

## Equivalent Frame Method

Dr. Samel Zafino

### a) Basis of Analysis :

- derived with the assumption that the analysis would be done using the moment distribution method.
- For vertical loading, each floor with its columns may be analyzed separately, with the columns assumed to be fixed at the floors above and below.

### b) Moment of Inertia :

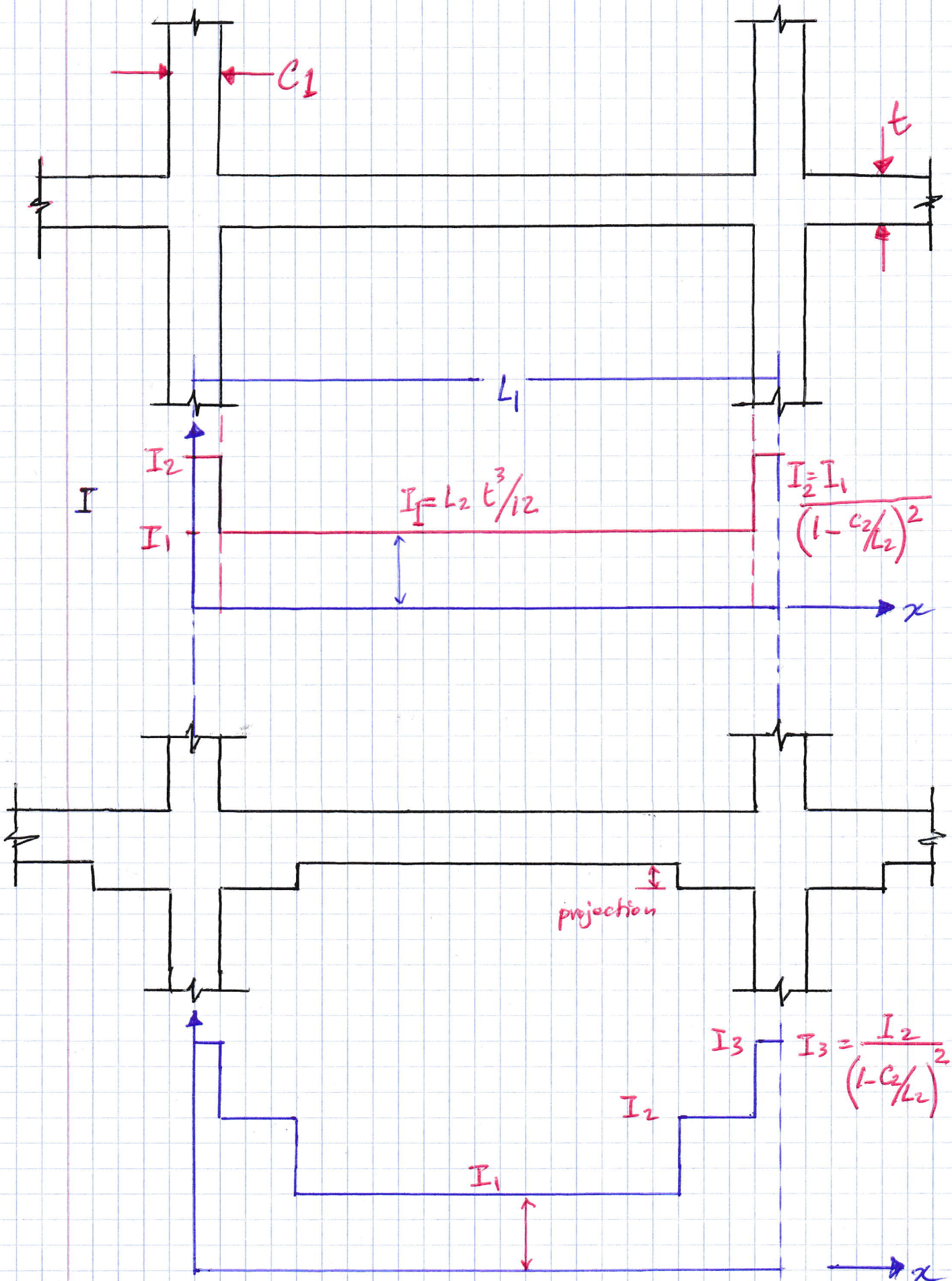
- based on the concrete x-section
- starting with the moment of inertia at midspan, changes occur at:
  - \* edge of drop panels
  - \* edge of the column or capitalfrom the center of the column to the face of the column (or capital)

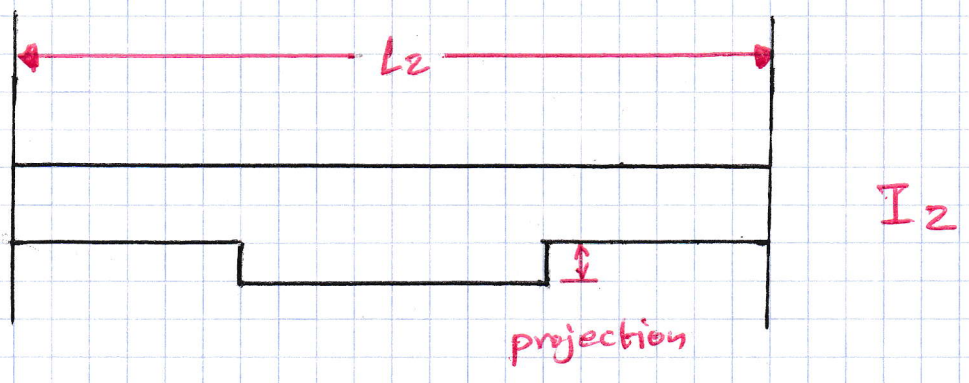
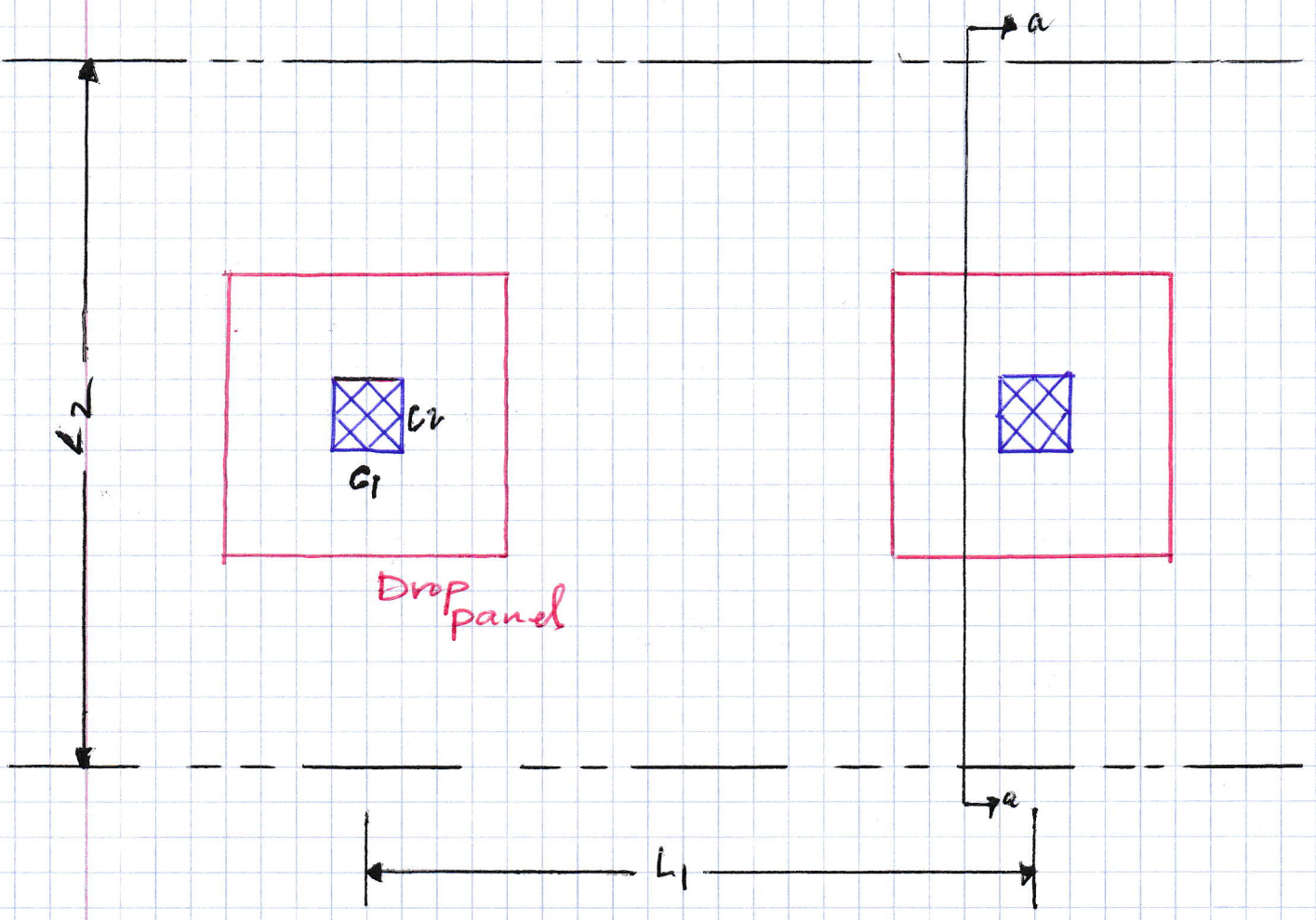
$$I_{\text{slab}} = \frac{I_{\text{slab at the face}}}{(1 - c_2/l_2)^2}$$

∴ moment of inertia varies in a stepwise manner

### c) The equivalent column

torsional deformation of the transverse beams reduces the effective flexural stiffness provided by the actual column





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⇒ equivalent column is assumed with a stiffness less than that of the actual column, for which:

total flexibility of the equivalent column  
(inverse of stiffness)

equals the sum of the flexibilities of the actual column and beam

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{K_b}$$

In computing  $K_c$ , the moment of inertia of the actual column is assumed to be infinite from the top of the slab to the bottom of the slab beam

⇒ stiffness values are tabulated

$$K_b = \sum \frac{9 E_c s C}{l_2 (1 - c_2/l_2)^3} \left( \times \frac{I_{sb}}{I_s} \right) \begin{matrix} \text{(slab beam)} \\ \text{(slab)} \\ \text{if a beam is used} \\ \text{in the longitudinal direction} \end{matrix}$$

summation = slab beams on both sides of the column

$l_2$  measured (transverse) center-to-center of the supports and may have different values in each of the summation terms

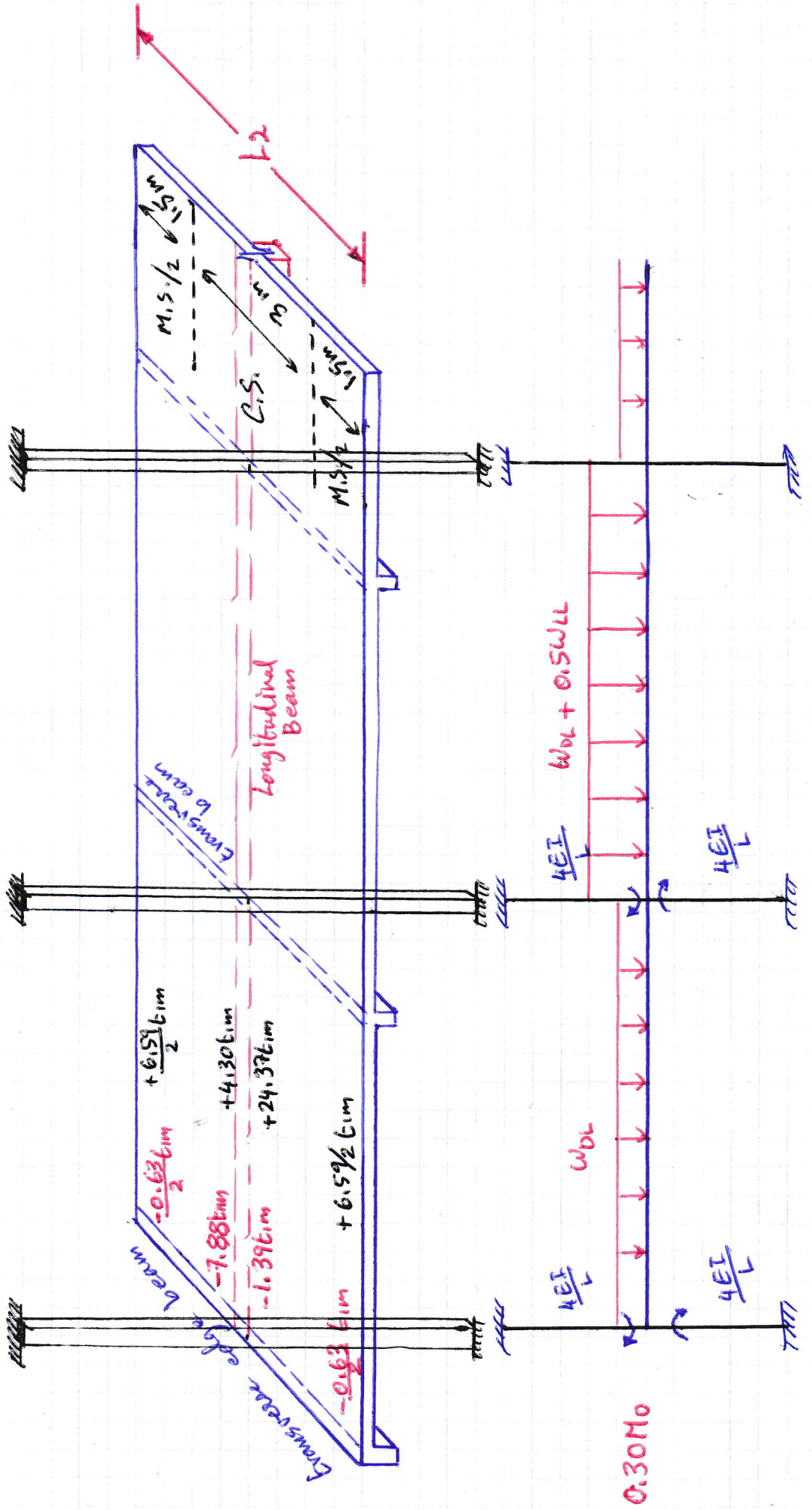
$I_{sb}$  = Moment of inertia of the total slab width including the beam drop

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$L_1$

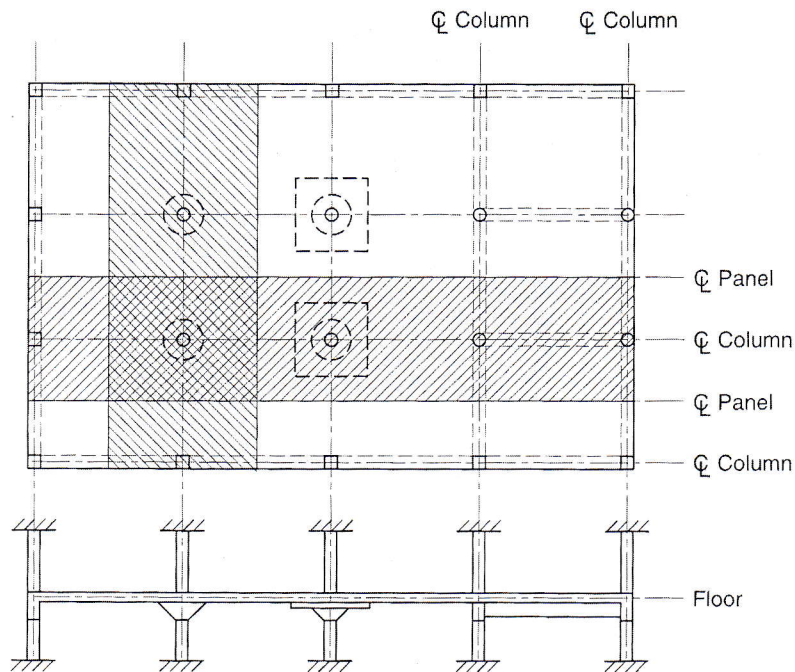
$+ 35.26$   
t/m

$- 9.90$   
t/m



$0.30M_0$

FIGURE 13.17  
 Slab idealization for  
 equivalent frame analysis.



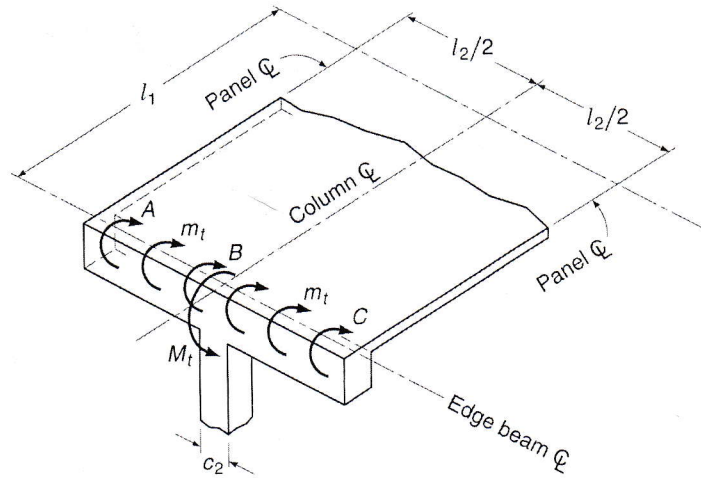
For the beam strips, the first change from the midspan moment of inertia normally occurs at the edge of drop panels, if they are used. The next occurs at the edge of the column or column capital. While the stiffness of the slab strip could be considered infinite within the bounds of the column or capital, at locations close to the panel centerlines (at each edge of the slab strip), the stiffness is much less. According to ACI Code 13.7.3, from the center of the column to the face of the column or capital, the moment of inertia of the slab is taken equal to the value at the face of the column or capital, divided by the quantity  $(1 - c_2/l_2)^2$ , where  $c_2$  and  $l_2$  are the size of the column or capital and the panel width, respectively, both measured transverse to the direction in which moments are being determined.

Accounting for these changes in moments of inertia results in a member, for analysis, in which the moment of inertia varies in a stepwise manner. The stiffness factors, carryover factors, and uniform-load fixed-end moment factors needed for moment distribution analysis (see Chapter 12) are given in Table A.13a of Appendix A for a slab without drop panels and in Table A.13b for a slab with drop panels with a depth equal to 1.25 times the slab depth and a total length equal to one-third the span length.

### c. The Equivalent Column

In the equivalent frame method of analysis, the columns are considered to be attached to the continuous slab beam by torsional members that are transverse to the direction of the span for which moments are being found; the torsional member extends to the panel centerlines bounding each side of the slab beam under study. Torsional deformation of these transverse supporting members reduces the effective flexural stiffness provided by

**FIGURE 13.18**  
Torsion at a transverse supporting member illustrating the basis of the equivalent column.



the actual column at the support. This effect is accounted for in the analysis by use of what is termed an *equivalent column* having stiffness less than that of the actual column.

The action of a column and the transverse torsional member is easily explained with reference to Fig. 13.18, which shows, for illustration, the column and transverse beam at the exterior support of a continuous slab-beam strip. From Fig. 13.18, it is clear that the rotational restraint provided at the end of the slab spanning in the direction  $l_1$  is influenced not only by the flexural stiffness of the column but also by the torsional stiffness of the edge beam AC. With distributed torque  $m_t$  applied by the slab and resisting torque  $M_t$  provided by the column, the edge-beam sections at A and C will rotate to a greater degree than the section at B, owing to torsional deformations of the edge beam. To allow for this effect, the actual column and beam are replaced by an equivalent column, so defined that the total flexibility (inverse of stiffness) of the equivalent column is the sum of the flexibilities of the actual column and beam. Thus

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{K_t} \tag{13.9}$$

- where  $K_{ec}$  = flexural stiffness of equivalent column
- $K_c$  = flexural stiffness of actual column
- $K_t$  = torsional stiffness of edge beam

all expressed in terms of moment per unit rotation. In computing  $K_c$ , the moment of inertia of the actual column is assumed to be infinite from the top of the slab to the bottom of the slab beam, and  $I_g$  is based on the gross concrete section elsewhere along the length. Stiffness factors for such a case are given in Table A.13c.

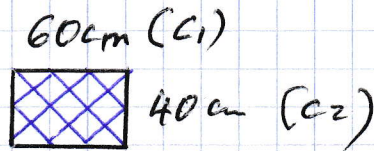
The effective cross section of the transverse torsional member, which may or may not include a beam web projecting below the slab, as shown in Fig. 13.18, is the same as defined earlier in Section 13.6c. The torsional constant  $C$  is calculated by Eq. (13.6) based on the effective cross section so determined. The torsional stiffness  $K_t$  can then be calculated by the expression

$$K_t = \sum \frac{9E_{cs}C}{l_2(1 - c_2/l_2)^3} \tag{13.10}$$

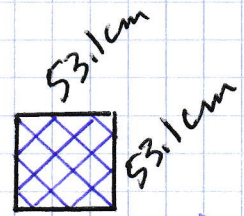
# Equivalent Column

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Interior Column



$$\frac{40 \times 60^3}{12} = 720000 \text{ cm}^4$$



$$\frac{53.1 \times 53.1^3}{12} = 662500 \text{ cm}^4$$

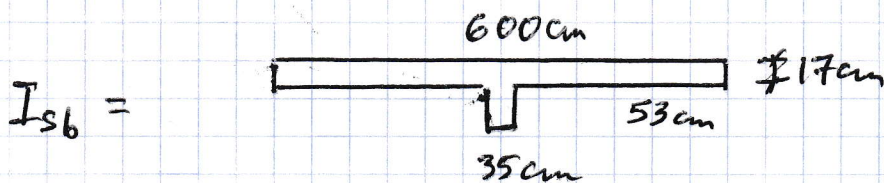
Frame A

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{K_t}$$

$$\begin{aligned} \sum K_c &= 2 \left( \frac{4 E_c \left( \frac{40 \times 60^3}{12} \right)}{370} \right) \\ &= 15568 E_c \end{aligned}$$

(B5)

$$\begin{aligned} K_t &= 2 \left( \frac{9 E_c \left( 4.76 \times 10^5 \right)}{600 \left( 1 - 40/600 \right)^3} \right) \left( \frac{2602581}{600 \times 17^3 / 12} \right) \\ &= 186082 E_c \end{aligned}$$



$I_{sb} =$

56111 cm

$$I_{sb} = 2602581$$

$$\frac{1}{K_{ec}} = \frac{1}{15568 E_c} + \frac{1}{186082 E_c}$$

$$K_{ec} = 14366 E_c = \frac{(2)(4 E_c (b)(b^3)/12)}{370}$$

$$b = 53.1 \text{ cm}$$



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$I_s =$  moment of inertia of the slab alone

#### d. Moment analysis

if  $LL \leq \frac{3}{4} DL$  (service)

maximum moment assumed to occur at all critical sections when the full load acts everywhere (ACI 13.7.6)

$\Rightarrow$  otherwise, use cases of loading when using cases of loading, only  $(\frac{3}{4} LL)$  factored is used for maximum load on a panel.

Note: factored moments must not be taken less than that obtained when full factored LL is placed on ALL panels.

#### e. Computers

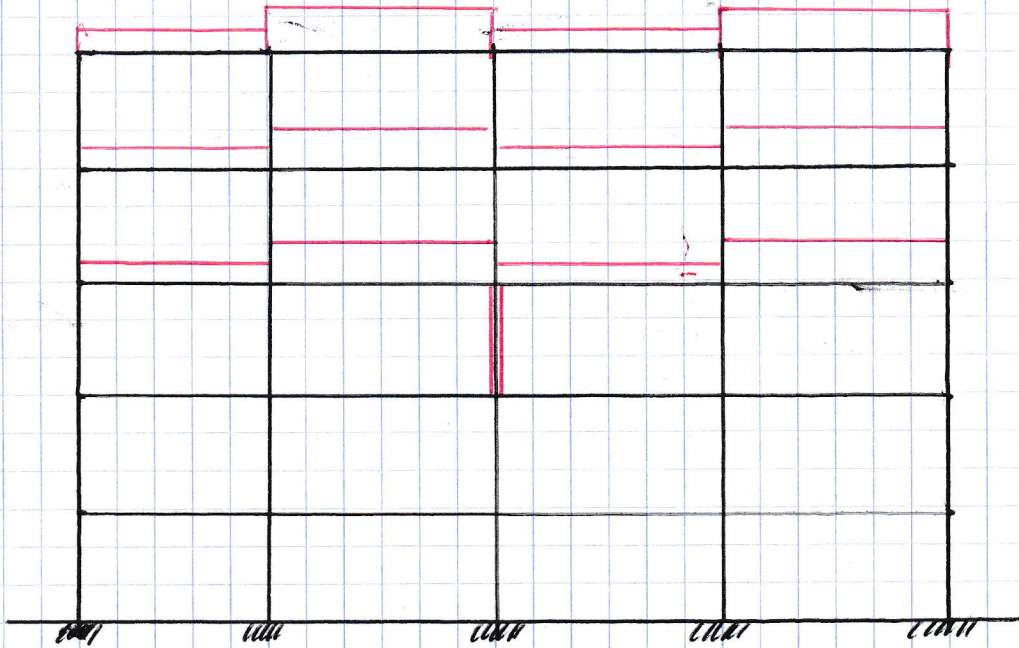
General purpose programs based on the direct stiffness method.

Variable  $I$  values require nodal points (continuous joints) between sections with constant  $I$ .

Also, compute  $K_{ec}$  for each column, then compute  $I_{equivalent}$

Alternately, a 3-dimensional frame analysis may be used in which

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$$LL \leq \frac{3}{4} DL \quad (\text{Service})$$

A)

full load acts everywhere

$$\max = (1.2 DL + 1.6 LL) \quad \downarrow \text{ case of loading only}$$

B)

$$LL > \frac{3}{4} DL \quad (\text{Service}) \quad \text{Multiple cases of loading}$$

$$\max = 1.2 DL + 1.6 (0.75 LL)$$

$$\min = 1.2 DL$$

The results must also be compared to those obtained in case A - the larger values are to be used.

the torsional properties of the transverse beams may be included directly.

Or, use special computer programs developed for slab systems.