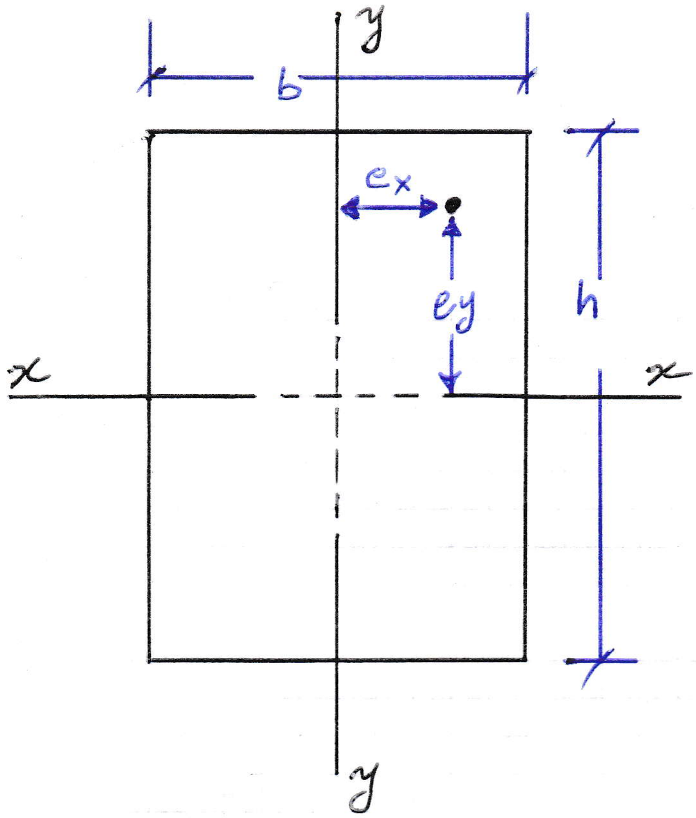
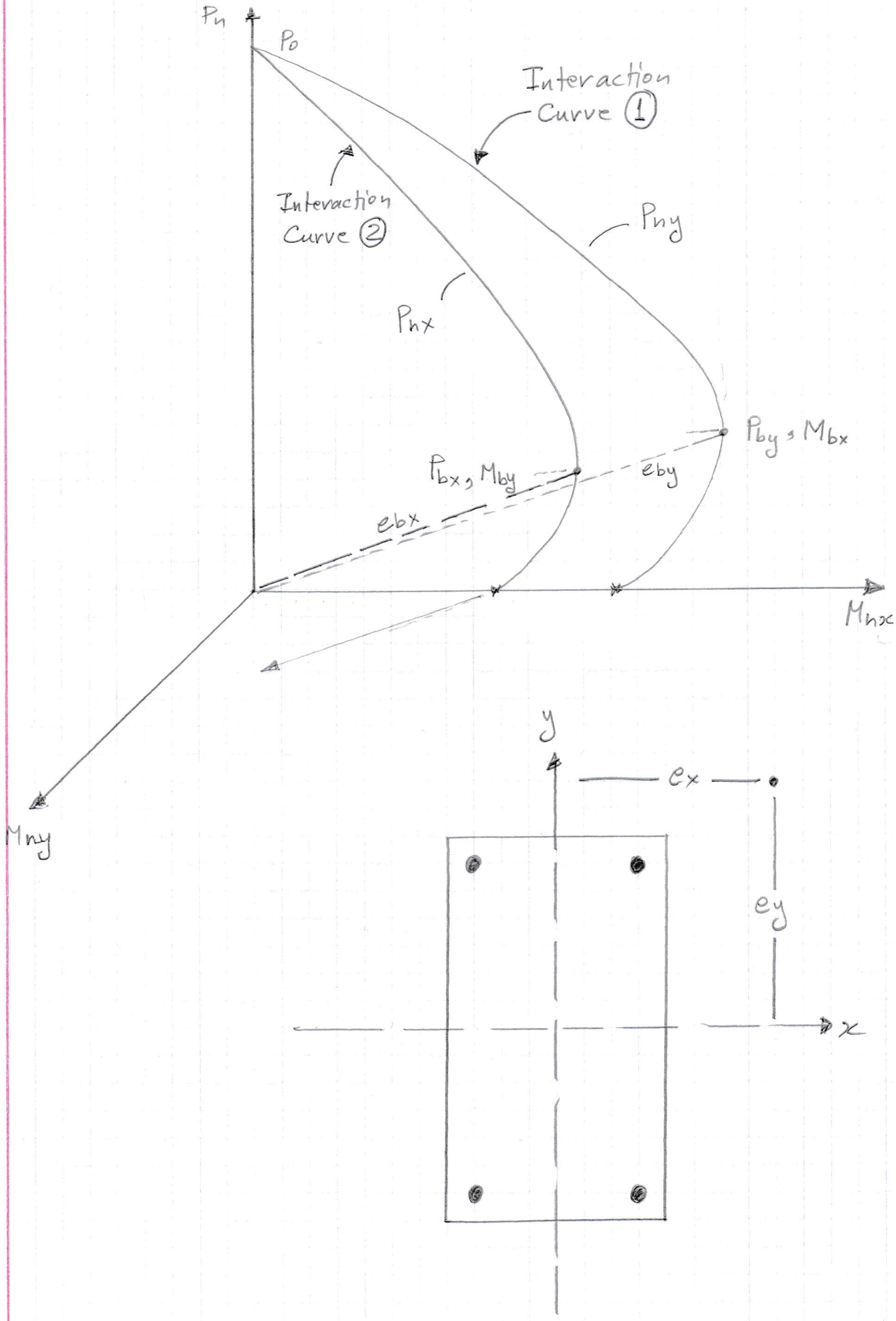
# Biaxial Bending and Compression:




$$M_{nx} = P_n e_y$$
$$M_{ny} = P_n e_x$$

$$\frac{1}{P_i} = \frac{1}{P_x} + \frac{1}{P_y} - \frac{1}{P_o} \qquad \text{(Bresler)}$$

Dr. Jamal Zalatimo



Pn
Po
Interaction Curve 1
Interaction Curve 2
Pny
Pnx
Pby , Mbx
Pbx , Mby
eby
ebx
Mnx
Mny
y
ex
ey
x

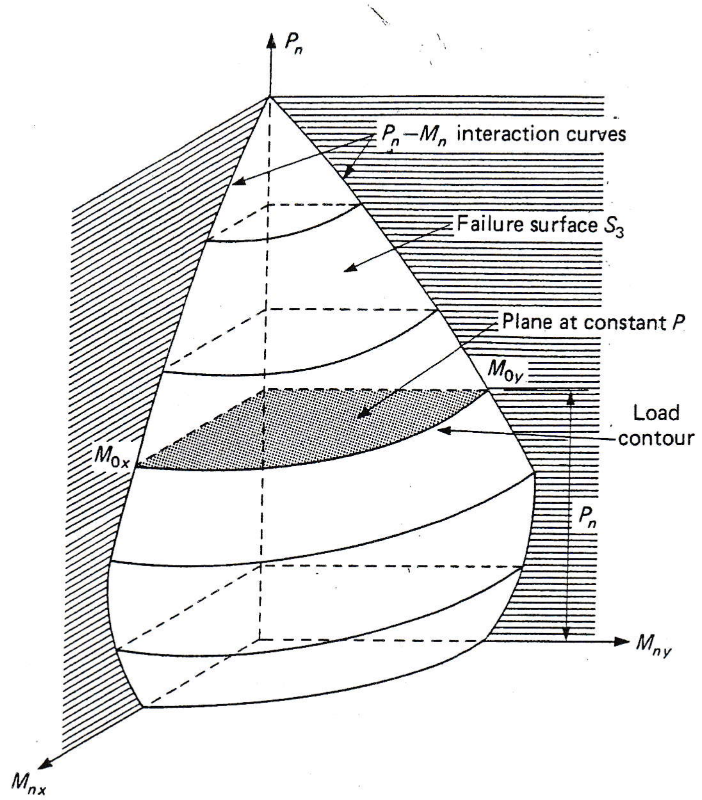



**Figure 13.21.7** Load contours for constant $P_n$ on failure surface $S_3$ (from Bresler [13.55]).

Bresler [13.55] suggests that it is acceptable to take $\alpha_1 = \alpha_2 = \alpha$; then

$$\left(\frac{M_{nx}}{M_{0x}}\right)^{\alpha} + \left(\frac{M_{ny}}{M_{0y}}\right)^{\alpha} = 1 \qquad \textbf{(13.21.8)}$$

which is shown graphically in Fig. 13.21.8.

In using Eq. (13.21.8) or Fig. 13.21.8, however, it is still necessary to have a value of $\alpha$ that is applicable to the particular column under investigation. Bresler [13.55] reported the calculated values of $\alpha$ to vary from 1.15 to 1.55.

For practical purposes, it seems satisfactory to take $\alpha$ as 1.5 for rectangular sections and between 1.5 and 2.0 for square sections.

**Load Contour Method—Parme Approach.*** The approach described herein has been developed by Parme et al. [13.58] as an extension of the Bresler load contour method. The Bresler interaction equation (13.21.8) is assumed to be the basic strength criterion to define the typical load contour representing the intersection of the failure surface $S_3$ (Fig. 13.21.7) with a horizontal plane at a

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*This approach may also be referred to as the Portland Cement Association (PCA) method since Reference 13.58 is also available as PCA *Advanced Engineering Bulletin No. 18.*

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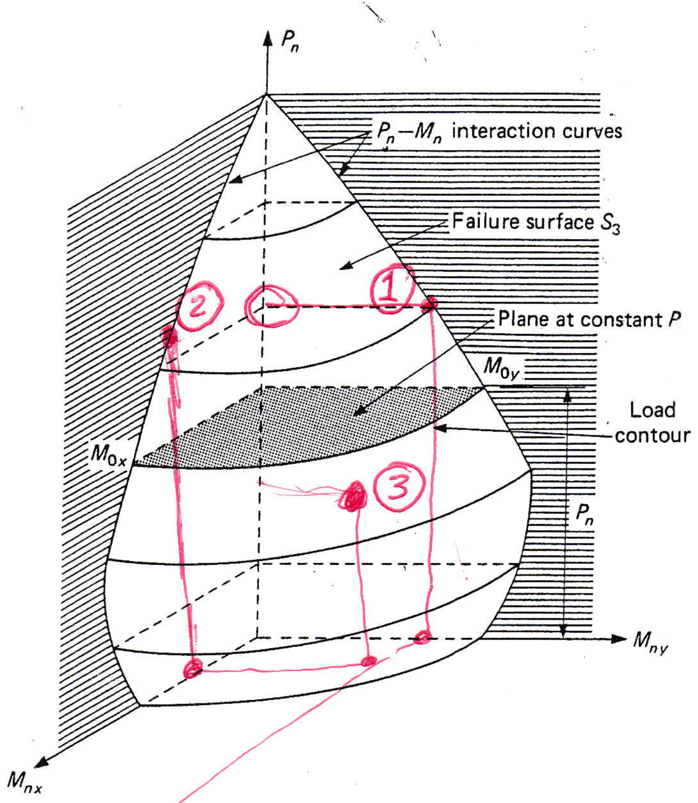


**Figure 13.21.7** Load contours for constant $P_n$ on failure surface $S_3$ (from Bresler [13.55]).

Note points
1
2
3

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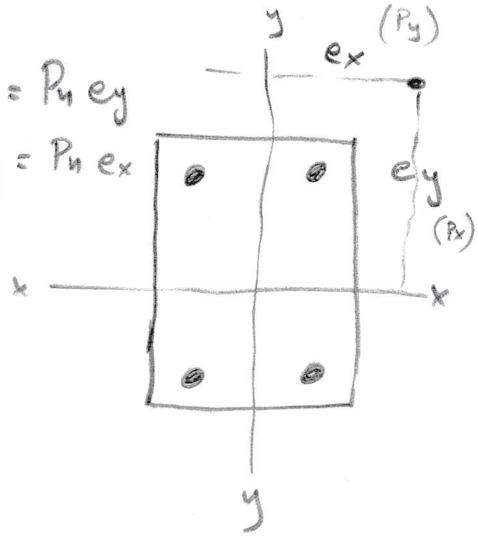
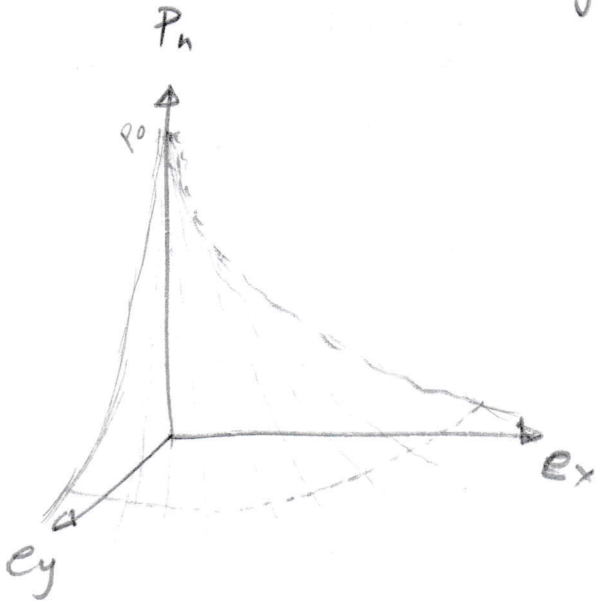
$S_1$

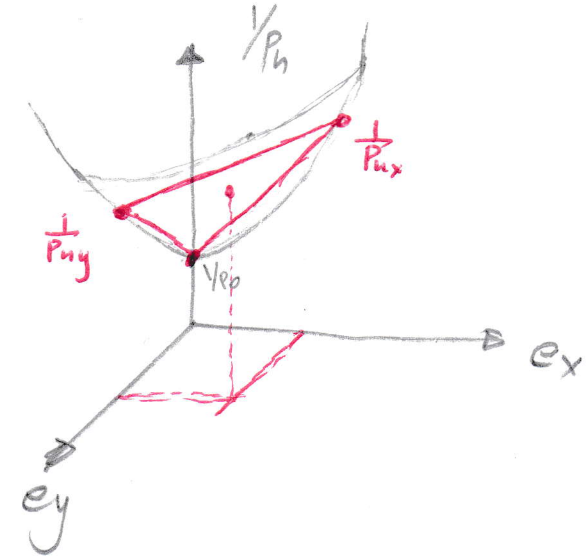
$M_{nx} = P_n e_y$

$M_{ny} = P_n e_x$

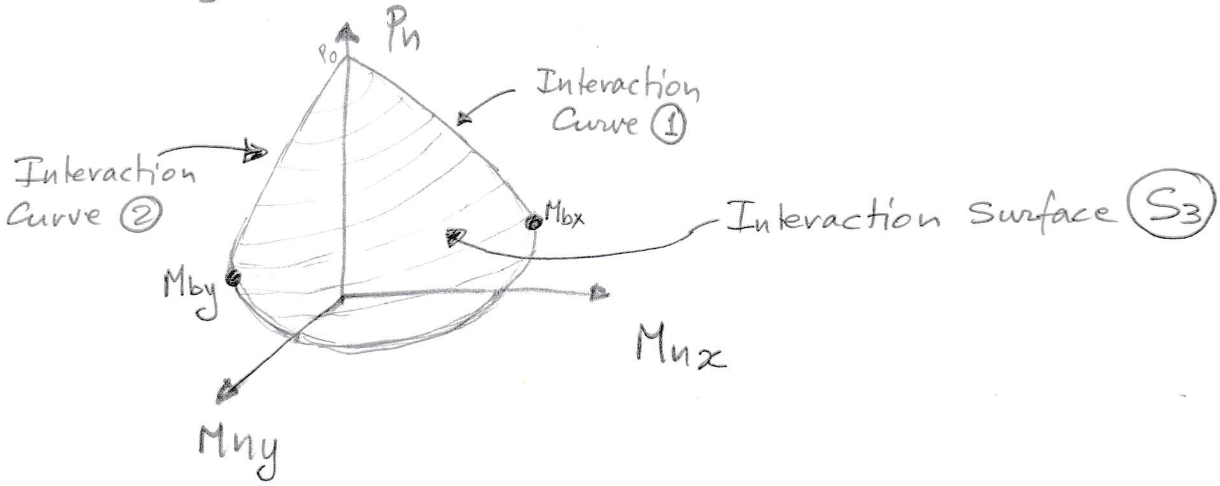


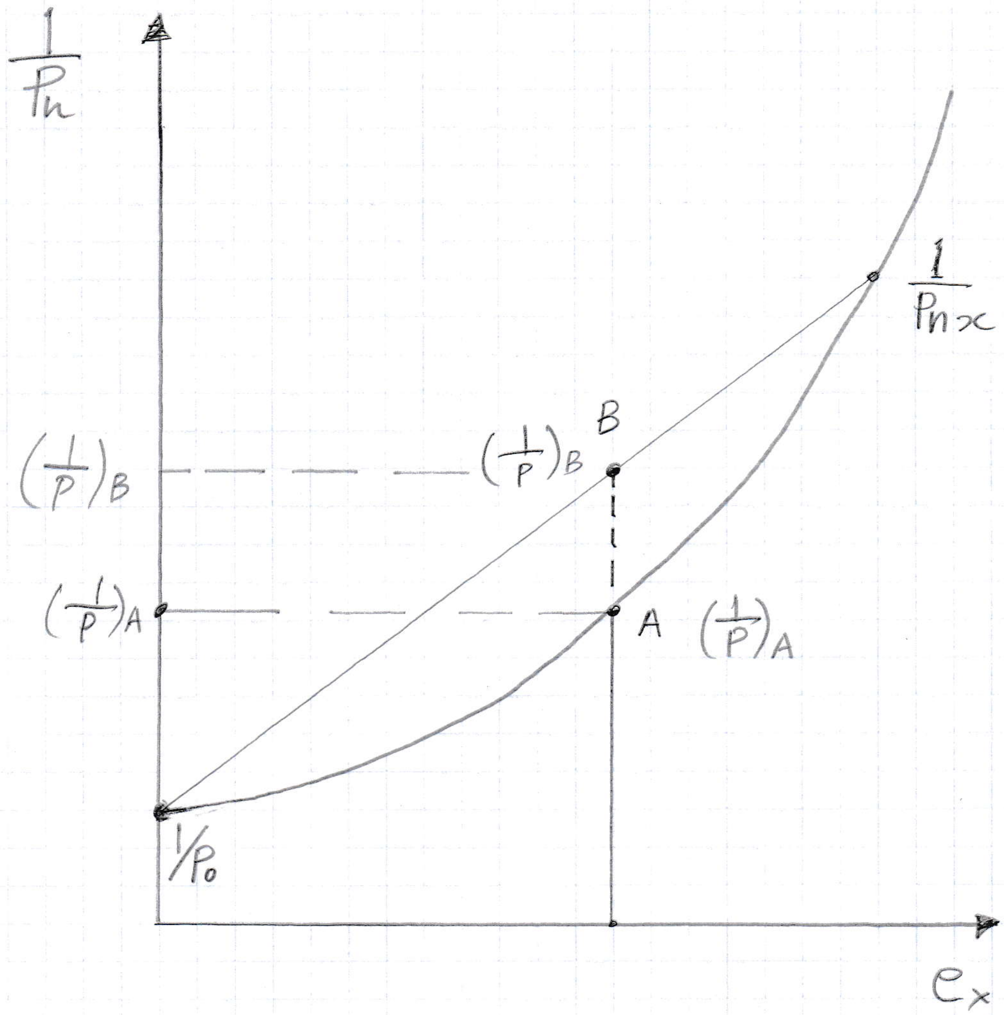

$S_2$




$S_3$



Failure Surface ($S_2$)




A is associated with the true capacity

B is a close approximation and is conservative

$$\left(\frac{1}{P}\right)_B > \left(\frac{1}{P}\right)_A \Rightarrow P_A > P_B$$

$\therefore$ $P_A$, the true capacity is larger than $P_B$, the estimated approximate capacity

$$\frac{1}{P_i} = \frac{1}{P_{x(n)}} + \frac{1}{P_{y(n)}} - \frac{1}{P_o} \approx \frac{1}{P_n} \quad \text{(Bresler)}$$

## Example 13.21.1 :

Determine the adequacy of a 40 x 40 cm square tied column containing 8 $\phi$ 30 bars. $f_c' = 20$ MPa, $f_y = 280$ MPa.

$P_u = 67$ t

$M_{ux} = 17$ t.m

$M_{uy} = 7$ t.m

$$e_y = \frac{M_{ux}}{P_u} = \frac{17}{67} = 0.254 \text{ m} = 25.4 \text{ cm}$$

$$e_x = \frac{M_{uy}}{P_u} = \frac{7}{67} = 0.104 \text{ m} = 10.4 \text{ cm}$$

Note: to determine the values of $P_x$ and $P_y$, the $P_n - M_{nx}$ and $P_n - M_{ny}$ interaction diagrams in uniaxial bending are needed for bending about the x and y axes.

$$P_o = 0.85(0.2)(1600 - 56.56) + 2.8(56.56)$$
$$= 420 \text{ t}$$

JZ

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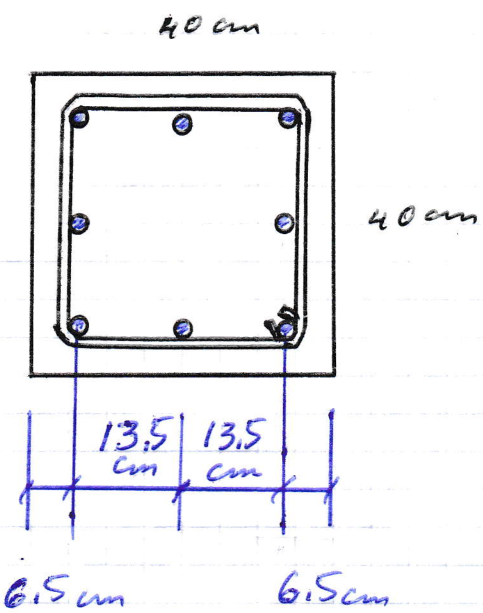
$f'_c = 20\ MPa$

$f_y = 280\ MPa$

$E_s = 200000\ MPa$

8 $\phi$ 30 bars

$\phi 30 = 7.07\ cm^2$




$\varepsilon_y = \frac{280}{200000}$

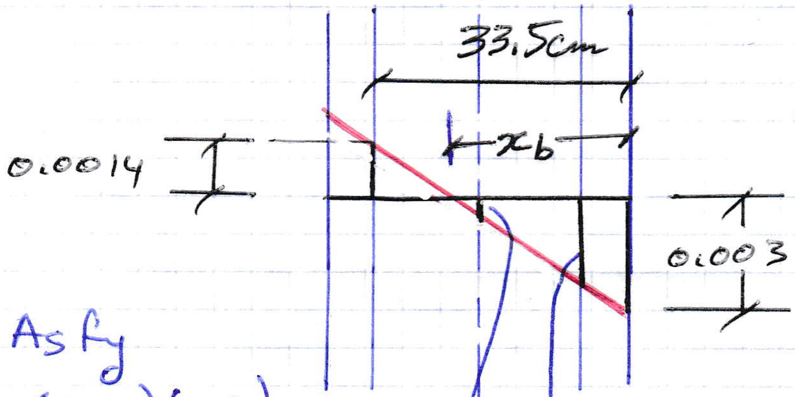
$= 0.0014$

$x_b = \frac{0.003}{0.0044}(33.5)$

$= 22.84\ cm$

$a_b = 19.41\ cm$

$C_c = 132\ t$




$T = A_s f_y$

$= 3(7.07)(2.8)$

$= 59.4\ t$

$\varepsilon'_{s1} = \frac{22.84 - 6.5}{22.84} \times 0.003 = 0.00215$

$\therefore f'_{s1} = f_y = 280\ MPa$

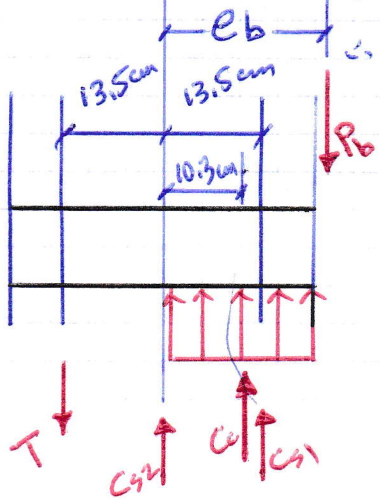
$P_b = C_c + C_{s1} + C_{s2} - T$

$= 132 + 55.8 + 8.1 - 59.4$

$= 136.5\ t$

$\varepsilon'_{s2} = \frac{2.84}{22.84} \times 0.003 = 0.000373$

$\therefore f'_{s2} = 74.6\ MPa$




$C_{s1} = (3 \times 7.07)(2.8 - 0.85(0.2))$

$= 55.8\ t$

$C_{s2} = (2 \times 7.07)(0.746 - 0.85(0.2))$

$= 8.14\ t$

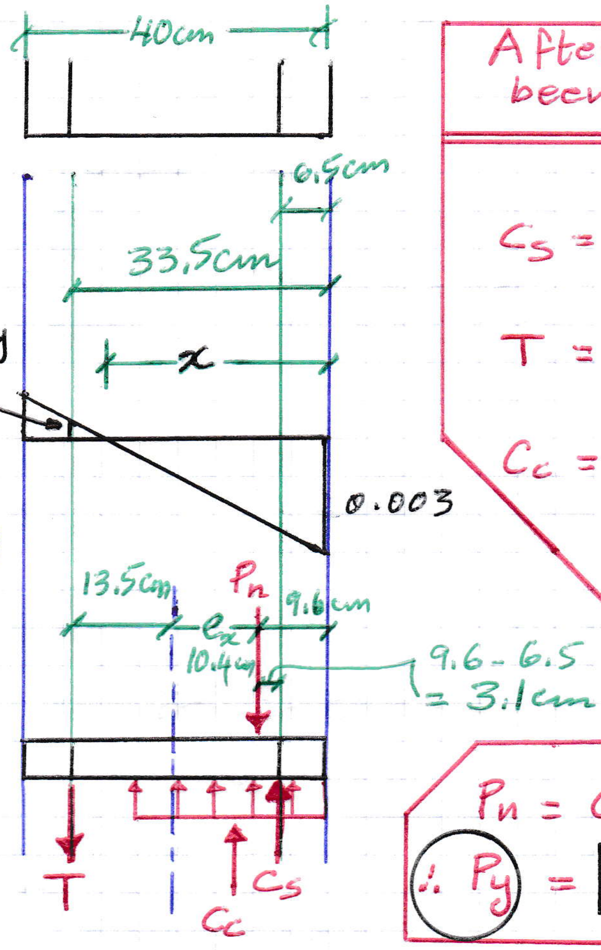
For $e_x = 10.4$ cm $< e_b = 21.4$ cm

$\epsilon_s' > \epsilon_y$ and ignoring middle steel

$C_s = A_s'(f_y - 0.85f_c')$
$= (3 \times 7.07)(2.63)$
$= 55.8t$

$T = A_s f_s$

$C_c = 0.85 f_c' b (0.85x)$
$= 0.85(0.2)(40)(0.85x)$
$= 5.78x$




After $x$ has been determined:

$x = 29.9$ cm

$C_s = 55.8t$

$T = 15.3t$

$C_c = 172.8t$

$P_n = C_c + C_s - T$

$\therefore P_y = \boxed{213.3t}$

$$f_s = (2000)(0.003(33.5 - x)/x) = 6(33.5 - x)/x$$

$$T = A_s f_s = (3 \times 7.07)(6(33.5 - x)/x)$$

$$= \frac{4263}{x} - 127.3$$

Taking moments about $P_n$, $+\circlearrowleft$

$$C_c\left(\frac{0.85x}{2} - 9.6\right) - C_s(3.1) - T(23.9) = 0$$

$$(5.78x)(0.425x - 9.6) - (55.8)(3.1) - \left(\frac{4263}{x} - 127.3\right)(23.9) = 0$$

$$2.46x^2 - 55.49x - 172.98 - \frac{101886}{x} + 3042.5 = 0$$

$$x^3 - 22.56x^2 + 1166.5x - 41417 = 0$$

By trial and error, $x = 29.9$ cm

$(a = 0.85x = 25.4 \text{ cm})$

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#1 Dr. Jamal

For $e_y = 25.4$ cm $> e_b = 21.4$ cm

$\epsilon_s > \epsilon_y$

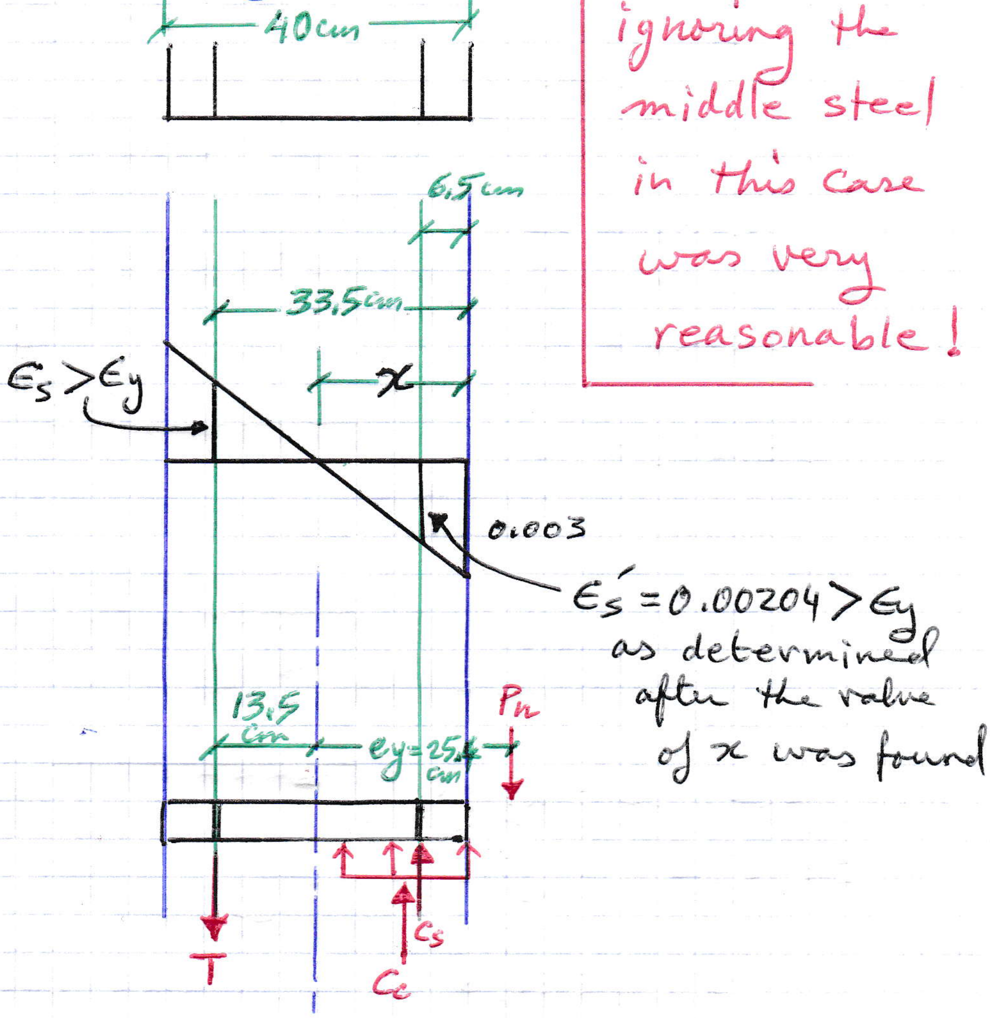
assume $\epsilon_s' > \epsilon_y$

$T = A_s f_y$
$= 59.4\,t$

$C_c = 5.78x$

$C_s = 55.8\,t$

$\epsilon_y = 0.0014$




Note that ignoring the middle steel in this case was very reasonable!

$$P_n = C_c + C_s - T = 5.78x + 55.8 - 59.4 = 5.78x - 3.6$$

Taking moments about tension steel, +↻

$$P_n(38.9) - C_c\left(33.5 - \frac{0.85x}{2}\right) - C_s(33.5 - 6.5) = 0$$

$$(38.9)(5.78x - 3.6) - (5.78x)(33.5 - 0.425x) - (55.8)(27) = 0$$

$$224.8x - 140 - 193.6x + 2.46x^2 - 1507 = 0$$

$$x^2 + 12.68x - 670 = 0$$

$\therefore x = 20.31$ cm $\quad$ ($a = 17.3$ cm)

Check assumption: $\epsilon_s' = \frac{(20.31 - 6.5)}{20.31}(0.003) = 0.00204 > \epsilon_y$ ✓

$\therefore$ assumption is correct

$C_c = 117.4\,t$, $P_n = 113.8\,t$

$$\boxed{\therefore P_x = 113.8\,t}$$

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Bresler Equation

$$\frac{1}{P_n} \approx \frac{1}{P_i} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_o}$$

$$= \frac{1}{113.8} + \frac{1}{213.3} - \frac{1}{420.0}$$

$$P_n \approx P_i = 90.13\,t$$

$$\phi P_n = 0.65(90.13) = 58.6\,t$$

$$< P_u = 67\,t$$

$\therefore$ The column is NOT adequate

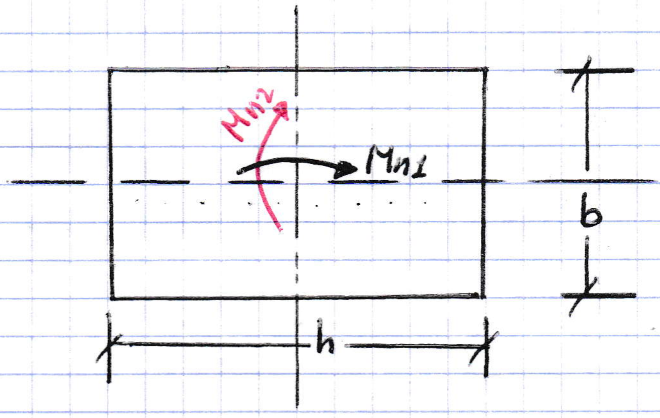
## Square columns:

Design for an equivalent uni-axial moment

$$M_{n\,req} = M_{n1} + 0.55\,M_{n2}$$

$$M_{n1} > M_{n2}$$

## Rectangular columns:




$$M_{n\,req} = M_{n1} + 0.55\left(\frac{h}{b}\right) M_{n2}$$

$$M_{n1} > M_{n2}$$

$M_{n\,req}$ is in the direction of $M_{n1}$

Design the section as a uni-axially loaded column

Check the design using the Bresler equation

Modify the design as necessary (trial and error)

**Birzeit University**
**Faculty of Engineering and Technology**
**Department of Civil and Environmental Engineering**

**ENCE 436** **Reinforced Concrete Design II**

**<u>Biaxial Bending – Basic Mechanics</u>**

**Homework 2** **Due whenever we meet again, InshAllah**

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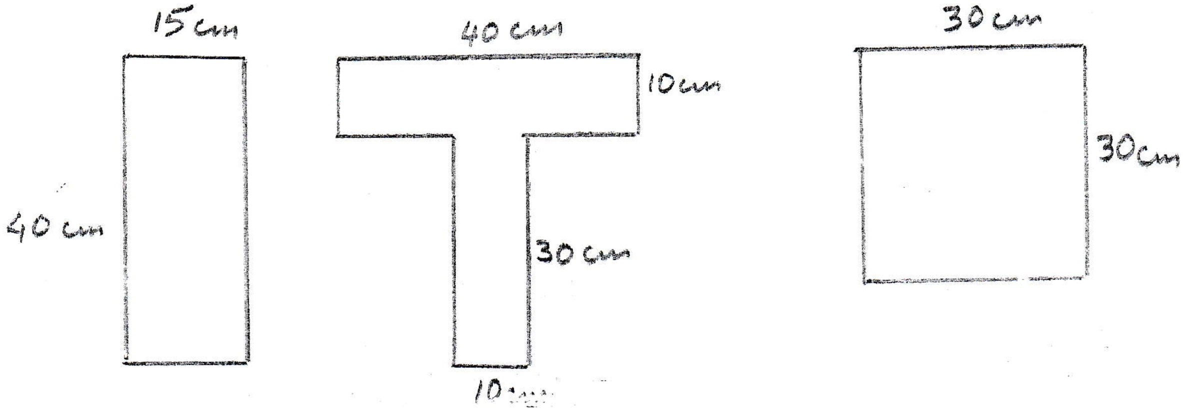
1. A beam extends over a simple span of 10 m and supports a uniformly distributed load of 5 tons per meter. The load is applied at an angle of 30 degrees from the vertical and passes through the centroid of the x-section. For a section at midspan and neglecting self-weight, determine the location of the neutral axis, the angle it makes with the vertical, and the maximum stress (in $kg/cm^2$ or $t/cm^2$) for each shape.

The shapes are as shown.

a. b. c.




2. Repeat your work if the beam is additionally acted on by an axial force of 370 ton.

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## Biaxial Bending – Design

**Homework 3 Due whenever we meet again, InshAllah**

**Design a square, tied, short column to support an axial load Pu = 390 t, in addition to two biaxial moments Mux = 95 t.m, and Muy = 35 t.m.**

**Use fc' = 35 MPa (note that $\beta_1 = 0.80$)**
**fy = 420 MPa**
**Φ 30 longitudinal bars distributed along all four faces**
**Φ 10 ties**

**Use the charts for the design.**
**Check your final design using statical equilibrium and the Bresler formula.**