

Torsion Constant of the Transverse Beam:

$$C = \sum \left(1 - 0.63 \frac{x}{y}\right) \frac{x^3 y}{3} \quad \text{for component rectangles}$$

$x$  = shorter dimension of rectangular part of x-section.

$y$  = longer dimension.

Note: Transverse Distribution of Longitudinal Moment:

- \* If the two adjacent transverse spans, each =  $L_2$  then the width of the column strip =  $\frac{L_2}{2}$  or  $\frac{L_1}{2}$  (smaller)
- \* Determine the ratio  $\beta_t = \frac{E_{cb} C}{2E_{cs} I_s}$  of edge beam torsional rigidity to slab flexural rigidity.
- \* Determine the ratio  $\alpha_1$  of longitudinal beam flexural stiffness to slab flexural stiffness, note:  $\alpha_1 = \frac{I_b}{I_s}$  for  $E_{cb} = E_{cs}$ .

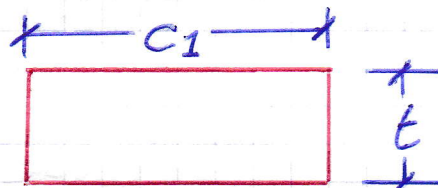
Percentage of Longitudinal Moment in Column strip:

$L_2/L_1$		0.50	1.0	2.0	
Negative Moment at Exterior Support	$\alpha_1 L_2/L_1 = 0$	$\beta_t = 0$	100	100	100
		$\beta_t \geq 2.5$	75	75	75
	$\alpha_1 L_2/L_1 \geq 1.0$	$\beta_t = 0$	100	100	100
		$\beta_t \geq 2.5$	90	75	45
Positive Moment	$\alpha_1 L_2/L_1 = 0$	60	60	60	
	$\alpha_1 L_2/L_1 \geq 1.0$	90	75	45	
Negative Moment at Interior Support	$\alpha_1 L_2/L_1 = 0$	75	75	75	
	$\alpha_1 L_2/L_1 \geq 1.0$	90	75	45	

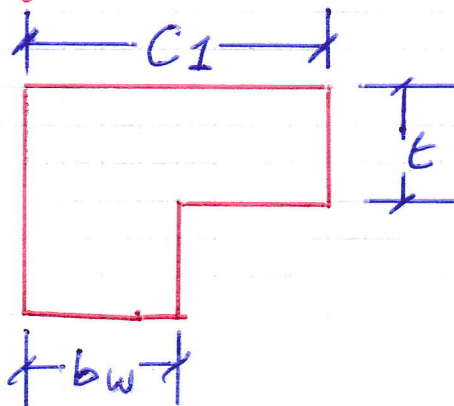
Use the combination that produces the largest C

Note: Even if there is no such beam (as defined by projection above or below the slab) actually visible, imagine that there is a beam made of a portion of the slab having a width equal to that of the column, bracket, or capital in the direction of the span for which moments are being determined,

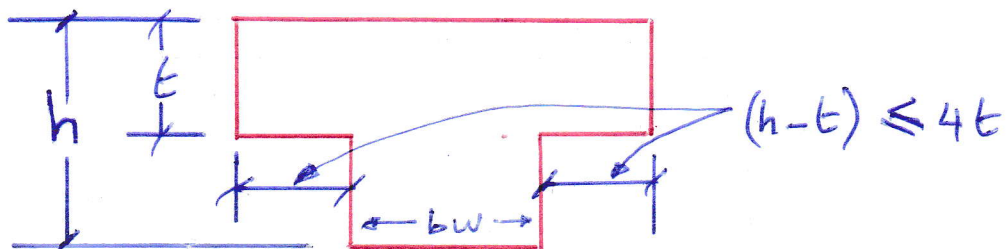
ACI -



When there is an actual web, include the portion of slab within the width of column, bracket, or capital plus the projection of web, ACI -



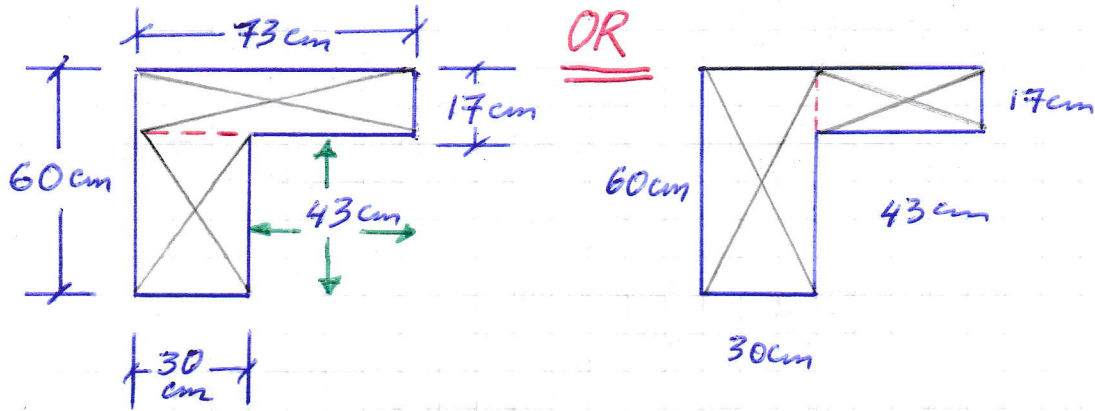
OR, include a portion on each side of web =  $(h-t) \leq 4t$ , ACI



Example 16.11.1 :

Compute the torsional constant  $C$  for the edge and interior beams.

Short direction : Beams B7, B8 - edge

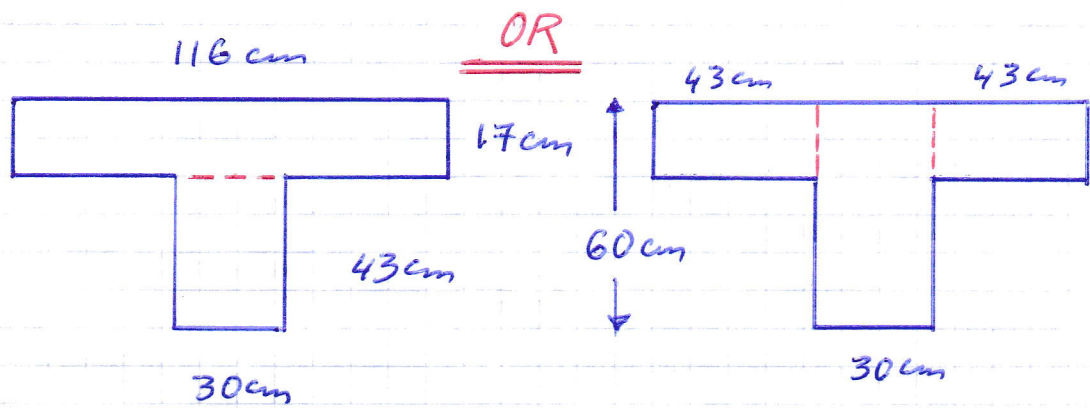


$$\begin{aligned}
 C &= \left[ 1 - 0.63 \left( \frac{17}{73} \right) \right] \times \left[ \frac{(17)^3 (73)}{3} \right] + \left[ 1 - 0.63 \left( \frac{30}{43} \right) \right] \times \left[ \frac{(30)^3 (43)}{3} \right] \\
 &= 1.02 \times 10^5 \text{ cm}^4 + 2.17 \times 10^5 \text{ cm}^4 \\
 &= 3.19 \times 10^5 \text{ cm}^4
 \end{aligned}$$

OR

$$\begin{aligned}
 C &= \left[ 1 - 0.63 \left( \frac{17}{43} \right) \right] \times \left[ \frac{(17)^3 (43)}{3} \right] + \left[ 1 - 0.63 \left( \frac{30}{60} \right) \right] \times \left[ \frac{(30)^3 (60)}{3} \right] \\
 &= 5.29 \times 10^4 + 3.70 \times 10^5 \\
 &= \underline{\underline{4.23 \times 10^5 \text{ cm}^4}}
 \end{aligned}$$



Beams B5, B6 - interior

$$C = \left[ 1 - 0.63 \left( \frac{17}{116} \right) \right] \times \left[ \frac{(17)^3 (116)}{3} \right] + \left[ 1 - 0.63 \left( \frac{30}{43} \right) \right] \times \left[ \frac{(30)^3 (43)}{3} \right]$$

$$= 3.89 \times 10^5 \text{ cm}^4$$

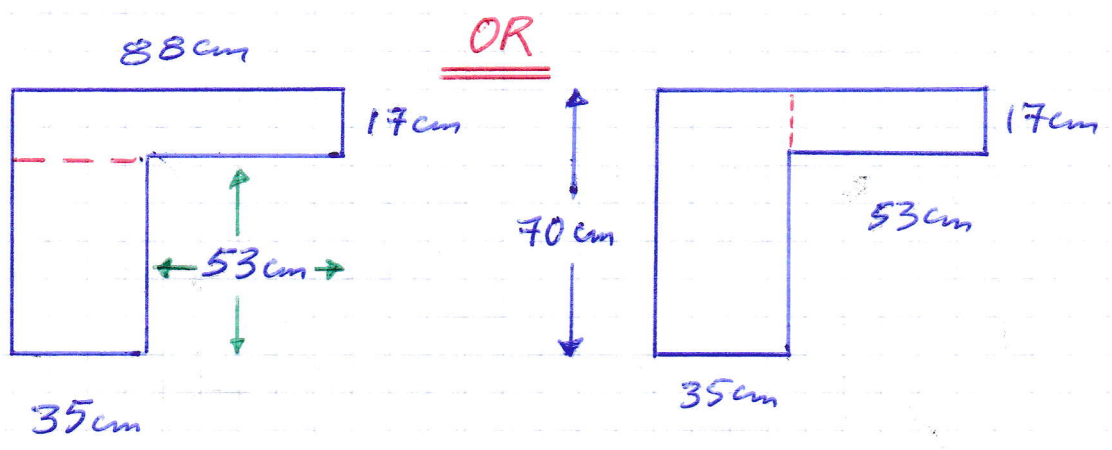
OR

$$C = 2 (5.29 \times 10^4) + (3.70 \times 10^5)$$

$$= \underline{\underline{4.76 \times 10^5 \text{ cm}^4}}$$

Long direction =

Beams B3, B4 - edge



$$C = \left[ 1 - 0.63 \left( \frac{17}{88} \right) \right] \times \left[ \frac{(17)^3 (88)}{3} \right] + \left[ 1 - 0.63 \left( \frac{35}{53} \right) \right] \times \left[ \frac{(35)^3 (53)}{3} \right]$$

$$= 1.27 \times 10^5 + 4.42 \times 10^5$$

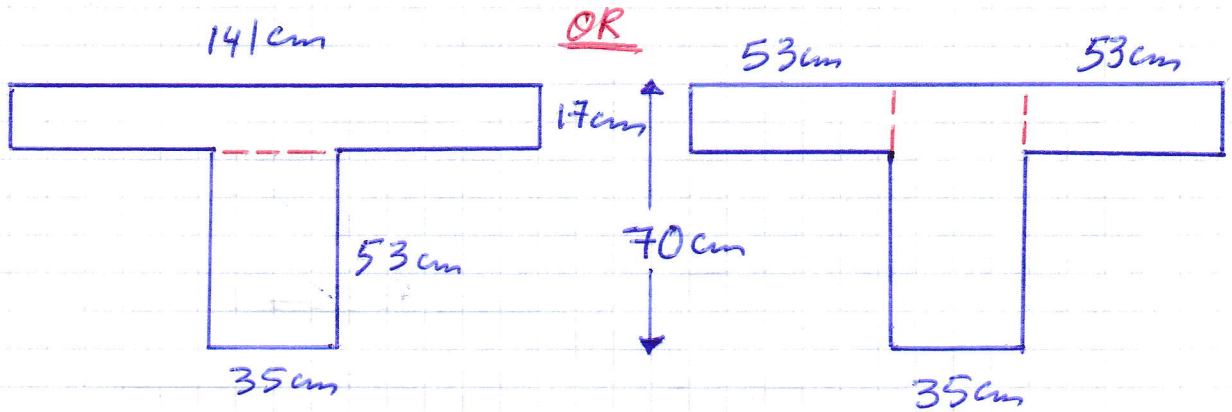
$$= 5.69 \times 10^5 \text{ cm}^4$$

OR

$$C = \left[ 1 - 0.63 \left( \frac{17}{53} \right) \right] \times \left[ \frac{(17)^3 (53)}{3} \right] + \left[ 1 - 0.63 \left( \frac{35}{70} \right) \right] \times \left[ \frac{(35)^3 (70)}{3} \right]$$

$$= 6.93 \times 10^4 + 6.85 \times 10^5$$

$$= \underline{\underline{7.54 \times 10^5 \text{ cm}^4}}$$

Beams B1, B2 - interior

$$C = \left[ 1 - 0.63 \left( \frac{17}{141} \right) \right] \times \left[ \frac{(17)^3 (141)}{3} \right] + \left[ 1 - 0.63 \left( \frac{35}{53} \right) \right] \times \left[ \frac{(35)^3 (53)}{3} \right]$$

$$= 2.13 \times 10^5 + 4.42 \times 10^5$$

$$= 6.55 \times 10^5 \text{ cm}^4$$

OR

$$C = \left[ 2 (6.93 \times 10^4) \right] + \left[ 6.85 \times 10^5 \right]$$

$$= \underline{\underline{8.24 \times 10^5 \text{ cm}^4}}$$

## Distribution of Moment in Column Strip to Beam and Slab:

When a longitudinal beam exists:

$\frac{\alpha L_2}{L_1} \geq 1.0 \Rightarrow 85\%$  of the column strip moment is taken by the beam

$0 \leq \frac{\alpha L_2}{L_1} < 1.0 \Rightarrow$  Linear interpolation between 0% and 85% for the moment to be taken by the beam.

### Example 16.12.1:

Distribute the longitudinal moments computed for frames A, B, C, and D, into 3 parts:

Column strip: ① longitudinal beam  
② Column strip slab

Middle strip ③ Middle strip slab



a) Negative moment at face of exterior support.

For Frame A:  $= -9.90 \text{ t.m}$

$$L_2/L_1 = 6/7.6 = 0.79$$

$$\alpha_1 = 7.22$$

$$\alpha_1 L_2/L_1 = 5.70$$

$$C = 4.23 \times 10^5 \text{ cm}^4 \quad (\text{Beams B7, B8. Edge/short})$$

$$I_s = \frac{(600)(17^3)}{12} = 2.457 \times 10^5 \text{ cm}^4 \quad (\text{Example 16.4.1a})$$

$$\beta_t = C/2 I_s = 0.86$$

$L_2/L_1$	0.5	0.79	1.0
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$$\alpha_1 L_2/L_1 = 5.70 \geq 1.0$$

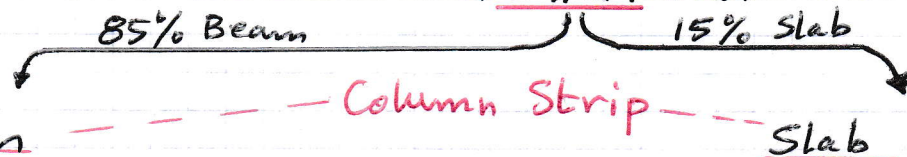
$\beta_t = 0$	100%	100%	100%
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$\beta_t = 0.86$		93.6%	
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$\beta_t \geq 2.50$	90%	81.3%	75%
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$$\therefore 93.6\% \text{ Column strip} = \frac{93.6}{100} \times -9.90$$

$$= \underline{-9.27 \text{ t.m}}$$



$$\frac{85}{100} \times 9.27 = \underline{-7.88 \text{ t.m}}$$

$$\frac{15}{100} \times 9.27 = \underline{-1.39 \text{ t.m}}$$

$$6.4\% \text{ Middle strip} = \frac{6.4}{100} \times -9.90 = \underline{-0.63 \text{ t.m}}$$



## Frame A

Total width = 6m , Column strip width = 3m , Middle strip width = 3m

	Exterior Span			Interior Span	
	Ex. Negative	+ve	Int. -ve	-ve	+ve
Total Moment	-9.90	+35.26	-43.30		
Beam	-7.88	+24.37	-29.92		
Column strip slab	-1.39	+4.30	-5.28		
Middle strip slab	-0.63	+6.59	-8.10		

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### e. Design of Columns

Columns in two-way construction must be designed to resist the moments found from analysis of the slab-beam system. The column supporting an edge beam must provide a resisting moment equal to the moment applied from the edge of the slab (see Table 13.4). At interior locations, slab negative moments are found, assuming that dead and full live loads act. For the column design, a more severe loading results from partial removal of the live load. Accordingly, ACI Code 13.6.9 requires that interior columns resist a moment

$$M_u = 0.07[(q_{Du} + 0.5q_{Lu})l_2l_n^2 - q'_{Du}l_2'(l'_n)^2] \quad (13.7)$$

In Eq. (13.7),  $q_{Du}$  and  $q_{Lu}$  are, respectively, the factored dead and live loads per unit area. The primed quantities refer to the shorter of the two adjacent spans (assumed to carry dead load only), and the unprimed quantities refer to the longer span (assumed to carry dead load and half live load). In all cases, the moment is distributed to the upper and lower columns in proportion to their relative flexural stiffness.

## 13.7 FLEXURAL REINFORCEMENT FOR COLUMN-SUPPORTED SLABS

Consistent with the assumptions made in analysis, flexural reinforcement in two-way slab systems is placed in an orthogonal grid, with bars parallel to the sides of the panels. Bar diameters and spacings may be found as described in Section 13.2. Straight bars are generally used throughout, although in some cases positive-moment steel is bent up where no longer needed, to provide for part or all of the negative requirement. To provide for local concentrated loads, as well as to ensure that tensile cracks are narrow and well distributed, a maximum bar spacing at critical sections of 2 times the total slab thickness is specified by ACI Code 13.3.2 for two-way slabs. At least the minimum steel required for temperature and shrinkage crack control (see Section 13.3) must be provided. For protection of the steel against damage from fire or corrosion, at least 20 mm concrete cover must be maintained.

Because of the stacking that results when bars are placed in perpendicular layers, the inner steel will have an effective depth 1 bar diameter less than the outer steel. For flat plates and flat slabs, the stacking problem relates to middle-strip positive steel and column-strip negative bars. In two-way slabs with beams on the column lines, stacking occurs for the middle-strip positive steel, and in the column strips is important mainly for the column-line beams, because slab moments are usually very small in the region where column strips intersect.

In the discussion of the stacking problem for two-way slabs supported by walls or stiff edge beams, in Section 13.4 it was pointed out that, because curvatures and moments in the short direction are greater than in the long direction of a rectangular panel, short-direction bars are normally placed closer to the top or bottom surface of the slab, with the larger effective depth  $d$ , and long-direction bars are placed inside these, with the smaller  $d$ . For two-way beamless flat plates, or slabs with relatively flexible edge beams, things are not so simple.

Consider a rectangular interior panel of a flat plate floor. If the slab column strips provided unyielding supports for the middle strips spanning in the perpendicular direction, the short-direction middle-strip curvatures and moments would