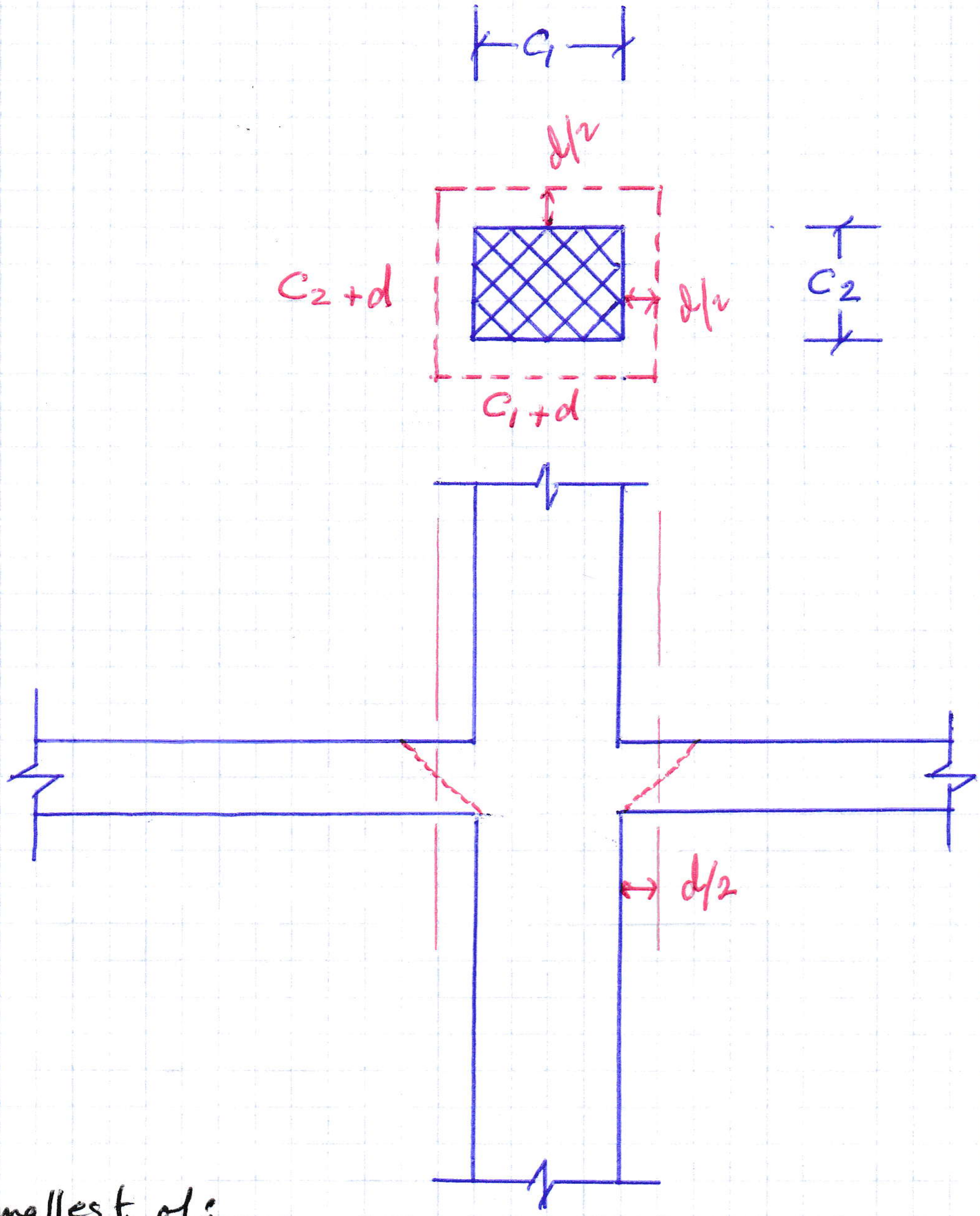



Smallest of:

$$\phi V_c = 0.75\,(0.17)\left(1+\frac{2}{\beta_c}\right)\lambda\sqrt{f'_c}\;b_o d$$

$$= 0.75\,(0.083)\left(\frac{\alpha_s d}{b_o}+2\right)\lambda\sqrt{f'_c}\;b_o d$$

$$= 0.75\,(0.33)\,\lambda\sqrt{f'_c}\;b_o d$$

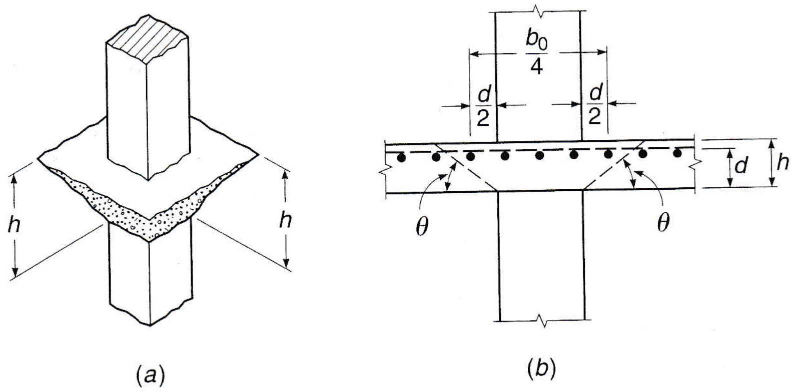
$\beta_c$ = ratio of the longside to short side of the column or capital

$\alpha_s$ = 40 for interior columns

= 30 for the edge columns

= 20 for corner columns

**FIGURE 13.21**
Failure surface defined by punching shear.




At such a section, in addition to the shearing stresses and horizontal compressive stresses due to negative bending moment, vertical or somewhat inclined compressive stress is present, owing to the reaction of the column. The simultaneous presence of vertical and horizontal compression increases the shear strength of the concrete. For slabs supported by columns having a ratio of long to short sides not greater than 2, tests indicate that the nominal shear strength may be taken equal to

$$V_c = 4\sqrt{f'_c}\,b_o d \tag{13.11a}$$

according to ACI Code 11.12.2, where $b_o$ = the perimeter along the critical section.

However, for slabs supported by very rectangular columns, the shear strength predicted by Eq. (13.11*a*) has been found to be unconservative. According to tests reported in Ref. 13.15, the value of $V_c$ approaches $2\sqrt{f'_c}\,b_o d$ as $\beta_c$, the ratio of long to short sides of the column, becomes very large. Reflecting this test data, ACI Code 11.12.2 states further that $V_c$ in punching shear shall not be taken greater than

$$V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f'_c}\,b_o d \tag{13.11b}$$

The variation of the shear strength coefficient, as governed by Eqs. (13.11*a*) and (13.11*b*) is shown in Fig. 13.22 as a function of $\beta_c$.

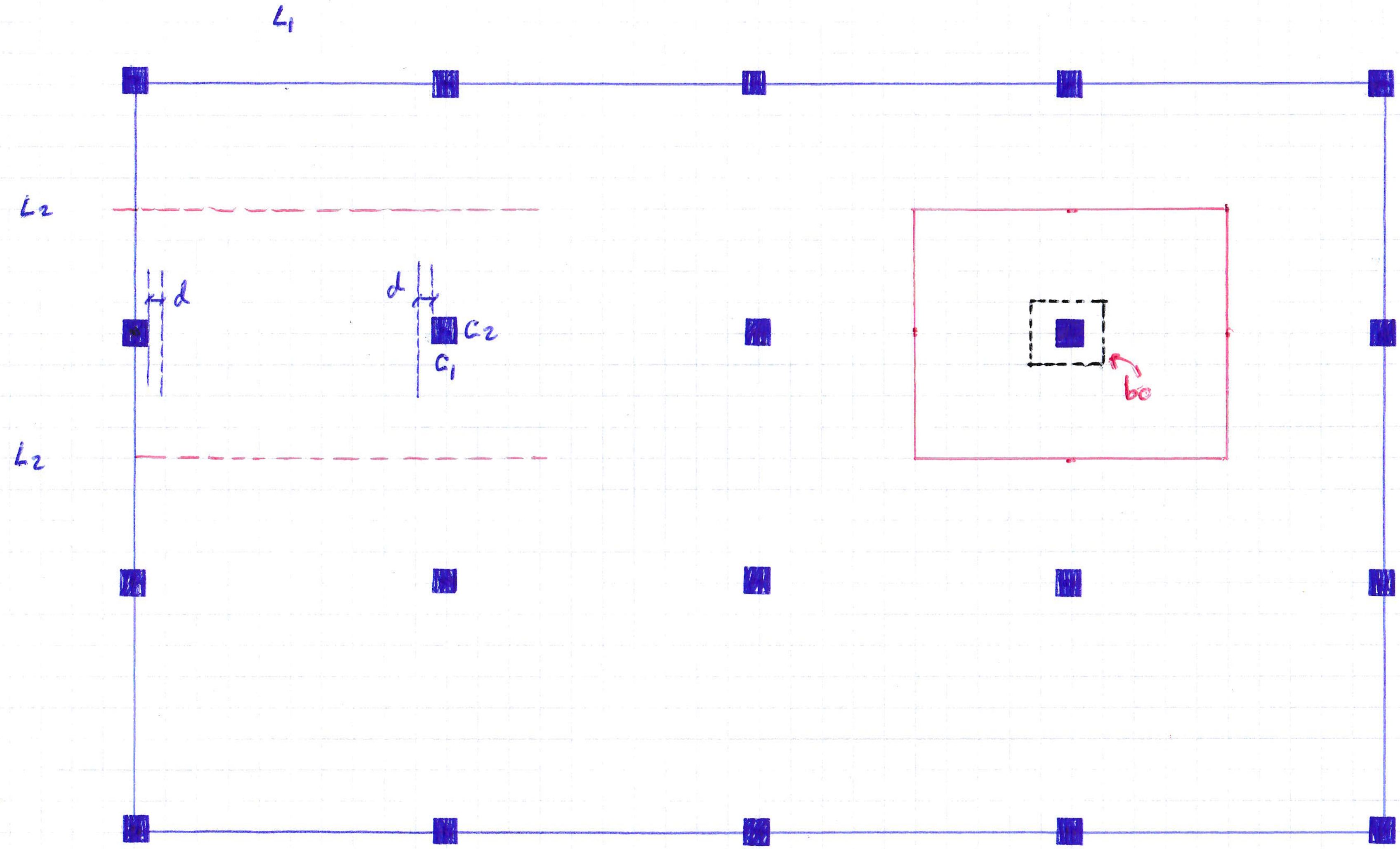
Further tests, reported in Ref. 13.16, have shown that the shear strength $V_c$ decreases as the ratio of critical perimeter to slab depth $b_o/d$ increases. Accordingly, ACI Code 11.12.2 states that $V_c$ in punching shear must not be taken greater than

$$V_c = \left(\frac{\alpha_s d}{b_o} + 2\right)\sqrt{f'_c}\,b_o d \tag{13.11c}$$

where $\alpha_s$ is 40 for interior columns, 30 for edge columns, and 20 for corner columns, i.e., columns having critical sections with 4, 3, or 2 sides, respectively.

Thus, according to the ACI Code, the punching shear strength of slabs and footings is to be taken as the smallest of the values of $V_c$ given by Eqs. (13.11*a*), (13.11*b*), and (13.11*c*). The design strength is taken as $\phi V_c$ as usual, where $\phi = 0.75$ for shear. The basic requirement is then $V_u \leq \phi V_c$.

For columns with nonrectangular cross sections the ACI Code indicates that the perimeter $b_o$ must be of minimum length, but need not approach closer than $d/2$ to the perimeter of the reaction area. The manner of defining the critical perimeter $b_o$ and the ratio $\beta_c$ for such irregular support configurations is illustrated in Fig. 13.23.




One-way Shear Action

(Beam)

$$V_u = w_u\left(\frac{1.15}{2}\right)\left[(L_2)(L_1 - c_1 - 2d)\right]$$

$$\phi V_c = 0.75(0.17)(\sqrt{f_c'})(\lambda)(L_2)(d)$$

$$V_u = w_u\left[L_1 L_2 - \text{Area within } b_o\right]$$

Two-way shear Action

(Punching Shear)

# Transfer of Moment and Shear (Monolithic Joints) Flat Plates

Final Exam

$$M_u = \underbrace{M_{ub}}_{\text{flexure transfer of } M_u} + \underbrace{M_{uv}}_{\text{shear transfer of } M_u}$$

(ACI 11.12.6)

$$M_{ub} = \gamma_f M_u = \left[\frac{1}{1+\frac{2}{3}\sqrt{\frac{b_1}{b_2}}}\right] M_u$$

$b_1$ = critical section dimension in the longitudinal direction

$= c_1 + d/2$ for exterior columns

$= c_1 + d$ for interior columns

$b_2$ = critical section dimension in the transverse direction

$= c_2 + d$

Simplifications: (ACI - 13.5.3.3)

if, for exterior supports: $V_u \le 0.75\,\phi V_c$ edge

$V_u \le 0.50\,\phi V_c$ corner

$\Rightarrow$ neglect interaction between shear and moment

i.e., the full exterior moment is transfered through flexure ($\gamma_f = 1.0$)

therefore, consider punching shear only

for interior supports: $V_u \le 0.4\,\phi V_c$

$\Rightarrow$ increase $\gamma_f$ by 25%

$$v_{u_1} = \frac{V_u}{A_c} - \frac{M_{uv}\, x_1}{J_c}$$

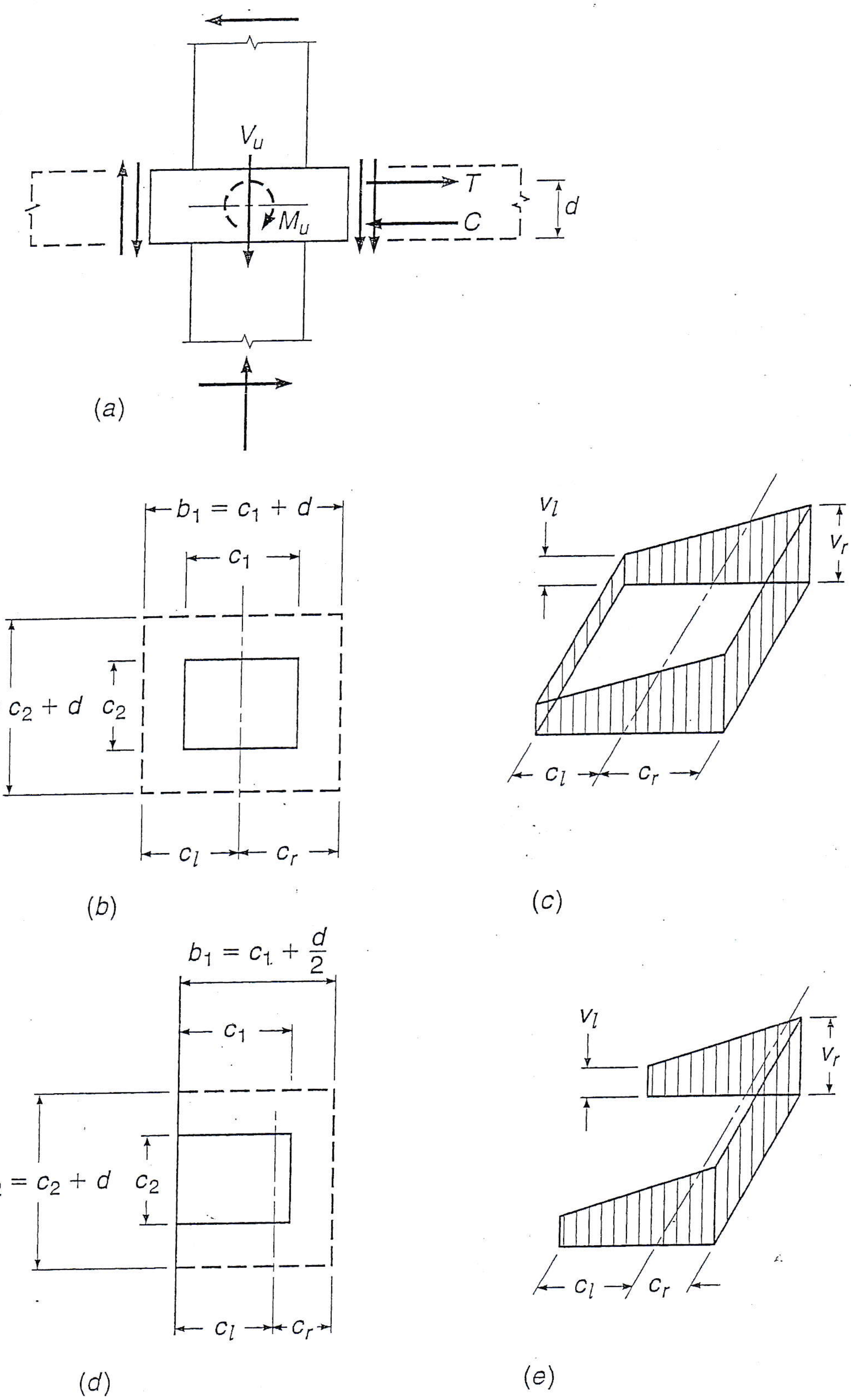
$$v_{u_2} = \frac{V_u}{A_c} + \frac{M_{uv}\, x_2}{J_c}$$

$$\phi V_n = \frac{\phi V_c}{b_0 d}$$

$$\phi V_n \geq v_{u_2}$$

stres          stress

E 13.31
of moment from
lumn: (*a*) forces
from vertical load
anced moment;
l section for an
olumn; (*c*) shear
ribution for an
olumn; (*d*) critical
r an edge column;
stress distribution
e column.




columns, it is reasonable to suppose that the portion transferred by flexure increases as the width of the critical section that resists the moment increases, i.e., as $c_2 + d$ becomes larger relative to $c_1 + d$ in Fig. 13.31*b*. According to ACI Code 13.5.3, the moment considered to be transferred by flexure is

$$M_{ub} = \gamma_f M_u \tag{13.16a}$$

Tests indicate that for square columns about 60 percent of the unbalanced moment is transferred by flexure (forces $T$ and $C$ in Fig. 13.33$a$) and about 40 percent by shear stresses on the faces of the critical section (Ref. 13.24). For rectangular columns, it is reasonable to suppose that the portion transferred by flexure increases as the width of the critical section that resists the moment increases, i.e., as $c_2 + d$ becomes larger relative to $c_1 + d$ in Fig. 13.33$b$. According to ACI Code 13.5.3, the moment considered to be transferred by flexure is

$$M_{ub} = \gamma_f M_u \tag{13.16a}$$

where

$$\gamma_f = \frac{1}{1 + \frac{2}{3}\sqrt{b_1/b_2}} \tag{13.16b}$$

and $b_1$ = width of critical section for shear measured in direction of span for which moments are determined

$b_2$ = width of critical section for shear measured in direction perpendicular to $b_1$

The value of $\gamma_f$ may be modified if certain conditions are met: For unbalanced moments about an axis parallel to the edge of exterior supports, $\gamma_f$ may be increased to 1.0, provided that the factored shear $V_u$ at the edge support does not exceed $0.75\phi V_c$ or at a corner support does not exceed $0.5\phi V_c$. For unbalanced moments at interior supports and about an axis perpendicular to the edge at exterior supports, $\gamma_f$ may be increased up to 1.25 times the value in Eq. (13.16$b$), provided that $V_u \leq 0.4\phi V_c$. In all of these cases, the net tensile strain $\epsilon_t$ calculated for the section within 1.5$h$ on either side of the column or column capital must be at least 0.010.

The moment assumed to be transferred by shear, by ACI Code 11.11.7, is

$$M_{ub} = (1 - \gamma_f)M_u = \gamma_v M_u \tag{13.16c}$$

It is seen that for a square column Eqs. (13.16$a$), (13.16$b$), and (13.16$c$) indicate that 60 percent of the unbalanced moment is transferred by flexure and 40 percent by shear, in accordance with the available data. If $b_2$ is very large relative to $b_1$, nearly all of the moment is transferred by flexure.

The moment $M_{ub}$ can be accommodated by concentrating a suitable fraction of the slab column-strip reinforcement near the column. According to ACI Code 13.5.3, this steel must be placed within a width between lines 1.5$h$ on each side of the column or capital, where $h$ is the total thickness of the slab or drop panel.

The moment $M_{uv}$, together with the vertical reaction delivered to the column, causes shear stresses assumed to vary linearly with distance from the centroid of the critical section, as indicated for an interior column by Fig. 13.33$c$. The stresses can be calculated from

$$v_l = \frac{V_u}{A_c} - \frac{M_{uv}c_l}{J_c} \tag{13.17a}$$

$$v_r = \frac{V_u}{A_c} + \frac{M_{uv}c_r}{J_c} \tag{13.17b}$$

where $A_c$ = area of critical section $= 2d[(c_1 + d) + (c_2 + d)]$

$c_l$, $c_r$ = distances from centroid of critical section to left and right faces of section, respectively

$J_c$ = property of critical section analogous to polar moment of inertia

Types

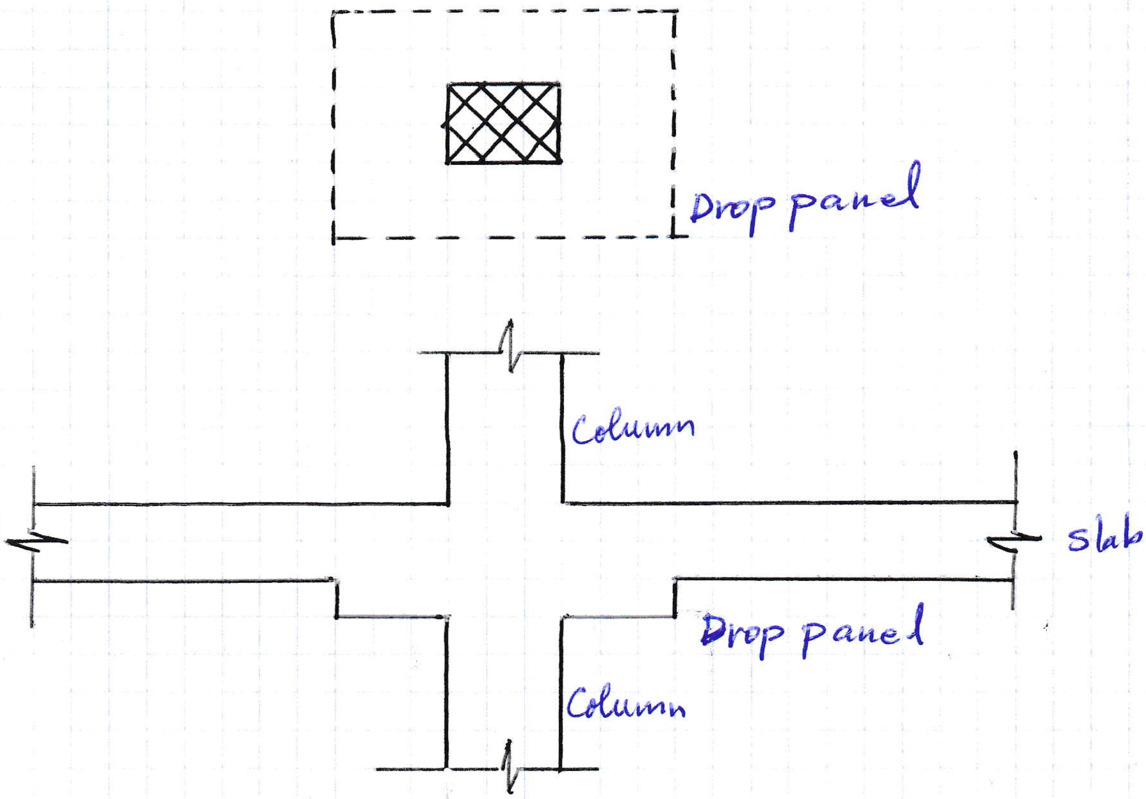
Dr Jamal Zalatimo

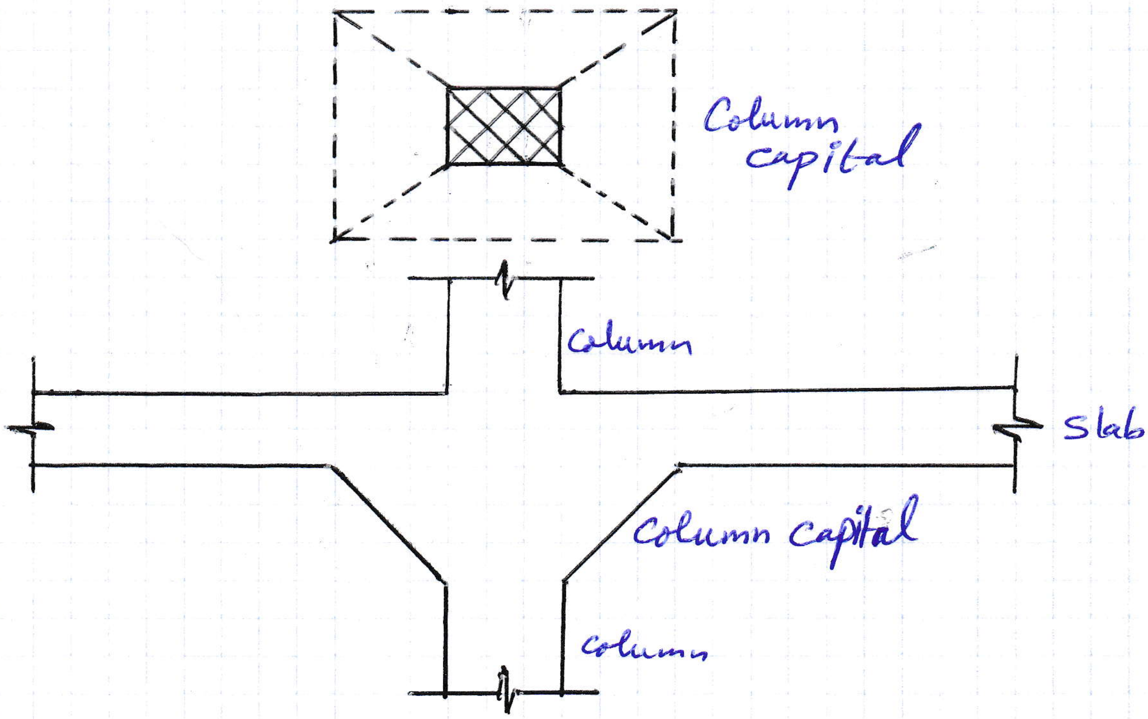
A) Two-way Slab on Beams

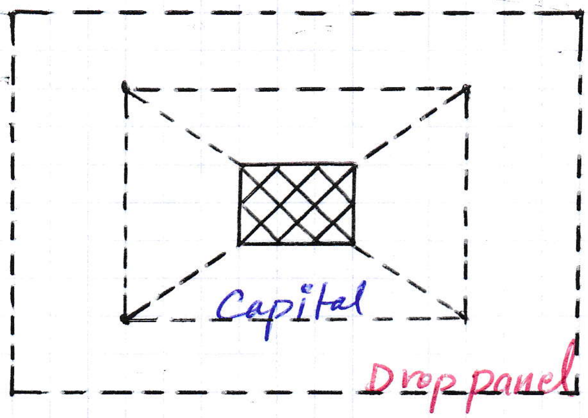
Solid, Ribbed, Waffle

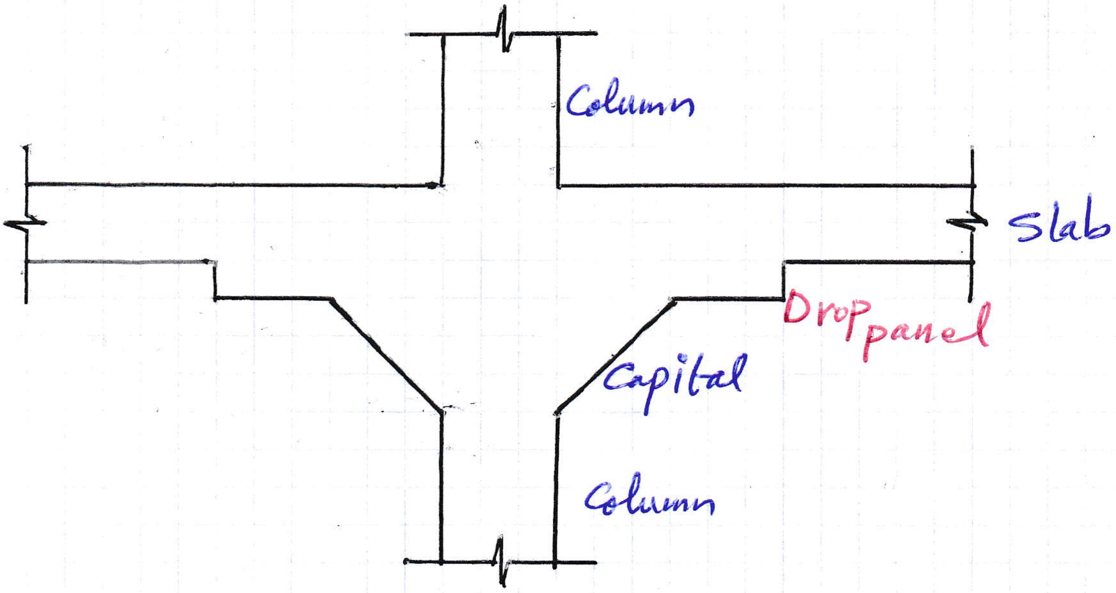
B) Flat Slab

Solid, Ribbed, Waffle






c) Flat Plate

Solid