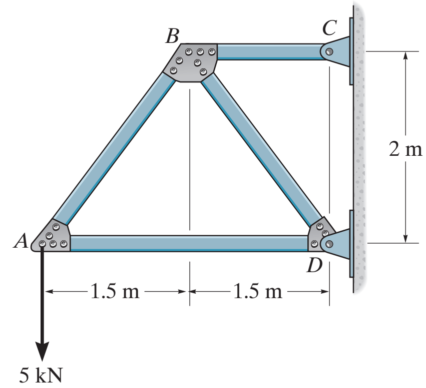
9–1. Determine the vertical displacement of joint A. Each bar is made of steel and has a cross-sectional area of 600 mm^2. Take $E = 200$ GPa. Use the method of virtual work.



The virtual forces and real forces in each member are shown in Fig. *a* and *b*, respectively.

Member	n(kN)	N(kN)	L(m)	nNL (kN^2 • m)
AB	1.25	6.25	2.50	19.531
AD	−0.75	−3.75	3	8.437
BD	−1.25	−6.25	2.50	19.531
BC	1.50	7.50	1.50	16.875
			Σ	64.375

$$1\text{ kN}\cdot\Delta_{A_v} = \sum\frac{nNL}{AE} = \frac{64.375\text{ kN}^2\cdot\text{m}}{AE}$$

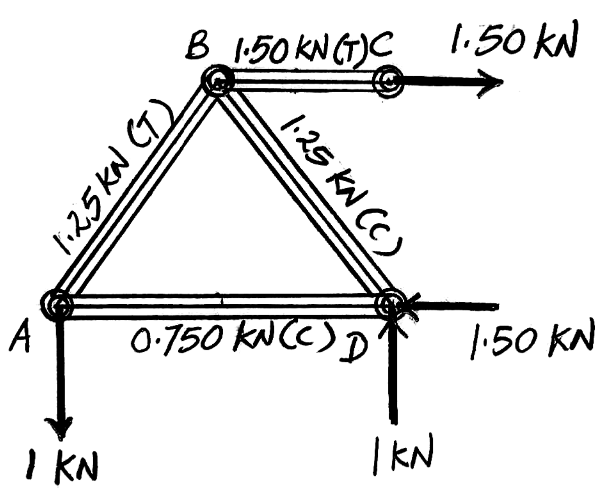
$$\Delta_{A_v} = \frac{64.375\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{64.375(10^3)\text{ N}\cdot\text{m}}{\left[0.6(10^{-3})\text{ m}^2\right]\left[200(10^9)\text{ N/m}^2\right]}$$

$$= 0.53646\,(10^{-3})\text{ m}$$

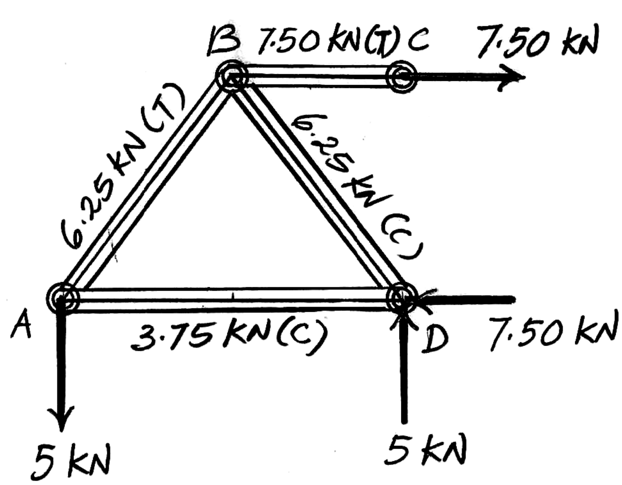
$$= 0.536\text{ mm}\downarrow$$

Ans.



Virtual Forces n

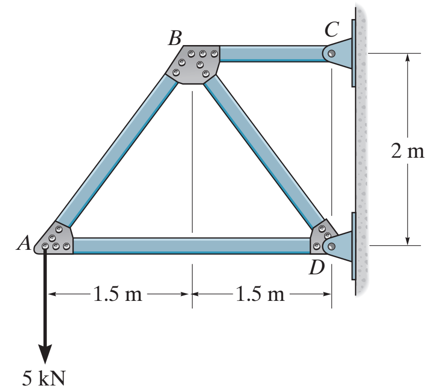
(a)



Real Forces N

(b)

9–2. Solve Prob. 9–1 using Castigliano's theorem.



Member	$N(kN)$	$\frac{\partial N}{\partial P}$	$N(P = 5\text{kN})$	L(m)	$N\left(\frac{\partial N}{\partial P}\right)L(\text{kN}\cdot\text{m})$
AB	1.25 P	1.25	6.25	2.5	19.531
AD	–0.750 P	–0.75	–3.75	3	8.437
BD	–1.25 P	–1.25	–6.25	2.5	19.531
BC	1.50 P	1.50	7.50	1.5	16.875
				Σ	64.375

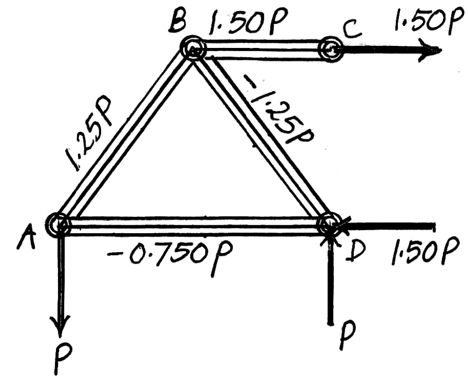
$$\Delta_{A_v} = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$= \frac{64.375\ \text{kN}\cdot\text{m}}{AE}$$

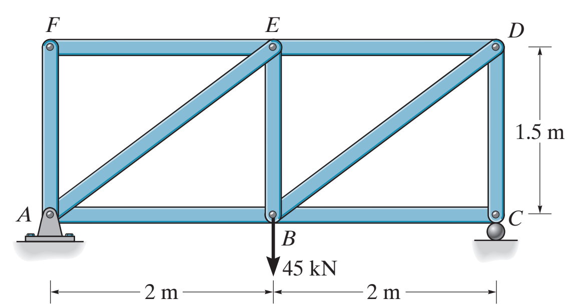
$$= \frac{64\cdot 375(10^3)\ \text{N}\cdot\text{m}}{[0.6(10^{-3})\,\text{m}^2][200(10^9)\,\text{N/m}^2]}$$

$$= 0.53646\ (10^{-3})\ \text{m}$$

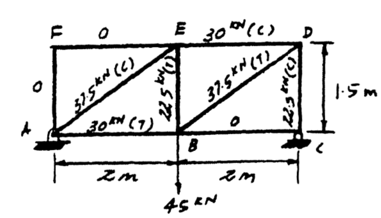
$$= 0.536\ \text{mm}\downarrow$$ **Ans.**



***9–3.** Determine the vertical displacement of joint B. For each member $A = 400\ \text{mm}^2$, $E = 200$ GPa. Use the method of virtual work.

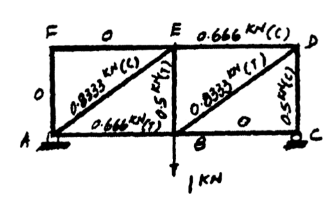


Member	n	N	L	nNL
AF	0	0	1.5	0
AE	–0.8333	–37.5	2.5	78.125
AB	0.6667	30.0	2.0	40.00
EF	0	0	2.0	0
EB	0.50	22.5	1.5	16.875
ED	–0.6667	–30.0	2.0	40.00
BC	0	0	2.0	0
BD	0.8333	37.5	2.5	78.125
CD	–0.5	–22.5	1.5	16.875
				$\Sigma = 270$

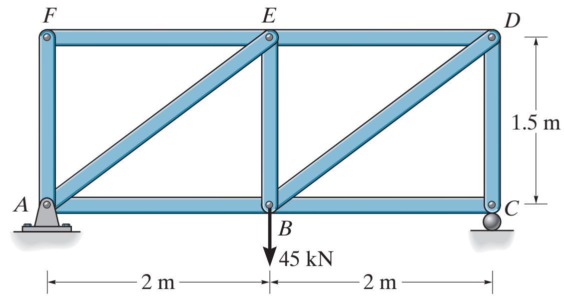


$$1\cdot\Delta_{B_v} = \sum\frac{nNL}{AE}$$

$$\Delta_{B_v} = \frac{270(10^3)}{400(10^{-6})(200)(10^9)} = 3.375(10^{-3})\,\text{m} = 3.38\ \text{mm}\downarrow$$ **Ans.**

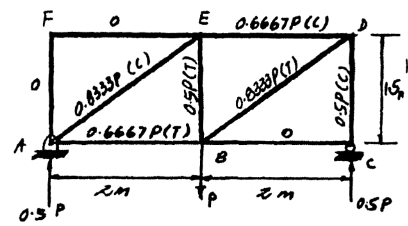


***9–4.** Solve Prob. 9–3 using Castigliano's theorem.



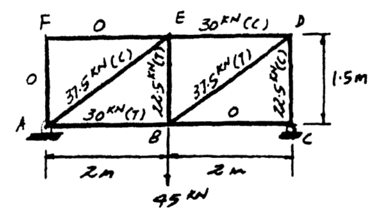
Member	N	$\frac{\partial N}{\partial P}$	$N(P = 45)$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AF	0	0	0	1.5	0
AE	$-0.8333P$	-0.8333	-37.5	2.5	78.125
AB	$0.6667P$	0.6667	30.0	2.0	40.00
BE	$0.5P$	0.5	22.5	1.5	16.875
BD	$0.8333P$	0.8333	37.5	2.5	78.125
BC	0	0	0	2.0	0
CD	$-0.5P$	-0.5	-22.5	1.5	16.875
DE	$0.6667P$	-0.6667	-30.0	2.0	40.00
EF	0	0	0	2.0	0

$\Sigma = 270$

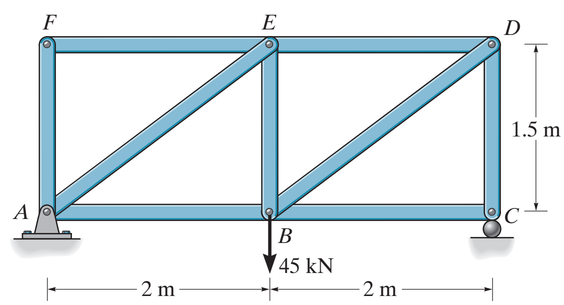


$$\delta_{B_v} = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{270}{AE}$$

$$= \frac{270(10^3)}{400(10^{-6})(200)(10^9)} = 3.375(10^{-3})\,\text{m} = 3.38\text{ mm}$$ **Ans.**

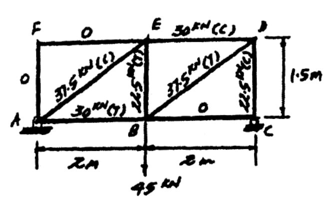


9–5. Determine the vertical displacement of joint E. For each member $A = 400\text{ mm}^2$, $E = 200$ GPa. Use the method of virtual work.



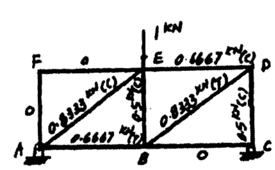
Member	n	N	L	nNL
AF	0	0	1.5	0
AE	-0.8333	-37.5	2.5	78.125
AB	0.6667	30.0	2.0	40.00
EF	0	0	2.0	0
EB	0.50	22.5	1.5	-16.875
ED	-0.6667	30.0	2.0	40.00
BC	0	0	2.0	0
BD	0.8333	37.5	2.5	78.125
CD	-0.5	-22.5	1.5	16.875

$\Sigma = 236.25$

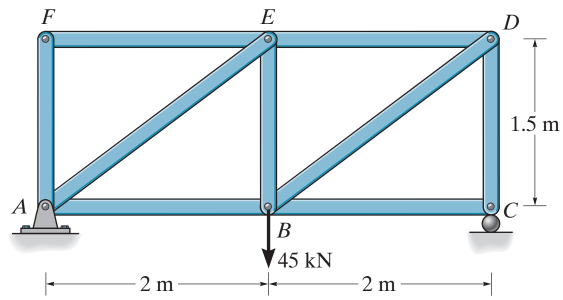


$$1 \cdot \Delta E_v = \sum \frac{nNL}{AE}$$

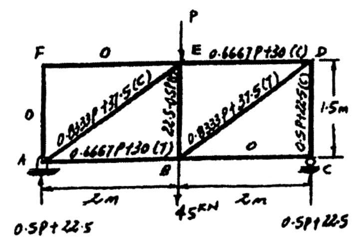
$$\Delta E_v = \frac{236.25(10^3)}{400(10^{-6})(200)(10^9)} = 2.95(10^{-3})\,\text{m} = 2.95\text{ mm}\downarrow$$ **Ans.**



9–6. Solve Prob. 9–5 using Castigliano's theorem.



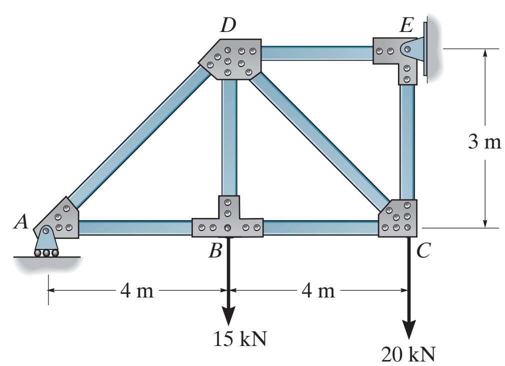
Member	N	$\frac{\partial N}{\partial P}$	$N(P = 45)$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AF	0	0	0	1.5	0
AE	$-(0.8333P + 37.5)$	-0.8333	-37.5	2.5	78.125
AB	$0.6667P + 30$	0.6667	30.0	2.0	40.00
BE	$22.5 - 0.5P$	-0.5	22.5	1.5	-16.875
BD	$0.8333P + 37.5$	0.8333	37.5	2.5	78.125
BC	0	0	0	2.0	0
CD	$-(0.5P + 22.5)$	-0.5	-22.5	1.5	16.875
DE	$-(0.6667P + 30)$	-0.6667	-30.0	2.0	40.00
EF	0	0	0	2.0	0
					$\Sigma = 236.25$



$$\Delta_{E_v} = \sum N \frac{\partial N}{\partial P} \frac{L}{AE} = \frac{236.25}{AE}$$

$$= \frac{236.25(10^3)}{400(10^{-6})(200)(10^9)} = 2.95(10^{-3})\,\text{m} = 2.95\text{ mm} \downarrow \qquad \textbf{Ans.}$$

9–7. Determine the vertical displacement of joint D. Use the method of virtual work. AE is constant. Assume the members are pin connected at their ends.



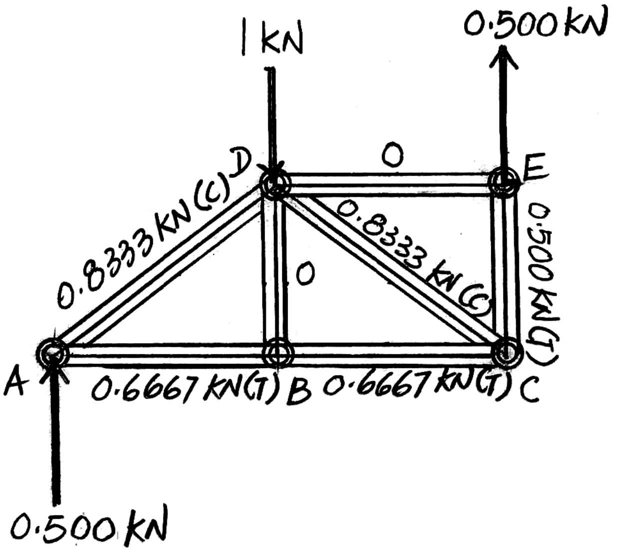
The virtual and real forces in each member are shown in Fig. *a* and *b*, respectively

Member	n(kN)	N(kN)	L(m)	$nNL(\text{kN}^2 \cdot \text{m})$
AB	0.6667	10.0	4	26.667
BC	0.6667	10.0	4	26.667
AD	-0.8333	-12.5	5	52.083
BD	0	15.0	3	0
CD	-0.8333	-12.5	5	52.083
CE	0.500	27.5	3	41.25
DE	0	0	4	0
			Σ	198.75

$$1\text{ kN} \cdot \Delta_{D_v} = \sum \frac{nNL}{AE} = \frac{198.75\text{ kN}^2 \cdot \text{m}}{AE}$$

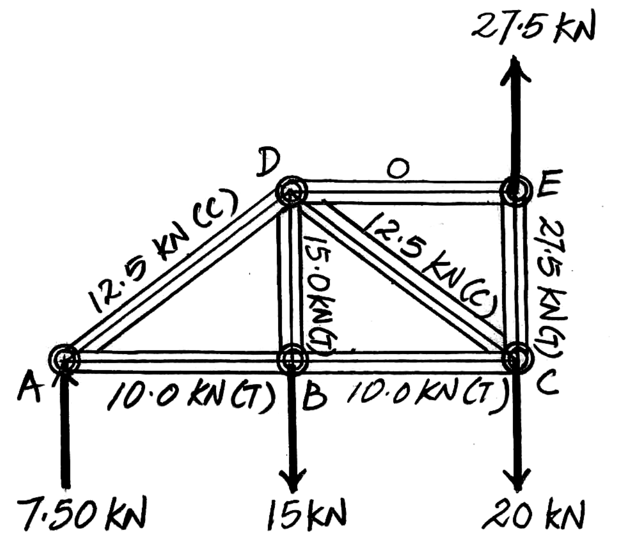
$$\Delta_{D_v} = \frac{198.75\text{ kN} \cdot \text{m}}{AE} = \frac{199\text{ kN} \cdot \text{m}}{AE} \downarrow \qquad \textbf{Ans.}$$

9–7. Continued



Virtual Forces n

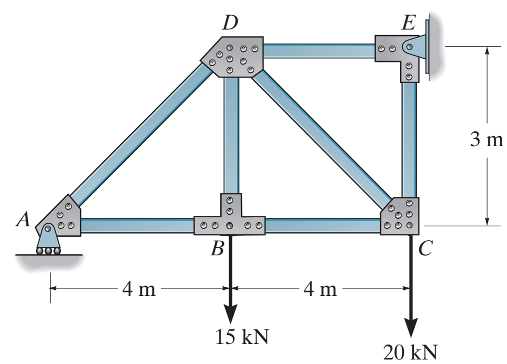
(a)



Real Forces N

(b)

***9–8.** Solve Prob. 9–7 using Castigliano's theorem.

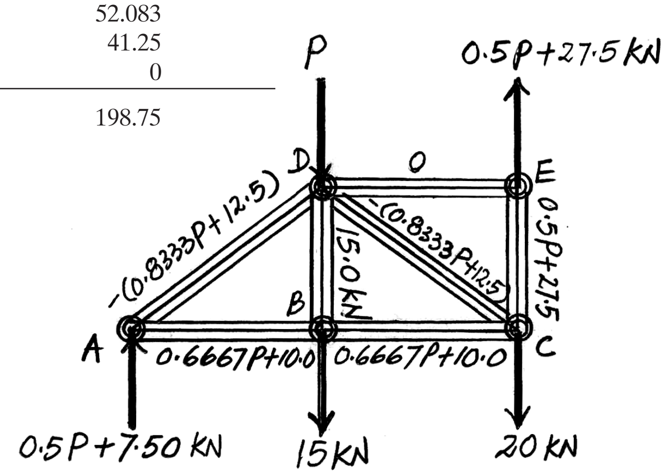


Member	N(kN)	$\frac{\partial N}{\partial P}$	$N(P=0)$ kN	L(m)	$N\left(\frac{\partial N}{\partial P}\right)L$ (kN · m)
AB	$0.6667P + 10.0$	0.6667	10.0	4	26.667
BC	$0.6667P + 10.0$	0.6667	10.0	4	26.667
AD	$-(0.8333P + 12.5)$	−0.8333	−12.5	5	52.083
BD	15.0	0	15.0	3	0
CD	$-(0.8333P + 12.5)$	−0.8333	−12.5	5	52.083
CE	$0.5P + 27.5$	0.5	27.5	3	41.25
DE	0	0	0	4	0
				Σ	198.75

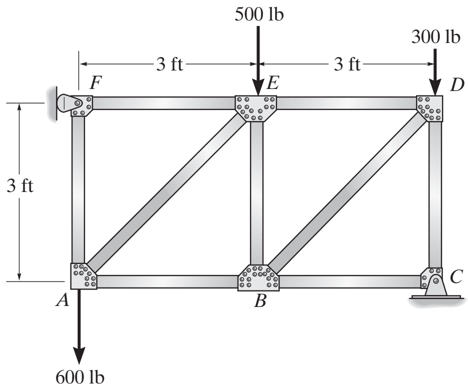
$$\Delta_{D_v} = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

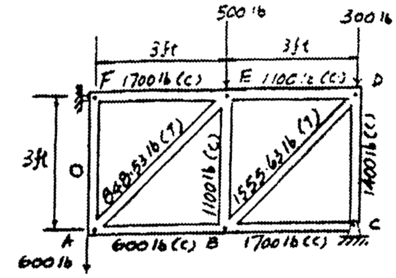
$$= \frac{198.75 \text{ kN} \cdot \text{m}}{AE} = \frac{199 \text{ kN} \cdot \text{m}}{AE} \downarrow$$

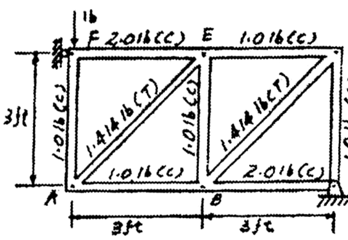
Ans.



9–9. Determine the vertical displacement of the truss at joint F. Assume all members are pin connected at their end points. Take $A = 0.5\ \text{in}^2$ and $E = 29(10^3)$ ksi for each member. Use the method of virtual work.



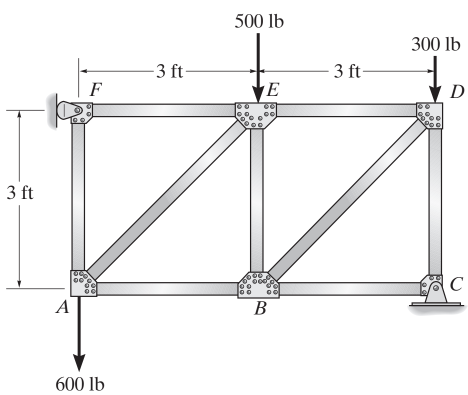




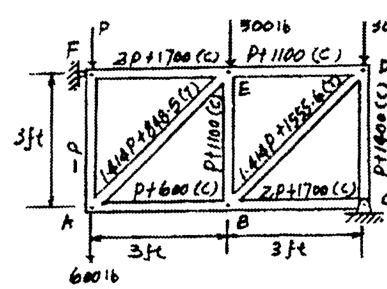
$$\Delta_{F_v} = \sum \frac{nNL}{AE} = \frac{L}{AE}[(-1.00)(-600)(3) + (1.414)(848.5)(4.243) + (-1.00)(0)(3)$$
$$+ (-1.00)(-1100)(3) + (1.414)(1555.6)(4.243) + (-2.00)(-1700)(3)$$
$$+ (-1.00)(-1400)(3) + (-1.00)(-1100)(3) + (-2.00)(-1700)(3)](12)$$

$$= \frac{47425.0(12)}{0.5(29)(10^6)} = 0.0392 \text{ in.} \downarrow \qquad \textbf{Ans.}$$

9–10. Solve Prob. 9–9 using Castigliano's theorem.



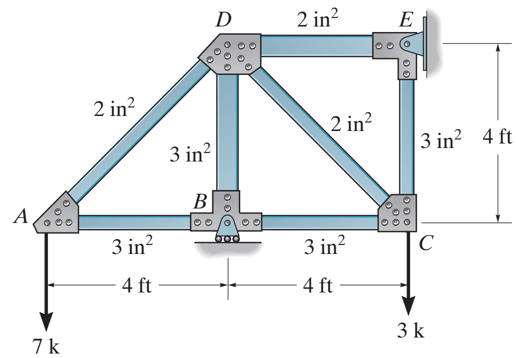
$$\Delta_{F_v} = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{1}{AE}\ [-(P + 600)](-1)(3) + (1.414P + 848.5)(1.414)(4.243)$$
$$+ (-P)(-1)(3) + (-(P + 1100))(-1)(3)$$
$$+ (1.414P + 1555.6)(1.414)(4.243) + (-(2P + 1700))\ (-2)(3)$$
$$+ (-(P + 1400)(-1)(3) + (-(P + 1100))(-1)(3)$$
$$+ (-(2P + 1700))(-2)(3)](12) = \frac{(55.97P + 47.425.0)(12)}{(0.5(29(10)^6)}$$



Set $P = 0$ and evaluate

$$\Delta_{F_v} = 0.0392 \text{ in.} \downarrow \qquad \textbf{Ans.}$$

9–11. Determine the vertical displacement of joint A. The cross-sectional area of each member is indicated in the figure. Assume the members are pin connected at their end points. $E = 29\,(10)^3$ ksi. Use the method of virtual work.



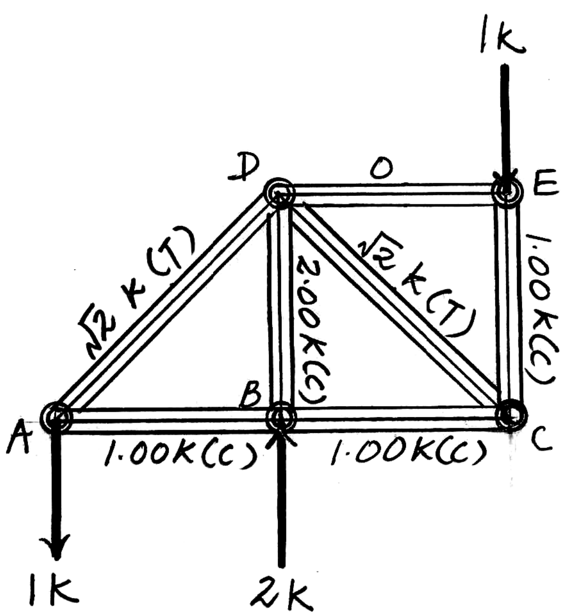
The virtual force and real force in each member are shown in Fig. *a* and *b*, respectively.

Member	$n(k)$	$N(k)$	L(ft)	$nNL(\text{k}^2\cdot\text{ft})$
AB	−1.00	−7.00	4	28
BC	−1.00	−7.00	4	28
AD	$\sqrt{2}$	$7\sqrt{2}$	$4\sqrt{2}$	$56\sqrt{2}$
BD	−2.00	−14.00	4	112
CD	$\sqrt{2}$	$7\sqrt{2}$	$4\sqrt{2}$	$56\sqrt{2}$
CE	−1.00	−4.00	4	16
DE	0	0	4	0

$$1\,k\cdot\Delta_{A_v} = \sum\frac{nNL}{AE}$$

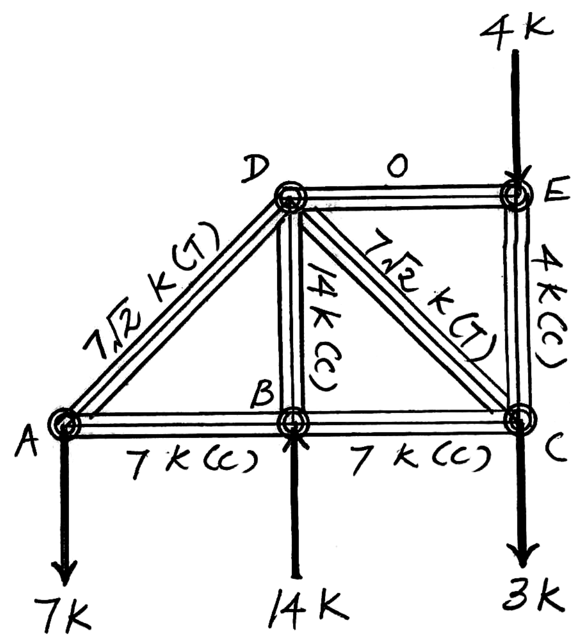
$$1\,k\cdot\Delta_{A_v} = \frac{(29 + 28 + 112 + 16)\text{k}^2\cdot\text{ft}}{(3\text{in}^2)[29(10^3)\,\text{k/in}^2]} + \frac{(56\sqrt{2} + 56\sqrt{2})\,\text{k}^2\cdot\text{ft}}{(2\text{in}^2)[29(10^3)\,\text{k/in}^2]}$$

$$\Delta_{A_v} = 0.004846\text{ ft}\left(\frac{12\text{ in}}{1\text{ ft}}\right) = 0.0582\text{ in.}\downarrow$$ **Ans.**



Virtual Forces n

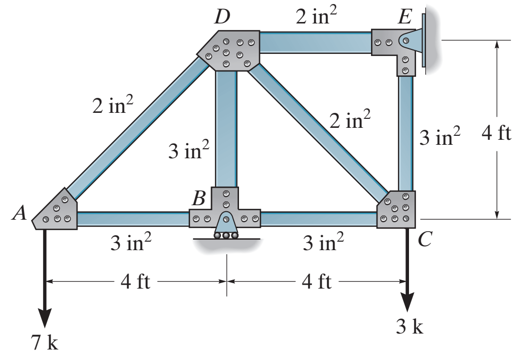
(a)



Real Forces N

(b)

***9–12.** Solve Prob. 9–11 using Castigliano's theorem.

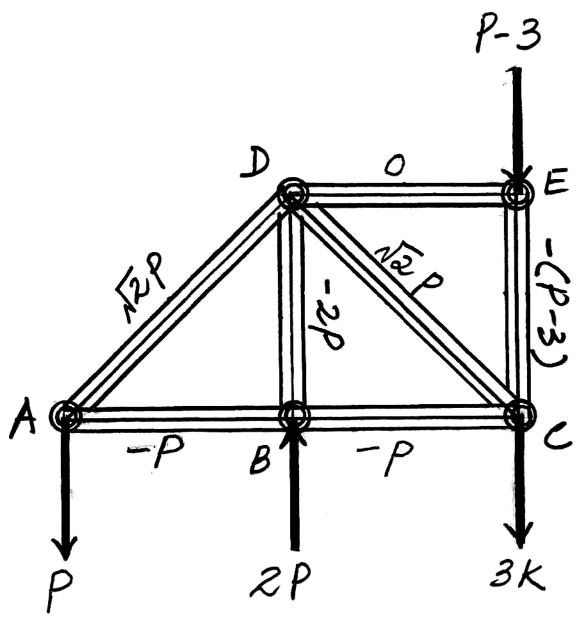


Member	$N(k)$	$\frac{\partial N}{\partial P}$	$N(P=7k)$	L(ft)	$N\left(\frac{\partial N}{\partial P}\right)L$(k·ft)
AB	$-P$	-1	-7	4	28
BC	$-P$	-1	-7	4	28
AD	$\sqrt{2}P$	$\sqrt{2}$	$7\sqrt{2}$	$4\sqrt{2}$	$56\sqrt{2}$
BD	$-2P$	-2	-14	4	112
CD	$\sqrt{2}P$	$\sqrt{2}$	$7\sqrt{2}$	$4\sqrt{2}$	$56\sqrt{2}$
CE	$-(P-3)$	-1	-4	4	16
DE	0	0	0	4	0

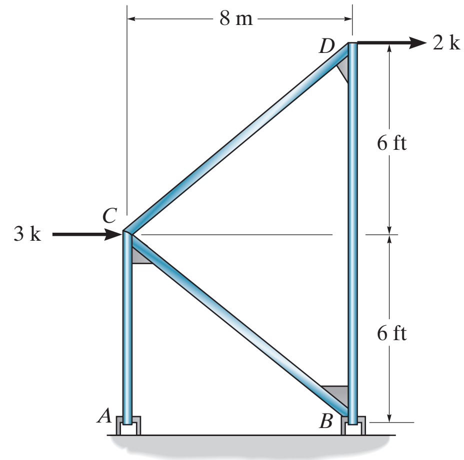
$$\Delta_{A_v} = \sum N\left(\frac{\delta N}{\delta P}\right)\frac{L}{AE}$$

$$= \frac{(28 + 28 + 112 + 16)\ \text{k}\cdot\text{ft}}{(3\ \text{in}^2)[29(10^3)\text{k/m}^2]} + \frac{56\sqrt{2} + 56\sqrt{2}\ \text{k}^2\cdot\text{ft}}{(2\ \text{in}^2)[29(10^3)\text{k/in}^2]}$$

$$= 0.004846\ \text{ft}\left(\frac{12\ \text{in}}{1\ \text{ft}}\right) = 0.0582\ \text{in}\downarrow$$ **Ans.**



9–13. Determine the horizontal displacement of joint D. Assume the members are pin connected at their end points. AE is constant. Use the method of virtual work.



The virtual force and real force in each member are shown in Fig. a and b, respectively.

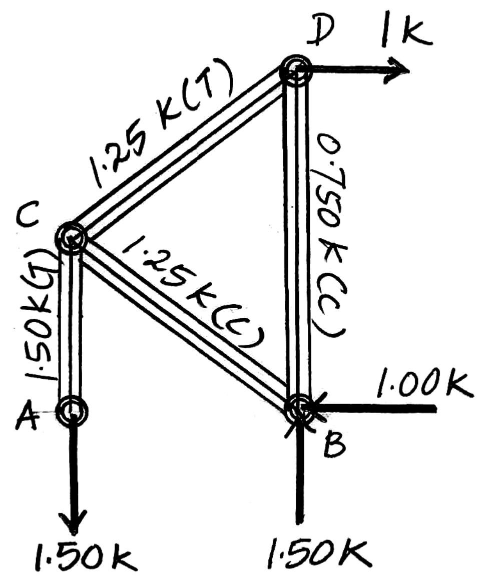
Member	$n(k)$	$N(k)$	L(ft)	$nNL(\text{k}^2 \cdot \text{ft})$
AC	1.50	5.25	6	47.25
BC	−1.25	−6.25	10	78.125
BD	−0.75	−1.50	12	13.50
CD	1.25	2.50	10	31.25
			Σ	170.125

$$1\text{k} \cdot \Delta_{D_h} = \sum \frac{nNL}{AE}$$

$$1\text{k} \cdot \Delta_{D_h} = \frac{170.125 \text{ k}^2 \cdot \text{ft}}{AE}$$

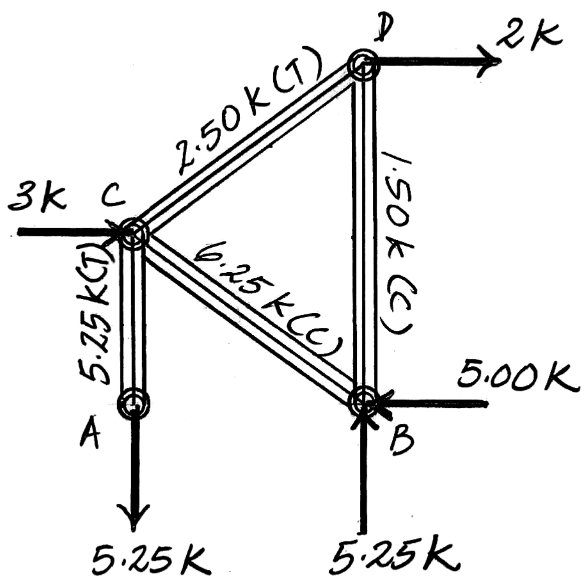
$$\Delta_{D_h} = \frac{170 \text{ k} \cdot \text{ft}}{AE} \rightarrow$$

Ans.



Virtual Forces n

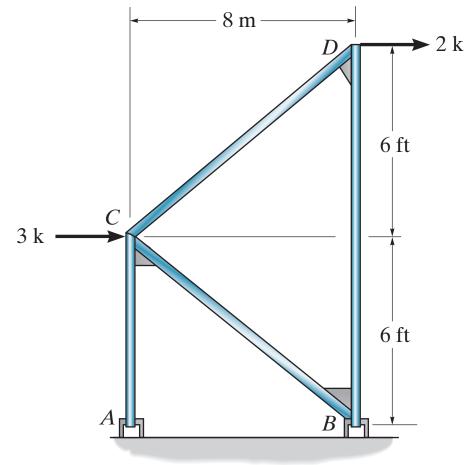
(a)



Real Forces N

(b)

9–14. Solve Prob. 9–13 using Castigliano's theorem.

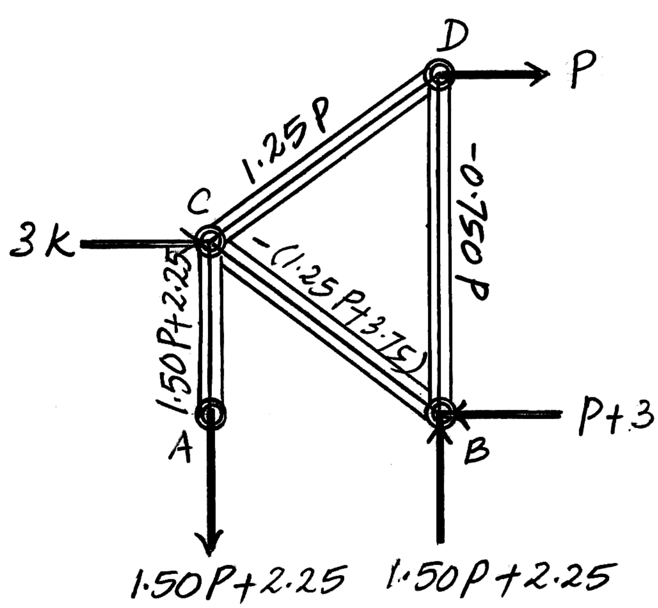


Member	$N(k)$	$\frac{\partial N}{\partial P}$	$N\ (P = 2\text{k})$	$L(\text{ft})$	$N\left(\frac{\partial N}{\partial P}\right)L(\text{k}\cdot\text{ft})$
AC	$1.50P + 2.25$	1.50	5.25	6	47.25
BC	$-(1.25P + 3.75)$	−1.25	−6.25	10	78.125
BD	$-0.750P$	−0.750	−1.50	12	13.5
CD	$1.25P$	1.25	2.50	10	31.25
				Σ	170.125

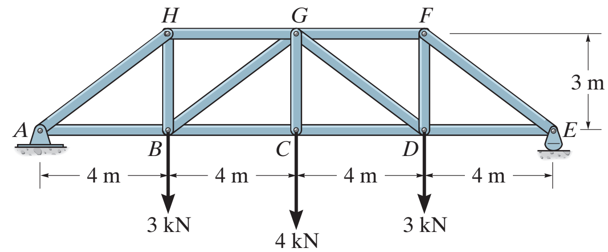
$$\Delta_{D_h} = \sum N\left(\frac{\delta N}{\delta P}\right)\frac{L}{AE}$$

$$= \frac{170.125\ \text{k}\cdot\text{ft}}{AE}$$

$$= \frac{170\ \text{k}\cdot\text{ft}}{AE} \rightarrow$$ **Ans.**



9–15. Determine the vertical displacement of joint C of the truss. Each member has a cross-sectional area of $A = 300\ \text{mm}^2$. $E = 200$ GPa. Use the method of virtual work.



The virtual and real forces in each member are shown in Fig. *a* and *b* respectively.

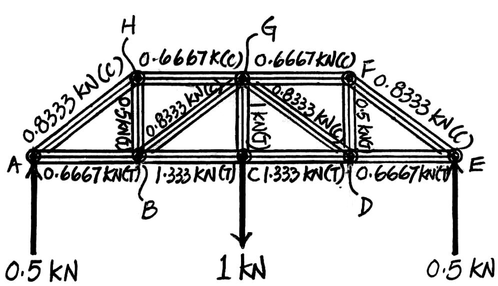
Member	$n(kN)$	$N(kN)$	L(m)	$nNL(\text{kN}^2 \cdot \text{m})$
AB	0.6667	6.667	4	17.78
DE	0.6667	6.667	4	17.78
BC	1.333	9.333	4	49.78
CD	1.333	9.333	4	49.78
AH	−0.8333	−8.333	5	34.72
EF	−0.8333	−8.333	5	34.72
BH	0.5	5	3	7.50
DF	0.5	5	3	7.50
BG	−0.8333	−3.333	5	13.89
DG	−0.8333	−3.333	5	13.89
GH	−0.6667	−6.6667	4	17.78
FG	−0.6667	−6.6667	4	17.78
CG	1	4	3	12.00
				$\Sigma = 294.89$

$$1\text{kN} \cdot \Delta_{C_v} = \sum \frac{nNL}{AE} = \frac{294.89\ \text{kN}^2 \cdot \text{m}}{AE}$$

$$\Delta_{C_v} = \frac{294.89\ \text{kN} \cdot \text{m}}{AE}$$

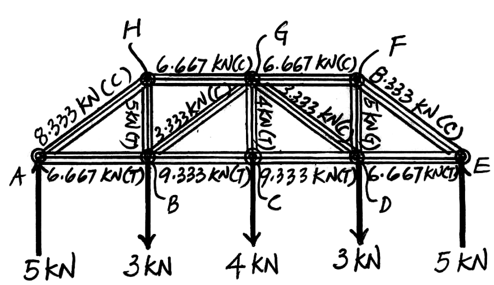
$$= \frac{294.89(10^3)\ \text{N} \cdot \text{m}}{[0.3(10^{-3})\ \text{m}^2][200(10^9)\ \text{N/m}^2]}$$

$$= 0.004914\,\text{m} = 4.91\,\text{mm} \quad \downarrow$$ **Ans.**



Virtual Forces n

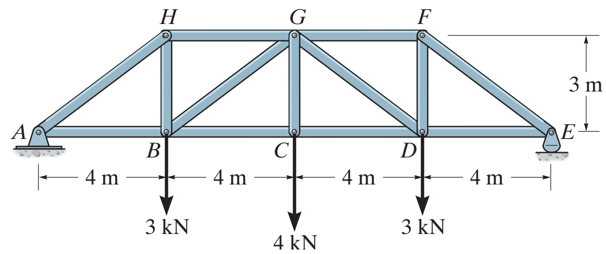
(a)



Real Forces N

(b)

***9–16.** Solve Prob. 9–15 using Castigliano's theorem.



Member	$N(kN)$	$\frac{\partial N}{\partial P}$	$N(P = 4\text{ kN})$	L(m)	$N\left(\frac{\partial N}{\partial P}\right)L(\text{k}\cdot\text{m})$
AB	$0.6667P + 4$	0.6667	6.667	4	17.78
DE	$0.6667P + 4$	0.6667	6.667	4	17.78
BC	$1.333P + 4$	1.333	9.333	4	49.78
CD	$1.333P + 4$	1.333	9.333	4	49.78
AH	$-(0.8333P + 5)$	−0.8333	−8.333	5	34.72
EF	$-(0.8333P + 5)$	−0.8333	−8.333	5	34.72
BH	$0.5P + 3$	0.5	5	3	7.50
DF	$0.5P + 3$	0.5	5	3	7.50
BG	$-0.8333P$	−0.8333	−3.333	5	13.89
DG	$-0.8333P$	−0.8333	−3.333	5	13.89
GH	$-(0.6667P + 4)$	−0.6667	−6.667	4	17.78
FG	$-(0.6667P + 4)$	−0.6667	−6.667	4	17.78
CG	P	1	4	3	12.00
				Σ	294.89

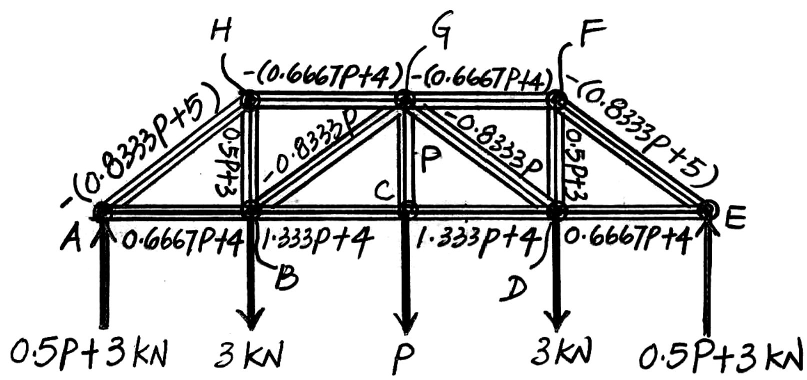
$$\Delta_{C_v} = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{294.89\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{294.89(10^3)\text{ N}\cdot\text{m}}{[0.3(10^{-3})\text{ m}^2][200(10^9)\text{ N/m}^2]}$$

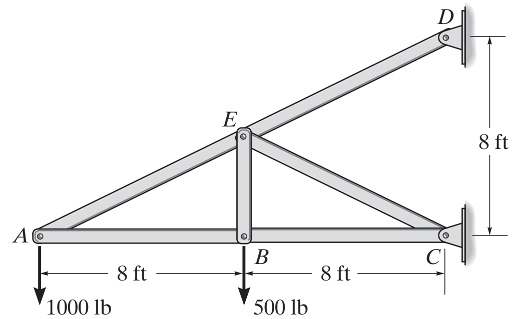
$$= 0.004914\text{ m}$$

$$= 4.91\text{ mm} \quad \downarrow$$

Ans.



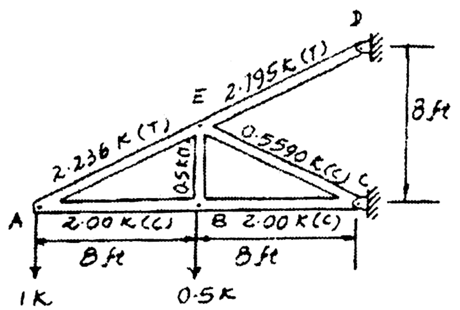
9–17. Determine the vertical displacement of joint A. Assume the members are pin connected at their end points. Take $A = 2\text{ in}^2$ and $E = 29\,(10^3)$ for each member. Use the method of virtual work.

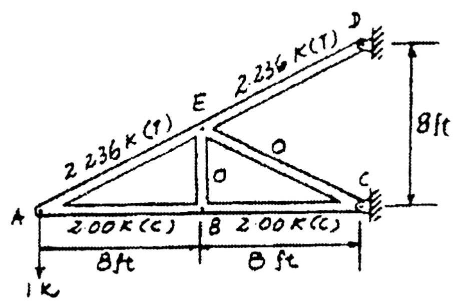


$$\Delta_{A_v} = \sum \frac{nNL}{AE} = \frac{1}{AE}[2(-2.00)(-2.00)(8) + (2.236)(2.236)(8.944) + (2.236)(2.795)(8.944)]$$

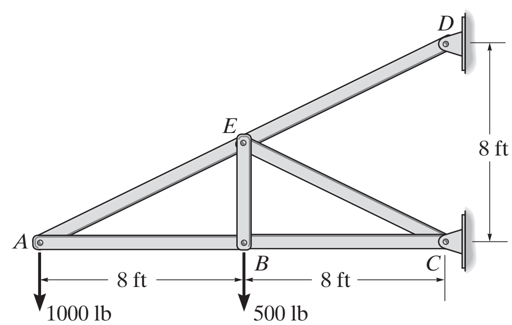
$$= \frac{164.62(12)}{(2)(29)(10^3)} = 0.0341\text{ in.}\ \downarrow$$

Ans.





9–18. Solve Prob. 9–17 using Castigliano's theorem.



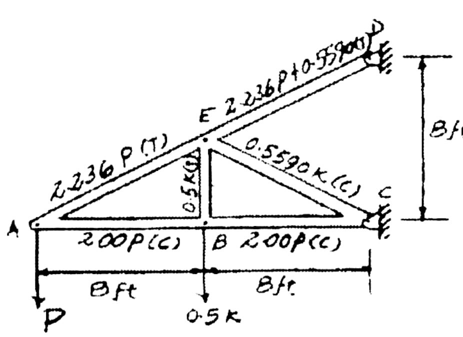
$$\Delta_{A_v} = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{1}{AE}[-2P(-2)(8) + (2.236P)(2.236)(8.944)$$

$$+ (-2P)(-2)(8) + (2.236P + 0.5590)(2.236)(8.944)](12)$$

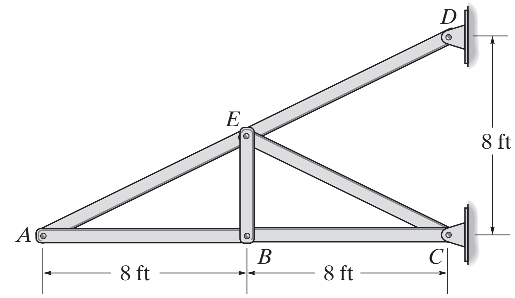
Set $P = 1$ and evaluate

$$\Delta_{A_v} = \frac{164.62(12)}{(2)(29)(10^3)} = 0.0341\text{ in.}\ \downarrow$$

Ans.

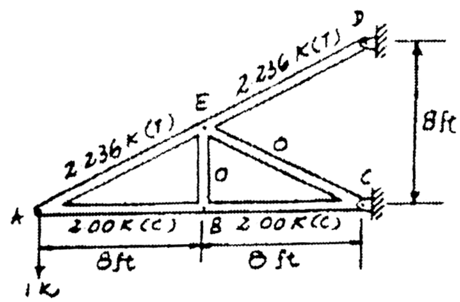


9–19. Determine the vertical displacement of joint A if members AB and BC experience a temperature increase of $\Delta T = 200°\text{F}$. Take $A = 2\text{ in}^2$ and $E = 29(10^3)$ ksi. Also, $\alpha = 6.60\,(10^{-6})/°\text{F}$.

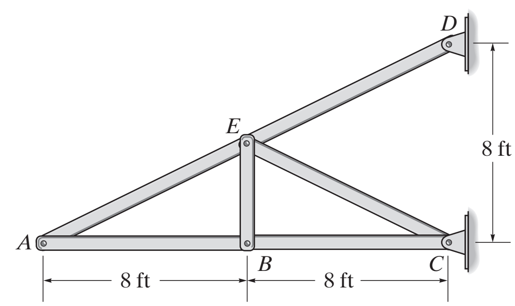


$$\Delta_{A_v} = \sum n\alpha\Delta TL = (-2)(6.60)(10^{-6})(200)(8)(12) + (-2)(6.60)(10^{-6})(200)(8)(12)$$

$$= -0.507\text{ in.} = 0.507\text{ in.}\uparrow \qquad \textbf{Ans.}$$

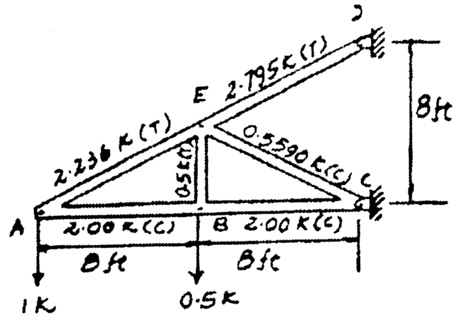


***9–20.** Determine the vertical displacement of joint A if member AE is fabricated 0.5 in. too short.

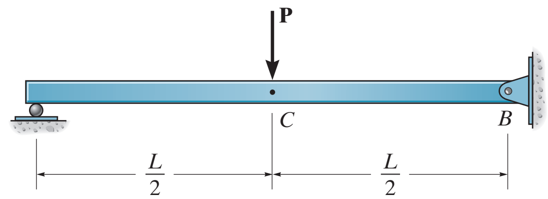


$$\Delta_{A_v} = \sum n\Delta L = (2.236)(-0.5)$$

$$= -1.12\text{ in} = 1.12\text{ in.}\uparrow \qquad \textbf{Ans.}$$



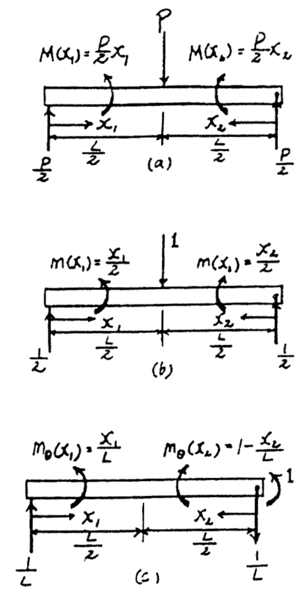
9–21. Determine the displacement of point C and the slope at point B. EI is constant. Use the principle of virtual work.



Real Moment function M(x): As shown on figure (a).

Virtual Moment Functions m(x) and $\mathbf{m}_\theta$(x): As shown on figure (b) and (c).

Virtual Work Equation: For the displacement at point C,



$$1 \cdot \Delta = \int_0^L \frac{mM}{EI} dx$$

$$1 \cdot \Delta_C = 2\left[\frac{1}{EI}\int_0^{\frac{L}{1}} \left(\frac{x_1}{2}\right)\left(\frac{P}{2}x_1\right)dx_1\right]$$

$$\Delta_C = \frac{PL^3}{48\,EI} \quad \downarrow$$ **Ans.**

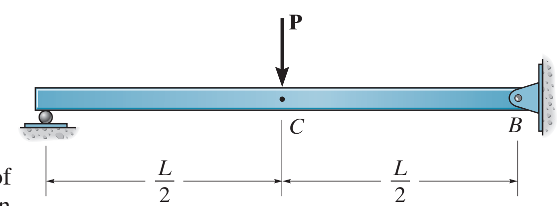
For the slope at B,

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI} dx$$

$$1 \cdot \theta_B = \frac{1}{EI}\left[\int_0^{\frac{L}{1}} \left(\frac{x_1}{L}\right)\left(\frac{P}{2}x_1\right)dx_1 + \int_0^{\frac{L}{2}} \left(1 - \frac{x_2}{L}\right)\left(\frac{P}{2}x_2\right)dx_2\right]$$

$$\theta_B = \frac{PL^2}{16EI} \quad \measuredangle$$ **Ans.**

9–22. Solve Prob. 9–21 using Castigliano's theorem.



Internal Moment Function M(x): The internal moment function in terms of the load **P**' and couple moment **M**' and externally applied load are shown on figures (a) and (b), respectively.

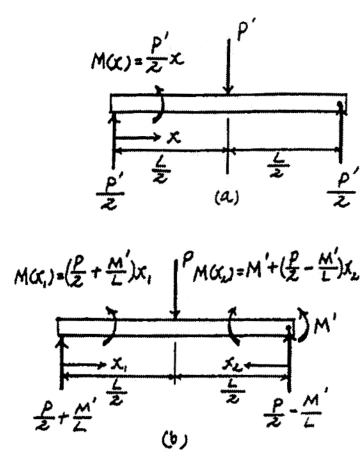
Castigliano's Second Theorem: The displacement at C can be determined with $\frac{\partial M(x)}{\partial P'} = = \frac{x}{2}$ and set $P' = P$.

$$\Delta = \int_0^L M\left(\frac{\partial M}{\partial P'}\right)\frac{dx}{EI}$$

$$\Delta_C = 2\left[\frac{1}{EI}\int_0^{\frac{L}{1}} \left(\frac{P}{2}x\right)\left(\frac{x}{2}\right)dx\right]$$

$$= \frac{PL^3}{48EI} \quad \downarrow$$ **Ans.**

To determine the slope at B, with $\frac{\partial M(x_1)}{\partial M'} = \frac{x_1}{L}, \frac{\partial M(x_2)}{\partial M'} = 1 - \frac{x_2}{L}$ and setting $M' = 0$.

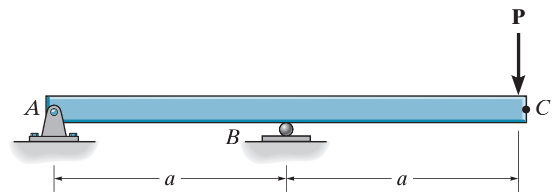


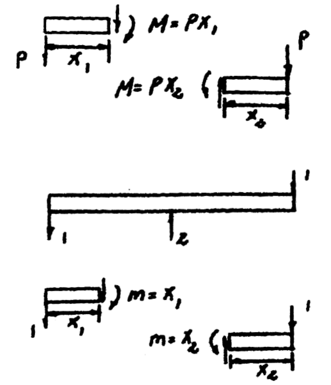
$$\theta = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$\theta_B = \frac{1}{EI}\int_0^{\frac{L}{1}} \left(\frac{P}{2}x_1\right)\left(\frac{x_1}{L}\right)dx_1 + \frac{1}{EI}\int_0^{\frac{L}{2}} \left(\frac{P}{2}x_2\right)\left(1 - \frac{x_2}{L}\right)dx_2$$

$$= \frac{PL^2}{16EI} \quad \measuredangle$$ **Ans.**

9–23. Determine the displacement at point C. EI is constant. Use the method of virtual work.





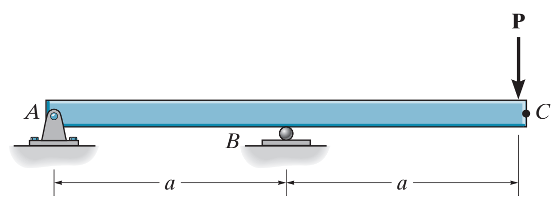
$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_C = \frac{1}{EI}\left[\int_o^a (x_1)(Px_1)dx_1 + \int_0^a (x_2)(Px_2)dx_2\right]$$

$$= \frac{2Pa^3}{3EI} \downarrow$$

Ans.

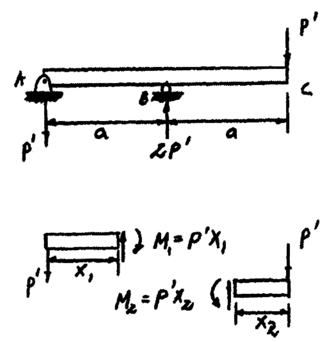
***9–24.** Solve Prob. 9–23 using Castigliano's theorem.



$$\frac{\partial M_1}{\partial P'} = x_1 \qquad \frac{\partial M_2}{\partial P'} = x_2$$

Set $P = P'$

$$M_1 = Px_1 \qquad M_2 = Px_2$$

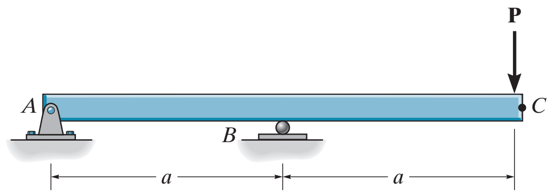


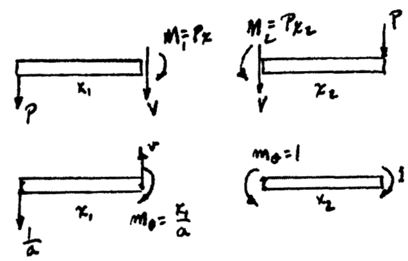
$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P'}\right)dx = \frac{1}{EI}\left[\int_0^a (Px_1)(x_1)dx_1 + \int_0^a (Px_2)(x_2)dx_2\right]$$

$$= \frac{2Pa^3}{3EI}$$

Ans.

9–25. Determine the slope at point C. EI is constant. Use the method of virtual work.





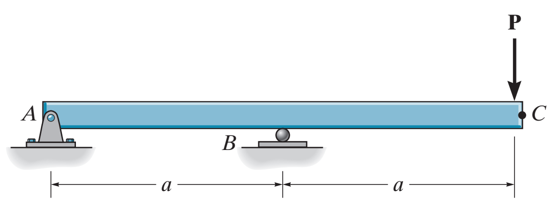
$$1 \cdot \theta_C = \int_0^L \frac{m_\theta M dx}{EI}$$

$$\theta_C = \int_0^a \frac{(x_1/a)Px_1 dx_1}{EI} + \int_0^a \frac{(1)Px_2 dx_2}{EI}$$

$$= \frac{Pa^2}{3EI} + \frac{Pa^2}{2EI} = \frac{5Pa^2}{6EI}$$

Ans.

9–26. Solve Prob. 9–25 using Castigliano's theorem.



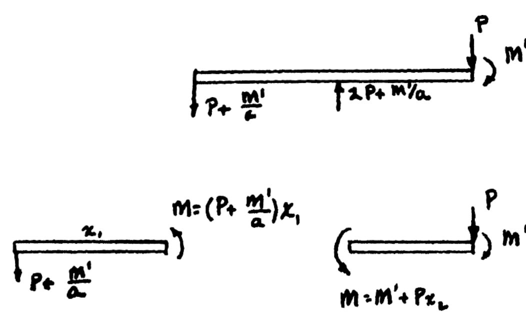
Set $M' = 0$

$$\theta_C = \int_0^L M\left(\frac{\delta M}{\delta M'}\right)\frac{dx}{EI}$$

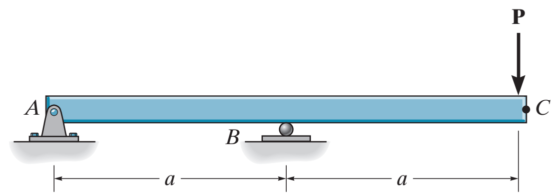
$$= \int_0^a \frac{(Px_1)(\frac{1}{a}x_1)dx_1}{EI} + \int_0^a \frac{(Px_2)(1)dx_2}{EI}$$

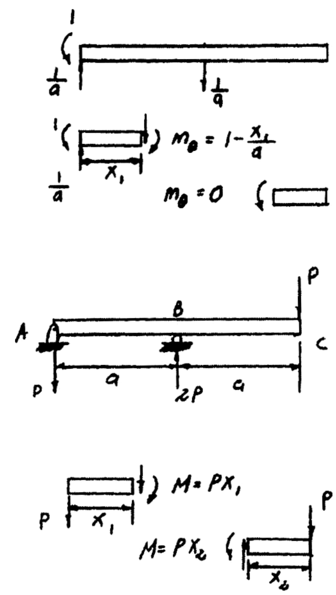
$$= \frac{Pa^2}{3EI} + \frac{Pa^2}{2EI} = \frac{5Pa^2}{6EI}$$

Ans.



9–27. Determine the slope at point A. EI is constant. Use the method of virtual work.



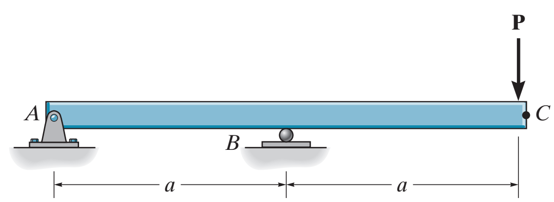


$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_A = \frac{1}{EI}\left[\int_0^a \left(1 - \frac{x_1}{a}\right)(Px_1)dx_1 + \int_0^a (0)(Px_2)dx_2\right] = \frac{Pa^2}{6EI} \quad \triangledown$$

Ans.

***9–28.** Solve Prob. 9–27 using Castigliano's theorem.



$$\frac{\partial M_1}{\partial M'} = 1 - \frac{x_1}{a} \qquad \frac{\partial M_2}{\partial M'} = 0$$

Set $M' = 0$

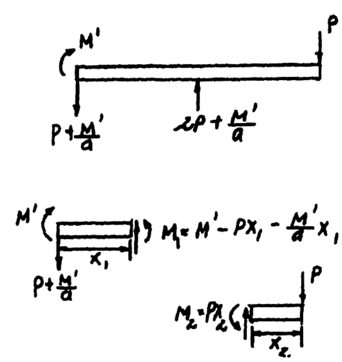
$$M_1 = -Px_1 \qquad M_2 = Px_2$$

$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \frac{1}{EI}\left[\int_0^a (-Px_1)\left(1 - \frac{x_1}{a}\right)dx_1 + \int_0^a (Px_2)(0)dx_2\right]$$

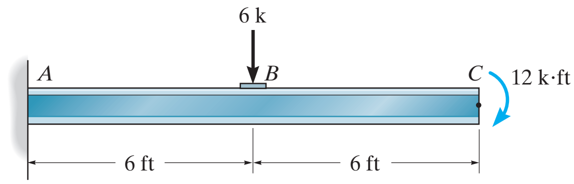
$$= \frac{-Pa^2}{6EI}$$

$$= \frac{Pa^2}{6EI}$$

Ans.



9–29. Determine the slope and displacement at point C. Use the method of virtual work. $E = 29(10^3)$ ksi, $I = 800\ \text{in}^4$.



Referring to the virtual moment functions indicated in Fig. *a* and *b* and the real moment function in Fig. *c*, we have

$$1\text{k}\cdot\text{ft}\cdot\theta_c = \int_0^L \frac{m_\theta M}{EI}dx = \int_0^{6\text{ ft}} \frac{(-1)(-12)}{EI}dx_1 + \int_0^{6\text{ ft}} \frac{(-1)[-(6x_2+12)]}{EI}dx_2$$

$$= \frac{252\ \text{k}^2\cdot\text{ft}^3}{EI}$$

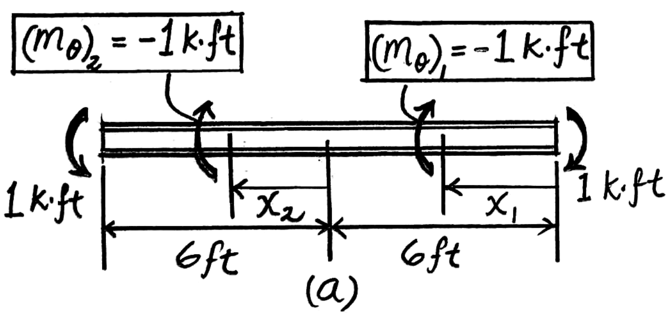
$$\theta_c = \frac{252\ \text{k}\cdot\text{ft}^2}{EI} = \frac{252(12^2)\ \text{k}\cdot\text{in}^2}{[29(10^3)\ \text{k/in}^2](800\ \text{in}^4)} = 0.00156\ \text{rad}$$ **Ans.**

and

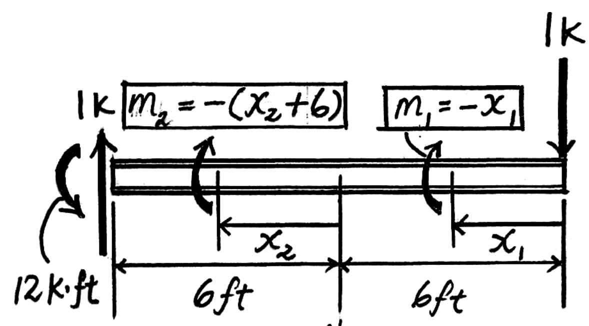
$$1\text{k}\cdot\Delta_C = \int_0^L \frac{mM}{EI}dx = \int_0^{6\text{ ft}} \frac{(-x_1)(-12)}{EI}dx_1 + \int_0^{6\text{ ft}} \frac{([-(x_2+6)][-(6x_2+12)]}{EI}dx_2$$

$$= \frac{1944\ \text{k}^2\cdot\text{ft}^3}{EI}$$

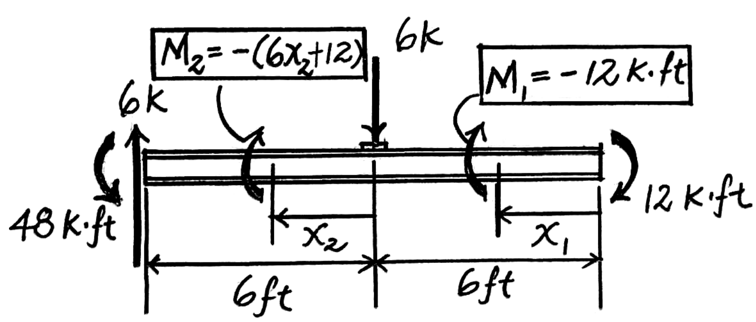
$$\Delta_C = \frac{1944\ \text{k}\cdot\text{ft}^3}{EI} = \frac{1944(12^3)\ \text{k}\cdot\text{in}^3}{[29(10^3)\ \text{k/in}^2](800\ \text{in}^4)} = 0.415\ \text{in}\ \downarrow$$ **Ans.**



(a)

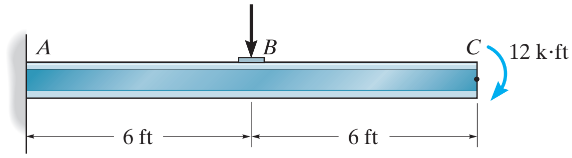


(b)



(c)

9–30. Solve Prob. 9–29 using Castigliano's theorem.



For the slope, the moment functions are shown in Fig. *a*. Here, $\frac{\partial M_1}{\partial M'} = -1$ and $\frac{\partial M_2}{\partial M'} = -1$. Also, set $M' = 12$ kft, then $M_1 = -12$ k· ft and $M_2 = -(6x_2 + 12)$ k·ft. Thus,

$$\theta_c = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^{6\text{ ft}} \frac{(-12)(-1)}{EI}dx_2 + \int_0^{6\text{ ft}} \frac{-(6x_2 + 12(-1)}{EI}dx_2$$

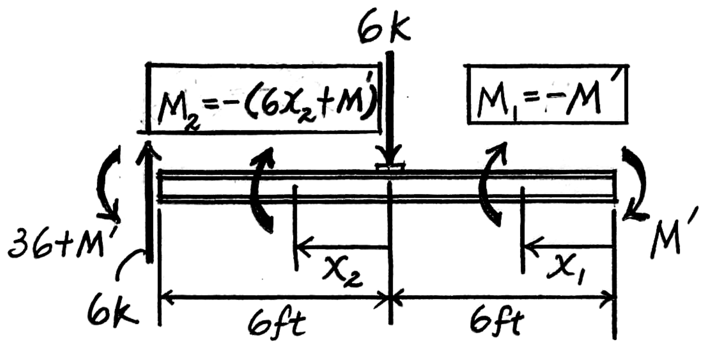
$$\theta_c = \frac{252\text{ k}\cdot\text{ft}^2}{EI} = \frac{252(12^2)\text{ k}\cdot\text{in}^2}{[29(10^3)\text{ k/in}^2](800\text{ in}^4)} = 0.00156\text{ rad}$$ **Ans.**

For the displacement, the moment functions are shown in Fig. *b*. Here, $\frac{\partial M_1}{\partial P} = -x_1$ and $\frac{\partial M_2}{\partial P} = -(x_2 + 6)$. Also set, $P = 0$, then $M_1 = -12$ k·ft and $M_2 = -(6x_2 + 12)$ k·ft. Thus,

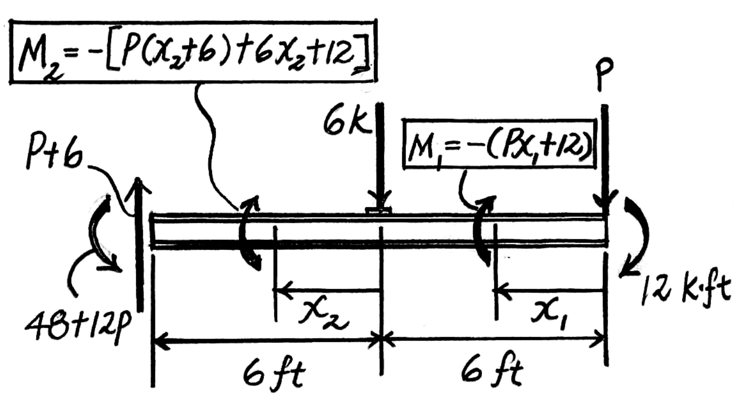
$$\Delta_C = \int_0^L \left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{6\text{ ft}} \frac{(-12)(-x_1)}{EI}dx_1 + \int_0^{6\text{ ft}} \frac{-(6x_2 + 12)[-(x_2 + 6)]}{EI}dx_2$$

$$= \frac{1944\text{ k}\cdot\text{ft}^3}{EI}$$

$$= \frac{1944(12^3)\text{ k}\cdot\text{in}^3}{[29(10^3)\text{ k/in}^2](800\text{ in}^4)} = 0.145\text{ in} \downarrow$$ **Ans.**

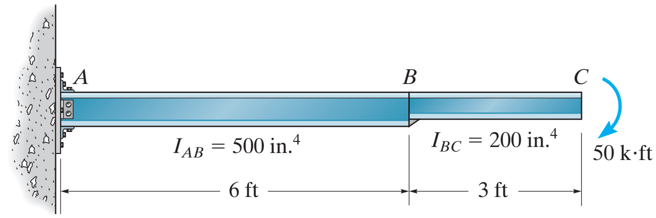


(a)



(b)

9–31. Determine the displacement and slope at point C of the cantilever beam. The moment of inertia of each segment is indicated in the figure. Take $E = 29(10^3)$ ksi. Use the principle of virtual work.



Referring to the virtual moment functions indicated in Fig. *a* and *b* and the real moment function in Fig. *c*, we have

$$1\text{k}\cdot\text{ft}\cdot\theta_c = \int_0^L \frac{m_0 M}{EI}dx = \int_0^{3\text{ ft}} \frac{(-1)(-50)}{EI_{BC}}dx_1 + \int_0^{6\text{ ft}} \frac{(-1)(-50)}{EI_{AB}}dx_2$$

$$1\text{k}\cdot\text{ft}\cdot\theta_c = \frac{150\text{ k}^2\cdot\text{ft}^3}{EI_{BC}} + \frac{300\text{ k}^2\cdot\text{ft}^3}{EI_{AB}}$$

$$\theta_c = \frac{150\text{ k}\cdot\text{ft}^2}{EI_{BC}} + \frac{300\text{ k}\cdot\text{ft}^2}{EI_{AB}}$$

$$= \frac{150(12^2)\text{ k}\cdot\text{in}^2}{[29(10^3)\text{ k/in}^2](200\text{ in}^4)} + \frac{300(12^2)\text{ k}\cdot\text{in}^2}{[29(10^3)\text{ k/in}^2](500\text{ in}^4)}$$

$$= 0.00670\text{ rad}$$ **Ans.**

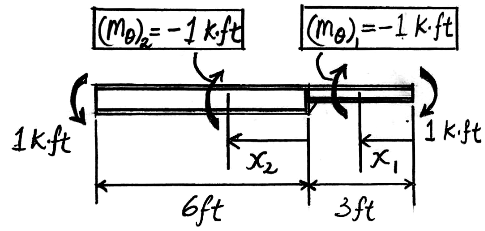
And

$$1\text{ k}\cdot\Delta_C = \int_0^L \frac{mM}{EI}dx = \int_0^{3\text{ ft}} \frac{-x_1(-50)}{EI_{BC}}dx_1 + \int_0^{6\text{ ft}} \frac{-(x_2+3)(-50)}{EI_{AB}}dx_2$$

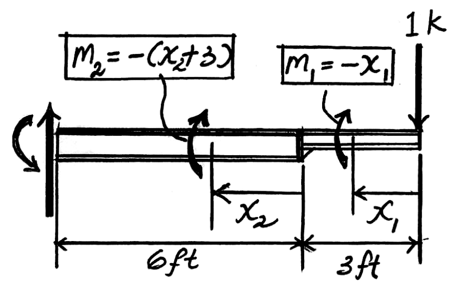
$$1\text{ k}\cdot\Delta_C = \frac{225\text{ k}^2\cdot\text{ft}^3}{EI_{BC}} + \frac{1800\text{ k}^2\cdot\text{ft}^3}{EI_{AB}}$$

$$\Delta_C = \frac{225\text{ k}\cdot\text{ft}^3}{EI_{BC}} + \frac{1800\text{ k}^2\cdot\text{ft}^3}{EI_{AB}}$$

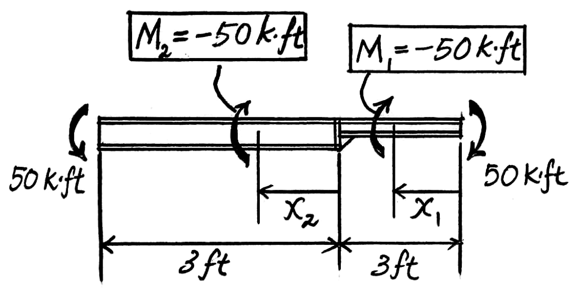
$$= \frac{225(12^3)\text{ k}\cdot\text{in}^3}{[29(10^3)\text{ k/in}^2](200\text{ in}^4)} + \frac{1800(12^3)\text{ k}\cdot\text{in}^3}{[29(10^3)\text{ k/in}^2](500\text{ in}^4)} = 0.282\text{ in}\downarrow$$ **Ans.**



(a)

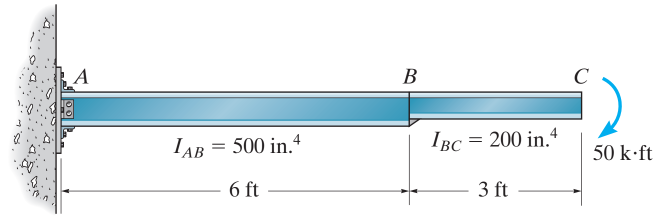


(b)



(c)

***9–32.** Solve Prob. 9–31 using Castigliano's theorem.



For the slope, the moment functions are shown in Fig. *a*. Here, $\dfrac{\partial M_1}{\partial M'} = \dfrac{\partial M_2}{\partial M'} = -1$.

Also, set $M' = 50$ k · ft, then $M_1 = M_2 = -50$ k · ft.

Thus,

$$\theta_C = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^{3\text{ ft}} \frac{-50(-1)dx}{EI_{BC}} + \int_0^{6\text{ ft}} \frac{-50(-1)dx}{EI_{AB}}$$

$$= \frac{150\text{ k}\cdot\text{ft}^2}{EI_{BC}} + \frac{300\text{ k}\cdot\text{ft}^2}{EI_{AB}}$$

$$= \frac{150(12^2)\text{ k}\cdot\text{in}^2}{[29(10^3\text{ k/in}^2)](200\text{ in}^4)} + \frac{300(12^2)\text{ k}\cdot\text{in}^2}{[29(10^3)\text{ k/in}^2](500\text{ in}^4)}$$

$$= 0.00670 \;\text{rad}$$ **Ans.**

For the displacement, the moment functions are shown in Fig, *b*. Here, $\dfrac{\partial M_1}{\partial P} = -x_1$

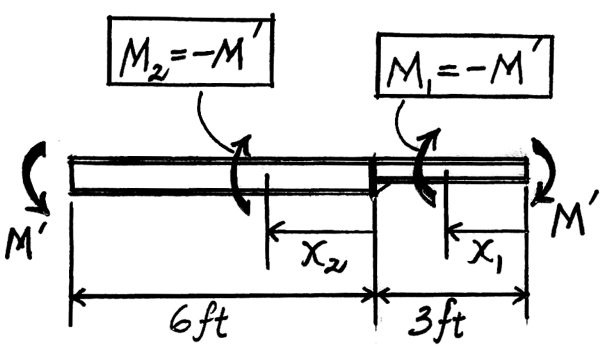
and $\dfrac{\partial M_2}{\partial P} = -(x_2 + 3)$. Also, set $P = 0$, then $M_1 = M_2 = -50$ k · ft. Thus,

$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{3\text{ ft}} \frac{(-50)(-x)dx}{EI_{BC}} + \int_0^{6\text{ ft}} \frac{(-50)[-(x_2+3)]dx}{EI_{AB}}$$

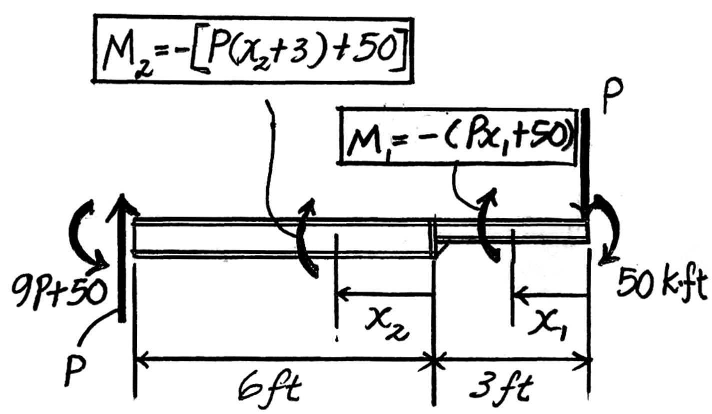
$$= \frac{225\text{ k}\cdot\text{ft}^3}{EI_{BC}} + \frac{1800\text{ k}\cdot\text{ft}^3}{EI_{AB}}$$

$$= \frac{225(12^3)\text{ k}\cdot\text{in}^3}{[29(10^3)\text{ k/in}^2](200\text{ in}^4)} + \frac{1800(12^3)\text{ k}\cdot\text{in}^3}{[29(10^3)\text{ k/in}^2](500\text{ in}^4)}$$

$$= 0.282\text{ in}\downarrow$$ **Ans.**

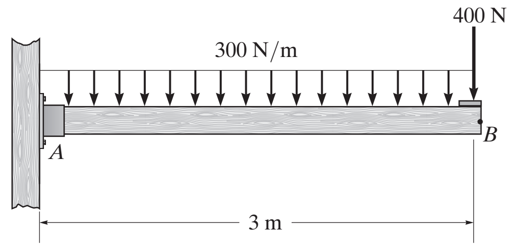


(a)



(b)

9–33. Determine the slope and displacement at point B. EI is constant. Use the method of virtual work.



Referring to the virtual moment function indicated in Fig. a and b, and real moment function in Fig. c, we have

$$1\ \text{N}\cdot\text{m}\cdot\theta_B = \int_0^L \frac{m_0 M}{EI}dx = \int_0^{3\text{ m}} \frac{(-1)[-(150x^2 + 400x)]}{EI}dx$$

$$1\ \text{N}\cdot\text{m}\cdot\theta_B = \frac{3150\ \text{N}^2\cdot\text{m}^3}{EI}$$

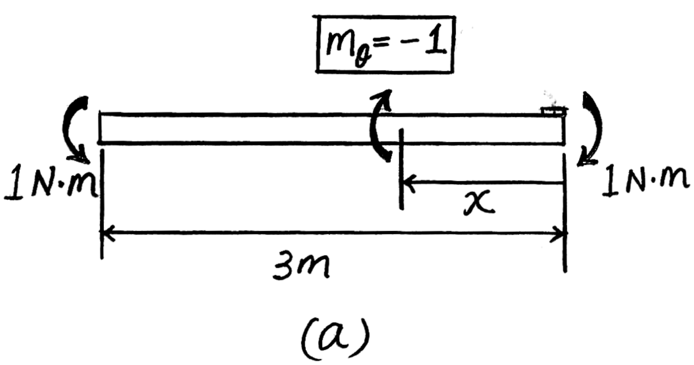
$$\theta_B = \frac{3150\ N\cdot\text{m}^2}{EI}$$ **Ans.**

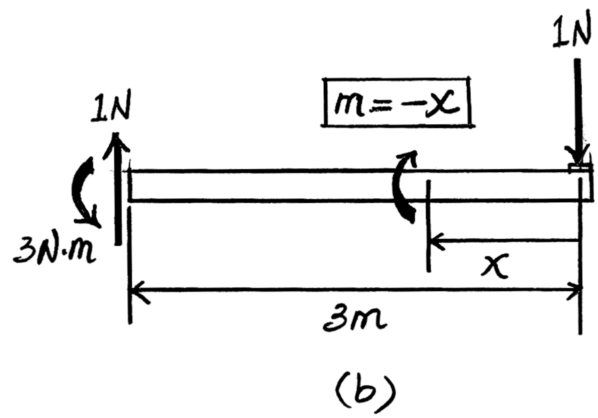
And

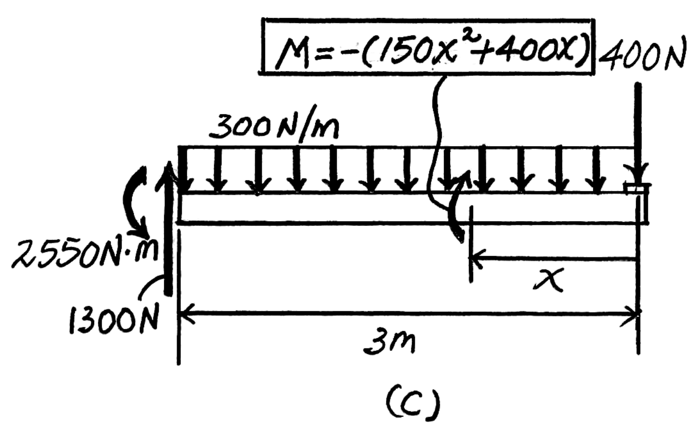
$$1\ \text{N}\cdot\Delta_B = \int_0^L \frac{mM}{EI}dx = \int_0^{3\text{ m}} \frac{(-x)[-(150x^2 + 400x)]}{EI}$$

$$1\ \text{N}\cdot\Delta_B = \frac{6637.5\ \text{N}^2\cdot\text{m}^3}{EI}$$

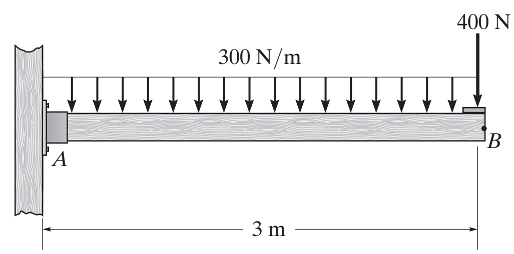
$$\Delta_B = \frac{6637.5\ \text{N}\cdot\text{m}^3}{EI} \downarrow$$ **Ans.**







9–34. Solve Prob. 9–33 using Castigliano's theorem.



For the slope, the moment function is shown in Fig. *a*. Here, $\frac{\partial M}{\partial M'} = -1$.
Also, set $M' = 0$, then $M = -(150x^2 + 400x)\ \text{N}\cdot\text{m}$. Thus,

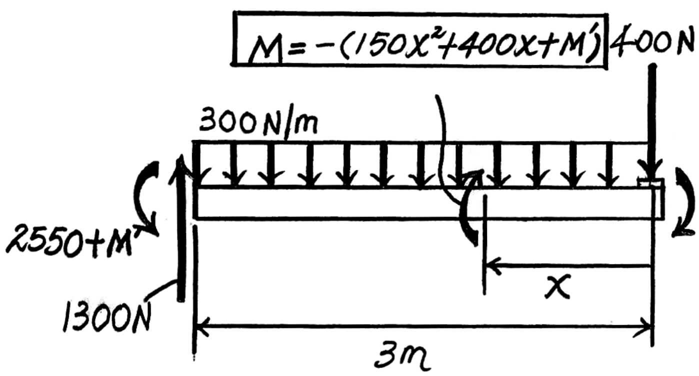
$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^{3\text{ m}} \frac{-(150x^2 + 400x)(-1)}{EI}dx$$

$$= \frac{3150\ \text{N}\cdot\text{m}^2}{EI} \quad \text{Ans.}$$

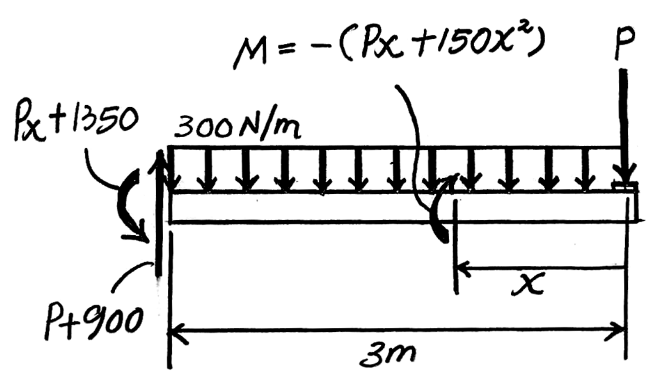
For the displacement, the moment function is shown in Fig. *b*. Here, $\frac{\partial M}{\partial P} = -x$.
Also, set $P = 400$ N, then $M = (400x + 150x^2)\ \text{N}\cdot\text{m}$. Thus,

$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{3\text{ m}} \frac{-(400x + 150x^2)(-x)}{EI}dx$$

$$= \frac{6637.5\ \text{N}\cdot\text{m}^3}{EI}\downarrow \quad \text{Ans.}$$

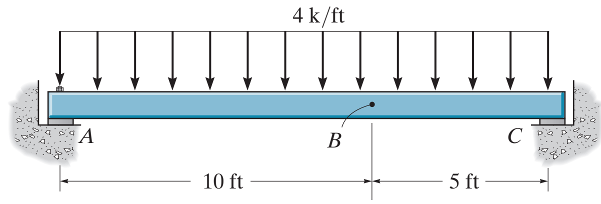


(a)



(b)

9–35. Determine the slope and displacement at point B. Assume the support at A is a pin and C is a roller. Take $E = 29(10^3)$ ksi, $I = 300$ in^4. Use the method of virtual work.

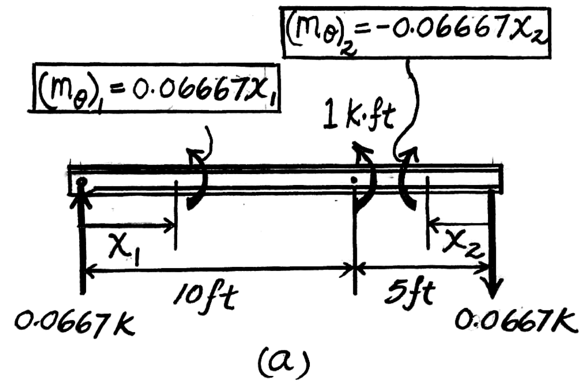


Referring to the virtual moment functions shown in Fig. a and b and the real moment function shown in Fig. c,

$$1\text{ k}\cdot\text{ft}\cdot\theta_B = \int_0^L \frac{m_\theta M}{EI}dx = \int_0^{10\text{ ft}} \frac{(0.06667x_1)(30x_1 - 2x_1^2)dx_1}{EI} + \int_0^{5\text{ ft}} \frac{(-0.06667x_2)(30x_2 - 2x_2^2)}{EI}dx_2$$

$$1\text{ k}\cdot\text{ft}\cdot\theta_B = \frac{270.83\text{ k}^2\cdot\text{ft}^3}{EI}$$

$$\theta_B = \frac{270.83\text{ k}\cdot\text{ft}^2}{EI} = \frac{270.83(12^2)\text{ k}\cdot\text{in}^2}{[29(10^3)\text{ k/in}^2](300\text{ in}^4)} = 0.00448\text{ rad} \quad \measuredangle \quad \textbf{Ans.}$$

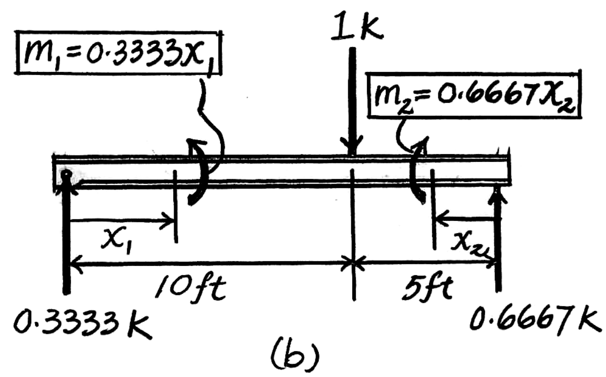


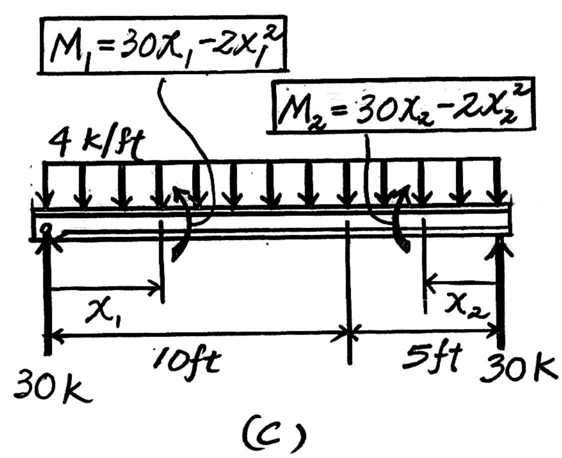
And

$$1\text{ k}\cdot\Delta_B = \int_0^L \frac{mM}{EI}dx = \int_0^{10\text{ ft}} \frac{(0.3333x_1)(30x_1 - 2x_1^2)dx_1}{EI} + \int_0^{5\text{ ft}} \frac{(0.6667x_2)(30x_2 - 2x_2^2)dx_2}{EI}$$

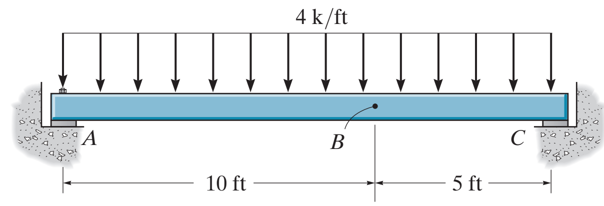
$$1\text{ k}\cdot\Delta_B = \frac{2291.67\text{ k}\cdot\text{ft}^3}{EI}$$

$$\Delta_B = \frac{2291.67\text{ k}\cdot\text{ft}^3}{EI} = \frac{2291.67(12^3)\text{ k}\cdot\text{in}^3}{[29(10^3)\text{ k/in}^2](300\text{ in}^4)} = 0.455\text{ in} \downarrow \quad \textbf{Ans.}$$





***9–36.** Solve Prob. 9–35 using Castigliano's theorem.



For the slope, the moment functions are shown in Fig. *a*. Here,

$\frac{\partial M_1}{\partial M'} = 0.06667x_1$ and $\frac{\partial M_2}{\partial M'} = 0.06667x_2$. Also, set $M' = 0$, then

$M_1 = (30x_1 - 2x_1^2)$ k · ft and $M_2 = (30x_2 - 2x_2^2)$ k · ft. Thus,

$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^{10\text{ ft}} \frac{(30x_1 - 2x_1^2)(0.06667x_1)dx_1}{EI} + \int_0^{5\text{ ft}} \frac{(30x_2 - 2x_2^2)(0.06667x_2)dx_2}{EI}$$

$$= \frac{270.83\text{ k}\cdot\text{ft}^2}{EI} = \frac{270.83(12^2)\text{ k}\cdot\text{in}^2}{[29(10^3)\text{k/in}^2](300\text{ in}^4)} = 0.00448\text{ rad}$$ **Ans.**

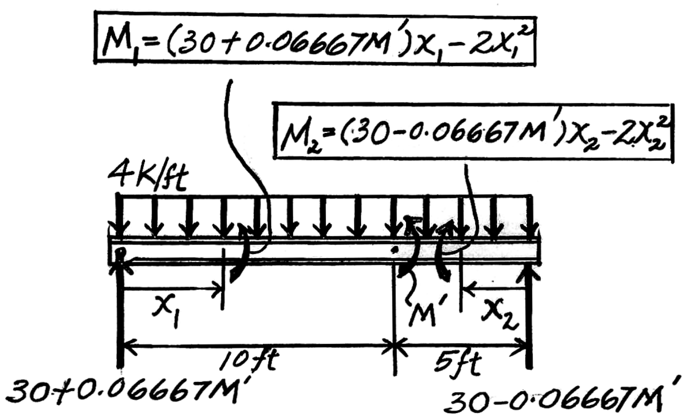
For the displacement, the moment fractions are shown in Fig. *b*. Here,

$\frac{\partial M_1}{\partial p} = 0.3333x_1$ and $\frac{\partial M_2}{\partial P} = 0.6667x_2$. Also, set $P = 0$, then

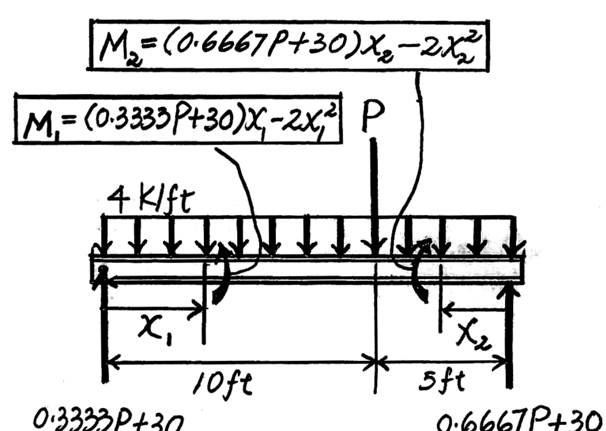
$M_1 = (30x_1 - 2x_1^2)$ k · ft and $M_2 = (30x_2 - 2x_2^2)$ k · ft. Thus

$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{10\text{ ft}} \frac{30x_1 - 2x_1^2(0.3333x_1)dx_1}{EI} + \int_0^{5\text{ ft}} \frac{(30x_2 - 2x_2^2)(0.6667x_2)dx_2}{EI}$$

$$= \frac{2291.67\text{ k}\cdot\text{ft}^3}{EI} = \frac{2291.67(12^3)\text{ k}\cdot\text{in}^3}{[29(10^3)\text{k/in}^2](300\text{ in}^4)} = 0.455\text{ in}\downarrow$$ **Ans.**

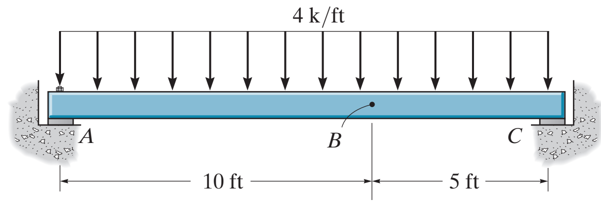


(a)



(b)

9–37. Determine the slope and displacement at point B. Assume the support at A is a pin and C is a roller. Account for the additional strain energy due to shear. Take $E = 29(10^3)$ ksi, $I = 300\text{ in}^4$, $G = 12(10^3)$ ksi, and assume AB has a cross-sectional area of $A = 7.50\text{ in}^2$. Use the method of virtual work.



The virtual shear and moment functions are shown in Fig. *a* and *b* and the real shear and moment functions are shown in Fig. *c*.

$$1\text{ k}\cdot\text{ft}\cdot\theta_B = \int_0^L \frac{m_\theta M}{EI}dx + \int_0^L \text{k}\left(\frac{\nu V}{GA}\right)dx$$

$$= \int_0^{10\text{ ft}} \frac{0.06667x_1(30x_1 - 2x_1^2)}{EI}dx_1 + \int_0^{10\text{ ft}} 1\left[\frac{0.06667(30 - 4x_1)}{GA}\right]dx_1$$

$$+ \int_0^{5\text{ ft}} \frac{(-0.06667x_2(30x_2 - 2x_2^2)}{EI}dx_2 + \int_0^{5\text{ ft}} 1\left[\frac{0.06667(4x_2 - 30)}{GA}\right]dx_2$$

$$= \frac{270.83\text{ k}^2\cdot\text{ft}^3}{EI} + 0$$

$$\theta_B = \frac{270.83\text{ k}\cdot\text{ft}^2}{EI} = \frac{270.83(12^2)\text{ k}\cdot\text{in}^2}{[29(10^3)\text{k/in}^2(300\text{ in}^4)]} = 0.00448\text{ rad}$$ **Ans.**

And

$$1\text{ k}\cdot\Delta_B = \int_0^L \frac{mM}{EI}dx + \int_0^L \text{k}\left(\frac{\nu V}{GA}\right)dx$$

$$= \int_0^{10\text{ ft}} \frac{(0.3333x_1)(30x_1 - 2x_1^2)}{EI}dx_1 + \int_0^{10\text{ ft}} 1\left[\frac{0.3333(30 - 4x_1)}{GA}\right]dx_1$$

$$+ \int_0^{5\text{ ft}} \frac{(0.6667x_2)(30x_2 - 2x_2^2)}{EI}dx_2 + \int_0^{5\text{ ft}} 1\left[\frac{(-0.6667)(4x_2 - 30)}{GA}\right]dx_2$$

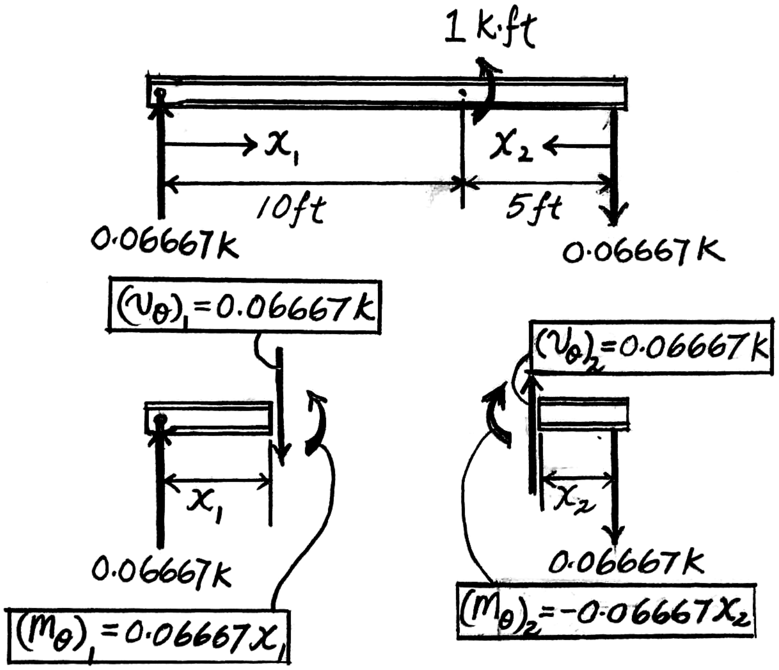
$$= \frac{2291.67\text{ k}^2\cdot\text{ft}^3}{EI} + \frac{100\text{ k}^2\cdot\text{ft}}{GA}$$

$$\Delta_B = \frac{2291.67\text{ k}\cdot\text{ft}^3}{EI} + \frac{100\text{ k}\cdot\text{ft}}{GA}$$

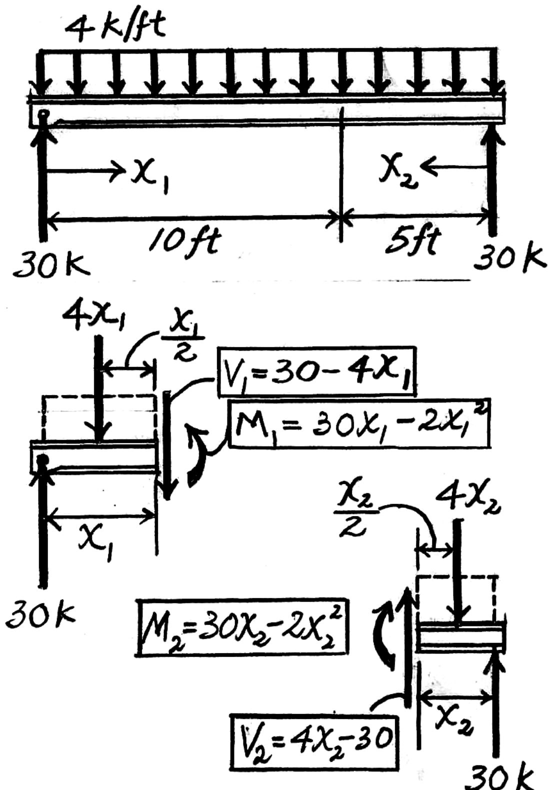
$$= \frac{2291.67(12^3)\text{k}\cdot\text{in}^3}{[29(10^3)\text{ k/in}^2](300\text{ in}^4)} + \frac{100(12)\text{ k}\cdot\text{in}}{[12(10^3)\text{ k/in}^2](7.50\text{ in}^2)}$$

$$= 0.469\text{ in}\downarrow$$ **Ans.**

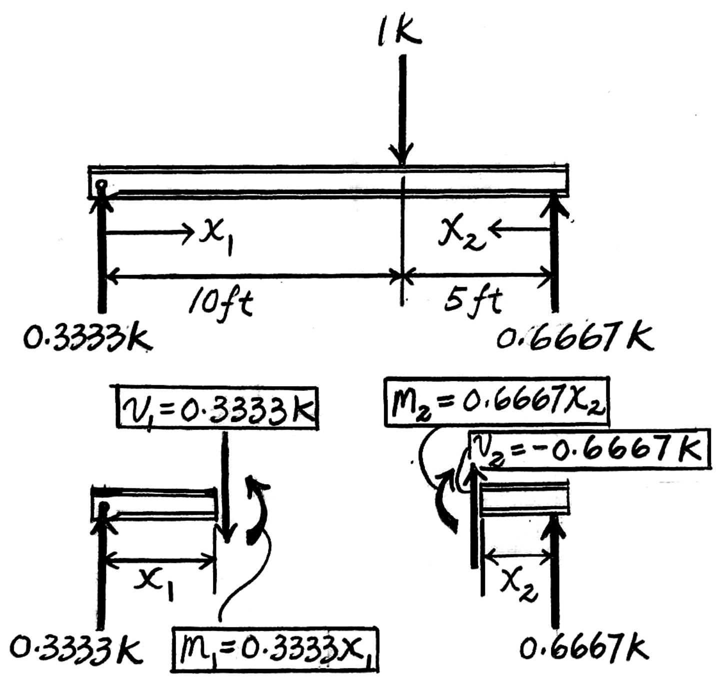
9–37. Continued



(a)

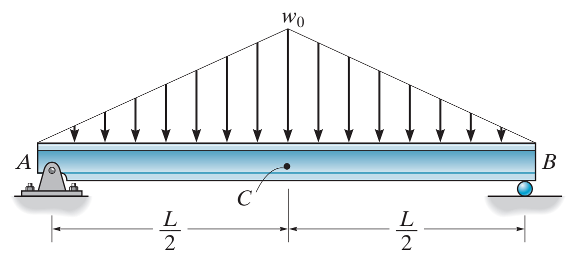


(c)



(b)

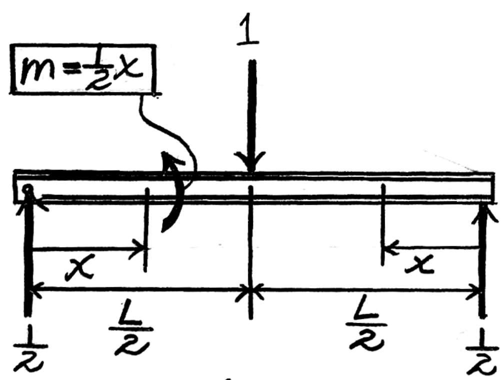
9–38. Determine the displacement of point C. Use the method of virtual work. EI is constant.



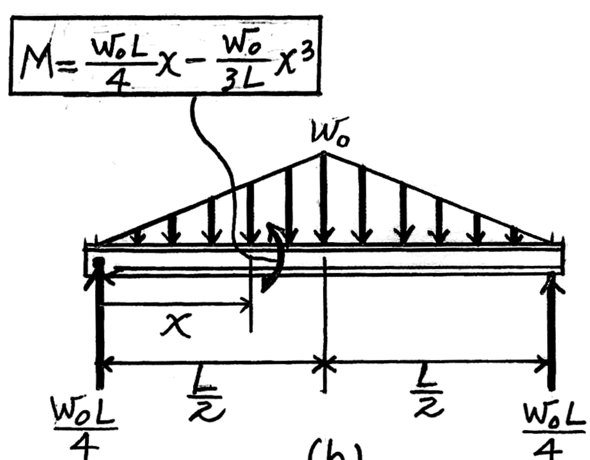
Referring to the virtual and real moment functions shown in Fig. *a* and *b*, respectively,

$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI} dx = 2\int_0^{\frac{L}{2}} \frac{\left(\frac{1}{2}\right)\left(\frac{w_0 L}{4}x - \frac{w_0}{3L}x^3\right)}{EI} dx$$

$$\Delta_C = \frac{w_0 L^4}{120 EI} \downarrow$$ **Ans.**

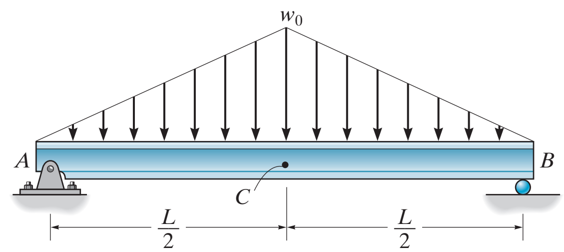


(a)



(b)

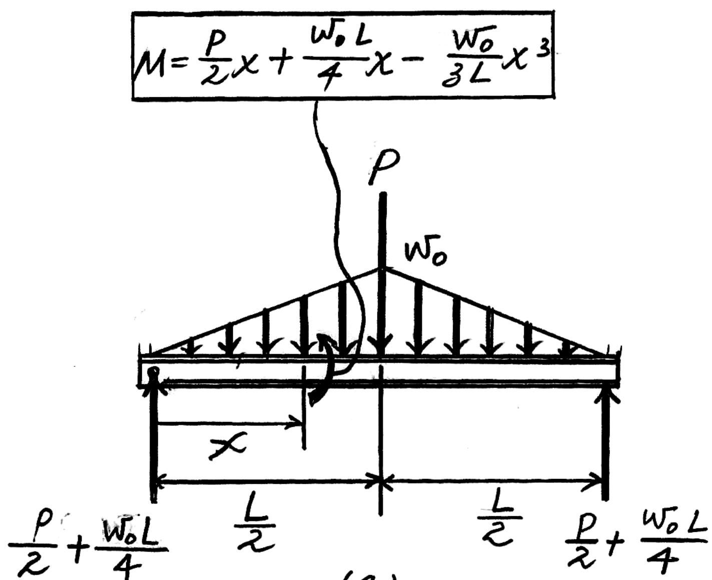
9–39. Solve Prob. 9–38 using Castigliano's theorem.



The moment function is shown in Fig. *a*. Here $\frac{\partial M}{\partial P} = \frac{1}{2}x$. Also, set $P = 0$, then $M = \frac{w_0 L}{4}x - \frac{w_0}{3L}x^3$. Thus

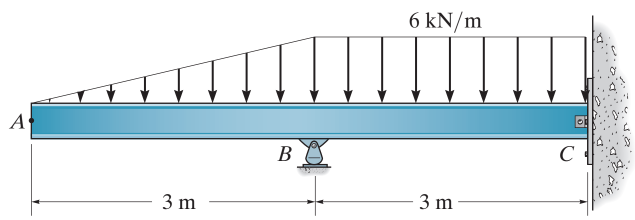
$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = 2\int_0^{\frac{L}{2}} \frac{\left(\frac{w_0 L}{4}x - \frac{w_0}{3L}x^3\right)\left(\frac{1}{2}x\right)}{EI}\,dx$$

$$= \frac{w_0 L^4}{120EI} \downarrow$$ **Ans.**



(a)

***9–40.** Determine the slope and displacement at point *A*. Assume *C* is pinned. Use the principle of virtual work. *EI* is constant.



Referring to the virtual moment functions shown in Fig. *a* and *b* and the real moment functions in Fig. *c*, we have

$$1\text{ kN}\cdot\text{m}\cdot\theta_A = \int_0^L \frac{m_\theta M}{EI}dx = \int_0^{3\text{ m}} \frac{(-1)(-0.3333x_1^3)}{EI}dx_1 + \int_0^{3\text{ m}} \frac{(0.3333x_2)(6x_2 - 3x_2^2)}{EI}dx_2$$

$$1\text{ kN}\cdot\text{m}\cdot\theta_A = \frac{9\text{ kN}^2\cdot\text{m}^3}{EI}$$

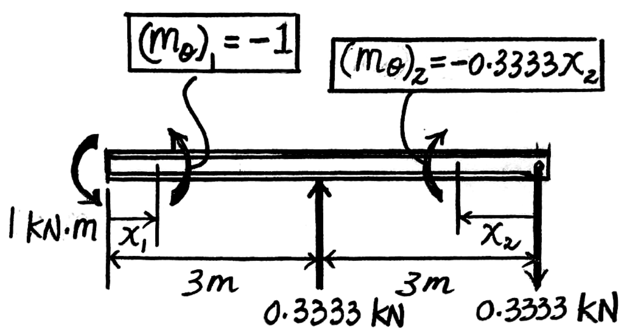
$$\theta_A = \frac{9\text{ kN}\cdot\text{m}^2}{EI}$$ **Ans.**

And

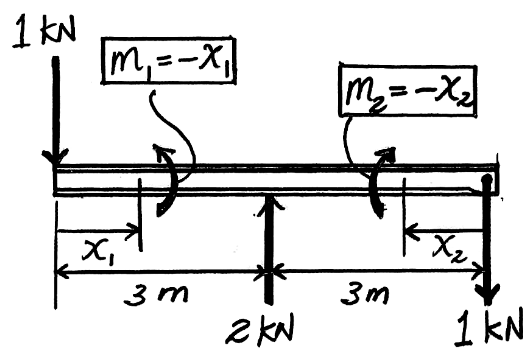
$$1\text{ kN}\cdot\Delta_A = \int_0^L \frac{mM}{EI}dx = \int_0^{3\text{ m}} \frac{(-x_1)(-0.3333x_1^3)}{EI}dx_1 + \int_0^{3\text{ m}} \frac{(-x_2)(6x_2 - 3x_2^2)}{EI}dx_2$$

$$1\text{ kN}\cdot\Delta_A = \frac{22.95\text{ kN}^2\cdot\text{m}^3}{EI}$$

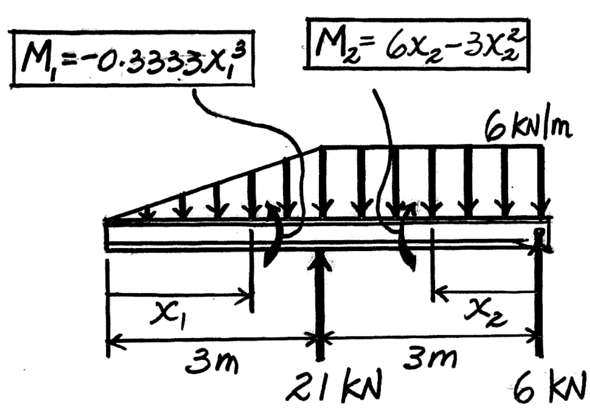
$$\Delta_A = \frac{22.95\text{ kN}\cdot\text{m}^3}{EI}\ \downarrow$$ **Ans.**



(a)

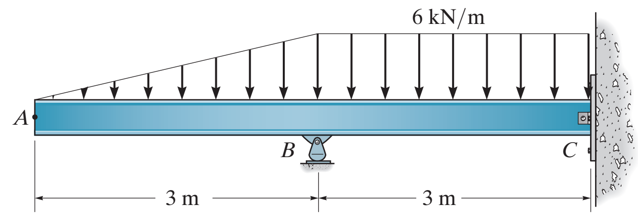


(b)



(c)

9–41. Solve Prob. 9–40 using Castigliano's theorem.



The slope, the moment functions are shown in Fig. *a*. Here, $\dfrac{\partial M_1}{\partial M'} = -1$ and $\dfrac{\partial M_2}{\partial M'} = -0.3333x_2$. Also, set $M' = 0$, then $M_1 = -0.3333x_1^3$ and $M_2 = 6x_2 - 3x_2^2$. Thus

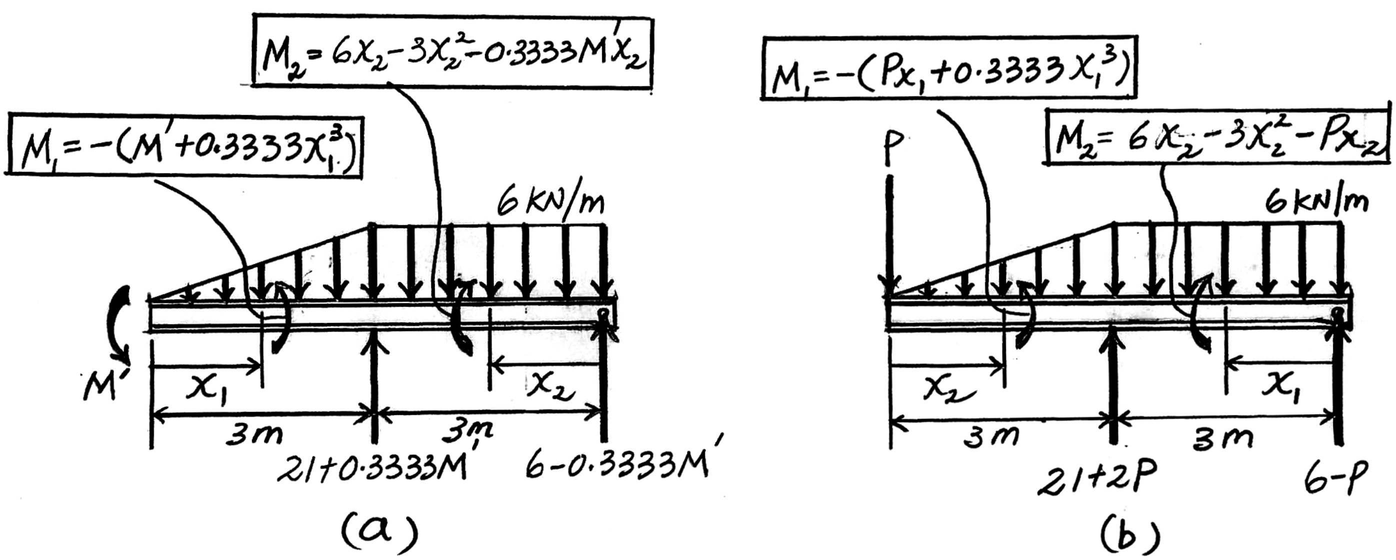
$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^{3\text{ m}} \frac{(-0.3333x_1^3)(-1)}{EI}dx_1 + \int_0^{3\text{ m}} \frac{(6x_2 - 3x_2^2)(0.3333x_2)}{EI}dx_2$$

$$\theta_A = \frac{9\text{ kN}\cdot\text{m}^2}{EI} \quad \text{Ans.}$$

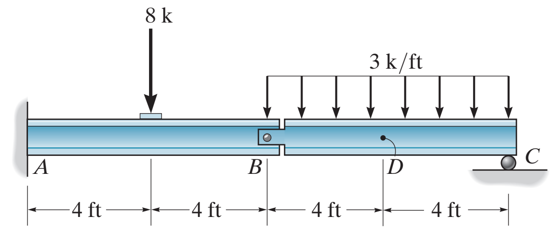
The displacement, the moment functions are shown in Fig. *b*. Here, $\dfrac{\partial M_1}{\partial P} = -x_1$ and $\dfrac{\partial M_2}{\partial P} = -x_2$. Also, set $P = 0$, then $M_1 = -0.3333x_1^3$ and $M_2 = 6x_2 - 3x_2^2$. Thus

$$\Delta_A = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{3\text{ m}} \frac{(-0.3333x_1^3)(-x_1)}{EI}dx_1 + \int_0^{3\text{ m}} \frac{(6x_2 - 3x_2^2)(-x_2)}{EI}dx_2$$

$$= \frac{22.95\text{ kN}\cdot\text{m}^3}{EI}\downarrow \quad \text{Ans.}$$



9–42. Determine the displacement at point D. Use the principle of virtual work. EI is constant.



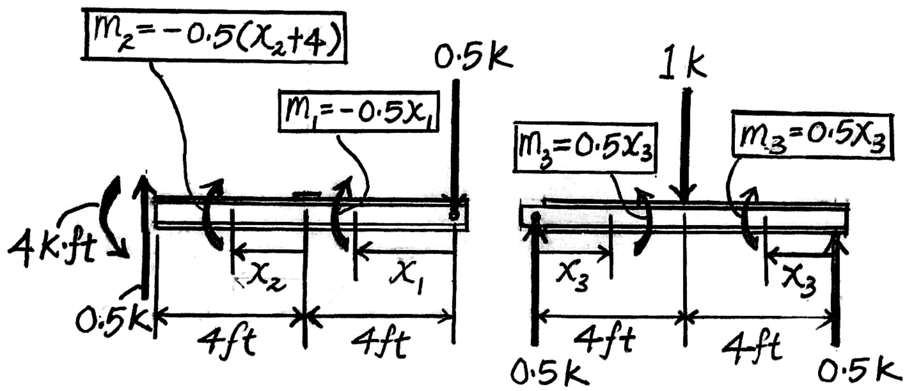
Referring to the virtual and real moment functions shown in Fig. a and b, respectively,

$$1\text{ k}\cdot\Delta_D = \int_0^L \frac{mM}{EI}dx = \int_0^{4\text{ ft}} \frac{(-0.5x_1)(-12x_1)}{EI}dx_1 + \int_0^{4\text{ ft}} \frac{[-0.5(x_2+4)][-(20x_2+48)]}{EI}dx_2$$

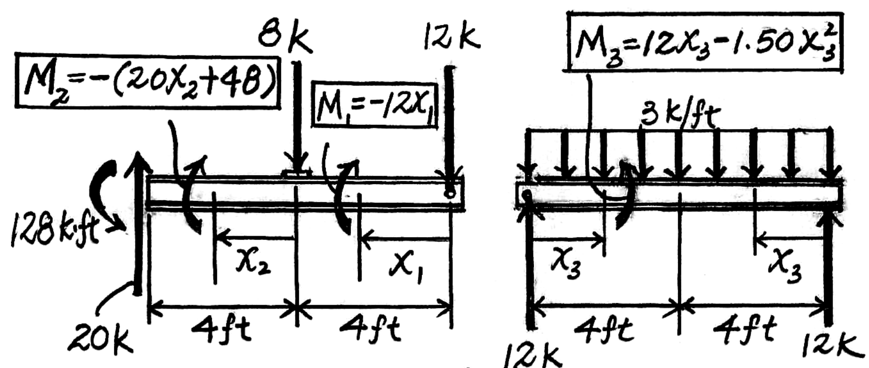
$$+\,2\int_0^{4\text{ ft}} \frac{(-0.5x_3)(12x_3-1.50x_3^2)}{EI}dx_3$$

$$1\text{ k}\cdot\Delta_D = \frac{1397.33\text{ k}^2\cdot\text{ft}^3}{EI}$$

$$\Delta_D = \frac{1397\text{ k}\cdot\text{ft}^3}{EI}\downarrow$$ **Ans.**

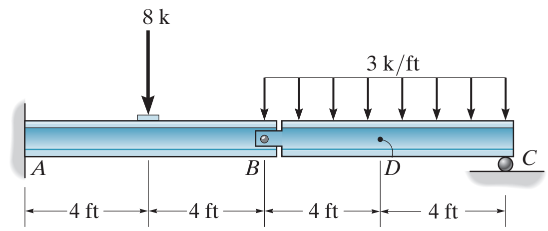


(a)



(b)

9–43. Determine the displacement at point D. Use Castigliano's theorem. EI is constant.



The moment functions are shown in Fig. a. Here, $\frac{\partial M_1}{\partial P} = -0.5x$, $\frac{\partial M_2}{\partial P} = -(0.5x_2 + 2)$ and $\frac{\partial M_3}{\partial P} = 0.5x_3$. Also set $P = 0$, $M_1 = -12x_1$, $M_2 = -(20x_2 + 48)$ and $M_3 = 12x_3 - 1.50x_3^2$. Thus,

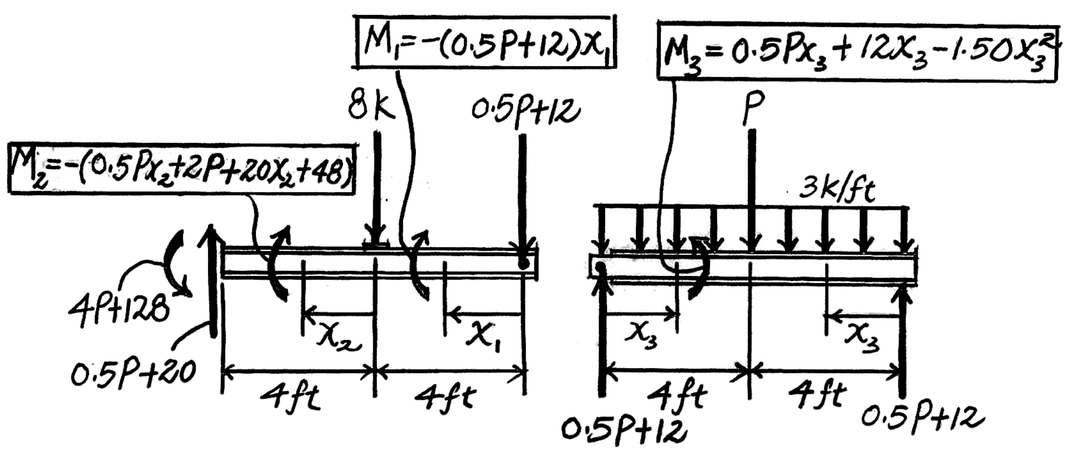
$$\Delta_D = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{4\text{ ft}} \frac{(-12x_1)(-0.5x_1)}{EI}dx_1$$

$$+ \int_0^{4\text{ ft}} \frac{[-(20x_2 + 48)][-(0.5x_2 + 2)]}{EI}dx_2$$

$$+ 2\int_0^{4\text{ ft}} \frac{(12x_3 - 1.50x_3^2)(0.5x_3)}{EI}dx_3$$

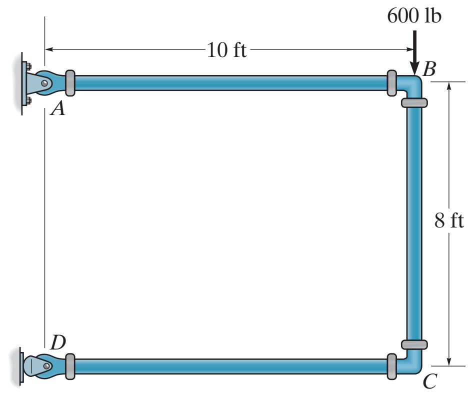
$$= \frac{1397.33\text{ k}\cdot\text{ft}^3}{EI} = \frac{1397\text{ k}\cdot\text{ft}^3}{EI}\downarrow$$

Ans.



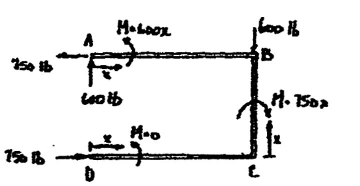
(a)

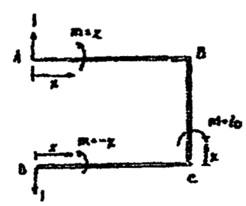
***9–44.** Use the method of virtual work and determine the vertical deflection at the rocker support D. EI is constant.



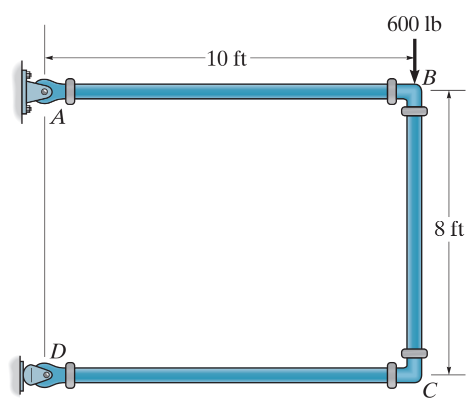
$$(\Delta_D)_x = \int_0^L \frac{mM}{EI}dx = \int_0^{Lo} \frac{(x)(600x)dx}{EI} + \int_0^L \frac{(10)(750x)dx}{EI} + 0$$

$$= \frac{440 \text{ k} \cdot \text{ft}^3}{EI} \downarrow$$ **Ans.**





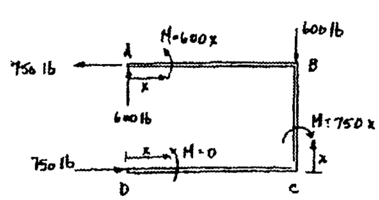
9–45. Solve Prob. 9–44 using Castigliano's theorem.

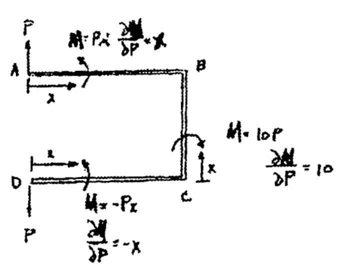


Set $P = 0$,

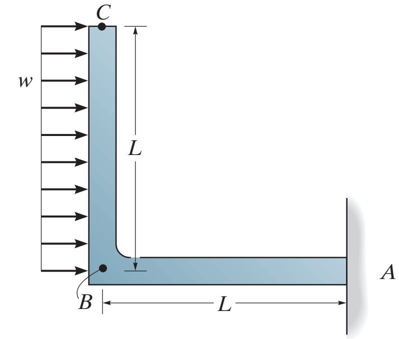
$$(\Delta_D)_v = \int_0^L \frac{M}{EI}\left(\frac{\partial M}{\partial P}\right)dx = \int_0^{10} \frac{(600x)(x)dx}{EI} + \int_0^1 \frac{(750x)(10)dx}{EI} + 0$$

$$= \frac{440 \text{ k} \cdot \text{ft}^3}{EI} \downarrow$$ **Ans.**





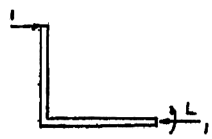
9–46. The L-shaped frame is made from two segments, each of length L and flexural stiffness EI. If it is subjected to the uniform distributed load, determine the horizontal displacement of the end C. Use the method of virtual work.

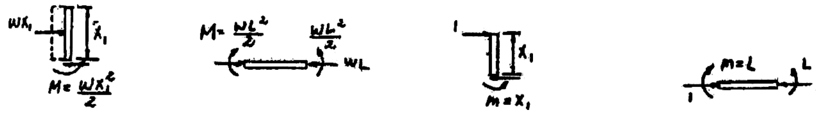


$$1 \cdot \Delta_{C_h} = \int_0^L \frac{mM}{EI} dx$$

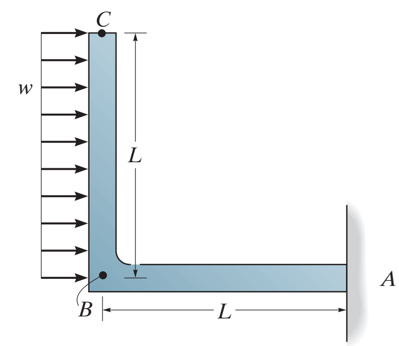
$$\Delta_{C_h} = \frac{l}{EI}\left[\int_0^L (1x_1)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^L (1L)\left(\frac{wL^2}{2}\right)dx_2\right]$$

$$= \frac{5wL^4}{8EI}$$ **Ans.**





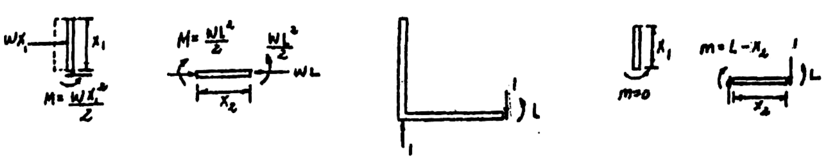
9–47. The L-shaped frame is made from two segments, each of length L and flexural stiffness EI. If it is subjected to the uniform distributed load, determine the vertical displacement of point B. Use the method of virtual work.



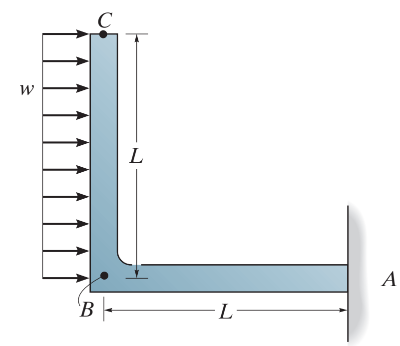
$$l \cdot \Delta_{B_v} = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_{B_v} = \frac{l}{EI}\left[\int_0^L (0)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^L (L - x_2)\left(\frac{wL^2}{2}\right)dx_2\right]$$

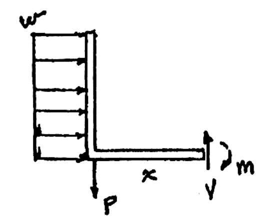
$$= \frac{wL^4}{4EI}$$ **Ans.**



***9–48.** Solve Prob. 9–47 using Castigliano's theorem.



P does not influence moment within vertical segment.



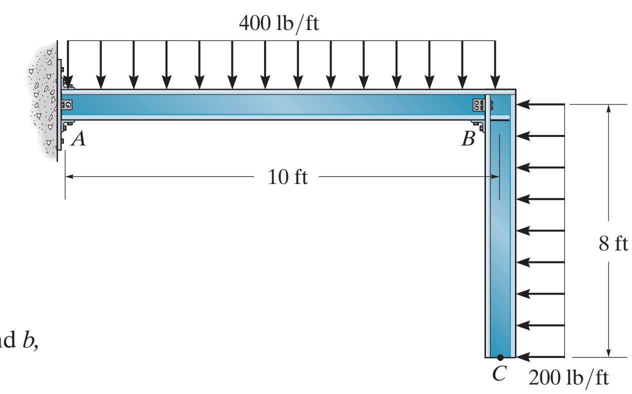
$$M = Px - \frac{wL^2}{2}$$

$$\frac{\partial M}{\partial P} = x$$

Set $P = 0$

$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^L \left(-\frac{wL^2}{2}\right)(x)\frac{dx}{EI} = \frac{wL^4}{4EI}$$ **Ans.**

9–49. Determine the horizontal displacement of point *C*. *EI* is constant. Use the method of virtual work.

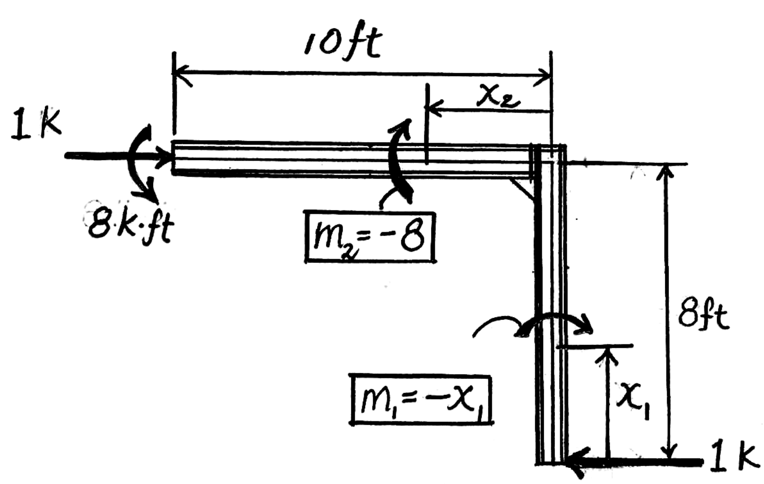


Referring to the virtual and real moment functions shown in Fig. *a* and *b*, respectively,

$$1\text{ k}\cdot\Delta_{C_h} = \int_0^L \frac{mM}{EI}dx = \int_0^{8\text{ ft}} \frac{(-x_1)(-0.1x_1^2)}{EI}dx_1 + \int_0^{10\text{ ft}} \frac{(-8)[-(0.2x_2^2 + 6.40)]}{EI}dx_2$$

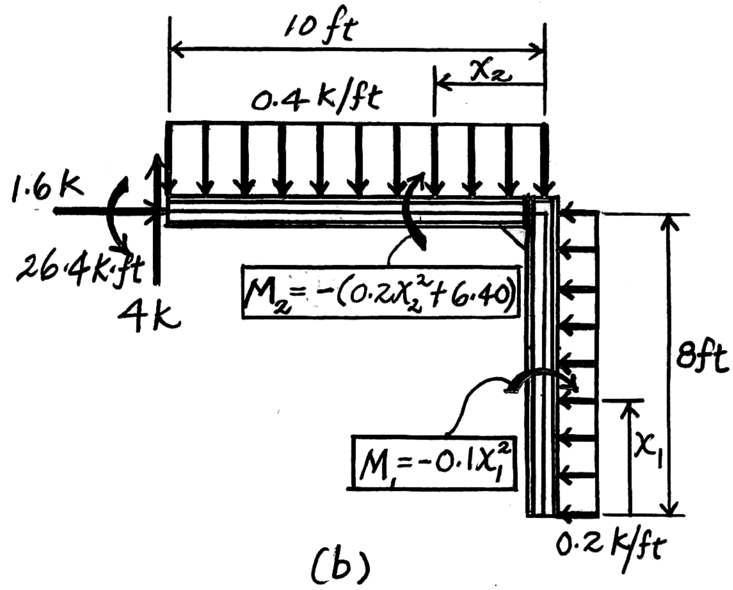
$$1\text{ k}\cdot\Delta_{C_h} = \frac{1147.73\text{ k}^2\cdot\text{ft}^3}{EI}$$

$$\Delta_{C_h} = \frac{1148\text{ k}\cdot\text{ft}^3}{EI} \leftarrow$$ **Ans.**



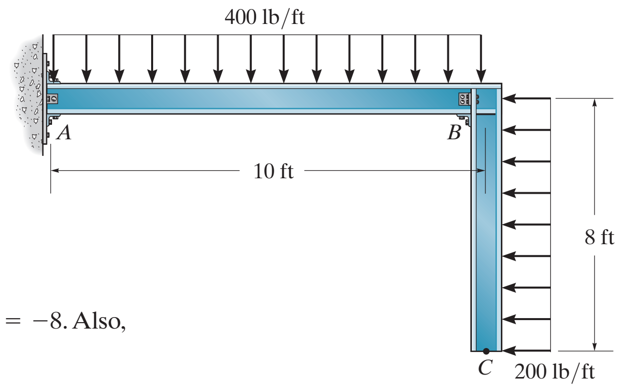
(a)

9–49. Continued



(b)

9–50. Solve Prob. 9–49 using Castigliano's theorem.



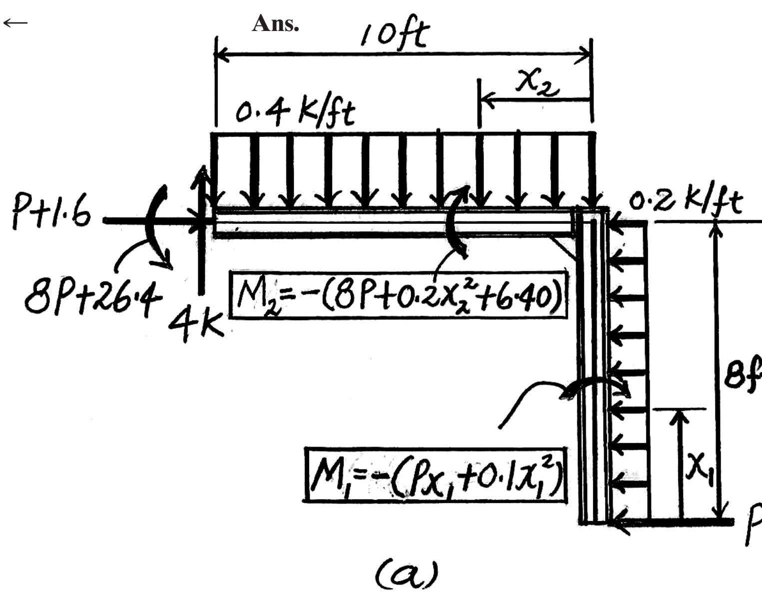
The moment functions are shown in Fig. *a*. Here, $\frac{\partial M_1}{\partial P} = -x_1$ and $\frac{\partial M_2}{\partial P} = -8$. Also, set $P = 0$, then $M_1 = -0.1x_1^2$ and $M_2 = -(0.2x_2^2 + 6.40)$.

Thus,

$$\Delta_{C_h} = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{8\text{ ft}} \frac{(-0.1x_1^2)(-x_1)}{EI}dx_1 + \int_0^{10\text{ ft}} \frac{[-(0.2x_2^2 + 6.40)](-8)}{EI}dx_2$$

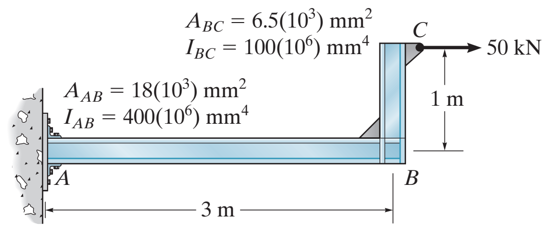
$$= \frac{1147.73 \text{ k}\cdot\text{ft}^3}{EI} = \frac{1148 \text{ k}\cdot\text{ft}^3}{EI} \leftarrow$$

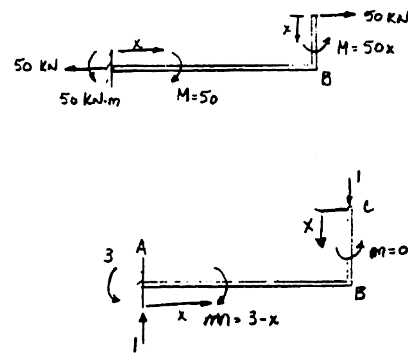
Ans.



(a)

9–51. Determine the vertical deflection at C. The cross-sectional area and moment of inertia of each segment is shown in the figure. Take $E = 200$ GPa. Assume A is a fixed support. Use the method of virtual work.





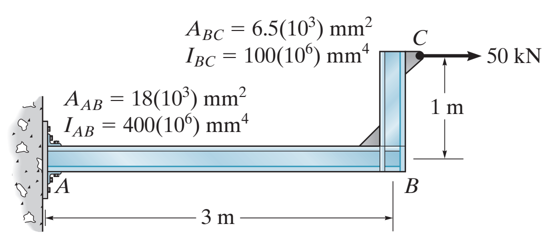
$$(\Delta_C)_v = \int_0^L \frac{mM}{EI}dx = \int_0^3 \frac{(3-x)(50)(10^3)dx}{EI_{AB}} + 0$$

$$= \frac{[150(10^3)x - 25(10^3)x^2]_0^3}{EI_{AB}}$$

$$= \frac{225(10^3)}{200(10^9)(400)(10^6)(10^{-12})}$$

$$= 2.81 \text{ mm} \downarrow$$ **Ans.**

***9–52.** Solve Prob. 9–51, including the effect of shear and axial strain energy.

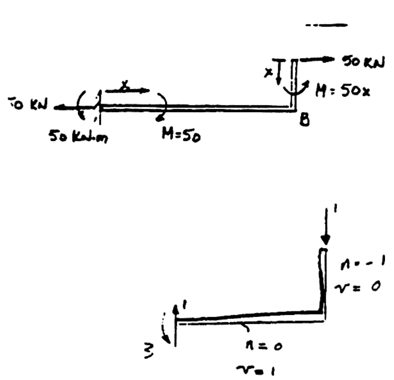


See Prob. 9–51 for the effect of bending.

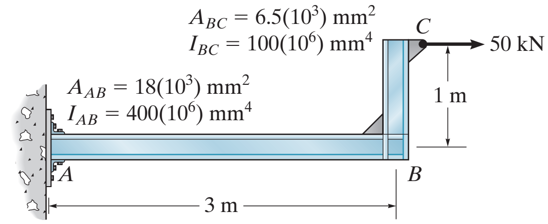
$$U = \sum \frac{nNL}{AE} + \int_0^L K\left(\frac{vV}{GA}\right)dx$$

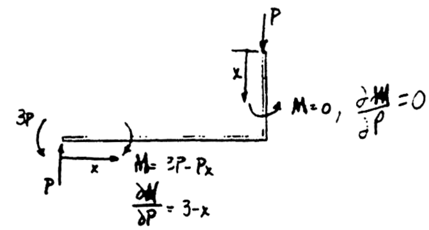
Note that each term is zero since n and N or v and V do not occur simultaneously in each member. Hence,

$$(\Delta_C)_v = 2.81 \ mm \downarrow$$ **Ans**



9–53. Solve Prob. 9–51 using Castigliano's theorem.





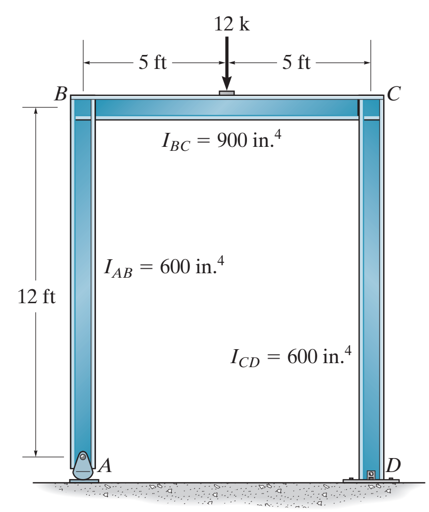
$$(\Delta_C)_v = \int_0^L \frac{M}{EI}\left(\frac{\partial M}{\partial P}\right)dx = \int_0^3 \frac{(50)(10^3)(3-x)dx}{EI_{AB}} + 0$$

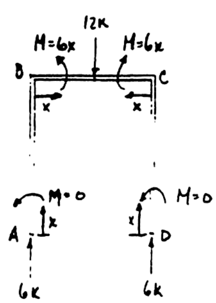
$$= \frac{[150(10^3)x - 25(10^3)x^2]_0^3}{EI_{AB}}$$

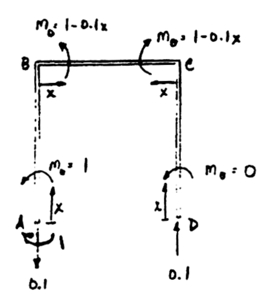
$$= \frac{225(10^3)}{200(10^9)(400)(10^6)(10^{-12})}$$

$$= 2.81 \text{ mm } \downarrow$$ **Ans.**

9–54. Determine the slope at A. Take $E = 29(10^3)$ ksi. The moment of inertia of each segment of the frame is indicated in the figure. Assume D is a pin support. Use the method of virtual work.



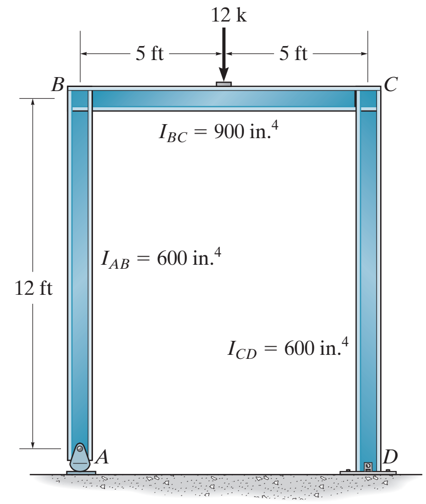


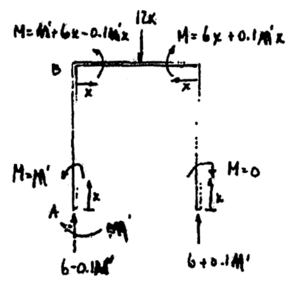


$$\theta_A = \int_0^L \frac{m_\theta M}{EI}dx = \int_0^5 \frac{(1-0.1x)(6x)dx}{EI_{BC}} + \int_0^5 \frac{(0.1x)(6x)dx}{EI_{BC}} + 0 + 0$$

$$= \frac{(75 - 25 + 25)}{EI_{BC}} = \frac{75(144)}{29(10^3)(900)} = 0.414(10^{-3})\text{ rad}$$ **Ans.**

9–55. Solve Prob. 9–54 using Castigliano's theorem.



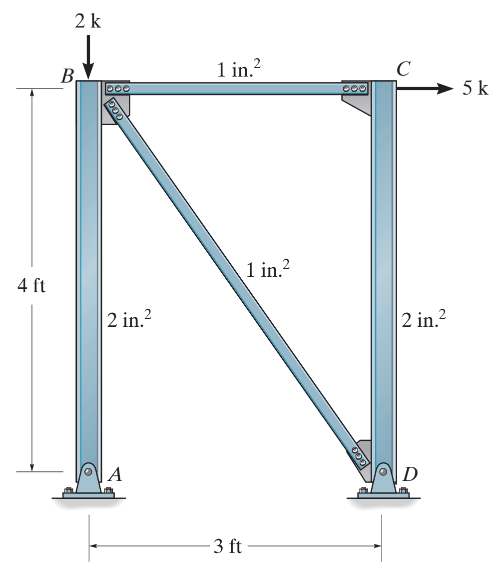


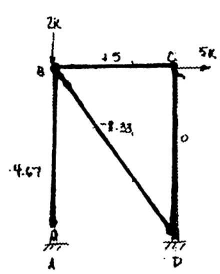
Set $M' = 0$,

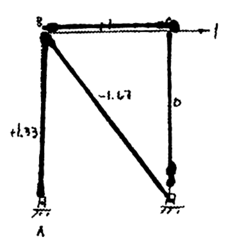
$$\theta_A = \int_0^L \frac{M}{EI}\left(\frac{\partial M}{\partial M'}\right)dx = \int_0^5 \frac{(6x)(1 - 0.1x)dx}{EI_{BC}} + \int_0^5 \frac{(6x)(0.1x)dx}{EI_{BC}} + 0 + 0$$

$$= \frac{(75 - 25 + 25)}{EI_{\partial C}} = \frac{75(144)}{29(10^3)(900)} = 0.414(10^{-3}) \text{ rad} \quad \textbf{Ans.}$$

***9–56.** Use the method of virtual work and determine the horizontal deflection at C. The cross-sectional area of each member is indicated in the figure. Assume the members are pin connected at their end points. $E = 29(10^3)$ ksi.



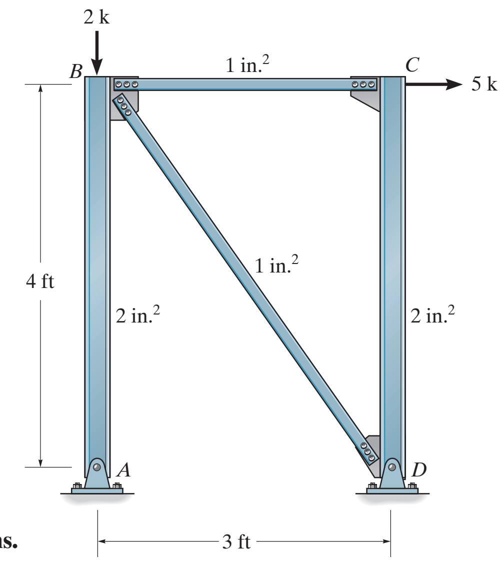




$$(\Delta_c)_h = \sum \frac{nNL}{AE} = \frac{1.33(4.667)(4)(12)}{2(29)(10^3)} + \frac{(1)(5)(3)(12)}{(1)(29)(10^3)} + 0 + \frac{(-8.33)(-1.667)(5)(12)}{(1)(29)(10^3)}$$

$$= 0.0401 \text{ in.} \rightarrow \qquad \textbf{Ans.}$$

9–57. Solve Prob. 9–56 using Castigliano's theorem.

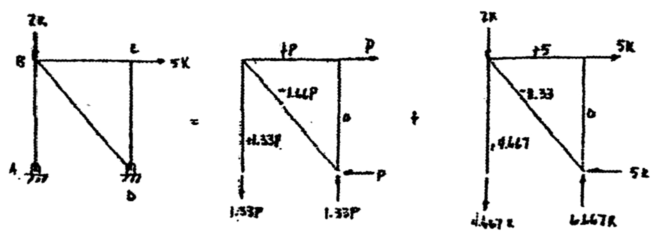


Member	N force	$\frac{\partial N}{\partial P}$
AB	$1.33P + 4.667$	1.33
BC	$P + 5$	1
BD	$-1.667P - 8.33$	-1.667
CD	0	0

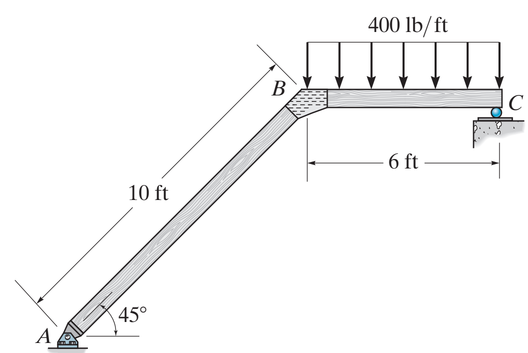
Set $P = 0$,

$$(\Delta_c)_h = N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{(4.667)(1.33)(4)(12)}{2(29)(10^3)} + \frac{(5)(1)(3)(12)}{(1)(29)(10^3)} + 0 + \frac{(-8.33)(-1.667)(5)(12)}{(1)(29)(10^3)}$$

$$= 0.0401 \text{ in.} \rightarrow$$ **Ans.**



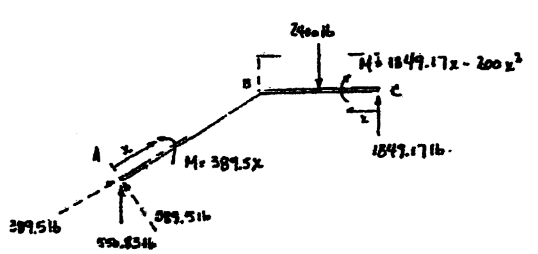
9–58. Use the method of virtual work and determine the horizontal deflection at C. E is constant. There is a pin at A, and assume C is a roller and B is a fixed joint.

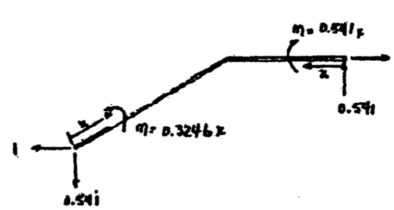


$$(\Delta_c)_h = \int_0^L \frac{mM}{EI}dx = \int_0^4 \frac{(0.541x)(1849.17x - 200x^2)dx}{EI} + \int_0^{10} \frac{(0.325x)(389.5x)dx}{EI}$$

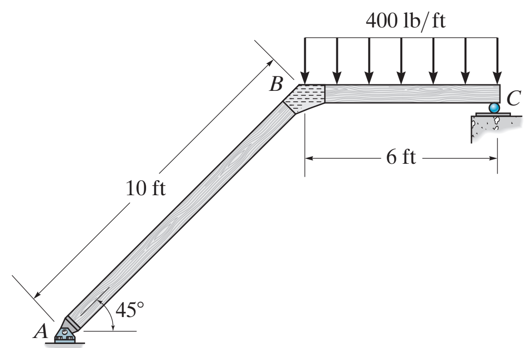
$$= \frac{1}{EI}\left[(333.47x^3 - 27.05x^4)|_0^6 + (42.15x^3)|_0^{10}\right]$$

$$= \frac{79.1\text{k.ft}^3}{EI} \rightarrow$$ **Ans.**





9–59. Solve Prob. 9–58 using Castigliano's theorem.



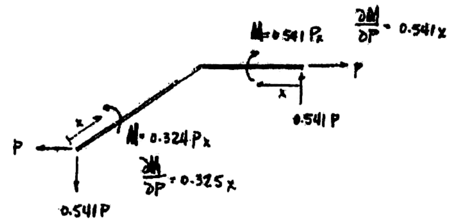
Set $P = 0$.

$$(\Delta_c)_h = \int_0^L \frac{M}{EI}\left(\frac{\partial M}{\partial P}\right)dx = \int_0^4 \frac{(1849.17x - 200x^2)(0.541x)dx}{EI} + \int_0^{10} \frac{(389.5x)(0.325x)dx}{EI}$$

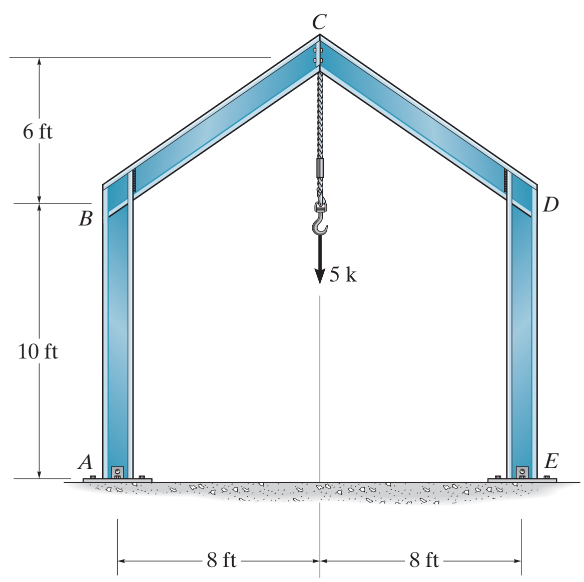
$$= \frac{1}{EI}\left[(333.47x^3 - 27.5x^4)\Big|_0^6 + (42.15x^3)\Big|_0^{10}\right]$$

$$= \frac{79.1\text{k.ft}^3}{\text{EI}} \rightarrow$$

Ans.



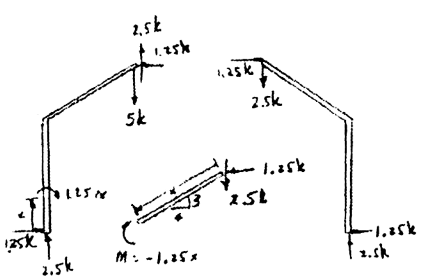
***9–60.** The frame is subjected to the load of 5 k. Determine the vertical displacement at C. Assume that the members are pin connected at A, C, and E, and fixed connected at the knee joints B and D. EI is constant. Use the method of virtual work.

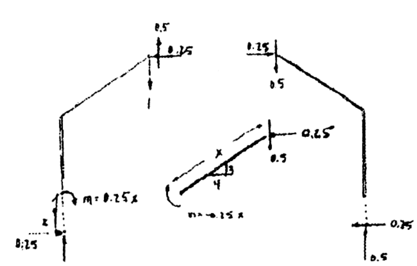


$$(\Delta_c)_v = \int_0^L \frac{mM}{EI}dx = 2\left[\int_0^{10} \frac{(0.25x)(1.25x)dx}{EI} + \int_0^{10} \frac{(-0.25x)(-1.25x)dx}{EI}\right]$$

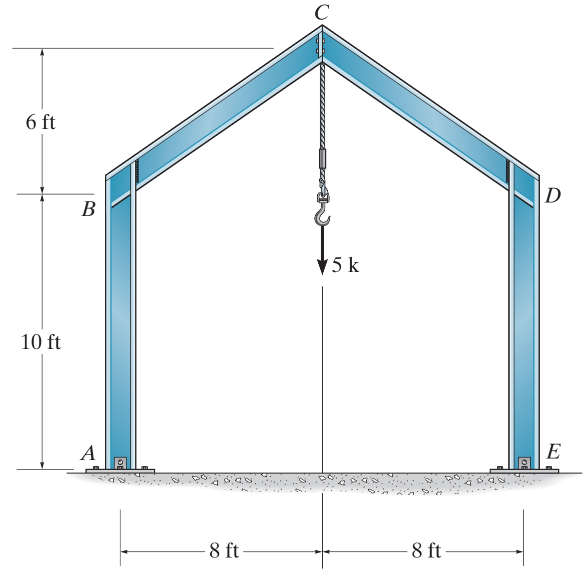
$$= \frac{1.25(10^3)}{3EI} = \frac{4.17\ \text{k}\cdot\text{ft}^3}{EI} \downarrow$$

Ans.





9–61. Solve Prob. 9–60 using Castigliano's theorem.



Set $P = 5$ k.

$$(\Delta_c)_v = \int_0^L \frac{M}{EI}\left(\frac{\partial M}{\partial P}\right)dx = 2\left[\int_0^{10}\frac{(1.25x)(0.25x)\,dx}{EI} + \int_0^{10}\frac{(-1.25x)(-0.25x)\,dx}{EI}\right]$$

$$= \frac{1.25(10^3)}{3EI} = \frac{4.17\text{ k}\cdot\text{ft}^3}{EI}\downarrow$$

Ans.