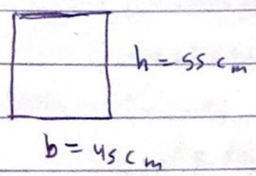
- Homework - Slender Columns - ENCE 436

HW #4 - Mohanad Moayad Shannak - 1181401.

Beams

5 @ 6 m = 30 m

b = 45 cm, h = 55 cm

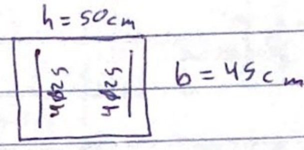


Columns

5 @ 8 m = 40 m

b = 45 cm, h = 50 cm

4 ϕ 25 along each end face.



" braced frame ".

$f_c' = 28$ MPa , $f_y = 420$ MPa.

Service loads

Dead	Live
P = 200 t	P = 100 t
$M_2 = 3$ t.m	$M_2 = 45$ t.m
$M_1 = -3$ t.m.	$M_1 = 17$ t.m.

* Check the capacity of this interior third floor column.
Show that under the magnified eccentricity
$\phi P_n \geq P_u$.

Solution:

$L_c = 8\ m$, $L_u = 8 - 0.55 = 7.45\ m$

$L_g = 6\ m.$

$P_u = 1.2(200) + 1.6(100) = 400\ t$

M_u: $M_2 = 75.6\ t.m$

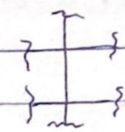
$M_1 = 1.2(-3) + 1.6(17) = 23.6\ t.m$

} Single Curvature.

$e_{actual} = 18.9\ cm.$

$$\Psi_A = \Psi_B = \frac{\Sigma[E_c I_c / L_c]}{\Sigma[E_g I_g / L_g]}$$; $I_c = 0.7\ I_{gross, column}$

$I_g = 0.35\ I_{gross\ of\ beam}$



$I_c = (0.7)\frac{1}{12}BH^3 = 328125\ cm^4$

~~$I_g = (0.35)\frac{1}{12}BH^3 = 218\ 367\ cm^4$~~

~~$E_c = E_g$~~

$$\Rightarrow \Psi_A = \Psi_B = \frac{(2)[(328\ 125)/800]}{(2)[(218367)/600]} = 1.127 \xrightarrow{\text{alignment chart:}} K = 0.78$$

- Check Slenderness affects: $r = 0.3h$

$$\frac{KL_u}{r} = \frac{(0.78)(7.45)}{(0.3)(0.5)} = 38.74$$

$$\left(\frac{KL_u}{r}\right)_{Limit} = \min\begin{cases} 34 + 12\left(\frac{M_{1ns}}{M_{2ns}}\right) = 34 - 12\left(\frac{23.6}{75.6}\right) = 30.25 \\ 40 \end{cases}$$

∴ So Slenderness affects must be Considered.

$$M_{max} = \left(\frac{C_m}{1-\alpha}\right) M_m$$

$\rightarrow C_m = 0.6 - 0.4\frac{M_{1ns}}{M_{2ns}} \geq 0.4$

$= 0.6 - 0.4(-0.312) = 0.725 \geq 0.4$

$M_{m,min.} = P_u(e_{min})$, $e_{min} = 15 + 0.03h$ (mm, mm)

$= (400)(15 + 0.03(500)) = 12\ t.m.$

$= 12000\ t.mm.$

< 75.7

M_{min} not controlled.

$\rightarrow \alpha = P_u / 0.75\ P_{cr}$

$\rightarrow P_{cr} = \frac{\pi^2 EI}{(KL_u)^2}$

$\rightarrow EI =$ largest of $\left\{ \frac{0.2 E_c I_g + E_s I_{se}}{1 + \beta_{dns}} , \frac{0.4 E_c I_g}{1 + \beta_{dns}} \right.$

$\rightarrow \beta_{dns} = \frac{1.2\ DL}{P_u} = 0.6$

$\Rightarrow$ $E_{concrete\,(MPa)} = 4700 \sqrt{f_c'}_{(MPa)} = 24\,870 \text{ MPa} = 248.7 \ t/cm^2$

$I_{gross} = \frac{1}{12} BH^3 = \frac{1}{12} (45)(50)^3 = 470\,000 \ cm^4$

$A_{bar} = 4.9 \ cm^2$ $\quad I_{sec} = (2)\underset{bar}{(4)}\underset{area\ b}{(4.9)}\underset{d^2}{(18.75)^2} = 13\,800 \ cm^4$

$EI = \text{largest} = \begin{cases} \dfrac{0.2 E_c I_g + E_s I_{se}}{1+\beta_{dns}} = \dfrac{(0.2)(248.7)(470\,000) + (2000)(13\,800)}{1+0.6} = 31.86 \times 10^6 \ t.cm^2 \\ \dfrac{0.4 E_c I_g}{1+\beta_{dns}} = \dfrac{(0.4)(248.7)(470\,000)}{1+0.6} = 29.2 \times 10^6 \ t.cm^2 \end{cases}$

$EI = 31.86 \times 10^6 \ t.cm^2$

$P_{cr} = \dfrac{\pi^2 EI}{(KL_u)^2} = \dfrac{\pi^2 (31.86)(10^6)}{((0.78)(745))^2} = \dfrac{\pi^2 (52.37) 10^6}{(745)^2} = 930 \ t$

$\alpha = P_u / 0.75 P_{cr} = \dfrac{400}{(0.75)(930)} = 0.573$

$\delta_{ns} = \dfrac{C_m}{1-\alpha} = 1.7 < 1.4$ - upper limit

$\delta_{ns} = 1.4$

$M_{max} = \delta_{ns} M_m$

$= (1.4)(75.6) = 105.84 \ t.m$

Check for $e = P_u / M_{max} = 26.46$ cm.

$P_u = 400 \ t \Rightarrow P_n = \dfrac{400}{0.65} = 615.4 \ t.$

$e_{balanced} = ?$

$P_{balanced} = ?$

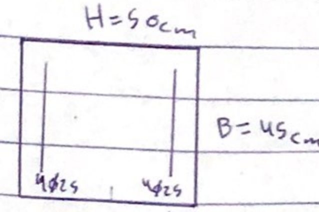
$f_c' = 28\,MPa$

$f_y = 420\,MPa$

$B_1 = 0.85$

tie $\phi 10$

$A_{bar} = 4.9\,cm^2$



$X = \frac{0.003}{0.003+0.0021}(d) = 25.74\,cm$

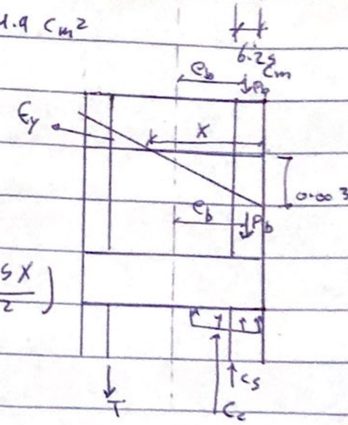
$\epsilon_s' = 0.00227 > \epsilon_y$

$\therefore$ Steel in Comp. Yield

$T = (4)(4.9)(4.2) = 82.32\,t$

$C_s = (4)(4.9)(4.2 - 0.85(0.28)) = 77.66\,t$

$C_c = (0.85)(0.28)(0.85)(25.74)(45) = 234.32\,t$



$\Sigma F_y = 0 \Rightarrow P_b = C_c + C_s - T = 229.66\,t$

$\Sigma M_{P.C} = 0 \Rightarrow P_b(e_b) = (18.75)(T + C_s) + C_c\left(25 - \frac{0.85X}{2}\right)$

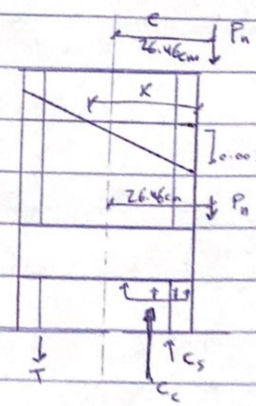
$e_b = 27.4\,cm.$

$e = 26.46\,cm$, $P_n = ?$

Compression Control

assume :- Steel in tension + Not Yield.

Steel in Compression will Yield



$T = (4)(4.9)(f_s)$, $f_s = \frac{262.5}{X} - 6$

$T = \frac{5145}{X} - 117.6$

$C_s = (4)(4.9)(4.2 - 0.85(0.28)) = 77.66\,t$

$C_c = 9.1\,X$

$P_n = C_c + C_s - T = 9.1X + 195.26 - \frac{5145}{X}$

$\Sigma M_{P.C} = 0 \Rightarrow P_n(e) = 18.75(T + C_s) + C_c\left[25 - \frac{0.85}{2}X\right]$

$241X + 5167 - \frac{136136}{X} = -749 + \frac{96469}{X} + 227.5X - 3.87X^2$

$3.87X^2 + 13.5X + 5916 - \frac{232605}{X} = 0 \Rightarrow X = 26.11\,cm$

Check assumption: $f_s = \frac{262.5}{X} - 6 = 4.05\,t/cm^2 < f_y$ $\therefore$ The assumption is Correct.

$\Rightarrow P_n = 235.8\,t \Rightarrow \phi P_n = 153\,t < P_u = 400\,t$

$\therefore$ The Column is not adequate.