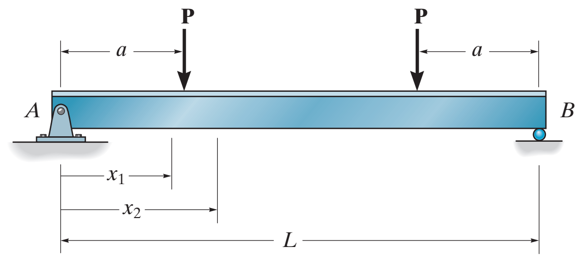
***8–1.** Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify the slope at A and the maximum deflection. EI is constant.



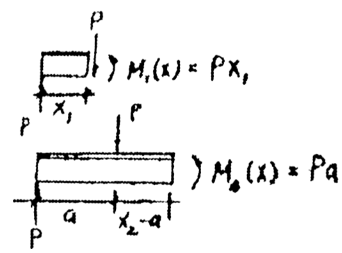
$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = Px_1$

$$EI\frac{d^2v_1}{dx_1^2} = Px_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{2} + C_1 \qquad (1)$$

$$EIv_1 = \frac{Px_1^2}{6}C_1x_1 + C_1 \qquad (2)$$



For $M_1(x) = Pa$

$$EI\frac{d^2v_1}{dx_1^2} = Pa$$

$$EI\frac{dv_1}{dx_1} = Pax_1 + C_1 \qquad (3)$$

$$EIv_1 = \frac{Pax_1^2}{2} = C_3x_1 + C_4 \qquad (4)$$

Boundary conditions:

$$v_1 = 0 \quad \text{at} \quad x = 0$$

From Eq. (2)

$$C_2 = 0$$

Due to symmetry:

$$\frac{dv_1}{dx_1} = 0 \quad \text{at} \quad x_1 = \frac{L}{2}$$

From Eq. (3)

$$0 = Pa\frac{L}{2} + C_3$$

$$C_3 = \frac{PaL}{2}$$

Continuity conditions:

$$v_1 = v_2 \quad \text{at} \quad x_1 = x_2 = a$$

$$\frac{Pa^3}{6} + C_1a = \frac{Pa^3}{2} - \frac{Pa^3L}{2} + C_4$$

$$C_1a \cdot C_4 = \frac{Pa^3}{2} - \frac{Pa^3L}{2} \qquad (5)$$

$$\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2} \quad \text{at} \quad x_1 = x_2 = a$$

8–1. Continued

$$\frac{Pa^3}{2} + C_1 = Pa^3 - \frac{PaL}{2}$$

$$C_1 = \frac{Pa^2}{2} - \frac{PaL}{2}$$

Substitute C_1 into Eq. (5)

$$C_a = \frac{Pa^3}{6}$$

$$\frac{dv_1}{dx_1} = \frac{P}{2EI}(x_1^2 + a^2 - aL)$$

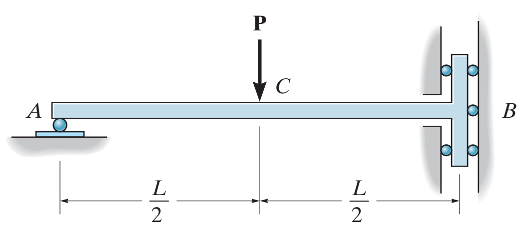
$$\theta_A = \frac{d_{v1}}{d_{x1}}\bigg|_{x_1=0} = \frac{Pa(a-L)}{2EI}$$ **Ans.**

$$v_1 = \frac{Px_1}{6EI}[x_1^2 + 3a(a-L)]$$ **Ans.**

$$v_2 = \frac{Pa}{6EI} + (3x_2(x_2 - L) + a^2)$$ **Ans.**

$$V_{x=1} = V_2\bigg|_{x=\frac{1}{2}} = \frac{Pa}{24EI}(4a^2 - 3L^2)$$ **Ans.**

8–2. The bar is supported by a roller constraint at B, which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope at A and the deflection at C. EI is constant.



$$EI\frac{d^2v_1}{dx_1^2} = M_1 = Px_1$$

$$EI\frac{dv_1}{dx_2} = \frac{Px_1^2}{2} + C_1$$

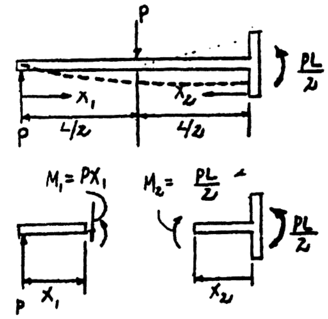
$$EIv_1 = \frac{Px_1^2}{6} + C_1x_1 + C_1$$

$$EI\frac{d^2v_2}{dx_2} = M_2 = \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{PL}{2}x_2 + C_3$$

$$EI\,v_2 = \frac{PL}{4}x_2^2 + C_3x_3 + C_4$$

8–2. Continued



Boundary conditions:

At $x_1 = 0, \quad v_1 = 0$

$0 = 0 + 0 + C_2; \quad C_2 = 0$

At $x_2 = 0, \quad \dfrac{dv_2}{dx_2} = 0$

$0 + C_3 = 0; \quad C_3 = 0$

At $x_1 = \dfrac{L}{2}, \quad x_2 = \dfrac{L}{2}, \quad v_1 = v_2, \quad \dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$

$$\frac{P\left(\frac{L}{2}\right)^2}{6} + C_1\left(\frac{L}{2}\right) = \frac{PL\left(\frac{L}{2}\right)^2}{4} + C_4$$

$$\frac{P\left(\frac{L}{2}\right)^2}{2} + C_1 = -\frac{P\left(\frac{L}{2}\right)}{2}; \quad C_1 = -\frac{3}{8}PL^3$$

$$C_4 = -\frac{11}{48}PL^3$$

At $x_1 = 0$

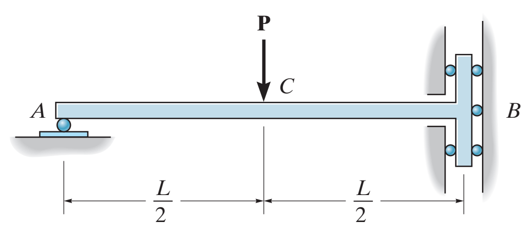
$$\frac{dv_1}{dx_1} = \theta_A = -\frac{3}{8}\frac{PL^2}{EI}$$ **Ans.**

At $x_1 = \dfrac{L}{2}$

$$v_c = \frac{P\left(\frac{L}{2}\right)^3}{6EI} - \left(\frac{3}{8}PL^2\right)\left(\frac{L}{2}\right) + 0$$

$$v_c = \frac{-PL^3}{6EI}$$ **Ans.**

8–3. Determine the deflection at B of the bar in Prob. 8–2.



$$EI\frac{d^2v_1}{dx_1^2} = M_1 = Px_1$$

$$EI\frac{dv_2}{dx_1} = \frac{Px_1^2}{2} + C_1$$

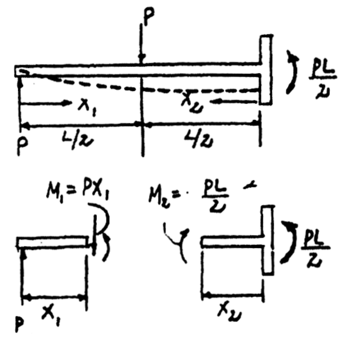
$$EIv_1 = \frac{Px_1^2}{6} + C_1x_1 + C_2$$

$$EI\frac{d^2v_2}{dx_2} = M_2 = \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2}\frac{PL}{2}x_2 + C_3$$

$$EI\,v_2 = \frac{PL}{4}x_2^2 + C_3x_2 + C_4$$

8–3. Continued



Boundary conditions:

At $x_1 = 0, \quad v_1 = 0$

$0 = 0 + 0 + C_2; \quad C_2 = 0$

At $x_2 = 0, \quad \dfrac{dv_2}{dx_2} = 0$

$0 + C_3 = 0; \quad C_3 = 0$

At $x_1 = \dfrac{L}{2}, \quad x_2 = \dfrac{L}{2}, \quad v_1 = v_2, \quad \dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$

$$\frac{P\left(\frac{L}{2}\right)^3}{6} + C_1\left(\frac{L}{2}\right) = \frac{PL\left(\frac{L}{2}\right)^2}{4} + C_4$$

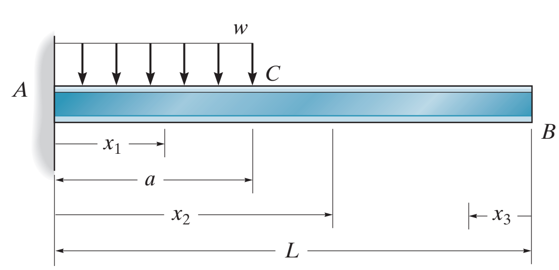
$$\frac{P\left(\frac{L}{2}\right)^3}{2} + C_1 = -\frac{P\left(\frac{L}{2}\right)}{2}; \quad C_1 = -\frac{3}{8}PL^2$$

$$C_4 = \frac{11}{48}PL^3$$

At $x_2 = 0,$

$$v_B = -\frac{11PL^3}{48EI}$$ **Ans.**

***8–4.** Determine the equations of the elastic curve using the coordinates x_1 and x_2, specify the slope and deflection at B. EI is constant.



$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = -\dfrac{w}{2}x_1^2 + wax_1 - \dfrac{wa^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$$

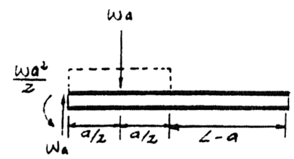
$$EI\frac{dv_1}{dx_1} = -\frac{w}{6}x_1^3 + \frac{wa}{2}x_1^2 - \frac{wa^2}{2}x_1 + C_1 \qquad (1)$$

8–4. Continued

$$EIv_1 = -\frac{w}{24}x_1^4 + \frac{wa}{6}x_1^3 - \frac{wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = 0; \quad EI\frac{d^2v_2}{dx_2^3} = 0$

$$EI\frac{dv_2}{dx_2} = C_3 \qquad (3)$$

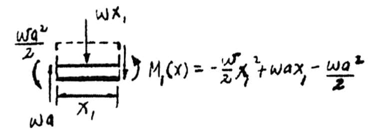


$$EI\,v_2 = C_3x_2 + C_4 \qquad (4)$$

Boundary conditions:

At $x_1 = 0, \quad \frac{dv_1}{dx_1} = 0$

From Eq. (1), $\quad C_1 = 0$

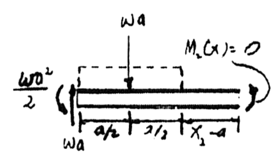


At $x_1 = 0, \quad v_1 = 0$

From Eq. (2): $\quad C_2 = 0$

Continuity conditions:

At $x_1 = a, \quad x_2 = a; \quad \frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$



From Eqs. (1) and (3),

$$-\frac{wa^3}{6} + \frac{wa^3}{2} - \frac{wa^3}{2} = C_3; \quad C_3 = -\frac{wa^3}{6}$$

From Eqs. (2) and (4),

At $x_1 = a, \quad x_2 = a \quad v_1 = v_2$

$$-\frac{wa^4}{24} + \frac{wa^4}{6} - \frac{wa^4}{4} = -\frac{wa^4}{6} + C_4; \quad C_4 = \frac{wa^4}{24}$$

The slope, from Eq. (3),

$$\theta_B = \frac{dv_2}{dx_2} = \frac{wa^3}{6EI} \qquad \textbf{Ans.}$$

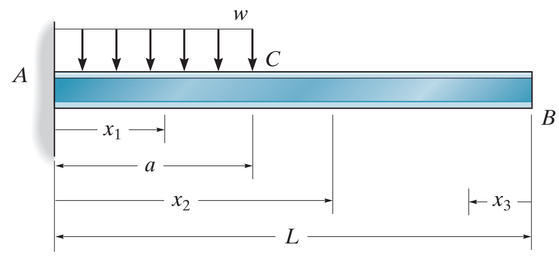
The elastic curve:

$$v_1 = \frac{w}{24EI}\left(-x_1^4 + 4ax_1^3 - 6a^2x_1^2\right) \qquad \textbf{Ans.}$$

$$v_2 = \frac{wa^3}{24EI}\left(-4x_2 + a\right) \qquad \textbf{Ans.}$$

$$v_1 = v_2\Big|_{x_3 = L} = \frac{wa^3}{24EI}\left(-4L + a\right) \qquad \textbf{Ans.}$$

8–5. Determine the equations of the elastic curve using the coordinates x_1 and x_3, and specify the slope and deflection at point B. EI is constant.



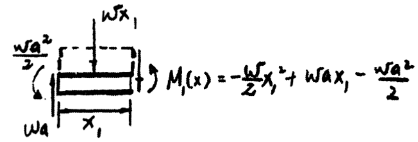
$$EI\frac{d^2v}{dx_2} = M(x)$$

For $M_1(x) = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = -\frac{w}{6}x_1^3 + \frac{wa}{2}x_1^2 - \frac{wa^2}{2}x_1 + C_1 \qquad (1)$$

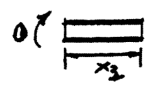
$$EIv_1 = -\frac{w}{24}x_1^4 + \frac{wa}{6}x_1^3 - \frac{wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad (2)$$



For $M_2(x) = 0$; $\quad EI\frac{d^2v_3}{dx_3^2} = 0$

$$EI\frac{dv_3}{dx_3} = C_3 \qquad (3)$$

$$EI\,v_3 = C_3x_3 + C_4 \qquad (4)$$



Boundary conditions:

At $x_1 = 0$, $\quad \frac{dv_1}{dx_1} = 0$

From Eq. (1),

$$0 = -0 + 0 - 0 + C_1; \quad C_1 = 0$$

At $x_1 = 0$, $\quad v_1 = 0$

From Eq. (2),

$$0 = -0 - 0 - 0 + 0 + C_2; \quad C_2 = 0$$

Continuity conditions:

At $x_1 = a$, $\quad x_3 = L - a$; $\quad \frac{dv_1}{dx_1} = \frac{dv_3}{dx_3}$

$$-\frac{wa^3}{6} + \frac{wa^3}{2} - \frac{-wa^3}{2} = -C_3; \quad C_3 = +\frac{wa^3}{6}$$

At $x_1 = a$, $\quad x_3 = L - a \quad v_1 = v_2$

$$-\frac{wa^4}{24} + \frac{wa^4}{6} - \frac{wa^4}{4} = \frac{wa^3}{6}(L - a) + C_4; \quad C_4 = \frac{wa^4}{24} - \frac{wa^3L}{6}$$

The slope

$$\frac{dv_3}{dx_3} = \frac{wa^3}{6EI}$$

$$\theta_B = \frac{d_{v3}}{d_{x3}}\bigg|_{x_3 = 0} = \frac{wa^3}{6EI} \qquad \textbf{Ans.}$$

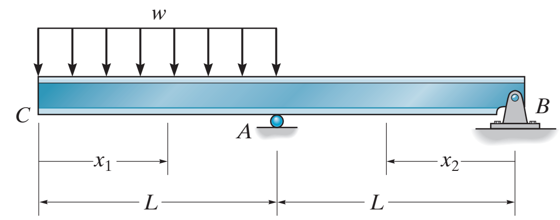
The elastic curve:

$$v_1 = \frac{wx_1^2}{24EI}\left(-x_1^2 + 4ax_1 - 6a^2\right) \qquad \textbf{Ans.}$$

$$v_3 = \frac{wa^3}{24EI}\left(4x_3 + a - 4L\right) \qquad \textbf{Ans.}$$

$$V_2 = V_3\bigg|_{x_3 = 0} = \frac{wa^3}{24EI}\left(a - 4L\right) \qquad \textbf{Ans.}$$

8–6. Determine the maximum deflection between the supports A and B. EI is constant. Use the method of integration.

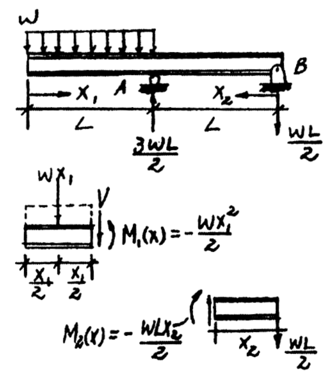


Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = \dfrac{-wx_1^2}{2}$

$$EI\frac{d^1v_1}{dx_1^2} = \frac{-wx_1^2}{2}$$



$$EI\frac{dv_1}{dx_1} = \frac{-wx_1^3}{6} + C_1 \qquad (1)$$

$$EIv_1 = -\frac{wx_1^4}{24} + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = \dfrac{-wLx_2}{2}$

$$EI\frac{d^2v_2}{dx_3^2} = \frac{-wLx_2}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{-wLx_2^2}{4} + C_3 \qquad (3)$$

$$EIv_2 = \frac{-wLx_2^3}{412} + C_3x_3 + C_4 \qquad (4)$$

Boundary conditions:

$v_2 = 0$ at $x_2 = 0$

From Eq. (4):

$C_4 = 0$

$v_2 = 0$ at $x_2 = L$

From Eq. (4):

$$0 = \frac{-wL^4}{12} + C_3L$$

$$C_3 = \frac{wL^3}{12}$$

$v_1 = 0$ at $x_1 = L$

From Eq. (2):

$$0 = -\frac{wL^4}{24} + C_1L + C_2 \qquad (5)$$

8–6. Continued

Continuity conditions:

$$\frac{dv_1}{dx_1} = \frac{dv_2}{-dx_2} \quad \text{at} \quad x_1 = x_2 = L$$

From Eqs. (1) and (3)

$$-\frac{wL^3}{6} + C_1 = -\left(-\frac{wL^3}{4} + \frac{wL^3}{12}\right)$$

$$C_1 = \frac{wL^3}{3}$$

Substitute C_1 into Eq. (5)

$$C_2 = \frac{7wL^4}{24}$$

$$\frac{dv_1}{dx_1} = \frac{w}{6EI}(2L^3 - x_1^3)$$

$$\frac{dv_2}{dx_2} = \frac{w}{12EI}(L^3 - 3Lx_2^2) \qquad (6)$$

$$\theta_A = \frac{d_{v1}}{d_{x1}}\bigg|_{x_1 = L} = -\frac{dv_2}{dv_3}\bigg|_{x_3 = L} = \frac{wL^3}{6EI}$$

$$v_1 = \frac{w}{24EI}(-x_1^4 + 8L^3x_1 - 7L^4)$$

$$(v_1)_{\max} = \frac{-7wL^4}{24EI}(x_1 = 0)$$

The negative sign indicates downward displacement

$$v_2 = \frac{wL}{12EI}(L^2x_2 - x_2^3) \qquad (7)$$

$(v_2)_{\max}$ occurs when $\frac{dv_2}{dx_2} = 0$

From Eq. (6)

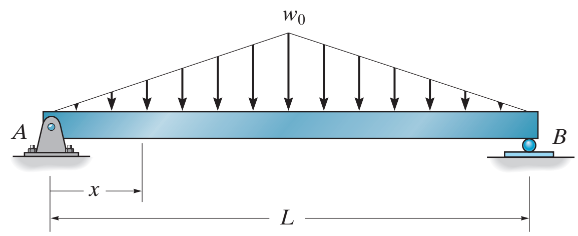
$$L^3 - 3Lx_2^2 = 0$$

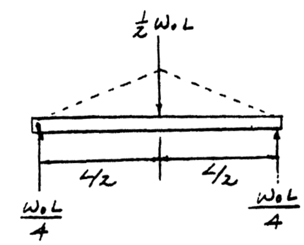
$$x_2 = \frac{L}{\sqrt{3}}$$

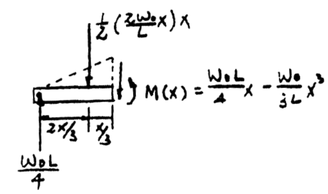
Substitute x_2 into Eq. (7),

$$(v_2)_{\max} = \frac{wL_4}{18\sqrt{3EI}} \qquad \textbf{Ans.}$$

8–7. Determine the elastic curve for the simply supported beam using the x coordinate $0 \le x \le L/2$. Also, determine the slope at A and the maximum deflection of the beam. EI is constant.







$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{w_o L}{4}x - \frac{w_o}{3L}x^3$$

$$EI\frac{dv}{dx} = \frac{w_o L}{8}x^2 - \frac{w_o}{12L}x^4 + C_1 \qquad (1)$$

$$EIv = \frac{w_o L}{24}x^3 - \frac{w_o}{60L}x^5 + C_1 x + C_2 \qquad (2)$$

Boundary conditions:

Due to symmetry, at $x = \frac{L}{2}, \frac{dv}{dx} = 0$

From Eq. (1),

$$0 = \frac{w_o L}{8}\left(\frac{L^2}{4}\right) - \frac{w_o}{12L}\left(\frac{L^4}{16}\right) + C_1; \qquad C_1 = -\frac{5w_o L^3}{192}$$

At $x = 0, \quad v = 0$

From Eq. (2),

$$0 = 0 - 0 + 0 + C_2; \quad C_2 = 0$$

From Eq. (1),

$$\frac{dv}{dx} = \frac{w_o}{192EIL}(24L^2x^2 - 16x^4 - 5L^4)$$

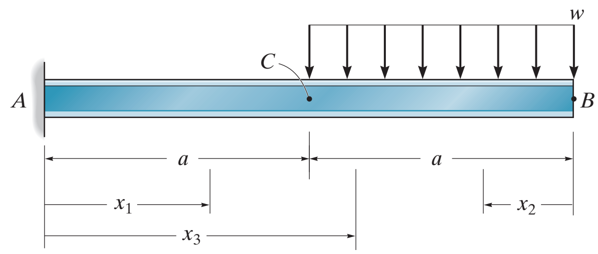
$$\theta_A = \frac{d_v}{d_x}\bigg|_{x=0} = -\frac{5w_o L^3}{192EI} = \frac{5w_o L^3}{192EI} \qquad \textbf{Ans.}$$

From Eq. (2),

$$v = \frac{w_o x}{960EIL}(40L^2x^2 - 16x^4 - 25L^4) \qquad \textbf{Ans.}$$

$$v_{\max} = v\bigg|_{x=\frac{L}{3}} = -\frac{w_o L^4}{120EI} = \frac{w_o L^4}{120EI} \qquad \textbf{Ans.}$$

***8–8.** Determine the equations of the elastic curve using the coordinates x_1 and x_2, and specify the slope at C and displacement at B. EI is constant.



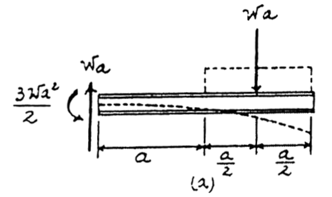
***Support Reactions and Elastic Curve*:** As shown on FBD(a).

***Moment Function*:** As shown on FBD(c) and (c).

***Slope and Elastic Curve*:**

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = wax_1 - \dfrac{3wa^2}{2}$,

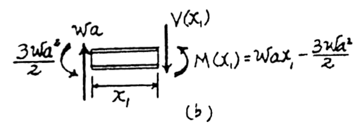


$$EI\frac{d^2v_1}{dx_1^2} = wax_1 - \frac{3wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{wa}{2}x_1^2 - \frac{3wa^2}{2}x_1 + C_1 \qquad (1)$$

$$EIv_1 = \frac{wa}{6}x_1^3 - \frac{3wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad (2)$$

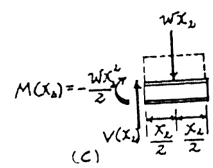
For $M(x_2) = -\dfrac{w}{2}x_2^2$,



$$EI\frac{d^2v_2}{dx_2^2} = -\frac{w}{2}x_2^2$$

$$EI\frac{dv_2}{dx_2} = -\frac{w}{6}x_2^3 + C_3 \qquad (3)$$

$$EIv_2 = \frac{w}{24}x_2^4 + C_3x_2 + C_4 \qquad (4)$$



***Boundary Conditions*:**

$\dfrac{dv_1}{dx_1} = 0$ at $x_1 = 0$, From Eq. [1], $C_1 = 0$

$v_1 = 0$ at $x_1 = 0$ From Eq. [2], $C_2 = 0$

***Continuity Conditions*:**

At $x_1 = a$ and $x_2 = a$, $\dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$ From Eqs. [1] and [3],

$$\frac{wa^3}{2} - \frac{3wa^3}{2} = -\left(-\frac{wa^3}{6} + C_3\right) \qquad C_3 = \frac{7wa^3}{6}$$

At $x_1 = a$ and $x_2 = a$, $v_1 = v_2$. From Eqs. [2] and [4],

$$\frac{wa^4}{6} - \frac{3wa^4}{4} = -\frac{wa^4}{24} + \frac{5wa^4}{6} + C_4 \qquad C_4 = -\frac{41wa^4}{8}$$

***The Slope*:** Substituting into Eq. [1],

$$\frac{dv_1}{dx_1} = \frac{wax_1}{2EI}(x_1 - 3a)$$

$$\theta_C = \left.\frac{dv_2}{dx_2}\right|_{x_1 = a} = -\frac{wa^3}{EI} \qquad \textbf{Ans.}$$

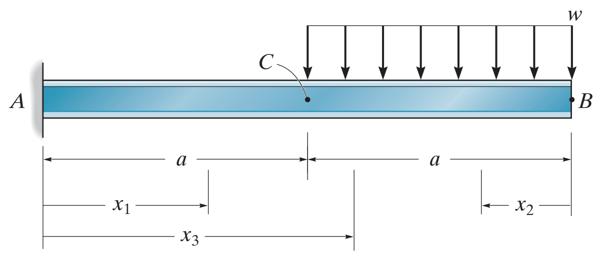
***The Elastic Curve*:** Substituting the values of C_1, C_2, C_3, and C_4 into Eqs. [2] and [4], respectively

$$v_1 = \frac{wax_1}{12EI}(2x_1^2 - 9ax_1) \qquad \textbf{Ans.}$$

$$v_2 = \frac{w}{24EI}(-x_2^4 + 28a^3x_2 - 41a^4) \qquad \textbf{Ans.}$$

$$v_B = \left.v_2\right|_{x_2 = 0} = -\frac{41wa^4}{24EI} \qquad \textbf{Ans.}$$

8–9. Determine the equations of the elastic curve using the coordinates x_1 and x_3, and specify the slope at B and deflection at C. EI is constant.



Support Reactions and Elastic Curve: As shown on FBD(a).
Moment Function: As shown on FBD(b) and (c).
Slope and Elastic Curve:

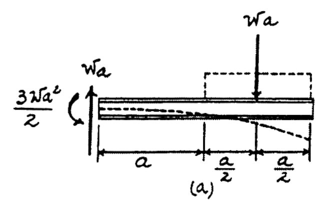
$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = wax_1 - \frac{3wa^2}{2}$,

$$EI\frac{d^2v_1}{dx_1^2} = wax_1 - \frac{3wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{wa}{2}x_1^2 - \frac{3wa^2}{2}x_1 + C_1 \qquad (1)$$

$$EIv_1 = \frac{wa}{6}x_1^3 - \frac{3wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad (2)$$

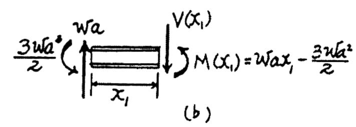


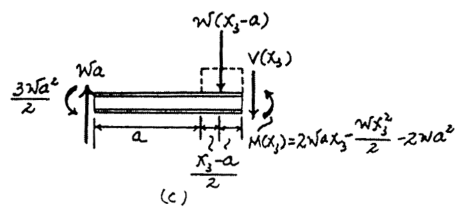
For $M(x_3) = 2wax_3 - \frac{w}{2}x_2^3 - 2wa^2$,

$$EI\frac{d^2v_3}{dx_3^2} = 2wax_3 - \frac{w}{2}x_3^2 - 2wa^2$$

$$EI\frac{dv_3}{dx_3} = wax_3^2 - \frac{w}{6}x_3^3 - 2wa^2x_3 + C_3 \qquad (3)$$

$$EIv_3 = \frac{wa}{3}x_3^3 - \frac{w}{24}x_3^4 - wa^2x_3^2 + C_3x_3 + C_4 \qquad (4)$$





Boundary Conditions:

$\frac{dv_1}{dx_1} = 0$ at $x_1 = 0$, From Eq. [1], $C_1 = 0$

$v_1 = 0$ at $x_1 = 0$, From Eq. [2], $C_2 = 0$

Continuity Conditions:

At $x_1 = a$ and $x_3 = a$, $\frac{dv_1}{dx_1} = \frac{dv_3}{dx_3}$ From Eqs. [1] and [3],

$$\frac{wa^3}{2} - \frac{3wa^3}{2} = wa^3 - \frac{wa^3}{6} - 2wa^3 + C_3 \qquad C_3 = \frac{wa^3}{6}$$

At $x_1 = a$ and $x_3 = a$, $v_1 = v_3$, From Eqs.[2] and [4],

$$\frac{wa^4}{6} - \frac{3wa^4}{4} = \frac{wa^4}{3} - \frac{wa^4}{24} - wa^4 + \frac{wa^4}{6} + C_4 \qquad C_4 = -\frac{wa^4}{24}$$

The Slope: Substituting the value of C_3 into Eq. [3],

$$\frac{dv_3}{dx_3} = \frac{w}{2EI}(6ax_3^2 - x_3^3 - 12a^2x_3 + a^3)$$

$$\theta_B = \left.\frac{dv_3}{dx_3}\right|_{x_3 = 2a} = -\frac{7wa^3}{6EI} \qquad \textbf{Ans.}$$

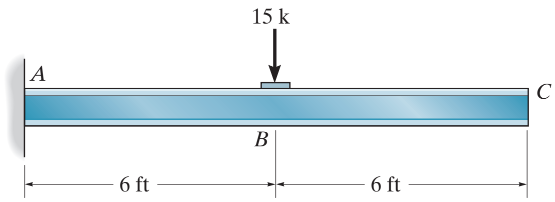
The Elastic Curve: Substituting the values of C_1, C_2, C_3, and C_4 into Eqs. [2] and [4], respectively,

$$v_1 = \frac{wax_1}{12EI}(2x_1^2 - 9ax_1) \qquad \textbf{Ans.}$$

$$v_C = v_1\Big|_{x_1 = a} = -\frac{7wa^4}{12EI} \qquad \textbf{Ans.}$$

$$v_3 = \frac{w}{24EI}(-x_3^4 + 8ax_3^3 - 24a^2x_3^2 + 4a^3x_3 - a^4) \qquad \textbf{Ans.}$$

8–10. Determine the slope at B and the maximum displacement of the beam. Use the moment-area theorems. Take $E = 29(10^3)$ ksi, $I = 500\ \text{in}^4$.



Using the $\dfrac{M}{EI}$ diagram and the elastic curve shown in Fig. a and b, respectively, Theorem 1 and 2 give

$$\theta_B = |\theta_{B/A}| = \frac{1}{2}\left(\frac{90\ \text{k}\cdot\text{ft}}{EI}\right)(6\ \text{ft})$$

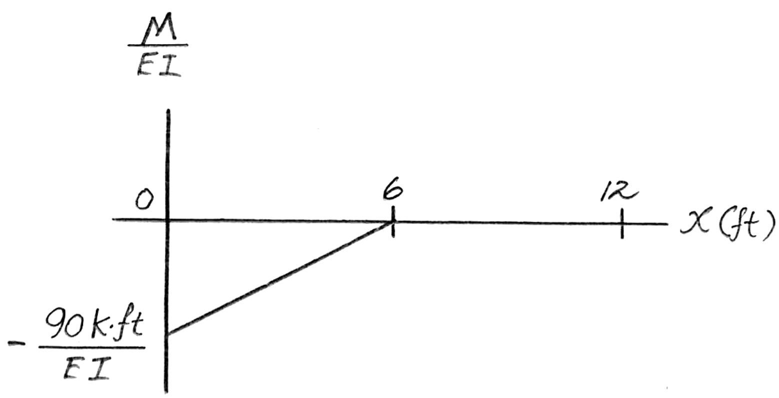
$$= \frac{270\ \text{k}\cdot\text{ft}^2}{EI} = \frac{270\ (144)\ \text{k}\cdot\text{in}^2}{\left[29(10^3)\frac{\text{k}}{\text{in}^2}\right](500\ \text{in}^4)} = 0.00268\ \text{rad} \qquad \textbf{Ans.}$$

$$\Delta_{max} = \Delta_C = |t_{B/A}| = \left[\frac{1}{2}\left(\frac{90\ \text{k}\cdot\text{ft}}{EI}\right)(6\ \text{ft})\right]\left[6\ \text{ft} + \frac{2}{3}(6\ \text{ft})\right]$$

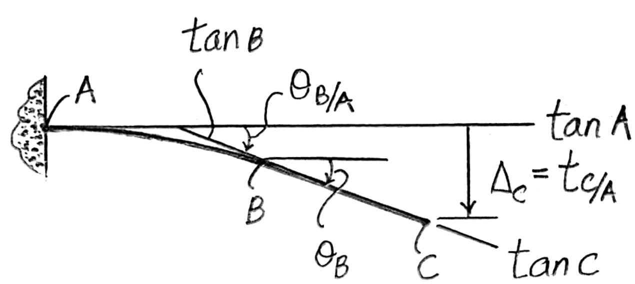
$$= \frac{2700\ \text{k}\cdot\text{ft}^3}{EI}$$

$$= \frac{2700\ (1728)\ \text{k}\cdot\text{in}^3}{\left[29(10^3)\frac{\text{k}}{\text{in}^2}\right](500\ \text{in}^4)}$$

$$= 0.322\ \text{in} \downarrow \qquad \textbf{Ans.}$$

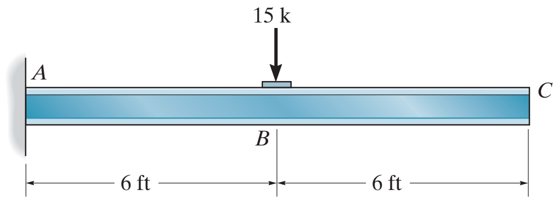


(a)



(b)

8–11. Solve Prob. 8–10 using the conjugate-beam method.

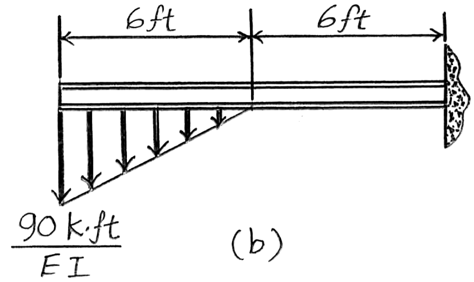


The real beam and conjugate beam are shown in Fig. *b* and *c*, respectively. Referring to Fig. *c*,

$$+\uparrow \sum F_y = 0; \qquad -V'_B - \frac{1}{2}\left(\frac{90\ \text{k}\cdot\text{ft}}{EI}\right)(6\ \text{ft}) = 0$$

$$\theta_B = V'_B = -\frac{270\ \text{k}\cdot\text{ft}^2}{EI}$$

$$= \frac{270\,(12^2)\ \text{k}\cdot\text{in}^2}{\left[29(10^3)\frac{\text{k}}{\text{in}^2}\right](500\ \text{in}^4)} = 0.00268\ \text{rad} \qquad \textbf{Ans.}$$



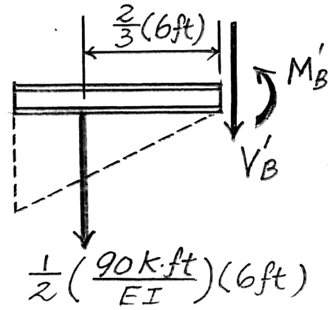
(b)

Referring to Fig. *d*,

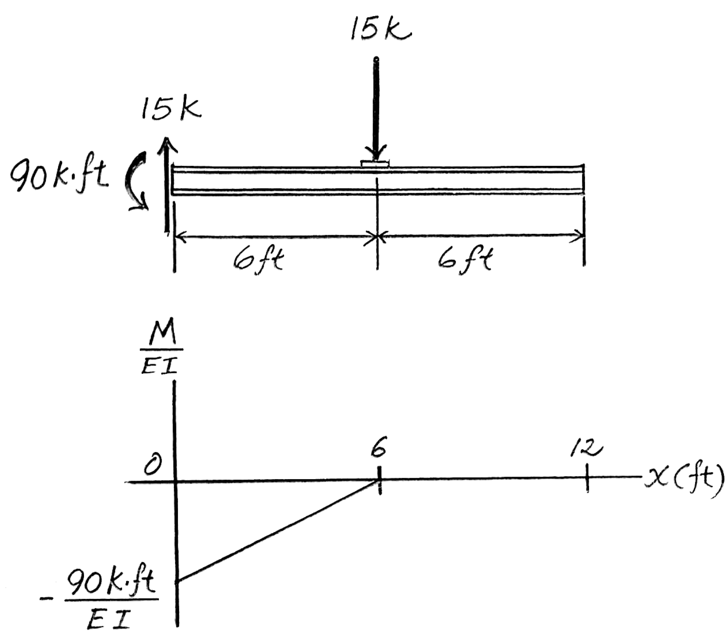
$$\circlearrowleft + \sum M_C = 0; \qquad M'_C + \left[\frac{1}{2}\left(\frac{90\ \text{k}\cdot\text{ft}}{EI}\right)(6\ \text{ft})\right]\left[6\ \text{ft} + \frac{2}{3}(6\ \text{ft})\right] = 0$$

$$\Delta_{max} = \Delta_C = M'_C = -\frac{2700\ \text{k}\cdot\text{ft}^3}{EI}$$

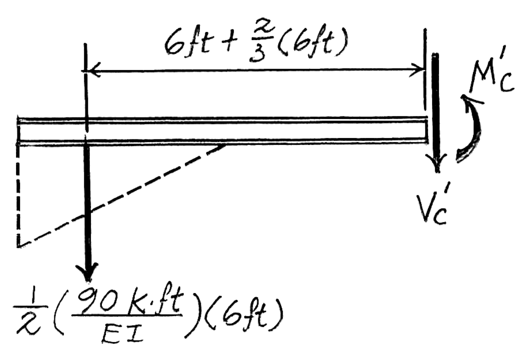
$$= \frac{2700\,(12^3)\ \text{k}\cdot\text{in}^3}{\left[29(10^3)\frac{\text{k}}{\text{in}^2}\right](500\ \text{in}^4)} = 0.322\ \text{in} \downarrow \qquad \textbf{Ans.}$$



(c)

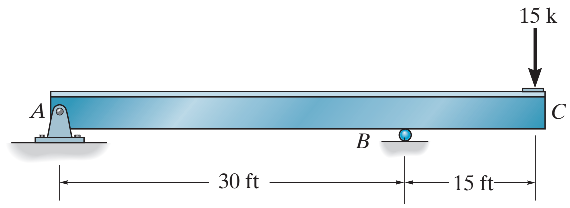


(a)



(d)

***8–12.** Determine the slope and displacement at *C*. *EI* is constant. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and the elastic curve shown in Fig. *a* and *b*, respectively, Theorem 1 and 2 give

$$\theta_{C/A} = \frac{1}{2}\left(-\frac{225\text{ k}\cdot\text{ft}}{EI}\right)(45\text{ ft}) = -\frac{5062.5\text{ k}\cdot\text{ft}^2}{EI} = \frac{5062.5\text{ k}\cdot\text{ft}^2}{EI}$$

$$|t_{B/A}| = \left[\frac{1}{2}\left(\frac{225\text{ k}\cdot\text{ft}}{EI}\right)(30\text{ ft})\right]\left[\frac{1}{3}(30\text{ ft})\right] = \frac{33750\text{ k}\cdot\text{ft}^3}{EI}$$

$$|t_{C/A}| = \left[\frac{1}{2}\left(\frac{225\text{ k}\cdot\text{ft}}{EI}\right)(30\text{ ft})\right]\left[15\text{ ft} + \frac{1}{3}(30\text{ ft})\right] + \left[\frac{1}{2}\left(\frac{225\text{ k}\cdot\text{ft}}{EI}\right)(15\text{ ft})\right]\left[\frac{2}{3}(15\text{ ft})\right]$$

$$= \frac{101250\text{ k}\cdot\text{ft}^3}{EI}$$

Then,

$$\Delta' = \frac{45}{30}(t_{B/A}) = \frac{45}{30}\left(\frac{33750\text{ k}\cdot\text{ft}^3}{EI}\right) = \frac{50625\text{ k}\cdot\text{ft}^3}{EI}$$

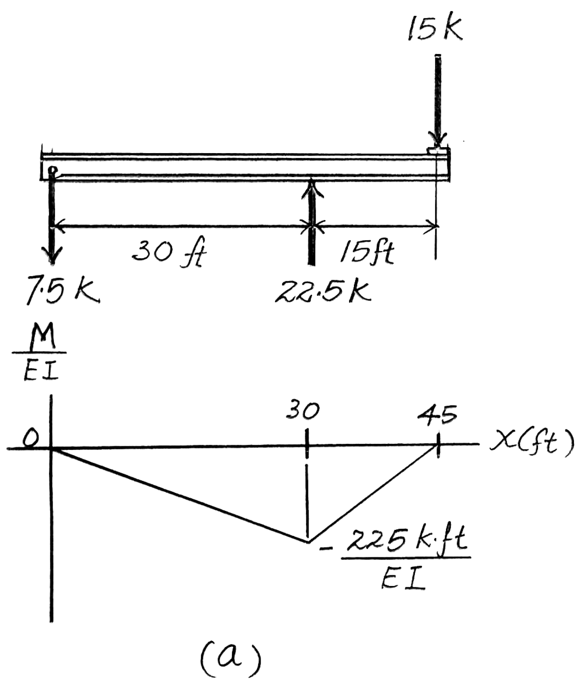
$$\theta_A = \frac{|t_{B/A}|}{L_{AB}} = \frac{33750\text{ k}\cdot\text{ft}^3/EI}{30\text{ ft}} = \frac{1125\text{ k}\cdot\text{ft}^2}{EI}$$

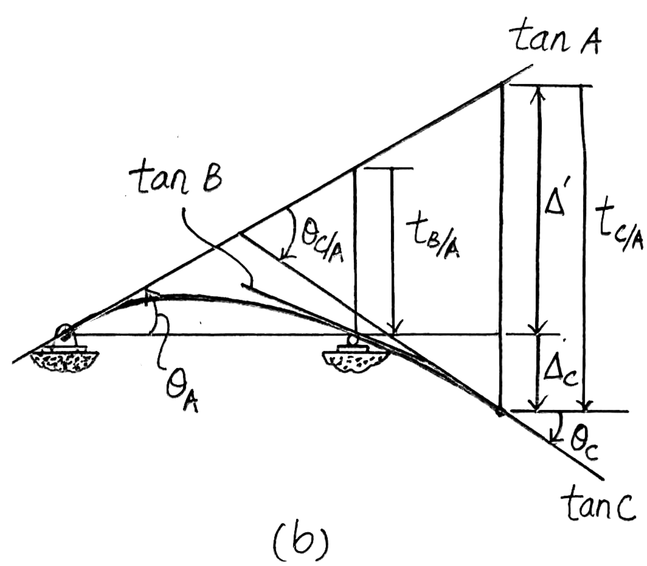
$$+\circlearrowleft\ \theta_C = \theta_A + \theta_{C/A}$$

$$\theta_C = \frac{-1125\text{ k}\cdot\text{ft}^2}{EI} + \frac{5062.5\text{ k}\cdot\text{ft}^2}{EI} = \frac{3937.5\text{ k}\cdot\text{ft}^2}{EI}$$ **Ans.**

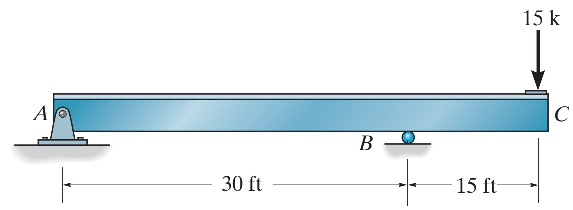
$$\Delta_C = \left|t_{C/A}\right| - \Delta' = \frac{101250\text{ k}\cdot\text{ft}^3}{EI} - \frac{50625\text{ k}\cdot\text{ft}^3}{EI}$$

$$= \frac{50625\text{ k}\cdot\text{ft}^3}{EI}\ \downarrow$$ **Ans.**





8–13. Solve Prob. 8–12 using the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*,

$$\zeta+ \sum M_A = 0; \quad B'_y(30\text{ ft}) - \left[\frac{1}{2}\left(\frac{225\text{ k}\cdot\text{ft}}{EI}\right)(30\text{ ft})\right](20\text{ ft})$$

$$B'_y = \frac{2250\text{ k}\cdot\text{ft}^2}{EI}$$

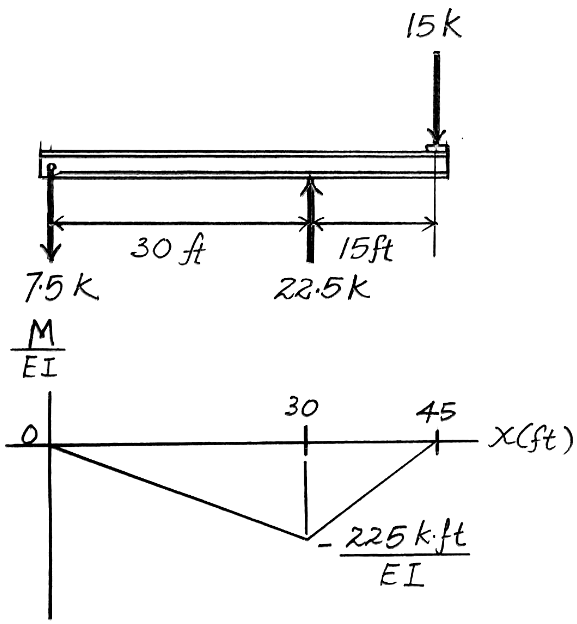
Referring to Fig. *d*,

$$+\uparrow \sum F_y = 0; \quad -V'_C - \frac{1}{2}\left(\frac{225\text{ k}\cdot\text{ft}}{EI}\right)(15\text{ ft}) - \frac{2250\text{ k}\cdot\text{ft}}{EI}$$

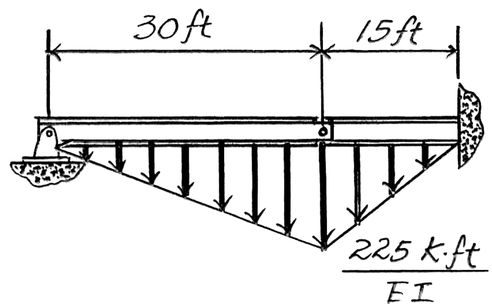
$$\theta_C = V'_C = -\frac{3937.5\text{ k}\cdot\text{ft}^2}{EI} = \frac{3937.5\text{ k}\cdot\text{ft}^2}{EI} \quad \triangledown \qquad \textbf{Ans.}$$

$$\zeta+ \sum M_C = 0; \quad M'_C + \left[\frac{1}{2}\left(\frac{225\text{ k}\cdot\text{ft}}{EI}\right)(15\text{ ft})\right](10\text{ ft}) + \left(\frac{2250\text{ k}\cdot\text{ft}^2}{EI}\right)(15\text{ ft})$$

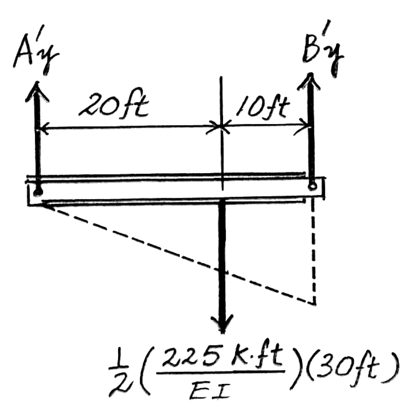
$$\Delta_C = M'_C = \frac{50625\text{ k}\cdot\text{ft}^3}{EI} = \frac{50625\text{ k}\cdot\text{ft}^3}{EI} \quad \downarrow \qquad \textbf{Ans.}$$



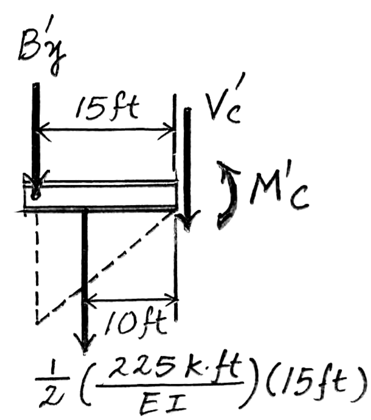
(a)



(b)

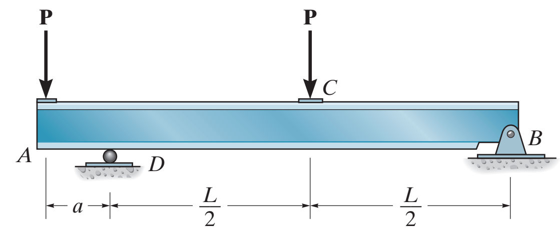


(c)



(d)

8–14. Determine the value of a so that the slope at A is equal to zero. EI is constant. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and the elastic curve shown in Fig. a and b, respectively, Theorem 1 and 2 give

$$\theta_{A/B} = \frac{1}{2}\left(\frac{PL}{4EI}\right)(L) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a + L)$$

$$= \frac{PL^2}{8EI} - \frac{Pa^2}{2EI} - \frac{PaL}{2EI}$$

$$t_{D/B} = \left[\frac{1}{2}\left(\frac{PL}{4EI}\right)(L)\right]\left(\frac{L}{2}\right) + \left[\frac{1}{2}\left(-\frac{Pa}{EI}\right)(L)\right]\left(\frac{L}{3}\right)$$

$$= \frac{PL^3}{16EI} - \frac{PaL^2}{6EI}$$

Then

$$\theta_B = \frac{t_{D/B}}{L} = \frac{PL^2}{16EI} - \frac{PaL}{6EI}$$

Here, it is required that

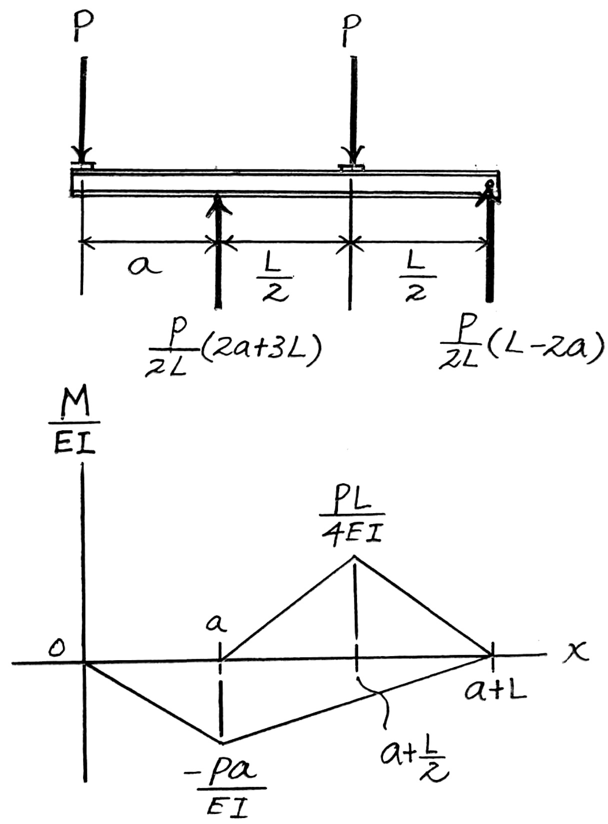
$$\theta_B = \theta_{A/B}$$

$$\frac{PL^2}{16EI} - \frac{PaL}{6EI} = \frac{PL^2}{8EI} - \frac{Pa^2}{2EI} - \frac{PaL}{2EI}$$

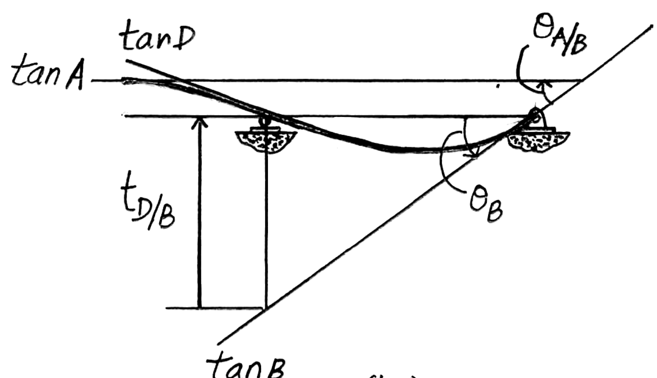
$$24a^2 + 16La - 3L^2 = 0$$

Choose the position root,

$$a = 0.153\,L$$ **Ans.**

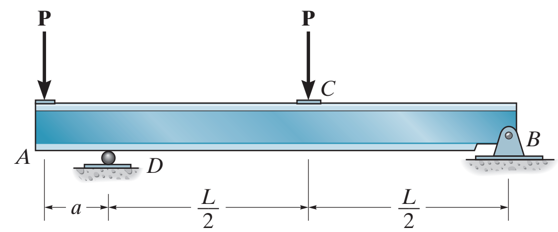


(a)



(b)

8–15. Solve Prob. 8–14 using the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *d*,

$$\zeta + \sum M_B = 0; \quad D_y'(L) + \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(L)\right]\left(\frac{2}{3}L\right) - \left[\frac{1}{2}\left(\frac{PL}{4EI}\right)(L)\right]\left(\frac{1}{2}\right) = 0$$

$$D_y' = \frac{PL^2}{16EI} - \frac{PaL}{3EI}$$

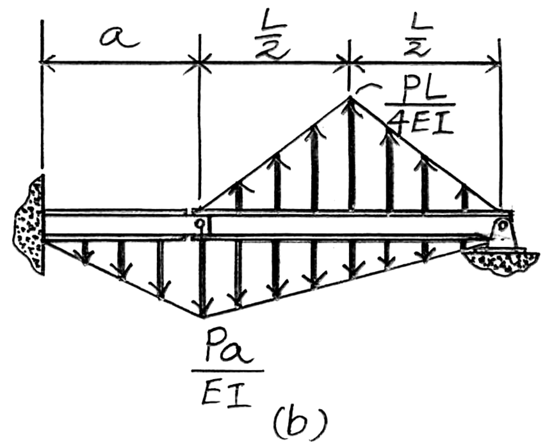
It is required that $V_A' = \theta_A = 0$, Referring to Fig. *c*,

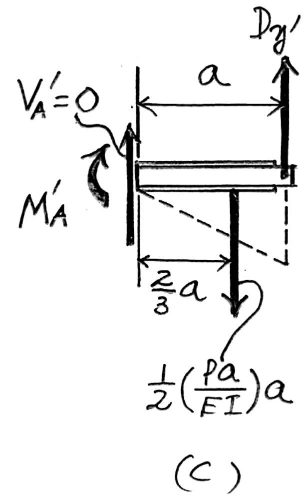
$$\uparrow + \sum F_y = 0; \quad \frac{PL^2}{16EI} - \frac{PaL}{3EI} - \frac{Pa^2}{2EI} = 0$$

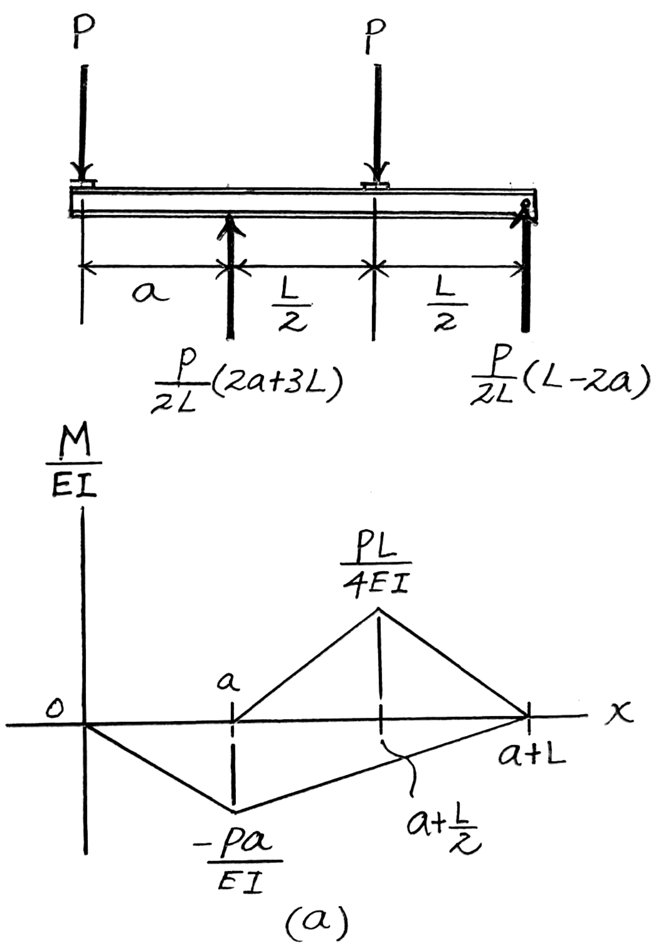
$$24a^2 + 16La - 3L^2 = 0$$

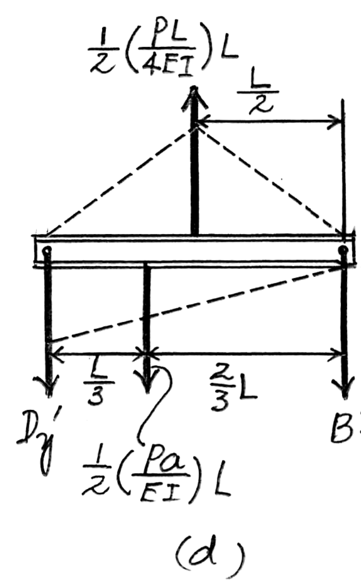
Choose the position root,

$$a = 0.153\text{ L}$$ **Ans.**

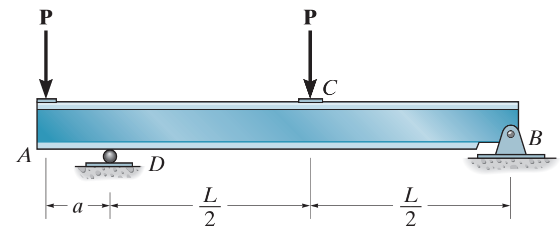








***8–16.** Determine the value of a so that the displacement at C is equal to zero. EI is constant. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and the elastic curve shown in Fig. a and b, respectively, Theorem 2 gives

$$t_{D/B} = \left[\frac{1}{2}\left(\frac{PL}{4EI}\right)(L)\right]\left(\frac{L}{2}\right) + \left[\frac{1}{2}\left(-\frac{Pa}{EI}\right)(L)\right]\left(\frac{L}{3}\right)$$

$$= \frac{PL^3}{16EI} - \frac{PaL^2}{6EI}$$

$$T_{C/B} = \left[\frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{2}\right)\right]\left[\frac{1}{3}\left(\frac{L}{2}\right)\right] + \left[\frac{1}{2}\left(-\frac{Pa}{2EI}\right)\left(\frac{L}{2}\right)\right]\left[\frac{1}{3}\left(\frac{L}{2}\right)\right]$$

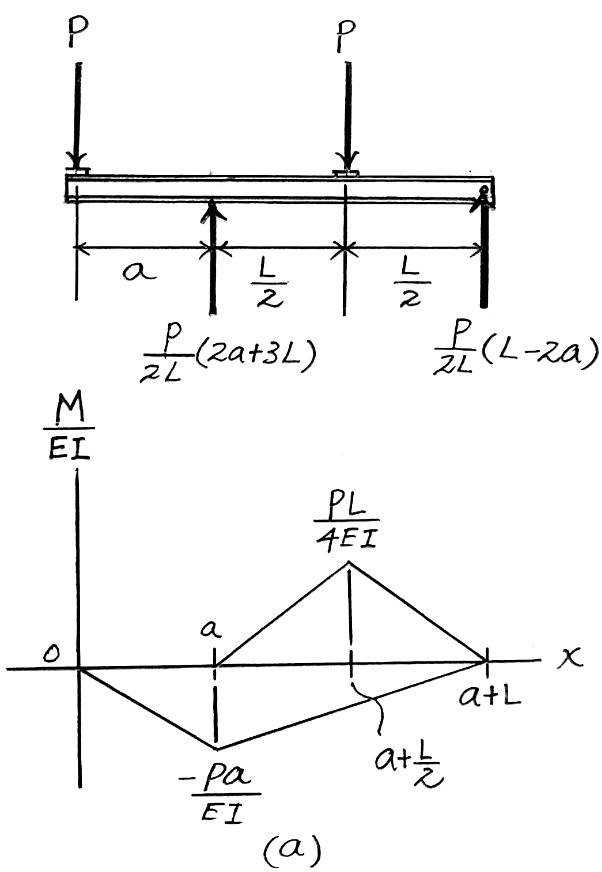
$$= \frac{PL^3}{96EI} - \frac{PaL^2}{48EI}$$

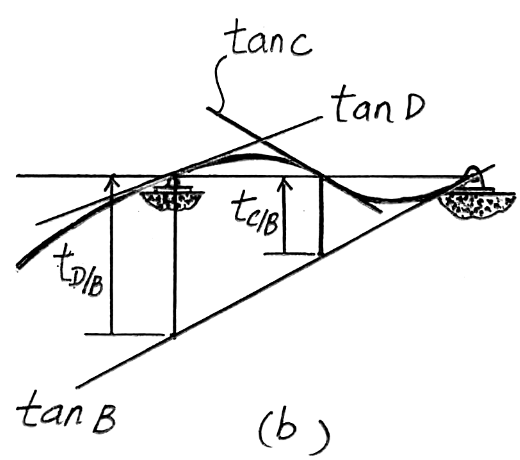
It is required that

$$t_{C/B} = \frac{1}{2}t_{D/B}$$

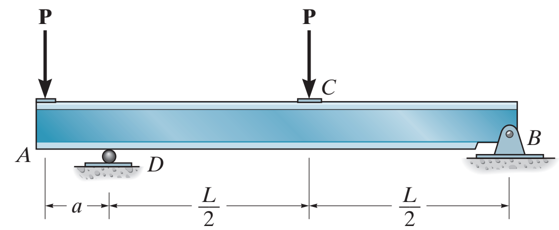
$$\frac{PL^3}{96EI} - \frac{PaL^2}{48EI} = \frac{1}{2}\left[\frac{PL^3}{16EI} - \frac{PaL^2}{6EI}\right]$$

$$a = \frac{L}{3}$$ **Ans.**





8–17. Solve Prob. 8–16 using the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*,

$$\circlearrowleft + \sum M_D = 0; \qquad \left[\frac{1}{2}\left(\frac{PL}{4EI}\right)(L)\right]\left(\frac{L}{2}\right) - \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(L)\right]\left(\frac{L}{3}\right) - B'_y(L) = 0$$

$$-B'_y = \frac{PL^2}{16EI} - \frac{PaL}{6EI}$$

Here, it is required that $M'_C = \Delta_C = 0$. Referring to Fig. *d*,

$$\circlearrowleft + \sum M_C = 0; \qquad \left[\frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{2}\right)\right]\left[\frac{1}{3}\left(\frac{L}{2}\right)\right]$$

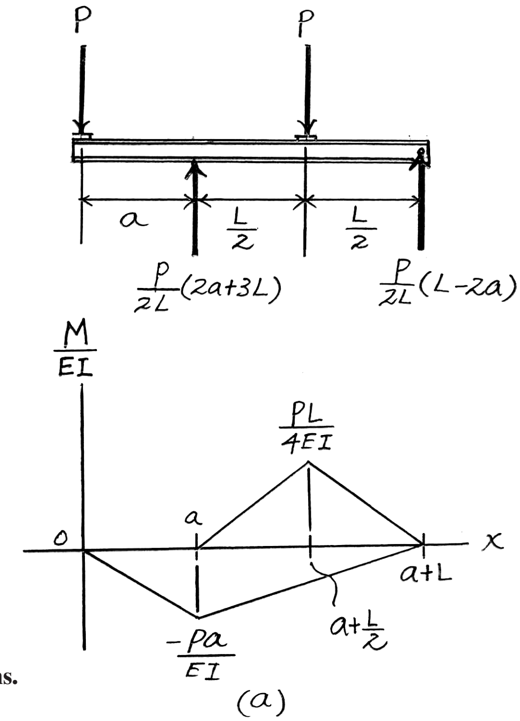
$$-\left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)\left(\frac{L}{2}\right)\right]\left[\frac{1}{3}\left(\frac{L}{2}\right)\right]$$

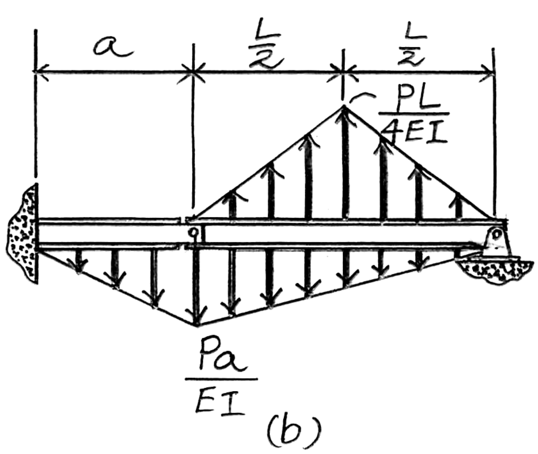
$$-\left[\frac{PL^2}{16EI} - \frac{PaL}{6EI}\right]\left(\frac{L}{2}\right) = 0$$

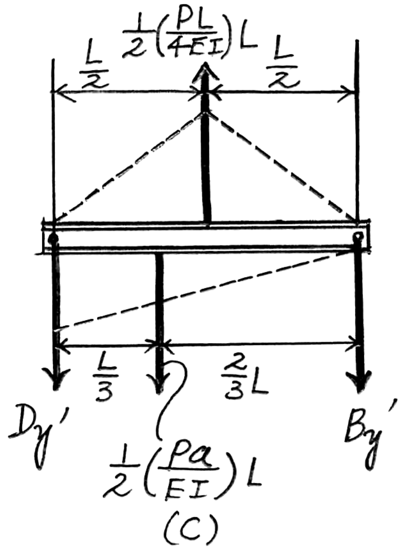
$$\frac{PL^3}{96EI} - \frac{PaL^2}{48EI} - \frac{PL^3}{32EI} + \frac{PaL^2}{12EI} = 0$$

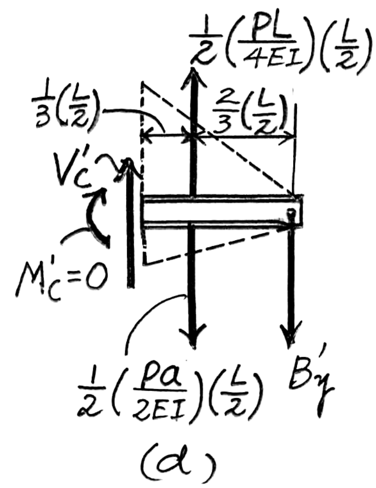
$$\frac{L}{96} - \frac{a}{48} - \frac{L}{32} + \frac{a}{12} = 0$$

$$a = \frac{L}{3}$$ **Ans.**

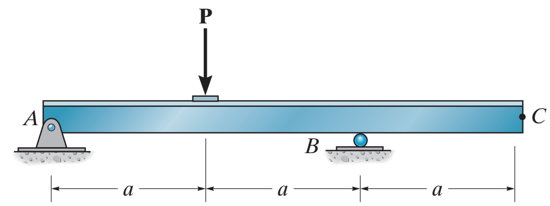








8–18. Determine the slope and the displacement at C. EI is constant. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and the elastic curve shown in Fig. a and b, respectively, Theorem 1 and 2 give

$$t_{B/D} = \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)\right]\left(\frac{2}{3}a\right) = \frac{Pa^3}{6EI}$$

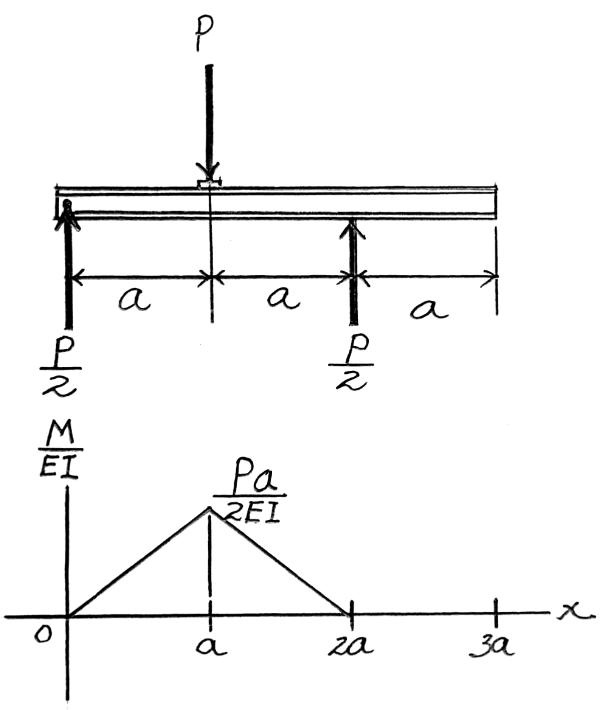
$$t_{C/D} = \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)\right]\left(a + \frac{2}{3}a\right) = \frac{5Pa^3}{12EI}$$

$$\theta_{C/D} = \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a) = \frac{Pa^2}{4EI}$$

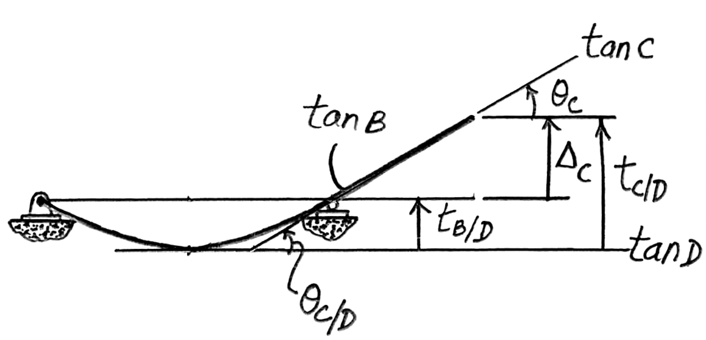
Then,

$$\theta_C = \theta_{C/D} = \frac{Pa^2}{4EI} \quad \measuredangle \qquad \textbf{Ans.}$$

$$\Delta_C = t_{C/D} - t_{B/D} = \frac{5Pa^3}{12EI} - \frac{Pa^3}{6EI} = \frac{Pa^3}{4EI} \quad \uparrow \qquad \textbf{Ans.}$$

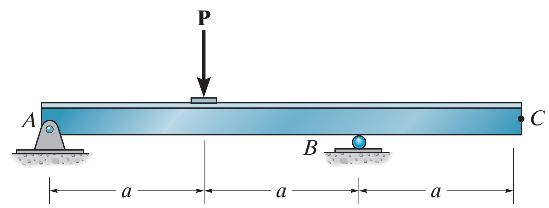


(a)



(b)

8–19. Solve Prob. 8–18 using the conjugate-beam method.



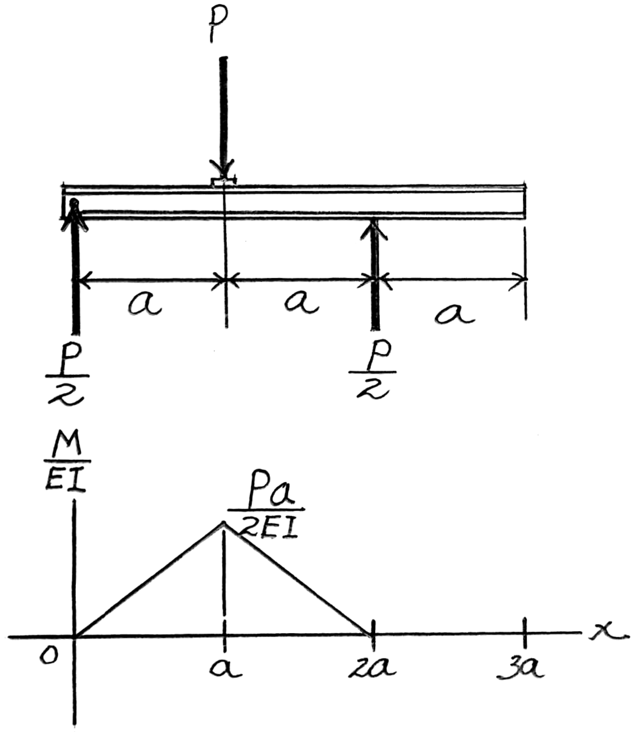
The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*,

$$\circlearrowleft + \sum M_A = 0; \qquad \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(2a)\right](a) - B'_y(2a) = 0 \quad B'_y = \frac{Pa^2}{4EI}$$

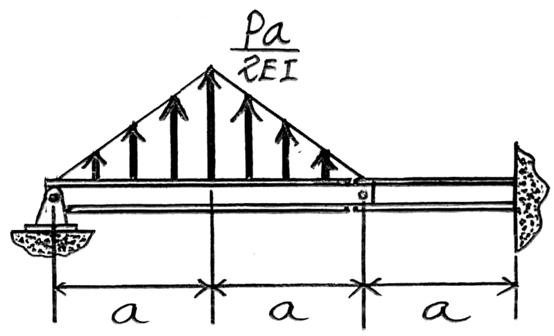
Referring to Fig. *d*

$$+\uparrow \sum F_y = 0; \qquad \frac{Pa^2}{4EI} - V'_C = 0 \quad \theta_C = V'_C = \frac{Pa^2}{4EI} \quad \measuredangle \qquad \textbf{Ans.}$$

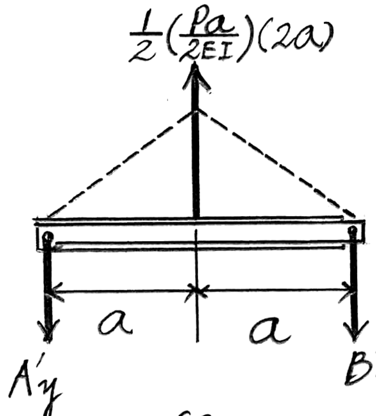
$$\circlearrowleft + \sum M_C = 0; \quad M'_C - \frac{Pa^2}{4EI}(a) = 0 \quad \Delta_C = M'_C = \frac{Pa^3}{4EI} \quad \uparrow \qquad \textbf{Ans.}$$



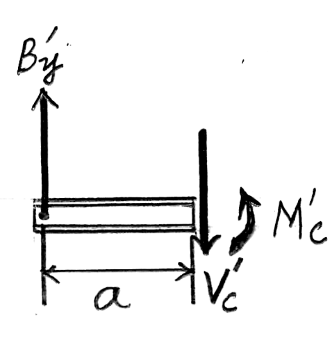
(a)



(b)

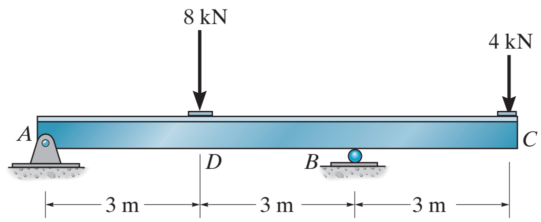


(c)



(d)

***8–20.** Determine the slope and the displacement at the end C of the beam. $E = 200$ GPa, $I = 70(10^6)$ mm^4. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and the elastic curve shown in Fig. a and b, respectively, Theorem 1 and 2 give

$$\theta_{C/A} = \frac{1}{2}\left(\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(6\text{ m}) + \frac{1}{2}\left(-\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(9\text{ m})$$

$$= -\frac{18\text{ kN}\cdot\text{m}}{EI} = \frac{18\text{ kN}\cdot\text{m}}{EI} \quad ⦩$$

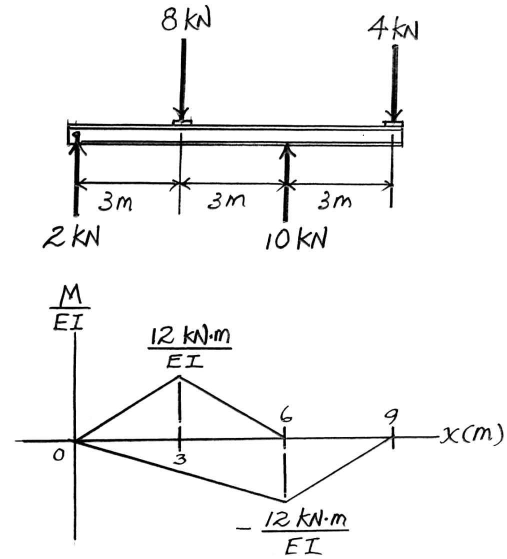
$$t_{B/A} = \left[\frac{1}{2}\left(\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(6\text{ m})\right](3\text{ m}) + \left[\frac{1}{2}\left(-\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(6\text{ m})\right]\left[\frac{1}{3}(6\text{ m})\right]$$

$$= \frac{36\text{ kN}\cdot\text{m}^3}{EI}$$

$$t_{C/A} = \left[\frac{1}{2}\left(\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(6\text{ m})\right](6\text{ m}) + \left[\frac{1}{2}\left(-\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(6\text{ m})\right]\left[3\text{ m} + \frac{1}{3}(6\text{ m})\right]$$

$$+ \left[\frac{1}{2}\left(-\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(3\text{ m})\right]\left[\frac{2}{3}(3\text{ m})\right]$$

$$= 0$$



(a)

Then

$$\theta_A = \frac{t_{B/A}}{L_{AB}} = \frac{36\text{ kN}\cdot\text{m}^3/EI}{6\text{ m}} = \frac{6\text{ kN}\cdot\text{m}^2}{EI} \quad ⦩$$

$$\Delta' = \frac{9}{6}t_{B/A} = \frac{9}{6}\left(\frac{36\text{ kN}\cdot\text{m}^3}{EI}\right) = \frac{54\text{ kN}\cdot\text{m}^3}{EI}$$

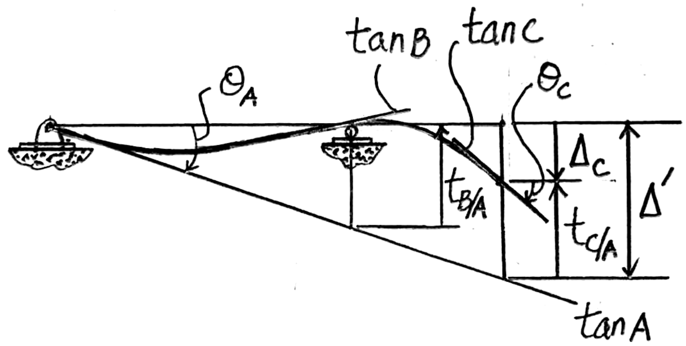
$$+↺\ \theta_C = \theta_A + \theta_{C/A} = \frac{6\text{ kN}\cdot\text{m}^2}{EI} + \frac{18\text{ kN}\cdot\text{m}^2}{EI}$$

$$= \frac{24\text{ kN}\cdot\text{m}^2}{EI} = \frac{24(10^3)\text{ N}\cdot\text{m}^2}{[200(10^9)\text{ N/m}^2][70(10^{-6})\text{ m}^4]} = 0.00171\text{ rad} \quad ⦩$$ **Ans.**

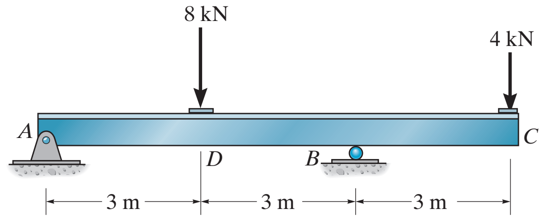
$$\Delta_C = \Delta' - t_{C/A} = \frac{54\text{ kN}\cdot\text{m}^3}{EI} - 0$$

$$= \frac{54\text{ kN}\cdot\text{m}^3}{EI} = \frac{54(10^3)\text{N}\cdot\text{m}^3}{[200(10^9)\text{ N/m}^2][70(10^{-6})\text{ m}^4]} = 0.00386\text{ m}$$

$= 3.86$ mm ↓ **Ans.**



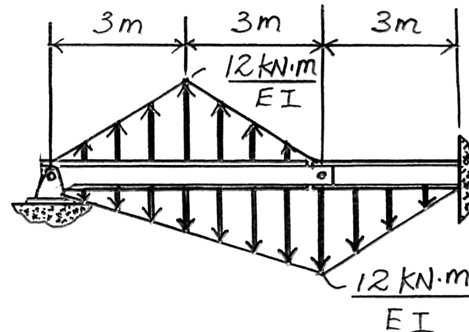
8–21. Solve Prob. 8–20 using the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*

$$\circlearrowleft + \sum M_A = 0; \quad B'_y(6\text{ m}) + \left[\frac{1}{2}\left(\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(6\text{ m})\right](3\text{ m}) - \left[\frac{1}{2}\left(\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(6\text{ m})\right]\left[\frac{2}{3}(6\text{ m})\right] = 0$$

$$B'_y = \frac{6\text{ kN}\cdot\text{m}^2}{EI}$$



(b)

Referring to Fig. *d*,

$$\circlearrowleft + \sum Fy = 0; \quad -V'_C - \frac{6\text{ kN}\cdot\text{m}^2}{EI} - \frac{1}{2}\left(\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(3\text{ m}) = 0$$

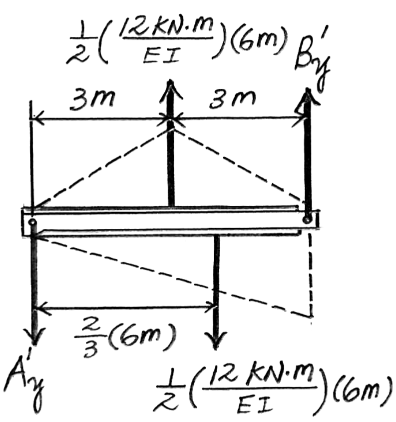
$$\theta_C = V'_C = -\frac{24\text{ kN}\cdot\text{m}^2}{EI} = \frac{24(10^3)\text{ N}\cdot\text{m}^2}{[(200(10^9)\text{ N/m}^2)][(70(10^{-6})\text{ m}^4]}$$

$$= 0.00171\text{ rad} \quad \textbf{Ans.}$$

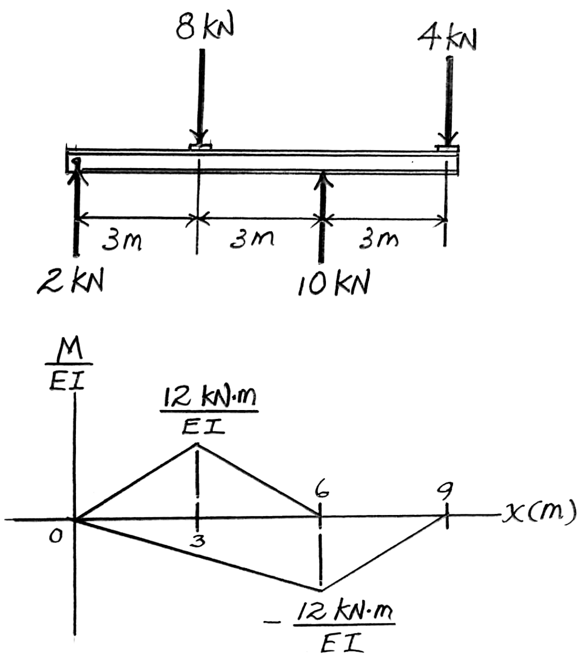
$$\circlearrowleft + \sum M_C = 0; \quad M'_C + \left[\frac{1}{2}\left(\frac{12\text{ kN}\cdot\text{m}^2}{EI}\right)(3\text{ m})\right]\left[\frac{2}{3}(3\text{ m})\right] + \left(\frac{6\text{ kN}\cdot\text{m}^2}{EI}\right)(3\text{ m}) = 0$$

$$\Delta_C = M'_C = -\frac{54\text{ kN}\cdot\text{m}^3}{EI} = \frac{54\,(10^3)\text{ N}\cdot\text{m}^3}{[200(10^9)\text{N/m}^2]\,[70(10^{-6})\text{m}^4]}$$

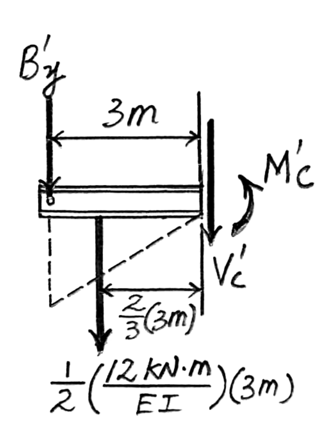
$$= 0.00386\text{ m} = 3.86\text{ mm} \downarrow \quad \textbf{Ans.}$$



(c)

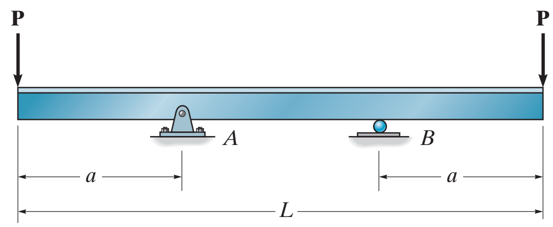


(a)



(d)

8–22. At what distance a should the bearing supports at A and B be placed so that the displacement at the center of the shaft is equal to the deflection at its ends? The bearings exert only vertical reactions on the shaft. EI is constant. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and the elastic curve shown in Fig. a and b, respectively. Theorem 2 gives

$$t_{B/C} = \left(-\frac{Pa}{EI}\right)\left(\frac{L-2a}{2}\right)\left(\frac{L-2a}{4}\right) = -\frac{Pa}{8EI}(L-2a)^2$$

$$t_{D/C} = \left(-\frac{Pa}{EI}\right)\left(\frac{L-2a}{2}\right)\left(a + \frac{L-2a}{4}\right) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{2}{3}a\right)$$

$$= -\left[\frac{Pa}{8EI}(L^2-4a^2) + \frac{Pa^3}{3EI}\right]$$

It is required that

$$t_{D/C} = 2\,t_{B/C}$$

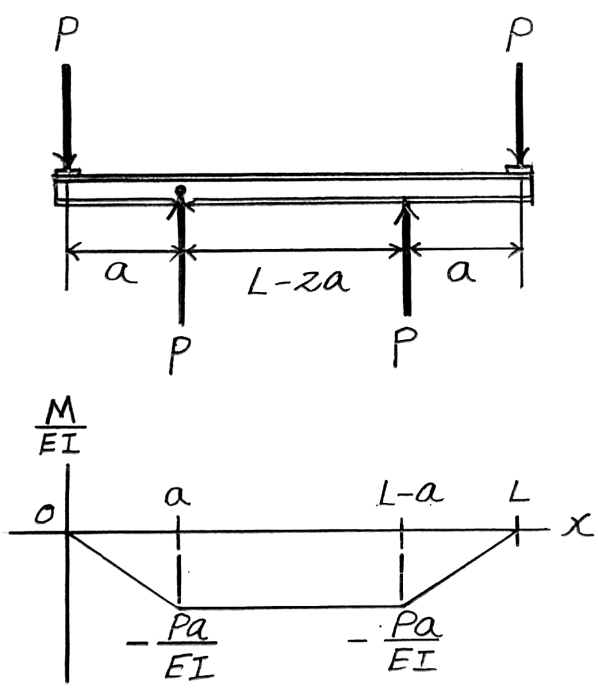
$$\frac{Pa}{8EI}(L^2-4a^2) + \frac{Pa^3}{3EI} = 2\left[\frac{Pa}{8EI} - (L-2a)^2\right]$$

$$\frac{7Pa^3}{6EI} - \frac{Pa^2L}{EI} + \frac{PaL^2}{8EI} = 0$$

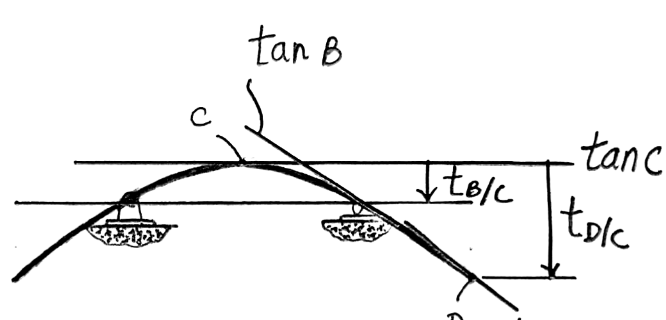
$$56a^2 - 48La + 6L^2 = 0$$

Choose

$$a = 0.152\ L$$ **Ans.**

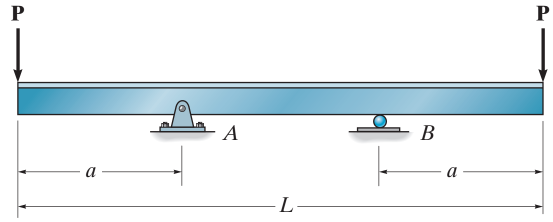


(a)



(b)

8–23. Solve Prob. 8–22 using the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*,

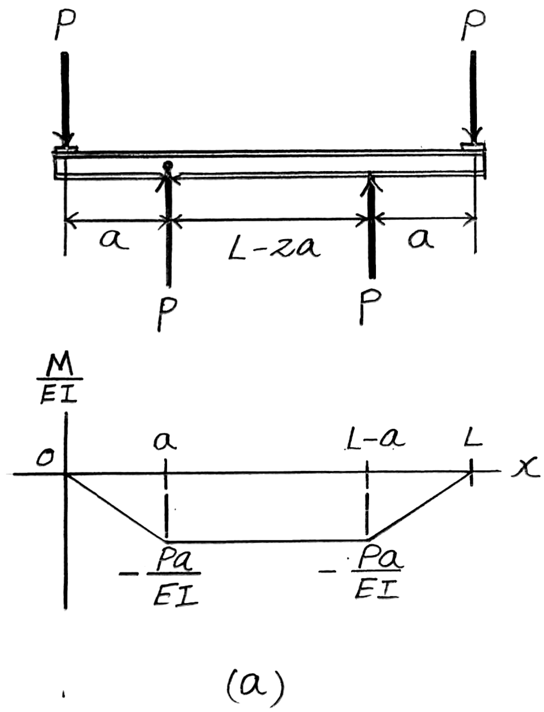
$$\circlearrowleft + \sum M_A = 0; \quad B'_y(L-2a) - \left[\frac{Pa}{EI}(L-2a)\right]\left(\frac{L-2a}{2}\right) = 0$$

$$B'_y = \frac{Pa}{2EI}(L-2a)$$

Referring to Fig. *d*,

$$M'_D + \frac{Pa}{2EI}(L-2a)(a) + \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\right]\left(\frac{2}{3}a\right) = 0$$

$$\Delta_D = M'_D = -\left[\frac{Pa^2}{2EI}(L-2a) + \frac{Pa^3}{3EI}\right]$$



Referring to Fig. *e*,

$$\frac{Pa}{2EI}(L-2a)\left(\frac{L-2a}{2}\right) - \frac{Pa}{EI}\left(\frac{L-2a}{2}\right)\left(\frac{L-2a}{4}\right) - M'_C = 0$$

$$\Delta_C = M'_C = \frac{Pa}{8EI}(L-2a)^2$$

It is required that

$$|\Delta_D| = \Delta_C$$

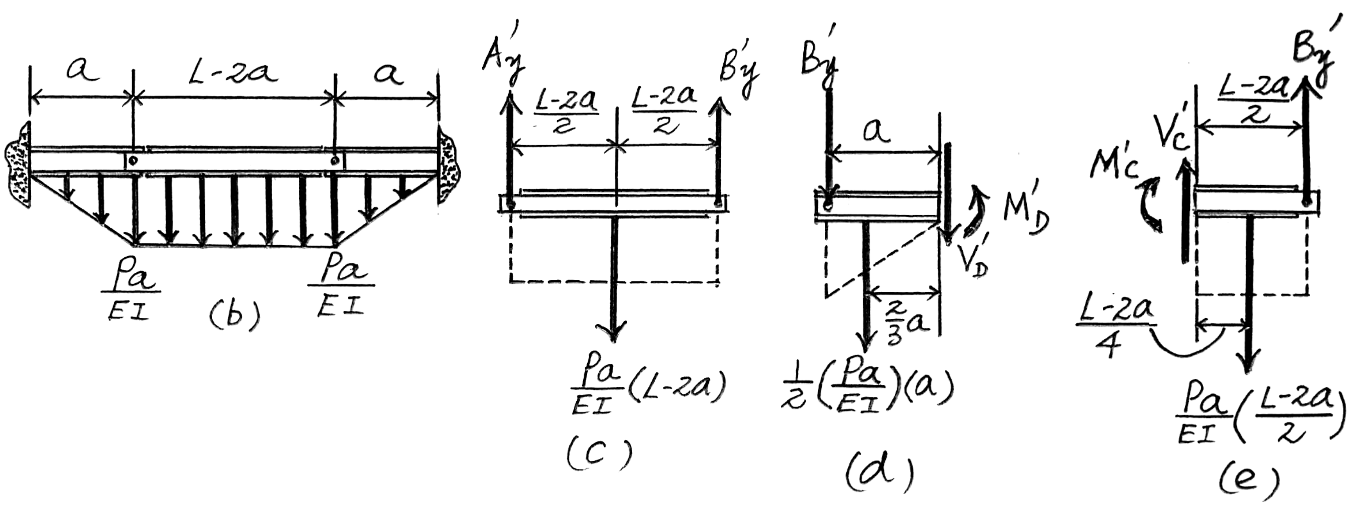
$$\frac{Pa^2}{2EI}(L-2a) + \frac{Pa^3}{3EI} = \frac{Pa}{8EI}(L-2a)^2$$

$$\frac{7Pa^3}{6EI} - \frac{Pa^2L}{EI} + \frac{PaL^2}{8EI} = 0$$

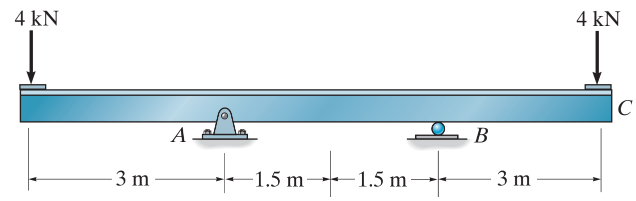
$$56a^2 - 48La + 6L^2 = 0$$

Choose

$$a = 0.152\,L \qquad \textbf{Ans.}$$



***8–24.** Determine the displacement at C and the slope at B. EI is constant. Use the moment-area theorems.



Using the $\dfrac{M}{EI}$ diagram and elastic curve shown in Fig. *a* and *b*, respectively, Theorem 1 and 2 give

$$\theta_{B/D} = \left(-\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(1.5\text{ m}) = -\frac{18\text{ kN}\cdot\text{m}^2}{EI}$$

$$t_{B/D} = \left[\left(-\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(1.5\text{ m})\right]\left[\frac{1}{2}(1.5\text{ m})\right] = \frac{13.5\text{ kN}\cdot\text{m}^3}{EI}$$

$$t_{C/D} = \left[\left(-\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(1.5\text{ m})\right]\left[\frac{1}{2}(1.5\text{ m}) + 3\text{ m}\right] + \left[\frac{1}{2}\left(-\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(3\text{ m})\right]\left[\frac{2}{3}(3\text{ m})\right]$$

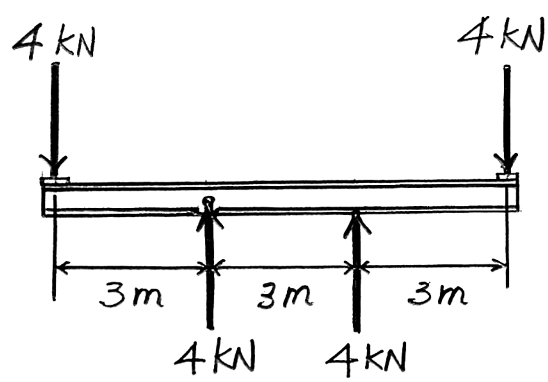
$$= \frac{103.5\text{ kN}\cdot\text{m}^3}{EI}$$

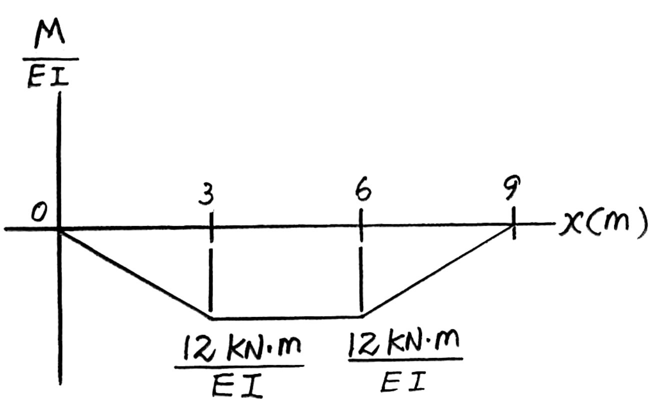
Then,

$$\theta_B = |\theta_{B/D}| = \frac{18\text{ kN}\cdot\text{m}^2}{EI} \quad \triangledown$$ **Ans.**

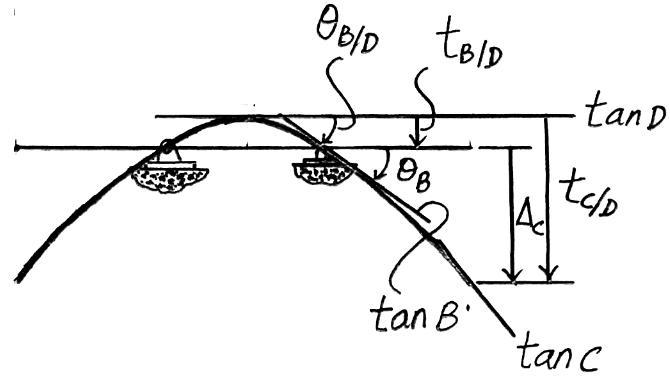
$$\Delta_C = |t_{C/D}| - |t_{B/D}| = \frac{103.5\text{ kN}\cdot\text{m}^3}{EI} - \frac{13.5\text{ kN}\cdot\text{m}^2}{EI}$$

$$= \frac{90\text{ kN}\cdot\text{m}^3}{EI} \downarrow$$ **Ans.**



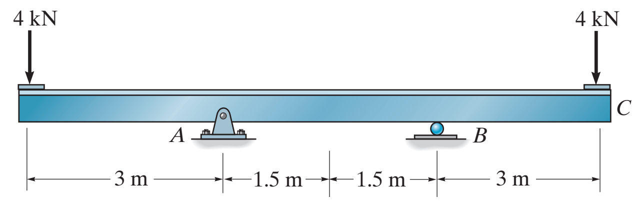


(a)



(b)

8–25. Solve Prob. 8–24 using the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*,

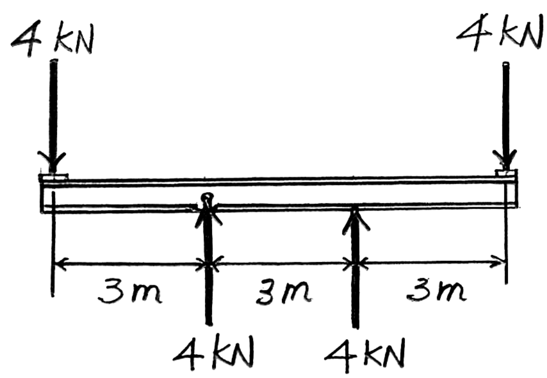
$$\circlearrowleft + \sum M_A = 0; \qquad B'_y\,(3\text{ m}) - \left(\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(3\text{ m})(1.5\text{ m}) = 0$$

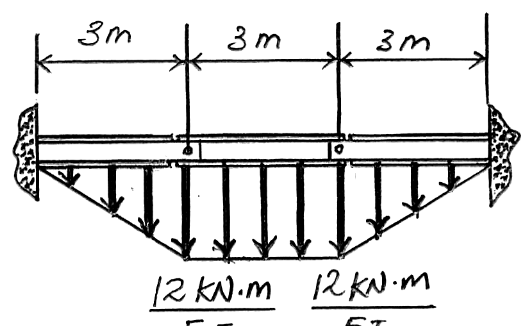
$$B'_y = \theta_B = \frac{18\text{ kN}\cdot\text{m}^2}{EI} \qquad \textbf{Ans.}$$

Referring to Fig. *d*,

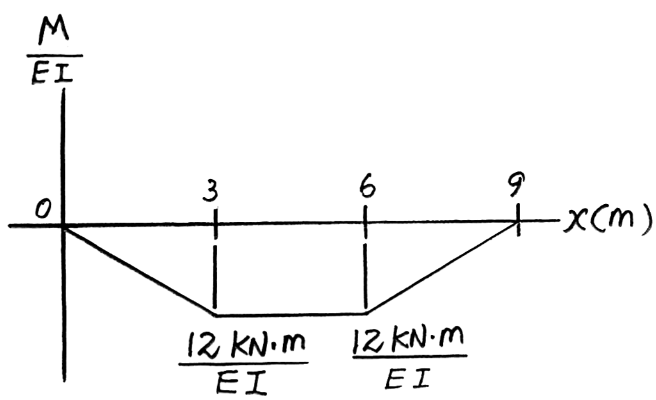
$$\circlearrowleft + \sum M_C = 0; \quad M'_C + \left(\frac{18\text{ kN}\cdot\text{m}^2}{EI}\right)(3\text{ m}) + \left[\frac{1}{2}\left(\frac{12\text{ kN}\cdot\text{m}}{EI}\right)(3\text{ m})\right]\left[\frac{2}{3}(3\text{ m})\right] = 0$$

$$\Delta_C = M'_C = -\frac{90\text{ kN}\cdot\text{m}^3}{EI} = \frac{90\text{ kN}\cdot\text{m}^3}{EI} \downarrow \qquad \textbf{Ans.}$$

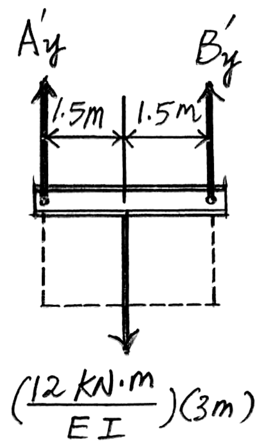




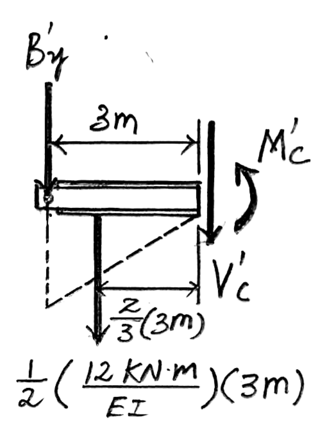
(b)



(a)

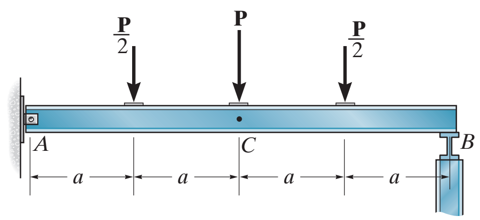


(c)



(d)

8–26. Determine the displacement at *C* and the slope at *B*. *EI* is constant. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and elastic curve shown in Fig. *a* and *b*, respectively, Theorem 1 and 2 give

$$\theta_{B/C} = \frac{1}{2}\left(\frac{Pa}{EI}\right)(a) + \left(\frac{Pa}{EI}\right)(a) + \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a) = \frac{7Pa^2}{4EI}$$

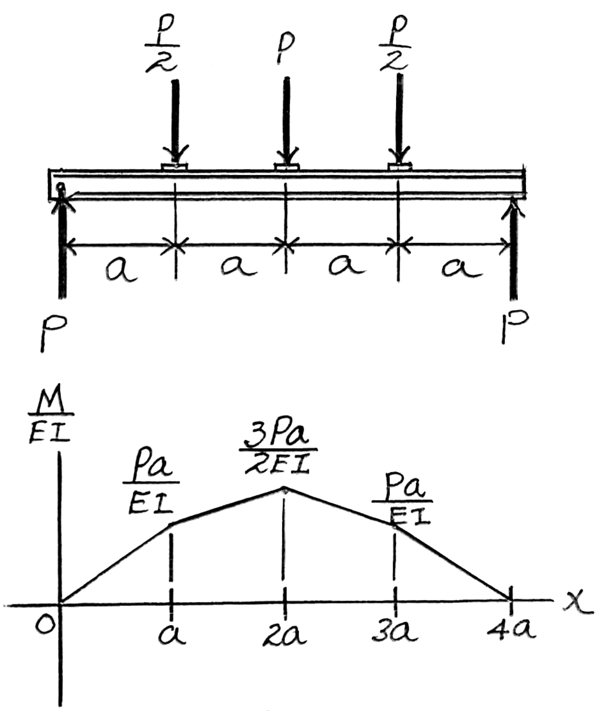
$$t_{B/C} = \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\right]\left(\frac{2}{3}a\right) + \left[\frac{Pa}{EI}(a)\right]\left(a + \frac{1}{2}a\right) + \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)\right]\left(a + \frac{2}{3}a\right)$$

$$= \frac{9Pa^3}{4EI}$$

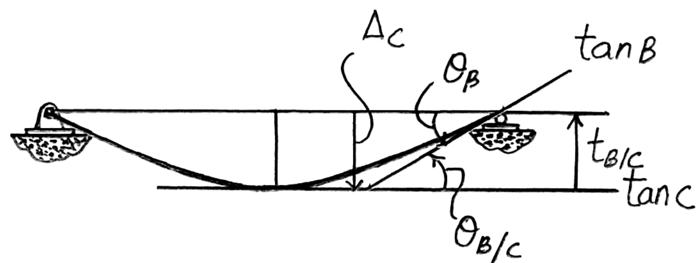
Then

$$\theta_B = \theta_{B/C} = \frac{7Pa^2}{4EI} \quad \measuredangle$$ **Ans.**

$$\Delta_C = t_{B/C} = \frac{9Pa^3}{4EI} \downarrow$$ **Ans.**

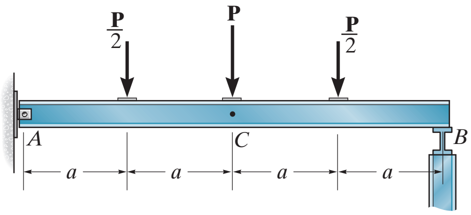


(a)



(b)

8–27. Determine the displacement at C and the slope at B. EI is constant. Use the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. a and b, respectively. Referring to Fig. c,

$$\zeta+\ \sum M_A = 0; \qquad \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\right]\left(\frac{2}{3}a\right) + \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)\right]\left(\frac{10}{3}a\right)$$

$$+ \left[\left(\frac{Pa}{EI}\right)(2a) + \frac{1}{2}\left(\frac{Pa}{2EI}\right)(2a)\right](2a)$$

$$-B'_y = (4a) = 0$$

$$\theta_B = B'_y = \frac{7Pa^2}{4EI} \quad \triangledown \qquad \textbf{Ans.}$$

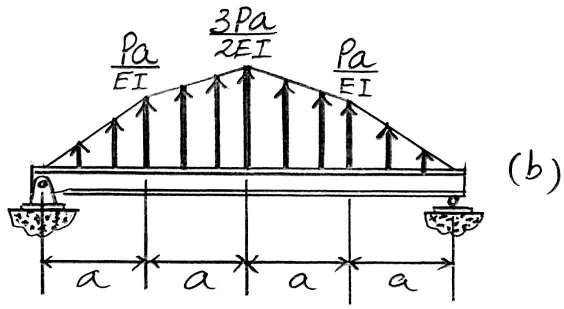
Referring to Fig. d,

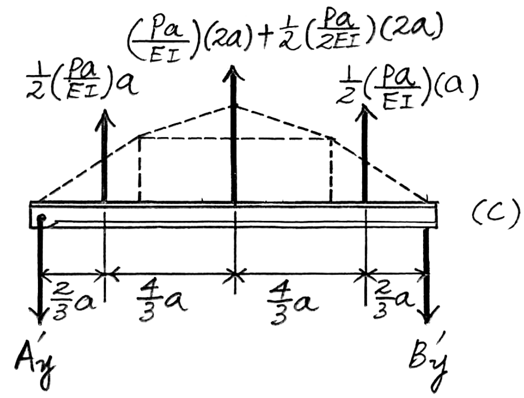
$$\zeta+\ \sum M_C = 0; \qquad \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)\right]\left(\frac{4}{3}a\right) + \left[\left(\frac{Pa}{EI}\right)(a)\right]\left(\frac{a}{2}\right)$$

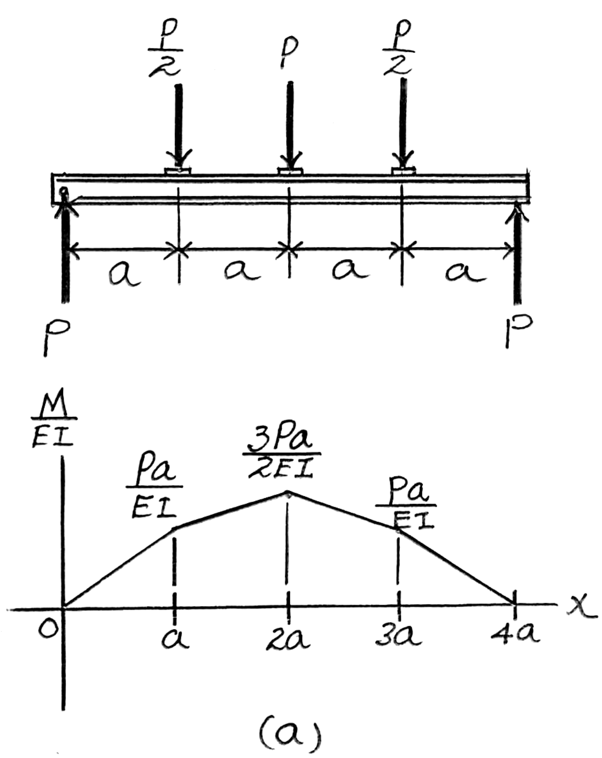
$$+ \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)\right]\left(\frac{a}{3}\right) - \frac{7Pa^2}{4EI}(2a)$$

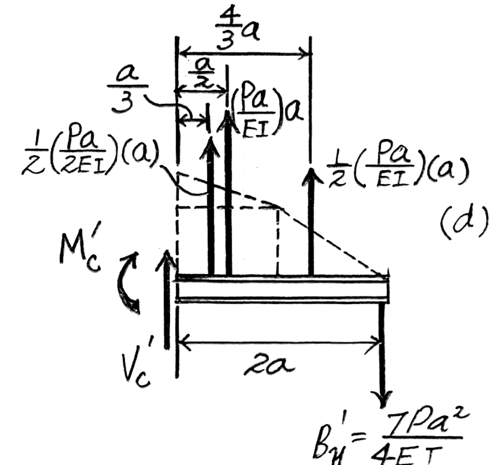
$$-M'_C = 0$$

$$\Delta_C = M'_C = -\frac{9Pa^3}{4EI} = \frac{9Pa^3}{4EI} \downarrow \qquad \textbf{Ans.}$$

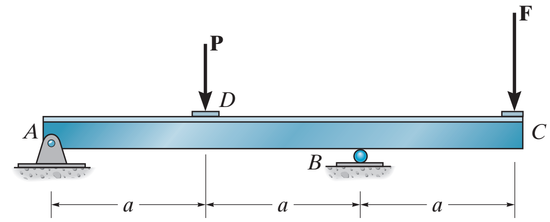








***8–28.** Determine the force **F** at the end of the beam C so that the displacement at C is zero. EI is constant. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and elastic curve shown in Fig. a and b, respectively, Theorem 2 gives

$$t_{B/A} = \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(2a)\right](a) + \left[\frac{1}{2}\left(-\frac{Fa}{EI}\right)(2a)\right]\left[\frac{1}{3}(2a)\right] = \frac{Pa^3}{2EI} - \frac{2Fa^3}{3EI}$$

$$t_{C/A} = \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(2a)\right](2a) + \left[\frac{1}{2}\left(-\frac{Fa}{EI}\right)(2a)\right]\left[\frac{1}{3}(2a) + a\right]$$

$$+ \left[\frac{1}{2}\left(-\frac{Fa}{EI}\right)(a)\right]\left[\frac{2}{3}(a)\right]$$

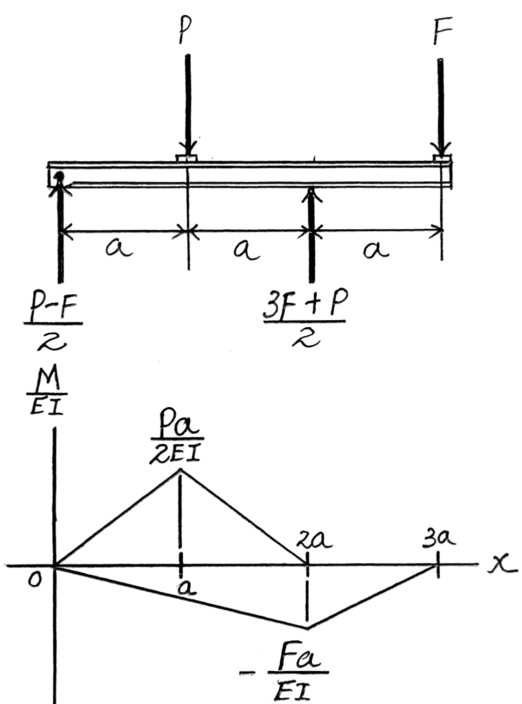
$$= \frac{Pa^3}{2EI} - \frac{2Fa^3}{3EI}$$

It is required that

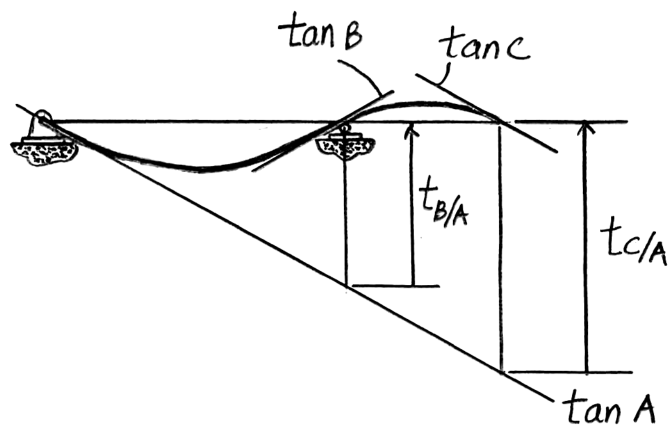
$$t_{C/A} = \frac{3}{2}t_{B/A}$$

$$\frac{Pa^3}{EI} - \frac{2Fa^3}{EI} = \frac{3}{2}\left[\frac{Pa^3}{2EI} - \frac{2Fa^3}{3EI}\right]$$

$$F = \frac{P}{4}$$ **Ans.**

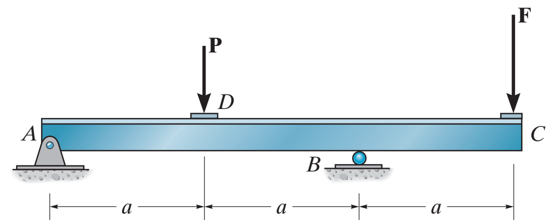


(a)



(b)

8–29. Determine the force **F** at the end of the beam C so that the displacement at C is zero. EI is constant. Use the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*,

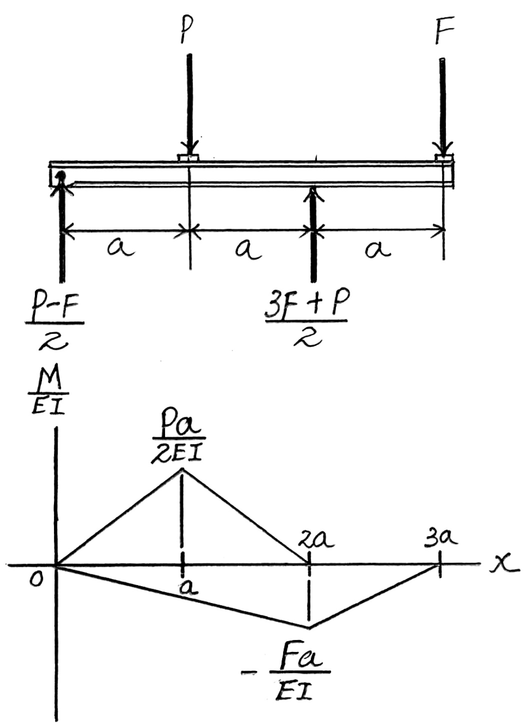
$$\zeta+\sum M_A = 0;\quad \left[\frac{1}{2}\left(\frac{Pa}{2EI}\right)(2a)\right](a) - \left[\frac{1}{2}\left(\frac{Fa}{EI}\right)(2a)\right]\left[\frac{2}{3}(2a)\right] - B'_y(2a) = 0$$

$$B'_y = \frac{Pa^2}{4EI} - \frac{2Fa^2}{3EI}$$

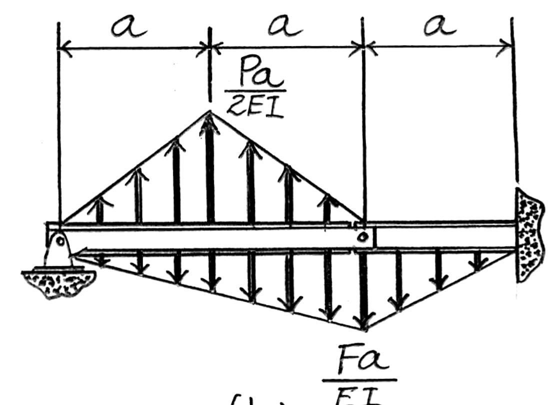
Here, it is required that $\Delta_C = M'_C = 0$. Referring to Fig. *d*,

$$\zeta+\sum M_C = 0;\quad \left[\frac{1}{2}\left(\frac{Fa}{EI}\right)(a)\right]\left[\frac{2}{3}(a)\right] - \left(\frac{Pa^2}{4EI} - \frac{2Fa^2}{3EI}\right)(a) = 0$$

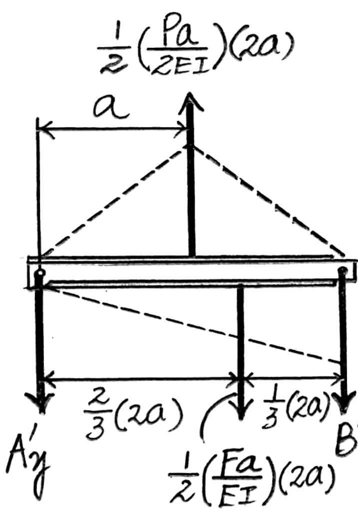
$$F = \frac{P}{4}$$ **Ans.**



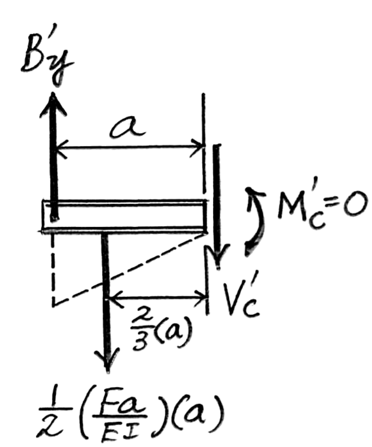
(a)



(b)

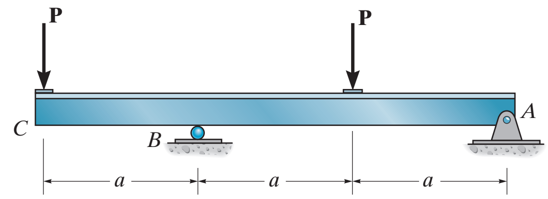


(c)



(d)

8–30. Determine the slope at B and the displacement at C. EI is constant. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and elastic curve shown in Fig. *a* and *b*, Theorem 1 and 2 give

$$\theta_{B/A} = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a) = -\frac{Pa^2}{2EI} = \frac{Pa^2}{2EI}$$

$$t_{B/A} = \left[\frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\right]\left[\frac{1}{3}(a)\right] = -\frac{Pa^3}{6EI}$$

$$t_{C/A} = \left[\frac{1}{2}\left(-\frac{Pa}{EI}\right)(2a)\right](a) = -\frac{Pa^3}{EI}$$

Then

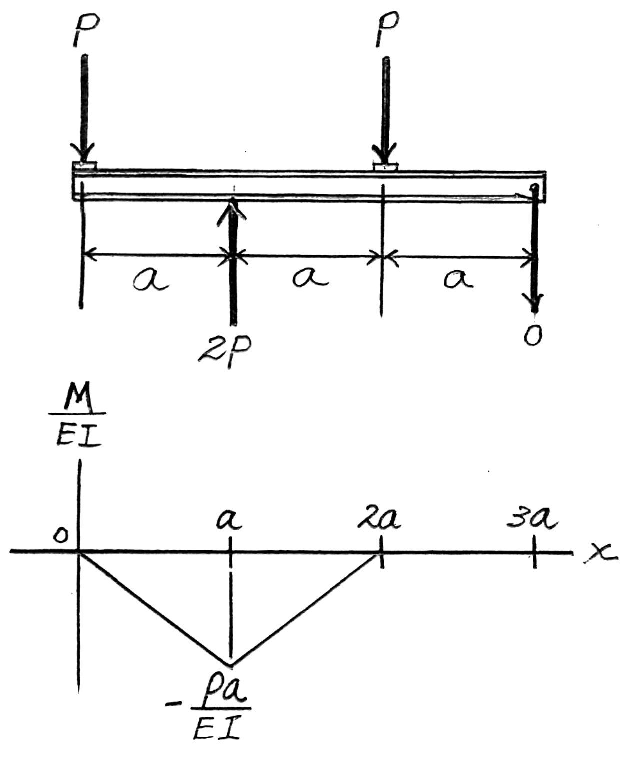
$$\theta_A = \frac{|t_{B/A}|}{L_{AB}} = \frac{Pa^3/6EI}{2a} = \frac{Pa^2}{12EI}$$

$$\Delta' = \frac{3}{2}|t_{B/A}| = \frac{3}{2}\left(\frac{Pa^3}{6EI}\right) = \frac{Pa^3}{4EI}$$

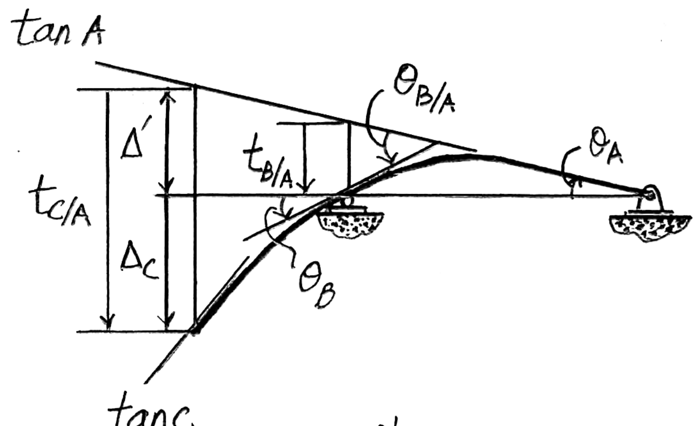
$$\theta_B = \theta_A + \theta_{B/A}$$

$$\circlearrowleft + \ \theta_B = -\frac{Pa^2}{12EI} + \frac{Pa^2}{2EI} = \frac{5Pa^2}{12EI} \qquad \textbf{Ans.}$$

$$\Delta_C = |t_{C/A}| - \Delta' = \frac{Pa^3}{EI} - \frac{Pa^3}{4EI} = \frac{3Pa^3}{4EI} \downarrow \qquad \textbf{Ans.}$$

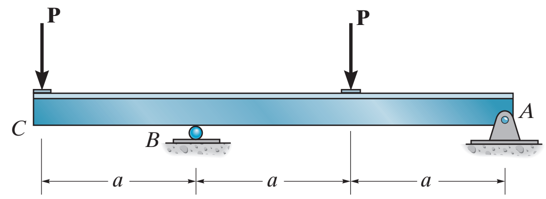


(a)



(b)

8–31. Determine the slope at B and the displacement at C. EI is constant. Use the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. c and d, respectively. Referring to Fig. d,

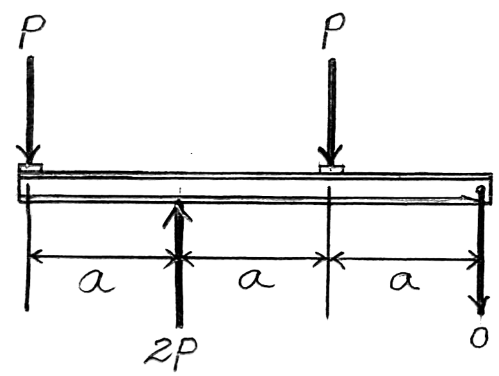
$$\zeta + \sum M_A = 0; \quad \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\right]\left(a + \frac{2}{3}a\right) - B'_y\,(2a) = 0$$

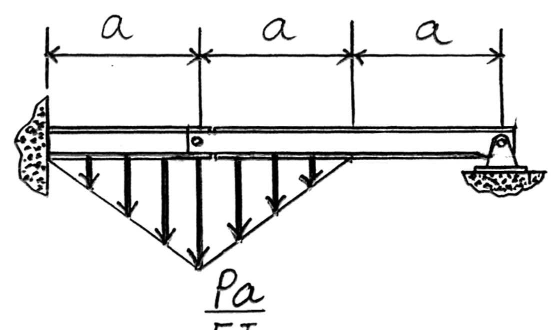
$$\theta_B = B'_y = \frac{5Pa^2}{12EI} \qquad \text{Ans.}$$

Referring to Fig. c,

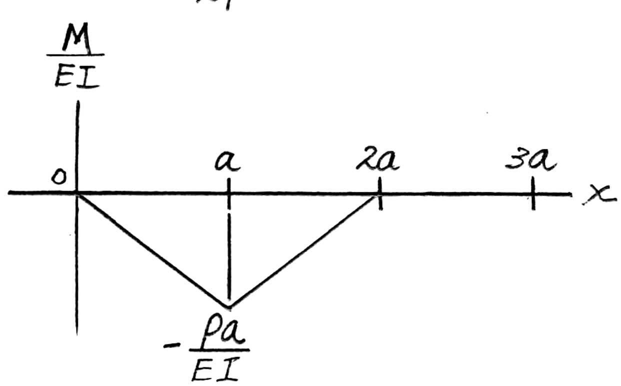
$$\zeta + \sum M_C = 0; \quad -M'_C - \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\right]\left(\frac{2}{3}a\right) - \left(\frac{5Pa^2}{12EI}\right)(a) = 0$$

$$\Delta_C = M'_C = -\frac{3Pa^3}{4EI} = \frac{3Pa^3}{4EI} \quad \downarrow \qquad \text{Ans.}$$

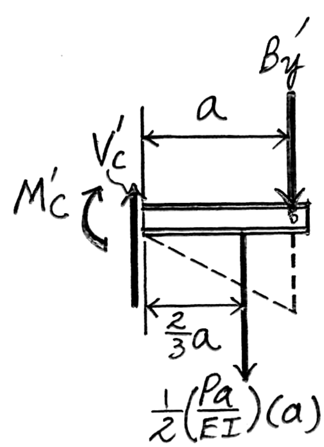




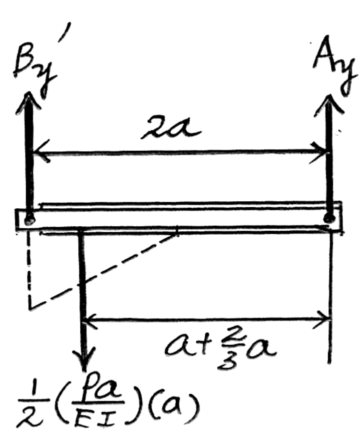
(b)



(a)

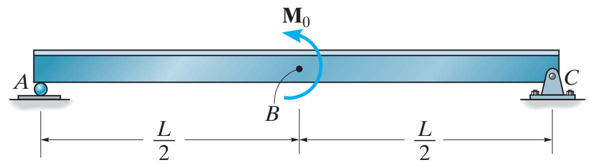


(c)



(d)

***8–32.** Determine the maximum displacement and the slope at A. EI is constant. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and the elastic curve shown in Fig. a and b, respectively, Theorem 1 and 2 give

$$\theta_{D/A} = \frac{1}{2}\left(\frac{M_0}{EIL}x\right)(x) = \frac{M_0}{2EIL}x^2$$

$$t_{B/A} = \left[\frac{1}{2}\left(\frac{M_0}{2EI}\right)\left(\frac{L}{2}\right)\right]\left[\frac{1}{3}\left(\frac{L}{2}\right)\right] = \frac{M_0L^2}{48EI}$$

Then,

$$\theta_A = \frac{|t_{B/A}|}{L_{AB}} = \frac{M_0L^2/48EI}{L/2} = \frac{M_0L}{24EI} \qquad \textbf{Ans.}$$

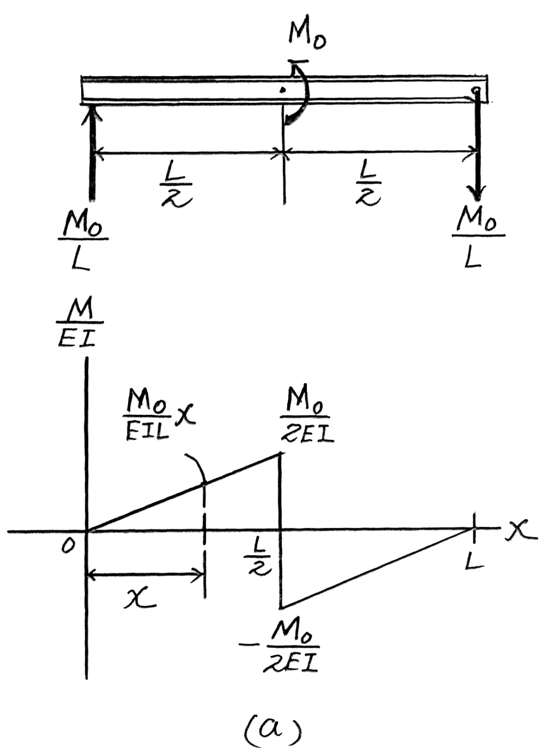
Here $\theta_D = 0$. Thus,

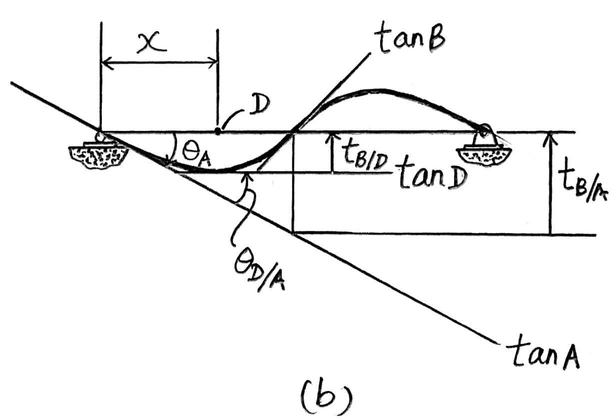
$$\theta_D = \theta_A + \theta_{D/A}$$

$$\circlearrowleft + \quad 0 = -\frac{M_0L}{24EI} + \frac{M_0}{2EIL}x^2 \qquad x = \frac{L}{\sqrt{12}} = 0.2887L$$

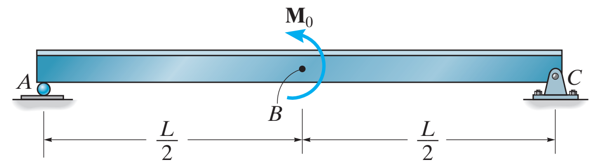
$$\Delta_{max} = \Delta_D = t_{B/D} = \left[\frac{1}{2}\left(\frac{0.2113M_0}{EI}\right)(0.2113L)\right]\left[\frac{1}{3}(0.2113L)\right] + \left[\left(\frac{0.2887M_0}{EI}\right)(0.2113L)\right]\left[\frac{1}{2}(0.2113L)\right]$$

$$= \frac{0.00802M_0L^2}{EI} \downarrow \qquad \textbf{Ans.}$$

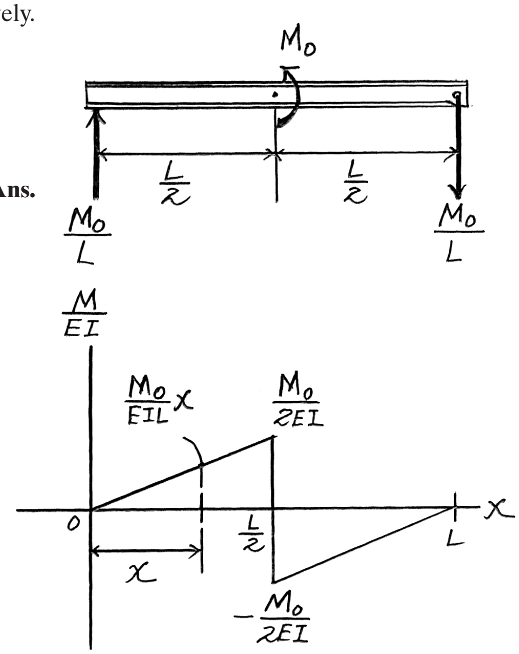




8–33. Determine the maximum displacement at B and the slope at A. EI is constant. Use the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*



(a)

$$\curvearrowleft + \sum M_B = 0; \quad A'_y(L) - \left[\frac{1}{2}\left(\frac{M_0}{2EI}\right)\left(\frac{L}{2}\right)\right]\left(\frac{L}{3}\right) = 0$$

$$A'_y = \theta_A = \frac{M_0 L}{24EI} \qquad \textbf{Ans.}$$

Here it is required that $\theta_D = V'_D = 0$. Referring to Fig. *d*,

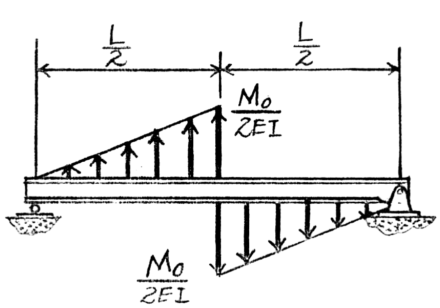
$$\uparrow \sum F_y = 0; \quad \frac{1}{2}\left(\frac{M_0}{EIL}x\right)(x) - \frac{M_0 L}{24EI} = 0$$

$$x = \frac{L}{\sqrt{12}}$$

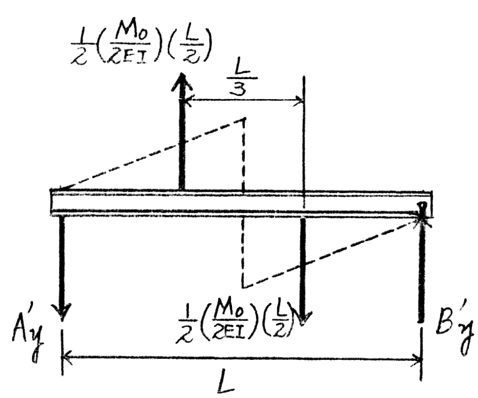
$$\curvearrowleft + \sum M_D = 0; \quad M'_D + \left(\frac{M_0 L}{24EI}\right)\left(\frac{L}{\sqrt{12}}\right) - \frac{1}{2}\left(\frac{M_0}{EIL}\right)\left(\frac{L}{\sqrt{12}}\right)\left(\frac{L}{\sqrt{12}}\right)\left[\frac{1}{3}\left(\frac{L}{\sqrt{12}}\right)\right] = 0$$

$$\Delta_{max} = \Delta_D = M'_D = -\frac{0.00802 M_0 L^2}{EI}$$

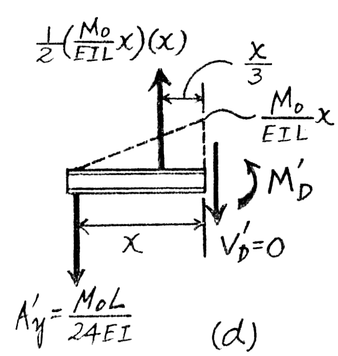
$$= \frac{0.00802 M_0 L^2}{EI} \downarrow \qquad \textbf{Ans.}$$



(b)

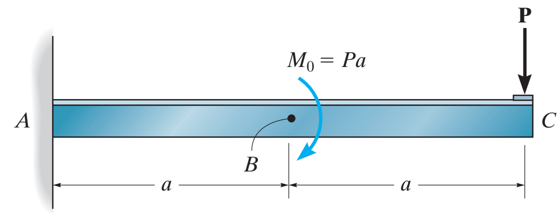


(c)



(d)

8–34. Determine the slope and displacement at C. EI is constant. Use the moment-area theorems.

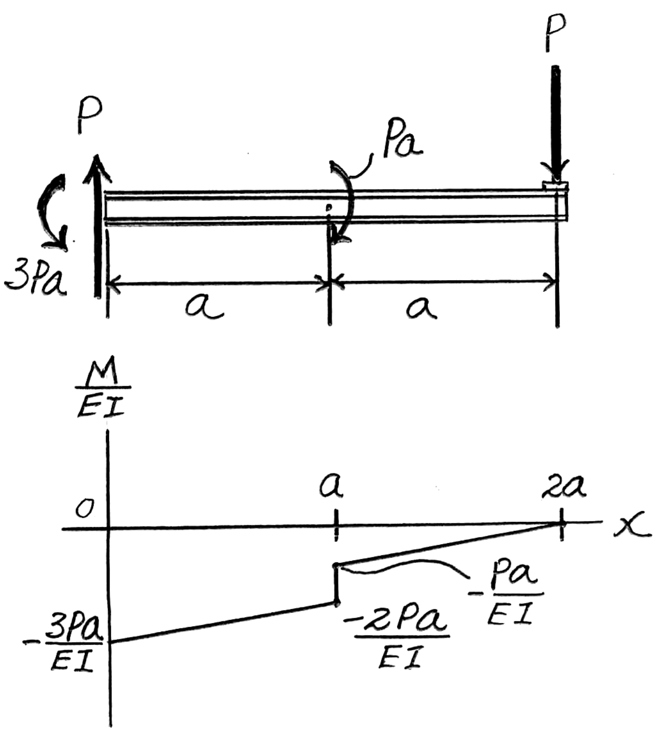


Using the $\frac{M}{EI}$ diagram and the elastic curve shown in Fig. *a* and *b*, respectively, Theorem 1 and 2 give

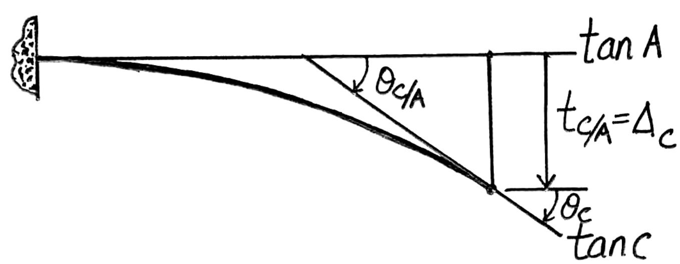
$$\theta_C = |\theta_{C/A}| = \frac{1}{2}\left(\frac{Pa}{EI}\right)(a) + \left(\frac{2Pa}{EI}\right)(a) + \frac{1}{2}\left(\frac{Pa}{EI}\right)(a) = \frac{3Pa^2}{EI} \quad \text{Ans.}$$

$$\Delta_C = |t_{C/A}| = \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\right]\left(a + \frac{2}{3}a\right) + \left[\left(\frac{2Pa}{EI}\right)(a)\right]\left(a + \frac{a}{2}\right) + \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\right]\left(\frac{2}{3}a\right) = 0$$

$$= \frac{25Pa^3}{6EI} \downarrow \quad \text{Ans.}$$

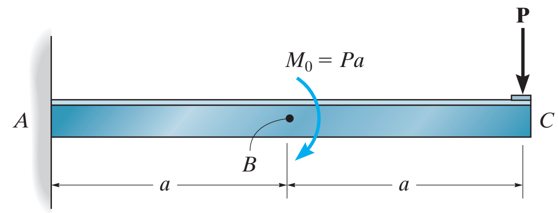


(a)



(b)

8–35. Determine the slope and displacement at C. EI is constant. Use the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*,

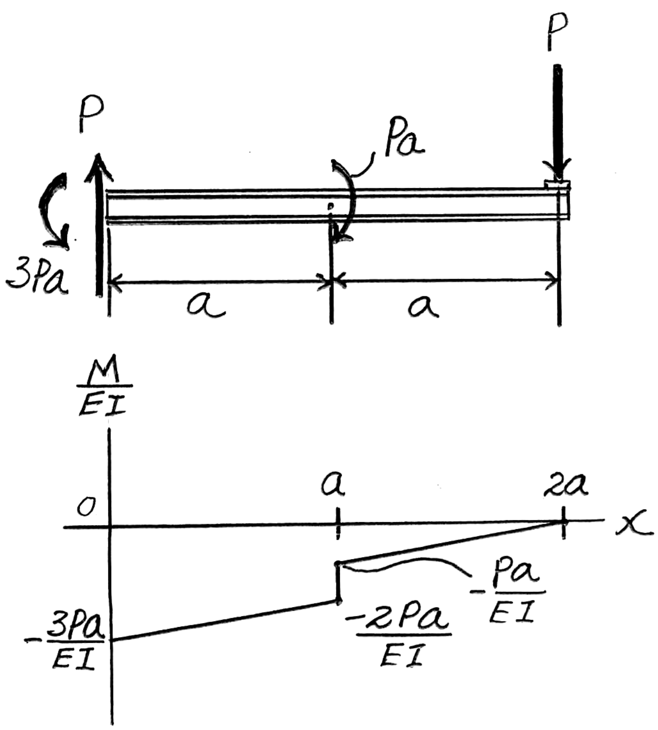
$$+\uparrow \sum F_y = 0; \quad -V'_C - \frac{1}{2}\left(\frac{Pa}{EI}\right)(a) - \left(\frac{2Pa}{EI}\right)(a) - \frac{1}{2}\left(\frac{Pa}{EI}\right)(a) = 0$$

$$\theta_C = V'_C = -\frac{3Pa^2}{EI} = \frac{3Pa^2}{EI} \qquad \textbf{Ans.}$$

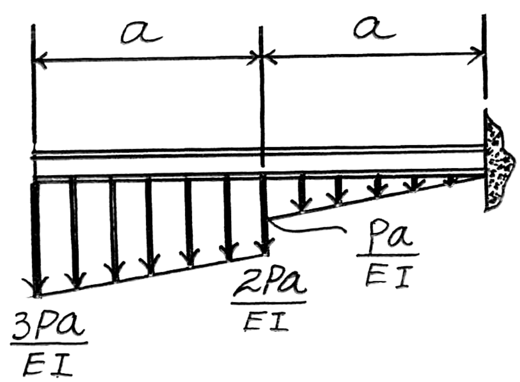
$$\circlearrowleft + \sum M_C = 0; \quad M'_C + \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\right]\left(\frac{2}{3}a\right) + \left[\left(\frac{2Pa}{EI}\right)(a)\right]\left(a + \frac{a}{2}\right)$$

$$+ \left[\frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\right]\left(a + \frac{2}{3}a\right) = 0$$

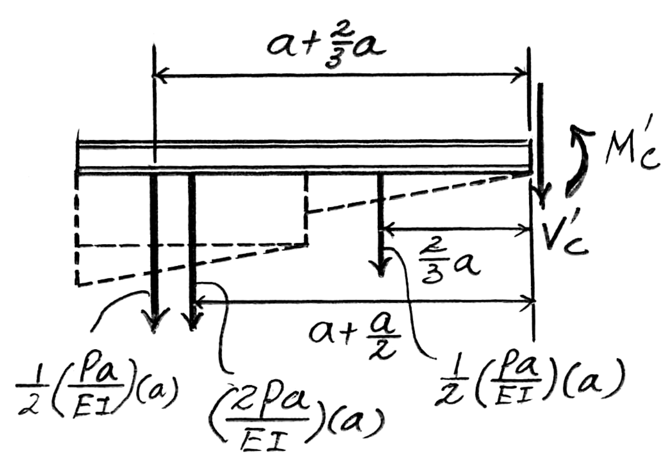
$$\Delta_C = M'_C = -\frac{25Pa^3}{6EI} = \frac{25Pa^3}{6EI} \downarrow \qquad \textbf{Ans.}$$



(a)

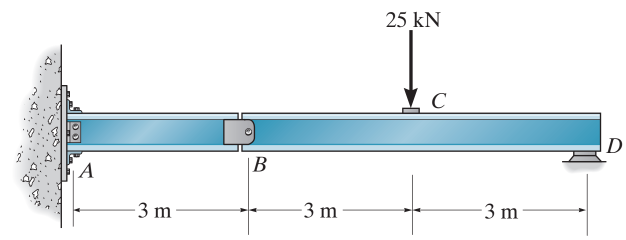


(b)



(c)

***8–36.** Determine the displacement at C. Assume A is a fixed support, B is a pin, and D is a roller. EI is constant. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and the elastic curve shown in Fig. a and b, respectively, Theorem 1 and 2 give

$$\Delta_B = |t_{B/A}| = \left[\frac{1}{2}\left(\frac{37.5\text{ kN}\cdot\text{m}}{EI}\right)(3\text{ m})\right]\left[\frac{2}{3}(3\text{ m})\right] = \frac{112.5\text{ kN}\cdot\text{m}^3}{EI}\ \downarrow$$

$$t_{C/D} = \left[\frac{1}{2}\left(\frac{37.5\text{ kN}\cdot\text{m}}{EI}\right)(3\text{ m})\right]\left[\frac{1}{3}(3\text{ m})\right] = \frac{56.25\text{ kN}\cdot\text{m}^3}{EI}$$

$$t_{B/D} = \left[\frac{1}{2}\left(\frac{37.5\text{ kN}\cdot\text{m}}{EI}\right)(6\text{ m})\right](3\text{ m}) = \frac{337.5\text{ kN}\cdot\text{m}^3}{EI}$$

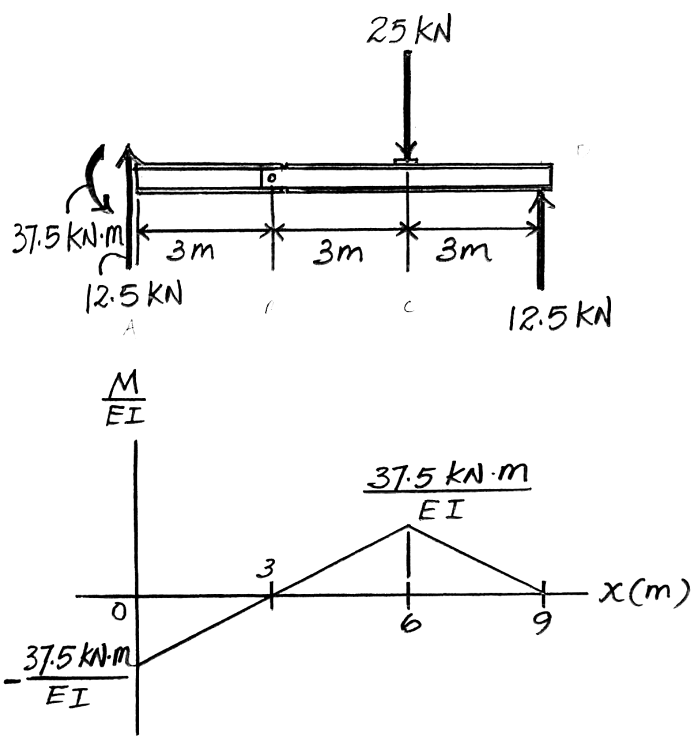
Then

$$\theta_D = \frac{\Delta_B + t_{B/D}}{L_{B/D}} = \frac{112.5\text{ kN}\cdot\text{m}^3/EI + 337.5\text{ kN}\cdot\text{m}^3/EI}{6\text{ m}} = \frac{75\text{ kN}\cdot\text{m}^2}{EI}\ \triangledown \qquad \textbf{Ans.}$$

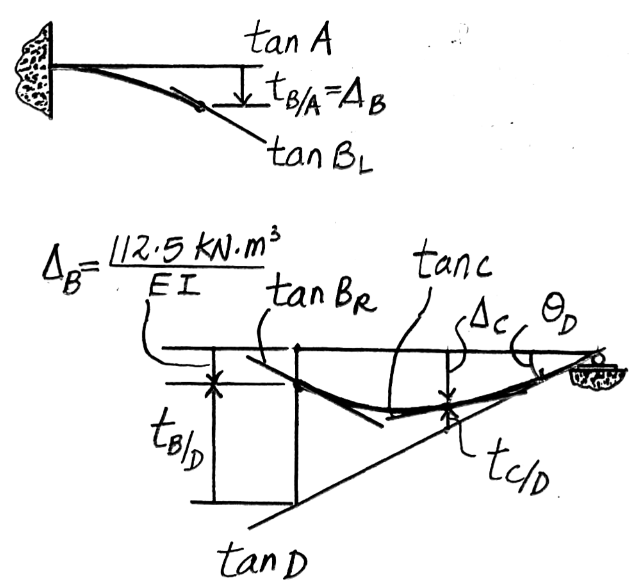
$$\Delta_C + t_{C/D} = \frac{1}{2}(\Delta_B + t_{B/D})$$

$$\Delta_C + \frac{56.25\text{ kN}\cdot\text{m}^3}{EI} = \frac{1}{2}\left(\frac{112.5\text{ kN}\cdot\text{m}^3}{EI} + \frac{337.5\text{ kN}\cdot\text{m}^3}{EI}\right)$$

$$\Delta_C = \frac{169\text{ kN}\cdot\text{m}^3}{EI}\ \downarrow \qquad \textbf{Ans.}$$

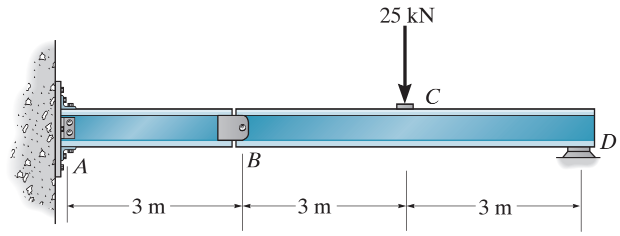


(a)



(b)

8–37. Determine the displacement at *C*. Assume *A* is a fixed support, *B* is a pin, and *D* is a roller. *EI* is constant. Use the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*,

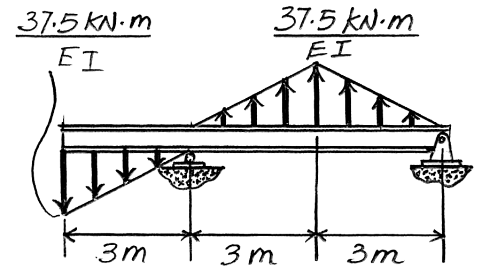
$$\zeta+ \sum M_B = 0; \quad \left[\frac{1}{2}\left(\frac{37.5\text{ kN}\cdot\text{m}}{EI}\right)(6\text{ m})\right](3\text{ m}) + \left[\frac{1}{2}\left(\frac{37.5\text{ kN}\cdot\text{m}}{EI}\right)(3\text{ m})\right](2\text{ m}) - D'_y(6\text{ m}) = 0$$

$$\theta_D = D'_y = \frac{75\text{ kN}\cdot\text{m}^2}{EI} \qquad \textbf{Ans.}$$

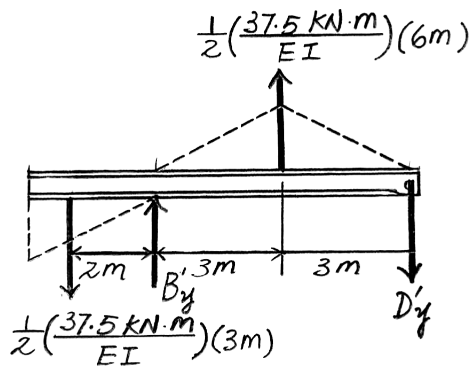
Referring to Fig. *d*,

$$\zeta+ \sum M_C = 0; \quad \left[\frac{1}{2}\left(\frac{37.5\text{ kN}\cdot\text{m}}{EI}\right)(3\text{ m})\right](1\text{ m}) - \left(\frac{75\text{ kN}\cdot\text{m}^2}{EI}\right)(3\text{ m}) - M'_C = 0$$

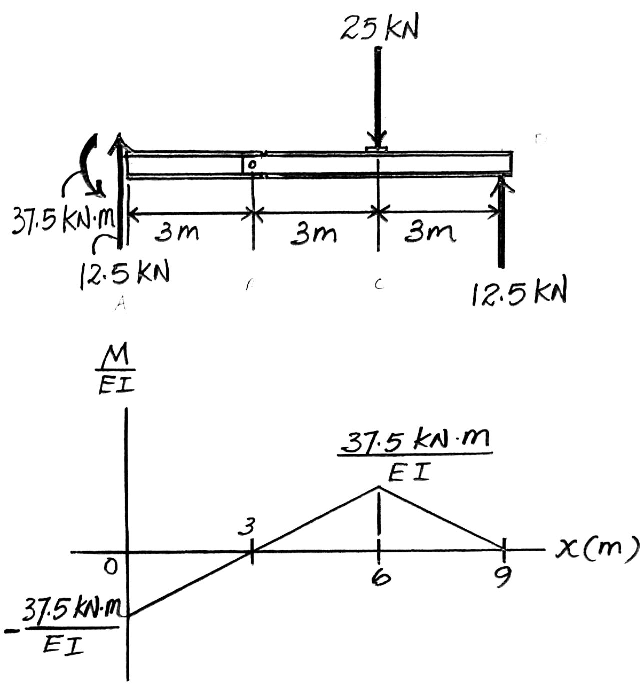
$$\Delta_C = -\frac{168.75\text{ kN}\cdot\text{m}^3}{EI} = \frac{168.75\text{ kN}\cdot\text{m}^3}{EI}\downarrow \qquad \textbf{Ans.}$$



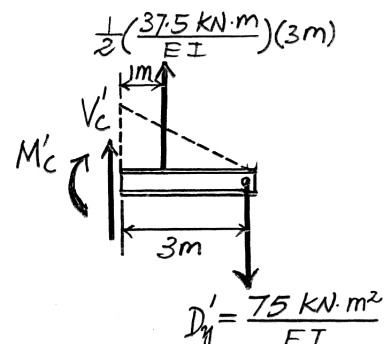
(b)



(c)

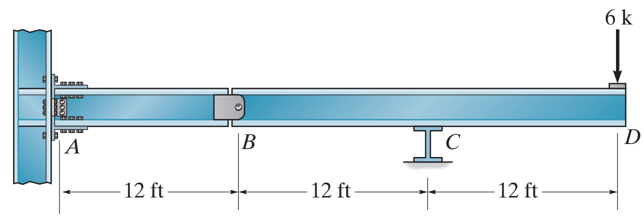


(a)



(d)

8–38. Determine the displacement at D and the slope at D. Assume A is a fixed support, B is a pin, and C is a roller. Use the moment-area theorems.



Using the $\frac{M}{EI}$ diagram and elastic curve shown in Fig. a and b, respectively, Theorem 1 and 2 give

$$\Delta_B = t_{B/A} = \left[\frac{1}{2}\left(\frac{72\text{ k}\cdot\text{ft}}{EI}\right)(12\text{ ft})\right]\left[\frac{2}{3}(12\text{ ft})\right] = \frac{3456\text{ k}\cdot\text{ft}^3}{EI} \uparrow$$

$$\theta_{D/B} = \frac{1}{2}\left(-\frac{72\text{ k}\cdot\text{ft}}{EI}\right)(24\text{ ft}) = -\frac{864\text{ k}\cdot\text{ft}^2}{EI} = \frac{864\text{ k}\cdot\text{ft}^2}{EI}$$

$$t_{C/B} = \left[\frac{1}{2}\left(-\frac{72\text{ k}\cdot\text{ft}}{EI}\right)(12\text{ ft})\right]\left[\frac{1}{3}(12\text{ ft})\right] = -\frac{1728\text{ k}\cdot\text{ft}^3}{EI}$$

$$t_{D/B} = \left[\frac{1}{2}\left(-\frac{72\text{ k}\cdot\text{ft}}{EI}\right)(24\text{ ft})\right](12\text{ ft}) = -\frac{10368\text{ k}\cdot\text{ft}^3}{EI}$$

Then,

$$\Delta' = 2(\Delta_B - |t_{C/B}| = 2\left(\frac{3456\text{ k}\cdot\text{ft}^3}{EI} - \frac{1728\text{ k}\cdot\text{ft}^3}{EI}\right) = \frac{3456\text{ k}\cdot\text{ft}^3}{EI}$$

$$\theta_{BR} = \frac{\Delta'}{L_{BD}} = \frac{3456\text{ k}\cdot\text{ft}^3/EI}{24\text{ ft}} = \frac{144\text{ k}\cdot\text{ft}^2}{EI}$$

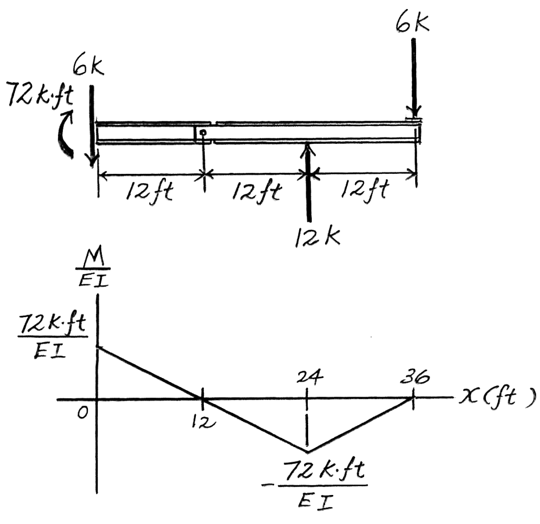
$$\theta_D = \theta_{BR} + \theta_{D/B}$$

$$+\circlearrowright\ \theta_D = \frac{144\text{ k}\cdot\text{ft}^2}{EI} + \frac{864\text{ k}\cdot\text{ft}^2}{EI} = \frac{1008\text{ k}\cdot\text{ft}^2}{EI}$$ **Ans.**

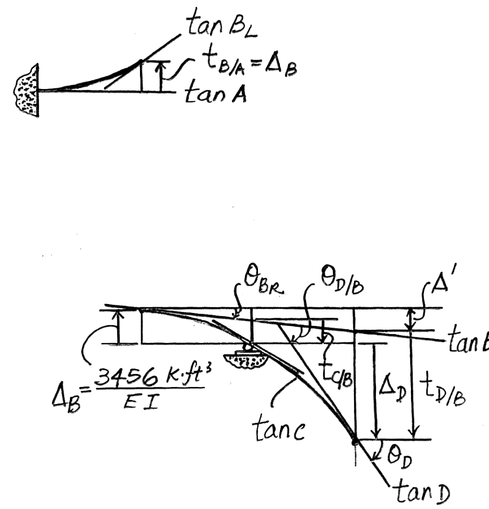
$$\Delta_D = |t_{D/B}| + \Delta' - \Delta_B$$

$$= \frac{10368\text{ k}\cdot\text{ft}^3}{EI} + \frac{3456\text{ k}\cdot\text{ft}^3}{EI} - \frac{3456\text{ k}\cdot\text{ft}^3}{EI}$$

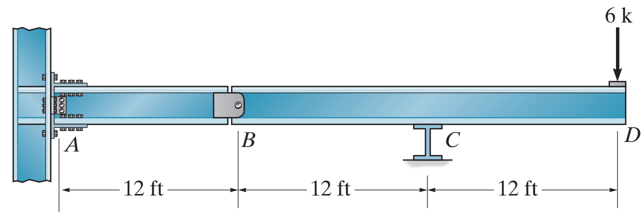
$$= \frac{10{,}368\text{ k}\cdot\text{ft}^3}{EI} \downarrow$$ **Ans.**



(a)



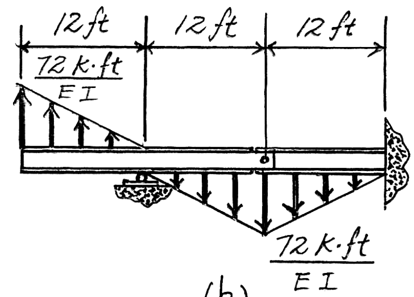
8–39. Determine the displacement at D and the slope at D. Assume A is a fixed support, B is a pin, and C is a roller. Use the conjugate-beam method.



The real beam and conjugate beam are shown in Fig. *a* and *b*, respectively. Referring to Fig. *c*,

$$\zeta+ \sum M_B = 0; \qquad C_y'(12\text{ ft}) - \left[\frac{1}{2}\left(\frac{72\text{ k}\cdot\text{ft}}{EI}\right)(12\text{ ft})\right](16\text{ ft}) = 0$$

$$C_y' = \frac{576\text{ k}\cdot\text{ft}^2}{EI}$$



(b)

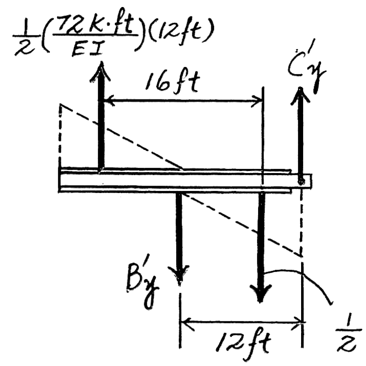
Referring to Fig. *d*,

$$+\uparrow \sum F_y = 0; \quad -V_D' - \frac{1}{2}\left(\frac{72\text{ k}\cdot\text{ft}}{EI}\right)(12\text{ ft}) - \frac{576\text{ k}\cdot\text{ft}^2}{EI} = 0$$

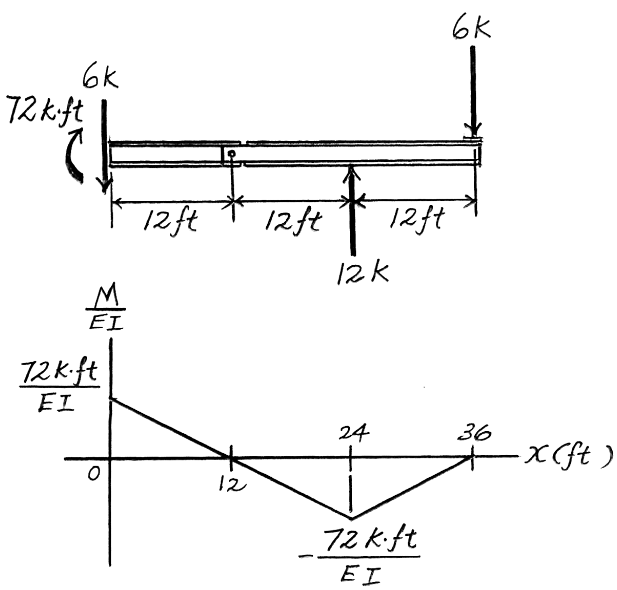
$$\theta_D = V_D' = -\frac{1008\text{ k}\cdot\text{ft}^2}{EI} = \frac{1008\text{ k}\cdot\text{ft}^2}{EI} \qquad \textbf{Ans.}$$

$$\zeta+ \sum M_C = 0; \quad M_D' + \left[\frac{1}{2}\left(\frac{72\text{ k}\cdot\text{ft}}{EI}\right)(12\text{ ft})\right](8\text{ ft}) + \left(\frac{576\text{ k}\cdot\text{ft}^2}{EI}\right)(12\text{ ft}) = 0$$

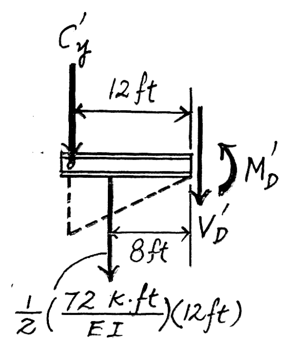
$$M_D' = \Delta_D = -\frac{10368\text{ k}\cdot\text{ft}^3}{EI} = \frac{10{,}368\text{ k}\cdot\text{ft}^3}{EI} \downarrow \qquad \textbf{Ans.}$$



(c)



(a)



(d)