- Home work #1 / ENCE 436

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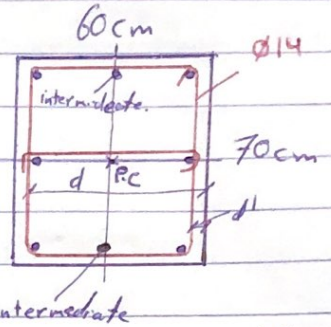


$f_c' = 35\ MPa = 0.35\ t/cm^2$

$f_y = 420\ MPa = 4.2\ t/cm^2$

$\beta_1 = 0.80$

$A_s = 76.96\ cm^2$

$A_g = 4200\ cm^2$

$\rho_g = 1.83\ \%$

$d = 52.85\ cm$

$d' = 7.15\ cm.$

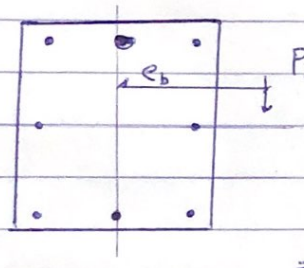
[1] $P_o = ?$ when load in P.C ($e=0$).

$P_o = 0.85 f_c' A_g + (f_y - 0.85 f_c') A_s$

$= 0.85 (0.35)(4200) + (4.2 - 0.85(0.35)) 76.96$

$= 1549.8\ t$

[2] $e_{balanced} = ?$, $P_b = ?$

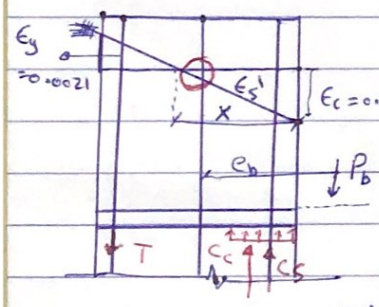


* ignore intermediate bars.

$X = \frac{0.003}{(0.003 + 0.0021)} (52.85)$

$X = 31.1\ cm$

$\Rightarrow \epsilon_s' = \frac{(31.1 - 7.15)}{31.1} (0.003) = 0.0023 > \epsilon_y$



$\therefore$ Steel in Comp. will yield $\therefore f_s' = f_y = 4.2\ t/cm.$

$T = (3)\left(\frac{3.5^2}{4}\right) 3.14\ (4.2) = 121.2\ t$

$C_s = A_s (f_s' - 0.85(f_c')) = 112.6\ t$

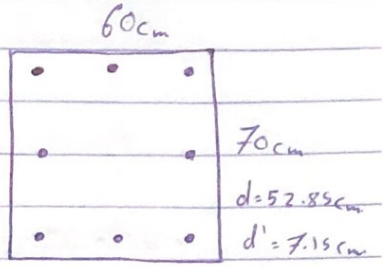
$C_c = 0.85 f_c' (70)(0.8)(31.1) = 518.1\ t$

$+\uparrow \Sigma f_y = 0 \Rightarrow P_b = C_c + C_s - T = 509.5\ t$

$+\circlearrowleft \Sigma M_{P.C} = 0 \Rightarrow e_b P_b = T(22.85) + C_s(22.85) + C_c(17.56)$

$e_b = 28.3\ cm \Rightarrow M_b = P_b * e_b = 144.2\ t.m$

[3] $e = 20$ cm , $P_n = ?$, $M_n = ?$

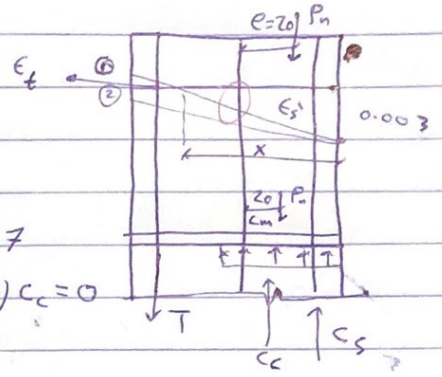


* assume Case ① tensioned in left steel.
* Steel in Comp. Side will yield.

$$\epsilon_t = \frac{52.85 - X}{X}\,(0.003)$$

$$f_s' = \frac{317.1}{X} - 6$$

$$T = (28.85)\left(\frac{317.1}{X} - 6\right) = \frac{9148.3}{X} - 173.1$$



$$C_s = (28.85)(4.2 - 0.85(0.35)) = 112.6\ t$$

$$C_c = 0.85\, f_c'\,(70)(0.8)\,X = 16.66\,X$$

$$P_n = C_c + C_s - T = 16.66X - \frac{9148.3}{X} + 285.7$$

$$\circlearrowleft \Sigma M_{P.C} = 0 \Rightarrow (20)P_n - 22.85T - 22.85C_s - \left(30 - \frac{0.8X}{2}\right)C_c = 0$$

$$20\left[16.66X - \frac{9148.3}{X} + 285.7\right] - 22.85\left[\frac{9148.3}{X} - 173.1\right]$$

$$- 22.85(112.6) - (30 - 0.4X)(16.66X) = 0$$

$$333.2X - \frac{182966}{X} + 5714 - \frac{209038}{X} + 3955.225 = 1382.425 - 499.8X + 6.664X^2 = 0$$

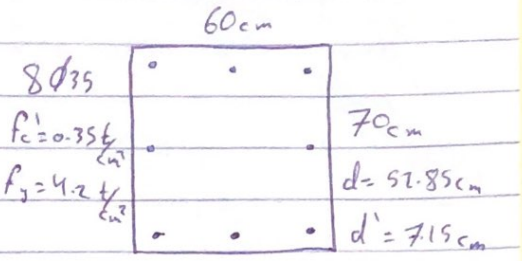
$$6.664X^3 - 166.6X^2 + 7096.425X - 392004 = 0$$

$X = 37.88$ cm $< d = 52.85$ cm as we assumed ✓

{check: $\Rightarrow \epsilon_t = \frac{52.85 - 37.88}{37.88}(0.003) = 0.001186$

$T = 68.4$ t
$C_s = 112.6$ t
$C_c = 631.1$ t
} $\Rightarrow P_n = 675.3$ t , $M_n = 135.06$ t.m.

[4] $e = 60$ cm , $P_n = ?$, $M_n = ?$



* Steel in tension will Yield.
* assume steel in Comp. not Yield.

$\epsilon_s' = \frac{x - 7.15}{x}(0.003) \Rightarrow f_s' = 6 - \frac{42.9}{x}$

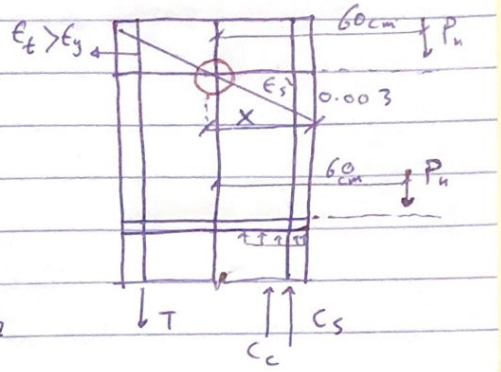
$C_s = A_s (f_s' - 0.85 f_c') = (28.85)(5.7025 - \frac{42.9}{x})$

$= 164.52 - \frac{1237.7}{x}$

$T = f_y (28.85) = 121.2$ t

$C_c = 0.85 f_c' (70) 0.8 x = 16.66 x$

$P_n = C_c + C_s - T = 16.66x - \frac{1237.7}{x} + 43.32$



$\circlearrowleft \Sigma M_{P.C} = 0 \Rightarrow 60 P_n - 22.85 T - 22.85 C_s - C_c [30 - \frac{0.8x}{2}] = 0$

$60[16.66x - \frac{1237.7}{x} + 43.32] - (22.85)(121.2) - 22.85(164.52 - \frac{1237.7}{x})$

$- (16.66x)[30 - 0.4x] = 0$

$999.6x - \frac{74267}{x} + 2599.2 - 2769.42 - 3759.3 + \frac{28281}{x} - 499.8x + 6.664x^2 = 0$

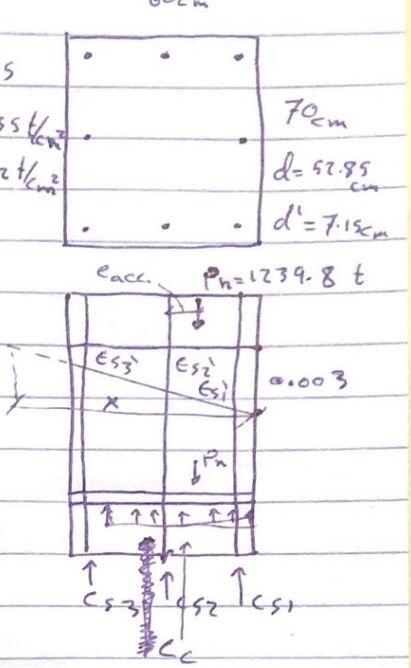
$6.664x^3 + 499.8x^2 - 3929.5x - 45981 = 0 \Rightarrow x = 12.83$ cm.

$\Rightarrow \epsilon_s' = 0.00133 < \epsilon_y$ ∴ not Yield ∴ The assumption is Correct.

$T = 121.2$ t
$C_c = 213.7$ t
$C_s = 68.1$ t

$\Rightarrow P_n = 160.6$ t , $M_n = 96.36$ t.m

[5] $e_{accidental} = ?$, when $P_n = 0.8\,P_o$ (Tied Column)



$P_n = 1239.8$ t

Steel in right will yield

∴assume all steel in compression and other steel not yield.

$\epsilon_{s3}' = \frac{0.003}{x}(x - 57.85) \Rightarrow f_{s3}' = 6 - \frac{317.1}{x}$

$\epsilon_{s2}' = \frac{0.003}{x}(x - 30) \Rightarrow f_{s2}' = 6 - \frac{180}{x}$

$C_{s1} = A_s(f_y - 0.85 f_c') = 28.85(4.2 - 0.85(0.35))$

$= 112.6$ t

$C_{s2} = A_s(f_{s2}' - 0.85 f_c') = (19.23)\left(6 - \frac{180}{x} - 0.85(0.35)\right)$

$= 109.7 - \frac{3461.4}{x}$

$C_{s3} = (28.85)(f_{s3}' - 0.85 f_c') = 28.85\left(5.7025 - \frac{317.1}{x}\right)$

$= 164.52 - \frac{9148.34}{x}$

$C_c = 0.85 f_c' (70)(0.8)x = 16.66\,x$

$P_n = C_{s1} + C_{s2} + C_{s3} + C_c \Rightarrow 1239.8 = 386.82 - \frac{12609.74}{x} + 16.66\,x$

$16.66\,x^2 - 853.02\,x - 12609.7 = 0$

$x^2 - 51.2\,x - 756.887 = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = 63.18$ cm.

$\Rightarrow a = 50.54$ cm.

Check ϵ_s' @ $X > h$ so all steel in compression.

② $\epsilon_{s3}' = 0.00049 < \epsilon_y = 0.0021$

③ $\epsilon_{s2}' = 0.00158 < \epsilon_y = 0.0021$

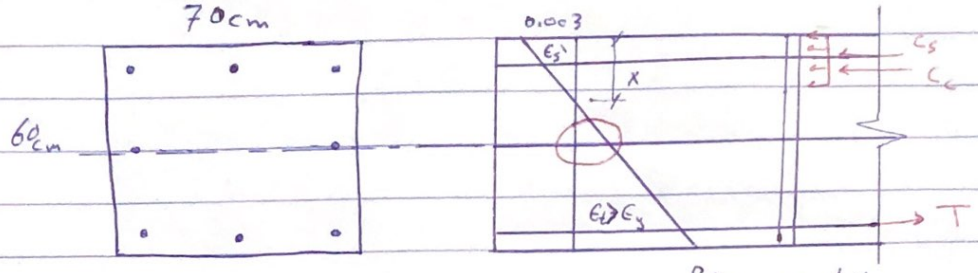
The assumption is correct.

$P_n = 1239.8$ t , $C_{s1} = 112.6$ t , $C_{s2} = 54.9$ t , $C_{s3} = 19.7$ t , $C_c = 1052.6$ t.

+) $\Sigma M_{P.C} = 0 \Rightarrow e_{acc.} P_n + (27.85)\,C_{s3} - (4.728)\,C_c + (0)\,C_{s2} - (27.85)\,C_{s1} = 0$

$\Rightarrow e_{accidental} = 5.73$ cm. , $M_n = 71.04$ t.m

6 $e = \infty$, $P_n = 0$, $M_o = ?$



$f_c' = 0.35\ t/cm^2$

$f_y = 4.2\ t/cm^2$

$T = 121.2\ t$

$C_s = A_s (f_s' - 0.85 f_c')$

$= 164.52 - \frac{1237.7}{x}$

$\epsilon_s' = \frac{0.003}{x}(x - 7.15)$

$C_c = 16.66\ x$

$T = C_s + C_c \Rightarrow 121.2 = 164.52 - \frac{1237.7}{x} + 16.66\ x$

$16.66\ x^2 + 43.32\ x - 1237.7 = 0$

$x^2 + 2.6\ x - 74.292 = 0$

$C_s = -2.286\ t$

$x = 7.42\ cm > d' = 7.15\ cm$

∴ The steel in Compresion will not resist the Compresion only Concrete.

So the assumption of strain line is Correct.

⇒ Repeat:

$T = 121.2\ t$, $C_c = 16.66\ x$

$T = C_c \Rightarrow 121.2 = 16.66\ x \Rightarrow x = 7.275\ cm. > d'$

$C_s = 0$

$\Rightarrow a = 5.82\ cm$

$C_c = 121.2\ t$

$M_o = T(d - a/2) = C_c (d - a/2)$

$M_o = 121.2\ (52.85 - 5.82/2)$

$M_o = 60.53\ t.m$

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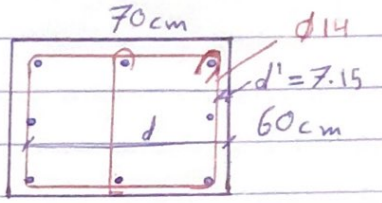


① $P_o = 1549.8\ t$ $(e=0)$.

② $e_{balanced} = ?$, $P_b = ?$, $M_b = ?$

$f'_c = 0.35\ t/cm^2$

$f_y = 4.2\ t/cm^2$

$\beta_1 = 0.8$

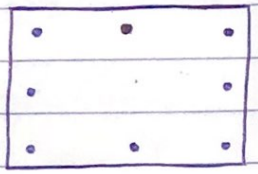
$A_s = 76.96\ cm^2$

$A_g = 4200\ cm^2$

$\rho_g = 1.83\ \%$

$d = 62.85\ cm$

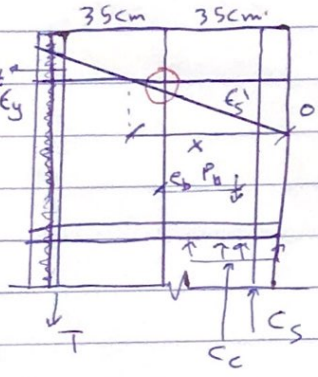
$d' = 7.15\ cm$.



* ignore indermidiate bars.

$x = \frac{d}{0.003+0.0021}(0.003) = 36.97\ cm$

$\Rightarrow a = 29.576\ cm$.



$\epsilon'_s = 0.00242 > \epsilon_y = 0.0021$

Yield.

$C_s = A_s(f_y - 0.85 f'_c)$

$C_s = 112.6\ t$

$C_c = 0.85 f'_c (60)(0.8) x$

$C_c = 527.9\ t$

$T = A_s f_y = 121.2\ t$

$+\uparrow \Sigma F_y = 0 \Rightarrow P_b = C_c + C_s - T = 519.3\ t$

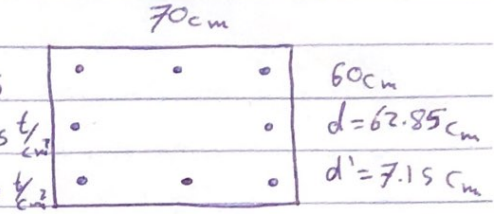
$+\circlearrowleft \Sigma M_{P.C} = 0 \Rightarrow P_b e_b - T(27.85) - C_s(27.85) - C_c(35 - a/2) = 0$

($C_c(35 - a/2) = 20.212$)

$\Rightarrow e_b = 33.085 \approx 33.1\ cm$.

$M_n = 171.9\ t.m$.

[3] $e = 20$ cm , $P_n = ?$, $M_n = ?$



$f_c' = 0.35\ t/cm^2$

$f_y = 4.2\ t/cm^2$

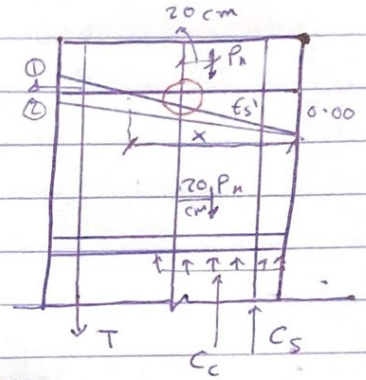
* assume Case ① (Tension in left steel).
* steel in Compresion side will yield.

$\Rightarrow \epsilon_t = \frac{d-x}{x}(0.003) = \frac{62.85-x}{x}(0.003)$

$f_s' = \frac{377.1}{x} - 6$

$T = f_s' A_s = \left[\frac{377.1}{x} - 6\right](28.85)$

$T = \frac{10879.3}{x} - 173.1$



$C_s = A_s (f_y - 0.85 f_c') = 112.6\ t$

$C_c = 0.85 f_c' (60)(0.8) x = 14.28\ x$

$+\uparrow \Sigma F_y = 0 \Rightarrow P_n = C_c + C_s - T$

$P_n = 14.28\ x + 285.7 - \frac{10879.3}{x}$

$+\circlearrowleft \Sigma M_{P_c} = 0 \Rightarrow 20 P_n - 27.85(T) - 27.85\ C_s - C_c\left[35 - \frac{0.8x}{2}\right] = 0$

$285.6\ x + 5714 - \frac{217586}{x} - \left[\frac{302988.5}{x} - 4820.8\right] - 3135.9 - 499.8\ x + 5.712 x^2 = 0$

$5.712\ x^3 - 214.2\ x^2 + 7398.9\ x - 520574.5 = 0$

$\Rightarrow x = 49.0113$ cm $< d = 62.85$ cm ∴ The assumption is correct.

$T = 48.88\ t \approx 48.9\ t$

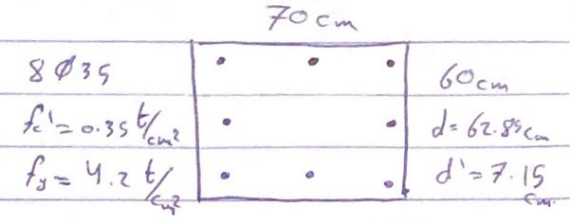
$C_s = 112.6\ t$

$C_c = 699.9\ t$

$\Rightarrow P_n = 763.6\ t$

$M_n = 152.72\ t \cdot m.$

(4) $e = 60$ cm , $P_n = ?$, $M_n = ?$



* Steel in tension will yield.
* assume steel in Comp. not yield.

$$\epsilon_s' = \frac{0.003}{x}(x - 7.15)$$

$$f_s' = 6 - \frac{42.9}{x}$$

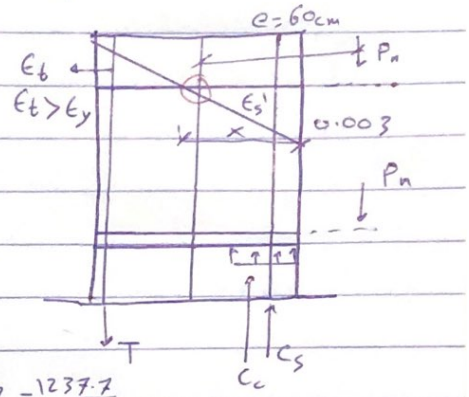
$$C_s = A_s(f_s' - 0.85 f_c')$$

$$C_s = 164.52 - \frac{1237.7}{x}$$

$$C_c = 0.85 f_c' (60)(0.8) x = 14.28 x$$

$$T = A_s f_y = 121.2 \text{ t}$$

$$P_n = C_c + C_s - T = 14.28 x + 43.32 - \frac{1237.7}{x}$$



$$\sum M_{P_c} = 0 \Rightarrow 60 P_n - 27.85 T - 27.85 C_s - C_c\left[35 - \frac{0.8x}{2}\right] = 0$$

$$856.8 x + 2599.2 - \frac{74262}{x} - 3375.42 - \left[4581.9 - \frac{34469.945}{x}\right] - 499.8 x + 5.712 x^2 = 0$$

$$5.712 x^3 + 357 x^2 - 5358.12 x - 39792.055 = 0$$

$x = 16.97$ cm

check $\epsilon_s' = 0.00174 < \epsilon_y$ The assumption is Correct ✓.

$T = 121.2$ t
$C_s = 91.6$ t
$C_c = 242.3$ t

$\Rightarrow P_n = 212.7$ t.

$\Rightarrow M_n = 127.62$ t.m

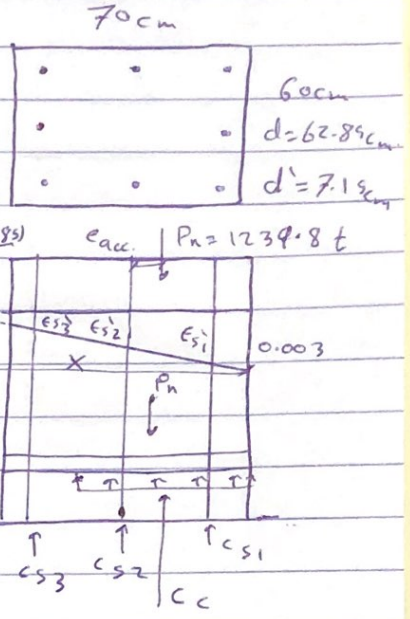
[5] $e_{accidental} = ?$, $M_n = ?$

When $P_n = 0.8\ P_o = 1239.8\ t$

Tied column.

Steel (1) in right will Yield.

* assume all steel in Compression and the rest not Yield.



$\epsilon_{s3}' = \frac{(X - 62.85)}{X}(0.003) \Rightarrow f_{s1}' = \frac{6(X-62.85)}{X}$

$\epsilon_{s2}' = \frac{(X - 35)}{X}(0.003) \Rightarrow f_{s2}' = \frac{6(X-35)}{X}$

$\epsilon_{s1}' > \epsilon_y$

$C_{s1} = A_s (f_y - 0.85 f_c') = (28.85)(4.2 - 0.85(0.35))$

$= 112.6\ t$

$C_{s2} = A_s (f_{s2}' - 0.85 f_c')$

$= (19.23)(5.7025 - \frac{210}{X})$

$C_{s2} = 109.7 - \frac{4038.3}{X}$

$C_{s3} = A_s (f_{s3}' - 0.85 f_c') = (28.85)(6 - \frac{377.1}{X} - 0.85 f_c')$

$= 164.52 - \frac{10879.3}{X}$

$C_c = 0.85 f_c' (60)(0.8) X = 14.28\ X$

$\Sigma f_y = 0 \Rightarrow P_n = C_c + C_{s1} + C_{s2} + C_{s3}$

$1239.8 = 14.28X + 112.6 + 109.7 - \frac{4038.3}{X} + 164.52 - \frac{10879.3}{X}$

$14.28X^2 - 852.98X - 14917.6 = 0 \Rightarrow X = 73.87\ cm$ ∴ the first assumption Correct,

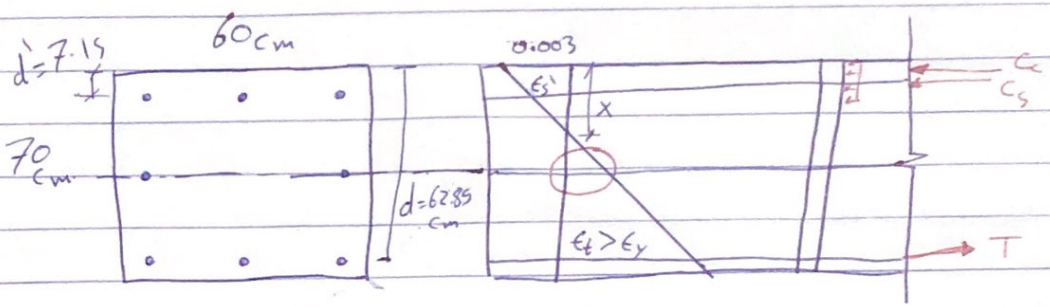
$\epsilon_{s3}' = 0.000447 < \epsilon_y$ and $\epsilon_{s2}' = 0.00158 < \epsilon_y$ ∴ the seand assumption is correct //.

$C_{s1} = 112.6\ t$, $C_{s2} = 55.1\ t$, $C_{s3} = 17.2\ t$, $C_c = 1054.9\ t$

$+\circlearrowleft \Sigma M_{P.C} = 0 \Rightarrow e_{acc.} P_n + 27.85\ C_{s3} + (0) C_{s2} - C_{s1}(27.85) - C_c(35 - 29.55) = 0$

$\Rightarrow e_{accidental} = 6.78\ cm \Rightarrow M_n = 84.06\ t.m$

[6] $M_o = ?$ $(e = \infty)$.



$T = 121.2\ t$

$C_s = A_s(f_s' - 0.85 f_c')$

$= 164.52 - \frac{1237.7}{x}$

$C_c = 14.28\,x$

$T = C_s + C_c \Rightarrow 121.2 = 14.28\,x + 164.52 - \frac{1237.7}{x}$

$0 = x^2 + 3.034\,x - \frac{1237.7}{14.28}$

$0 = x^2 + 3.034\,x - 86.67$

$x = 7.916\ cm. > d' = 7.15\ cm.$

$\Rightarrow \epsilon_s' = 0.00029 < \epsilon_y$

The assumption is correct.

$C_s = 8.17\ t$

$C_c = 113.03\ t$

$a = 0.8\,x = 6.333\ cm.$

$f_c' = 0.35\ t/cm^2$

$f_y = 4.2\ t/cm^2$

$\epsilon_s' = \frac{x - 7.15}{x}(0.003)$

* assume steel in comp. not yield

$M_o = T\,(d - a/2)$

$= 72.34\ t.m.$

Column (b x h)		e (cm)					
		e=0	$e_{accidental}$	e=20	e_b	e=60	e=∞
70 x 60	$P_n(t)$	1549.8	1239.8	675.3	509.5	160.6	0
	$M_n(t.m)$	0	71.04	135.06	144.2	96.36	60.53
60 x 70	$P_n(t)$	1549.8	1239.8	763.6	519.3	212.7	0
	$M_n(t.m)$	0	84.06	152.72	171.9	127.62	72.34

Column (b x h): 70 x 60

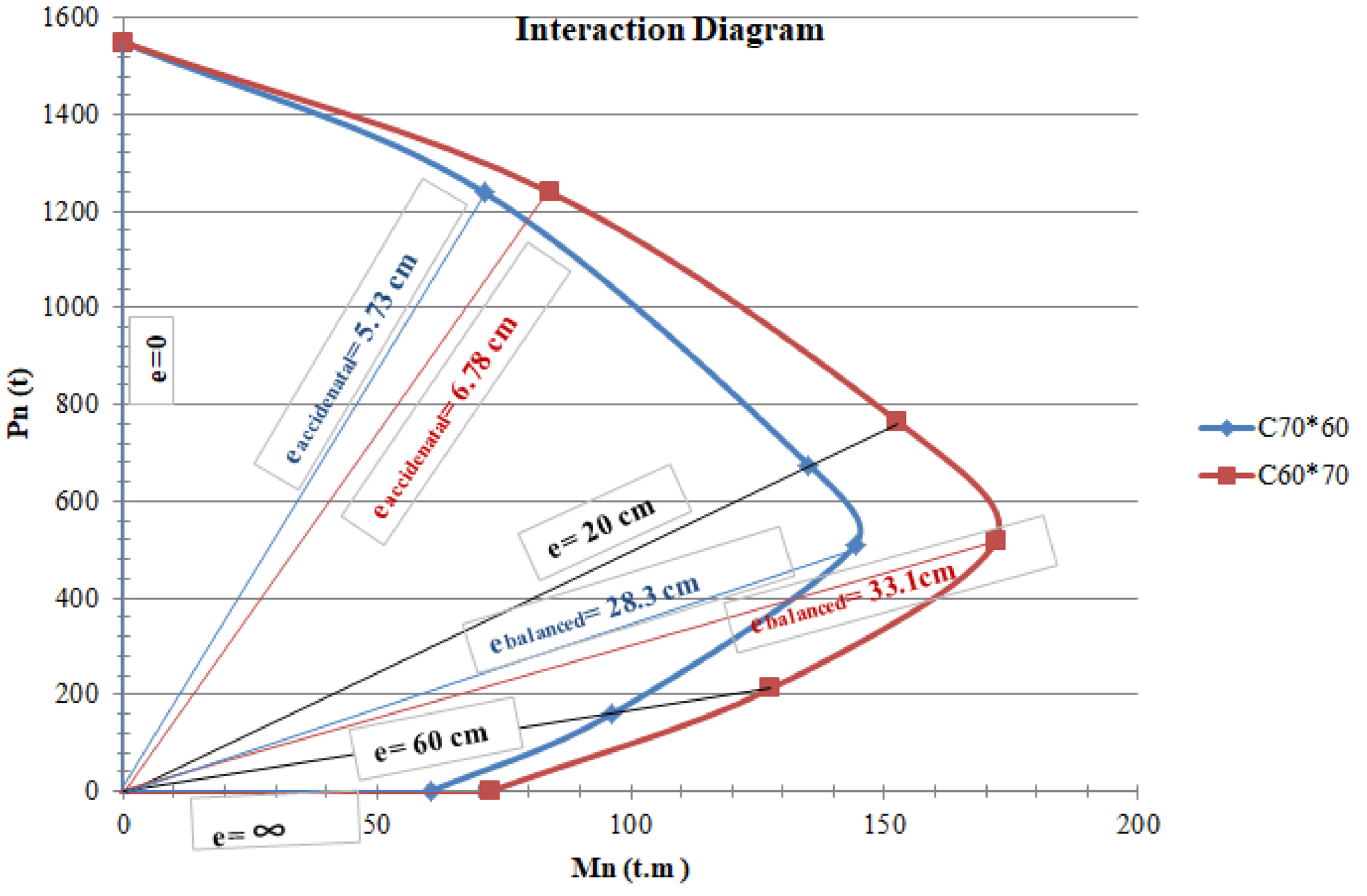
$e_{accidental} = 5.73$ cm

$e_b = 28.3$ cm

60 x 70

$e_{acc.} = 6.78$ cm

$e_b = 33.1$ cm


Interaction Diagram
Pn (t)
Mn (t.m)
1600
1400
1200
1000
800
600
400
200
0
0
50
100
150
200
e=0
e accidental= 5.73 cm
e accidental= 6.78 cm
e= 20 cm
e balanced= 28.3 cm
e balanced= 33.1cm
e= 60 cm
e= ∞
C70*60
C60*70