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# ACI Code - General Analysis Method

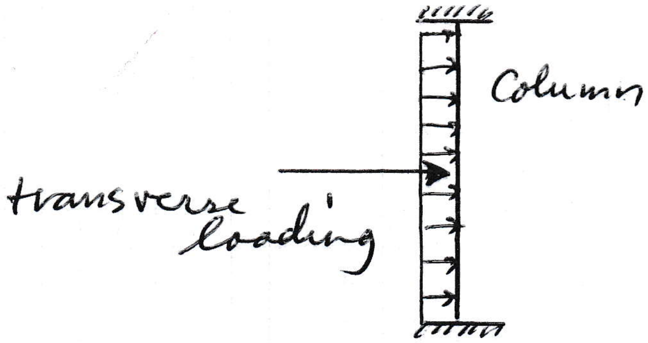
## Moment Magnifier Method - Slender Columns for Non-Sway, Braced, Frames

$$M_{max} = \left(\frac{C_m}{1-\alpha}\right) M_m$$

$$= \delta_{ns} \, M_m$$

$\delta_{ns} = \left(\frac{C_m}{1-\alpha}\right)$ is a magnification factor $\geq 1.0$

$M_m$ = Maximum primary moment, factored
Elastic frame analysis
may occur at either end,
or in the midspan region if there
is a transverse loading




$$\alpha = P_u / 0.75 P_{cr}$$

$C_m = 1.0$, if transverse loading exists

$$= \left(0.6 - 0.4 \frac{M_{1ns}}{M_{2ns}}\right) \geq 0.4$$

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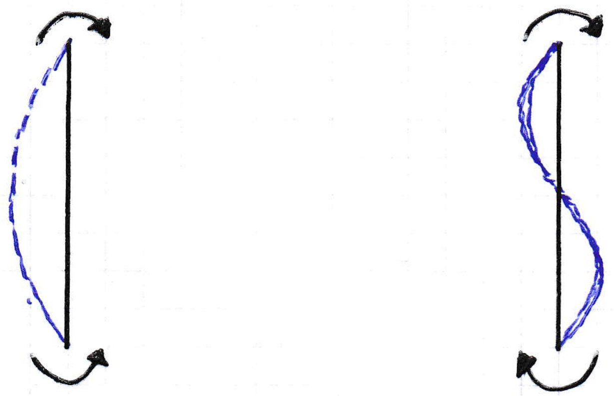
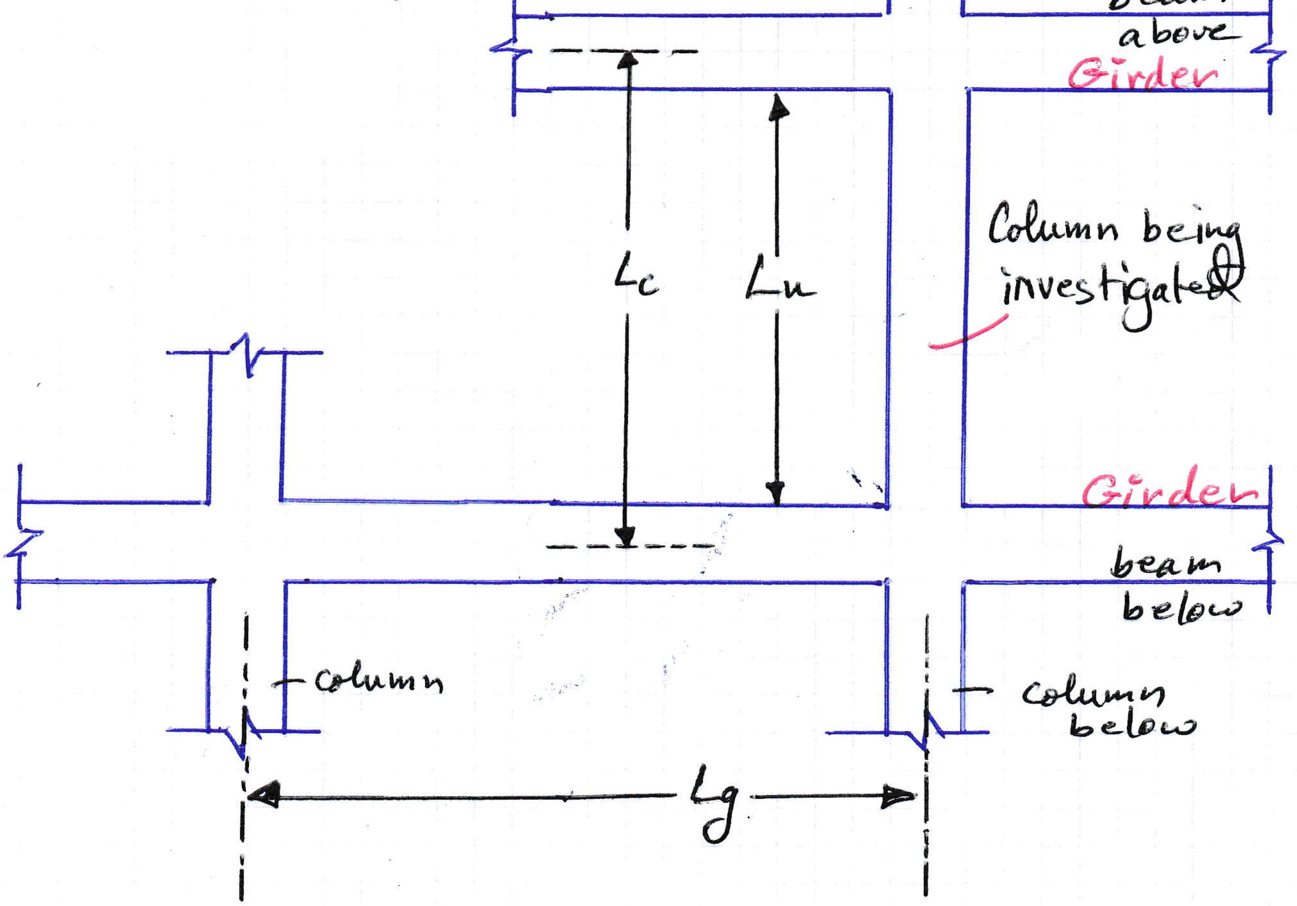
ns = nonsway

$M_{2ns} > M_{1ns}$ , (−ve) for single curvature



Single Curvature        Double Curvature

$$P_{cr} = \frac{\pi^2 EI}{(k\,l_u)^2}$$



$EI$ = larger of

$$= \frac{0.2 E_c I_g + E_s I_{se}}{1 + \beta_{dns}}$$

$$= \frac{0.4 E_c I_g}{1 + \beta_{dns}}$$

$I_g$ = gross moment of inertia of the column ignoring the steel reinforcement

$I_{se}$ = moment of inertia of the reinforcement

$$\beta_{dns} = \frac{\text{factored axial dead load}}{\text{factored axial total load}}$$

$e_{min}$ = 15 + 0.03h (mm units)

The required nominal strength of the column is:

$P_{n\,req} = P_u / \phi$

$M_{n\,req} = M_{max} / \phi$

$(M_m)$ must be larger than $P_u(e_{min})$

$$M_m \geq P_u (15 + 0.03h)$$

if $M_{1ns}$ and $M_{2ns}$ are less than $P_u (e_{min})$, this would indicate that $C_m$ must be taken as 1.0.

For braced frame members, the ACI-Code permits neglect of slenderness effects if:

$$\frac{k\,l_u}{r} \leq 34 + 12\,\frac{M_{1ns}}{M_{2ns}} \leq 40$$

$\frac{M_{1ns}}{M_{2ns}}$ is −ve for single curvature

+ve for double curvature

When the member is subject to large transverse loading, the ratio should be taken as (−1)

For sway frames, the limit is

$$\frac{k\,l_u}{r} \leq 22$$

# Effective Length Factors

$$\psi = \frac{\Sigma (E_c I_c / L_c)}{\Sigma (E_g I_g / L_g)}$$

$I_c$ = effective moment of inertia of column = 0.70 $I_{gross}$

$I_g$ = effective moment of inertia of girder = 0.35 $I_{gross}$

$L_c$ = Length of column, c/c

$L_g$ = Length of girder, c/c

$E_g$, $E_c$ = moduli of elasticity

Then use alignment charts to determine $k$

Given two identical frames, one braced and the other unbraced, the effective length of the columns will always be greater in the unbraced frame than in the braced frame. Since the strength of a column, like the stiffness of a structure, decreases as the effective length increases, the designer should ensure that bracing elements are incorporated into a structure.

**EXAMPLE 7.1.** Under factored gravity and wind loads, a first-order structural analysis determines that the third floor of a reinforced concrete building frame displaces laterally, with respect to the second floor, a distance $\Delta_0 = 0.48$ inches. The analysis produces the forces shown in Fig. 7.11. Verify if the columns in the story are considered members of a *braced* or *unbraced* frame using Eq. (7.7) to check the magnitude of the Stability Index, $Q$.

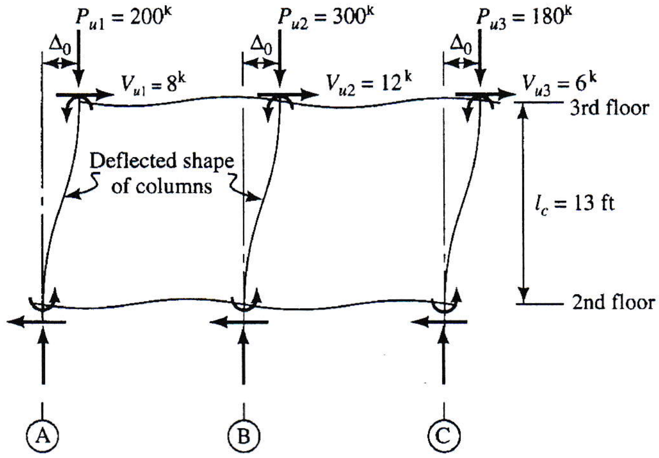



**FIGURE 7.11** Section of a reinforced concrete frame showing both the column forces and the relative lateral displacement between floors ($\Delta_0 = 0.48$ in) created by factored wind and gravity loads.

**Solution.** To be classified as a *braced* frame, $Q = \Sigma P_u \Delta_0/(V_u l_c)$ must not exceed 0.05.

$$Q = \frac{(200 + 300 + 180)0.48 \text{ in}}{(8 + 12 + 6)13 \times 12} = 0.08 > 0.05 \qquad \text{Frame classified as } \textit{unbraced}$$

## 7.4 EFFECTIVE-LENGTH FACTORS FOR COLUMNS OF RIGID FRAMES

In a reinforced concrete frame, columns are rigidly attached to girders and adjacent columns. The effective length of a particular column between stories will depend on how the frame is braced and on the bending stiffness of the girders. As a column bends in response to applied loads, the ends of the attached girders must rotate with the column because of the rigid joint. If the girders are stiff and do not bend significantly, they will provide full rotational restraint to the column, like a fixed support (Fig. 7.12*a*). If the girders are flexible and bend easily, as in Fig. 7.12*b*, they provide only a small degree of rotational restraint, and the end conditions for the column approach those of a pin support that allows unrestrained rotation.

The Jackson and Moreland alignment charts[2] (Fig. 7.13) can be used to evaluate the influence of girder bending stiffness on the effective-length factor of a column that is part of a rigid frame. The charts are entered with values of $\psi$ for the joints at each end of a column. For a rigid frame whose members are prismatic, $\psi$, the ratio of the sum of the relative bending stiffnesses of the columns to that of the girders, is defined as

$$\psi = \frac{\Sigma(E_c I_c / L_c)}{\Sigma(E_g I_g / L_g)} \tag{7.8}$$

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**6.2.4** Additional analysis methods that are permitted include 6.2.4.1 through 6.2.4.4.

**6.2.4.1** Two-way slabs shall be permitted to be analyzed for gravity loads in accordance with (a) or (b):

(a) Direct design method in 8.10
(b) Equivalent frame method in 8.11

**6.2.4.2** Slender walls shall be permitted to be analyzed in accordance with 11.8 for out-of-plane effects.

**6.2.4.3** Diaphragms shall be permitted to be analyzed in accordance with 12.4.2.

**6.2.4.4** A member or region shall be permitted to be analyzed and designed using the strut-and-tie method in accordance with Chapter 23.

**6.2.5** Slenderness effects shall be permitted to be neglected if (a) or (b) is satisfied:

(a) For columns not braced against sidesway

$$\frac{k\ell_u}{r} \leq 22 \qquad (6.2.5a)$$

(b) For columns braced against sidesway

$$\frac{k\ell_u}{r} \leq 34 + 12(M_1/M_2) \qquad (6.2.5b)$$

and

$$\frac{k\ell_u}{r} \leq 40 \qquad (6.2.5c)$$

where $M_1/M_2$ is negative if the column is bent in single curvature, and positive for double curvature.

If bracing elements resisting lateral movement of a story have a total stiffness of at least 12 times the gross lateral stiffness of the columns in the direction considered, it shall be permitted to consider columns within the story to be braced against sidesway.

**6.2.5.1** The radius of gyration, $r$, shall be permitted to be calculated by (a), (b), or (c):

(a) $$r = \sqrt{\frac{I_g}{A_g}} \qquad (6.2.5.1)$$

(b) 0.30 times the dimension in the direction stability is being considered for rectangular columns

## COMMENTARY

Finite element analysis was introduced in the 2014 Code to explicitly recognize a widely used analysis method.

**R6.2.5** Second-order effects in many structures are negligible. In these cases, it is unnecessary to consider slenderness effects, and compression members, such as columns, walls, or braces, can be designed based on forces determined from first-order analyses. Slenderness effects can be neglected in both braced and unbraced systems, depending on the slenderness ratio ($k\ell_u/r$) of the member.

The sign convention for $M_1/M_2$ has been updated so that $M_1/M_2$ is negative if bent in single curvature and positive if bent in double curvature. This reflects a sign convention change from the 2011 Code.

The primary design aid to estimate the effective length factor $k$ is the Jackson and Moreland Alignment Charts (Fig. R6.2.5), which provide a graphical determination of $k$ for a column of constant cross section in a multi-bay frame (ACI SP-17(09); Column Research Council 1966).

Equations (6.2.5b) and (6.2.5c) are based on Eq. (6.6.4.5.1) assuming that a 5 percent increase in moments due to slenderness is acceptable (MacGregor et al. 1970). As a first approximation, $k$ may be taken equal to 1.0 in Eq. (6.2.5b) and (6.2.5c).

The stiffness of the lateral bracing is considered based on the principal directions of the framing system. Bracing elements in typical building structures consist of shear walls or lateral braces. Torsional response of the lateral-force-resisting system due to eccentricity of the structural system can increase second-order effects and should be considered.

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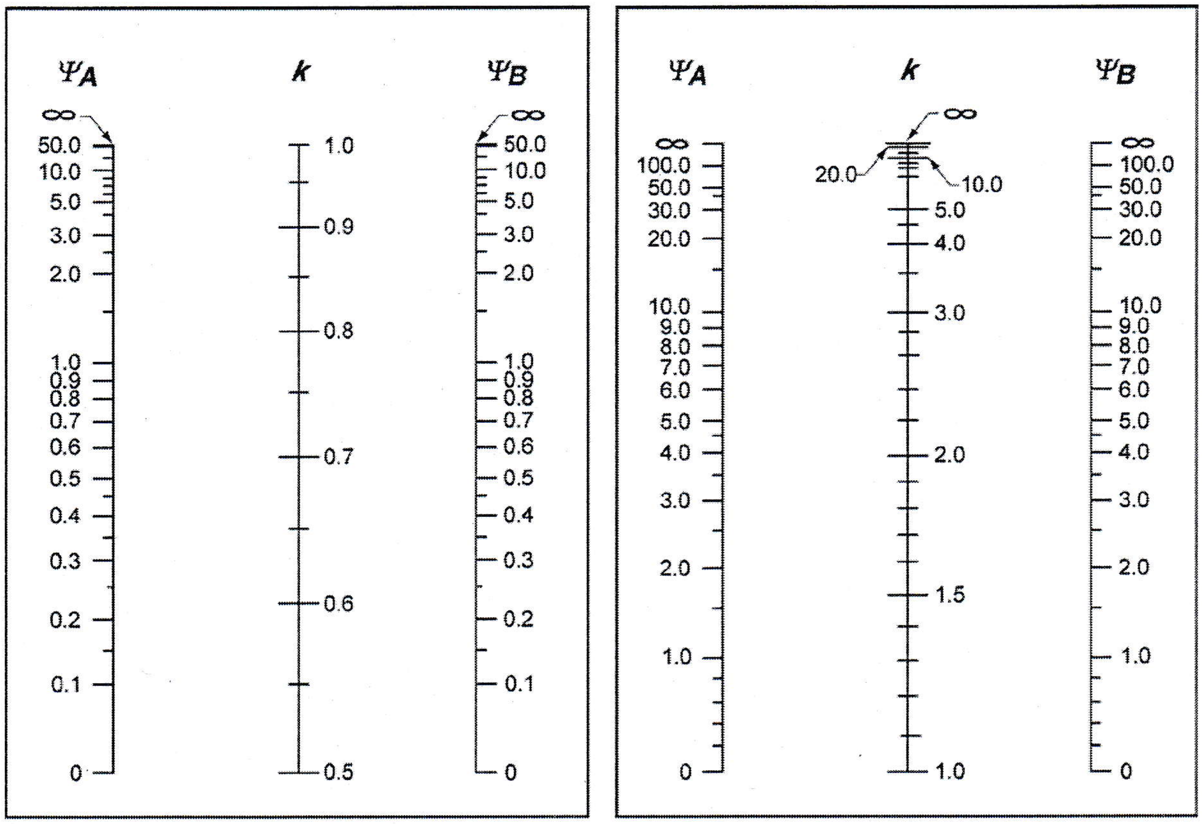


*(a)*
*Nonsway Frames*

*(b)*
*Sway Frames*

$\Psi$ = ratio of $\Sigma(EI/\ell_c)$ of columns to $\Sigma(EI/\ell)$ of beams in a plane at one end of a column

$\ell$ = span length of beam measured center to center of joints

***Fig. R6.2.5**—Effective length factor* k.

(c) 0.25 times the diameter of circular columns

**6.2.5.2** For composite columns, the radius of gyration, ***r***, shall not be taken greater than:

$$r = \sqrt{\frac{(E_c I_g/5) + E_s I_{sx}}{(E_c A_g/5) + E_s A_{sx}}} \qquad (6.2.5.2)$$

Longitudinal bars located within a concrete core encased by structural steel or within transverse reinforcement surrounding a structural steel core shall be permitted to be used in calculating $A_{sx}$ and $I_{sx}$.

**R6.2.5.2** Equation (6.2.5.2) is provided because the provisions in 6.2.5.1 for estimating the radius of gyration are overly conservative for concrete-filled tubing and are not applicable for members with enclosed structural shapes.

**6.2.6** Unless slenderness effects are neglected as permitted by 6.2.5, the design of columns, restraining beams, and other supporting members shall be based on the factored forces and moments considering second-order effects in accordance with 6.6.4, 6.7, or 6.8. $M_u$ including second-order effects shall not exceed $1.4M_u$ due to first-order effects.

**R6.2.6** Design considering second-order effects may be based on the moment magnifier approach (MacGregor et al. 1970; MacGregor 1993; Ford et al. 1981), an elastic second-order analysis, or a nonlinear second-order analysis. Figure R6.2.6 is intended to assist designers with application of the slenderness provisions of the Code.

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**6.6.3.1.1** Moment of inertia and cross-sectional area of members shall be calculated in accordance with Tables 6.6.3.1.1(a) or 6.6.3.1.1(b), unless a more rigorous analysis is used. If sustained lateral loads are present, $I$ for columns and walls shall be divided by $(1 + \beta_{ds})$, where $\beta_{ds}$ is the ratio of maximum factored sustained shear within a story to the maximum factored shear in that story associated with the same load combination.

### Table 6.6.3.1.1(a)—Moment of inertia and cross-sectional area permitted for elastic analysis at factored load level

| Member and condition | | Moment of Inertia | Cross-sectional area |
|---|---|---|---|
| Columns | | $0.70I_g$ | $1.0A_g$ |
| Walls | Uncracked | $0.70I_g$ | $1.0A_g$ |
| Walls | Cracked | $0.35I_g$ | $1.0A_g$ |
| Beams | | $0.35I_g$ | $1.0A_g$ |
| Flat plates and flat slabs | | $0.25I_g$ | $1.0A_g$ |

### Table 6.6.3.1.1(b)—Alternative moments of inertia for elastic analysis at factored load

| Member | Alternative value of $I$ for elastic analysis: Minimum | $I$ | Maximum |
|---|---|---|---|
| Columns and walls | $0.35I_g$ | $\left(0.80+25\frac{A_{st}}{A_g}\right)\left(1-\frac{M_u}{P_u h}-0.5\frac{P_u}{P_o}\right)I_g$ | $0.875I_g$ |
| Beams, flat plates, and flat slabs | $0.25I_g$ | $(0.10+25\rho)\left(1.2-0.2\frac{b_w}{d}\right)I_g$ | $0.5I_g$ |

Notes: For continuous flexural members, $I$ shall be permitted to be taken as the average of values obtained for the critical positive and negative moment sections. $P_u$ and $M_u$ shall be calculated from the load combination under consideration, or the combination of $P_u$ and $M_u$ that produces the least value of $I$.

**6.6.3.1.2** For factored lateral load analysis, it shall be permitted to assume $I = 0.5I_g$ for all members or to calculate $I$ by a more detailed analysis, considering the reduced stiffness of all members under the loading conditions.

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**R6.6.3.1.1** The values of $I$ and $A$ have been chosen from the results of frame tests and analyses, and include an allowance for the variability of the calculated deflections. The moments of inertia are taken from MacGregor and Hage (1977), which are multiplied by a stiffness reduction factor $\phi_K = 0.875$ (refer to R6.6.4.5.2). For example, the moment of inertia for columns is $0.875(0.80I_g) = 0.70I_g$.

The moment of inertia of T-beams should be based on the effective flange width defined in 6.3.2.1 or 6.3.2.2. It is generally sufficiently accurate to take $I_g$ of a T-beam as $2I_g$ for the web, $2(b_w h^3/12)$.

If the factored moments and shears from an analysis based on the moment of inertia of a wall, taken equal to $0.70I_g$, indicate that the wall will crack in flexure, based on the modulus of rupture, the analysis should be repeated with $I = 0.35I_g$ in those stories where cracking is predicted using factored loads.

The values of the moments of inertia were derived for nonprestressed members. For prestressed members, the moments of inertia may differ depending on the amount, location, and type of reinforcement, and the degree of cracking prior to reaching ultimate load. The stiffness values for prestressed concrete members should include an allowance for the variability of the stiffnesses.

The equations in Table 6.6.3.1.1(b) provide more refined values of $I$ considering axial load, eccentricity, reinforcement ratio, and concrete compressive strength as presented in Khuntia and Ghosh (2004a,b). The stiffnesses provided in these references are applicable for all levels of loading, including service and ultimate, and consider a stiffness reduction factor $\phi_K$ comparable to that for the moment of inertias included in Table 6.6.3.1.1(a). For use at load levels other than ultimate, $P_u$ and $M_u$ should be replaced with their appropriate values at the desired load level.

**R6.6.3.1.2** The lateral deflection of a structure under factored lateral loads can be substantially different from that calculated using linear analysis, in part because of the inelastic response of the members and the decrease in effective stiffness. Selection of the appropriate effective stiffness for reinforced concrete frame members has dual purposes: 1) to provide realistic estimates of lateral deflection; and 2) to determine deflection-imposed actions on the gravity system of the structure. A detailed nonlinear analysis of the structure would adequately capture these two effects. A simple way to estimate an equivalent nonlinear lateral deflection using linear analysis is to reduce the modeled stiffness of the concrete members in the structure. The type of lateral load analysis affects the selection of appropriate effective stiffness values. For analyses with wind loading, where it is desirable to prevent nonlinear action in the structure, effective stiffnesses representative of pre-yield behavior may be appropriate. For earthquake-induced loading, the level of nonlinear deformation depends on the intended structural performance and earthquake recurrence interval.

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tions of the story that any resulting lateral deflection is not large enough to affect the column strength substantially. If not readily apparent without calculations, 6.6.4.3 provides two possible ways of determining if sway can be neglected.

**6.6.4.2** The cross-sectional dimensions of each member used in an analysis shall be within 10 percent of the specified member dimensions in construction documents or the analysis shall be repeated. If the stiffnesses of Table 6.6.3.1.1(b) are used in an analysis, the assumed member reinforcement ratio shall also be within 10 percent of the specified member reinforcement in construction documents.

**6.6.4.3** It shall be permitted to analyze columns and stories in structures as nonsway frames if (a) or (b) is satisfied:

(a) The increase in column end moments due to second-order effects does not exceed 5 percent of the first-order end moments

(b) $\boldsymbol{Q}$ in accordance with 6.6.4.4.1 does not exceed 0.05

**R6.6.4.3** In 6.6.4.3(a), a story in a frame is classified as nonsway if the increase in the lateral load moments resulting from $\boldsymbol{P}\Delta$ effects does not exceed 5 percent of the first-order moments (MacGregor and Hage 1977). Section 6.6.4.3(b) provides an alternative method of determining if a frame is classified as nonsway based on the stability index for a story, $\boldsymbol{Q}$. In calculating $\boldsymbol{Q}$, $\sum \boldsymbol{P_u}$ should correspond to the lateral loading case for which $\sum \boldsymbol{P_u}$ is greatest. A frame may contain both nonsway and sway stories.

If the lateral load deflections of the frame are calculated using service loads and the service load moments of inertia given in 6.6.3.2.2, it is permissible to calculate $\boldsymbol{Q}$ in Eq. (6.6.4.4.1) using 1.2 times the sum of the service gravity loads, the service load story shear, and 1.4 times the first-order service load story deflections.

**6.6.4.4** *Stability properties*

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**6.6.4.4.1** The stability index for a story, $\boldsymbol{Q}$, shall be calculated by:

$$Q = \frac{\Sigma P_u \Delta_o}{V_{us} \ell_c} \qquad (6.6.4.4.1)$$

where $\sum \boldsymbol{P_u}$ and $\boldsymbol{V_{us}}$ are the total factored vertical load and horizontal story shear, respectively, in the story being evaluated, and $\Delta_o$ is the first-order relative lateral deflection between the top and the bottom of that story due to $\boldsymbol{V_{us}}$.

**6.6.4.4.2** The critical buckling load $\boldsymbol{P_c}$ shall be calculated by:

$$P_c = \frac{\pi^2 (EI)_{eff}}{(k\ell_u)^2} \qquad (6.6.4.4.2)$$

**R6.6.4.4.2** In calculating the critical axial buckling load, the primary concern is the choice of a stiffness $\boldsymbol{(EI)_{eff}}$ that reasonably approximates the variations in stiffness due to cracking, creep, and nonlinearity of the concrete stress-strain curve. Sections 6.6.4.4.4 and 6.6.4.4.5 may be used to calculate $\boldsymbol{(EI)_{eff}}$.

**6.6.4.4.3** The effective length factor $\boldsymbol{k}$ shall be calculated using $\boldsymbol{E_c}$ in accordance with 19.2.2 and $\boldsymbol{I}$ in accordance with 6.6.3.1.1. For nonsway members, $\boldsymbol{k}$ shall be permitted to be taken as 1.0, and for sway members, $\boldsymbol{k}$ shall be at least 1.0.

**R6.6.4.4.3** The effective length factor for a compression member, such as a column, wall, or brace, considering braced behavior, ranges from 0.5 to 1.0. It is recommended that a $\boldsymbol{k}$ value of 1.0 be used. If lower values are used, the calculation of $\boldsymbol{k}$ should be based on analysis of the frame using $\boldsymbol{I}$ values given in 6.6.3.1.1. The Jackson and More-

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land Alignment Charts (Fig. R6.2.5) can be used to estimate appropriate values of $k$ (ACI SP-17(09); Column Research Council 1966).

**6.6.4.4.4** For noncomposite columns, $(EI)_{eff}$ shall be calculated in accordance with (a), (b), or (c):

(a) $$(EI)_{eff} = \frac{0.4E_cI_g}{1+\beta_{dns}} \qquad (6.6.4.4.4a)$$

(b) $$(EI)_{eff} = \frac{(0.2E_cI_g + E_sI_{se})}{1+\beta_{dns}} \qquad (6.6.4.4.4b)$$

(c) $$(EI)_{eff} = \frac{E_cI}{1+\beta_{dns}} \qquad (6.6.4.4.4c)$$

where $\beta_{dns}$ shall be the ratio of maximum factored sustained axial load to maximum factored axial load associated with the same load combination and $I$ in Eq. (6.6.4.4.4c) is calculated according to Table 6.6.3.1.1(b) for columns and walls.

**R6.6.4.4.4** The numerators of Eq. (6.6.4.4.4a) to (6.6.4.4.4c) represent the short-term column stiffness. Equation (6.6.4.4.4b) was derived for small eccentricity ratios and high levels of axial load. Equation (6.6.4.4.4a) is a simplified approximation to Eq. (6.6.4.4.4b) and is less accurate (Mirza 1990). For improved accuracy, $(EI)_{eff}$ can be approximated using Eq. (6.6.4.4.4c).

Creep due to sustained loads will increase the lateral deflections of a column and, hence, the moment magnification. Creep effects are approximated in design by reducing the stiffness $(EI)_{eff}$ used to calculate $P_c$ and, hence, $\delta$, by dividing the short-term $EI$ provided by the numerator of Eq. (6.6.4.4.4a) through (6.6.4.4.4c) by $(1 + \beta_{dns})$. For simplification, it can be assumed that $\beta_{dns} = 0.6$. In this case, Eq. (6.6.4.4.4a) becomes $(EI)_{eff} = 0.25E_cI_g$.

In reinforced concrete columns subject to sustained loads, creep transfers some of the load from the concrete to the longitudinal reinforcement, increasing the reinforcement stresses. In the case of lightly reinforced columns, this load transfer may cause the compression reinforcement to yield prematurely, resulting in a loss in the effective $EI$. Accordingly, both the concrete and longitudinal reinforcement terms in Eq. (6.6.4.4.4b) are reduced to account for creep.

**6.6.4.4.5** For composite columns, $(EI)_{eff}$ shall be calculated by Eq. (6.6.4.4.4b), Eq. (6.6.4.4.5), or from a more detailed analysis.

$$(EI)_{eff} = \frac{(0.2E_cI_g)}{1+\beta_{dns}} + E_sI_{sx} \qquad (6.6.4.4.5)$$

**R6.6.4.4.5** For composite columns in which the pipe or structural shape makes up a large percentage of the cross section, the load transfer due to creep is insignificant. Accordingly, only the $EI$ of the concrete in Eq. (6.6.4.4.5) is reduced for sustained load effects.

**6.6.4.5** *Moment magnification method: Nonsway frames*

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**6.6.4.5.1** The factored moment used for design of columns and walls, $M_c$, shall be the first-order factored moment $M_2$ amplified for the effects of member curvature.

$$M_c = \delta M_2 \qquad (6.6.4.5.1)$$

**6.6.4.5.2** Magnification factor $\delta$ shall be calculated by:

$$\delta = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} \geq 1.0 \qquad (6.6.4.5.2)$$

**R6.6.4.5.2** The 0.75 factor in Eq. (6.6.4.5.2) is the stiffness reduction factor $\phi_K$, which is based on the probability of understrength of a single isolated slender column. Studies reported in Mirza et al. (1987) indicate that the stiffness reduction factor $\phi_K$ and the cross-sectional strength reduction $\phi$ factors do not have the same values. These studies suggest the stiffness reduction factor $\phi_K$ for an isolated column should be 0.75 for both tied and spiral columns. In the case of a multistory frame, the column and frame deflections depend on the average concrete strength, which is higher than the strength of the concrete in the critical

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single understrength column. For this reason, the value of $\phi_K$ implicit in $I$ values in 6.6.3.1.1 is 0.875.

**6.6.4.5.3** $C_m$ shall be in accordance with (a) or (b):

(a) For columns without transverse loads applied between supports

$$C_m = 0.6 - 0.4\frac{M_1}{M_2} \quad (6.6.4.5.3a)$$

where $M_1/M_2$ is negative if the column is bent in single curvature, and positive if bent in double curvature. $M_1$ corresponds to the end moment with the lesser absolute value.

(b) For columns with transverse loads applied between supports.

$$C_m = 1.0 \quad (6.6.4.5.3b)$$

**R6.6.4.5.3** The factor $C_m$ is a correction factor relating the actual moment diagram to an equivalent uniform moment diagram. The derivation of the moment magnifier assumes that the maximum moment is at or near midheight of the column. If the maximum moment occurs at one end of the column, design should be based on an equivalent uniform moment $C_mM_2$ that leads to the same maximum moment at or near midheight of the column when magnified (MacGregor et al. 1970).

The sign convention for $M_1/M_2$ has been updated to follow the right hand rule convention; hence, $M_1/M_2$ is negative if bent in single curvature and positive if bent in double curvature. This reflects a sign convention change from the 2011 Code.

In the case of columns that are subjected to transverse loading between supports, it is possible that the maximum moment will occur at a section away from the end of the member. If this occurs, the value of the largest calculated moment occurring anywhere along the member should be used for the value of $M_2$ in Eq. (6.6.4.5.1). $C_m$ is to be taken as 1.0 for this case.

**6.6.4.5.4** $M_2$ in Eq. (6.6.4.5.1) shall be at least $M_{2,min}$ calculated according to Eq. (6.6.4.5.4) about each axis separately.

$$M_{2,min} = P_u(15 + 0.03h) \quad (6.6.4.5.4)$$

If $M_{2,min}$ exceeds $M_2$, $C_m$ shall be taken equal to 1.0 or calculated based on the ratio of the calculated end moments $M_1/M_2$, using Eq. (6.6.4.5.3a).

**R6.6.4.5.4** In the Code, slenderness is accounted for by magnifying the column end moments. If the factored column moments are small or zero, the design of slender columns should be based on the minimum eccentricity provided in Eq. (6.6.4.5.4). It is not intended that the minimum eccentricity be applied about both axes simultaneously.

The factored column end moments from the structural analysis are used in Eq. (6.6.4.5.3a) in determining the ratio $M_1/M_2$ for the column when the design is based on the minimum eccentricity. This eliminates what would otherwise be a discontinuity between columns with calculated eccentricities less than the minimum eccentricity and columns with calculated eccentricities equal to or greater than the minimum eccentricity.

**6.6.4.6** *Moment magnification method: Sway frames*

**R6.6.4.6** *Moment magnification method: Sway frames*

**6.6.4.6.1** Moments $M_1$ and $M_2$ at the ends of an individual column shall be calculated by (a) and (b).

(a) $$M_1 = M_{1ns} + \delta_s M_{1s} \quad (6.6.4.6.1a)$$

(b) $$M_2 = M_{2ns} + \delta_s M_{2s} \quad (6.6.4.6.1b)$$

**R6.6.4.6.1** The analysis described in this section deals only with plane frames subjected to loads causing deflections in that plane. If the lateral load deflections involve significant torsional displacement, the moment magnification in the columns farthest from the center of twist may be underestimated by the moment magnifier procedure. In such cases, a three-dimensional second-order analysis should be used.

**6.6.4.6.2** The moment magnifier $\delta_s$ shall be calculated by (a), (b), or (c). If $\delta_s$ exceeds 1.5, only (b) or (c) shall be permitted:

(a) $$\delta_s = \frac{1}{1-Q} \geq 1 \quad (6.6.4.6.2a)$$

**R6.6.4.6.2** Three different methods are allowed for calculating the moment magnifier. These approaches include the $Q$ method, the sum of $P$ concept, and second-order elastic analysis.

(a) $Q$ method: