qquad (2)$$

Solving Eqs. (1) & (2)

$$\frac{63}{18} = \frac{5P}{102}$$

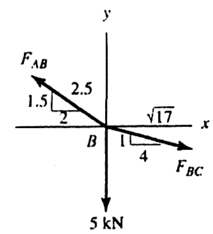
$$P = 71.4 \text{ lb}$$ **Ans.**

5–6. Determine the forces P_1 and P_2 needed to hold the cable in the position shown, i.e., so segment CD remains horizontal. Also find the maximum loading in the cable.



Method of Joints:

Joint *B*:



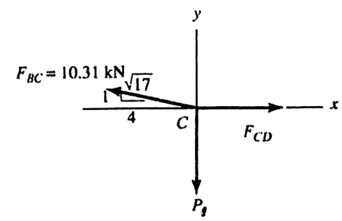
$\xrightarrow{+} \sum F_x = 0;\qquad F_{BC}\left(\frac{4}{\sqrt{17}}\right) - F_{AB}\left(\frac{2}{2.5}\right) = 0 \qquad [1]$

$+\uparrow \sum F_y = 0;\qquad F_{AB}\left(\frac{1.5}{2.5}\right) - F_{BC}\left(\frac{1}{\sqrt{17}}\right) - 5 = 0 \qquad [2]$

Solving Eqs. [1] and [2] yields

$F_{BC} = 10.31\text{ kN} \qquad F_{AB} = 12.5\text{ kN}$

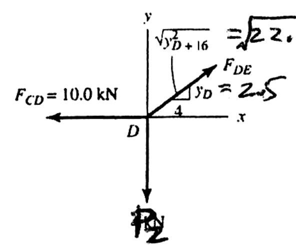
Joint *C*:



$\xrightarrow{+} \sum F_x = 0;\qquad F_{CD} - 10.31\left(\frac{4}{\sqrt{17}}\right) = 0 \quad F_{CD} = 10.00\text{ kN}$

$+\uparrow \sum F_y = 0;\qquad 10.31\left(\frac{1}{\sqrt{17}}\right) - P_1 = 0 \quad P_1 = 2.50\text{ kN} \qquad$ **Ans.**

Joint *D*:



$\xrightarrow{+} \sum F_x = 0;\qquad F_{DE}\left(\frac{4}{\sqrt{22.25}}\right) - 10 = 0 \qquad [1]$

$+\uparrow \sum F_y = 0;\qquad F_{DE}\left(\frac{25}{\sqrt{22.25}}\right) - P_2 = 0 \qquad [2]$

Solving Eqs. [1] and [2] yields

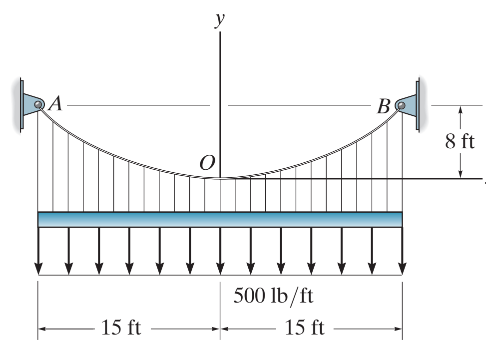
$P_2 = 6.25\text{ kN} \qquad$ **Ans.**

$F_{DE} = 11.79\text{ kN}$

Thus, the maximum tension in the cable is

$F_{max} = F_{AB} = 12.5\text{ kN} \qquad$ **Ans.**

5–7. The cable is subjected to the uniform loading. If the slope of the cable at point O is zero, determine the equation of the curve and the force in the cable at O and B.



From Eq. 5–9.

$$y = \frac{h}{L^2}x^2 = \frac{8}{(15)^2}x^2$$

$$y = 0.0356x^2 \qquad \textbf{Ans.}$$

From Eq. 5–8

$$T_o = F_H = \frac{w_o L^2}{2h} = \frac{500(15)^2}{2(8)} = 7031.25\text{ lb} = 7.03\text{ k} \qquad \textbf{Ans.}$$

From Eq. 5–10.

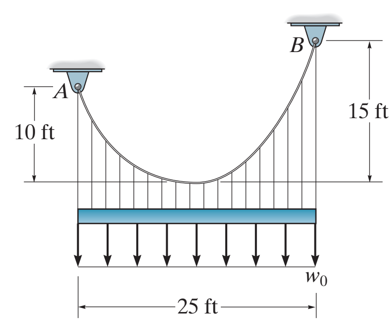
$$T_B = T_{\max} = \sqrt{(F_H)^2 + (w_o L)^2} = \sqrt{(7031.25)^2 + [(500)(15)]^2}$$

$$= 10\,280.5\text{ lb} = 10.3\text{ k} \qquad \textbf{Ans.}$$

Also, from Eq. 5–11

$$T_B = T_{\max} = w_o L\sqrt{1 + \left(\frac{L}{2h}\right)^2} = 500(15)\sqrt{1 + \left(\frac{15}{2(8)}\right)^2} = 10\,280.5\text{ lb} = 10.3\text{ k} \qquad \textbf{Ans.}$$

***5–8.** The cable supports the uniform load of $w_0 = 600$ lb/ft. Determine the tension in the cable at each support A and B.



$$y = \frac{w_o}{2\,F_H}x^2$$

$$15 = \frac{600}{2\,F_H}x^2$$

$$10 = \frac{600}{2\,F_H}(25 - x)^2$$

$$\frac{600}{2(15)}x^3 = \frac{600}{2(10)}(25 - x)^2$$

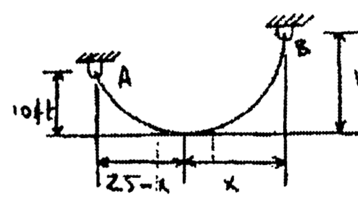
$$x^2 = 1.5(625 - 50x + x^2)$$

$$0.5x^2 - 75x + 937.50 = 0$$

Choose root < 25 ft

$$x = 13.76\text{ ft}$$

$$F_H = \frac{w_o}{2y}x^2 = \frac{600}{2(15)}(13.76)^2 = 3788\text{ lb}$$



5–8. Continued

At B:

$$y = \frac{w_o}{2\,F_H}x^2 = \frac{600}{2(3788)}x^2$$

$$\frac{dy}{dx} = \tan\theta_B = 0.15838\,x\Big|_{x=13.76} = 2.180$$

$$\theta_B = 65.36^\circ$$

$$T_B = \frac{F_H}{\cos\theta_B} = \frac{3788}{\cos\ 65.36^\circ} = 9085\text{ lb} = 9.09\text{ kip}$$ **Ans.**

At A:

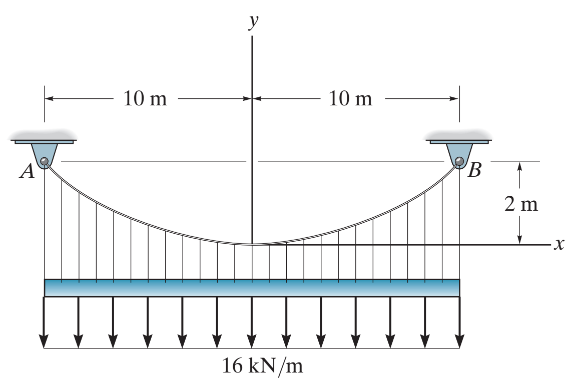
$$y = \frac{w_o}{2\,F_H}x^2 = \frac{600}{2(3788)}x^2$$

$$\frac{dy}{dx} = \tan\theta_A = 0.15838\,x\Big|_{x=(25-13.76)} = 1.780$$

$$\theta_{A=60.67^\circ}$$

$$T_A = \frac{F_H}{\cos\theta_A} = \frac{3788}{\cos 60.67^\circ} = 7734\text{ lb} = 7.73\text{ kip}$$ **Ans.**

5–9. Determine the maximum and minimum tension in the cable.



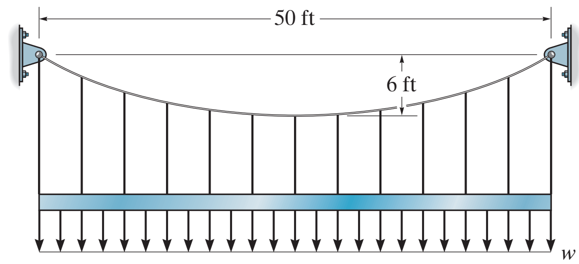
The minimum tension in the cable occurs when $\theta = 0^\circ$. Thus, $T_{\min} = F_H$. With $w_o = 16$ kN/m, $L = 10$ m and $h = 2$ m,

$$T_{\min} = F_H = \frac{w_o L^2}{2\,h} = \frac{(16\text{ kN/m})(10\text{ m})^2}{2(2\text{ m})} = 400\text{ kN}$$ **Ans.**

And

$$T_{\max} = \sqrt{F_{H^2} + (w_o L)^2}$$
$$= \sqrt{400^2 + [16(10)]^2}$$
$$= 430.81\text{ kN}$$
$$= 431\text{ kN}$$ **Ans.**

5–10. Determine the maximum uniform loading w, measured in lb/ft, that the cable can support if it is capable of sustaining a maximum tension of 3000 lb before it will break.

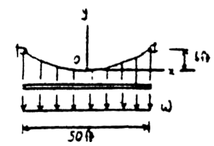


$$y = \frac{1}{F_H}\int\left(\int w\,dx\right)dx$$

At $x = 0$, $\dfrac{dy}{dx} = 0$

At $x = 0$, $y = 0$

$$C_1 = C_2 = 0$$



$$y = \frac{w}{2\,F_H}x^2$$

At $x = 25$ ft, $y = 6$ ft $\qquad F_H = 52.08\,w$

$$\left.\frac{dy}{dx}\right|_{max} = \tan\theta_{max} = \left.\frac{w}{F_H}x\right|_{x=25\text{ ft}}$$

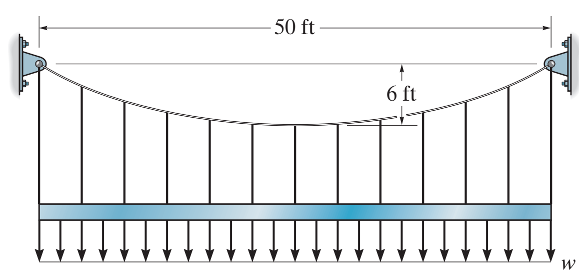
$$\theta_{max} = \tan^{-1}(0.48) = 25.64°$$

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = 3000$$

$$F_H = 2705\text{ lb}$$

$$w = 51.9\text{ lb/ft} \qquad \textbf{Ans.}$$

5–11. The cable is subjected to a uniform loading of $w = 250$ lb/ft. Determine the maximum and minimum tension in the cable.



$$F_H = \frac{w_o L^2}{8\,h} = \frac{250(50)^2}{8(6)} = 13\,021\text{ lb}$$

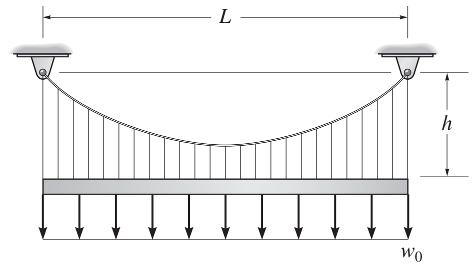
$$\theta_{max} = \tan^{-1}\left(\frac{w_o L}{2\,F_H}\right) = \tan^{-1}\left(\frac{250(50)}{2(13\,021)}\right) = 25.64°$$

$$T_{max} = \frac{F_H}{\cos_{\theta_{max}}} = \frac{13\,021}{\cos 25.64°} = 14.4\text{ kip} \qquad \textbf{Ans.}$$

The minimum tension occurs at $\theta = 0°$

$$T_{min} = F_H = 13.0\text{ kip} \qquad \textbf{Ans.}$$

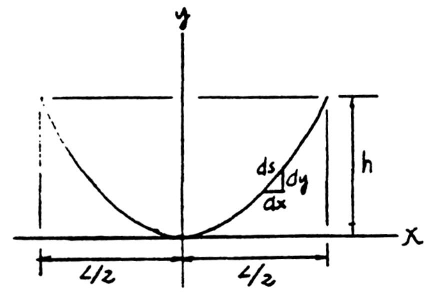
***5–12.** The cable shown is subjected to the uniform load w_0. Determine the ratio between the rise h and the span L that will result in using the minimum amount of material for the cable.



From Eq. 5–9,

$$y = \frac{h}{\left(\frac{L}{2}\right)^2}x^2 = \frac{4h}{L^2}x^2$$

$$\frac{dy}{dx} = \frac{8h}{L^2}x$$



From Eq. 5–8,

$$F_H = \frac{w_o\left(\frac{L}{2}\right)^2}{2h} = \frac{w_oL^2}{8h}$$

Since $F_H = T\left(\frac{dx}{ds}\right)$, then

$$T = \frac{w_oL^2}{8h}\left(\frac{ds}{dx}\right)$$

Let σ_{allow} be the allowable normal stress for the cable. Then

$$\frac{T}{A} = \sigma_{\text{allow}}$$

$$\frac{T}{\sigma_{\text{allow}}} = A$$

$$dV = A\,ds$$

$$dV = \frac{T}{\sigma_{\text{allow}}}ds$$

The volume of material is

$$V = \frac{2}{\sigma_{\text{allow}}}\int_0^{\frac{1}{2}} T\,ds = \frac{2}{\sigma_{\text{allow}}}\int_0^{\frac{1}{2}} \frac{w_oL^2}{8h}\left[\frac{(ds)^2}{dx}\right]$$

$$\frac{ds^2}{dx} = \frac{dx^2 + dy^2}{dx} = \left[\frac{dx^2 + dy^2}{dx^2}\right]dx = \left[1 + \left(\frac{dy}{dx}\right)^2\right]dx$$

$$= \int_0^{\frac{1}{2}} \frac{w_oL^2}{4h\sigma_{\text{allow}}}\left[1 + \left(\frac{dy}{dx}\right)^2\right]dx$$

$$= \frac{w_oL^2}{4h\sigma_{\text{allow}}}\int_0^{\frac{1}{2}}\left[1 + 64\left(\frac{h^2x^2}{L^4}\right)\right]dx$$

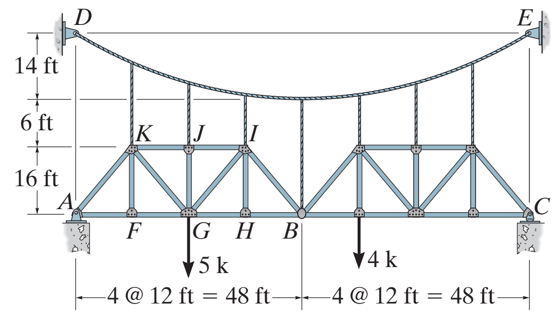
$$= \frac{w_oL^2}{4h\sigma_{\text{allow}}}\left[\frac{L}{2} + \frac{8h^2}{3L}\right] = \frac{w_oL^2}{8\,\sigma_{\text{allow}}}\left[\frac{L}{h} + \frac{16}{3}\left(\frac{h}{L}\right)\right]$$

Require,

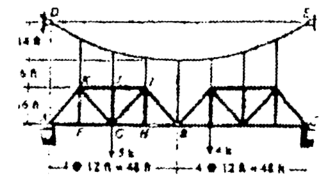
$$\frac{dV}{dh} = \frac{w_oL^2}{8\sigma_{\text{allow}}}\left[-\frac{L}{h^2} + \frac{16}{3L}\right] = 0$$

$h = 0.433\,L$ **Ans.**

5–13. The trusses are pin connected and suspended from the parabolic cable. Determine the maximum force in the cable when the structure is subjected to the loading shown.



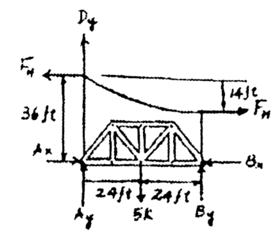
Entire structure:



$$\zeta+\sum M_C = 0; \quad 4(36) + 5(72) + F_H(36) - F_H(36) - (A_y + D_y(96) = 0$$

$$(A_y + D_y) = 5.25 \qquad (1)$$

Section *ABD*:



$$\zeta+\sum M_B = 0; \quad F_H(14) - (A_y + D_y)(48) + 5(24) = 0$$

Using Eq. (1):

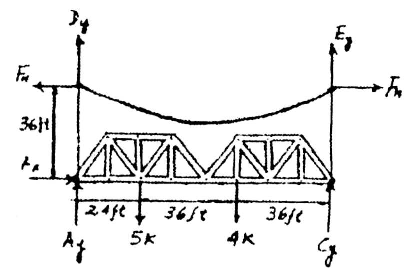
$$F_H = 9.42857 \text{ k}$$

From Eq. 5–8:

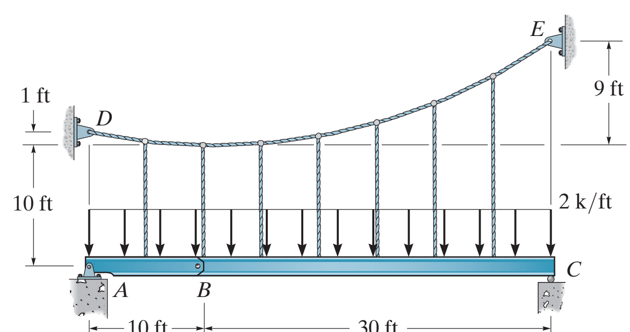
$$w_o = \frac{2F_H h}{L^2} = \frac{2(9.42857)(14)}{48^2} = 0.11458 \text{ k/ft}$$

From Eq. 5–11:

$$T_{\max} = w_o L\sqrt{1 + \left(\frac{L}{2h}\right)^2} = 0.11458(48)\sqrt{1 + \left[\frac{48}{2(14)}\right]^2} = 10.9 \text{ k} \qquad \textbf{Ans.}$$



5–14. Determine the maximum and minimum tension in the parabolic cable and the force in each of the hangers. The girder is subjected to the uniform load and is pin connected at *B*.

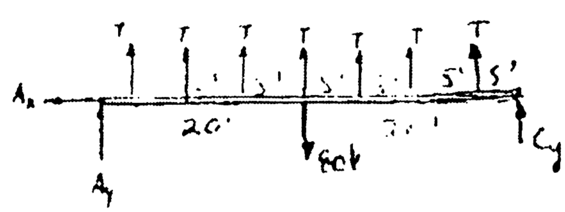


Member *BC*:

$$\xrightarrow{+}\sum F_x = 0; \qquad B_x = 0$$

Member *AB*:

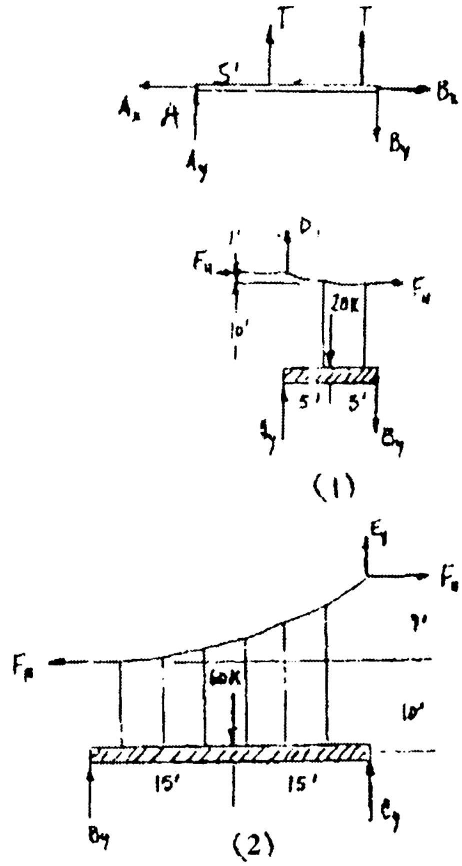
$$\xrightarrow{+}\sum F_x = 0; \qquad A_x = 0$$



FBD 1:

$$\zeta+\sum M_A = 0; \qquad F_H(1) - B_y(10) - 20(5) = 0$$

5–14. Continued



FBD 2:

$\circlearrowleft + \sum M_C = 0; \qquad -F_H(9) - B_y(30) + 60(15) = 0$

Solving,

$B_y = 0, \quad F_H = F_{min} = 100 \text{ k}$ **Ans.**

Max cable force occurs at E, where slope is the maximum.

From Eq. 5–8.

$$W_o = \frac{2F_H h}{L^2} = \frac{2(100)(9)}{30^2} = 2 \text{ k/ft}$$

From Eq. 5–11,

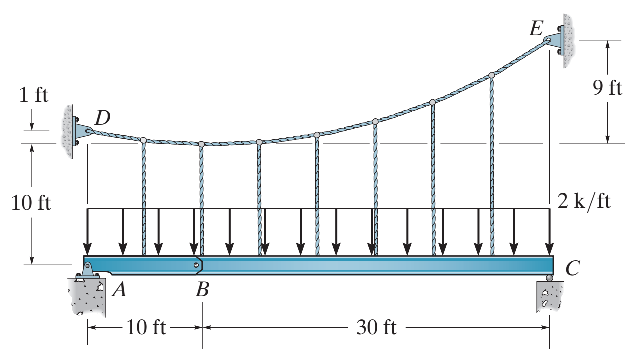
$$F_{max} = w_o L\sqrt{1 + \left(\frac{L}{2h}\right)^2} = 2(30)\sqrt{1 + \left(\frac{30}{2(9)}\right)^2}$$

$F_{max} = 117 \text{ k}$ **Ans.**

Each hanger carries 5 ft of w_o.

$T = (2 \text{ k/ft})(5 \text{ ft}) = 10 \text{ k}$ **Ans.**

5–15. Draw the shear and moment diagrams for the pin-connected girders *AB* and *BC*. The cable has a parabolic shape.



$\circlearrowleft + \sum M_A = 0; \quad T(5) + T(10) + T(15) + T(20) + T(25)$

$+ T(30) + T(35) + C_y(40) - 80(20) = 0$

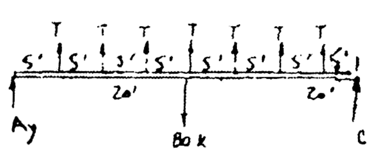
Set $T = 10$ k (See solution to Prob. 5–14)

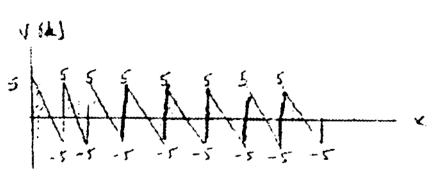
$C_y = 5 \text{ k}$

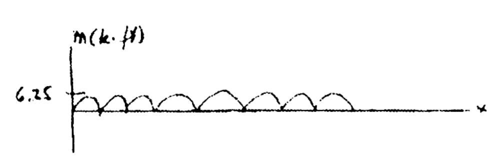
$+\uparrow \sum F_y = 0; \qquad 7(10) + 5 - 80 + A_y = 0$

$A_y = 5 \text{ k}$

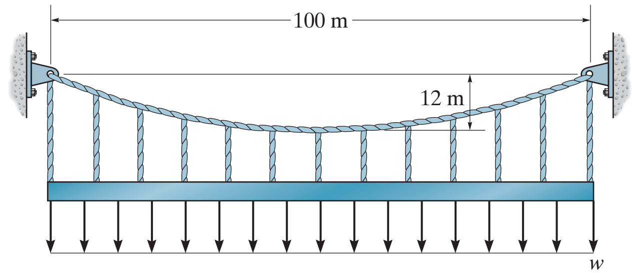
$M_{max} = 6.25 \text{ k} \cdot \text{ft}$ **Ans.**







***5–16.** The cable will break when the maximum tension reaches $T_{max} = 5000\text{ kN}$. Determine the maximum uniform distributed load w required to develop this maximum tension.



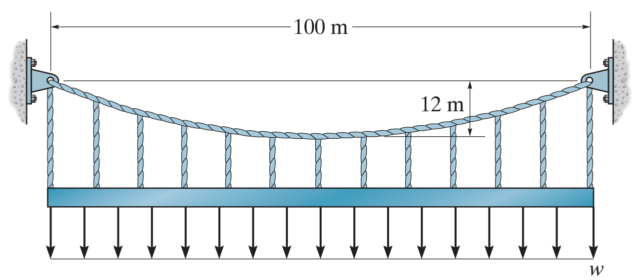
With $T_{max} = 80(10^3)$ kN, $L = 50$ m and $h = 12$ m,

$$T_{max} = w_o L\sqrt{1 + \left(\frac{L}{2h}\right)^2}$$

$$8000 = w_o(50)\left[\sqrt{1 + \left(\frac{50}{24}\right)^2}\right]$$

$$w_o = 69.24\text{ kN/m} = 69.2\text{ kN/m}$$ **Ans.**

5–17. The cable is subjected to a uniform loading of $w = 60\text{ kN/m}$. Determine the maximum and minimum tension in cable.



The minimum tension in cable occurs when $\theta = 0°$. Thus, $T_{min} = F_H$.

$$T_{min} = F_H = \frac{w_o L^2}{2h} = \frac{(60\text{ kN/m})(50\text{ m})^2}{2(12\text{ m})} = 6250\text{ kN}$$

$$= 6.25\text{ MN}$$ **Ans.**

And,

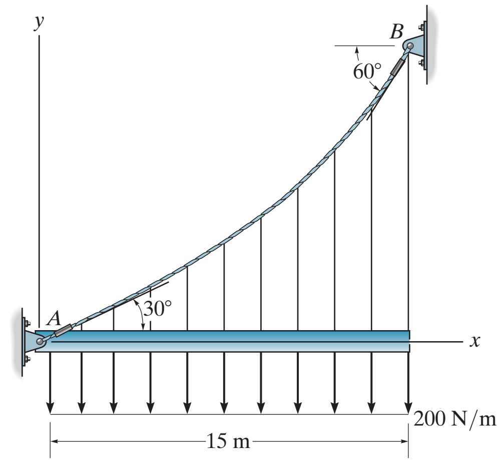
$$T_{max} = \sqrt{F_H^2 + (w_o L)^2}$$

$$= \sqrt{6250^2 + [60(50)]^2}$$

$$= 6932.71\text{ kN}$$

$$= 6.93\text{ MN}$$ **Ans.**

5–18. The cable AB is subjected to a uniform loading of 200 N/m. If the weight of the cable is neglected and the slope angles at points A and B are 30° and 60°, respectively, determine the curve that defines the cable shape and the maximum tension developed in the cable.



Here the boundary conditions are different from those in the text.

Integrate Eq. 5–2,

$$T\sin\theta = 200x + C_1$$

Divide by by Eq. 5–4, and use Eq. 5–3

$$\frac{dy}{dx} = \frac{1}{F_H}(200x + C_1)$$

$$y = \frac{1}{F_H}(100x^2 + C_1x + C_2)$$

At $x = 0$, $y = 0$; $C_2 = 0$

At $x = 0$, $\frac{dy}{dx} = \tan 30°$; $C_1 = F_H \tan 30°$

$$y = \frac{1}{F_H}(100x^2 + F_H \tan 30° x)$$

$$\frac{dy}{dx} = \frac{1}{F_H}(200x + F_H \tan 30°)$$

At $x = 15$ m, $\frac{dy}{dx} = \tan 60°$; $F_H = 2598$ N

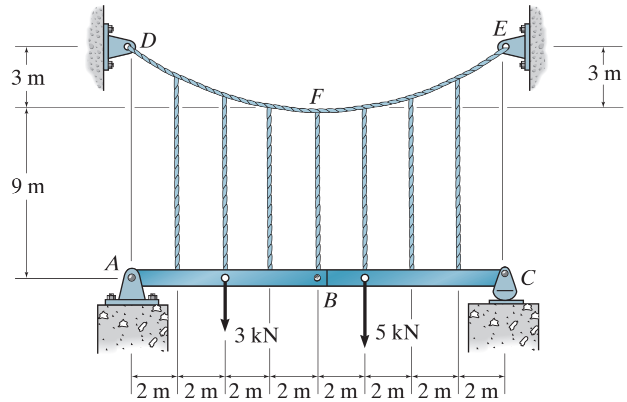
$y = (38.5x^2 + 577x)(10^{-3})$ m **Ans.**

$\theta_{max} = 60°$

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{2598}{\cos 60°} = 5196 \text{ N}$$

$T_{max} = 5.20$ kN **Ans.**

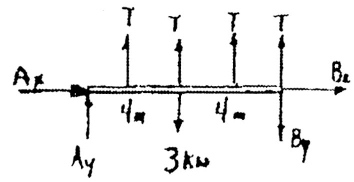
5–19. The beams AB and BC are supported by the cable that has a parabolic shape. Determine the tension in the cable at points D, F, and E, and the force in each of the equally spaced hangers.



$\xrightarrow{+} \sum F_x = 0;$ $\quad B_x = 0 \quad$ (Member BC)

$\zeta + \sum M_A = 0;$ $\quad F_F(12) - F_F(9) - B_y(8) - 3(4) = 0$

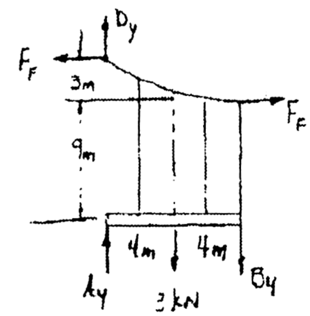
$$3F_F - B_y(8) = 12 \qquad (1)$$



$\xrightarrow{+} \sum F_x = 0;$ $\quad A_x = 0 \quad$ (Member AB)

$\zeta + \sum M_C = 0;$ $\quad -F_F(12) + F_F(9) - B_y(8) + 5(6) = 0$

$$-3F_F - B_y(8) = -30 \qquad (2)$$



Soving Eqs. (1) and (2),

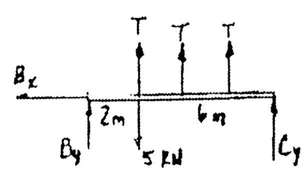
$$B_y = 1.125 \text{ kN}, \qquad F_F = 7.0 \text{ kN}$$ **Ans.**

From Eq. 5–8.

$$w_o = \frac{2F_H h}{L^2} = \frac{2(7)(3)}{8^2} = 0.65625 \text{ kN/m}$$

From Eq. 5–11,

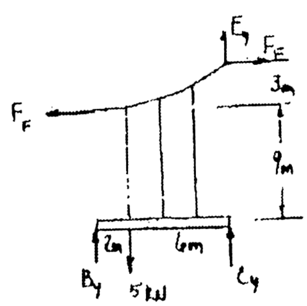
$$T_{\max} = w_o L\sqrt{1 + \left(\frac{L}{2h}\right)^2} = 0.65625(8)\sqrt{1 + \left(\frac{8}{2(3)}\right)^2}$$



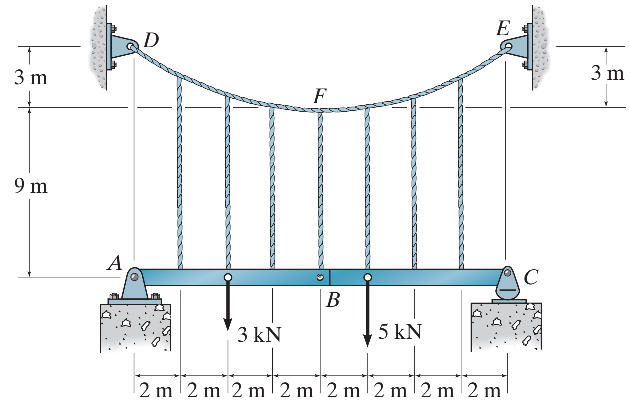
$T_{\max} = T_E = T_D = 8.75 \text{ kN}$ **Ans.**

Load on each hanger,

$T = 0.65625(2) = 1.3125 \text{ kN} = 1.31 \text{ kN}$ **Ans.**



***5–20.** Draw the shear and moment diagrams for beams *AB* and *BC*. The cable has a parabolic shape.



Member *ABC*:

$\circlearrowleft + \sum M_A = 0;$ $T(2) + T(4) + T(6) + T(8) + T(10)$

$+ T(12) + T(14) + C_y(16) - 3(4) - 5(10) = 0$

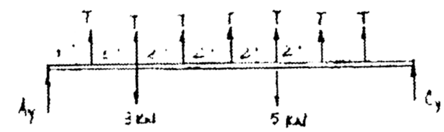
Set $T = 1.3125$ kN (See solution to Prob 5–19).

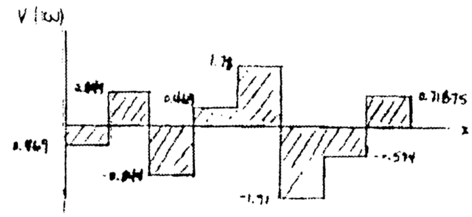
$C_y = -0.71875$ kN

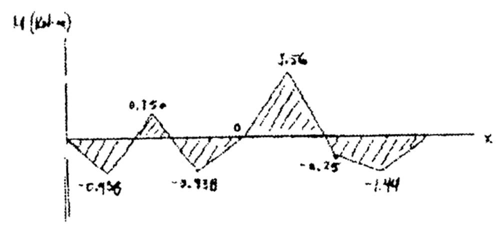
$+\uparrow \sum F_y = 0;$ $7(1.3125) - 8 - 0.71875 + A_y = 0$

$A_y = -0.46875$ kN

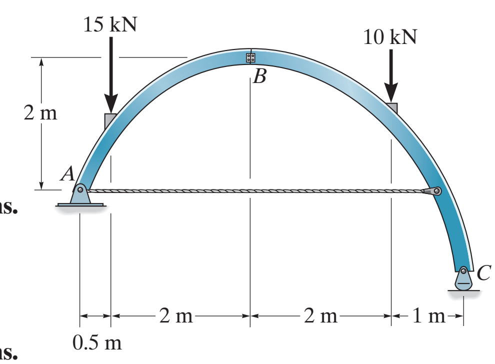
$M_{max} = 3056$ kN · m **Ans.**







5–21. The tied three-hinged arch is subjected to the loading shown. Determine the components of reaction at *A* and *C* and the tension in the cable.

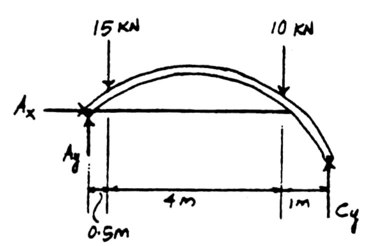


Entire arch:

$\xrightarrow{+} \sum F_x = 0;$ $A_x = 0$ **Ans.**

$\circlearrowleft + \sum M_A = 0;$ $C_y(5.5) - 15(0.5) - 10(4.5) = 0$

$C_y = 9.545 \text{ kN} = 9.55 \text{ kN}$ **Ans.**



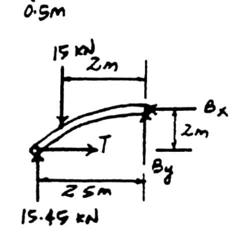
$+\uparrow \sum F_y = 0;$ $9.545 - 15 - 10 + A_y = 0$

$A_y = 15.45 \text{ kN} = 15.5 \text{ kN}$ **Ans.**

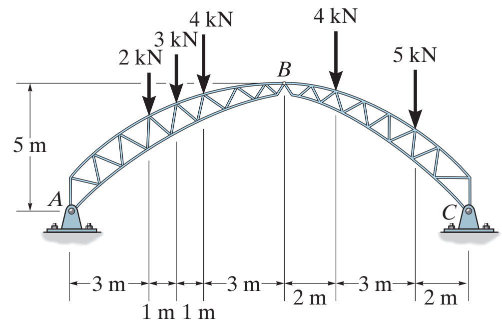
Section *AB*:

$\circlearrowleft + \sum M_B = 0;$ $-15.45(2.5) + T(2) + 15(2) = 0$

$T = 4.32$ kN **Ans.**

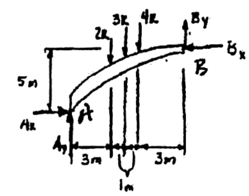


5–22. Determine the resultant forces at the pins A, B, and C of the three-hinged arched roof truss.



Member AB**:**

$\circlearrowleft + \sum M_A = 0;$ $\quad B_x(5) + B_y(8) - 2(3) - 3(4) - 4(5) = 0$



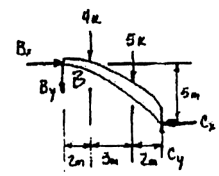
Member BC**:**

$\circlearrowleft + \sum M_C = 0;$ $\quad -B_x(5) + B_y(7) + 5(2) + 4(5) = 0$

Soving,

$B_y = 0.533 \text{ k}, \qquad B_x = 6.7467 \text{ k}$

Member AB**:**



$\xrightarrow{+} \sum F_x = 0;$ $\quad A_x = 6.7467 \text{ k}$

$+\uparrow \sum F_y = 0;$ $\quad A_y - 9 + 0.533 = 0$

$A_y = 8.467 \text{ k}$

Member BC**:**

$\xrightarrow{+} \sum F_x = 0;$ $\quad C_x = 6.7467 \text{ k}$

$+\uparrow \sum F_y = 0;$ $\quad C_y - 9 + 0.533 = 0$

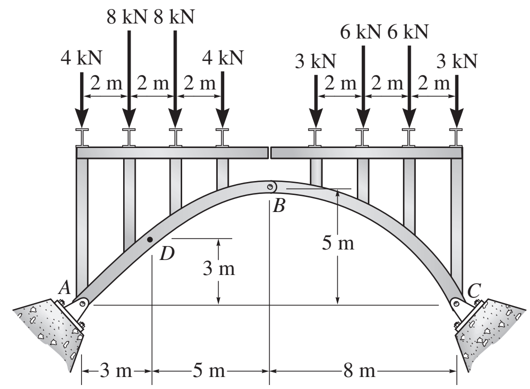
$C_y = 9.533 \text{ k}$

$F_B = \sqrt{(0.533)^2 + (6.7467)^2} = 6.77 \text{ k}$ **Ans.**

$F_A = \sqrt{(6.7467)^2 + (8.467)^2} = 10.8 \text{ k}$ **Ans.**

$F_C = \sqrt{(6.7467)^2 + (9.533)^2} = 11.7 \text{ k}$ **Ans.**

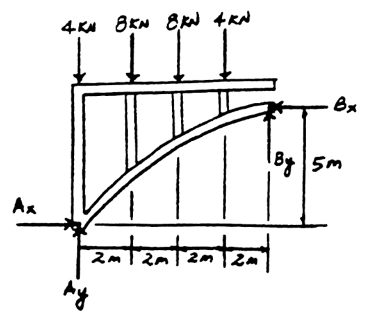
5–23. The three-hinged spandrel arch is subjected to the loading shown. Determine the internal moment in the arch at point D.



Member *AB*:

$\circlearrowleft + \sum M_A = 0; \qquad B_x(5) + B_y(8) - 8(2) - 8(4) - 4(6) = 0$

$$B_x + 1.6B_y = 14.4 \qquad (1)$$



Member *CB*:

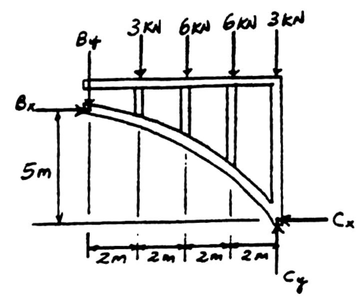
$\circlearrowleft + \sum M_C = 0; \qquad B_{(y)}(8) - B_x(5) + 6(2) + 6(4) + 3(6) = 0$

$$-B_x + 1.6B_y = -10.8 \qquad (2)$$

Soving Eqs. (1) and (2) yields:

$$B_y = 1.125 \text{ kN}$$

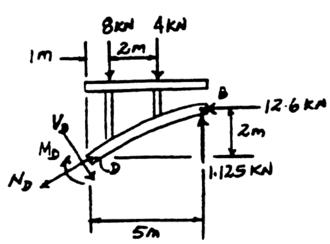
$$B_x = 12.6 \text{ kN}$$



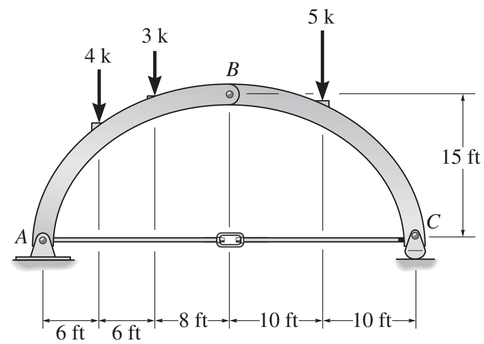
Segment *BD*:

$\circlearrowleft + \sum M_D = 0; \qquad -M_D + 12.6(2) + 1.125(5) - 8(1) - 4(3) = 0$

$$M_D = 10.825 \text{ kN}\cdot\text{m} = 10.8 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$



***5–24.** The tied three-hinged arch is subjected to the loading shown. Determine the components of reaction at A and C, and the tension in the rod



Entire arch:

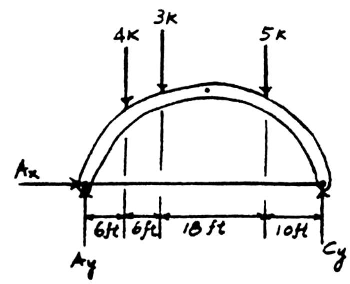
$\circlearrowleft + \sum M_A = 0;$ $\quad -4(6) - 3(12) - 5(30) + C_y(40) = 0$

$C_y = 5.25 \text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad A_y + 5.25 - 4 - 3 - 5 = 0$

$A_y = 6.75 \text{ k}$ **Ans.**

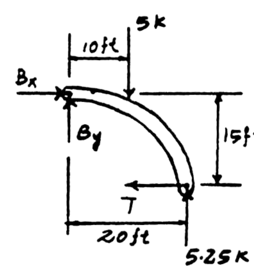
$\xrightarrow{+} \sum F_x = 0;$ $\quad A_x = 0$ **Ans.**



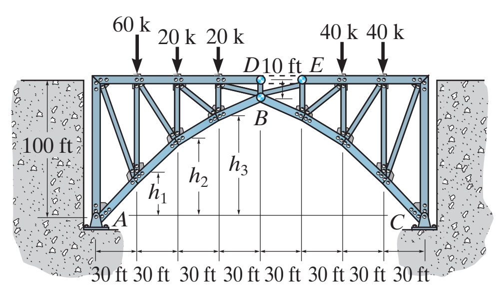
Section BC:

$\circlearrowleft + \sum M_B = 0;$ $\quad -5(10) - T(15) + 5.25(20) = 0$

$T = 3.67 \text{ k}$ **Ans.**



5–25. The bridge is constructed as a *three-hinged trussed arch*. Determine the horizontal and vertical components of reaction at the hinges (pins) at A, B, and C. The dashed member DE is intended to carry *no* force.



Member AB:

$\circlearrowleft + \sum M_A = 0;$ $\quad B_x(90) + B_y(120) - 20(90) - 20(90) - 60(30) = 0$

$9B_x + 12B_y = 480 \qquad (1)$

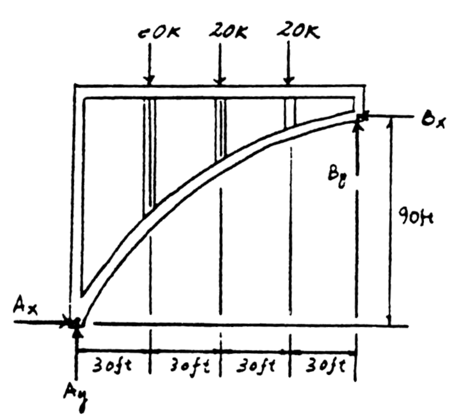
Member BC:

$\circlearrowleft + \sum M_C = 0;$ $\quad -B_x(90) + B_y(120) + 40(30) + 40(60) = 0$

$-9B_x + 12B_y = -360 \qquad (2)$

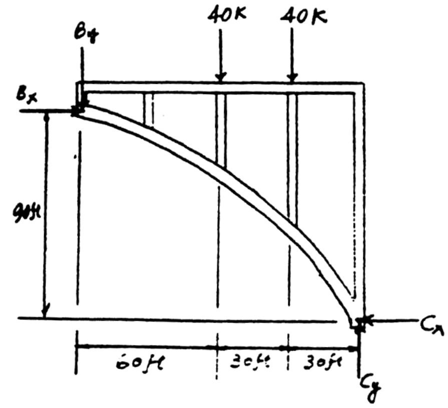
Soving Eqs. (1) and (2) yields:

$B_x = 46.67 \text{ k} = 46.7 \text{ k} \qquad B_y = 5.00 \text{ k}$ **Ans.**



5–25. Continued

Member *AB*:



$\xrightarrow{+} \sum F_x = 0;$ $\quad A_x - 46.67 = 0$

$A_x = 46.7 \text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad A_y - 60 - 20 - 20 + 5.00 = 0$

$A_y = 95.0 \text{ k}$ **Ans.**

Member *BC*:

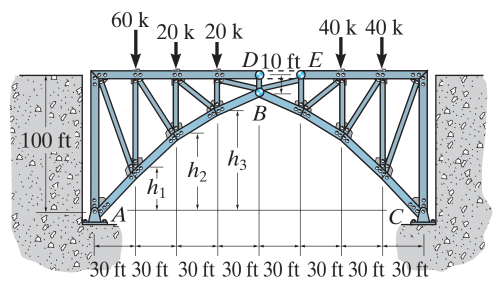
$\xrightarrow{+} \sum F_x = 0;$ $\quad -C_x + 46.67 = 0$

$C_x = 46.7 \text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad C_y - 5.00 - 40 - 40 = 0$

$C_y = 85 \text{ k}$ **Ans.**

5–26. Determine the design heights h_1, h_2, and h_3 of the bottom cord of the truss so the three-hinged trussed arch responds as a funicular arch.

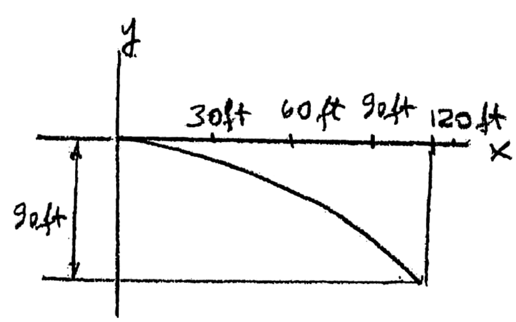


$y = -Cx^2$

$-100 = -C(120)^2$

$C = 0.0069444$

Thus,



$y = -0.0069444x^2$

$y_1 = -0.0069444(90 \text{ ft})^2 = -56.25 \text{ ft}$

$y_2 = -0.0069444(60 \text{ ft})^2 = -25.00 \text{ ft}$

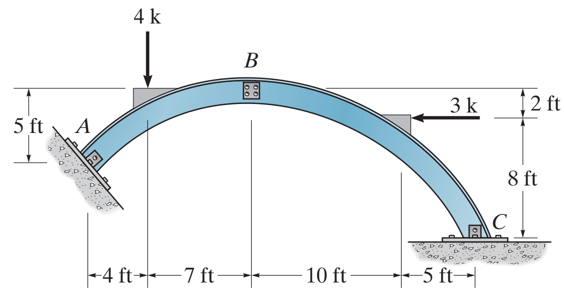
$y_3 = -0.0069444(30 \text{ ft})^2 = -6.25 \text{ ft}$

$h_1 = 100 \text{ ft} - 56.25 \text{ ft} = 43.75 \text{ ft}$ **Ans.**

$h_2 = 100 \text{ ft} - 25.00 \text{ ft} = 75.00 \text{ ft}$ **Ans.**

$h_3 = 100 \text{ ft} - 6.25 \text{ ft} = 93.75 \text{ ft}$ **Ans.**

5–27. Determine the horizontal and vertical components of reaction at *A*, *B*, and *C* of the three-hinged arch. Assume *A*, *B*, and *C* are pin connected.



Member *AB*:

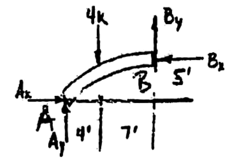
$\circlearrowleft + \sum M_A = 0;$ $\quad B_x(5) + B_y(11) - 4(4) = 0$

Member *BC*:

$\circlearrowleft + \sum M_C = 0;$ $\quad -B_x(10) + B_y(15) + 3(8) = 0$

Soving,

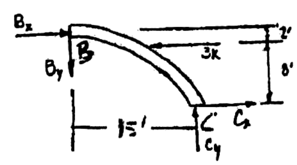
$B_y = 0.216\text{ k}, \quad B_x = 2.72\text{ k}$ **Ans.**



Member *AB*:

$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad A_x - 2.7243 = 0$

$A_x = 2.72\text{ k}$ **Ans.**



$+\uparrow \sum F_y = 0;$ $\quad A_y - 4 + 0.216216 = 0$

$A_y = 3.78\text{ k}$ **Ans.**

Member *BC*:

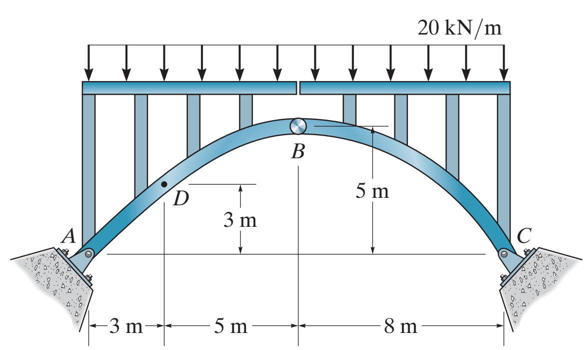
$\overset{+}{\rightarrow} \sum F_x = 0;$ $\quad C_x + 2.7243 - 3 = 0$

$C_x = 0.276\text{ k}$ **Ans.**

$+\uparrow \sum F_y = 0;$ $\quad C_y - 0.216216 = 0$

$C_y = 0.216\text{ k}$ **Ans.**

***5–28.** The three-hinged spandrel arch is subjected to the uniform load of 20 kN/m. Determine the internal moment in the arch at point *D*.



Member *AB*:

$\circlearrowleft + \sum M_A = 0;$ $\quad B_x(5) + B_y(8) - 160(4) = 0$

Member *BC*:

$\circlearrowleft + \sum M_C = 0;$ $\quad -B_x(5) + B_y(8) + 160(4) = 0$

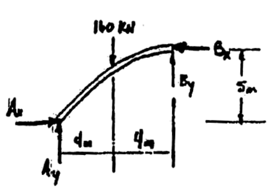
Solving,

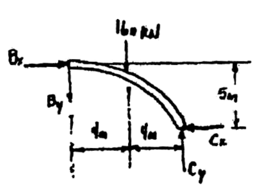
$B_x = 128\text{ kN}, \qquad B_y = 0$

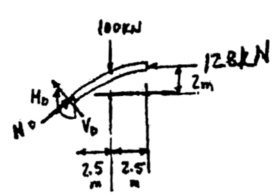
Segment *DB*:

$\circlearrowleft + \sum M_D = 0;$ $\quad 128(2) - 100(2.5) - M_D = 0$

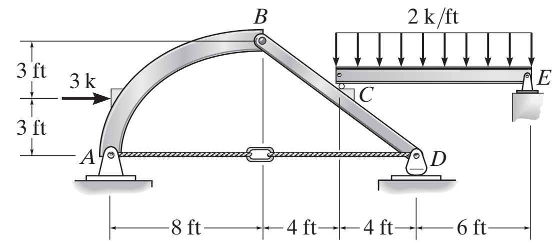
$M_D = 6.00\text{ kN}\cdot\text{m}$ **Ans.**







5–29. The arch structure is subjected to the loading shown. Determine the horizontal and vertical components of reaction at A and D, and the tension in the rod AD.



$\overset{+}{\rightarrow}\sum F_x = 0;$ $\quad -A_x + 3\text{ k} = 0; \quad A_x = 3\text{ k}$ **Ans.**

$\circlearrowleft + \sum M_A = 0;$ $\quad -3\text{ k}\,(3\text{ ft}) - 10\text{ k}\,(12\text{ ft}) + D_y(16\text{ ft}) = 0$

$D_y = 8.06\text{ k}$ **Ans.**

$+\uparrow\sum F_y = 0;$ $\quad A_y - 10\text{ k} + 8.06\text{ k} = 0$

$A_y = 1.94\text{ k}$ **Ans.**

$\circlearrowleft + \sum M_B = 0;$ $\quad 8.06\text{ k}\,(8\text{ ft}) - 10\text{ k}\,(4\text{ ft}) - T_{AD}(6\text{ ft}) = 0$

$T_{AD} = 4.08\text{ k}$ **Ans.**