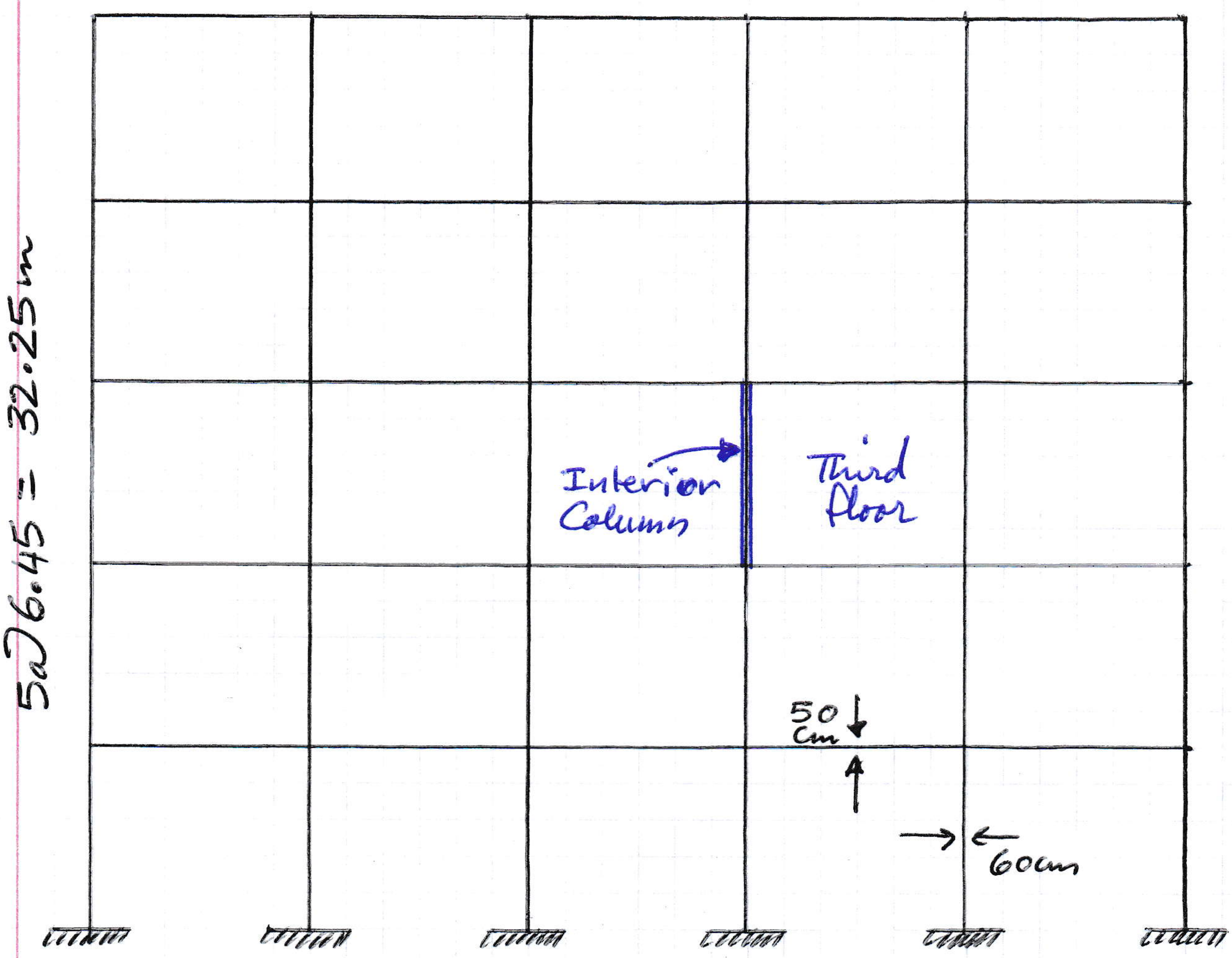
Example:

(e1)

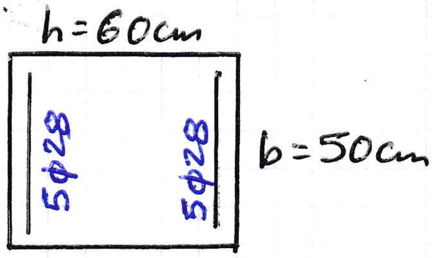
Braced Frame

5@8.5 = 42.5m

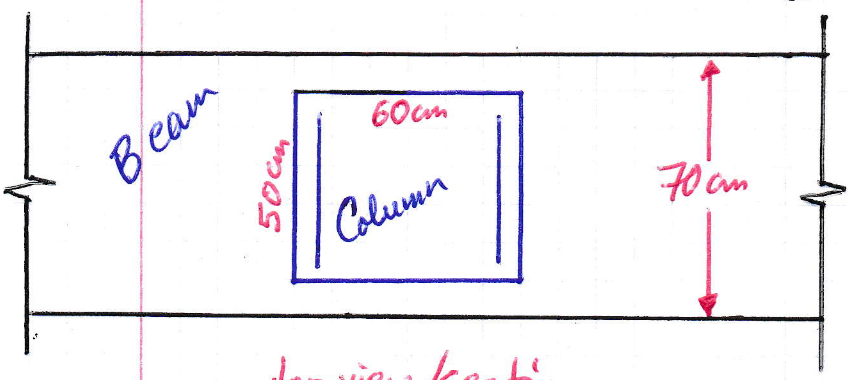


$l_c = 6.45$ m

Column



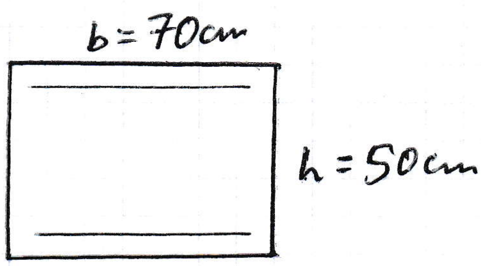
5φ28 along each end face



top view / section

$l_u = 6.45 - 0.50 = 5.95$m

Beam



C2

Service loads:

Dead	Live
$P = 170t$	$P = 110t$
$M_2 = 6\ t.m$	$M_2 = 28\ t.m$
$M_1 = -6\ t.m$	$M_1 = 21\ t.m$

$f_c' = 28\ MPa, \quad f_y = 420\ MPa$

$$P_u = 1.2(170) + 1.6(110) = 380t$$

$M_u =$

$$M_2 = 1.2(6) + 1.6(28) = 52\ t.m$$

$$M_1 = 1.2(-6) + 1.6(21) = 26.4\ t.m$$

$e_{actual} = 13.68\ cm$

Check the capacity of this interior third floor column.

3

$l_c = 6.45\,m$

$l = 8.5\,m$

$$\Psi_A = \Psi_B = \frac{\cancel{2}\left[(0.\cancel{70}^{\,2})\left(\frac{0.5\times 0.6^3}{\cancel{12}}\right)/6.45\right]}{\cancel{2}\left[(0.\cancel{35})\left(\frac{0.7\times 0.5^3}{\cancel{12}}\right)/8.5\right]}$$

$$= 3.25$$

$\therefore\ k = 0.9$

$$k l_u / r = \frac{(0.9)(595)}{(0.3)(60)} = 29.75$$

$$\left(\frac{k l_u}{r}\right)_{limit} = 34 + (12)\left[(-)\left(\frac{1.2\times -6 + 1.6\times 21}{1.2\times 6 * 1.6\times 28}\right)\right]$$

$$= 27.91 \qquad \text{ratio} = -0.508$$

$\therefore$ Slenderness effects must be considered

$$M_2\ min = \frac{380\left(1.5 + 0.03(600)\right)}{1000} = 12.54\ t.m$$

Does not control $(M_2 = 52\ t.m)$

$(e_{min} = 3.3\ cm)$

$$C_m = 0.6 - (0.4)(-0.508) = 0.803$$

$$\beta_{dns} = \frac{1.2(170)}{380} = 0.537$$

4

$$EI = \frac{0.2 E_c I_g + E_s I_{se}}{1 + \beta_{dns}}$$

$$= \frac{0.4 E_c I_g}{1 + \beta_{dns}}$$

$\phi 28 = 6.16 \text{ cm}^2 \qquad E_c = \frac{4700\sqrt{28}}{100} = 249 \text{ t/cm}^2$

$$EI = \frac{\overset{4.482\times10^7}{0.2(249)(50\times60^3/12)} + \overset{6.8617\times10^7}{(2000)(34309)}}{1 + 0.537}$$

$$= 7.3805\times10^7 \text{ t.cm}^2$$

$\left(I_{se} = 2(5\times6.16)(23.6)^2 = 34309 \text{ cm}^4 \right)$

(d)

or

$$EI = \frac{0.4(249)(50\times60^3/12)}{1.537}$$

$$= 5.8321\times10^7 \text{ t.cm}^2$$

$$\therefore EI = 7.3805\times10^7 \text{ t.cm}^2$$

$$P_c = \frac{\pi^2(EI)}{(k l_u)^2} = \frac{\pi^2 (7.3805\times10^7)\text{t.cm}^2}{(0.9\times595)^2 \text{ cm}^2}$$

$$= 2540 \text{ t}$$

$$\therefore \delta_{ns} = \frac{0.803}{1 - \left(\frac{380}{0.75\times2540}\right)} = 1.003 \le 1.4$$

ACI - 6.2.6

$$\therefore (M_u)_{design} = 1.003(52) = 52.16 \text{ t.m}$$

(P)

Homework - Slender Columns

Due - Whenever we meet again, Insh Allah

Beams: 5 @ 6 m = 30 m

b = 45 cm, h = 55 cm

Columns: 5 @ 8 m = 40 m

b = 45 cm, h = 50 cm

4 ϕ 25 along each end face

Braced frame

$f_c' = 28$ MPa, $f_y = 420$ MPa

Service Loads:

Dead	Live
P = 200 t	P = 100 t
M_2 = 3 t.m	M_2 = 45 t.m
M_1 = − 3 t.m	M_1 = 17 t.m

* Check the capacity of this interior third-floor column.

Show that under the magnified eccentricity, $\phi P_n \geq P_u$

Ⓟ

Homework - Slender Columns

Due - Whenever we meet again, Insh Allah

Beams: 5 @ 6 m = 30 m

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Service Loads:

Dead	Live
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$M_2 = 3\,t.m$	$M_2 = 45\,t.m$
$M_1 = -3\,t.m$	$M_1 = 17\,t.m$

* Check the capacity of this interior third-floor column.

Show that under the magnified eccentricity, $\phi P_n \geq P_u$

9 Slender Columns

9.1 INTRODUCTION

The material presented in Chapter 8 pertained to concentrically or eccentrically loaded *short columns*, for which the strength is governed entirely by the strength of the materials and the geometry of the cross section. Most columns in present-day practice fall in that category. However, with the increasing use of high-strength materials and improved methods of dimensioning members, it is now possible, for a given value of axial load, with or without simultaneous bending, to design a much smaller cross section than in the past. This clearly makes for more slender members. It is because of this, together with the use of more innovative structural concepts, that rational and reliable design procedures for slender columns have become increasingly important.

A column is said to be *slender* if its cross-sectional dimensions are small compared with its length. The degree of slenderness is generally expressed in terms of the slenderness ratio l/r, where l is the unsupported length of the member and r is the radius of gyration of its cross section, equal to $\sqrt{I/A}$. For square or circular members, the value of r is the same about either axis; for other shapes r is smallest about the minor principal axis, and it is generally this value that must be used in determining the slenderness ratio of a freestanding column.

It has long been known that a member of great slenderness will collapse under a smaller compression load than a stocky member with the same cross-sectional dimensions. When a stocky member, say with $l/r = 10$ (e.g., a square column of length equal to about 3 times its cross-sectional dimension h), is loaded in axial compression, it will fail at the load given by Eq. (8.3), because at that load both concrete and steel are stressed to their maximum carrying capacity and give way, respectively, by crushing and by yielding. If a member with the same cross section has a slenderness ratio $l/r = 100$ (e.g., a square column hinged at both ends and of length equal to about 30 times its section dimension), it may fail under an axial load equal to one-half or less of that given by Eq. (8.3). In this case, collapse is caused by buckling, i.e., by sudden lateral displacement of the member between its ends, with consequent overstressing of steel and concrete by the bending stresses that are superimposed on the axial compressive stresses.

Most columns in practice are subjected to bending moments as well as axial loads, as was made clear in Chapter 8. These moments produce lateral deflection of a member between its ends and may also result in relative lateral displacement of joints. Associated with these lateral displacements are *secondary moments* that add to the primary moments and that may become very large for slender columns, leading to failure. A practical definition of a slender column is one for which there is a significant reduction in axial load capacity because of these secondary moments. In the development of ACI Code column provisions, for example, any reduction greater than about 5 percent is considered significant, requiring consideration of slenderness effects.

The ACI Code and Commentary contain detailed provisions governing the design of slender columns. ACI Code 10.10.5, 10.10.6, and 10.10.7 present approximate methods for accounting for slenderness through the use of *moment magnification factors*. The provisions are quite similar to those used for many years for steel columns designed under the American Institute of Steel Construction (AISC) Specification. Alternatively, in ACI Code 10.10.3 and 10.10.4, a more fundamental approach is endorsed, in which the effect of lateral displacements is accounted for directly in the frame analysis. The latter approach, known as *second-order analysis*, is often incorporated as a feature in commercially available structural analysis software.

As noted, most columns in practice continue to be short columns. Simple expressions are included in the ACI Code to determine whether slenderness effects must be considered. These will be presented in Section 9.4 following the development of background information in Sections 9.2 and 9.3 relating to column buckling and slenderness effects.

9.2 CONCENTRICALLY LOADED COLUMNS

The basic information on the behavior of straight, concentrically loaded slender columns was developed by Euler more than 200 years ago. In generalized form, it states that such a member will fail by buckling at the critical load

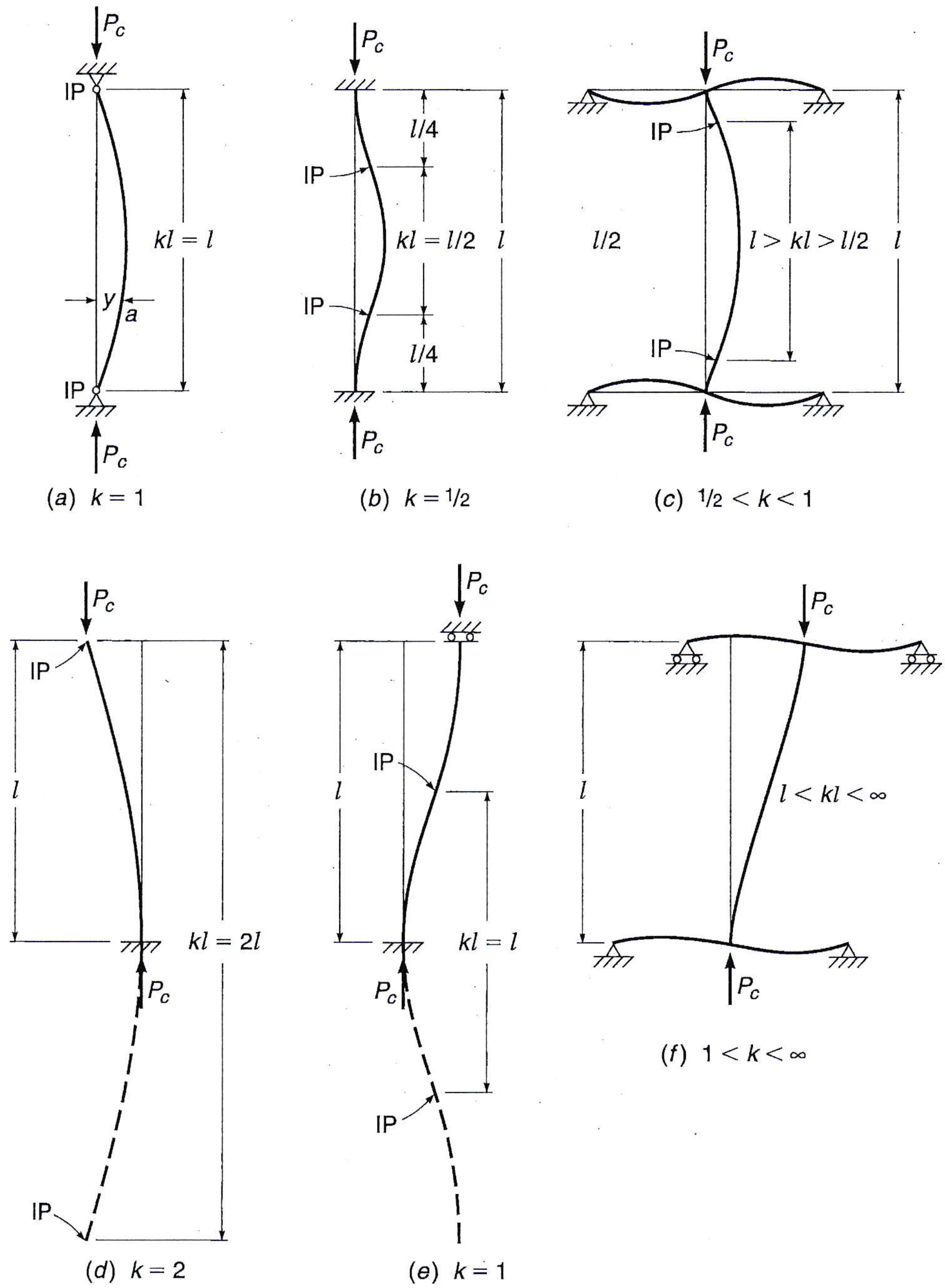
$$P_c = \frac{\pi^2 E_t I}{(kl)^2} \tag{9.1}$$

It is seen that the buckling load decreases rapidly with increasing *slenderness ratio* kl/r (Ref. 9.1).

For the simplest case of a column hinged at both ends and made of elastic material, E_t simply becomes Young's modulus and kl is equal to the actual length l of the column. At the load given by Eq. (9.1), the originally straight member buckles into a half sine wave, as shown in Fig. 9.1*a*. In this bent configuration, bending moments Py act at any section such as a; y is the deflection at that section. These deflections continue to increase until the bending stress caused by the increasing moment, together with the original compression stress, overstresses and fails the member.

If the stress-strain curve of a short piece of the given member has the shape shown in Fig. 9.2*a*, as it would be for reinforced concrete columns, E_t is equal to Young's modulus, provided that the buckling stress P_c/A is below the proportional limit f_p. If the strain is larger than f_p, buckling occurs in the inelastic range. In this case, in Eq. (9.1), E_t is the tangent modulus, i.e., the slope of the tangent to the stress-strain curve. As the stress increases, E_t decreases. A plot of the buckling load vs. the slenderness ratio, the so-called column curve, therefore has the shape given in Fig. 9.2*b*, which shows the reduction in buckling strength with increasing slenderness. For very stocky columns, the value of the buckling load, calculated from Eq. (9.1), exceeds the direct crushing strength of the stocky column P_n, given by Eq. (8.3). This is also shown in Fig. 9.2*b*. Correspondingly, there is a limiting slenderness ratio $(kl/r)_{\text{lim}}$. For values smaller than this, failure occurs by simple crushing, regardless of

FIGURE 9.1
Buckling and effective length of axially loaded columns.



kl/r; for values larger than $(kl/r)_{\text{lim}}$, failure occurs by buckling, the buckling load or stress decreasing for greater slenderness.

If a member is fixed against rotation at both ends, it buckles in the shape of Fig. 9.1*b*, with inflection points (IPs) as shown. The portion between the inflection points is in precisely the same situation as the hinge-ended column of Fig. 9.1*a*, and thus, the *effective length kl* of the fixed-fixed column, i.e., the distance between inflection points, is seen to be $kl = l/2$. Equation (9.1) shows that an elastic column fixed at both ends will carry 4 times as much load as when hinged.

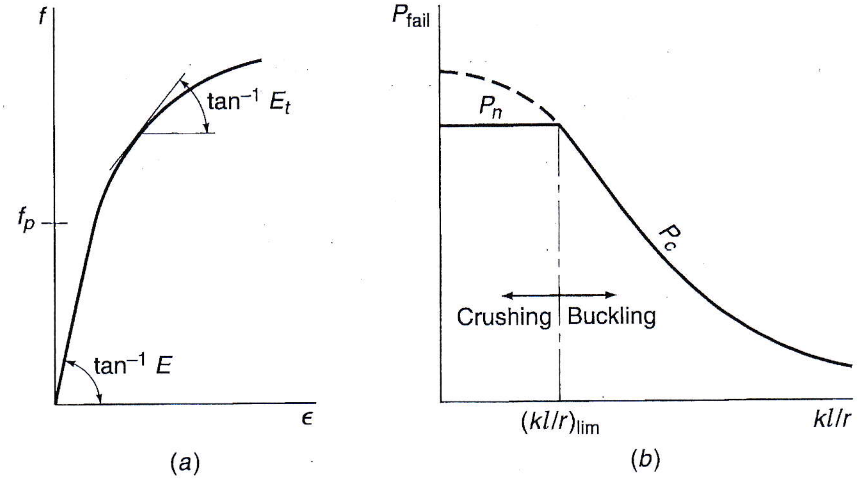


FIGURE 9.2
Effect of slenderness on strength of axially loaded columns.

Columns in real structures are rarely either hinged or fixed but have ends partially restrained against rotation by abutting members. This is shown schematically in Fig. 9.1*c*, from which it is seen that for such members the effective length kl, i.e., the distance between inflection points, has a value between l and $l/2$. The precise value depends on the degree of end restraint, i.e., on the ratio of the stiffness EI/l of the column to the sum of stiffnesses EI/l of the restraining members at both ends.

In the columns of Fig. 9.1*a* to *c*, it was assumed that one end was prevented from moving laterally relative to the other end, by horizontal bracing or otherwise. In this case, it is seen that the effective length kl is always smaller than (or at most it is equal to) the real length l.

If a column is fixed at one end and entirely free at the other (cantilever column or flagpole), it buckles as shown in Fig. 9.1*d*. That is, the upper end moves laterally with respect to the lower, a kind of deformation known as *sidesway*. It buckles into a quarter of a sine wave and is therefore analogous to the upper half of the hinged column in Fig. 9.1*a*. The inflection points, one at the end of the actual column and the other at the imaginary extension of the sine wave, are a distance $2l$ apart, so that the effective length is $kl = 2l$.

If the column is rotationally fixed at both ends but one end can move laterally with respect to the other, it buckles as shown in Fig. 9.1*e*, with an effective length $kl = l$. If one compares this column, fixed at both ends but free to sidesway, with a fixed-fixed column that is braced against sidesway (Fig. 9.1*b*), one sees that the effective length of the former is twice that of the latter. By Eq. (9.1), this means that the buckling strength of an elastic fixed-fixed column that is free to sidesway is only one-quarter that of the same column when braced against sidesway. This is an illustration of the general fact that *compression members free to buckle in a sidesway mode are always considerably weaker than when braced against sidesway.*

Again, the ends of columns in actual structures are rarely hinged, fixed, or entirely free but are usually restrained by abutting members. If sidesway is not prevented, buckling occurs as shown in Fig. 9.1*f*, and the effective length, as before, depends on the degree of restraint. If the cross beams are very rigid compared with the column, the case of Fig. 9.1*e* is approached and kl is only slightly larger than l. On the other hand, if the restraining members are extremely flexible, a hinged condition is approached at both ends. Evidently, a column hinged at both ends and free to sidesway is unstable. It will simply topple, being unable to carry any load whatever.

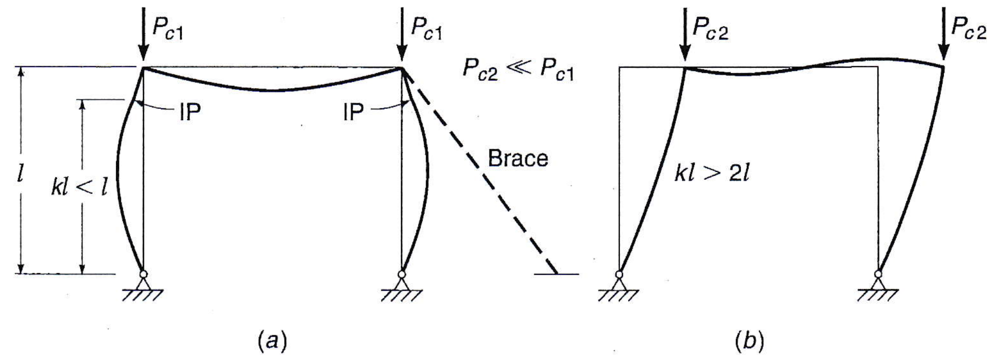


FIGURE 9.3
Rigid-frame buckling: (*a*) laterally braced; (*b*) unbraced.

In reinforced concrete structures, one is rarely concerned with single members but rather with rigid frames of various configurations. The manner in which the relationships just described affect the buckling behavior of frames is illustrated by the simple portal frame shown in Fig. 9.3, with loads applied concentrically to the columns. If sidesway is prevented, as indicated schematically by the brace in Fig. 9.3*a*, the buckling configuration will be as shown. The buckled shape of the column corresponds to that in Fig. 9.1*c*, except that the lower end is hinged. It is seen that the effective length kl is smaller than l. On the other hand, if no sidesway bracing is provided to an otherwise identical frame, buckling occurs as shown in Fig. 9.3*b*. The column is in a situation similar to that shown in Fig. 9.1*d*, upside down, except that the upper end is not fixed but only partially restrained by the girder. It is seen that the effective length kl exceeds $2l$ by an amount depending on the degree of restraint. The buckling strength depends on kl/r in the manner shown in Fig. 9.2*b*. As a consequence, even though they are dimensionally identical, the unbraced frame will buckle at a radically smaller load than the braced frame.

In summary, the following can be noted:

1. The strength of concentrically loaded columns decreases with increasing slenderness ratio kl/r.
2. In columns that are *braced against sidesway* or that are parts of frames braced against sidesway, the effective length kl, i.e., the distance between inflection points, falls between $l/2$ and l, depending on the degree of end restraint.
3. The effective lengths of columns that are *not braced against sidesway* or that are parts of frames not so braced are always larger than l, the more so the smaller the end restraint. In consequence, the buckling load of a frame not braced against sidesway is always substantially smaller than that of the same frame when braced.

9.3 COMPRESSION PLUS BENDING

Most reinforced concrete compression members are also subject to simultaneous flexure, caused by transverse loads or by end moments owing to continuity. The behavior of members subject to such combined loading also depends greatly on their slenderness.

Figure 9.4*a* shows such a member, axially loaded by P and bent by equal end moments M_e. If no axial load were present, the moment M_0 in the member would be constant throughout and equal to the end moments M_e. This is shown in Fig. 9.4*b*. In this situation, i.e., in simple bending without axial compression, the member deflects as shown by the dashed curve of Fig. 9.4*a*, where y_0 represents the deflection at any

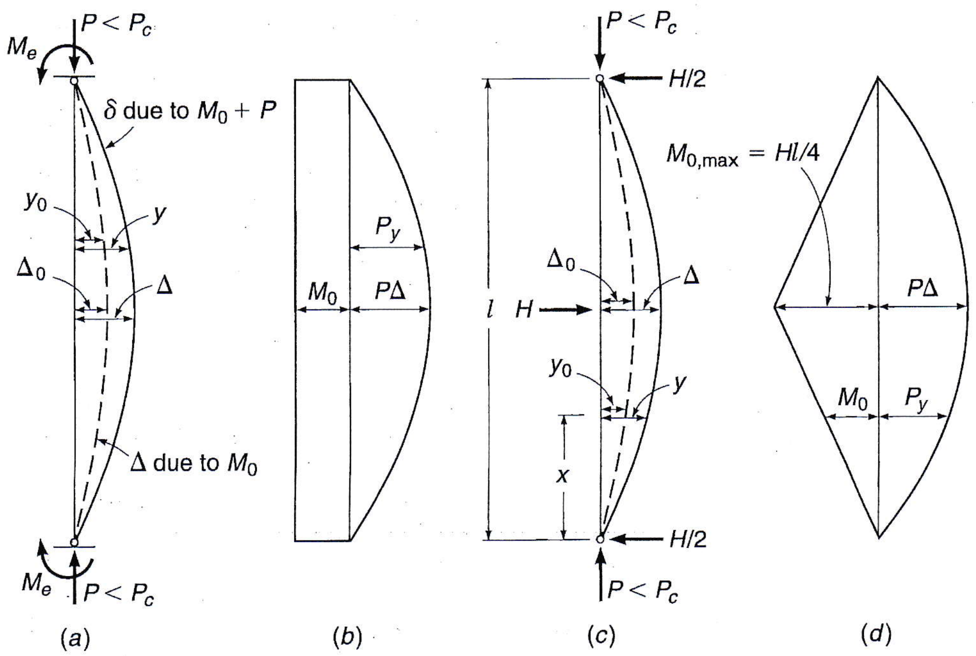


FIGURE 9.4
Moments in slender members with compression plus bending, bent in single curvature.

point caused by bending only. When P is applied, the moment at any point increases by an amount equal to P times its lever arm. The increased moments cause additional deflections, so that the deflection curve under the simultaneous action of P and M_0 is the solid curve of Fig. 9.4*a*. At any point, then, the total moment is now

$$M = M_0 + Py \tag{9.2}$$

i.e., the total moment consists of the moment M_0 that acts in the presence of P and the additional moment caused by P, equal to P times the deflection. This is one illustration of the so-called P-Δ effect.

A similar situation is shown in Fig. 9.4*c*, where bending is caused by the transverse load H. When P is absent, the moment at any point x is $M_0 = Hx/2$, with a maximum value at midspan equal to $Hl/4$. The corresponding M_0 diagram is shown in Fig. 9.4*d*. When P is applied, additional moments Py are caused again, distributed as shown, and the total moment at any point in the member consists of the same two parts as in Eq. (9.2).

The deflections y of elastic columns of the type shown in Fig. 9.4 can be calculated from the deflections y_0, that is, from the deflections of the corresponding beam without axial load, using the following expression (see, for example, Ref. 9.1).

$$y = y_0 \frac{1}{1 - P/P_c} \tag{9.3}$$

If Δ is the deflection at the point of maximum moment M_{max}, as shown in Fig. 9.4, M_{max} can be calculated using Eqs. (9.2) and (9.3).

$$M_{max} = M_0 + P\Delta = M_0 + P\Delta_0 \frac{1}{1 - P/P_c} \tag{9.4}$$

Dr. Bayan Zalatimo

$$M_1 = M_{1ns} + \delta_s M_{1s}$$
$$M_2 = M_{2ns} + \delta_s M_{2s}$$

M_1 = smaller factored end moment
M_2 = larger

M_{1ns} (M_{2ns}) "gravity loads" = factored end moment at end at which M_1 acts due to loads that cause no sidesway - calculated using a first-order elastic analysis

δ_s = moment magnification factor for sway frames, to reflect lateral drift

M_{1s} (M_{2s}) "lateral loads" = factored end moment at end at which M_1 acts due to loads that cause sidesway - calculated using a first order elastic analysis

Method 1: $\delta_s = \dfrac{1}{1-Q} \geq 1.0$

may be used only if $\dfrac{1}{1-Q} \leq 1.5$

Otherwise, use an elastic second-order analysis (ACI 10.10.4) or use Method 2.

Method 2: $\delta_s = \dfrac{1}{1 - \dfrac{\Sigma P_u}{0.75 \Sigma P_c}} \geq 1.0$

ΣP_u = total axial load on all columns

ΣP_c = total critical buckling load for all columns

0.75 = a stiffness reduction factor to provide a conservative estimate of the critical buckling loads P_c

β_{ds} is defined differently than β_{dns}

$$= \frac{\text{maximum factored sustained shear}}{\text{maximum factored shear}}$$

within a story.

∴ for most applications $\beta_{ds} = 0$

$\beta_{ds} \neq 0$ when, for example, the building is subjected to soil pressure

Difference in Design Steps from those for Nonsway Frames:

* Loads are separated into gravity loads which are assumed to produce no sway, and horizontal loads producing sway
* Separate frame analyses are required
* Different equivalent length factors k and creep coefficients β_{dns} and β_{ds}

Although the ACI moment magnifier method works well for nonsway frames, its application to sway frames is complicated, with many opportunities for error, especially when the equation: $\delta_s = \dfrac{1}{1 - \dfrac{\Sigma P_u}{0.75 \Sigma P_c}} \geq 1$ is used. (Method 2)

However, with the wide availability of computers and software, it is best to perform a second-order frame analysis (P-Δ) in which the effects of lateral deflection on moments, axial forces, and again on lateral deflections are computed directly (Method 3).

The resulting moments and deflections include the effects of slenderness.