Design of Reinforced Concrete Structures

A Practical Approach

Ibrahim Mohammad Arman

Third Edition

2021

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تنويه

هذه النسخة من الكتاب نسخة مجانية عن روحي فلا تباع. سيستفيد منها طالب الهندسة والمهندس في سوق العمل.

Ibrahim Mohammad Arman
March 20, 2021

ابراهيم محمد عرمان
اذار 20، 2021

Preface

This book is designed as a reference for reinforced concrete design of structures. This book has seventeen chapters which cover the main aspects of reinforced concrete design in a simple way. The chapters of this reference discuss the analysis and design of reinforced concrete sections for flexure, shear, torsion and axial forces. Also, they include the analysis and design of one-way and two-way slab systems including solid, ribbed, waffle and voided slabs. This reference illustrates the analysis and design of footing systems, retaining walls, water tanks and shells. Spherical domes and conical shells are illustrated. Also, this reference introduces the main principles for seismic design of reinforced concrete structures based on the international codes IBC, ASCE and UBC. The analysis and design of reinforced concrete sections is based on the American concrete Institute ACI.

This reference is very valuable for design engineers in the design industry, as it contains practical examples on the analysis and design of reinforced concrete elements and systems like beams, columns, slabs, footings, retaining walls, water tanks, and shell structures.

Ibrahim Mohammad Arman
12-1-2019

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Chapter 1: Introduction

This chapter introduces the main structural properties of concrete and reinforcing bars. In addition, it illustrates concrete cover and bars spacings in the structural members. Also, it discusses the loads and the main reinforced concrete design methods. The following points illustrate these subjects.

1. **Definition of Reinforced concrete:** It is a mixture of concrete and steel. Such a mixture combines the best properties of both materials to overcome their deficiencies.
2. **Main concrete properties:**

    A. Good in compression
    B. Low tensile strength
    C. Good in fire resistance

3. **Reinforcing steel:**

    A. High tensile strength
    B. Low fire resistance

4. **Concrete compressive strength:**

The compressive strength, f'c, is based on using uniaxial compression test, which 150mm diameter and 300mm height concrete cylinder is tested after cured under standard laboratory conditions and tested at a specified rate of loading at 28 days of age. From this test, the stress-strain curves of concrete cylinders are obtained.

Figures 1.1 and 1.2 show the typical concrete stress-strain curves for concrete in compression. From these figures, the following notes can be stated:

A. The lower the strength of concrete, the higher the failure strain
B. The length of the initial relatively linear portion increases with the increase in the compressive strength
C. The first portion of the curve to about 40% of the ultimate strength, f'c, can be considered linear for all practical purposes
D. There is apparent reduction in ductility with increased strength
E. After approximately 70% of the ultimate stress, the material loses a large portion of its stiffness, thereby increasing the curvilinearity of the stress-strain diagram
F. Usually concrete can reach a strain of 0.003 or larger before actual crushing occurs

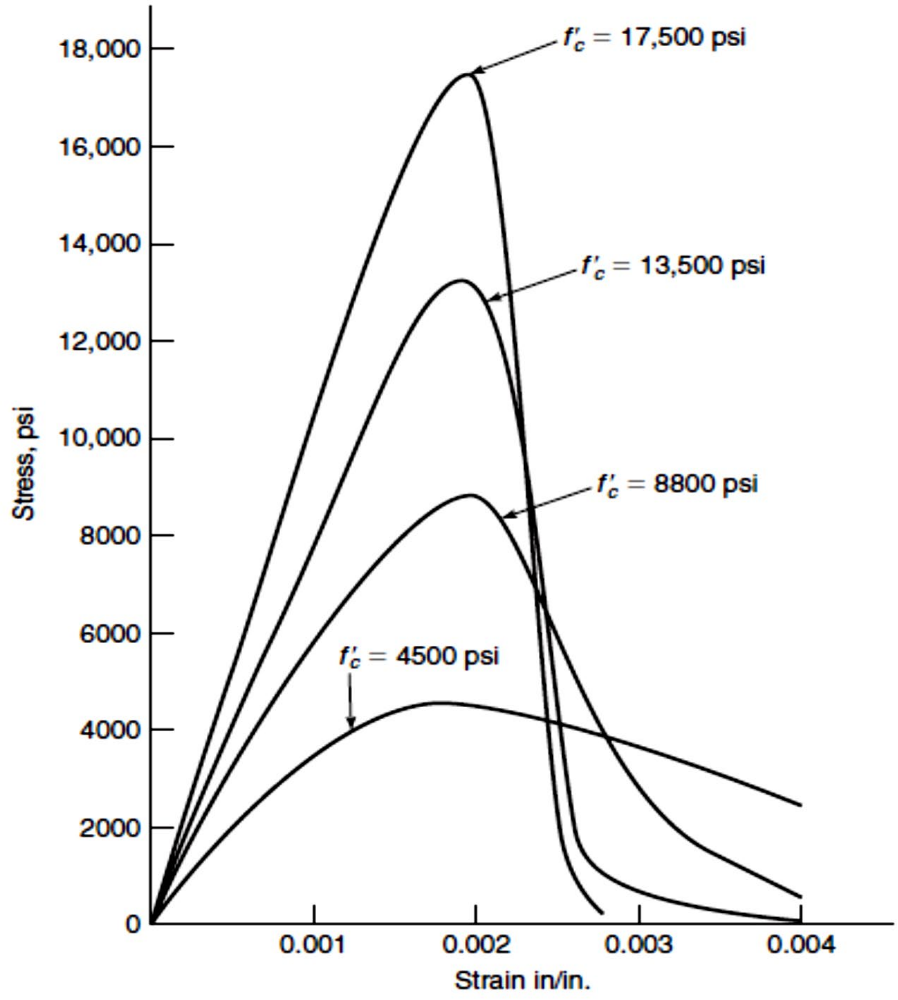


Figure 1.1: Typical concrete stress-strain curves in compression "REINFORCED CONCRETE Mechanics and Design by JAMES K. WIGHT and JAMES G. MACGREGOR". 1.0 psi=0.007MPa.

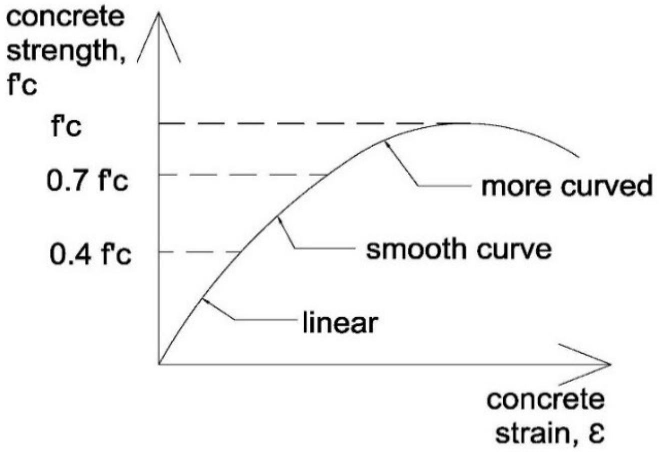


Figure 1.2: Typical stress-strain curve for concrete in compression

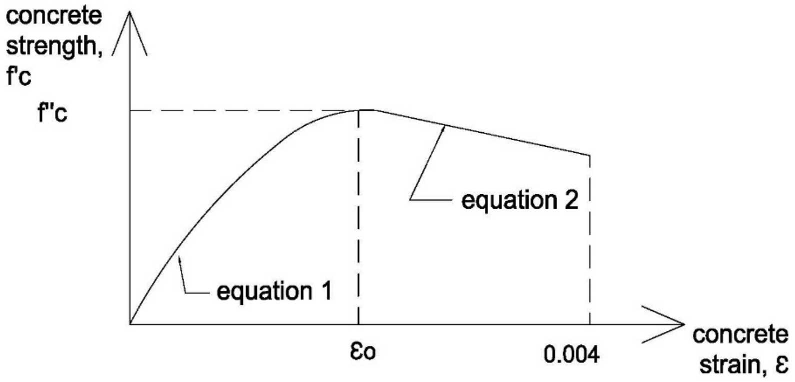


Figure 1.3: Simplified stress-strain curve for concrete in compression "Park and Pauly"

Equation 1:

$$f_c = f''_c\left(\frac{2 \in c}{\in o} - \left(\frac{\in c}{\in o}\right)^2\right)$$

Equation 2:

$$f_c = f''_c\big(1 - 100(\in c{-}\in o)\big)$$

f''c= f'c in the concrete stress-strain curve and it equals 0.85 f'c for concrete member to account for the differences between cylinder strength and member strength. These differences result from different curing and placing, which give rise to different water-gain effects due to vertical migration of bleed water, and differences between the strengths of rapidly loaded cylinders and the strength of the same concrete loaded more slowly

Based on ACI code, the maximum strain in concrete "ultimate crushing strain" is equal to 0.003

Based on British Standards, the concrete strength is based on testing a cube of 150mm side length at 28 days. The cylinder compressive strength equals about 0.80 the cube compressive strength.

Minimum concrete strength:

Special moment frames and special structural walls with Grade 60 or 80 reinforcement:

$$f'_c \geq 21MPa$$

Special structural walls with Grade 100 reinforcement:

$$f'_c \geq 35MPa$$

Other structures:

$$f'_c \geq 17.5MPa$$

5. Concrete modulus of elasticity, Ec:

The concrete has no well-defined modulus of elasticity. It depends on concrete compressive strength and varies with time.

The ACI 318-19 code "American Concrete Institute" gives the following equation for Ec:

$$E_C = 0.043{w_c}^{1.5}\sqrt{f'_c}$$

Where:

w_c= concrete density in kg/m^3

w_c varies between 1440 to 2560 kg/m^3

E_c and f'_c are in MPa

For normal weight concrete, substitute wc=2300kg/m^3 in the above equation, this gives:

$$E_C = 4700\sqrt{f'_c}$$

6. Concrete tensile strength:

The tensile strength of concrete is relatively low. It is approximately equals to about 0.1 f’c.

Based on ACI 209R-92: the tensile strength for pure tension is given by:

$$f_t = 0.33\lambda\sqrt{f'_c} \qquad in\ MPa$$

The tensile strength of concrete can be determined using the split cylinder test. Figure 1.4 summarizes the split cylinder test.

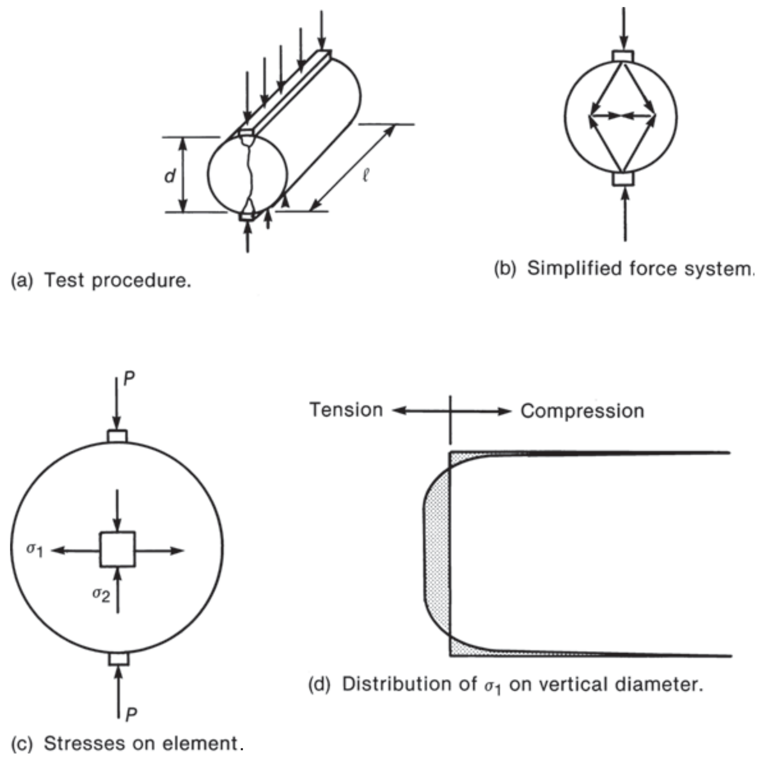


Figure 1.4: Split cylinder test

For members subjected to bending, the tensile strength of concrete can be determined by bending test on plain concrete beam. The tensile capacity then can be found using the following equation:

$$f_r = 0.62\lambda\sqrt{f'_c} \qquad in\ MPa$$

The factor λ is used to consider the type of concrete. λ= 1 for normal weight concrete.

λ is less than 1.0 for light weight concrete.

From fr, the cracked moment, M_{cr}, of a concrete cross section can be computed as follows:

$$\sigma = \frac{My_t}{I_g} = f_r \ \rightarrow M = \frac{f_r I_g}{y_t} = M_{cr}$$

I_g= the uncracked moment of inertia of the cross section. Reinforcing steel can be neglected.

y_t= distance from extreme tension fibers to centroidal axis.

7. Reinforcing steel strength:

- The modulus of elasticity of steel is equal to 200 000 MPa
- Steel grade 60. ASTM A615 and ASTM A706
- The main difference between steel A615 and A706, is that the steel A706 is more ductile and they both have the same yield strength

- Grade 40: fy= 40ksi= 280MPa
- Grade 60: fy= 60ksi= 420MPa
- Grade 80: fy= 80ksi= 560MPa
- Grade 100: fy= 100ksi= 700MPa
- Based on ACI 318-19, the ratio of actual tensile strength to actual yield strength, minimum: 1.10 for steel A615 and 1.17 for steel A706.
- There are limitations on steel yield strength for seismic design especially in special moment resisting frames to provide ductility provisions (Refer to ACI 318-19 chapter 20).
- In steel bars, sometimes, the ultimate strength of steel, fu= 1.25-1.5 fy.
- Stress – strain of steel is as follows, Figure 1.5:

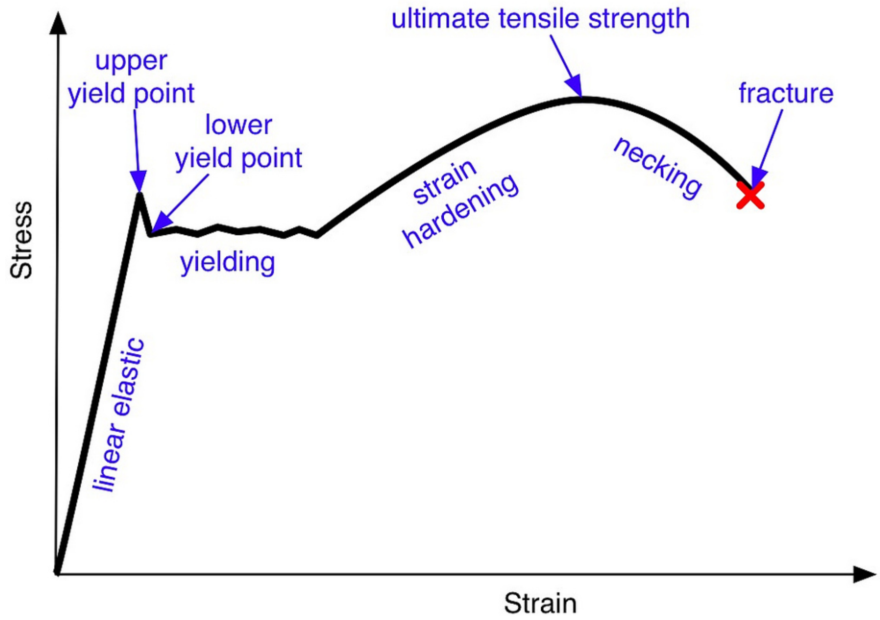


Figure 1.5: Stress-strain curve of reinforcing steel- typical

8. Reinforcing steel bars:

- The bar area is given by:

$$A_b = \frac{\pi}{4} {d_b}^2 \qquad\qquad A_b: mm^2,\ \ d_b: mm$$

- Bar weight "mass" per meter:

$$W = bar\ area\ x\ one\ meter\ x\ steel\ density = \frac{\pi}{4} {d_b}^2 (10)^{-6}(1)\left(\frac{7850kg}{m^3}\right) = \frac{{d_b}^2}{162} = 0.0062{d_b}^2$$

- Bar types: plain, deformed
- Yield strength of undeformed bars, fy= 250MPa
- Bar diameter in mm: 6, 8, 10, 12, 14, 16, 18, 20, 22, 25, 32

9. Bar spacing and concrete cover:

- It is necessary to guard against honeycombing and ensure that the concrete mix passes through the reinforcing steel without separation.
- Cover protect steel from corrosion.
- Concrete protect steel from fire.
- Codes specify a minimum required concrete cover.
- Some of the major requirements of ACI 318-19 (Section 25.2) code are:

  - In beams, clear distance between parallel bars in a layer shall be not less than the maximum of the bar diameter, db, 25mm and 4/3 times the maximum size of the aggregate.
  - In beams, clear distance between layer of bars shall be not less than 25mm.
  - In columns, clear distance between parallel bars in a layer shall be not less than the maximum of 1.5 times the bar diameter, 40mm and 4/3 times the maximum size of the aggregate.
  - Minimum clear cover shall be as follows:

    ** columns, beams: 40mm

    ** slabs, walls, joists, shells: 20mm for bars with diameter less than 35mm

    ** members cast against soil: 75mm

    ** members subjected to drink water: 50mm

10.Loads:

- Gravity loads:

  - Dead: own weight of structural elements
  - Superimposed dead: weight of nonstructural elements
  - Live: people + furniture
  - Snow
  - Soil weight
  - Water weight

- lateral loads:

  - Earthquake
  - Wind
  - Water
  - Soil

- Blast

- Static loads
- Dynamic loads

Weight of beam per meter= cross section area x unit weight of concrete

Weight of column per meter= cross section area x unit weight of concrete

Weight of solid slab- m^2= thickness of slab x unit weight of concrete

Weight of solid concrete wall- m^2= thickness of wall x unit weight of concrete

Superimposed dead loads:

- Brick wall: w= thickness of bricks x unit weight of bricks + thickness of plastering x unit weight of plain concrete
- Tiles: w= thickness of each layer x unit weight of the layer
- Unit weights:

  - Reinforced concrete: 25kN/m^3
  - Plain concrete: 23kN/m^3
  - Masonry stone: 27kN/m^3
  - Concrete blocks: 12-15kN/m^3 , take 15kN/m^3
  - Fill under tiles: 18kN/m^3
  - Plastering: 23kN/m^3

For example:

- Brick wall: 200mm bricks + 20mm plastering + 20mm plastering: w= 3.92kN/m^2
- Brick wall: 100mm bricks + 20mm plastering + 20mm plastering: w= 2.42kN/m^2
- Masonry wall: 50mm stone + 120mm plain concrete + 0.03 insulation + 100mm bricks + 20mm plastering: w= 6.07kN/m^2
- Tiles: 30mm marble tiles + 20mm plain concrete + 100mm fill + 10mm plastering at bottom surface of slab: w= 3.3kN/m^2
- One can add 1-1.5kN/m^2 to account for 100mm brick partitions, this value can be added to the tiles loads to have a complete superimposed dead load on the slab.

Live loads:

Codes: IBC-2012, IBC-2015, IBC-2018, ASCE 7-10, ASCE 7-16, UBC 97 (IBC: International Building Code. ASCE: American Society of Civil Engineers. UBC: Uniform Building Code)

11.Design methods:

- **Working design method: allowable stress method**

- Here, in this textbook, it will be used to check stress only.
- Its principles are:

  - use of actual, working, unfactored or service loads
  - use of allowable stresses in concrete and steel:

    Concrete allowable strength; compression: $f_{c,all} = 0.45f'_c$

    Steel allowable strength; tension: $f_{s,all} = 0.4f_y$

- **Ultimate strength method:**

  - Use of ultimate loads: use load combinations: magnified loads: load factors
  - Use of section capacity.

Required strength: required internal forces to be resisted by using load combinations (like M_u).

Nominal strength: strength of a member or a section calculated in accordance with provisions and assumptions of the strength design method of code (like M_n).

Design strength: nominal strength x strength reduction factor (like $\emptyset M_n$)

Ø= 0.9 flexure- tension controlled

Ø= 0.75 shear

Ø= 0.75 torsion

Ø= 0.65 axial- tied column

Ø= 0.75 axial- spiral column

Purpose of the strength reduction factor:

- To allow slight variations of material strengths.
- To allow slight variations in dimensions.
- To allow inaccuracies in the design equations.
- To reflect the degree of ductility.
- To reflect the importance of the member in the structure.

Purpose of load factors:

- To account for inaccuracy in load calculations.
- To account for slight variations in loads during lifetime of the structure.
- To account for variability is structural analysis.

In design, use the critical load combinations.

12.Structural analysis and modeling:

- One dimensional structural modeling: beam, truss.
- Two-dimensional structural modeling: plane frame.
- Three-dimensional structural modeling: space frame.

Slabs: one way, two way.

Slabs: solid, waffle, ribbed, voided.

In general, for very stiff supports, when L/B > 2, the solid slab can be assumed to be one-way.

L and B are the panel dimensions.

Figures 1.6 and 1.7 show general components of a building structure.

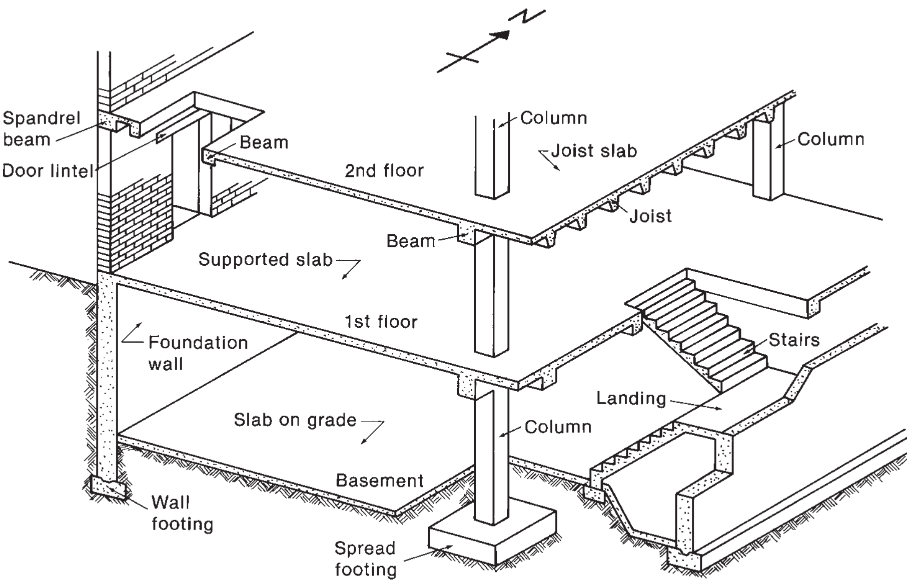


Figure 1.6: Building components-1

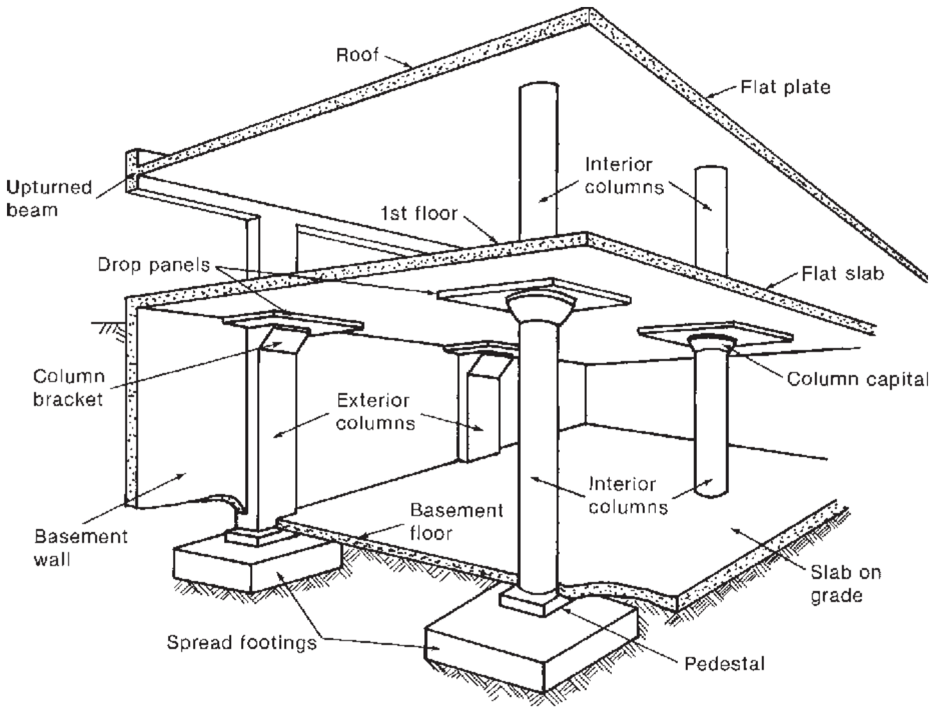


Figure 1.7: Building components-2

Notes on one-way slab system:

- The beams in one-way slab system are distributed in one direction.
- Align beams in the long direction (maximum column spacings) and align the slab in the short direction to have a smaller slab thickness.
- The loads on the beams are determined from slab reactions or an approximate method can be used which is the tributary area or distance method. In this method, each beam carries the direct load on it plus the load from half the distance to the next beam.
- Grid and space frame structural models are the best way to analyze the slab systems.

Figure 1.8 shows a plan of a reinforced concrete building. It is required to draw the structural models of slab strips and beams.

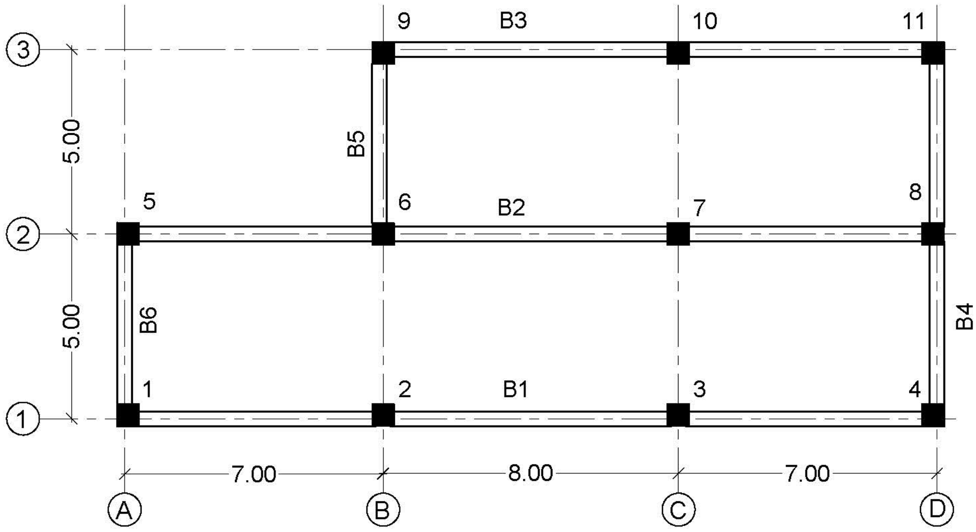


Figure 1.8: Beams layout

Given:

- The slab is solid with thickness, h= 200mm
- Superimposed dead load on slab, W_{SD}= 3kN/m^2
- Live load on slab, W_L= 4kN/m^2
- All beams have width, bw= 400mm and thickness, h= 600mm
- Perimeter wall weight= 10 kN/m

The beams are aligned in x-direction and the slab is aligned in y-direction.

Slab own weight = 0.20 (25)= 5kN/m^2

Slab ultimate load, Wu= 1.2(5+3)+1.6(4)= 16kN/m^2

There are two slab strips

- Strip 1, S1: between gridlines A and B
- Strip 2, S2: between gridlines B and D

Load on beam B1, Wu1= weight of beam + weight of wall + loads from slab

Wu1= (0.4x0.60x25x1.2) + (10)(1.2) + (5/2)(16)= 59.2kN/m

Load on beam B2, Wu2A= weight of beam + weight of wall + loads from slab

Wu2A= (0.4x0.60x25x1.2) + (10)(1.2) + (5/2)(16)= 59.2kN/m

Load on beam B2, Wu2B= weight of beam + loads from slab

Wu2B= (0.4x0.60x25x1.2) + (5)(16)= 87.2kN/m

Load on beam B3, Wu3= weight of beam + weight of wall + loads from slab

Wu3= (0.4x0.60x25x1.2) + (10)(1.2) + (5/2)(16)= 59.2kN/m

Load on beams B4, B5 and B6= weight of beam + weight of wall

Wu4= Wu5= Wu6= (0.4x0.60x25x1.4) + (10)(1.4)= 22.4kN/m

Figures 1.9 and 1.10 show the slab strips and beams structural models.

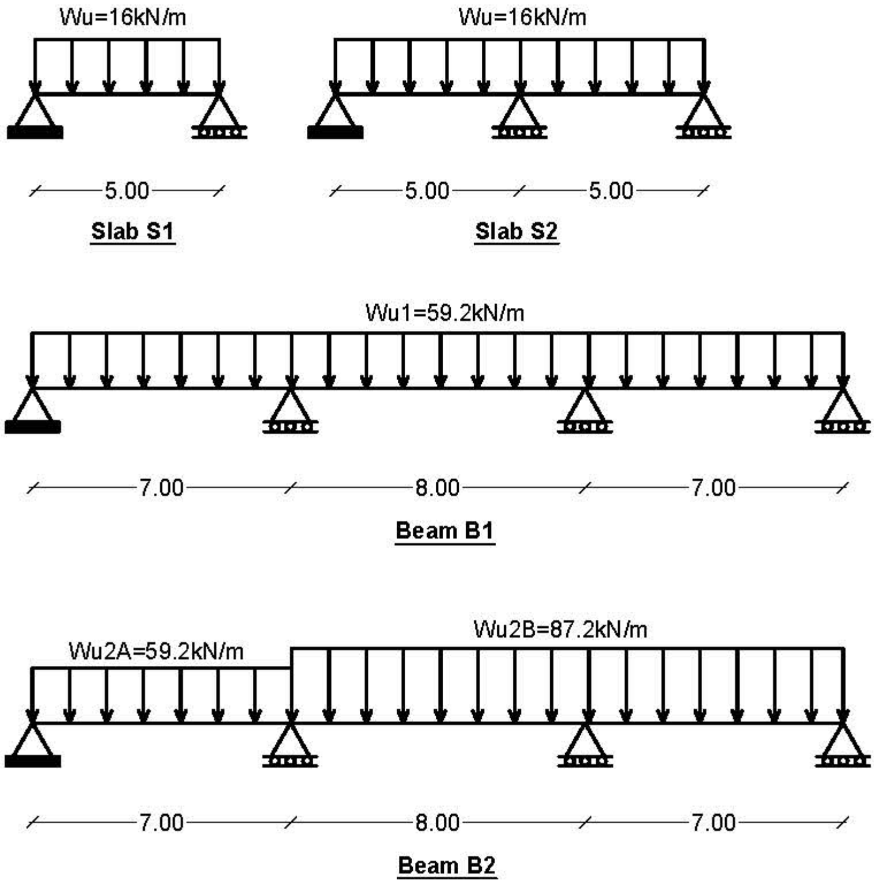
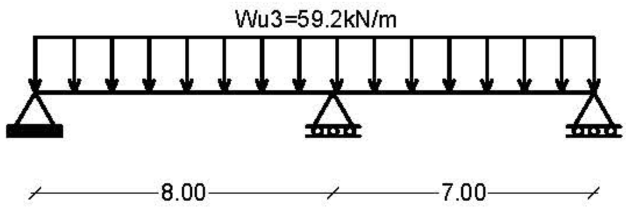
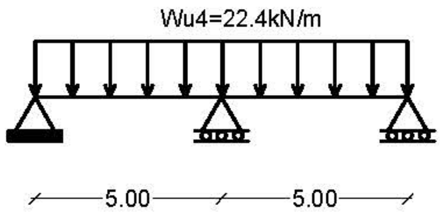


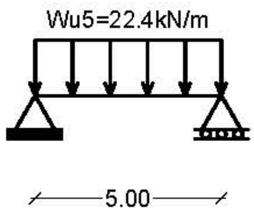
Figure 1.9: Slabs and beams structural models -1



Beam B3



Beam B4



Beams B5, B6

Figure 1.10: Slabs and beams structural models-2

Figure 1.11: Reinforcing bars-picture 1

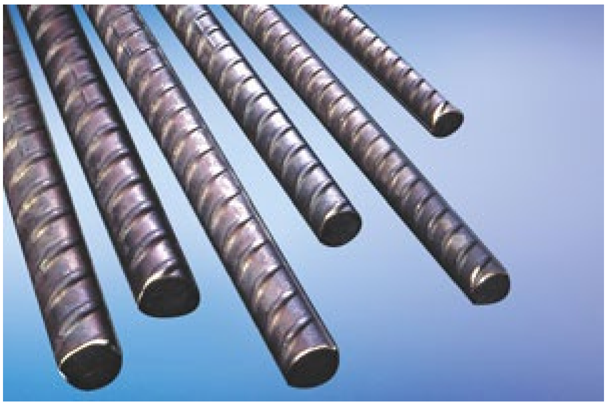


Figure 1.12: Reinforcing bars- picture 2

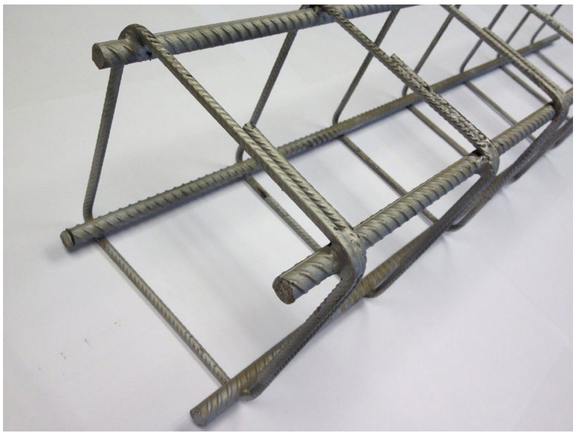


Figure 1.13: Steel reinforcement- picture 1

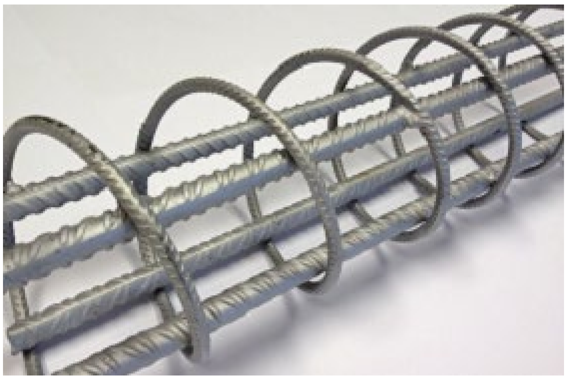


Figure 1.14: Steel reinforcement- picture 2

Figure 1.15: Steel reinforcement- picture 3

Figure 1.16: Steel reinforcement- picture 4

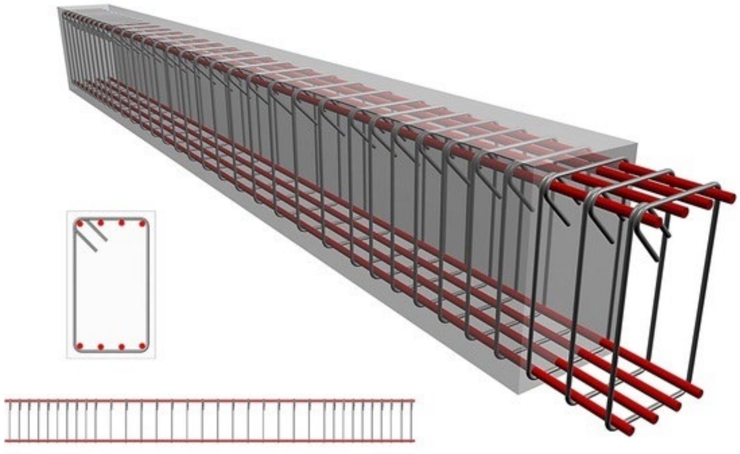


Figure 1.17: Steel reinforcement- picture 5

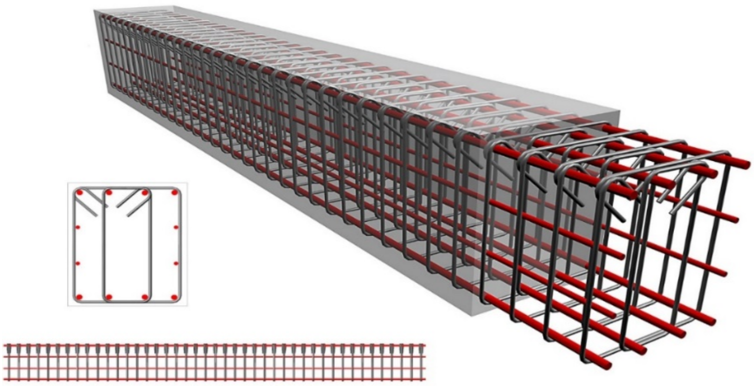


Figure 1.18: Steel reinforcement- picture 6

Chapter 2: Working Design Method

The importance of this chapter is as follows:

1. Determine the stresses in concrete and steel due to service loads, this is very important in the design of water tanks to control cracks of sections designed by the ultimate design method
2. Determine the cracked moment of inertia of a member, this is very important in calculating the deflections in beams

By this method, structural elements are designed assuming linear stress-strain behavior, such that, at service loads the stresses in steel and concrete do not exceed a specified working stress. This working stress is taken as a certain proportion of the ultimate strength of concrete or yield strength of steel.

In this method (working design method: ASD, allowable stress design), the allowable stresses in concrete and reinforcing steel are:

$f_{s,all}$= 0.4 f_y

$f_{c,all}$= 0.45 f'_c

The concrete sections are classified into:

- Uncracked sections
- Cracked sections

For uncracked sections, the stresses in concrete and steel are calculated using the formula,

$$\sigma = \frac{My}{I}$$

Where:

M: the applied service moment

y: distance from the centroid of section to the point at which the stresses are to be calculated

I: gross moment of inertia

So,

$$f_c = \frac{My_c}{I_g} \qquad f_t = \frac{My_t}{I_g} \qquad f_s = \frac{My_s}{I_g}n$$

Where:

Yc: distance from the extreme fibers at the compression edge to the centroid of the section

Yt: distance from the extreme fibers at the tension edge to the centroid of the section

Ys: distance from centroid of tension reinforcement to the centroid of the section

Ig: gross moment of inertia

Here, the strain and stress diagrams are linear as follows: Figure 2.1

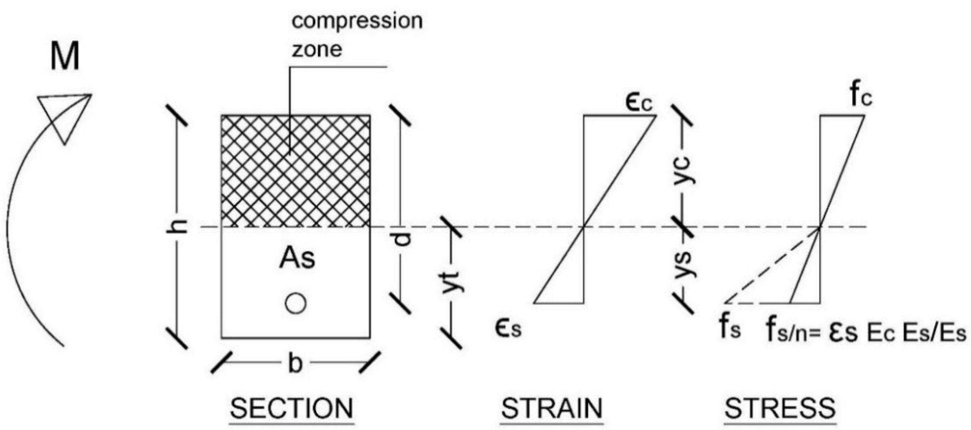


Figure 2.1: uncracked section

If the moment of inertia, Ig is computed neglecting reinforcing steel, yc=h/2=yt in a rectangular section.

For cracked section: Figure 2.2. Note: kd can be denoted by x.

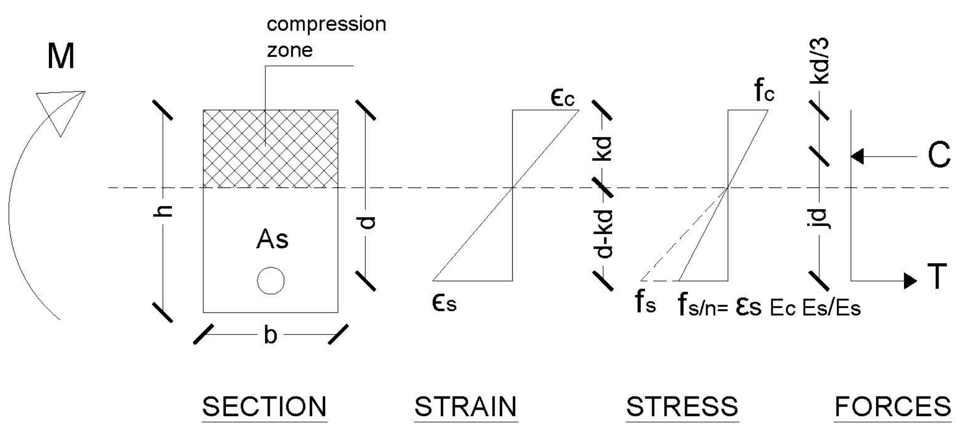


Figure 2.2: Stress, strain and forces in concrete section based on the working design method- cracked section

n: modular ratio= Es/Ec

The neutral axis or line of zero strain is located by considering the equilibrium of forces acting on cross section, C=T,

$$bkd\frac{f_c}{2} = A_s f_s$$

To determine the unknown stresses fc and fs, utilize the relationship between the stresses, from similar triangles in the stress diagram,

$$\frac{f_c}{kd} = \frac{f_s/n}{d-kd}$$

Let,

$$\rho = \frac{A_s}{bd}$$

From the three equations, above,

$$k = -n\rho + \sqrt{(n\rho)^2 + 2n\rho}$$

For a given bending moment, M, the maximum concrete and steel stresses are:

$$f_c = \frac{Mkd}{I_{cr}}$$

$$f_s = \frac{M(d-kd)}{I_{cr}}n$$

The moment of inertia, Icr is given by: neglecting concrete in tension zone.

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d-kd)^2$$

fc and fs can be determined using another procedure, as follows:

$$M = Cjd = Tjd$$

$$jd = d - \frac{kd}{3} \qquad j = 1 - \frac{k}{3}$$

$$M = Cjd = bkd\left(\frac{f_c}{2}\right)jd \qquad \rightarrow \quad f_c = \frac{2M}{bd^2kj}$$

And

$$M = Tjd = A_s f_s jd \quad \rightarrow \quad f_s = \frac{M}{A_s jd}$$

Example 2.1:

Given: F'c= 24MPa fy= 420MPa

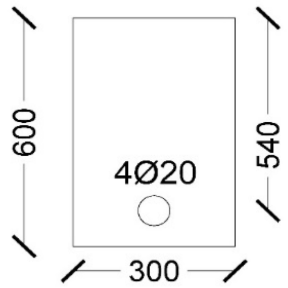


Figure 2.3: Section for example 1

Rectangular section: b= 300mm, h= 600mm, d= 540mm, bottom bars: 4Φ20 (1256mm^2)

1. Determine stresses in concrete and steel for service moment, M= 50 kN.m
2. Determine stresses in concrete and steel for service moment, M= 160kN.m
3. Determine the service moment that the section can carry.

Solution:

1. For M= 50kN.m:

Check cracked section:

Es= 200000MPa $E_c = 4700\sqrt{f'_c} = 4700\sqrt{24} = 23000MPa$

n= Es/Ec= 200000/23000= 8.7

neglect reinforcement, Ig= bh^3/12= 300(600)^3/12= 5.4x10^9 mm^4

$$f_r = 0.62\lambda\sqrt{f'_c} = 0.62(1)\sqrt{24} = 3.04MPa$$

Tensile stress in concrete, ft:

$$f_t = \frac{My_t}{I_g} = \frac{(50x10^6)(300)}{5.4x10^9} = 2.78MPa < f_r \qquad uncracked\ section$$

So, ft= 2.78MPa and from symmetry, fc= 2.78MPa

Tensile stress in steel, fs:

$$f_s = \frac{My_s}{I_g}n = \frac{(50x10^6)(300-60)}{5.4x10^9}x8.7 = 19.3MPa$$

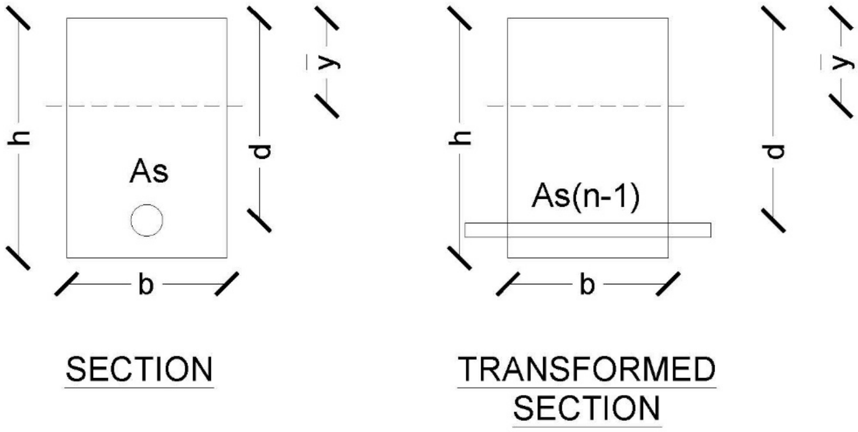


Figure 2.4: Transformed section

If reinforcing steel is considered (Take the reference line from the top face of section):

$$y^{-} = \frac{\sum A_i y_i}{\sum A_i} = \frac{bh\left(\frac{h}{2}\right) + (n-1)(A_s)(d)}{bh + (n-1)(A_s)} = 312mm$$

$$I_g = \frac{bh^3}{12} + bh\left(y^{-} - \frac{h}{2}\right)^2 + (n-1)(A_s)(d - y^{-})^2 = 5.93x10^9 mm^4$$

$y_c = 312mm, y_t = 600 - 312 = 288mm, y_s = 288 - 60 = 228mm.$

So,

fc= 2.63MPa Ratio= 2.63/2.78= 0.95

ft= 2.43MPa Ratio= 2.43/2.78= 0.87

fs= 16.7MPa Ratio= 16.7/19.3= 0.87

2. For M= 160MPa

Check cracked section:

Tensile stress in concrete, ft:

$$f_t = \frac{My_t}{I_g} = \frac{(160x10^6)(300)}{5.4x10^9} = 8.89MPa > f_r \qquad cracked\ section$$

So,

$$f_c = \frac{2M}{bd^2kj} \ or \quad f_c = \frac{Mkd}{I_{cr}}$$

$$f_s = \frac{M}{A_s jd} \quad or \quad f_s = \frac{M(d-kd)}{I_{cr}} n$$

$$k = -n\rho + \sqrt{(n\rho)^2 + 2n\rho}$$

$$j = 1 - \frac{k}{3}$$

$$\rho = \frac{A_s}{bd}$$

As= 1256mm^2

b=300mm

d= 540mm ρ= 0.007753

n= 8.7 k= 0.306 j= 0.898 fc= 13.3MPa fs= 262.7MPa

“note: assume linear stress up to 0.7 f’c”

3. Compute M:

Fc,all= 0.45 f’c= 0.45 x 24= 10.8MPa

Fs,all= 0.40 fy= 0.40 x 420= 168MPa

$$f_c = \frac{2M}{bd^2kj} \quad \rightarrow \quad 10.8 = \frac{2Mx10^6}{300(540)^2(0.306)(0.898)} \rightarrow M = 130kN.m$$

$$f_s = \frac{M}{A_s jd} \quad \rightarrow 168 = \frac{Mx10^6}{1256(0.898)(540)} \rightarrow M = 102.3kN.m$$

Take, M= 102.3 kN.m

Example 2.2:

Given: F'c= 28MPa fy= 420MPa

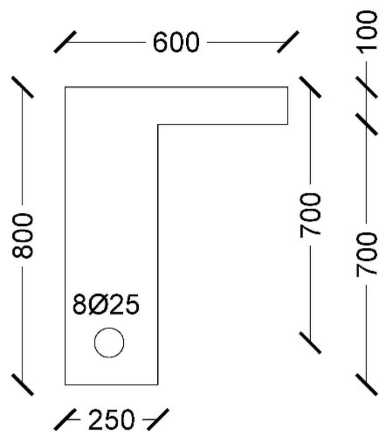


Figure 2.5: Section for example 2.2

Compute fc and fs for service moment, M= 200kN.m

Solution:

As= 8 x 491 = 3928 mm^2

Check cracked section:

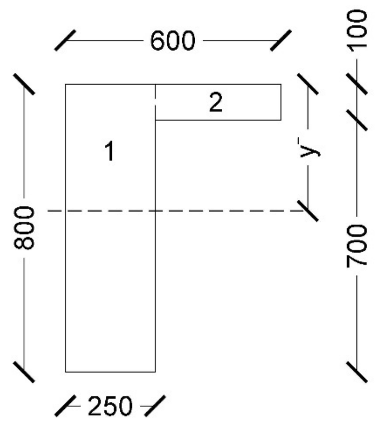


Figure 2.6: Section for example 2.2- Ig

At first, ft shall be computed at the bottom edge of section.

$$y^{-} = \frac{(250)(800)(400) + (350)(100)(50)}{(250)(800) + (350)(100)} = 348mm\ from\ top\ face\ of\ section.$$

$$I_g = \frac{(250)(800)^3}{12} + (250)(800)(400-348)^2 + \frac{(350)(100)^3}{12} + (350)(100)(348-50)^2$$
$$= 1.43x10^{10}mm^4$$

$$f_t = \frac{My_t}{I_g} = \frac{200x10^6x(800-348)}{1.43x10^{10}} = 6.32MPa$$

$$f_r = 0.62(1)\sqrt{28} = 3.28MPa < 6.32MPa \qquad \boldsymbol{cracked\ section}$$

Now compute the cracked moment of inertia, Icr:

At first, determine the location of the neutral axis:

Assume that the depth of the neutral axis, x from the top edge of section is equal to the flange thickness. So,

The moment of area of the zone above the neutral axis (flange):

$$M_c = (600)(100)(50) = 3x10^6mm^3$$

The moment of area of the zone below the neutral axis:

$$M_T = nA_s(d-x) = 8(3928)(700-100) = 18.9x10^6mm^4$$

M_T > M_C, so X >100mm

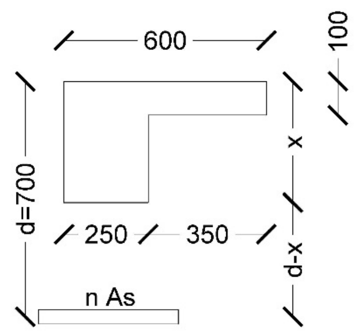


Figure 2.7: Section for example 2.2- Icr

Compute X:

Moment of area above the neutral axis= moment of area below the neutral axis

$$(350)(100)(x-50) + 250\frac{x^2}{2} = (8)(3928)(700-x) \rightarrow x = 245mm$$

$$I_{cr} = \left(\frac{1}{12}\right)(350)(100)^3 + (350)(100)(245-50)^2 + \frac{(250)(245)^3}{3} + (8)(3928)(700-245)^2 = 9.09x10^9 mm^4$$

Stress in concrete, fc:

$$f_c = \frac{MX}{I_{cr}} = \frac{(200x10^6)(245)}{9.09x10^9} = 5.4MPa$$

Stress in steel, fs:

$$f_s = \frac{M(d-X)}{I_{cr}} n = \frac{(200x10^6)(700-245)}{9.09x10^9}(8) = 80.1MPa$$

Chapter 3: Ultimate Design Method: Flexure in Beams

This chapter introduces beam section analysis and design; doubly and singly using the ultimate strength method. Based on this method, structural elements are designed taking inelastic strains into account to reach the maximum strength (concrete at ultimate strength and steel at yielding).

Some of the reasons for the trend towards ultimate strength design are as follows:

1. Reinforced concrete sections behave inelastically at high loads. Thus, the working stress method based on elastic stress-strain curve, cannot give a reliable prediction of the ultimate strength of the member.
2. Ultimate strength allows separate load factors to different types of service loads.
3. Ultimate strength design does not require a knowledge of the modular ratio. The concrete modulus of elasticity is not predicted well.

Structures should be designed for:

1. Adequate strength at ultimate loads.
2. Limited and accepted deflections at service loads.
3. Limited crack widths.
4. Ductility provisions: the deflection at ultimate loads should be large enough to give warning of failure so that the total collapse could be prevented. To ensure ductile behavior, the designers should give special attention to reinforcement ratios and details.

The following assumptions are made in defining the behavior of beam section:

1. Strain distribution is assumed to be linear. This assumption is based on Bernoulli's hypothesis that plane sections before bending remain plane and perpendicular to the neutral axis after bending.
2. Strain in the steel and the surrounding concrete is the same prior to cracking of the concrete or yielding of the steel.
3. Concrete is weak in tension. It cracks at early stages of loading. Consequently, concrete in tension zone of the section is neglected in the flexural analysis and design computations, and the tension reinforcement is assumed to take the total tensile force.

3.1Analysis of singly reinforced beam sections:

Figure 3.1 shows section, strain, stress and forces in a general reinforced concrete section using the ultimate design method.

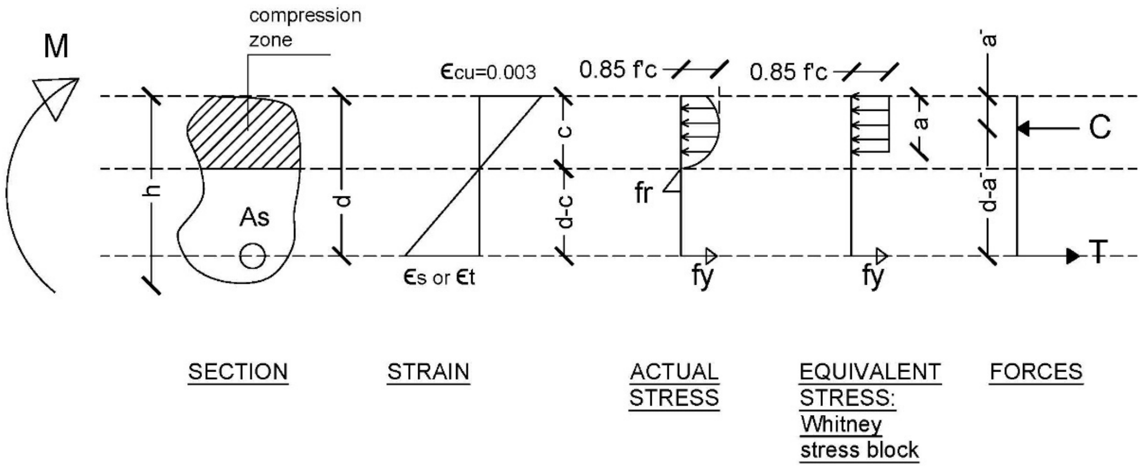


Figure 3.1: Section, strain, stress and forces- ultimate design method

The location of the compressive force, C, is the centroid of the concrete area that is subjected to the stress 0.85 f'c and has a depth a.

The depth of the compressive force from the extreme compression fibers is a^- for a general section and it is a/2 for a rectangular compression zone and for a rectangular beam section.

The tension force, T = The compression force, C so,

$$A_s f_y = 0.85 f'_c A_{cc}$$

Acc: area of the compression zone and can be computed from the above equation (T=C)

The nominal resisting moment, Mn will be:

$$M_n = T \text{ or } C\ (d - a^-)$$

So,

$$M_n = A_s f_y (d - a^-) \quad \text{or } M_n = 0.85 f'_c A_{cc} (d - a^-)$$

For rectangular sections:

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) \quad \text{or } M_n = 0.85 f'_c ba \left(d - \frac{a}{2}\right)$$

Where b is the width of the compression zone.

The depth of the compression zone, a in rectangular sections can be computed as above; T=C, so:

$$A_s f_y = 0.85 f'_c ba \rightarrow a = \frac{A_s f_y}{0.85 f'_c b}$$

The design moment, $\emptyset M_n$ is computed by multiplying the strength reduction factor, Φ by the nominal flexural strength, Mn.

The strength reduction factor, Ø can be computed using the figure below, Figure 3.2

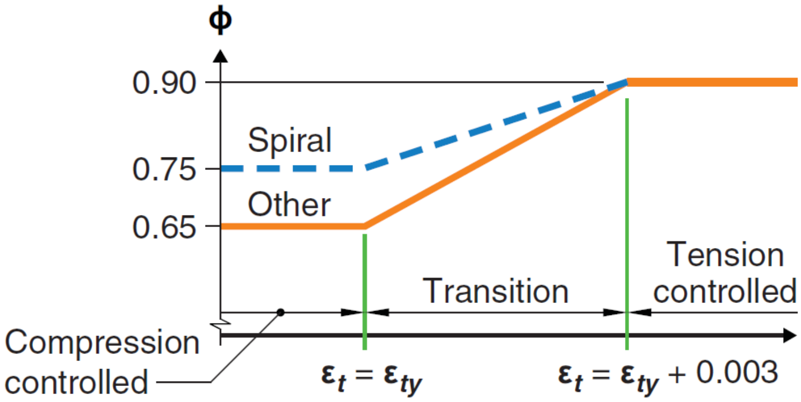


Figure 3.2: Strength reduction factor (ACI 318-19), Ø,

Figure 3.2 can be done using the ratio C/d instead of ε_t as follows:

For ε_t=0.002 for fy= 420MPa:

For strain 0.002, c/d= 0.6 For strain 0.005, c/d= 0.375

These values can be obtained using Figure 3.3 by applying equations for similar triangles.

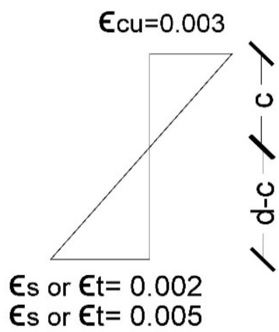


Figure 3.3: Strain diagram for flexure

For beams, the ACI 318-14 specifies that the minimum strain in steel at crushing of concrete is not less than 0.004 ($\emptyset = 0.817$). But it is recommended to use a strain of 0.005 instead of 0.004 to have larger ductility and to simply use $\phi = 0.9$. In ACI 318-19, the minimum strain in steel at crushing of concrete shall be not less than 0.005.

In section analysis, ϕ can be between 0.65 and 0.9, but in design, do not use a strain that gives ϕ not equal to 0.9.

Interpolation can be used to determine the values of Ø between 0.65 and 0.90, or the following table can be used. (ACI 318-19 Table 21.2.2).

Table 3.1: ACI 318-19 Table 21.2.2—Strength reduction factor ϕ for moment, axial force, or combined moment and axial force

Net tensile stain ε_t	**Classification**	ϕ			
		Type of transverse reinforcement			
		Spirals conforming to 25.7.3		Other	
$\varepsilon_t \le \varepsilon_{ty}$	Compression-controlled	0.75	(a)	0.65	(b)
$\varepsilon_{ty} < \varepsilon_t < \varepsilon_{ty} + 0.003$	Transition[1]	$0.75 + 0.15\dfrac{(\epsilon_t - \epsilon_{ty})}{(0.003)}$	(c)	$0.65 + 0.25\dfrac{(\epsilon_t - \epsilon_{ty})}{(0.003)}$	(d)
$\varepsilon_t \ge \varepsilon_{ty} + 0.003$	Tension-controlled	0.90	(e)	0.90	(f)

[1] For sections classified as transition, it shall be permitted to use ϕ corresponding to compression-controlled sections.

The equivalent depth of compression zone a is given by:

$$a = \beta_1 c$$

$$\beta_1 = 0.85 \quad for \quad 17MPa \le f'_c \le 28MPa$$

$$\beta_1 = 0.85 - 0.05\frac{f'_c - 28}{7} \quad for \quad 28MPa < f'_c < 56MPa$$

$$\beta_1 = 0.65 \quad for \quad 56MPa \le f'_c$$

Reinforced concrete beam sections can be classified into:

- Under reinforced sections: yielding of steel occurs before crushing of concrete: strain in steel at crushing of concrete > yield strain of steel.
- Balanced sections: yielding of steel occurs at the same time of crushing of concrete: strain in steel at crushing of concrete = yield strain of steel.
- Over reinforced sections: crushing of concrete occurs without yielding of steel: strain in steel at crushing of concrete < yield strain of steel.

Example 3.1:

Determine the design moment, $\emptyset Mn$ for the rectangular section shown in Figure 3.4 below.

Given: f’c= 24 MPa fy= 420MPa.

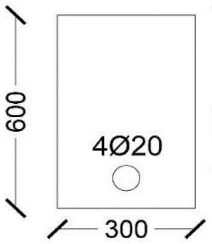


Figure 3.4: Section for Example 3.1

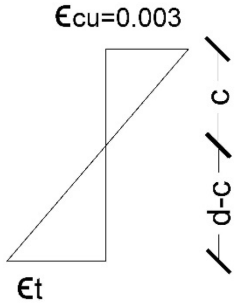
Solution:

As= 4 x 314= 1256 mm^2

d= 540mm

$$T = C \rightarrow a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(1256)(420)}{0.85(24)(300)} = 86.2mm$$

$$c = \frac{a}{\beta_1} = \frac{86.2}{0.85} = 101.4mm$$



From similar triangles:

$$\frac{0.003}{c} = \frac{\epsilon_t}{d-c} \rightarrow \epsilon_t = 0.013 > 0.005 \qquad \emptyset = 0.9$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = \frac{(1256)(420)\left(540 - \frac{86.2}{2}\right)}{10^6} = 262kN.m$$

The design moment, ϕM_n= 0.90 x 262= 236kN.m

Example 3.2:

Determine the design moment, ϕM_n for the T- section shown in Figure 3.5 below.

Given: f'c= 28 MPa fy= 420MPa

As: 8Φ25

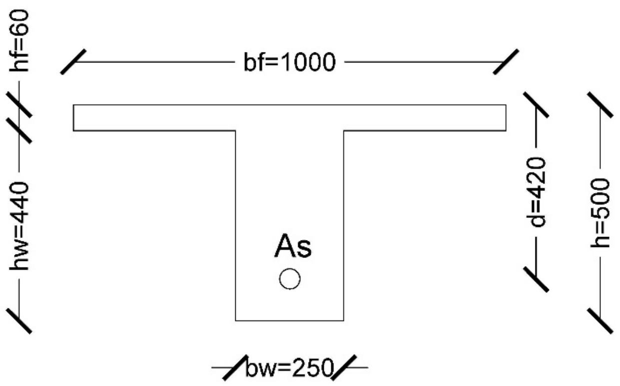


Figure 3.5: Section for Example 3.2

Solution:

As= 8 x 491= 3928mm^2

Determine the depth of the compression zone, a:

Tension force, T= As fy = 3928(420)/1000= 1649.76kN

Assume that a= 60mm, then,

Compression force, C= 0.85 f'c x area of the flange= (0.85)(28)(1000)(60)/1000= 1428kN < 1649.76kN, so a>60mm

Compression force, C1= 0.85(28)(1000-250)(60)/1000= 1071kN

Compression force, C2= T-C1= 1649.76-1071= 578.76kN

C2= (0.85)(28)(250)(a)/1000= 578.76kN so, a= 97.3kN

The nominal moment can be computed by taking the moments of forces C1 and C2 about the location of steel; at T force.

Distance from C1 force to to T force= d- hf/2

Distance from C2 force to T force= d- a/2

$$M_n = C_1\left(d - \frac{h_f}{2}\right) + C_2\left(d - \frac{a}{2}\right) = \frac{\left[1071\left(420 - \frac{60}{2}\right) + 578.76\left(420 - \frac{97.3}{2}\right)\right]}{1000}$$
$$= 632.6kN.m$$

$$c = \frac{a}{\beta_1} = \frac{97.3}{0.85} = 114.47mm$$

$$\frac{c}{d} = \frac{114.47}{420} = 0.27 < 0.375 \qquad \emptyset = 0.9$$

The design moment, $\emptyset M_n$= 0.90 x 632.6= 569.34kN.m

Another solution:

$$T = C \rightarrow A_s f_y = 0.85 f'_c A_{cc} \rightarrow (3928)(420) = 0.85(28)(A_{cc}) \rightarrow A_{cc} = 69317.6mm^2$$

$$Area\ of\ flange = (1000)(60) = 60000mm^2 < A_{cc}$$

$$So, A_{cc} = (1000 - 250)(60) + 250a \rightarrow a = 97.3mm$$

Determine centroid of A_{cc}:

$$a^- = \frac{(750)(60)(30) + (250)(97.3)(97.3/2)}{(750)(60) + (250)(97.3)} = 36.54mm$$

$$M_n = A_s f_y (d - a^-) = (3928)(420)(420 - 36.54)/10^6 = 632.6kN.m$$

$$\emptyset M_n = (0.9)(632.6) = 569.34kN.m$$

<u>Example 3.3:</u>

<u>Given:</u> f'c= 28MPa fy= 420MPa As= 600mm^2

Compute ϕM_n for the beam section in Figure 3.6 below.

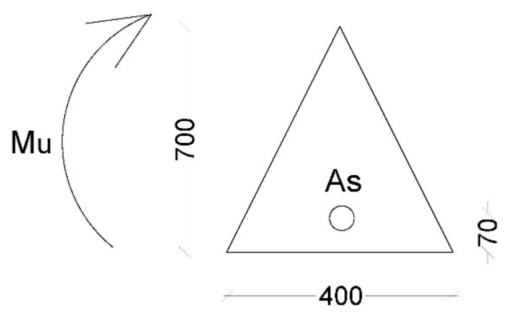


Figure 3.6: Reinforced concrete section for Example 3.3

Solution:

Depth of compression zone is a.

From similar triangles:

$$\frac{a}{x1} = \frac{700}{200} \qquad \rightarrow x1 = 0.286a$$

Tension, T = Compression, C

(600)(420)=0.85(28)(0.286a^2)

a = 192.4mm

c= 192.4/0.85 = 226.4mm

c/d= 226.4/630 = 0.36 < 0.375, so Φ= 0.9

$$\emptyset M_n = 0.9 A_s f_y \left(d - \frac{2}{3}a\right) = \frac{0.9(600)(420)\left(630 - \frac{2}{3}(192.4)\right)}{10^6} = 113.8 kN.m$$

3.2 Design of singly reinforced concrete rectangular beam sections:

For design,

$$\emptyset M_n \geq M_u$$

$$\emptyset A_s f_y \left(d - \frac{a}{2}\right) = M_u \qquad a = \frac{A_s f_y}{0.85 f'_c b}$$

This gives:

$$bd^2 = \frac{M_u}{\emptyset \rho f_y \left(1 - \frac{\rho f_y}{1.7 f'_c}\right)}$$

And

$$\rho = \frac{0.85 f'_c}{f_y}\left(1 - \sqrt{1 - \frac{2.35 M_u}{\emptyset b d^2 f'_c}}\right), \quad \rho = \frac{0.85 f'_c}{f_y}\left(1 - \sqrt{1 - \frac{2.61 M_u}{b d^2 f'_c}}\right) \quad here, \emptyset = 0.9$$

Where:

ρ: steel ratio. $\rho = \frac{A_s}{bd}$

Mu: ultimate applied bending moment, N.mm

b: width of compression zone, width of section, mm

d= effective depth of section, mm

f'c: compressive strength of concrete, cylinder test, at 28 days, MPa

fy: yield strength of reinforcing steel, MPa

3.3 Minimum thickness of beams and one way slabs:

Note: Beam width:

The width of beam shall be determined using the following hints:

- Section width, b ≥ 200mm to have a space for reinforcement
- Section width, b ≈ L/20
- Section width, b ≈ (0.3-0.5)h, where h is the thickness of cross section
- For seismic design, beams of special moment resisting frame: the section width shall be not less than the minimum of 250mm and 0.3 h.

- For seismic design, beams of special moment resisting frame: Projection of the beam width beyond the width of the Supporting column on each side shall not exceed the lesser of c2 and 0.75 c1. Where c1 is the width of column in direction of beam and c2 is the width of column transverse to beam.

Beam and one way slab thickness:

This section gives preliminary thickness of beams and one-way slabs based on ACI 318-19. The designed sections shall be adequate for shear and torsion strength in addition to be checked for deflection.

Table 3.2: ACI 318-19 Table 7.3.1.1—Minimum thickness of solid nonprestressed one-way slabs

Support condition	Minimum *h*[1]
Simply supported	L/20
One end continuous	L/24
Both ends continuous	L/28
Cantilever	L/10

[1] Expression applicable for normal weight concrete and *fy* = 420 MPa. For other cases, minimum *h* shall be modified in accordance with 7.3.1.1.1 through 7.3.1.1.3, as appropriate.

[1] Expression applicable for normal weight concrete and fy = 420 MPa. For other cases, minimum h shall be modified in accordance with 7.3.1.1.1 through 7.3.1.1.3, as appropriate.

ACI 318-19 section 7.3.1.1.1: For fy other than 420 MPa, the expressions in Table 7.3.1.1 shall be multiplied by (0.4 + fy/700).

ACI 318-19 section 7.3.1.1.2: For nonprestressed slabs made of lightweight concrete having Wc in the range of 1440 to 1840 kg/m^3, the expressions in Table 7.3.1.1 shall be multiplied by the greater of (a) and (b):

(a) 1.65 – 0.0003Wc

(b) 1.09

Table 3.3: ACI 318-19 Table 9.3.1.1—Minimum depth of nonprestressed beams

Support condition	Minimum *h*[1]
Simply supported	L/16
One end continuous	L/18.5
Both ends continuous	L/21
Cantilever	L/8

[1] Expressions applicable for normal weight concrete and Grade 420 reinforcement. For other cases, minimum *h* shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

[1] Expressions applicable for normal weight concrete and Grade 420 reinforcement. For other cases, minimum h shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

ACI 318-19 section 9.3.1.1.1: For fy other than 420 MPa, the expressions in Table 9.3.1.1 shall be multiplied by (0.4 + fy/700).

ACI 318-19 section 9.3.1.1.2: For nonprestressed beams made of lightweight concrete having Wc in the range of 1440 to 1840 kg/m^3, the expressions in Table 9.3.1.1 shall be multiplied by the greater of (a) and (b):

(a) 1.65 – 0.0003Wc

(b) 1.09

3.4 Minimum reinforcement of flexural members:

Based on ACI 318-19, the following points can be stated:

1. At every section of a flexural member where tensile reinforcement is required by analysis, except as provided in 2, 3 and 4, As provided shall not be less than given by:

$$A_{s,min} = \frac{0.25\sqrt{f'_c}}{f_y} b_w d \ \geq \frac{1.4}{f_y} b_w d \quad or$$

$$\rho_{min} = \frac{0.25\sqrt{f'_c}}{f_y} \geq \frac{1.4}{f_y}$$

2. For statically determinate members with a flange in tension, As,min shall not be less than the value given in 1, except that b_w is replaced by either 2 b_w or width of the flange, bf whichever is smaller.
3. The requirements of 1 and 2 need not be applied if As provided is at least 4/3 As required by analysis.

4. For structural slabs and footings of uniform thickness, As,min shall be not less than $0.0018A_g$.

In slabs, the maximum spacing between bars is the smaller of three times the slab thickness and 450mm.

For shrinkage steel, the maximum spacing between bars is the smaller of five times the slab thickness and 450mm.

To control cracks, the maximum spacing of bars in beams and slabs is given by (ACI 318-19 section 24.3.2):

$$s \leq 380\left(\frac{280}{f_s}\right) - 2.5C_c$$

$$s \leq 300\left(\frac{280}{f_s}\right)$$

fs = tensile stress in reinforcement at service loads, MPa. fs can be taken equal to 2/3 times fy

Cc = clear cover of reinforcement, mm

For fy= 420MPa, and for slabs, cover, Cc=20mm, Smax= 300mm

For fy= 420MPa, and for beams, cover, Cc= 52mm, Smax= 250mm (Clear cover= 40mm + 12mm stirrup)

One can use the following criteria for center to center bars,

Slabs: Smax= 250mm Smin= 100mm

Beams: Smax= 150mm Smin= 65mm

In general, the minimum area of steel is required to resist a moment equal at least the cracking moment of the concrete section (moment capacity of unreinforced section, using maximum concrete stress in tension equal to modulus of rupture, fr= $0.62\lambda\sqrt{f'_c}$), *so*

$$M_{cr} = \frac{f_r I_g}{y_t} \approx A_s f_y d \rightarrow A_s = \frac{f_r I_g}{y_t d f_y}$$

$$I_g = \frac{bh^3}{12} \qquad y_t = \frac{h}{2}$$

$$\frac{I_g}{y_t} = \frac{2bh^3}{12h} = \frac{bh^2}{6} \approx \frac{bd^2}{6}$$

$$A_s = \frac{f_r I_g}{y_t d f_y} = \frac{f_r b d^2}{6 d f_y} = \frac{0.62\sqrt{f'_c} bd}{6 f_y}$$

Using factor of safety of 2.4:

$$A_s = \frac{0.62\sqrt{f'_c} bd}{6 f_y} x2.4 = \frac{0.25\sqrt{f'_c}}{f_y} bd$$

3.5 Maximum steel ratio for singly reinforced beam sections:

From previous sections, the minimum strain in tensile steel at crushing of concrete is 0.005 which corresponds to C/d= 0.375 (Strain in concrete= 0.003 and strain in steel= 0.005).

$$c_{max} = 0.375d$$

Compression force= Tension force

$$0.85 f'_c A_{cc} = A_{s,max} f_y \rightarrow A_{s,max} = \frac{0.85 f'_c A_{cc}}{f_y}$$

Acc is computed using:

$$a_{max} = \beta_1 c_{max}$$

For rectangular sections:

$$0.85 f'_c b a_{max} = A_{s,max} f_y = 0.85 f'_c b \beta_1 0.375d$$

Divide the above equation by bd,

$$\rho_{max,singly,\in_t=0.005} = 0.375\beta_1 \frac{0.85 f'_c}{f_y}$$

Using the previous procedure, the balanced steel ratio and steel area can be determined using strain in steel at crushing of concrete equals to $\in_y = \frac{f_y}{E_s}$.

The balanced steel ratio can be determined by assuming that the tensile strain in steel at crushing of concrete equals to the yield strain which is equal to 0.002 for steel yield strength, $f_y = 420MPa$, so:

$$\rho_{balanced,singly,\in_t=0.002} = 0.6\beta_1 \frac{0.85 f'_c}{f_y}$$

The maximum area of steel for any beam section can be determined using the depth of the compression zone given by:

$$a_{max} = \beta_1 c_{max} = \beta_1 0.375d$$

From this formula, the area of the compression zone is determined and then:

$$Compression\ force = Tension\ force$$

$$0.85f'_c A_{cc} = A_{s,max} f_y \qquad here\ Acc\ is\ computed\ using\ a_{max}$$

Notes:

1. ***For design, it is recommended to have the steel ratio, $\rho = (0.01 - 0.013)$ for economical purpose and to control deflection.***
2. ***It is not recommended to have the steel ratio, $\rho > 0.025$ even using compression steel for fy= 420MPa and it is not recommended to have a steel ratio, $\rho > 0.02$ for fy= 560Mpa.***
3. ***The designer can choose a steel ratio, then the sections dimensions will be determined using the equation of bd^2.***
4. ***In previous codes, the maximum steel ratio for singly reinforced sections was 0.75 time the balanced steel ratio***
5. ***To have a specified strain in tension steel, the value of a can be determined from the strain diagram, then the steel ratio or the steel area can be derived from compression force equals to tension force.***

Example 3.4 shows the analysis and design of one-way solid slab. The slab is considered as singly reinforced rectangular beam section of 1000mm width.

The one-way solid slab can be designed using strips perpendicular to beams with 1000mm width.

When the slab strip modeled as a continuous beam structure (line element) with pin-ends, the beams will be designed for flexure and shear, and when the continuity between the slab strips and the beams is considered, the beam shall be designed for flexure, shear and torsion.

Example 3.4:

Given:

f'c= 28Mpa

fy= 420Mpa

All columns are 400mm x400mm

Live load on slab= 16kN/m^2

Design the beams and the slab (one-way solid) for the plan shown in Figure 3.7. Assume that beams have rectangular sections.

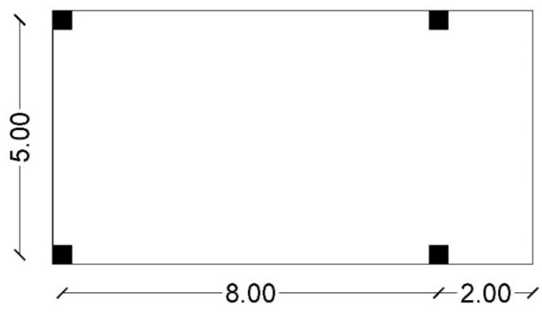


Figure 3.7: Plan for Example 3.4

Solution:

There are two options for one-way slab system as shown below in Figure 3.8.

Option 2 will be adopted as the slab thickness will be minimum.

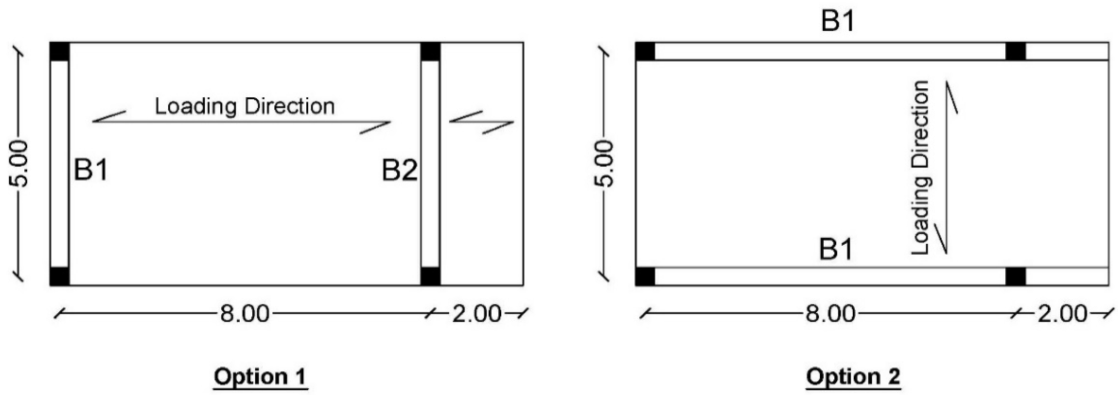


Figure 3.8: Slab options for Example 3.4

Design of slab: see Figure 3.9: slab structural model

Minimum slab thickness, h= L/20= 5/20= 0.25m

Slab own weight, W_D= 0.25 x 25 = 6.25kN/m^2

The slab is a strip of one-meter width, so, the line load will be, W_D= 6.25kN/m

The live load is given, WL= 16kN/m

The ultimate loads are:

Wu1= 1.4(6.25) = 8.75kN/m

Wu2= 1.2(6.25) +1.6(16) = 33.1kN/m

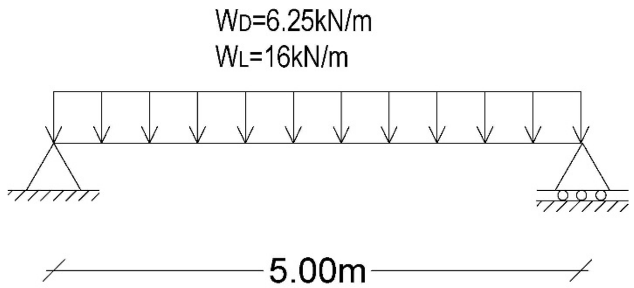


Figure 3.9: Structural model of slab

Reaction due to dead load= 15.6kN.

Reaction due to live load= 40kN.

The maximum ultimate bending moment at mid span, Mu= WuL^2/8= 33.1(5)^2/8= 103.4kN.m

Cross section: rectangle: b= 1000mm, h= 250mm, d= 250-40= 210mm

The steel ratio is given by:

$$\rho = \frac{0.85f'_c}{f_y}\left(1-\sqrt{1-\frac{2.61M_u}{bd^2f'_c}}\right) = \frac{0.85(28)}{420}\left(1-\sqrt{1-\frac{2.61(103.4x10^6)}{1000(210)^2(28)}}\right) = 0.0066$$

$$\rho_{max,singly,\epsilon_t=0.005} = 0.375\beta_1\frac{0.85f'_c}{f_y} = 0.375(0.85)\frac{0.85(28)}{420} = 0.01806 > 0.0066 \quad ok$$

Required area of steel, As= 0.0066(1000)(210)= 1386mm^2

Minimum area of steel, As, min= 0.0018(1000)(250)= 450mm^2 < 1386mm^2 ok

Take, As= 1386mm^2 (7Φ16/m)

The shrinkage steel in the transverse direction; x-direction, As= 450mm^2, 4Φ12/m

Design of beam: Figure 3.10

Assume that width of beam, b= 400mm

Minimum thickness of beam, h:

h1= L/18.5= 8/18.5= 0.43m

h2= 2/8= 0.25m

Try, h= 700mm d= 640mm

Weight of beam, W_{D1}= (0.4)(0.7)(25)= 7kN/m

Dead load on beam from the slab, W_{D2}= (5/2)(6.25)= 15.6kN.m

Live load on beam from slab, W_L= (5/2)(16)= 40kN/m

Ultimate load on beam:

Wu1= 1.4(7+15.6)= 31.64kN/m

Wu2= 1.2(7+15.6)+1.6(40)= 91.12kN/m

Take Wu= 91.12kN/m

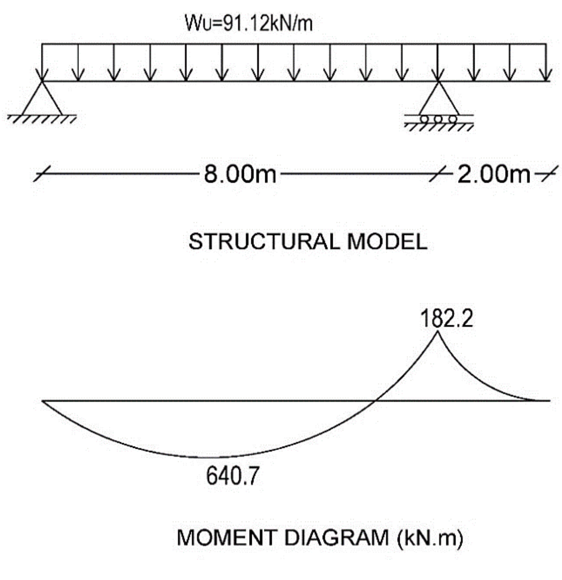


Figure 3.10: Structural model and bending moment diagram of the beam

Left reaction= 341.7kN. Right reaction= 569.5kN.

For maximum positive moment, Mu= 640.7kN.m:

$$\rho = \frac{0.85f'_c}{f_y}\left(1-\sqrt{1-\frac{2.61M_u}{bd^2f'_c}}\right) = \frac{0.85(28)}{420}\left(1-\sqrt{1-\frac{2.61(640.7x10^6)}{400(640)^2(28)}}\right) = 0.0115$$

Maximum steel ratio= 0.01806 > 0.0115 ok

Minimum steel ratio:

$$\rho_{min} = \frac{0.25\sqrt{f'_c}}{f_y} \geq \frac{1.4}{f_y} \qquad \rho_{min} = \frac{0.25\sqrt{28}}{420} = 0.00315 \geq \frac{1.4}{420} = 0.00333$$

Use ρ_{min}= 0.00333 <0.0115 ok

So, As= 0.0115(400)(640)= 2944mm^2 $6\phi25$

For maximum negative moment, Mu= 182.2kN.m:

$$\rho = \frac{0.85f'_c}{f_y}\left(1-\sqrt{1-\frac{2.61M_u}{bd^2f'_c}}\right) = \frac{0.85(28)}{420}\left(1-\sqrt{1-\frac{2.61(182.2x10^6)}{400(640)^2(28)}}\right) = 0.00302$$

Maximum steel ratio= 0.01806 > 0.00302 ok

Minimum steel ratio:

$$\rho_{min} = \frac{0.25\sqrt{f'_c}}{f_y} \geq \frac{1.4}{f_y} \qquad \rho_{min} = \frac{0.25\sqrt{28}}{420} = 0.00315 \geq \frac{1.4}{420} = 0.00333$$

Use ρ_{min}= 0.00333 > 0.00302

So, use the minimum of 0.00333 and 4/3 x 0.00302= 0.00403

Use ρ= 0.00333

So, As= 0.00333(400)(640)= 852mm^2 $4\phi18$

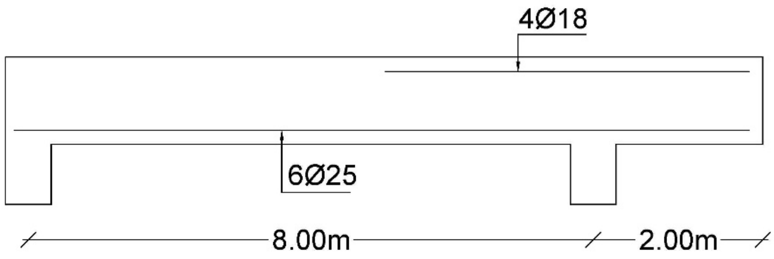


Figure 3.11: Bars layout in the beam

3.6 Design of singly reinforced concrete irregular beam sections:

The T and L sections are commonly found in slabs. Based on ACI 318-14, the width of the flange, bf or be, is given as below.

ACI 318-14 section 6.3.2.1: For nonprestressed T-beams supporting monolithic or composite slabs, the effective flange width ***bf*** shall include the beam web width ***bw*** plus an effective overhanging flange width in accordance with Table 6.3.2.1, where ***h*** is the slab thickness and ***Sw*** is the clear distance to the adjacent web.

Isolated nonprestressed T-beams in which the flange is used to provide additional compression area shall have a flange thickness greater than or equal to $0.5b_w$ and an effective flange width less than or equal to $0.4b_w$.

Table 3.4: ACI 318-19 Table 6.3.2.1—Dimensional limits for effective overhanging flange width for T-beam

	Effective overhanging flange width, beyond face	
Each side of web	Least of:	8h
		sw/2
		Ln/8
One side of web	Least of:	6h
		sw/2
		Ln/12

Ln: clear span length

Sw: clear distance to adjacent web (beam)

h: slab thickness= thickness of flange

<u>Example 3.5:</u>

<u>Given:</u>

f'c= 21MPa

fy= 420MPa

Mu= 840kN.m

Determine the required area of steel to resist Mu.

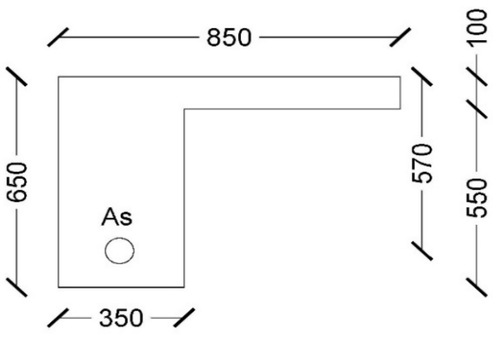


Figure 3.12: Reinforced concrete section- Example 3.5

Solution:

Assume that depth of compression block, a= 100mm, then compute the moment capacity of the flange, ΦMnf as follows:

$$\emptyset M_{nf} = \emptyset 0.85 f'_c b_f a\left(d - \frac{a}{2}\right) = \frac{0.9(0.85)(21)(850)(100)(570 - 50)}{10^6} = 710kN.m < Mu = 840kN.m$$

So, a>100mm, then divide the compression zone to two zones; one within the flange and the other within the web, see Figure 3.13.

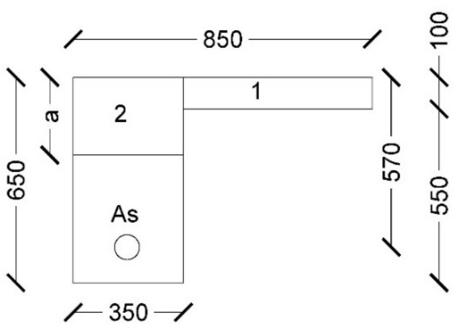


Figure 3.13: Reinforced concrete section- Example 3.5- compression zones

The moment capacity of zone 1 is given by:

$$\emptyset M_{n1} = \emptyset 0.85 f'_c (b_f - b_w) h_f \left(d - \frac{h_f}{2}\right) = \frac{0.9(0.85)(21)(850 - 350)(100)(570 - 50)}{10^6} = 417.69kN.m$$

The needed area of steel for $\phi Mn1$ is given by:

$$\emptyset M_{n1} = \emptyset 0.85f'_c(b_f - b_w)h_f\left(d - \frac{h_f}{2}\right) = \emptyset A_{s1}f_y(d - \frac{h_f}{2})$$

$$A_{s1} = \frac{\emptyset M_{n1}}{\emptyset f_y\left(d - \frac{h_f}{2}\right)} = 2125mm^2$$

$$\emptyset M_{n2} = Mu - \emptyset M_{n1} = 840 - 417.69 = 422.31kN.m$$

The moment ϕM_{n2}= M_{u2} and will be resisted by compression zone 2. The required area of steel is determined by applying the formula for steel ratio ρ for a rectangular section.

$$\rho = \frac{0.85f'_c}{f_y}\left(1 - \sqrt{1 - \frac{2.61M_u}{bd^2f'_c}}\right) = \frac{0.85(21)}{420}\left(1 - \sqrt{1 - \frac{2.61(422.31x10^6)}{350(570)^2(21)}}\right)$$
$$= 0.01131$$

This value of ρ is less than maximum ρ for singly reinforced section, or a<a_{max}

$$\rho_{max,singly,\epsilon_t=0.005} = 0.375\beta_1\frac{0.85f'_c}{f_y} = 0.375(0.85)\frac{0.85(21)}{420} = 0.0135 > 0.01131 \quad ok$$

Or:

As2= 0.01131(350)(570)= 2256mm^2

$$a = \frac{A_sf_y}{0.85f'_cb} = \frac{2256(420)}{0.85(21)(350)} = 152mm$$

$$a_{max} = \beta_1 0.375d = 0.85(0.375)(570) = 181.7mm > 152mm$$

Total area of steel, As= As1+As2= 2125 + 2256 = 4381 mm^2

Additional checks:

Minimum area of steel:

$$A_{s,min} = 0.00333b_wd = 0.00333(350)(570) = 664mm^2 < 4381mm^2 \quad ok$$

Maximum area of steel is checked above. If maximum steel area is needed, it will be calculated as follows:

$$a_{max} = \beta_1 0.375d = 0.85(0.375)(570) = 181.7mm$$

Compression force = Tension force

$$0.85f'_c A_{cc} = A_{s,max} f_y$$

$$0.85(21)\big((850-350)(100)+181.7(350)\big) = A_{s,max}(420) \rightarrow A_{s,max} = 4828mm^2 > 4381mm^2 \quad ok$$

Example 3.6:

Given:

f'c= 28MPa

fy= 420MPa

Mu= 180kN.m

Determine the required area of bottom steel to resist Mu.

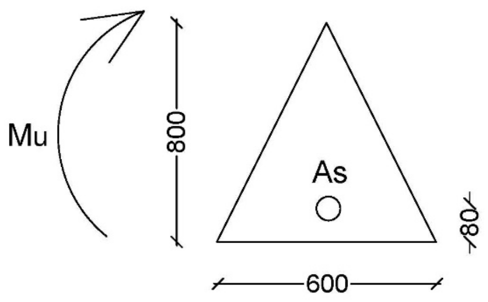


Figure 3.14: Reinforced concrete section- Example 3.6

Solution:

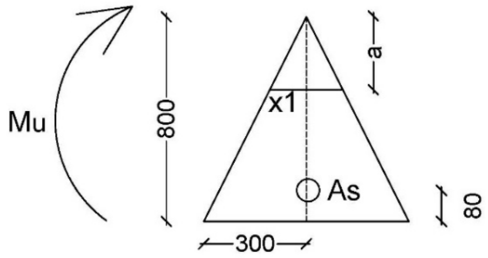


Figure 3.15: Compression zone of depth, a

Check singly or doubly:

The maximum compression block depth, a_{max} is given by:

$$a_{max} = \beta_1 0.375d = (0.85)(0.375)(720) = 229.5mm$$

From similar triangles:

$$\frac{a}{x1} = \frac{800}{300} \rightarrow x1 = 0.375a$$

So, x1= 0.375(229.5)=86mm. The compression force in concrete, Cc is given by:

$$C_c = \frac{0.85(28)(86)(229.5)}{1000} = 469.7kN$$

The compression force= the tension force

$$469.7x1000 = A_{s,max} f_y = A_{s,max}(420) \rightarrow A_{s,max} = 1118mm^2$$

The design moment, $\emptyset M_n$, is given by:

$$\emptyset M_n = \emptyset C_c\left(d - \frac{2}{3}a\right) = \frac{0.9(469.7)\left(720 - \left(\frac{2}{3}\right)(229.5)\right)}{1000} = 239.7kN.m$$
$$> 180kN.m \quad singly\ reinforced\ section$$

<u>Compute area of tension steel:</u>

$$M_u = \emptyset M_n$$

$$180x10^6 = 0.9(0.85)(28)(0.375a)(a)(720 - \frac{2}{3}a)$$

Simplifying,

$$a^3 - 1080a^2 + 33.6x10^6 = 0.0 \rightarrow a = 194.8mm$$

The compression force= the tension force

$$0.85(28)(0.375)(194.8)^2 = A_s(420) \rightarrow A_s = 806mm^2$$

<u>Example 3.7:</u>

<u>Given:</u>

f'c= 28MPa

fy= 420MPa

Mu= 700kN.m

Determine the required area of bottom steel to resist Mu.

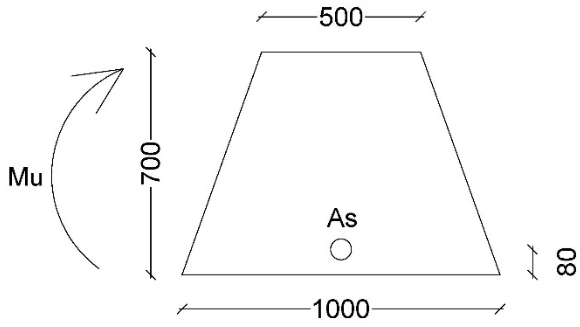


Figure 3.16: Reinforced concrete section- Example 3.7

Solution:

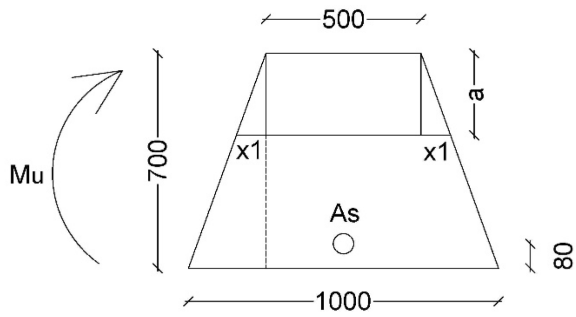


Figure 3.17: Compression zone of depth, a

Check singly or doubly:

The maximum compression block depth, a_{max} is given by:

$$a_{max} = \beta_1 0.375d = (0.85)(0.375)(620) = 197.6mm$$

From similar triangles:

$$\frac{a}{x1} = \frac{700}{250} \rightarrow x1 = 0.357a$$

The compression force in concrete, Cc1 is given by:

$$C_{c1} = \frac{0.85(28)(500)(197.6)}{1000} = 2351.4kN$$

The compression force in concrete, Cc2 is given by:

$$C_{c1} = \frac{0.85(28)(0.357)(197.6)^2}{1000} = 331.8kN$$

The total compression force, Cc= Cc1 + Cc2=2683.2kN

The compression force= the tension force

$$2683.2x1000 = A_{s,max}f_y = A_{s,max}(420) \rightarrow A_{s,max} = 6389mm^2$$

The design moment, $\emptyset M_n$, is given by:

$$\emptyset M_n = \frac{\emptyset}{1000}\left[C_{c1}\left(620 - \frac{197.6}{2}\right) + C_{c2}(620 - (\frac{2}{3})(197.6)\right] = 1249kN.m$$
$$> 700kN.m \quad singly\ reinforced\ section$$

Compute area of tension steel:

$$M_u = \emptyset M_n$$

$$700x10^6 = 0.9\left[(0.85)(28)(0.357a)(a)\left(620 - \frac{2}{3}a\right) + (0.85)(28)(500a)(620 - \frac{a}{2})\right]$$

Simplifying,

$$5.664a^3 + 682a^2 - 7378000a + 777.78x10^6 = 0.0 \rightarrow a = 107.4mm$$

The compression force= the tension force

$$0.85(28)(0.357)(194.8)^2 + 0.85(28)(500)(194.8) = A_s(420) \rightarrow A_s = 3276mm^2$$

3.7 Analysis of doubly reinforced concrete beam sections:

The basic principle of analysis of doubly reinforced concrete beam section is based on assuming a trial value of compression depth, a, then checking that the compression force, C is equal to the tension force T, then the moment capacity, ϕM_n will be determined.

- Assume a value for a.
- Compute the compression force in concrete: Cc=0.85 f'c Acc.
- Compute the compression force in compression steel: Cs= As' fs'.
- Compute the tension force in tension steel: T= As fs.
- The values of fs' and fs are computed by multiplying the strain values by the steel modulus of elasticity, Es.
- The values of strains in tension steel and compression steel are computed from the strain diagram based on the value of C= a/β1.
- If T is approximately equal Cc+ Cs, stop, so the value of a is appropriate.

- Compute the moment capacity by multiplying Cc and Cs with their distances from the tension steel location.

$$0.85f'_c A_{cc} + A_s' f_s' = A_s f_s$$

For rectangular section:

$$0.85f'_c ba + A_s' f_s' = A_s f_s$$

$$a = \frac{A_s f_s - A_s' f_s'}{0.85f'_c b} = \frac{T - C_s}{0.85f'_c b}$$

Assuming than the tension and compression steel yield, then:

$$a = \frac{(A_s - A_s')f_y}{0.85f'_c b}$$

For section analysis, one can start with this value. If a is not appropriate, the new value of a will be:

$$a = \frac{T - C_s}{0.85f'_c b} \qquad T \text{ and } C_s \text{ are computed from the previous step}$$

Example 3.8:

Given: f'c= 24MPa fy= 420MPa Compute ΦMn.

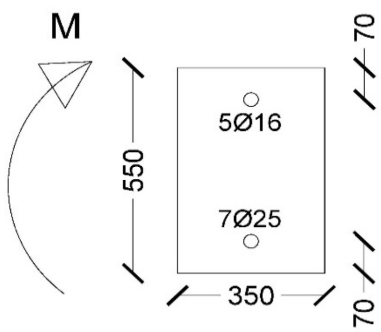


Figure 3.18: Reinforced concrete section for example 3.8

Solution:

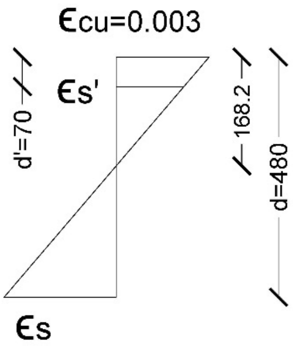
As= 7x491= 3437mm^2

As'= 5x201= 1005mm^2

Assume that both steel yield, so

$$a = \frac{(A_s - A_s')f_y}{0.85f'_c b} = \frac{(3437 - 1005)(420)}{0.85(24)(350)} = 143.1mm$$

$$c = \frac{a}{\beta_1} = \frac{143.1}{0.85} = 168.2mm$$



From similar triangles:

$$\epsilon_s = 0.00556 \qquad \epsilon_s' = 0.00175$$

$$f_s = \epsilon_s E_s = 0.00556(200000) = 1112MPa \quad > f_y \quad use\ f_s = f_y = 420MPa$$

$$f_s' = \epsilon_s' E_s = 0.00175(200000) = 350MPa$$

$$compression\ in\ concrete, C_c = 0.85f'_c ba = \frac{0.85(24)(350)(143.1)}{1000} = 1021.7kN$$

$$compression\ in\ steel, C_s = A_s' f_s' = \frac{1005(350)}{1000} = 351.8kN$$

$$tension\ in\ steel, T = A_s f_s = \frac{3437(420)}{1000} = 1443.5kN$$

Total compression force= 1021.7 + 351.8 = 1373.5kN

$$\frac{T}{C} = \frac{1443.5}{1373.5} = 1.05 \qquad error\ 5\%$$

Try a new value of a:

$$a = \frac{T - C_s}{0.85f'_c b} = \frac{1443.5 - 351.8}{0.85(24)(350)} = 152.9mm$$

Resolve for a= 152.9mm:

C= 179.9mm

Cc= 1091.7kN

$\epsilon_s{}' = 0.00183$ $\quad f_s{}' = 366MPa$ $\quad \epsilon_s = 0.005$

Cs= 367.8kN C= 1459.5kN T= 1443.5kN C/T=1.01

Error 1%, accepted.

Since $\epsilon_s = 0.005, \phi = 0.9$

$$\phi M_n = \phi \left[C_c\left(d - \frac{a}{2}\right) + C_s(d - d')\right] = \frac{0.9\left[1091.7\left(480 - \frac{152.9}{2}\right) + 367.8(480 - 70)\right]}{10^6} = 528.8kN.m$$

Note: ***the procedure in Example 3.8 can be applied for any section shape other than rectangular sections. Students are responsible to solve for any shape especially flanged sections (T, L and I). For flanged sections, the compression zone of depth a may be composed of parts; each part has its own compression force.***

Example 3.9:

Given: f'c= 20MPa fy=420MPa

As= 7111mm^2 As'= 1570mm^2

Concrete cover to tension steel = 80mm

Concrete cover to compression steel = 60mm

Compute the design moment, ΦMn for the section shown below in Figure 3.19.

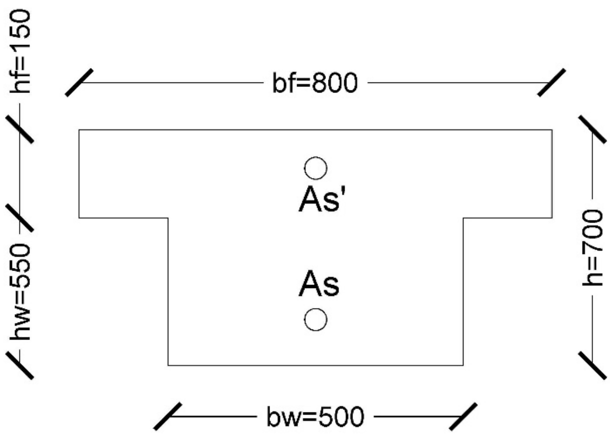


Figure 3.19: Reinforced section for Example 3.9

Solution:

If the tension reinforcement yields, then

Tension, T= As fy = 7111(420)/1000= 2986.62kN

If the depth of compression zone, a= 150mm and the compression steel yields, then

Compression, C= [0.85 (20)(800)(150) + 1570(420)] / 1000 = 2699.4kN

So, a > 150mm

T= Cc + Cs

2986.62 = 0.85(20)(300)(150)/1000 + 0.85(20)(500)(a)/1000 + 1570 (420)/1000

So, a= 184mm

Check a and stresses in tension and compression steel:

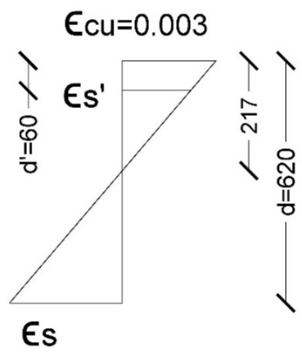
Depth of neutral axis = a/β1 = 184/0.85 = 217mm

From similar triangles in strain diagram:

Strain in compression steel = 0.00217 > yield strain = 420/200000 = 0.0021 so, compression steel yield

Stain in tension steel= 0.0056 > yield strain, so tension steel yield

So, a= 184mm



Tension force, T= 2987kN

Compression force in flange, Cc1= 0.85(20)(300)(150)/1000 = 765kN

Compression force in web, Cc2= 0.85(20)(500)(184)/1000 = 1564kN

Compression force in compression steel, Cs= 1570(420)/1000 = 659kN

The design moment, ϕM_n is given by:

ϕM_n = 0.90{Cc1(d-hf/2) + Cc2(d-a/2) + Cs(d-d') = 1450kN.m

3.8 Design of doubly reinforced concrete beam sections:

- The main use of the compression steel is to keep the strain in the tensile steel not less than 0.005 to ensure ductility provisions for large values of moments. So, the maximum depth of the compression zone is $a_{max} = \beta_1 C_{max} = \beta_1 0.375d$.
- The first step is to compute the needed steel area As1 which is equivalent to the maximum value of steel ratio or the maximum value of a. See Figure 3.13. For rectangular sections, the maximum steel ratio or a value less than it, can be used to determine As1. For non-rectangular sections, a_{max} or a value less than it, can be used to determine As1.
- Then, compute the moment capacity, ΦMn1 for As1.
- Compute the moment to be resisted by As2 and As' which is ΦMn2= Mu-ΦMn1
- Compute As2 and As1 as follows:

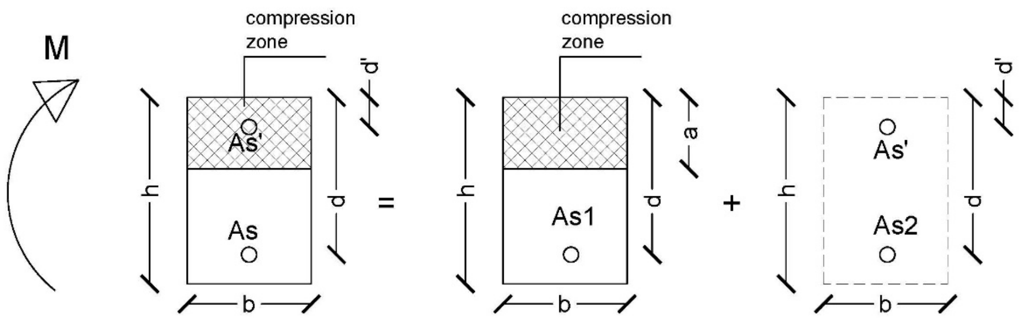


Figure 3.20: Doubly reinforced beam section

$\phi M_{n2} = \phi A_s{}' f_s{}'(d - d') = \phi A_{s2} f_y (d - d')$

The stress in compression steel, fs' is computed by multiplying the strain $\epsilon_s{}'$ by the modulus of elasticity of steel, Es.

The total area of tension steel in the section will be: As= As1 + As2

Example 3.10:

Given: f'c= 28MPa fy= 420MPa

Rectangular section: b= 350mm h= 700mm d= 640mm d'= 60mm

Determine the required area of steel to resist an ultimate bending moment, Mu= 1100kN.m.

Solution:

Determine steel ratio for Mu= 1100kN.m:

$$\rho = \frac{0.85f'_c}{f_y}\left(1 - \sqrt{1 - \frac{2.61M_u}{bd^2f'_c}}\right) = \frac{0.85(28)}{420}\left(1 - \sqrt{1 - \frac{2.61(1100x10^6)}{350(640)^2(28)}}\right) = 0.026$$

$$\rho_{max,singly,\epsilon_t=0.005} = 0.375\beta_1\frac{0.85f'_c}{f_y} = 0.375(0.85)\frac{0.85(28)}{420} = 0.01806 < 0.026$$

So, there is a need for compression steel.

The maximum steel ratio, ρ= 0.01806 requires As= 0.01806(350)(640)= 4045.4mm^2

$$a = \frac{4045.4(420)}{0.85(28)(350)} = 204mm$$

$$\phi M_{n1} = \phi A_s f_y\left(d - \frac{a}{2}\right) = \frac{0.9(4045.4)(420)\left(640 - \frac{204}{2}\right)}{10^6} = 822.7kN.m$$

$$\phi M_{n2} = 1100 - 822.7 = 277.3kN.m$$

$$\phi M_{n2} = \phi A_s{}' f_s{}'(d - d') = \phi A_{s2} f_y (d - d')$$

The depth of neutral axis, C= 204/0.85= 240mm

From similar triangles, the strain in compression steel is equal to 0.00225

So, $f_s{}' = 0.00225(200000) = 450MPa > f_y, \quad use\ f_s{}' = 420MPa$

Substitute in the above equation, As2= As'= 1265mm^2

As= 4045.4 + 1265= 5311mm^2 7Ø32

As'= 1265mm^2 4Ø20

Example 3.11:

Given: f'c= 20MPa fy=420MPa

Concrete cover to tension steel= 80mm

Concrete cover to compression steel if needed= 60mm

Determine the required reinforcement for the section shown below in Figure 3.21 to resist a positive moment, Mu= 1450kN.m.

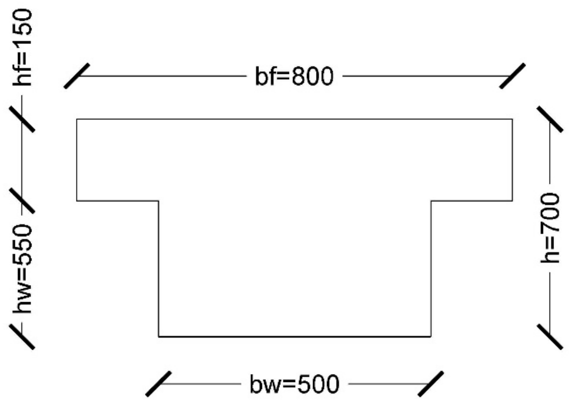


Figure 3.21: Reinforced section for Example 3.11

Solution:

Assume depth of compression zone, a= hf= 150mm, so, the resisting moment of the flange will be:

ΦMn= 0.9{0.85(20)(800)(150)(620-75)/10^6= 1000.62kN.m > Mu = 1450kN.m, then a > 150mm

The design moment of the overhangs is given by:

ΦMn1 = 0.9(0.85)(20)(300)(150)(620-75)/10^6= 375.2 kN.m, this moment requires an area of steel determined as follows:

ΦMn1= 375.2 kN.m = 0.90 As1 fy (d-hf/2)/10^6 , so

As1= 1821 mm^2

The design moment of the web will be 1450- 375.2 = 1074.8 kN.m= ΦMn2. This moment shall be resisted by a rectangular section of width, b= 500mm and thickness, h= 700mm, so:

The steel ratio will be, ρ = 0.01944 > ρ_{max} for singly reinforced section, then there is a need for compression steel.

$$\rho_{max,singly,\epsilon_t=0.005} = 0.375\beta_1 \frac{0.85f'_c}{f_y} = 0.375(0.85)\frac{0.85(20)}{420} = 0.0129 < 0.01944$$

For steel ratio, ρ = 0.012, the resisting moment can be computed as follows:

As2 = 0.012 (500) (620) = 3720 mm^2

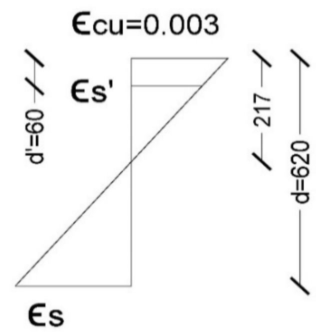
$$a = \frac{3720(420)}{0.85(20)(500)} = 183.8mm$$

ΦMn3= 0.9(3720)(420)(620-183.8/2)/10^6 = 742.5 kN.m

ΦMn4 = ΦMn2 – ΦMn3 = 1074.8 – 742.5 = 332.3 kN.m = 0.9 As3 fy (d-d') = 0.9 As' fs' (d-d')

The strain in compression steel is equal to 0.00217 > yielding strain, so, the compression steel yields.

C= a/β1 = 217mm



So, As3 = As' = 1570 mm^2

The total tension steel, As = As1 + As2 + As3 = 1821 + 3720 + 1570 = 7111 mm^2

The compression steel, As' = 1570mm^2

3.9 Load cases and moment envelope:

- The design should consider the load cases on a structure
- The live load is variable and movable
- Span can have dead load only

- Span can have dead plus live loads
- Moment envelope can be constructed by applying the dead loads to all spans and change the live loads from span to another. The maximum positive and negative moments can be found by using the following two principles – see Figure 3.22:

  - Load the two spans adjacent to the support to get the maximum negative moment at a support
  - Load the span itself and load alternative spans to get the maximum positive moment in a span

- If the arrangement of L is known, the slab system shall be analyzed for that arrangement

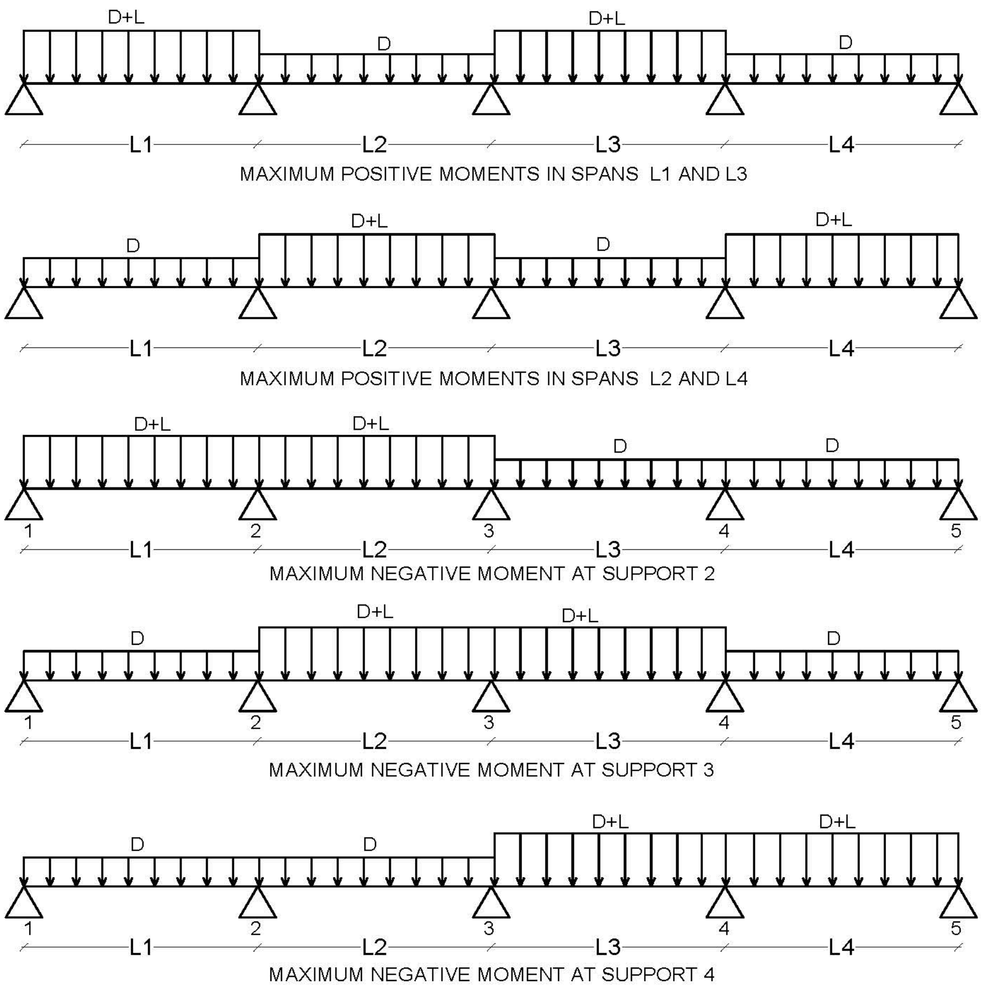


Figure 3.22: Load cases in a beam

3.10 Simplified method of analysis for nonprestressed continuous beams and one-way slabs: ACI coefficients

- It shall be permitted to calculate Mu and Vu due to gravity loads in accordance with this section for continuous beams and one-way slabs satisfying (a) through (e):

(a) Members are prismatic.
(b) Loads are uniformly distributed.
(c) L≤ 3D.
(d) There are at least two spans.
(e) The longer of two adjacent spans does not exceed the shorter by 20%.

- Mu due to gravity loads shall be calculated in accordance with Table 6.5.2 in ACI 318-19 as shown below:

Table 3.5: ACI 318-19 Table 6.5.2—Approximate moments for nonprestressed continuous beams and one-way slabs

Moment	Location	Condition	M_u
Positive	End span	Discontinuous end integral with support	$w_u\ell_n^2/14$
		Discontinuous end unrestrained	$w_u\ell_n^2/11$
	Interior spans	All	$w_u\ell_n^2/16$
Negative[1]	Interior face of exterior support	Member built integrally with supporting spandrel beam	$w_u\ell_n^2/24$
		Member built integrally with supporting column	$w_u\ell_n^2/16$
	Exterior face of first interior support	Two spans	$w_u\ell_n^2/9$
		More than two spans	$w_u\ell_n^2/10$
	Face of other supports	All	$w_u\ell_n^2/11$
	Face of all supports satisfying (a) or (b)	(a) slabs with spans not exceeding 10 ft (b) beams where ratio of sum of column stiffnesses to beam stiffness exceeds 8 at each end of span	$w_u\ell_n^2/12$

[1]To calculate negative moments, ℓ_n shall be the average of the adjacent clear span lengths.

- Vu due to gravity loads shall be calculated in accordance with Table 6.5.4 in ACI 318-19 as shown below

Table 3.6: ACI 318-19 Table 6.5.4—Approximate shears for nonpre- stressed continuous beams and one-way slabs

Location	V_u
Exterior face of first interior support	$1.15w_uL_n/2$
Face of all other supports	$w_uL_n/2$

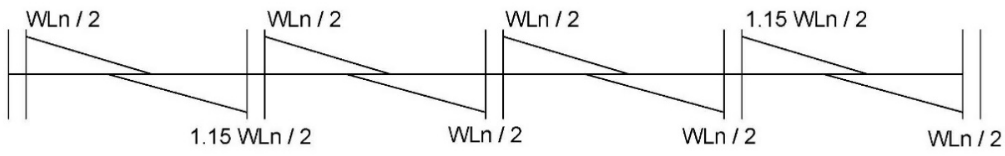


Figure 3.23: Shear envelope

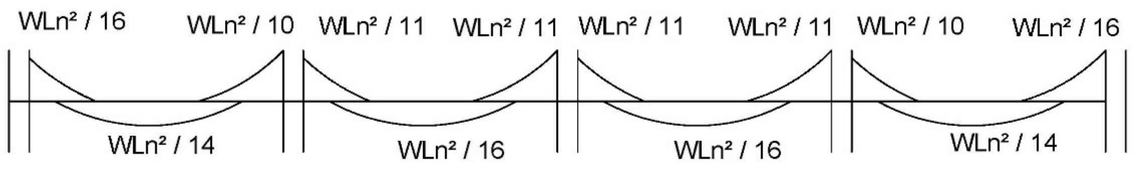


Figure 3.24: Moment envelope for a beam

Example 3.12:

Given:

F'c= 28MPa fy= 420MPa

Superimposed dead load, W_{SD}= 4kN/m^2

Live load, W_L= 6kN/m^2

Perimeter wall weight, W_{WALL}= 10kN/m

All columns are: 400mm x 400mm

All beams are: 400mm x550mm

Draw moment and shear forces envelopes for the slab and interior beam B2. See Figure 3.25 below.

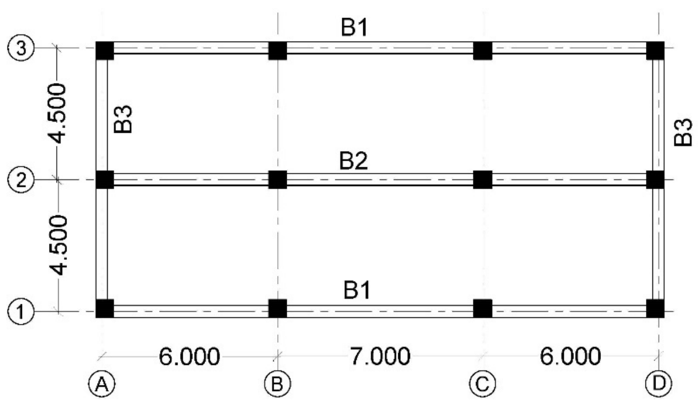


Figure 3.25: Slab and beams layout- Example 3.12

Solution:

Slab thickness:

The slab has two spans of 4.5m length. Ln=4.5-0.4=4.10m.

Minimum slab thickness, h= L/24 = 4.5/24= 0.19 m, try h= 0.20m

Own weight of slab, WD= 0.20(25)= 5kN/m^2

Wu1= 1.4(5+4)= 12.6kN/m^2

Wu2= 1.2(5+4)+1.6(6)= 20.4kN/m^2

Take, Wu= 20.4kN/m^2

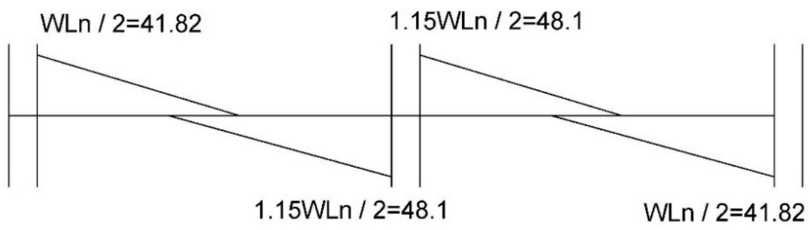


Figure 3.26: Shear envelope in slab- kN

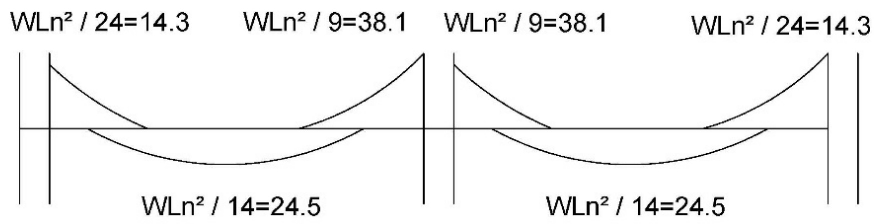


Figure 3.27: Bending moment envelope in slab- kN.m

The ultimate load on beam B2 is given by: Wu= 1.2(0.40 x 0.55)(25)+4.5(20.4)(1.15)= 112.17kN/m

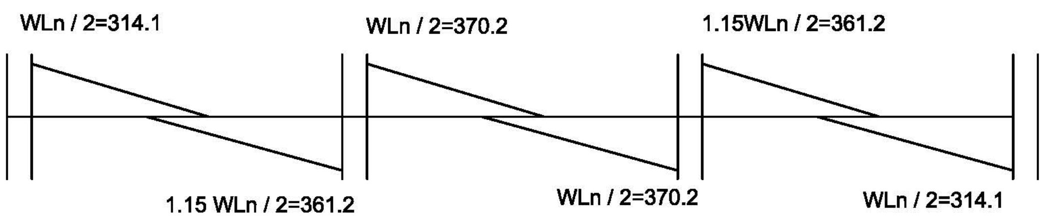


Figure 3.28: Shear envelope in beam B2- kN

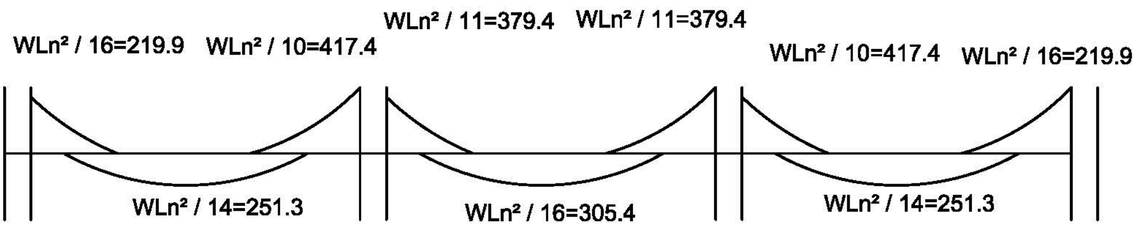


Figure 3.29: Bending moment envelope in beam B2- kN.m

3.11 Simplified flexural bars layout:

For beams analyzed using the ACI coefficients, a simplified bars layout can be used. The main points of bars layout are as follows:

1. In simple spans, at least 1/3 the bottom bars shall be extended into the supports with standard hooks. In continuous spans, at least ¼ the bottom bars shall be extended in the supports to develop fy at interior supports and will be with standard hooks at end supports.
2. Top bars at interior supports can be extended to 1/3 the larger clear span at each side, while at exterior supports, top bars can be extended to ¼ the clear span.

3.12 Beam section subjected to axial force:

If the beam section is subjected to tension force, Pu, in addition to the bending moment, Mu, the required reinforcement to resist this axial tension force can be calculated by dividing the force, Pu by $\emptyset f_y$, and this reinforcement can be distributed at top and bottom of section or at section perimeter (Top, bottom and sides).

If the beam section is subjected to compression force, Pu, in addition to the bending moment, Mu, the required reinforcement to resist both the bending moment and the compression force can be calculated using the previous principles but the compression force is not equal to the tension force. The compression force is larger than the tension force by the value of Pu as follows (Singly reinforced rectangular section):

$$0.85f_c'ba = A_s f_y + P_u \qquad So, A_s = \frac{0.85f_c'ba - P_u}{f_y}$$

The nominal moment capacity of the section, M_n is determined by summing the moment of the two forces C and T about point a in Figure 3.30.

$$M_n = 0.85f_c'ba\left(\frac{h}{2} - \frac{a}{2}\right) + A_s f_y\left(\frac{h}{2} - cov\right)$$

The following quadratic equation can be obtained by rearranging the above equations:

$$0.425f_c'ba^2 - 0.85f_c'bda + (M_n + 0.5P_u h - cov\,P_u) = 0.0$$

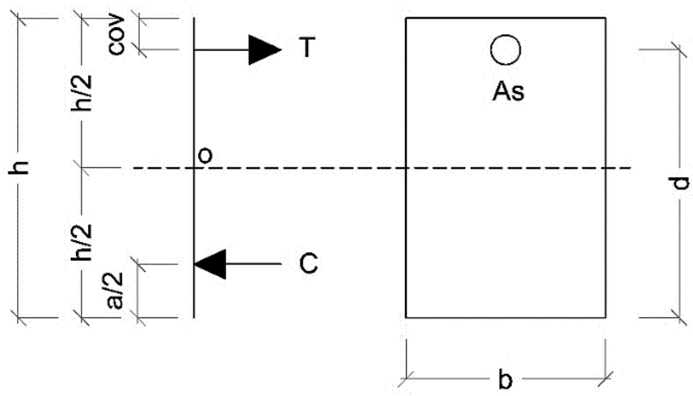


Figure 3.30: Internal forces in the beam section

Solving the above equations for a:

$$a = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$$
$$A = 0.425 f_c' b$$
$$B = -0.85 f_c' bd$$
$$C = M_n + 0.5 P_u h - cov\ P_u$$

Then, the required area of steel can be obtained.
Where:
b: width of beam cross section.
h: thickness of beam cross section.
d: effective depth of beam cross section.
f'_c: cylinder concrete compressive strength at 28 days.
f_y: steel yielding strength.
a: depth of the compression block.
cov: concrete cover to the centroid of tension reinforcing steel.

The above principle can be modified to determine the required reinforcing steel for the bending moment, Mu and the axial compression force, pu for other than rectangular beam sections like T or L shapes.

Also, a section analysis can be done when the tension reinforcing steel and one of the forces is known (The axial compression or the bending moment) to determine the other force (The bending moment or the axial compression force).
In general, it is recommended to design a beam as a column when subjected to an axial force especially when the compression force exceeds $0.1f'_c A_g$, where A_g is the area of the cross section.

Chapter 4: Design for Shear

This chapter illustrates the principles and the procedure for analysis and design of reinforced concrete sections for shear forces. In a later chapter, the design for torsion will be introduced and combined with the design for shear.

The design of cross sections subjected to shear shall be based on:

$$\emptyset V_n \geq Vu \quad or \quad V_n \geq \frac{V_u}{\emptyset}$$

Where:

V_n= nominal shear strength

V_u= ultimate shear force

∅= strength reduction factor= 0.75

$\emptyset V_n$= design shear strength

$$V_n = V_c + V_s$$

V_c= nominal shear strength provided by concrete

V_s= nominal shear strength provided by shear reinforcement

In determining Vc, the effect of any openings in members shall be considered.

4.1 Shear strength provided by concrete:

ACI 318-14:

For members without axial loads, V_c can be calculated as:

$$V_c = 0.17\lambda\sqrt{f'_c}b_w d = \frac{1}{6}\lambda\sqrt{f'_c}b_w d$$

More detailed calculation can be made in accordance with Table 4.1 (Table 22.5.5.1 in ACI 318-14).

Table 4.1: ACI 318-14 Table 22.5.5.1—Detailed method for calculating V_c

	V_c	
Least of (a), (b), and (c):	$\left(0.16\lambda\sqrt{f'_c} + 17\rho_w \frac{V_u d}{M_u}\right) b_w d$ [1]	(a)
	$(0.16\lambda\sqrt{f'_c} + 17\rho_w) b_w d$	(b)
	$0.29\lambda\sqrt{f'_c} b_w d$	(c)

[1] M_u occurs simultaneously with V_u at the section considered.

The value ρ_w is the flexural steel ratio in a beam section.

M_u: the ultimate bending moment occurs with the shear Vu at the same section.

For members subjected to axial compression force, V_c is given by:

$$V_c = \frac{1}{6}\left(1 + \frac{N_u}{14A_g}\right)\lambda\sqrt{f'_c} b_w d$$

N_u: axial compression force, positive sign, N.

For members subjected to axial tension force, V_c is given by:

$$V_c = \frac{1}{6}\left(1 + \frac{N_u}{3.5A_g}\right)\lambda\sqrt{f'_c} b_w d$$

N_u: axial tension force, negative sign, N. V_c shall be not less than zero.

A_g: cross sectional area of member, mm^2.

For circular members, the area used to compute V_c shall be taken as the product of the diameter and effective depth of the concrete section. It shall be permitted to take d as 0.8 times the diameter of the concrete section.

ACI 318-19:

V_c can be calculated by:

$$For\ A_v \geq A_{v.min}\ (or\ \frac{A_v}{s} \geq \left(\frac{A_v}{s}\right)_{min})\ use\ either\ of:$$

$$V_c = \left(0.17\lambda\sqrt{f'_c} + \frac{N_u}{6A_g}\right) b_w d \quad and \quad V_c = \left(0.66\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right) b_w d$$

$$For\ A_v < A_{v.min}\ \left(or\ \frac{A_v}{s} < \left(\frac{A_v}{s}\right)_{min}\right) use:$$

$$V_c = \left(0.66\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right)b_w d$$

Where A_v is the area of shear reinforcement within spacing s, mm^2.

And, V_c shall not be taken greater than:

$$V_c \leq 0.42\lambda\sqrt{f'_c}b_w d$$

$$\lambda_s = \sqrt{\frac{2}{1 + 0.004\,d}} \leq 1.0$$

For $d \leq 250mm, \lambda_s = 1.0$

$$\frac{N_u}{6A_g} \leq 0.05f'_c$$

Axial load, *Nu*, is positive for compression and negative for tension.

$$\rho_w = \frac{A_s}{b_w d}$$

The value of *As* to be used in the calculation of ρ_w may be taken as the sum of the areas of longitudinal bars located more than two thirds of the overall member depth away from the extreme Compression fiber.

The value of $\sqrt{f'c}$ used to calculate ***Vc*** for one-way shear shall not exceed 100 psi (8.3MPa), unless allowed in 22.5.3.2 ($A_v \geq A_{v,min}$).

Interaction of shear forces acting along orthogonal axes:

The interaction of shear forces acting along orthogonal axes shall be permitted to be neglected if (a) or (b) is satisfied:

$$(a) \quad \frac{V_{u,x}}{\emptyset V_{n,x}} \leq 0.5 \qquad (b) \quad \frac{V_{u,y}}{\emptyset V_{n,y}} \leq 0.5$$

$$If \ \frac{V_{u,x}}{\emptyset V_{n,x}} > 0.5 \ \ and \ \frac{V_{u,y}}{\emptyset V_{n,y}} > 0.5\,, \qquad then \ \frac{V_{u,x}}{\emptyset V_{n,x}} + \frac{V_{u,y}}{\emptyset V_{n,y}} \leq 1.5$$

4.2 Shear strength provided by shear reinforcement:

The values of steel yield strength f_{yt} in design of shear reinforcement shall not exceed 420MPa.

The shear reinforcement can be calculated using the following formula:

$$\frac{A_v}{s} = \frac{V_s}{f_{yt} d} \qquad V_s = \frac{V_u}{\phi} - V_c$$

A_v: the area of vertical legs of stirrups (shear reinforcement), mm^2

S: spacing of stirrups, mm

f_{yt}: yield strength of stirrups reinforcing bars, MPa

d: effective depth of cross section, mm

4.3 Spacing limits for shear reinforcement:

- $If\ V_s \le \frac{1}{3}\sqrt{f'_c} b_w d \ \rightarrow S_{max} = min\left[\frac{d}{2}, 600mm\right]$
- $If\ \frac{1}{3}\sqrt{f'_c} b_w d < V_s \le \frac{2}{3}\sqrt{f'_c} b_w d \ \rightarrow S_{max} = min\left[\frac{d}{4}, 300mm\right]$
- $If\ V_s > \frac{2}{3}\sqrt{f'_c} b_w d \ \rightarrow Incease\ section\ dimensions$

So, cross-sectional dimensions shall be selected to satisfy:

$$V_u \le \emptyset \left(V_c + 0.66\sqrt{f'_c} b_w d\right)$$

These limitations are summarized in Table 9.7.6.2.2 in ACI 318-19 as shown below.

Reduced stirrup spacing across the beam width provides a more uniform transfer of diagonal compression across the beam web, enhancing shear capacity. Laboratory tests of wide members with large spacing of legs of shear reinforcement across the member width indicate that nominal capacity is not always achieved. The intent of this provision is to provide multiple stirrup legs across wide beams and one-way slabs that require stirrups. In seismic design, the maximum spacing between bars restrained by legs of crossties or hoops is 350mm as shown in Figure 4.1.

Table 4.2: ACI 318-19 Table 9.7.6.2.2—Maximum spacing of shear reinforcement

Required Vs	**Maximum s, mm**				
		Nonprestressed beam		Prestressed beam	
		Along length	Across width	Along length	Across width
$\le 0.33\sqrt{f_c'} b_w d$	Lesser of:	d/2	d	3h/4	3h/2
		600			
$> 0.33\sqrt{f_c'} b_w d$	Lesser of:	d/4	d/2	3h/8	3h/4
		300			

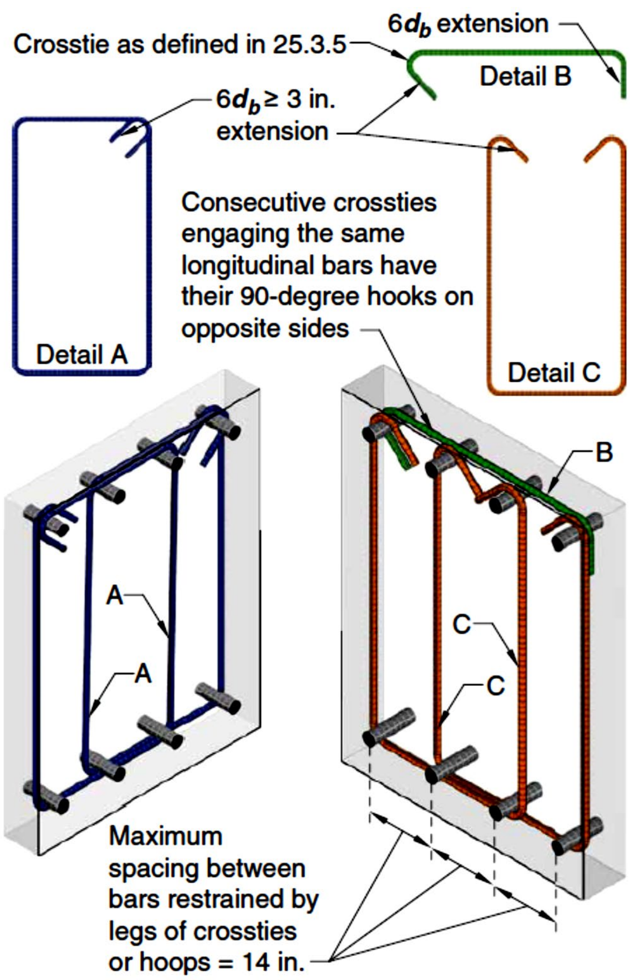


Figure 4.1: ACI 318-19 Fig. R18.6.

4.4 Minimum shear reinforcement:

A minimum shear reinforcement $(Av/s)_{min}$ shall be provided where $V_u > 0.5\ \emptyset\ V_c$, except in the following cases, ACI 318-19 Table 9.6.3.1 in addition to slabs and footings with uniform thickness.

Table 4.3: ACI 318-19: Table 9.6.3.1—Cases where $A_{v,min}$ is not required if $0.5\phi V_c < V_u \leq \phi V_c$

Beam type	Conditions
Shallow depth	$h \leq 250$ mm
Integral with slab	$h \leq$ greater of $2.5t_f$ or $0.5b_w$ and $h \leq 600$ mm
Constructed with steel fiber-reinforced normal weight concrete conforming to 26.4.1.5.1(a), 26.4.2.2(i), and 26.12.7.1(a) and with $f_c' \leq 40$ MPa	$h \leq 600$ mm and $V_u \leq \phi\ 0.17\sqrt{f_c'}\ b_w d$
One-way joist system	In accordance with 9.8

The minimum shear reinforcement is given by:

$$\left(\frac{A_v}{s}\right)_{min} = \max of \begin{bmatrix} 0.062\sqrt{f'_c}\dfrac{b_w}{f_{yt}} \\ 0.35\dfrac{b_w}{f_{yt}} \end{bmatrix}$$

4.5 Shear reinforcement details:

In general:

$If\ \frac{V_u}{\emptyset} > V_c: There\ is\ a\ need\ for\ shear\ reinforcement$

$If\ \frac{V_c}{2} < \frac{V_u}{\emptyset} \le V_c: Use\ minimum\ shear\ reinforceemnt\ except\ in\ the\ above\ cases$

$If\ \frac{V_u}{\emptyset} \le \frac{V_c}{2}: No\ need\ for\ shear\ reinforcement$

For beams built integrally with supports, Vu at the support shall be permitted to be calculated at the face of support. The critical section can be taken at distance d from face of support if:

(a) Support reaction, in direction of applied shear, introduces compression into the end region of the beam

(b) Loads are applied at or near the top surface of the beam

(c) No concentrated load occurs between the face of support and critical section

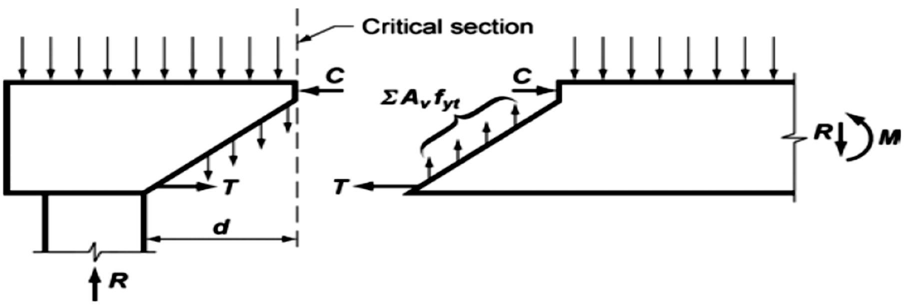


Figure 4.2: Critical sections for shear

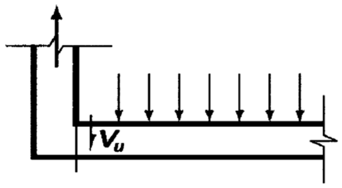
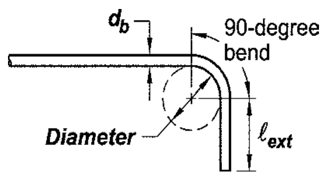
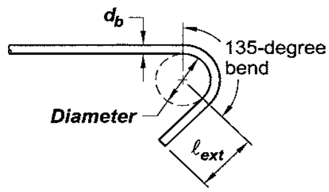
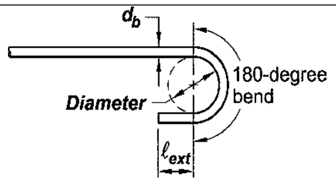


Figure 4.2 continued: Critical sections for shear

The hooks details for stirrups are as given in Table 4.4 (Table 25.3.2 in ACI 318-19).

Table 4.4: ACI 318-19 Table 25.3.2—Minimum inside bend diameters and standard hook geometry for stirrups, ties, and hoops

Type of standard hook	Bar size	Minimum inside bend diameter, mm	Straight extension[1] ℓ_{ext}, mm	Type of standard hook
90-degree hook	No. 10 through No. 16	$4d_b$	Greater of $6d_b$ and 75 mm	
	No. 19 through No. 25	$6d_b$	$12d_b$	
135-degree hook	No. 10 through No. 16	$4d_b$	Greater of $6d_b$ and 75 mm	
	No. 19 through No. 25	$6d_b$		
180-degree hook	No. 10 through No. 16	$4d_b$	Greater of 4 d_b and 65 mm	
	No. 19 through No. 25	$6d_b$		

[1] A standard hook for stirrups, ties, and hoops includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

Notes:

Note1: Seismic hooks used to anchor stirrups, ties, hoops, and crossties shall be in accordance with (a) and (b):

(a) Minimum bend of 90 degrees for circular hoops and 135 degrees for all other hoops.

(b) Hook shall engage longitudinal reinforcement and the extension shall project into the interior of the stirrup or hoop.

Note 2: Crossties shall be in accordance with (a) through e:

(a) Crosstie shall be continuous between ends.

(b) There shall be a seismic hook at one end.

(c) There shall be a standard hook at other end with minimum bend of 90 degrees.

(d) Hooks shall engage peripheral longitudinal bars.

(e) 90-degree hooks of two successive crossties engaging the same longitudinal bars shall be alternated end for end, unless crossties satisfy 18.6.4.3 or 25.7.1.6.1.

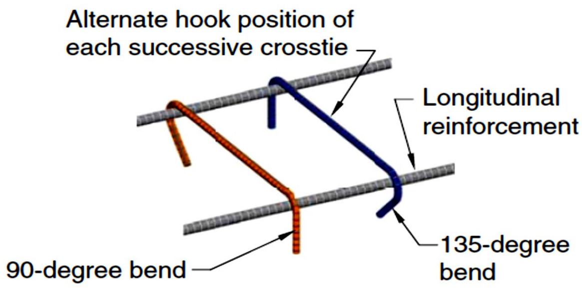


Fig. R25.3.5—Crosstie.

Figure 4.3: Crosstie Fig. R25.3.5 ACI 318-19

Note 3: Stirrups shall extend as close to the compression and tension surfaces of the member as cover requirements and proximity of other reinforcement permits and shall be anchored at both ends. Where used as shear reinforcement, stirrups shall extend a distance *d* from extreme compression fiber.

Note 4: Anchorage of stirrup deformed bar shall be in accordance with (a), (b), or (c):

(a) For Ø16 bar and smaller, and for Ø19 through Ø25 bars with $f_{yt} \leq 280MPa$, a standard hook around longitudinal reinforcement.
(b) For Ø19 through Ø25 bars with $f_{yt} > 280MPa$, a standard hook around a longitudinal bar plus an embedment between mid-height of the member and the outside end of the hook equal to or greater than $0.17 d_b f_{yt} / \lambda \sqrt{f'_c}$.
(c) In joist construction, for ϕ13 bar and smaller, a standard hook.

Note 5: Stirrups used for torsion or integrity reinforcement shall be closed stirrups perpendicular to the axis of the member. Where welded wire reinforcement is used, transverse wires shall be perpendicular to the axis of the member. Such stirrups shall be anchored by (a) or (b): refer to Figure 4.4.

(a) Ends shall terminate with 135-degree standard hooks around a longitudinal bar.

(b) In accordance with 25.7.1.3(a) or (b) or 25.7.1.4 (Points in Note 4 above), where the concrete surrounding the anchorage is restrained against spalling by a flange or slab or similar member.

Note 6: Except where used for torsion or integrity reinforcement, closed stirrups are permitted to be made using pairs of U-stirrups spliced to form a closed unit where lap lengths are at least $1.3l_{dt}$. In members with a total depth of at least 450mm, such splices with $A_b f_{yt} \leq 40kN$ per leg shall be considered adequate if stirrup legs extend the full available depth of member.

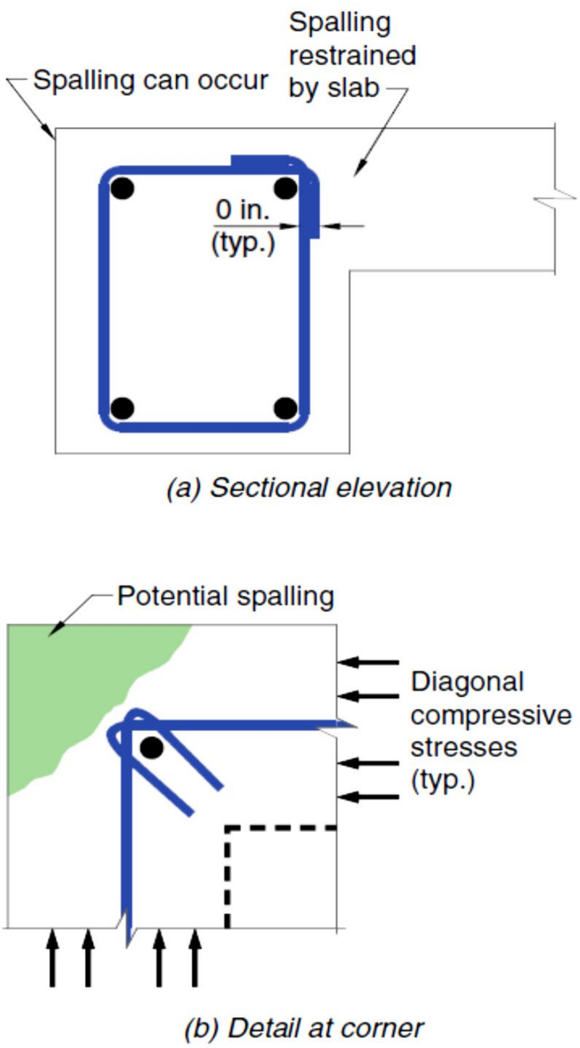


Fig. R25.7.1.6—Spalling of corners of beams subjected to torsion.

Figure 4.4: Corner of a stirrup for torsion or integrity reinforcement

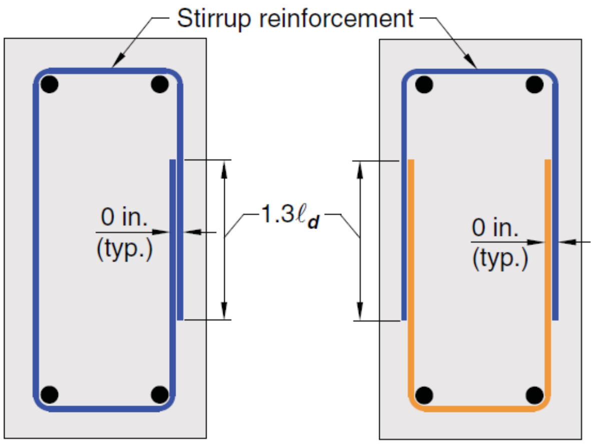


Figure 4.5: Splicing of closed stirrup

Example 4.1:

Given: f'c= 24MPa fy= 420MPa

Check slab thickness for shear. See Figure 4.4

Slab width, b= 1000mm

Slab thickness, h= 250mm

Slab effective depth, d= 210mm

Support (beam) width= 500mm

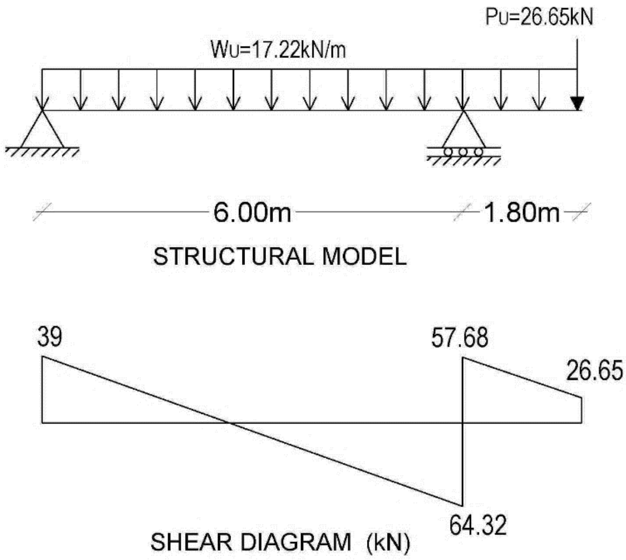


Figure 4.4: Concrete slab for Example 4.1

Solution:

The maximum ultimate shear force at distance d from face of support, Vu is given by:

Vu= 64.32 – 17.22(0.25+0.21) = 56.4kN

ACI 318-14: The concrete shear strength, $\emptyset V_c$ is given by:

$$\emptyset V_c = \emptyset \frac{1}{6}\lambda\sqrt{f'_c}b_w d = \frac{0.75\left(\frac{1}{6}\right)(1)\sqrt{24}(1000)(210)}{1000} = 128.6kN > 56.4kN \quad ok$$

ACI 318-19: The concrete shear strength, $\emptyset V_c$ is given by:

$$\emptyset V_c = \emptyset\left(0.66\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right)b_w d$$

$$Let\ \rho_w = 0.0018\left(\frac{h}{d}\right) = 0.0018\left(\frac{250}{210}\right) = 0.0021$$

So,

$$\emptyset V_c = \emptyset\left(0.66\lambda_s\lambda(\rho_w)^{\frac{1}{3}}\sqrt{f'_c} + \frac{N_u}{6A_g}\right)b_w d$$
$$= \frac{0.75\left(0.66(1)(1)(0.0021)^{\frac{1}{3}}\sqrt{24} + 0.0\right)(1000)(210)}{1000} = 65.2kN$$
$$> 56.4kN. OK$$

Example 4.2:

Given: f'c= 24MPa fy= 420MPa

Design the beam for the maximum ultimate shear force. See Figure 4.5

Beam width, b= 500mm

Beam thickness, h= 900mm

Beam effective depth, d= 810mm

Support (column) width= 500mm

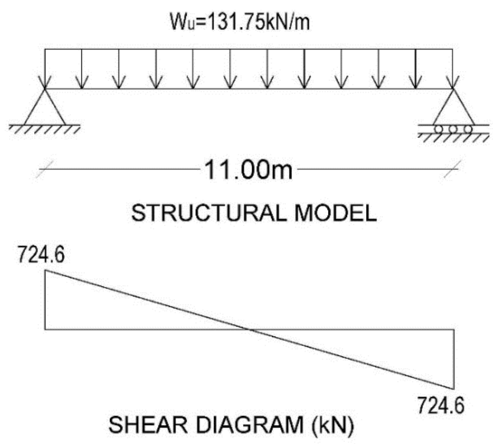


Figure 4.5: Concrete beam for Example 4.1

Solution:

The maximum ultimate shear force at distance d from face of support, Vu is given by:

Vu= 724.6 – 131.75(0.25+0.81) = 585kN $V_u/\emptyset$= 585/0.75 = 780kN

The concrete shear strength, Vc is given by ($\frac{A_v}{s} \geq \left(\frac{A_v}{s}\right)_{min}$), no axial loads exist:

$$V_c = \frac{1}{6}\lambda\sqrt{f'_c}b_w d = \frac{\left(\frac{1}{6}\right)(1)\sqrt{24}(500)(810)}{1000} = 330.7kN$$
$$< 780kN \quad shear\ reinforcemnt\ is\ required$$

Vs = 780 – 330.7= 449.3 kN

$$V_{s,max} = \frac{2}{3}\sqrt{f'_c}b_w d = \frac{\frac{2}{3}\sqrt{24}(500)(810)}{1000} = 1322.8kN > Vs = 449.3kN \quad ok$$

The shear reinforcement is given by:

$$\frac{A_v}{s} = \frac{V_s}{f_{yt}d} = \frac{449.3x1000}{420(810)} = \frac{1.3207mm^2}{mm}$$

Check minimum area of steel:

$$\left(\frac{A_v}{s}\right)_{min} = \max of \begin{bmatrix} 0.062\sqrt{f'_c}\frac{b_w}{f_{yt}} \\ 0.35\frac{b_w}{f_{yt}} \end{bmatrix} = \max\ of \begin{bmatrix} 0.062\sqrt{24}\frac{500}{420} = 0.36 \\ 0.35\frac{500}{420} = 0.42 \end{bmatrix} = \frac{0.42mm^2}{mm}$$
$$< \frac{1.3207mm^2}{mm} \quad ok$$

Assume using $\emptyset 12mm$ stirrups, Av= 2 x 113 = 226 mm^2 - two legs stirrup

So,

S=226/1.3207 = 170mm

Check maximum spacing of stirrups, S_{max}:

$$\frac{1}{3}\sqrt{f'_c}b_w d = \frac{\frac{1}{3}\sqrt{24}(500)(810)}{1000} = 661.4kN > Vs = 449.3kN$$

So,

S_{max} = min (d/2= 810/2= 405mm, 600mm)= 405mm > 170mm ok

Additional note:

If it is required to determine the distance at which stirrups at spacing of d/2= 400mm can be used, so, the following calculations can be done:

For stirrups: $1\emptyset 12/400mm$:

$$\frac{A_v}{s} = \frac{V_s}{f_{yt}d} = \frac{226}{400} = \frac{V_s x1000}{420(810)} \rightarrow V_s = 192.2kN$$

$$\emptyset V_n = \emptyset[V_c + V_s] = 0.75[330.7 + 192.2] = 392.18kN$$

Vu= ΦVn, from the shear force diagram, the value of Vu = 392.18 kN is located, then its location from the left support can be determined from similar triangles.

The distance from left or right support, X1= 2.5m

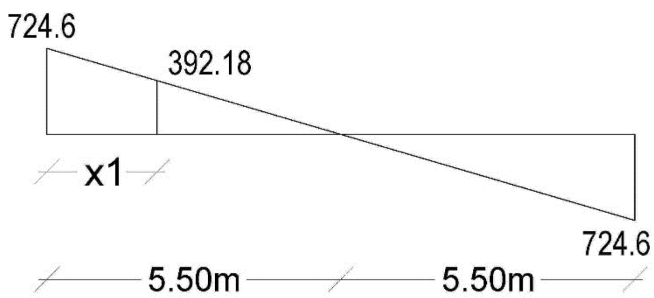


Figure 4.6: Shear force diagram of beam

So, the stirrups $\emptyset 12/400mm$ can be used in the middle zone of the beam (11-2x2.5)= 6m

Chapter 5: Development, Anchorage and Splicing of Reinforcement

In a reinforced concrete beam, the flexural compressive forces are resisted by concrete, while the flexural tensile forces are provided by reinforcement. For this process to exist, there must be a force transfer, or bond, between the two materials. For the bar to be in equilibrium, bond stresses must exist. If these disappear, the bar will pull out of the concrete and the tensile force, T, will drop to zero, causing the beam to fail.

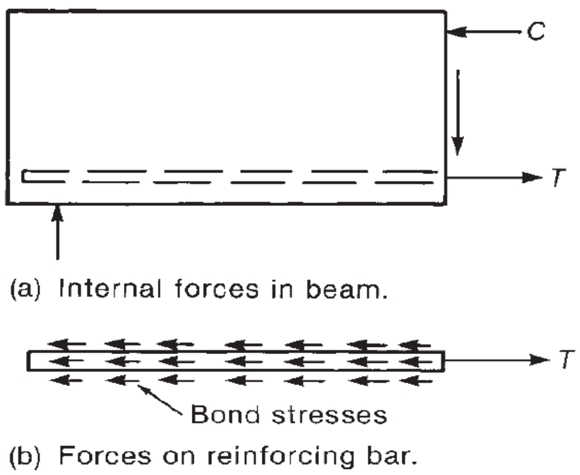


Figure 5.1: Forces in reinforcing bar

The development length of a bar depends on:

- Bar diameter
- Yield strength of steel
- Compressive strength of concrete
- Force in bar, tension or compression
- Spacing between bars
- Concrete cover
- Confinement of concrete

A smooth bar embedded in concrete develops bond by adhesion between the concrete and the bar and by a small amount of friction. Both of these effects are quickly lost when the bar is loaded in tension, particularly because the diameter of the bar decreases slightly, due to Poisson's ratio. For this reason, smooth bars are generally not used as reinforcement. In cases where smooth bars must be embedded in concrete (anchor bolts, stirrups made of small diameter bars, etc.), mechanical anchorage in the form of hooks, nuts, and washers on the embedded end (or similar devices) are used.

Although adhesion and friction are present when a deformed bar is loaded for the first time, these bond-transfer mechanisms are quickly lost, leaving the bond to be transferred by bearing on the deformations of the bar. Equal and opposite bearing stresses act on the concrete. The forces on the concrete have both a longitudinal and a radial component. The latter causes circumferential tensile stresses in the concrete around the bar. Eventually, the concrete will split parallel to the bar, and the resulting crack will propagate out toward the surface of the beam. The splitting cracks follow the reinforcing bars along the bottom or side surfaces of the beam. Once these cracks develop, the bond transfer drops rapidly unless reinforcement is provided to restrain the opening of the splitting crack.

The load at which splitting failure develops is a function of:

1. the minimum distance from the bar to the surface of the concrete or to the next bar—the smaller this distance, the smaller is the splitting load;

2. the tensile strength of the concrete; and

3. the average bond stress—as this increases, the wedging forces increase, leading to a splitting failure.

The development length, Ld, is the shortest length of bar in which the bar stress can increase from zero to the yield strength, fy. If the distance from a point where the bar stress equals fy to the end of the bar is less than the development length, the bar will pull out of the concrete. The development lengths are different in tension and compression, because a bar loaded in tension is subject to in-and-out bond stresses and hence requires a considerably longer development length. Also, for a bar in compression, bearing stresses at the end of the bar will transfer part of the compression force into the concrete.

5.1 Development of deformed bars in tension:

The values of $\sqrt{f'_c}$ used to calculate development length shall not exceed 8.3 MPa.

Table 5.1: ACI 318-19 Table 25.4.2.2—Development length for deformed bars and deformed wires in tension

Spacing and cover	No. 19 and smaller bars and deformed wires	No. 22 and larger bars
Clear spacing of bars or wires being developed or lap spliced not less than d_b, clear cover at least d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or lap spliced at least $2d_b$ and clear cover at least d_b	$\left(\frac{f_y\psi_t\psi_e\psi_g}{2.1\lambda\sqrt{f'_c}}\right)d_b$	$\left(\frac{f_y\psi_t\psi_e\psi_g}{1.7\lambda\sqrt{f'_c}}\right)d_b$
Other cases	$\left(\frac{f_y\psi_t\psi_e\psi_g}{1.4\lambda\sqrt{f'_c}}\right)d_b$	$\left(\frac{f_y\psi_t\psi_e\psi_g}{1.1\lambda\sqrt{f'_c}}\right)d_b$

Table 5.2: ACI 318-19 Table 25.4.2.5—Modification factors for development of deformed bars and deformed wires in tension

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Normal weight concrete	1.0
Reinforcement grade ψ_g	Grade 40 or Grade 60 (280MPa or 420MPa)	1.0
	Grade 80 (560MPa)	1.15
	Grade 100 (700MPa)	1.3
Epoxy[1] ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement with clear cover less than $3d_b$ or clear spacing less than $6d_b$	1.5
	Epoxy-coated or zinc and epoxy dual-coated reinforcement for all other conditions	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Size ψ_s	No. 7 and larger bars (22mm)	1.0
	No. 6 and smaller bars and deformed wires (19mm)	0.8
Casting position[1] ψ_t	More than 300mm of fresh concrete placed below horizontal reinforcement	1.3
	Other	1.0

[1] The product $\psi_t\ \psi_e$ need not exceed 1.7.

The minimum development length in tension shall be not less than 300mm.

The above values can be summarized as follows for normal concrete (ψ_t, ψ_e, ψ_g and λ are equal to 1.0) and f_y=420MPa:

Clear spacing of bars or wires being developed or lap spliced not less than db, clear cover at least db, and stirrups or ties throughout Ld not less than the Code minimum or:

Clear spacing of bars or wires being developed or lap spliced at least 2db and clear cover at least db:

$$l_{dt} = \frac{0.48 f_y}{\sqrt{f'_c}} d_b \qquad for\ bars\ less\ than\ 20mm$$

$$l_{dt} = \frac{0.59 f_y}{\sqrt{f'_c}} d_b \qquad for\ bars\ equals\ and\ larger\ than\ 20mm$$

In other cases:

$$l_{dt} = \frac{0.71 f_y}{\sqrt{f'_c}} d_b \qquad for\ bars\ less\ than\ 20mm$$

$$l_{dt} = \frac{0.91 f_y}{\sqrt{f'_c}} d_b \qquad for\ bars\ equals\ and\ larger\ than\ 20mm$$

In all cases, $l_{dt} \geq 300$mm.

Modification factors:

1. If there is more than 300 mm of fresh concrete placed below horizontal reinforcement, increase the above values by 30%.
2. The use of excess reinforcement: multiply the above values by:

   $A_{s,\ required} / A_{s,\ provided}$

5.2 Development of deformed bars in compression:

The development length in compression can be calculated as follows:

$$l_{dc} = \frac{0.24 f_y}{\lambda \sqrt{f'_c}} d_b$$

$$\geq 0.043 f_y d_b$$

$$\geq 200mm$$

Modification factors:

- If having spirals less or equals 100mm or having $\phi 13mm$ or larger ties with spacing less than or equals 100mm, multiply the above values by 0.75
- Excess reinforcement: multiply the above values by: $A_{s,\ required} / A_{s,\ provided}$

5.3 Development of standard hooks in tension:

The development length for hooked bars in tension can be calculated as follows:

$$l_{dh} \geq \frac{f_y \Psi_e \Psi_r \Psi_o \Psi_c}{23 \lambda \sqrt{f'_c}} {d_b}^{1.5} \qquad \geq 8 d_b \qquad \geq 150mm$$

Use:

$\Psi_e = 1.0,\ \Psi_r = 1.6, \Psi_o = 1.25, \Psi_c = 1.0$

Then:

$$l_{dh} \geq \frac{0.087 f_y}{\lambda\sqrt{f'_c}} {d_b}^{1.5}$$

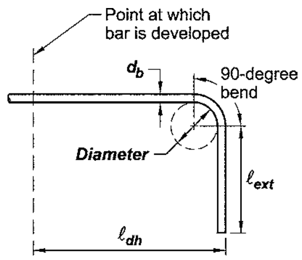
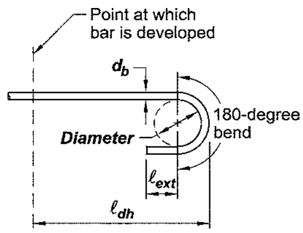
Refer to table below: ACI 318-19 Table 25.4.3.2.

Table 5.3: ACI 318-19 Table 25.4.3.2—Modification factors for development of hooked bars in tension

<table>
<tr><th>Modification factor</th><th>Condition</th><th>Value of factor</th></tr>
<tr><td rowspan="2">Lightweight λ</td><td>Lightweight concrete</td><td>0.75</td></tr>
<tr><td>Normal weight concrete</td><td>1.0</td></tr>
<tr><td rowspan="2">Epoxy ψ_e</td><td>Epoxy-coated or zinc and epoxy dual-coated reinforcement</td><td>1.2</td></tr>
<tr><td>Uncoated or zinc-coated (galvanized) reinforcement</td><td>1.0</td></tr>
<tr><td rowspan="2">Confining Reinforcement ψ_r</td><td>For No. 11 and smaller bars with $A_{th} \geq 0.4A_{hs}$ or s[1] $\geq 6d_b$[2]</td><td>1.0</td></tr>
<tr><td>Other</td><td>1.6</td></tr>
<tr><td rowspan="2">Location ψ_o</td><td>For No. 11 and smaller diameter hooked bars:
(1) Terminating inside column core with side cover normal to plane of hook ≥ 65mm, or
(2) With side cover normal to plane of hook $\geq 6d_b$</td><td>1.0</td></tr>
<tr><td>Other</td><td>1.25</td></tr>
<tr><td rowspan="2">Concrete Strength ψ_c</td><td>For f_c' < 6000 psi (42MPa)</td><td>f_c'/15,000 + 0.6 in psi: $\frac{f'_c}{105} + 0.6$ in MPa</td></tr>
<tr><td>For f_c' ≥2 6000 psi (42MPa)</td><td>1.0</td></tr>
</table>

[1] S is minimum center-to-center spacing of hooked bars.
[2] d_b is nominal diameter of hooked bar.
A_{th}: total cross-sectional area of ties or stirrups confining hooked bars, mm^2.
A_{hs}: total cross-sectional area of hooked or headed bars being developed at a critical section, mm^2.

Table 5.4: ACI 318-19 Table 25.3.1—Standard hook geometry for development of deformed bars in tension

<table>
<tr><th>Type of standard hook</th><th>Bar size</th><th>Minimum inside bend diameter, mm</th><th>Straight extension [1] ℓ_{ext}, mm</th><th>Type of standard hook</th></tr>
<tr><td rowspan="3">90-degree hook</td><td>No. 10 through No. 25</td><td>$6d_b$</td><td rowspan="3">$12d_b$</td><td rowspan="3"></td></tr>
<tr><td>No. 29 through No. 36</td><td>$8d_b$</td></tr>
<tr><td>No. 43 and No. 57</td><td>$10d_b$</td></tr>
<tr><td rowspan="3">180-degree hook</td><td>No. 10 through No. 25</td><td>$6d_b$</td><td rowspan="3">Greater of $4d_b$ and 65 mm</td><td rowspan="3"></td></tr>
<tr><td>No. 29 through No. 36</td><td>$8d_b$</td></tr>
<tr><td>No. 43 and No. 57</td><td>$10d_b$</td></tr>
</table>

[1] A standard hook for deformed bars in tension includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

5.4 Development of bundled bars:

Groups of parallel reinforcing bars bundled in contact to act as a unit shall be limited to four in any one bundle.

Development length for individual bars within a bundle, in tension or compression, shall be that of the individual bar, increased 20 percent for a three-bar bundle, and 33 percent for a four-bar bundle.

A unit of bundled bars shall be treated as a single bar with an area equivalent to that of the bundle and a centroid coinciding with that of the bundle. The diameter of the equivalent bar shall be used for db in spacing limitations, cover requirements and confinement.

Bundled bars shall be enclosed within transverse reinforcement. Bundled bars in compression members shall be enclosed by transverse reinforcement at least No. 4 in size (12mm). Bars larger than a No. 11 (35mm) shall not be bundled in beams.

Individual bars within a bundle terminated within the span of flexural members shall terminate at different points with at least **40*db*** stagger.

5.5 Splicing methods for reinforcing bars:

- Lap splice: for bars not larger than 36mm

- Welding: weld splice shall develop at least 1.25 fy of the bars

- Mechanical connection

5.6 Splicing of tension bars:

- Lap splice shall be not less than 300mm.

- It is recommended to use class B splice which equals to 1.3 times the development length of a bar.

- Splices shall be staggered at least 600mm.

- For contact lap splices, minimum clear spacing between the contact lap splice and adjacent splices or bars shall be in accordance with the requirements for individual bars in ACI 318-19 section 25.2.1 (clear spacing shall be at least the greatest of 25 mm, *db*, and (4/3)*d, aggregate*.

- For noncontact splices in flexural members, the transverse center-to-center spacing of spliced bars shall not exceed the lesser of one-fifth the required lap splice length and 150 mm.

5.7 Splicing of compression bars:

- for fy≤ 420 MPa or less:

$$splicing\ length, ls = 0.071 f_y d_b$$

- For $420MPa < f_y \leq 560\ MPa$:

$$splicing\ length, ls = \left(0.13 f_y - 24\right) d_b \geq 300mm$$

- For $f_y > 560MPa$:

$$The\ longer\ of: splicing\ length, ls = \left(0.13 f_y - 24\right) d_b,\ splicing\ length\ in\ tension\ per\ section\ 25.5.2.1\ in\ ACI\ 318-19$$

The splicing length shall be not less than 300mm.

Note: For f'c < 21 MPa, length of splicing shall be increased by one third.

5.8 Development of flexural reinforcement:

1. Reinforcement shall extend beyond the point at which it is no longer required to resist flexure for a distance equal to the greater of d and 12db, except at supports of simply-supported spans and at free ends of cantilevers

2. Continuing flexural tension reinforcement shall have an embedment length at least ℓd beyond the point where bent or terminated tension reinforcement is no longer required to resist flexure.

3. At simple supports, at least one-third of the maximum positive moment reinforcement shall extend along the beam bottom into the support at least 150 mm, except for precast beams where such reinforcement shall extend at least to the center of the bearing length. At other supports, at least one-fourth of the maximum positive moment reinforcement shall extend along the beam bottom into the support at least 150 mm and, if the beam is part of the primary lateral-load-resisting system, shall be anchored to develop fy at the face of the support.

4. At least one-third of the negative moment reinforcement at a support shall have an embedment length beyond the point of inflection at least the greatest of d, 12db, and Ln/16.

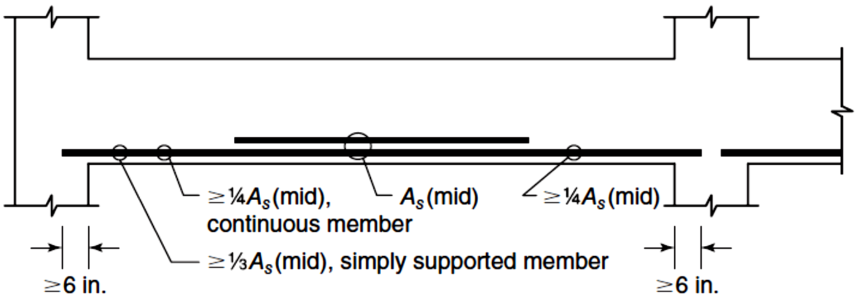


Figure 5.2: Continuity requirements for positive-moment reinforcement in continuous beams

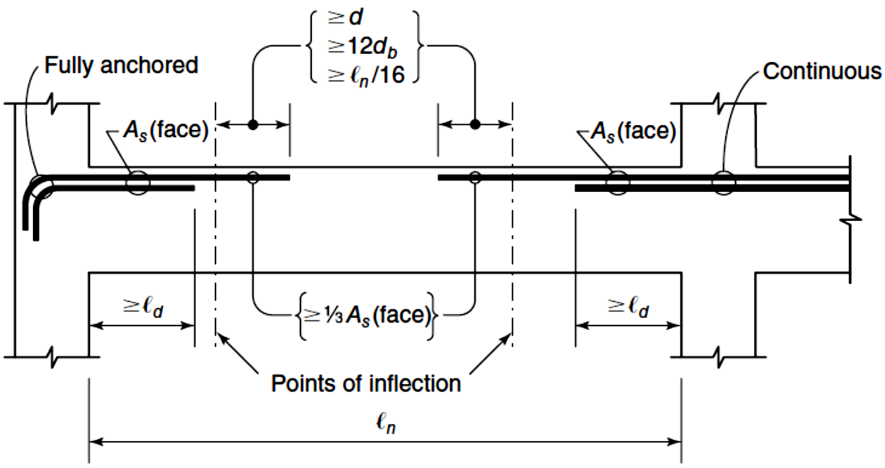


Figure 5.3: Continuity requirements for negative-moment reinforcement in continuous beams

Example 5.1:

Given:

- Materials: F'c= 28MPa fy= 420MPa
- Cross section: rectangle: b= 350mm, h= 700mm, d= 630mm
- Support width= 0.50m
- Structural model of beam is shown in Figure 5.4 below. The self-weight of beam is included in Wu.

Design the beam for flexure and compute the required bars lengths.

Solution:

From structural analysis:

- Reaction at Left support, R1= 424.11kN
- Reaction at Right support, R2= 787.62kN
- Distance to maximum positive moment from left support= 2.73m, shear is zero.
- Distance to inflection point = 5.46m, moment is zero at this point.

Let x is the distance from left support to maximum positive moment, so:

-155.35 X + 424.11=0.0, this gives x= 2.73m, then take the moment at location of x, so:

$Mu= 424.11(2.73)-155.35(2.73)^2/2= 578.92kN.m$

Determine location of inflection point:

$424.11X – 155.35 X^2/2 = 0$, this gives X= 5.46m

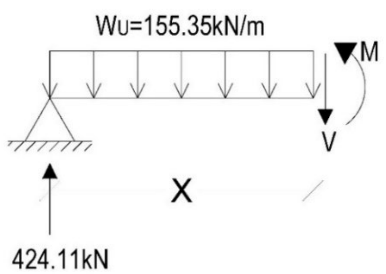


Figure 5.4: Beam for Example 5.1

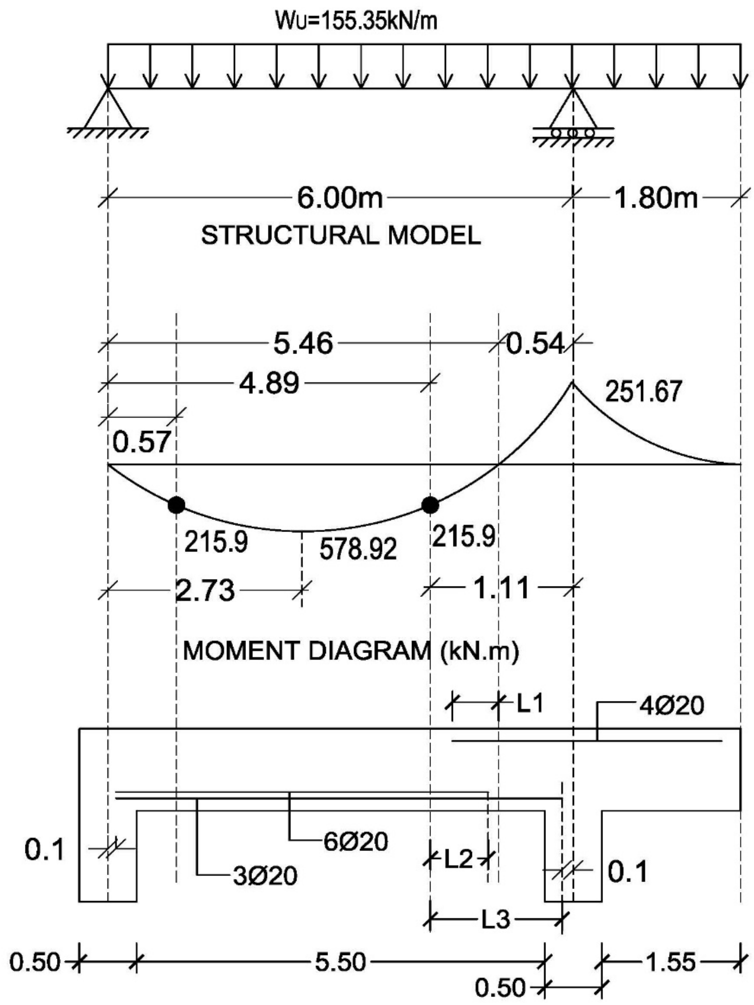


Figure 5.4 continued: Beam for Example 5.1

Section reinforcement:

- Maximum positive moment:

Mu= 578.92kN.m ρ= 0.01235 > ρ_{min}= 0.00333 and less than $\rho_{max,\ singly}$= 0.01548

As= 0.01235(350)(630)= 2723mm^2 9Ø20

- Maximum negative moment:

Mu= 251.67kN.m ρ= 0.005 > ρ_{min}= 0.00333 and less than $\rho_{max,\ singly}$= 0.01548

As= 0.005(350)(630)= 1103mm^2 4Φ20

Distance, L1:

L1 ≥ d= 0.63m

L1 ≥ 12 db = 12(20/1000)= 0.24m

L1 ≥ Ln/16 = 5.5/16 = 0.344m so, **L1= 0.63m**

Length of top bars= 1.8 + 0.54 – 0.04 + 0.63 = 2.93m (use 3m)

Distances L2 and L3 for bottom bars:

Three bars $\emptyset 20mm$ (1/3 As) shall be extended into the supports a distance not less than 150mm. The moment capacity of these three bars are calculated as follows:

$$A_s = 3(314) = 942mm^2$$

$$a = \frac{942(420)}{0.85(28)(350)} = 47.5mm \qquad \emptyset = 0.9$$

$$\emptyset M_n = \frac{0.90(942)(420)\left(630 - \frac{47.5}{2}\right)}{10^6} = 215.9kN.m$$

By applying summation of moment about a point with distance X from left support equals to 215.9kN.m, the value of X will be calculated as follows:

215.9= 424.11 X – 155.35 $X^2/2$, X_1= 0.57m, X_2= 4.89m

The length of extended bottom bars is = 5.50 + 2 (0.15) = 5.80m

The short bars (6Ø20) shall be extended beyond X1 and X2 a distance equals to the larger of 12db and d. The value of d is larger than the value of db, so, L2= d= 0.63m

The length of cut-off (short) bars is = 4.89 -0.10+0.63 = 5.42m, the left side of these bars starts extends 0.15m into the left support. So, these bars start from the same point as the extended bars (3Ø20).

The extended bars (3Ø20) shall be extended a distance L3 beyond the theoretical cut-off point which is located at a distance 4.89m from the left support.

L3 is equal to the development length of $\emptyset 20mm$ bars which is given by:

$$l_{dt} = \frac{0.59(420)}{\sqrt{28}}(20mm) = 937mm < 1.01m \quad ok.$$

The distance from the point of Mu= 215.9kN.m to the right support is equal to 6-4.89= 1.11m.

The extended bars (3Ø20) are extended into the support a distance equal to 0.15m, so the right end of these bars is located a distance 0.10m from the center of the right support, so, the available distance from the theoretical cut-off point for Mu= 215.9kN.m to the end of extended bars 3Φ20 will be 1.11-0.1=1.01m which is larger than the development length 0.937m and this is adequate based on code specifications. Structural integrity can be achieved by extending the 3Ø20 Ldt or Ldh into the supports.

5.9 Termination of flexural bars:

Flexural tension reinforcement shall not be terminated in a tension zone unless (a), (b) or (c) are satisfied:

(a) $V_u \leq (2/3)\emptyset V_n$ at the cut-off point.

(b) For $\emptyset 35mm$ bars and smaller, continuing reinforcement provides double the area required for flexure at the cut-off point and $V_u \leq (3/4)\emptyset V_n$.

(c) Stirrup or hoop area in excess of that required for shear and torsion is provided along each terminated bar over a distance $(3/4)d$ from the cut-off point. Excess stirrup or hoop area shall be at least $0.41 b_w S/f_{yt}$. Spacing S shall not exceed $d/8\beta_b$. Where β_b is ratio of area reinforcement cut-off to total area of tension reinforcement at section.

For simplicity. It is recommended to consider the first requirement in practice.

5.10: Reinforcement continuity and structural integrity requirements:

The primary purpose for both the continuity and structural integrity reinforcement requirements is to tie the structural elements together and prevent localized damage from spreading progressively to other parts of the structure. However, because of the limited amount of calculations required to select and detail this reinforcement, structures satisfying these requirements cannot be said to have been designed to resist progressive collapse.

Joists:

For joist construction, code requires that at least one bottom bar shall be continuous over all spans and through interior supports and shall be anchored to develop f_y at the face of exterior supports. Continuity of the bar shall be achieved with either a Class B tension lap splice or a mechanical or welded splice satisfying ACI Code specifications.

One-way slabs:

At least one-quarter of the maximum positive moment reinforcement shall be continuous. Reinforcement at noncontinuous supports shall be anchored to develop f_y at the face of the support.

Beams:

1. The continuous top reinforcement shall consist of at least one-sixth of the negative-moment (top) reinforcement required at the face of the support, but shall not be less than two bars.
2. The continuous bottom reinforcement shall consist of at least one-fourth of the positive-moment (bottom) reinforcement required at midspan, but not less than two bars.

3. At noncontinuous supports (corners), all of the continuous bars must be anchored to develop f_y at the face of the support.
4. All of the continuous longitudinal bars must be enclosed by closed transverse reinforcement, as specified for torsional or integrity transverse reinforcement (Ends shall terminate with 135-degree standard hooks) around a longitudinal bar, and placed over the full clear span at a spacing not exceeding $d/2$.
5. For interior beams, at least one-fourth of the positive-moment (bottom) reinforcement required at midspan, but not less than two bars shall be continuous. Longitudinal reinforcement shall be enclosed by closed stirrups in accordance with 25.7.1.6 or hoops along the clear span of the beam.
6. If splices are necessary in continuous structural integrity reinforcement, positive moment reinforcement shall be spliced at or near the support and negative moment reinforcement shall be spliced at or near midspan.

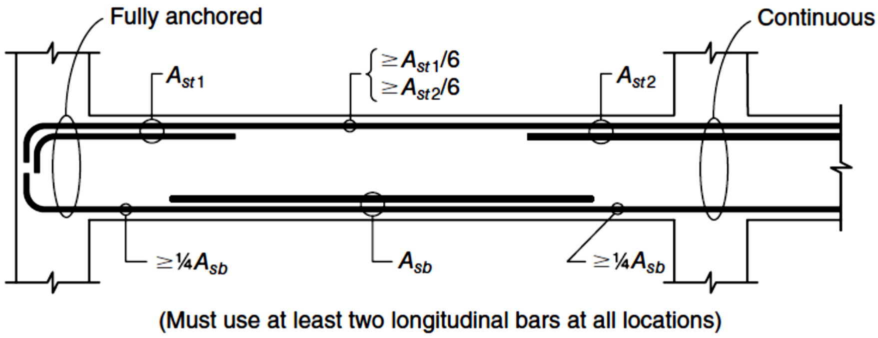


Figure 5.5: Requirements for longitudinal structural-integrity reinforcement in perimeter beams. (*Note:* required closed transverse reinforcement not shown.)

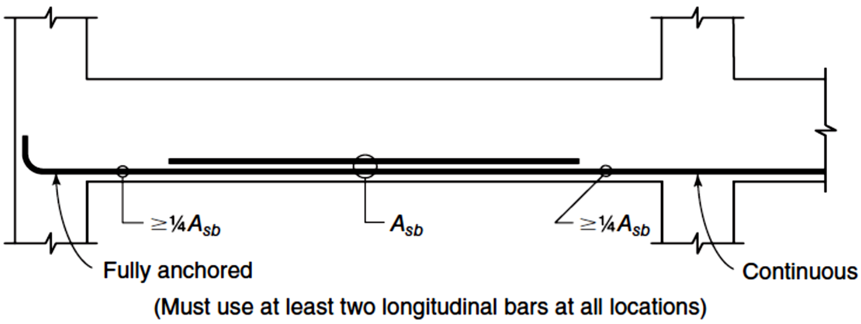


Figure 5.6: Requirements for longitudinal structural-integrity reinforcement for interior beams framing into columns. (*Note:* required closed transverse reinforcement not shown.)

Chapter 6: Combined Compression and Bending

A column is a vertical structural member supporting axial compressive loads, with or without moments. The cross-sectional dimensions of a column are generally considerably less than its height. Columns support vertical loads from the floors and roof and transmit these loads to the foundations.

The more general terms compression members and members subjected to combined axial load and bending are sometimes used to refer to columns, walls, and members in concrete trusses or frames.

6.1 Types of columns:

- **Form or shape:** Rectangle, circle, irregular, composite, ….

- **Position of loads:** concentric and eccentric

- **Mode of failure:** nonslender (short) and slender (long)

Short column: failure starts by crushing of concrete, yielding of steel or both.

Long column: failure starts by buckling.

The term that is used to differentiate between short and long columns is called the slenderness ratio, KLu/r, where:

K: effective length factor

Lu: unsupported height of column

R: radius of gyration which is the square root of the section moment of inertia divided by the section area.

6.2 Column reinforcement:

1. Longitudinal reinforcement shall be at least 0.01Ag but shall not exceed 0.08Ag.

2. Minimum number of bars in a rectangular tied column is four

3. Minimum number of bars in a circular column is six

4. Lateral reinforcement is required to prevent spalling of the concrete cover or local buckling of longitudinal bars. The vertical spacing of ties are the smaller of 48 ds, 16 db and the column least dimension.

Where:

ds: diameter of the tie.

db: diameter of the longitudinal bars.

The ties shall be so arranged that every corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie having an included angle of not more than 135 degrees, and no bar shall be farther than 150mm clear on either side from such a laterally supported bar. Longitudinal bars spaced more than 150mm apart should be supported by lateral ties.

The bottom tie or hoop shall be located not more than one-half the tie or hoop spacing above the top of footing or slab.

ACI 318-19 section 10.7.4 Offset bent longitudinal reinforcement:

ACI 318-19 section 10.7.4.1: The slope of the inclined portion of an offset bent longitudinal bar relative to the longitudinal axis of the column shall not exceed 1 in 6. Portions of bar above and below an offset shall be parallel to axis of column.

ACI 318-19 section 10.7.4.2: If the column face is offset 75 mm or more, longitudinal bars shall not be offset bent and separate dowels, lap spliced with the longitudinal bars adjacent to the offset column faces, shall be provided.

ACI 318-19 section 10.7.6.4: Lateral support of offset bent longitudinal bars:

ACI318-19 section 10.7.6.4.1: Where longitudinal bars are offset, horizontal support shall be provided by ties, hoops, spirals, or parts of the floor construction and shall be designed to resist 1.5 times the horizontal component of the calculated force in the inclined portion of the offset bar.

ACI 318-19 section 10.7.6.4.2: If transverse reinforcement is provided to resist forces that result from offset bends, ties, hoops, or spirals shall be placed not more than 150 mm from points of bend.

Bars in compression shall be enclosed by transverse ties, at least No. 10 in size for longitudinal bars No. 32 or smaller, and at least No. 13 in size for No. 36, No. 43, No. 57, and bundled longitudinal bars.

Note: It is recommended to have bars with spacing (clear) not larger than 150mm.

6.3 Strength of nonslender concentrically loaded column:

The column capacity is given by:

$$\emptyset P_n = \emptyset\lambda\left(0.85f'_c\left(A_g - A_s\right) + f_y A_s\right)$$

Where:

Ø: strength reduction factor. Ø=0.65 for tied column. Ø=0.75 for spiral column.

λ: factor to consider minimum eccentricity. λ= 0.8 for tied column. λ= 0.85 for spiral column.

A_g: gross section area, mm^2

A_s: area of longitudinal steel, mm^2

F'c and fy are in MPa

So, $\emptyset P_n$ will be in N. The value can be divided by 1000 to have $\emptyset P_n$ in kN.

Example 6.1:

Design a rectangular tied column of h=2b that can carry an ultimate axial compression force, Pu= 5000kN. F'c= 28MPa. Fy= 420MPa. b and h are the column sides. Assume that the steel ratio is around 0.02.

Solution:

Steel ratio, ρ= 0.02.

$$A_s = \rho A_g = 0.02A_g$$

$$P_u = \emptyset P_n = \emptyset\lambda\left(0.85f'_c\left(A_g - A_s\right) + f_y A_s\right)$$

$$5000(1000) = 0.65(0.80)\left(0.85(28)\left(A_g - 0.02A_g\right) + (420)\left(0.02A_g\right)\right)$$

So, Ag= 303.03x10^3 mm^2

A_g= bh= b (2b) , so, b= 389mm

So, use b= 400mm and h= 800mm, this gives steel ratio less than 0.02

Or, b= 350mm and h= 700mm and this gives steel ratio greater than 0.02

Try: b= 400mm and h= 800mm

$$5000(1000) = 0.65(0.80)\left(0.85(28)(400x800 - A_s) + (420)(A_s)\right)$$

This gives, As= 5047mm^2, steel ratio= 5047/(400x800)= 0.016 < 0.02 ok

Try: 14Ø22 As= 14 x 381= 5334mm², see Figure 6.1.

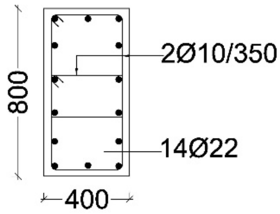


Figure 6.1: Column details- Example 6.1

Here, the spacing between bars can be calculated as follows:

Clear cover= 40mm

Tie diameter= 10mm

Distance from center of edge bar to edge of section= 40 + 10 + 11= 61mm

Distance available for bars center to center in short direction of column= 400-2(61)= 278mm

Distance between bars center to center= 278/2= 139mm < 150mm ok

Distance available for bars center to center in long direction of column= 800-2(61)= 678mm

Distance between bars center to center= 678/5= 136mm < 150mm ok

Ties:

Assume diameter of tie is 10mm

S ≤ 48 x 10 = 480mm

S ≤ 16 x 22 = 352mm

Least column dimension= 400mm

Use S= 350mm

And there is a need for two ties to fix bar after another with corners of ties.

6.4 Amount of spirals and spacing requirements:

The minimum spiral reinforcement required by the ACI code was chosen so that the second maximum load of the core and the longitudinal reinforcement would roughly equal the initial maximum load of the entire column before the shell spalled off.

Under a compressive load, the concrete in the column shortens longitudinally under the stress f1 and so, to satisfy poisson's ratio, it expands laterally. This lateral expansion is especially pronounced at stresses in excess of the cylinder strength. In the spiral column, the lateral expansion of the concrete inside the spiral (referred to as the core) is restrained by the spiral. This stresses the spirals in tension. For equilibrium, the concrete is subjected to compressive stresses f2. From experiment, the triaxial compression was shown to increase the strength of concrete by:

$$f1 = f'_c + 4.1f2 \qquad (1)$$

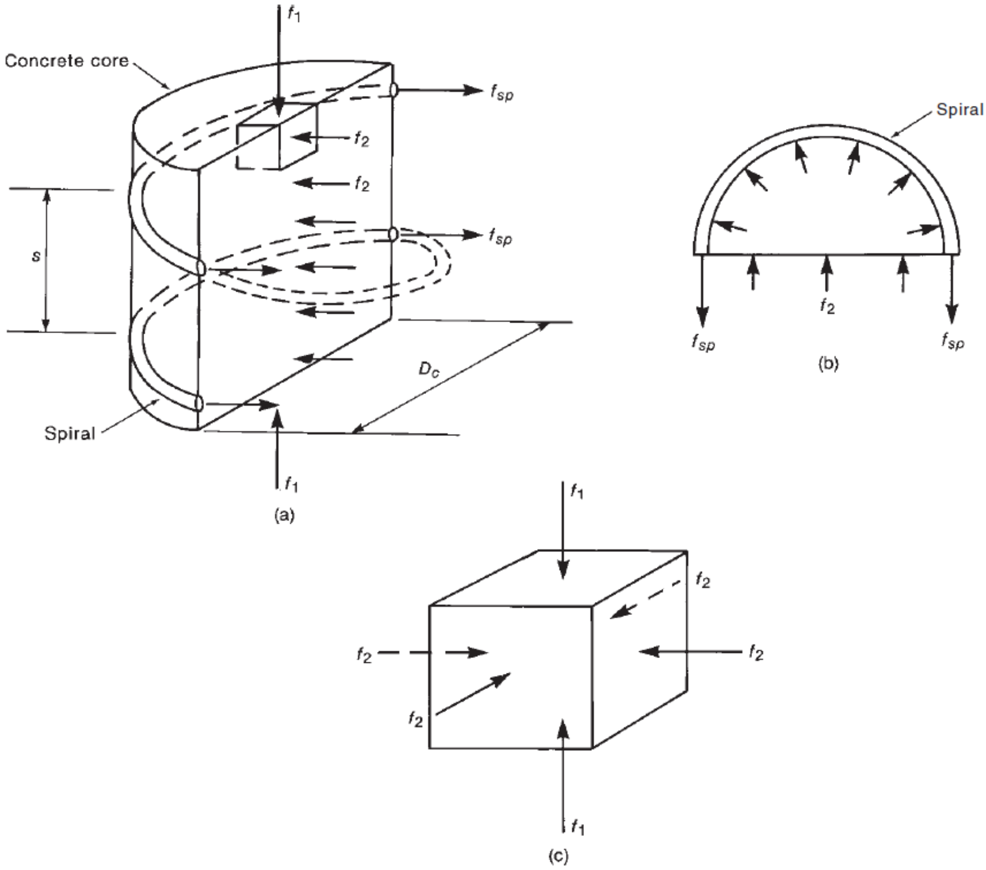


Figure 6.2: Triaxial stresses in core of spiral column

The amount of spiral reinforcement is defined by using a spiral reinforcement ratio, ρs equal to the ratio of the volume of the spiral reinforcement to the volume of the core, measured out-to-out of the spirals, enclosed by the spiral.

For one turn of the spiral,

$$\rho_s = \frac{A_{sp} l_{sp}}{A_{ch} l_c} \qquad (2)$$

Where:

Asp= area of the spiral bar= (3.14/4)$(dsp)^2$

dsp= diameter of the spiral bar

lsp= length of the turn of the spiral= 3.14 Dc

Dc= diameter of the core, out-to-out of the spirals

Ach= area of the core= 3.14 $(Dc)^2$ /4

Lc= spiral pitch= S

Thus,

$$\rho_s = \frac{A_{sp}\pi D_c}{\left(\frac{\pi {D_c}^2}{4}\right)S} \qquad (3)$$

Or

$$\rho_s = \frac{4A_{sp}}{SD_c} \qquad (4)$$

From the horizontal force equilibrium of the free body diagram,

$$2f_{sp}A_{sp} = f2D_cS \qquad (5)$$

From equations 4 and 5:

$$f2 = \frac{f_{sp}\rho_s}{2} \qquad (6)$$

The strength of a column at the first maximum load before the shell spalls off is:

$$P_o = \left(0.85f'_c\left(A_g - A_s\right) + f_yA_s\right) \qquad (7)$$

And the strength after the shell spalls is:

$$P_2 = \left(0.85f_1(A_{ch} - A_s) + f_yA_s\right) \qquad (8)$$

Thus, if P2= Po,

$$0.85f_1(A_{ch} - A_s) = 0.85{f'}_c\left(A_g - A_s\right)$$

Because As is small compared with Ag or Ach, it can be disregarded, so

$$f_1 = \frac{A_g f'_c}{A_{ch}} \qquad (9)$$

Substituting equations 1 and 6 into equation 9, taking fsp equal to the yield strength of the spiral bar, rearranging, and rounding down the coefficient gives:

$$\rho_s = 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f'_c}{f_{yt}} \qquad (10)$$

There is experimental evidence that more spiral reinforcement may be needed in high-strength concrete spiral columns than is given in this equation (10), to ensure that ductile behavior precedes any failure.

From equation 4,

Spiral spacing, S is given by:

$$s = \frac{4A_{sp}}{\rho_s D_c} \leq 75mm$$

Also, the clear spacing between the successive turns of the spiral must be spaced relatively close together.

Notes:

1. The clear spacing of the spirals is limited to 75mm
2. The minimum clear spacing of the spirals is not less than 25mm but not less than (4/3) times the nominal size of the coarse aggregate, whichever is greater
3. Spirals shall not be less than 10mm diameter
4. Anchorage of spiral reinforcement shall be provided by 1.5 turns of spiral bar at each end of a spiral unit
5. Spiral reinforcement shall be spliced if needed. Lap splices not less than the larger of 300mm and 48 times the spiral diameter for deformed uncoated bars should be used.

Example 6.2:

Design a spiral column that can carry an ultimate axial compression force, Pu= 5000kN. F’c= 28MPa. Fy= 420MPa. Assume that the steel ratio is around 0.02.

Solution:

Steel ratio, ρ= 0.02

$$P_u = \emptyset P_n = \emptyset \lambda\left(0.85f'_c\left(A_g - A_s\right) + f_y A_s\right)$$

$$5000(1000) = 0.75(0.85)\left(0.85(28)\left(A_g - 0.02A_g\right) + (420)\left(0.02A_g\right)\right)$$

So, Ag= 247230 mm^2

1.14 D^2/4 = 247230, so D= 561mm

Try column with D= 550mm

Resolve and determine As.

As= 5524mm^2

Steel ratio, ρ= 0.0233 12Φ25

Spirals:

$$\rho_s = 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f'_c}{f_{yt}} = 0.45\left(\frac{(\frac{3.14}{4})(550)^2}{(\frac{3.14}{4})(470)^2} - 1\right)\frac{28}{420} = 0.01108$$

$$s = \frac{4A_{sp}}{\rho_s D_c} = \frac{4(78.5)}{0.01108(470)} = 60.3mm \le 75mm$$

Use spirals Φ10mm/60mm

6.5 Interaction diagrams for reinforced concrete columns:

Interaction diagrams for columns are generally computed by assuming a series of strain distribution, each corresponding to a particular point on the interaction diagram, and computing the corresponding values of P and M. Once enough such points have been computed, the results are plotted as interaction diagram.

Significant points on the column interaction diagram:

1. Point A: pure axial load: This is the largest axial load the column can carry. The maximum usable axial load is limited to 0.8 and 0.85 times the pure axial load capacity for tied and spiral columns respectively.
2. Point B: zero tension: onset of cracking: The strain distribution at this point corresponds to axial load and moment at the onset of crushing of the concrete just as the strains in the concrete on the opposite face of the column reach zero. This represents the onset of cracking of the least compressed side of the column. Here, $\epsilon_t = 0.0 \ or \ \epsilon_{s1} = 0.0$

3. Point C: Balanced failure: compression – controlled limit strain: This point corresponds to a strain distribution with a maximum compressive strain of 0.003 on one face of the section, and a tensile strain equals to the yield strain, in the layer of reinforcement farthest from the compression face of the column. Here, $\epsilon_t = \epsilon_y or\ \epsilon_{s1} = \epsilon_y$
4. Point D: Tension – controlled limit: This point corresponds to a strain distribution with a maximum compressive strain of 0.003 on one face of the section, and a tensile strain equals to 0.005, in the layer of reinforcement farthest from the compression face of the column. Here, $\epsilon_t = 0.005 or\ \epsilon_{s1} = 0.005$
5. Point E: strain limit for beams: here, the column section will have an axial capacity of zero. So, the tension force will be equal to the compression force as the case in beams. By applying this principle, the moment capacity will be calculated. One can use , $\epsilon_t = 4\ \epsilon_y or\ \epsilon_{s1} = 4\epsilon_y$ as approximation method to get the moment capacity for axial load equals to zero. One can use the procedure that was used in doubly reinforced beam sections
6. Point F: column tension capacity:

$$\emptyset P_n = \emptyset A_s f_y \quad where\ A_s\ is\ the\ total\ area\ of\ steel\ in\ the\ cross\ section$$

6.6 Derivation of computation method for interaction diagrams:

The relationships needed to compute the various points on the interaction diagram are derived by using strain compatibility and mechanics.

The general case of computations involves the calculation of Pn acting at the centroid and Mn acting about the centroid of the gross cross section, for an assumed strain distribution with crushing strain in concrete, $\epsilon_{cu} = 0.003$.

Each layer of reinforcement in column section has an area of steel named As, so there are As1, As2, As3, …….. and each layer has a strain ϵ_s, so there are $\epsilon_{s1}, \epsilon_{s2}, \epsilon_{s3}, \dots \dots$

Layer 1 is closest to the “least compressed” surface and it is at a distance d1 from the “most compressed” surface. Layer 1 is called the extreme tension layer. It has a depth d1 and a strain ϵ_{s1}

The interaction diagram can be controlled by selecting a series of values for the neutral axis depth, c. Large values of c will give points high in the interaction diagram and low values of c will give points low in the interaction diagram.

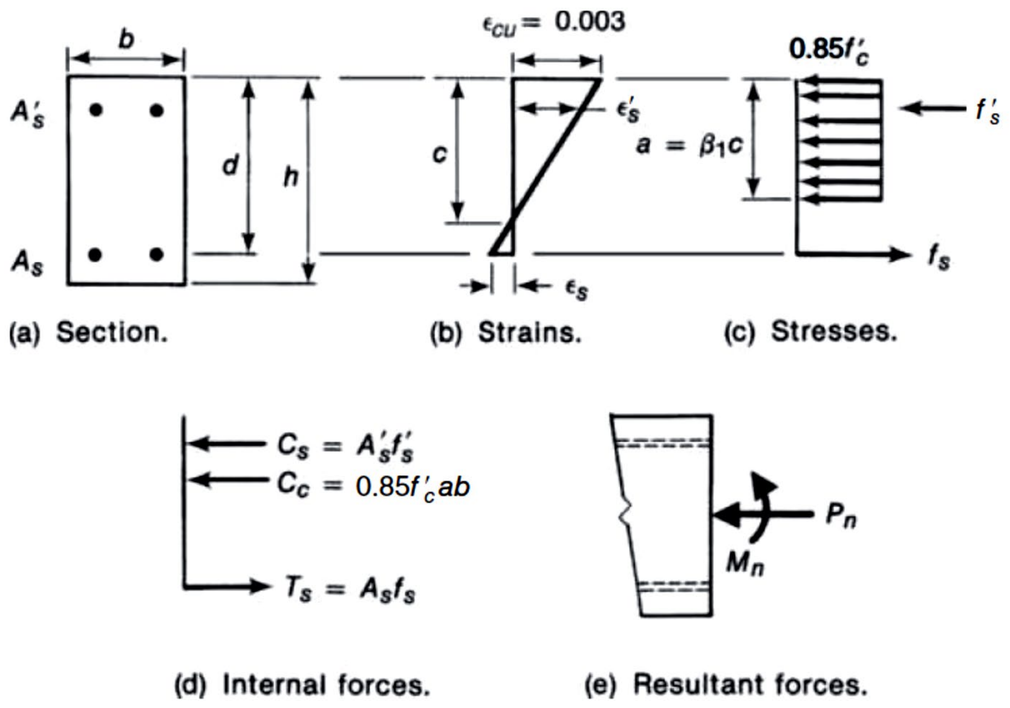


Figure 6.3: Column section forces

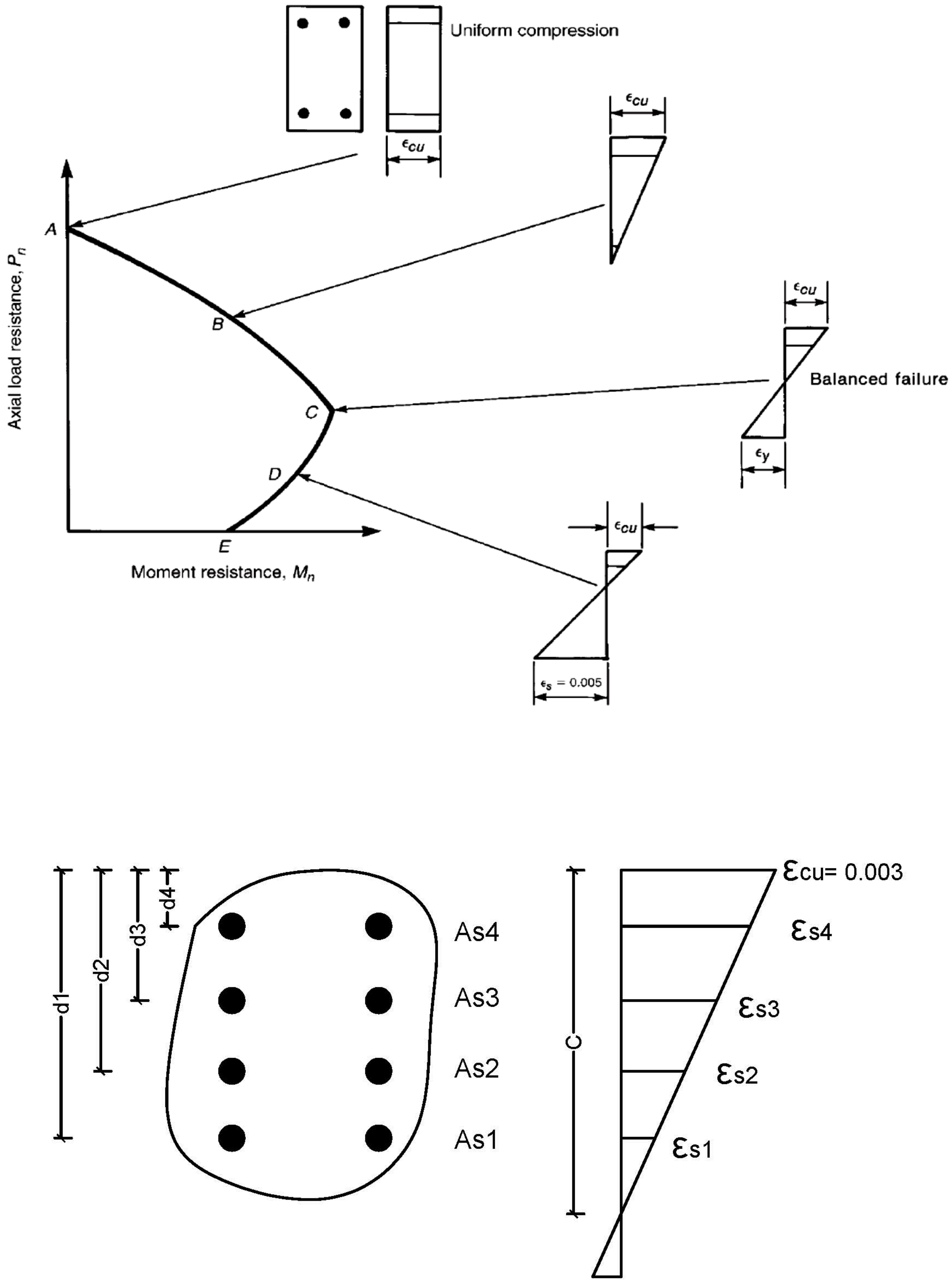


Figure 6.4: Computations of column interaction diagram points

From similar triangles:

$$\frac{0.003}{c} = \frac{\epsilon_{si}}{c - d_i}$$

$$c = \frac{0.003}{0.003 - \epsilon_{s1}} d_1$$

$$\epsilon_{si} = \frac{c - d_i}{c} 0.003$$

$$\epsilon_{s1}: poisitive\ for\ compression\ strain$$

$$\epsilon_{s1}: negative\ for\ tension\ strain$$

So, the strain in each layer of reinforcement is determined from the strain diagram.

Then, the stress in each layer of reinforcement is determined by:

$$f_{si} = \epsilon_{si} E_s$$

The maximum absolute value of fsi shall be not larger than fy.

The relation between the depth of the compression zone and the neutral axis of the section is given by:

$$a = \beta_1 c$$

$$\beta_1 = 0.85 \quad for \quad 17MPa \le f'_c \le 28MPa$$

$$\beta_1 = 0.85 - 0.05 \frac{f'_c - 28}{7} \quad for \quad 28MPa < f'_c < 56MPa$$

$$\beta_1 = 0.65 \quad for \quad 56MPa \le f'_c$$

The compressive force in the concrete is given by:

$$C_c = 0.85 f'_c A_{cc}$$

For a rectangular zone, Acc= b a

The force in each layer of reinforcement is determined as follows:

If the depth of the compression zone, a less than depth of reinforcement layer, di, then

$$F_{si} = f_{si} A_{si}$$

And if the depth of the compression zone, a is greater than or equals to the depth of reinforcement layer, di, then

$$F_{si} = (f_{si} - 0.85f'_c)A_{si}$$

The nominal axial load capacity, Pn for assumed stress distribution is the summation of the axial forces:

$$P_n = C_c + \sum_{i=1}^{n} F_{si}$$

The nominal moment capacity, Mn for the assumed strain distribution is found by summing the moments of all internal forces about the centroid of the column. So,

$$M_n = C_c(y^- - a^-) + \sum_{i=1}^{n} F_{si}(y^- - d_i)$$

Where y^- is the distance from the extreme compression fiber to the centroid of the section.

Example 6.3:

Draw the moment-axial force interaction diagram for the column shown in Figure 6.5 below.

Given: f'c= 28MPa fy= 420MPa

Cover to bars centroid= 60mm.

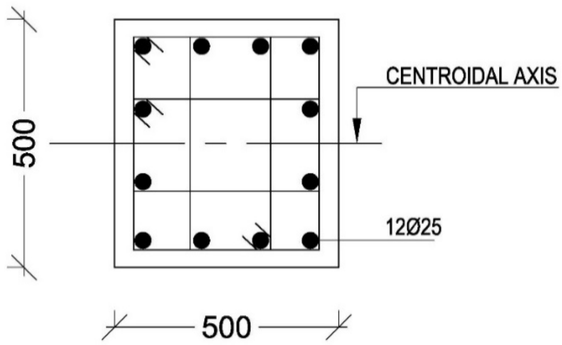


Figure 6.5: Column cross section for Example 6.3

Solution:

As= 12(491) = 5892 mm^2

Point A:

Pure compression axial force capacity.

$$P_n = \left(0.85f'_c\left(A_g - A_s\right) + f_yA_s\right) = \frac{[0.85(28)(500x500 - 5892) + 420(5892)]}{1000}$$
$$= 8284.4kN$$

$$\emptyset P_n = 0.65(8284.4) = 5384.86kN$$

$$\lambda\emptyset P_n = 0.8(5384.86) = 4307.89kN$$

Point B:

Pn and Mn at strain in tensile steel, $\epsilon_{s1} = 0.0$

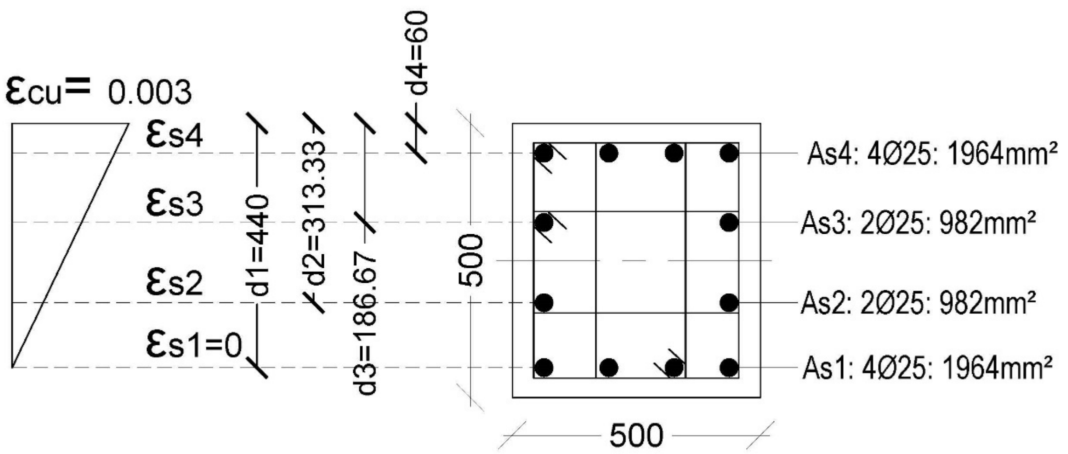


Figure 6.6: Section and strains for point B

C= 440mm

$$a = \beta_1 c = 0.85(440) = 374mm$$

$$C_c = \frac{0.85(28)(500)(374)}{1000} = 4450.6kN$$

Fs1= 0.0kN

Fs2:

$$\epsilon_{s2} = \frac{c - d_2}{c} 0.003 = \frac{440 - 313.33}{440} 0.003 = 0.0008636$$

$$f_{s2} = \epsilon_{s2}\, E_s = 0.0008636(200000) = 172.72 Mpa$$

Since a>d2:

$$F_{s2} = (f_{s2} - 0.85 f'_c) A_{s2} = (172.72 - 0.85x28)(982)/1000 = 146.24 kN$$

Fs3:

$$\epsilon_{s3} = \frac{c - d_3}{c} 0.003 = \frac{440 - 186.67}{440} 0.003 = 0.001727$$

$$f_{s3} = \epsilon_{s3}\, E_s = 0.001727(200000) = 345.4 Mpa$$

Since a>d3:

$$F_{s3} = (f_{s3} - 0.85 f'_c) A_{s3} = (345.4 - 0.85x28)(982)/1000 = 315.81 kN$$

Fs4:

$$\epsilon_{s4} = \frac{c - d_4}{c} 0.003 = \frac{440 - 60}{440} 0.003 = 0.002591$$

$$f_{s4} = \epsilon_{s4}\, E_s = 0.002591(200000) = 518.2 Mpa\,, use\ f_{s4} = 420 MPa$$

Since a>d4:

$$F_{s4} = (f_{s4} - 0.85 f'_c) A_{s4} = (420 - 0.85x28)(1962)/1000 = 778.1 kN$$

$$P_n = C_c + \sum_{i=1}^{n} F_{si} = 4450.60 + 0.0 + 146.24 + 315.81 + 778.1 = 5690.75 kN$$

$$M_n = C_c(y^- - a^-) + \sum_{i=1}^{n} F_{si}(y^- - d_i)$$
$$= C_c\left(\frac{h}{2} - \frac{a}{2}\right) + F_{s1}\left(\frac{h}{2} - d1\right) + F_{s2}\left(\frac{h}{2} - d2\right) + F_{s3}\left(\frac{h}{2} - d3\right) + F_{s4}\left(\frac{h}{2} - d4\right)$$

Mn= [4450.6(250-374/2)+ 0.0 + (146.24)(250-313.33)+315.81(250-186.67)+778.1(250-60)]= 439kN.m

$$\emptyset P_n = 0.65(5690.75) = 3699kN$$

$$\emptyset M_n = 0.65(439) = 285.35kN$$

Point C:

Pn and Mn at strain in tensile steel, $\epsilon_{s1} = -\epsilon_y = -0.0021$

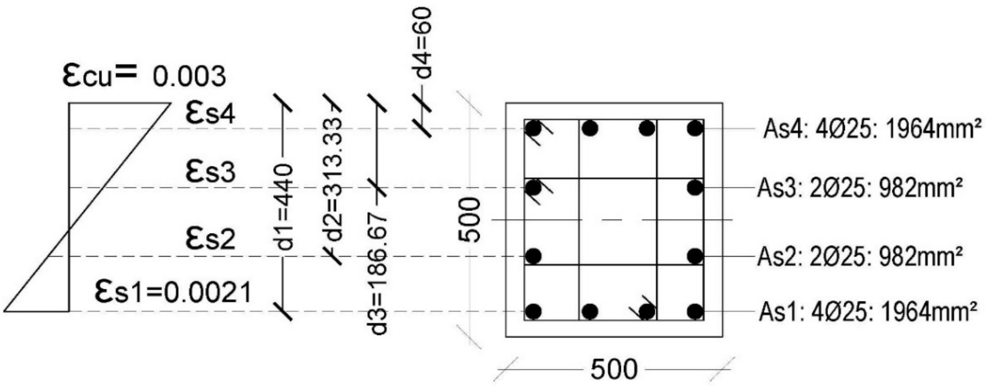


Figure 6.7: Section and strains for point C

$$c = \frac{0.003}{0.003 - \epsilon_{s1}} d_1 = \frac{0.003}{0.003 + 0.0021} 440 = 258.8kN$$

$$a = \beta_1 c = 0.85(258.8) = 220mm$$

$$C_c = \frac{0.85(28)(500)(220)}{1000} = 2618kN$$

Fs1:

$$\epsilon_{s1} = -0.0021$$

$$f_{s1} = -420MPa$$

$$F_{s1} = f_{s1} A_{s1} = -\frac{420(1964)}{1000} = -824.88kN$$

Fs2:

$$\epsilon_{s2} = \frac{c - d_2}{c} 0.003 = \frac{258.8 - 313.33}{258.8} 0.003 = -0.000632$$

$$f_{s2} = \epsilon_{s2} E_s = -0.000632(200000) = -126.4Mpa$$

Since a<d2:

$$F_{s2} = f_{s2}A_{s2} = \frac{(-126.4)(982)}{1000} = -124.1kN$$

Fs3:

$$\epsilon_{s3} = \frac{c - d_3}{c} 0.003 = \frac{258.8 - 186.67}{258.8} 0.003 = 0.000836$$

$$f_{s3} = \epsilon_{s3}\, E_s = 0.000836(200000) = 167.2Mpa$$

Since a>d3:

$$F_{s3} = (f_{s3} - 0.85f'_c)A_{s3} = \frac{(167.2 - 0.85x28)(982)}{1000} = 140.82kN$$

Fs4:

$$\epsilon_{s4} = \frac{c - d_4}{c} 0.003 = \frac{258.8 - 60}{258.8} 0.003 = 0.0023$$

$$f_{s4} = \epsilon_{s4}\, E_s = 0.0023(200000) = 460Mpa \quad use\ f_{s4} = 420MPa$$

Since a>d4:

$$F_{s4} = (f_{s4} - 0.85f'_c)A_{s4} = \frac{(420 - 0.85x28)(1964)}{1000} = 778.1kN$$

$$P_n = C_c + \sum_{i=1}^{n} F_{si} = 2618 + (-824.88) + (-124.1) + (140.82) + (778.1) = 2587.94kN$$

$$M_n = C_c(y^- - a^-) + \sum_{i=1}^{n} F_{si}(y^- - d_i)$$
$$= C_c\left(\frac{h}{2} - \frac{a}{2}\right) + F_{s1}\left(\frac{h}{2} - d1\right) + F_{s2}\left(\frac{h}{2} - d2\right) + F_{s3}\left(\frac{h}{2} - d3\right) + F_{s4}(\frac{h}{2} - d4)$$

Mn= [2618(250-220/2)+ (-824.88)(250-440) + (-124.1)(250-313.33)+140.82(250-186.67)+778.1(250-60)]/1000= 687.86kN.m

$$\emptyset P_n = 0.65(2587.94) = 1682.2kN$$

$$\emptyset M_n = 0.65(687.86) = 447.1kN$$

Point D:

Pn and Mn at strain in tensile steel, $\epsilon_{s1} = -0.005$

C= 165mm

A= 140.25mm

Cc= 1669kN

Fs1= -824.88kN

Fs2= -412.44kN

Fs3=-77.38kN

Fs4= 703.5kN

Pn= 1057.8kN

Mn= 611.8kN.m

$$\emptyset P_n = 0.9(1057.8) = 952.02kN$$

$$\emptyset M_n = 0.9(611.8) = 550.62kN$$

Point E:

Pn and Mn at strain in tensile steel, $\epsilon_{s1} = -4x0.0021 = -0.0084$

Pn= 94.58kN

Mn= 494.2kN.m

$$\emptyset P_n = 0.9(94.58) = 85.1kN$$

$$\emptyset M_n = 0.9(494.2) = 444.78kN$$

Point F:

Tensile capacity of the column:

$$\emptyset P_n = -\frac{0.9(5892)(420)}{1000} = -2227kN$$

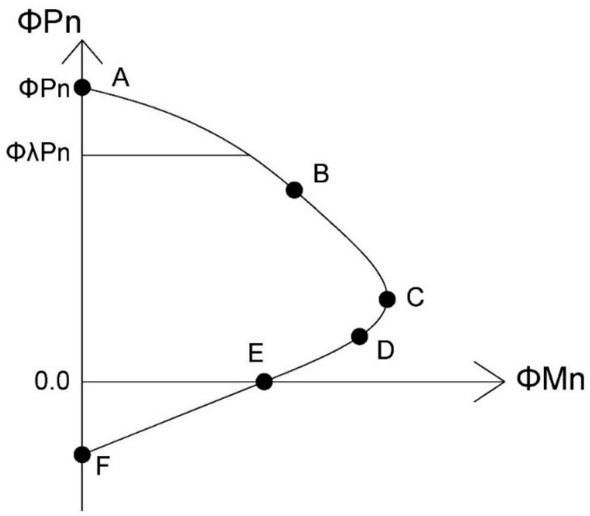


Figure 6.8: Moment- Axial force interaction diagram

6.7 Moment- Axial force interaction diagram sheets:

There are sheets for Moment- Axial force diagrams for rectangular and circular columns in appendices of many textbooks.

The main points of these sheets are as follows:

- Material strengths; f'c and fy
- Shape of section, rectangle or circle
- Type of reinforcement in rectangular column: distributed bars or bars at two edges only
- Factor gama, γ, which expresses the effectiveness of section to resist moment, this factor is given by:

$$\gamma = \frac{h - 2\ covers}{h}, the\ cover\ can\ be\ equal\ to: 40mm\ clear\ cover + 10mm\ diameter\ of\ tie + 10mm\ (half\ bar\ diameter) = 60mm$$

- Section dimensions
- The terms that determines the axial force and the bending moment which are:

$$\frac{\emptyset P_n}{bh} \quad and \quad \frac{\emptyset M_n}{bh^2}$$

The available sheets here are in units of ksi (kips per square inches), so to convert from Mpa to ksi, divide by 7.0.

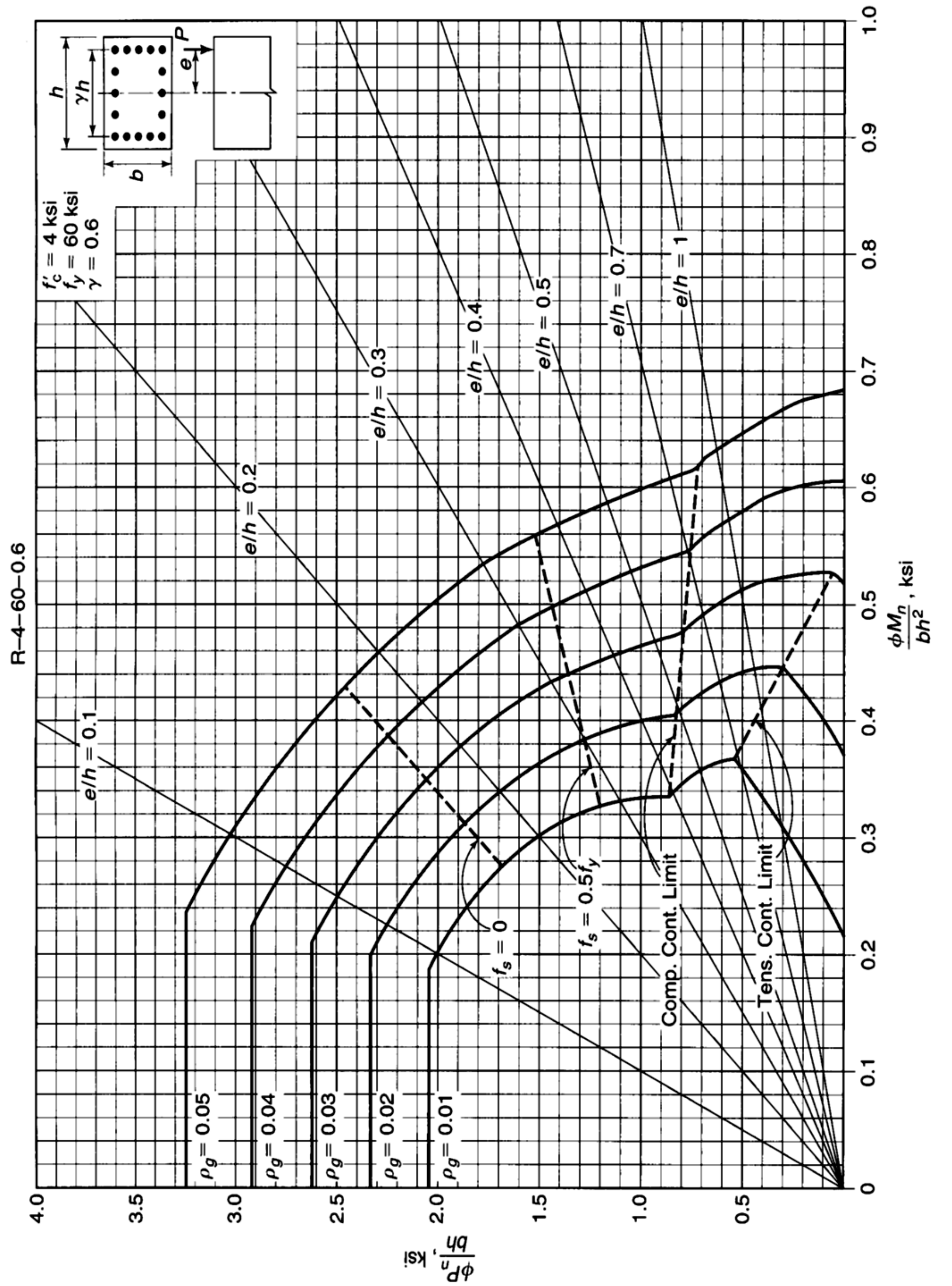


Figure 6.9: Moment- Axial force interaction diagram for gama= 0.60

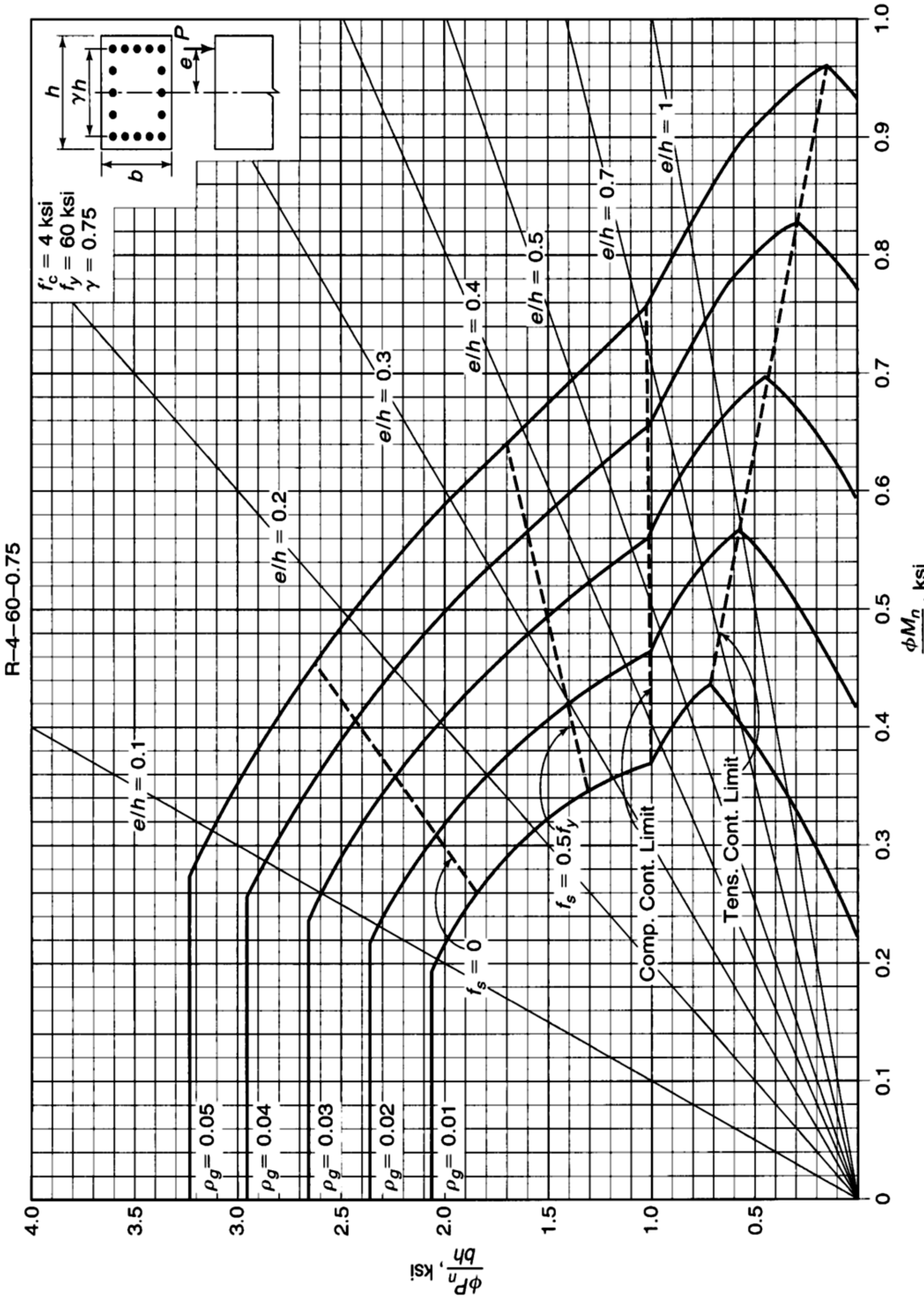


Figure 6.10: Moment- Axial force interaction diagram for gama= 0.75

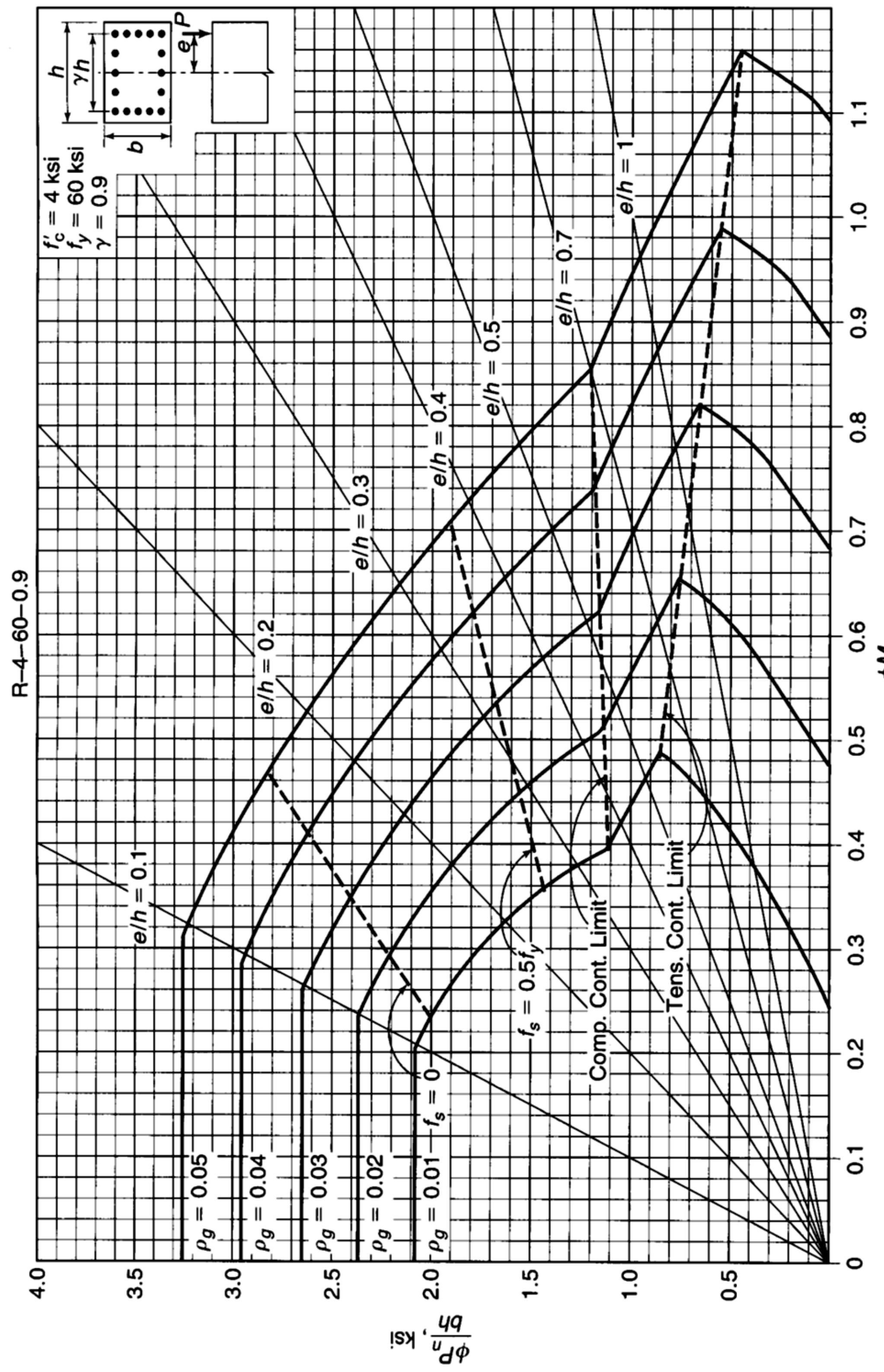


Figure 6.11: Moment- Axial force interaction diagram for gama= 0.90

Example 6.4:

Given:

F'c= 28MPa fy= 420MPa

Section: rectangle: b= 400mm h= 800mm

Loads: Pu= 383.3kN Mu= 690kN.m (The moment acts along h=800mm)

Determine the required area of longitudinal steel needed to resist these loads.

Solution:

$$\gamma = \frac{h - 2\,covers}{h} = \frac{800 - 2(60)}{800} = 0.85$$

$$\frac{\emptyset P_n}{bh} = \frac{383.3(1000)}{400(800)(7)} = 0.17ksi$$

$$\frac{\emptyset M_n}{bh^2} = \frac{690(10)^6}{400(800)^2(7)} = 0.39ksi$$

Using column moment- axial force interaction diagram sheets:

For $\gamma = 0.75 \rightarrow \rho = 0.017$

For $\gamma = 0.9 \rightarrow \rho = 0.014$

By interpolation for $\gamma = 0.85 \rightarrow \rho = 0.015$

As= 0.015(400)(800)= 4800mm^2 $16\emptyset 20$

Ties:

Spacing of ties:

$$S \leq 16d_b = 16(20) = 320mm$$

$$S \leq 48d_s = 48(10) = 480mm$$

$$S \leq least\ column\ section\ dimension = 400mm$$

So, use S= 300mm

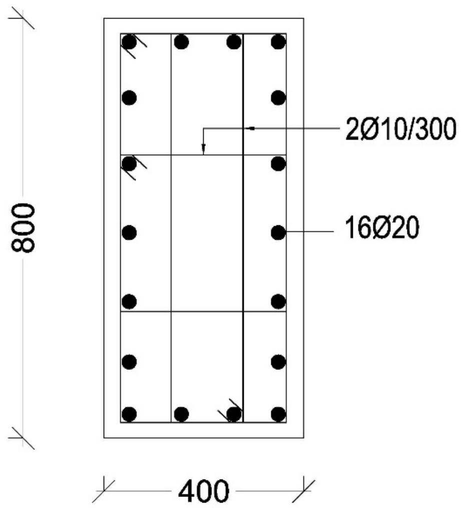


Figure 6.12: Reinforcement details for column in Example 6.4

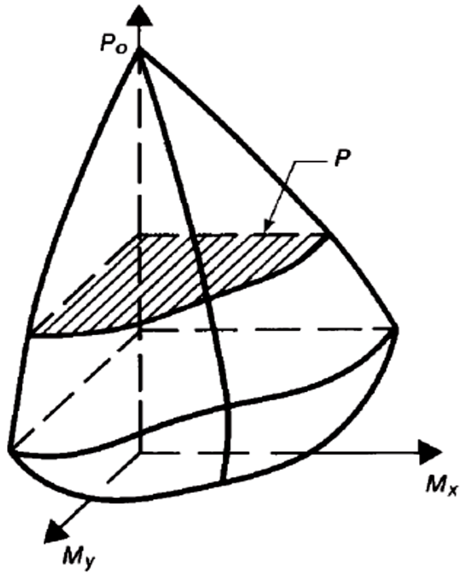
6.8 Design of biaxially loaded columns- simplified method:

It is not unusual for columns to support axial forces and bending about two perpendicular axes. One common example is a corner column in a structure.

For a given cross section and reinforcing pattern, one can draw an interaction diagram for axial load and bending about either principal axes. These two interaction diagrams form two edges of a three- dimensional interaction surface for axial load and bending about two axes. The calculation of each point on such a surface involves a double integration:

1. The strain gradient across the section is varied
2. The angle of the neutral axis is varied

A horizontal section through such a diagram resembles a quadrant of a circle or an ellipse at high axial loads, and depending on the arrangement of bars, it becomes considerably less circular near the balanced load.



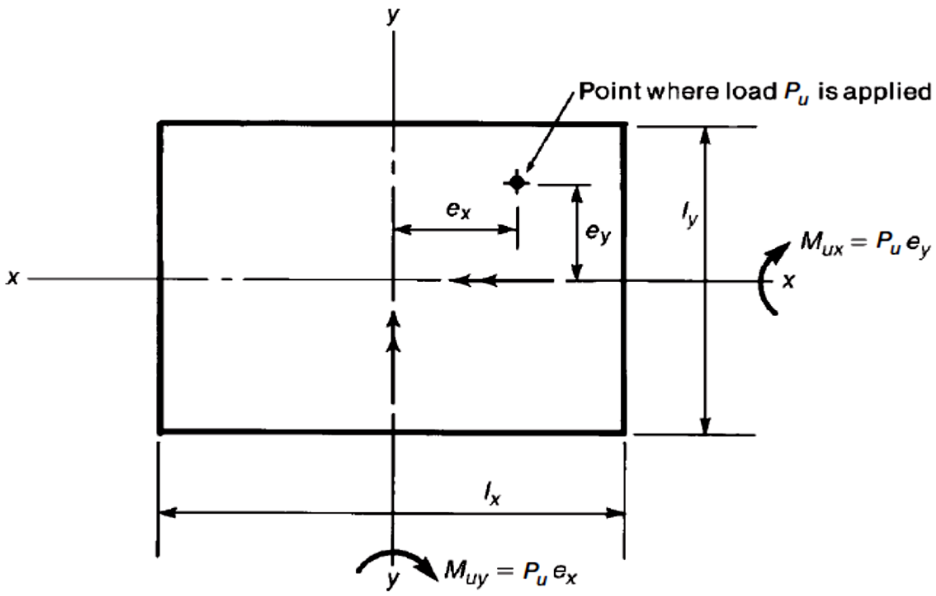


Figure 6.13: Interaction surface for axial load and biaxial bending.

A common simplified method for analysis and design of column section loaded biaxially, is Bresler Reciprocal Load Method.

ACI commentary gives the following equation, originally presented by Bresler, for calculating the capacity of a column under biaxial bending:

$$\frac{1}{\emptyset P_n} = \frac{1}{\emptyset P_{nx}} + \frac{1}{\emptyset P_{ny}} - \frac{1}{\emptyset P_{no}}$$

Definitions of variables:

Pu= factored axial load

ex= eccentricity of applied load measured parallel to the x- axis

ey= eccentricity of applied load measured parallel to y- axis

M_{ux}= factored moment about x-axis equals to Pu ey

M_{uy}= factored moment about x-axis equals to Pu ex

$\emptyset P_{nx}$ = reduced nominal axial load capacity for the moment about y- axis; Muy using e_x.

$\emptyset P_{ny}$ = reduced nominal axial load capacity for the moment about x- axis; Mux using e_y.

$\emptyset P_{no}$ = reduced nominal axial load capacity for ex and ey equal to zero

$\emptyset P_{n}$ = design axial load capacity for the moments about the two axes; Mux and Muy.

Lx= length of side of column section parallel to x- axis

Ly= length of side of column section parallel to y- axis

Example 6.5:

Given:

Column section: 400mm x 400mm

Longitudinal bars: 8Ø25

Loads: Pu= 1130kN Mux= 75kN.m Muy= 150kN.m

f'c= 28MPa fy= 420MPa

Concrete cover to bars centroid= 60mm

Check the adequacy of the column section to carry the applied loads

Solution:

Compute $\emptyset P_{no}$:

$$\emptyset P_{no} = \emptyset\lambda\left(0.85f'_c\left(A_g - A_s\right) + f_yA_s\right)$$
$$= \frac{(0.65)(0.8)[0.85(28)(400x400 - 3928) + 420(3928)]}{1000} = 2789kN$$

Compute $\emptyset P_{nx}$: Axial load capacity for e$_x$ (M_{uy})

Factored axial load capacity corresponding to e$_x$ and ρ.

$$\rho = \frac{3928}{400(400)} = 0.0246 = 0.025$$

$$\frac{e_x}{l_x} = \frac{M_{uy}}{P_u l_x} = \frac{150}{1130(0.4)} = 0.33$$

From the interaction diagram and for

$$\frac{e}{h} = \frac{e_x}{l_x} = 0.33 \; and \; \rho = 0.025$$

$$\frac{\emptyset P_n}{bh} = 1.3ksi \quad for\, \gamma = 0.6$$

$$\frac{\emptyset P_n}{bh} = 1.4ksi \quad for\, \gamma = 0.75$$

For

$$\gamma = \frac{400 - 120}{400} = 0.7 :$$

$$\frac{\emptyset P_n}{bh(7)} = 1.36ksi \quad \phi P_{nx} = \frac{bh(1.36)(7)}{1000} = 1523kN$$

Compute $\emptyset P_{ny}$: Axial load capacity for e$_y$ (M_{ux})

Factored axial load capacity corresponding to e$_y$ and ρ.

$$\rho = \frac{3928}{400(400)} = 0.0246 = 0.025$$

$$\frac{e_y}{l_y} = \frac{M_{ux}}{P_u l_y} = \frac{75}{1130(0.4)} = 0.166$$

From the interaction diagram and for

$$\frac{e}{h} = \frac{e_y}{l_y} = 0.166 \; and \; \rho = 0.025$$

$$\frac{\emptyset P_{ny}}{bh} = 2.1ksi \quad for \; \gamma = 0.6$$

$$\frac{\emptyset P_{ny}}{bh} = 2.2ksi \quad for \; \gamma = 0.75$$

For

$$\gamma = \frac{400 - 120}{400} = 0.7 \quad :$$

$$\frac{\emptyset P_{ny}}{bh(7)} = 2.16ksi \quad \phi P_{ny} = \frac{bh(2.16)(7)}{1000} = 2419kN$$

Apply the formula:

$$\frac{1}{\emptyset P_n} = \frac{1}{\emptyset P_{nx}} + \frac{1}{\emptyset P_{ny}} - \frac{1}{\emptyset P_{no}} = \frac{1}{1523} + \frac{1}{2419} - \frac{1}{2789}$$

So,

$$\phi P_n = 1406kN > 1130kN \qquad ok$$

So, the column section and reinforcement is adequate.

6.9 Loads on Columns:

The columns are subjected to axial forces and bending moments about the two principal axes of the cross section.

The column loads are computed using the following procedures:

1. Tributary area principle: the column carry axial load from the slab and from the direct loads which it is subjected to. The column carry dead loads from the slab, beams

weights, walls weights in addition to its self-weight. Also, it carries a live load from the slab. The tributary area is computed by taking half the distances to adjacent columns from all sides. This method is approximate and can be applied for approximate equal spans. It has large errors if the spans varied largely. The moments on the columns can be determined by using the ACI coefficients for moments.

2. Reactions from beams using ACI coefficients: the beams are supported by columns, so the reactions (vertical forces and moments) of the beams are loads on the columns. The vertical reaction at a support is equal to the summation of shear forces at the support. The end moments in the beam are moments on the columns.
3. Analysis of continuous beams: the reactions on the beam are loads on the columns. The end supports of the beam can be treated as hinges in a model and as fixed in another model and the average values of end moments will be applied to the columns.
4. Analysis of plane frames
5. Analysis of space frames

Many examples can be solved using the above principles.

Note:

Based on the course “Design of Reinforced Concrete I”, an additional chapter shall be covered which is the design of single concentric footings; footings which are subjected to axial compression force only.

Chapter 7: One Way Slab Systems

This chapter introduces the analysis and design of one-way slab systems; solid and ribbed in addition to beam- girder systems. Voided slabs are considered implicitly.

In one-way slabs, the load is assumed to be transferred in one direction. Usually, one-way slabs are concrete structures for which the ratio of the long span to the short span equals or exceeds a value of 2, when this ratio is less than 2, the floor panel becomes two-way slab. Pure one-way slabs are available when there are supports in one direction only or when the section moment of inertia in a direction is very large comparing with the section moment of inertia in the perpendicular direction, otherwise, all slabs can be treated as two way.

7.1 One-way solid slabs:

7.1.1 Basic principles:

A. A one-way solid slab is designed as singly reinforced 1000mm wide beam strip using the same design and analysis procedure for singly reinforced beams.
B. One-way solid slabs usually have a thickness that is adequate for shear strength; no shear reinforcement is used; $V_u \leq \emptyset V_c$. Generally, shear reinforcement can be used.
C. Transverse reinforcement has to be provided perpendicular to the direction of bending in order to resist shrinkage, temperature stresses and load distribution which is $0.0018A_g$, where A_g is the gross sectional area.
D. Preliminary thickness of one-way slabs and beams can be determined using ACI 318-19 code provisions (Minimum thickness of slabs and beams).
E. The exterior beams have L-sections and the interior beams have T-sections. The section effective width (flange width) for flexure is stated in ACI 318-19 code section 6.3.2.

F. Structural modeling:

Beams and slabs can be modeled as one-dimensional structures (line structure). Here, the slab and the beam are modeled as line (frame member) and with pined or hinged supports. In this model, the end moments (exterior negative moments) are equal to zero. Note that, here, torsion on beams is not considered, it will be discussed in next chapters.

The slab and the beams are casted monolithically, so torsion will develop in beams as the slab rotates under load, especially edge beams.

Column strips in one-way slabs exist when the slab is modeled as a three-dimensional structure; space frame. The slab strips which are aligned at column lines will have larger internal forces.

G. Beam size: For the initial analysis- design cycle, preliminary member sizes can be selected on prior experience with similar floor systems. Total beams depth, h, are typically in the range of L/18 to L/12, where L is the span length center to center of supports (other than cantilever). One may use b_w= 0.5 h for typical drop beams. Beam width shall be not less than 200mm for practical purposes. Beam width can be equal to about L/20, where L is the span length. In seismic special frames, width of beam, b_w, shall be at least 0.3 section thickness, h, and 250mm and its projection beyond the supporting column shall not exceed the lesser of c_2 and $0.75c_1$. C_1 is the width of column in direction of beam and c_2 is the transverse width of column.

Beam size is controlled by:

1. Deflection criteria (Code limitations, deflection calculations)
2. Flexural design (economical section, steel ratio = 0.01-0.014, singly reinforced section)
3. Shear design
4. Architectural purposes

Beams can be dropped or inverted with different shapes.

H. Distribution of flexural reinforcement in beams and one- way slabs:

The flexural reinforcement shall be distributed to control flexural cracking. The spacing of reinforcement closest to the tension face, s, shall not exceed that given by:

$$s = 380\left(\frac{280}{f_s}\right) - 2.5c_c \leq 300\left(\frac{280}{f_s}\right)$$

Where:

f_s = calculated tensile stress in reinforcement at service loads, MPa (refer to working design method). It shall be permitted to take $f_s = \frac{2}{3} f_y$.

C_c= the least distance from surface of flexural reinforcement to the tension face.

Generally, the maximum spacing of reinforcing bars is 250mm and 150mm in slabs and beams respectively.

According to ACI 318-19 Section (7.7.2), the maximum spacing between bars in one-way solid slabs is the smaller of 3h and 450mm and the maximum spacing of shrinkage steel shall be the smaller of 5h and 450mm. Where h is the slab thickness.

I. Structural integrity:

- Longitudinal structural integrity reinforcement consisting of at least one-quarter of the maximum positive moment reinforcement shall be continuous.

- Longitudinal structural integrity reinforcement at noncontinuous supports shall be anchored to develop f_y at the face of the support.
- If splices are necessary in continuous structural integrity reinforcement, the reinforcement shall be spliced near supports. Splices shall be mechanical or welded in accordance with 25.5.7 or Class B tension lap splices in accordance with 25.5.2.

More details on structural integrity can be found in chapter 5.

7.1.2 Example: One-way solid slab

Given:

Concrete, f'c= 24MPa.

Steel, fy= 420MPa.

Superimposed dead load, W_{SD}= 4.5kN/m^2.

Live load, W_L= 2.5kN/m^2.

Perimeter wall weight= 21kN/m.

All columns are 300x600mm.

Design the slab strip and draw the structural models for all required beams.

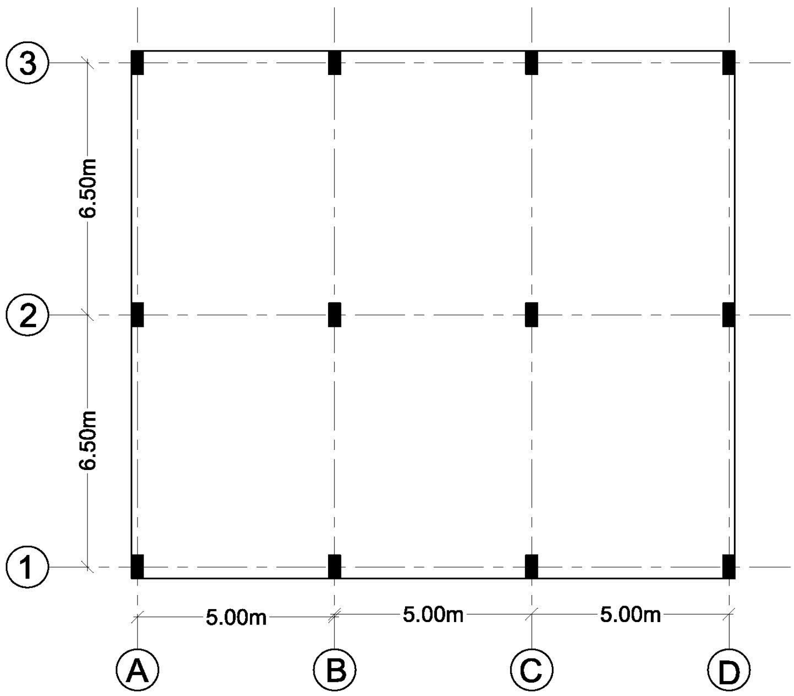


Figure 7.1: Columns layout

Solution:

Analysis and design steps for one-way slab systems:

1. Structural system and beams layout.
2. Slab thickness.
3. Loads (slab load kN/m^2, wall weight).
4. Analysis and design of slab strips.
5. Analysis of beams using preliminary dimensions.
6. Design of beams.
7. Structural drawings.

Notes: 1. Here, no torsion is considered.

2. It is considered that a strip represents the whole slab. In reality, a slab strip at column lines has larger moments than strips between column lines.

3. Beams parallel to slab strip carry no load from the slab.

Step 1: Slab system

- One-way solid slab.
- Beams are distributed in y- direction (to have the smallest slab thickness).

Step 2: Slab thickness

$$h_{min} = \frac{L}{24} = \frac{5.0}{24} = 0.21m$$

Try h= 200mm

Step 3: Loads

Slab own weight, W_D= 0.20(25kN/m^3)= **5kN/m^2**

ultimate load on the slab, W_{u1}= 1.4(W_D +W_{SD})

or W_{u2}= 1.2(W_D+W_{SD})+1.6W_L

which is larger

W_{u1}= 1.4(5+4.5)= **13.3kN/m^2**

W_{u2}= 1.2(5+4.5)+1.6(2.5)= **15.4kN/m^2**

Use W_u= 15.4kN/m^2

Step 4: Slab analysis and design

A one-meter strip perpendicular to the supporting beams can be taken to represent the whole slab.

Assume that width of supporting beams= 0.3m.

Clear span for the slab, $L_n = 5 - 0.3 = 4.7m$.

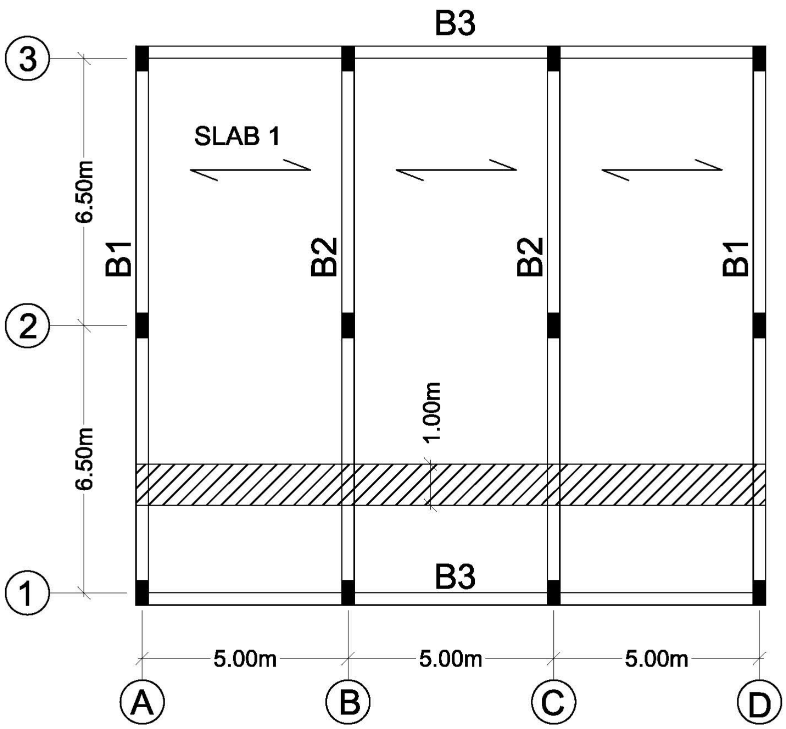


Figure 7.2: Beams layout

Main beams which are B1 and B2 are in y-direction.

B3; the perimeter beam is used to carry the external wall.

B1: Carries its own weight + load from slab + wall weight

B2: Carries its own weight + load from slab

B3: Carries its own weight + wall weight

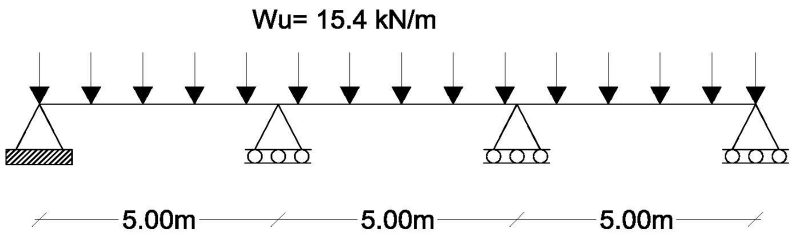


Figure 7.3: Structural model of slab

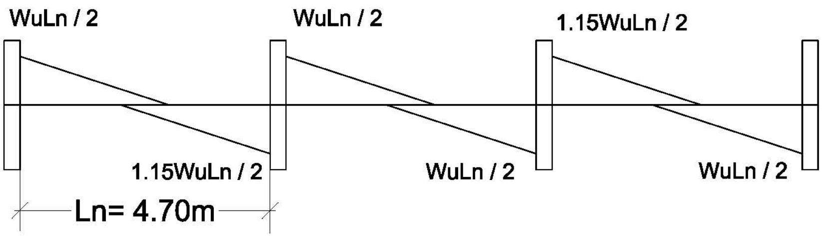


Figure 7.4: Shear envelope of slab

The slab thickness shall be adequate to resist shear.

Check slab for shear:

The maximum ultimate shear force at face of support (Beam) is given by:

$$V_u = 1.15 w_u \frac{L_n}{2} = 1.15(15.4)\frac{4.7}{2} = 41.6kN$$

ACI 318-19:

Effective depth of slab= 200-40=160mm.

V_c can be calculated by:

$$For\ A_v \geq A_{v.min}\ (or\ \frac{A_v}{s} \geq \left(\frac{A_v}{s}\right)_{min})\ use\ either\ of:$$

$$V_c = \left(0.17\lambda\sqrt{f'_c} + \frac{N_u}{6A_g}\right) b_w d \quad and \quad V_c = \left(0.66\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right) b_w d$$

$$For\ A_v < A_{v.min}\ \left(or\ \frac{A_v}{s} < \left(\frac{A_v}{s}\right)_{min}\right) use:$$

$$V_c = \left(0.66\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right)b_w d$$

Where A_v is the area of shear reinforcement within spacing s, mm^2.

And, V_c shall not be taken greater than:

$$V_c \leq 0.42\lambda\sqrt{f'_c}b_w d$$

$$Size\ factor, \qquad \lambda_s = \sqrt{\frac{2}{1 + 0.004\,d}} \leq 1.0$$

For $d \leq 250mm, \lambda_s = 1.0$

$$\frac{N_u}{6A_g} \leq 0.05f'_c$$

Axial load, *Nu*, is positive for compression and negative for tension.

$$\rho_w = \frac{A_s}{b_w d}$$

$$\emptyset V_c = \emptyset\left(0.66\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right)b_w d$$

$$Let\ \rho_w = 0.0018\left(\frac{h}{d}\right) = 0.0018\left(\frac{200}{160}\right) = 0.00225$$

So,

$$\emptyset V_c = \emptyset\left(0.66\lambda_s\lambda(\rho_w)^{\frac{1}{3}}\sqrt{f'_c} + \frac{N_u}{6A_g}\right)b_w d$$
$$= \frac{0.75\left(0.66(1)(1)(0.00225)^{\frac{1}{3}}\sqrt{24} + 0.0\right)(1000)(160)}{1000} = 50.85kN$$
$$> 41.6kN \qquad OK$$

ACI 318-14:

$$\emptyset V_c = \emptyset\frac{1}{6}\lambda\sqrt{f'_c}b_w d = \frac{0.75\left(\frac{1}{6}\right)(1)\sqrt{24}(1000)(160)}{1000} = 98kN > 41.6kN \qquad OK.$$

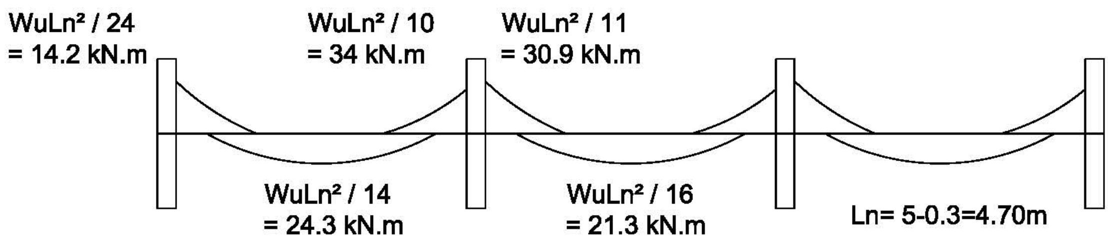


Figure 7.5: Moment envelope of slab

Flexural design:

The minimum area of steel in slabs is: 0.0018 b h Where: b= section width=1000mm and h= slab thickness=200mm.

Here, $A_{s,min}$= 0.0018 x 1000 x 200 = **360 mm²** $(1\Phi12/300mm)$

(Here, $1\Phi12/250$ is used instead of $1\Phi12/300$ to have better bars arrangement)

As an example: calculations for steel area, As for Mu=34kN.m:

b_w= 1000mm h=200mm d= 160mm f'c= 24MPa fy= 420MPa

$$\rho = \frac{0.85f'_c}{f_y}\left(1-\sqrt{1-\frac{2.61M_u}{bd^2f'_c}}\right) = \frac{0.85(24)}{420}\left(1-\sqrt{1-\frac{2.61(34x10^6)}{(1000)(160)^2(24)}}\right) = 0.00364$$

$$A_s = \rho b_w d = (0.00364)(1000)(160) = 582mm^2 > A_{s,min}$$

And this steel ratio is less than the maximum allowed for singly reinforced section, $\rho_{max,singly} = 0.375\beta_1 \frac{0.85f'_c}{f_y}$.

Number of bars per meter width= 582/154=3.78 bars , spacing= 1000/3.78= 264mm **Use** $1\Phi14/250mm$ or use $4\ \Phi14/m$

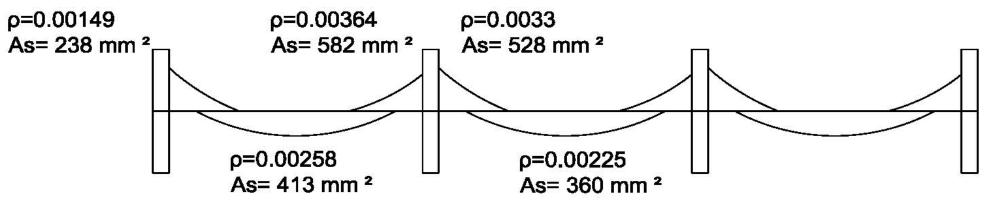


Figure 7.6: Flexural reinforcement in slab

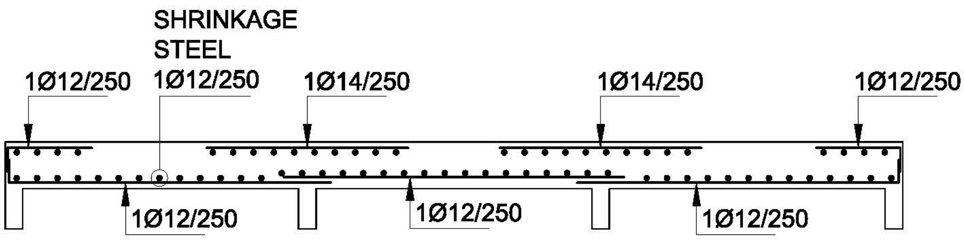


Figure 7.7: Bars layout in slab

Step 5: Analysis and design of beams:

Beam width, b_w= 300mm.

Generally, it is preferred to use beam width less than column width to facilitate bars layout.

According to ACI 318 -19, the minimum thickness of beam is L/18.5; one end continuous span.

L= 6.5m, then h= 6.5/18.5= **0.35m**

Loads on beams are usually large, so this depth will lead to have doubly reinforced section, or in general, not adequate section especially for deflection. So, it is recommended to increase it, say, h= 1.5 x 0.35 = 0.55m.

Try h=600mm and b_w= 300mm

B1 and B3 are L-section and B2 is T- section beams.

Use the ACI code limitations to compute width of flange (b_f or b_e).

B1:

$$b_e \leq b_w + \frac{1}{2} clear\ transverse\ span = 300 + 0.5(4700) = 2650mm$$

$$b_e \leq b_w + 6h_f = 300 + 6(200) = 1500mm$$

$$b_e \leq b_w + \frac{L_n}{12} = 300 + \frac{6500 - 600}{12} = 792mm$$

Use flange width, b_e= 800mm

B2:

$$b_e \leq b_w + 2\frac{1}{2} clear\ transverse\ span = 300 + 2(0.5)(4700) = 5000mm$$

$$b_e \leq b_w + 2(8)h_f = 300 + 16(200) = 3500mm$$

$$b_e \le b_w + 2\frac{L_n}{8} = 300 + \frac{6500 - 600}{4} = 1775mm$$

Use flange width, b_e= 1775mm

B3:

Use rectangular section: bw= 300mm, h= 600mm

Loads on beams: (*The thickness of slab is subtracted from beam thickness; 0.6-0.2=0.4m)*

Wu1= 0.3(0.4)(25)(1.2)+(5/2)(15.4)+(21)(1.2)= **67.3kN/m**

Wu2= (0.3)(0.4)(25)(1.2)+(5)(15.4)= **80.6kN/m** (Notice that the factor 1.15 is not used as an approximation)

Wu3= (0.3)(0.4)(25)(1.4)+(21)(1.4)= **33.6kN/m**

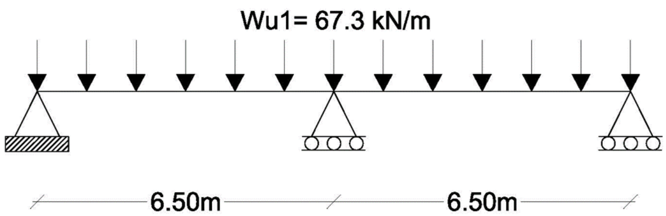


Figure 7.8: Structural model of beam B1

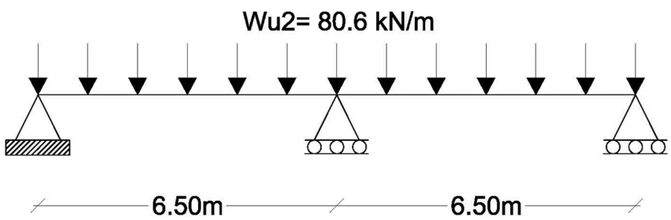


Figure 7.9: Structural model of beam B2

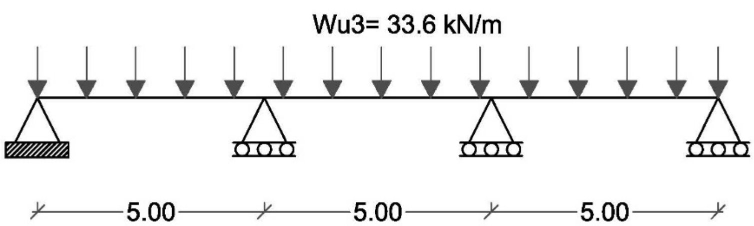


Figure 7.10: Structural model of beam B3

Design of beam B2 for flexure:

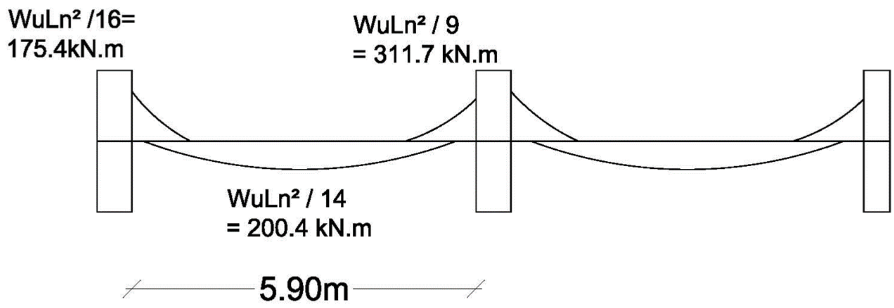


Figure 7.11: Bending moment envelope for beam B2

For Mu= 311.7kN.m:

b_w= 300mm h= 600mm

d= 540mm

→ ρ= 0.0106

→ As= 1717mm^2

$$\rho_{min} = max\left[\frac{1.4}{f_y}, \frac{0.25\sqrt{f'_c}}{f_y}\right] = 0.00333 < 0.0106 \quad ok.$$

This As is distributed in the flange width. 2/3 As in 300mm and 1/3 As in (1925-300)mm

$$\rho_{max,singly} = 0.375\beta_1\frac{0.85f'_c}{f_y} = 0.015548 > 0.0106 \quad ok.$$

For Mu= 200.4mm:

Assume rectangular compression zone; a< hf=200mm

Use the formula for ρ with b=1775mm

→ρ = 0.00103

→ $As = \rho bd = (0.00103)(1775)(540) = 988\,mm^2$ $(4\Phi 20)$

Check a: (a is less than a_{max}= $0.375\beta_1 d = 0.375(0.85)(540) = 172mm$)

$$a = \frac{A_s f_y}{0.85f'_c b} = \frac{(988)(420)}{0.85(24)(1775)} = 11.5mm < 200mm\ and\ < 172mm \qquad ok.$$

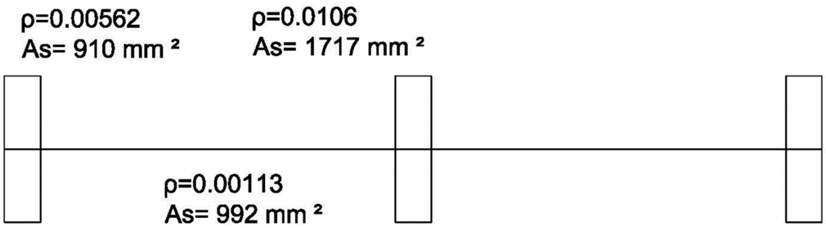


Figure 7.12: Flexural steel for beam B2

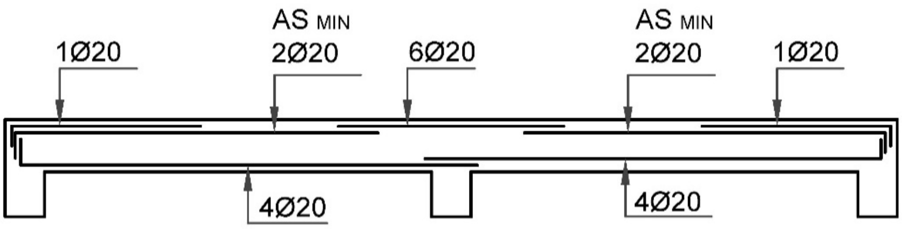


Figure 7.13: Flexural reinforcing bars in beam B2

Design of beam B2 for shear:

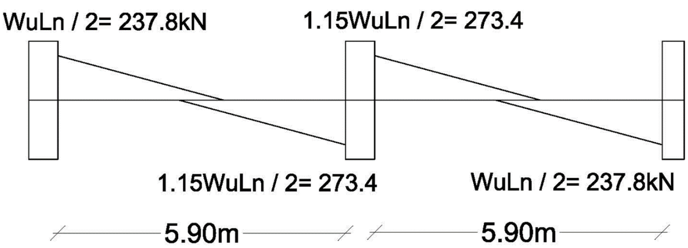


Figure 7.14: Shear envelope for beam B2

For Vu= 273.4kN:

V_u/Φ= 273.4/0.75= 364.5kN

$$V_c = \frac{1}{6}\lambda\sqrt{f'_c}b_w d = \frac{\frac{1}{6}(1)\sqrt{24}(300)(540)}{1000} = 132.3kN$$

$$V_s = V_u/\Phi - V_c = 232.2kN$$

$$V_{s,max} = \frac{2}{3}\sqrt{f'_c}b_w d = 529.1kN > 232.2kN \qquad Section\ size\ is\ adeqaute$$

$$\frac{A_v}{s} = \frac{V_s}{f_{yt}d} = 1.02mm^2/mm$$

$$\left(\frac{A_v}{s}\right)_{min} = max\left[\frac{0.062\sqrt{f'_c}b_w}{f_{yt}}, \frac{0.35b_w}{f_{yt}}\right] = max[0.22,0.25] = 0.25mm^2/mm$$

Av/s > (Av/s) min ok → use Av/s= 1.02 mm²/mm

Try Φ10mm stirrups, two legs → s= (2x78.5)/1.02= 154mm

Check stirrups spacing:

- $If\ V_s \le \frac{1}{3}\sqrt{f'_c}b_w d\ \rightarrow S_{max} = min\left[\frac{d}{2}, 600mm\right]$
- $If\ \frac{1}{3}\sqrt{f'_c}b_w d < V_s \le \frac{2}{3}\sqrt{f'_c}b_w d\ \rightarrow S_{max} = min\left[\frac{d}{4}, 300mm\right]$
- $If\ V_s > \frac{2}{3}\sqrt{f'_c}b_w d\ \rightarrow Incease\ section\ dimensions$

$$\frac{1}{3}\sqrt{f'_c}b_w d = 264.5kN > 232.2kN$$

$$S_{max} = min\left[\frac{d}{2}, 600mm\right] = min\left[\frac{540}{2} = 270mm, 600mm\right] = 270mm$$

270mm> 154mm → use s= 150mm 2 legs Φ10mm stirrups

Note:

The maximum transverse spacing of stirrup legs are given by (Section 9.7.6.2.2 in ACI 318-19):

- $If\ V_s \le \frac{1}{3}\sqrt{f'_c}b_w d\ \rightarrow S_{max} = min[d, 600mm]$
- $If\ \frac{1}{3}\sqrt{f'_c}b_w d < V_s \le \frac{2}{3}\sqrt{f'_c}b_w d\ \rightarrow S_{max} = min\left[\frac{d}{2}, 300mm\right]$

$$Here, V_s < \frac{1}{3}\sqrt{f'_c}b_w d \qquad so, S_{max} = min[d, 600mm] = 540mm$$

And the value 540mm is larger than the width of the beam, so, two - legs stirrup can be used for this cross section.

7.2 One-way ribbed (joist) slabs:

Figure 7.15: One-way ribbed slab picture

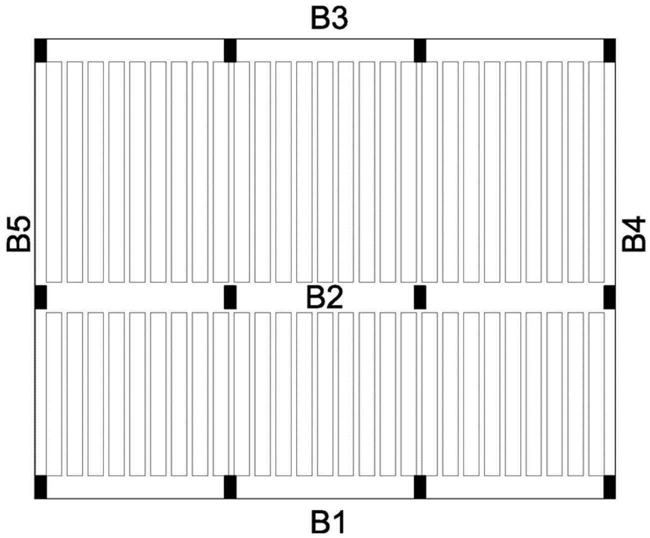


Figure 7.16: One-way ribbed slab layout

7.2.1 Basic principles:

A. Joist construction can be used as a slab system for light loads and it can be with drop or hidden beams.
B. Joist construction with blocks can be used for spans up to about 7 meters.
C. Joist construction with removable metal or plastic forms can be used for medium spans from 7 to 12 meters.

D. The most economical forming results if joists and supporting beams have the same depth. This will involve beams considerably wider than columns. Such a system is referred to as joist- band system.

E. Joist dimensions:

$b_w \geq 100$mm

h ≤ 3.5 b_w

S ≤ 750mm

$h_f \geq 50$mm

≥ S/12

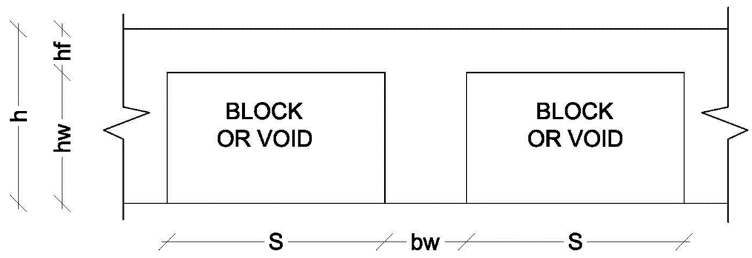


Figure 7.17: Section in rib

F. Ribbed slab not meeting the above requirements for the rib dimensions are designed as slabs and beams – Figure 7.18.

G. The rib shear capacity can be increased by 10% if the previous dimensions are used.

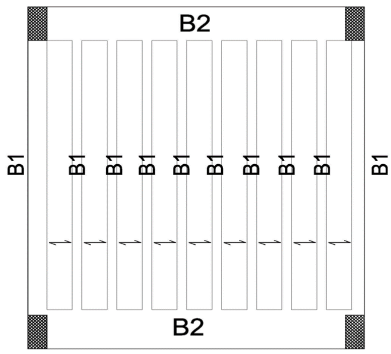


Figure 7.18: Slab layout

H. The overall depth of slab is governed by deflection and shear.

I. Load distributing rib – Figure 7.19

- span < 6m : no distributing ribs
- 6m ≤ span < 9m : one distributing rib
- 9m ≤ span : two distributing ribs

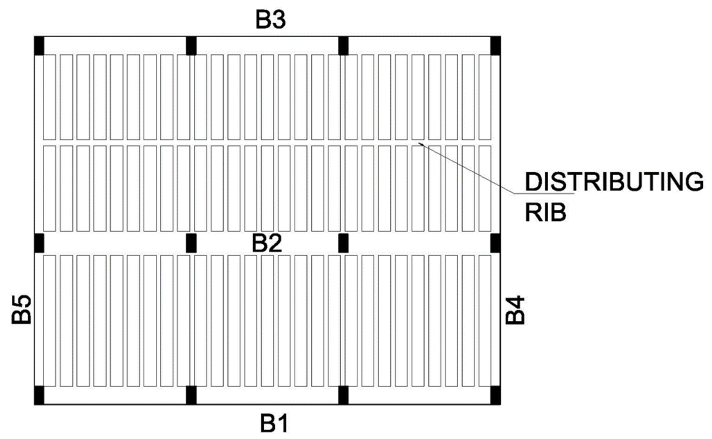


Figure 7.19: Load distributing rib

J. Block size:

* Normal weight concrete blocks:

S= 400mm

hw= 140, 170, 200, 240, 300, 320 b1= 200, 250

* Ytong blocks (Light weight):

S= 500, 550, 600

hw= as required.

b1= 300mm usually, or as required.

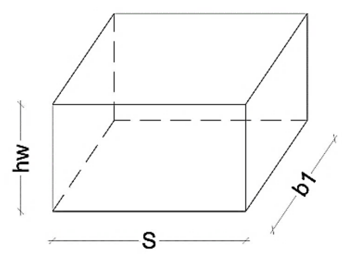


Figure 7.20: Block dimensions

* Light weight concrete blocks: as for normal concrete blocks

* Concrete blocks + polystyrene: concrete blocks, hw= 70mm, S= 400mm

* Polystyrene blocks: variable dimensions

  - Block unit weights:

    Normal weight concrete: 12kN/m^3

    Light weight concrete: 6kN/m^3

    Ytong: 5kN/m^3

    Polystyrene: 0.3kN/m^3

7.2.2 Example:

Resolve the Example 7.1.2 using one-way ribbed slab system with hidden beams. Use light weight concrete blocks, $\gamma = 6kN/m^3$.

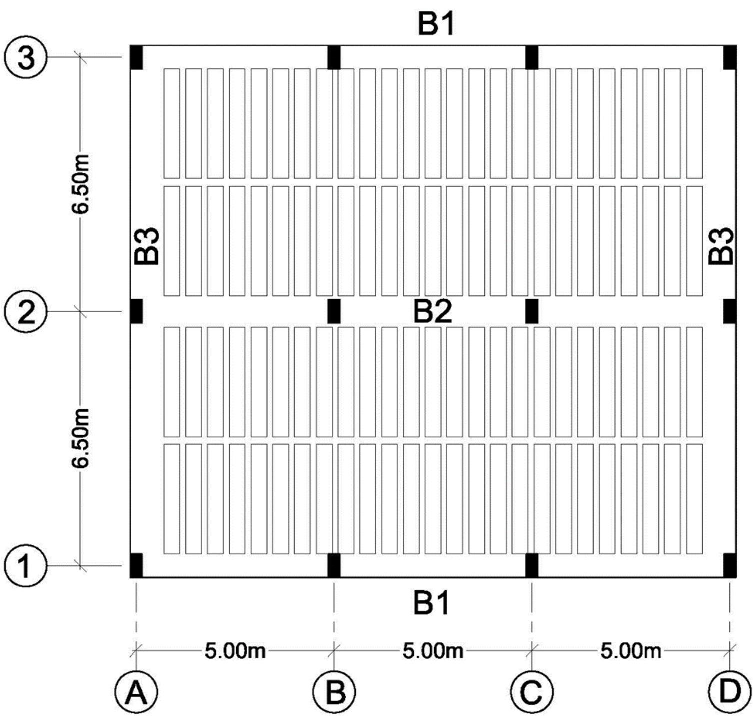


Figure 7.21: Slab layout for example 7.2.2.

Solution:

Step 1: Slab system:

- One-way ribbed slab.
- Main hidden beams are distributed in x- direction.

Step 2: Slab thickness:

6.5/18.5= 0.35m

Check rib dimensions:

b_w=150mm > 100mm ok

h=360 < 3.5 x 150= 525mm ok

s = 400mm < 750mm ok

Hf= 60mm > 50mm and > 400/12=33mm ok

Try h= 360mm - Figure 7.22

Step 3: Loads:

Slab own weight, W_D= {(0.55) (0.06) +(0.15) (0.30)} (25kN/m^3) + (0.40) (0.30) (6kN/m^3) = **2.67kN/rib** → W_D= 2.67/0.55= **4.90kN/m^2**

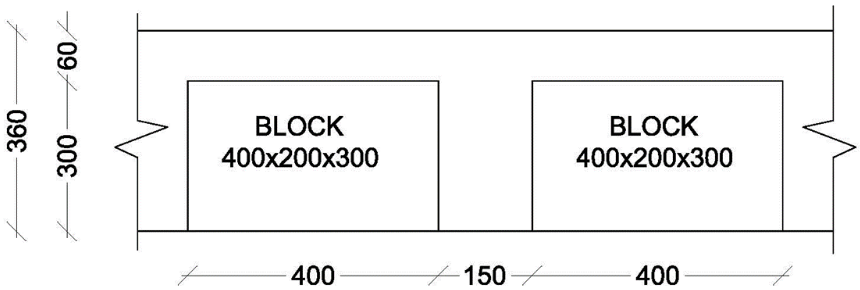


Figure 7.22: Section in slab

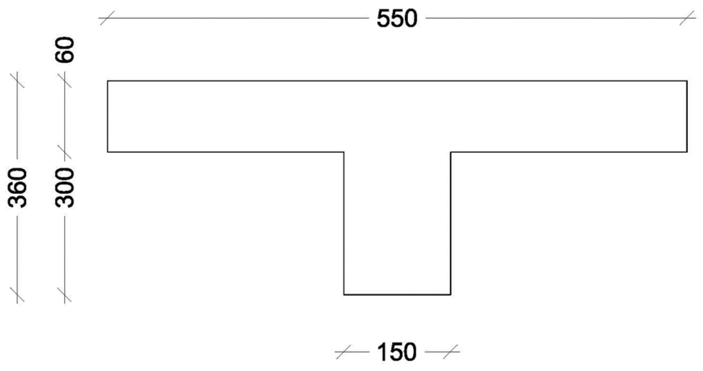


Figure 7.23: Section in rib

Ultimate load on the slab, W_{u1}= 1.4(W_D +W_{SD}) or W_{u2}= 1.2(W_D+W_{SD})+1.6W_L which is larger.

W_{u1}= 1.4(4.9+4.5)= **13.2kN/m^2**

W_{u2}= 1.2(4.9+4.5)+1.6(2.5)= **15.3kN/m^2**

Use W_u= 15.3kN/m^2

Or Wu= 15.3 x 0.55= **8.4kN/m** for a rib.

Step 4: Slab analysis and design:

A rib strip can be taken to represent the whole slab. Assume support widths are 0.60m which are the width of the main beams.

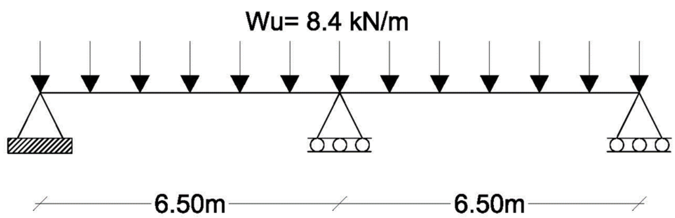


Figure 7.24: Structural model of the rib

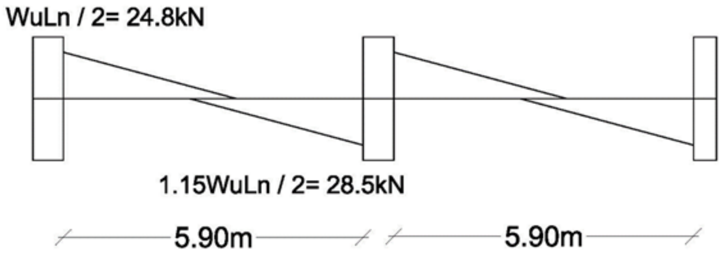


Figure 7.25: Shear envelope of the rib

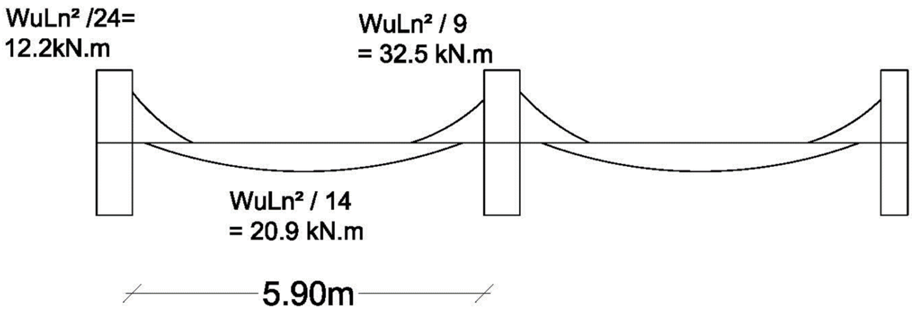


Figure 7.26: Bending moment envelope of the rib

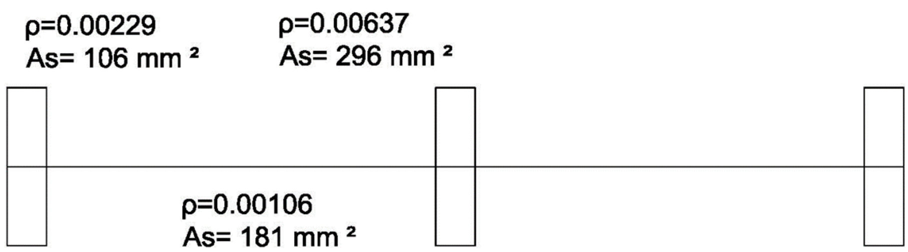


Figure 7.27: Flexural reinforcement of the rib

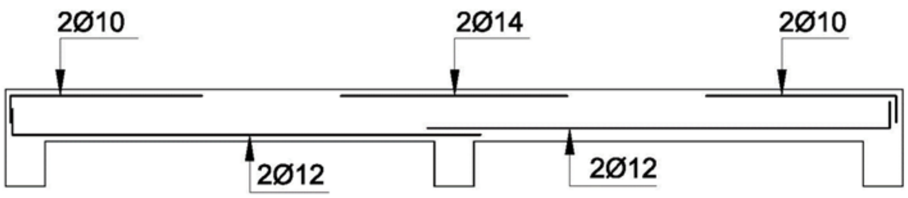


Figure 7.28: Bars layout in the rib

Check slab for shear: (Maximum shear force at interior support)

ACI 318-14:

$$V_{u,face} = 1.15 w_u \frac{L_n}{2} = 1.15(8.4)\frac{5.9}{2} = 28.5kN$$

$$V_{u\,at\,d} = 28.5 - w_u d = 28.5 - (8.4)(0.31) = 25.9kN$$

$$\emptyset V_c = 1.1\emptyset\lambda\frac{1}{6}\sqrt{f'_c}b_w d = 1.1\frac{(0.75)(1)\left(\frac{1}{6}\right)\sqrt{24}(150)(310)}{1000} = 31.3kN > 25.9kN \quad OK.$$

No need for shear reinforcement.

ACI 318-19:

$$If\ shear\ reinforcement\ is\ not\ used:\ \emptyset V_c = 1.1\emptyset\left(0.66\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right)b_w d$$

$$For\ minimum\ steel\ ratio, \rho_w = 0.00333:$$

$$\lambda_s = \sqrt{\frac{2}{1 + 0.004\,d}} \le 1.0 \rightarrow \sqrt{\frac{2}{1 + 0.004(310)}} = 0.94$$

$$\emptyset V_c = 1.1\emptyset\left(0.66\lambda_s\lambda(\rho_w)^{\frac{1}{3}}\sqrt{f'_c} + \frac{N_u}{6A_g}\right)b_w d$$

$$= 1.1\frac{0.75\left(0.66(0.94)(1)(0.00333)^{\frac{1}{3}}\sqrt{24} + 0.0\right)(150)(310)}{1000} = 17.4kN < 25.9kN \qquad N.G.$$

At interior support, $\rho = 0.00637, \emptyset V_c = 21.6kN < 25.9kN \quad N.G$

So, shear reinforcement shall be used, and the value of $\emptyset V_c$ is taken equal to 21.6kN/0.94=23.0kN (Notice that the factor λ_s is not used in V_c when shear reinforcement is used). The old equation of V_c can be used.

$$V_c = \frac{23.0}{0.75} = 30.67kN$$

$$V_s = \frac{V_u}{\emptyset} - V_c = \frac{25.9}{0.75} - 30.67 = 3.86kN$$

$$\frac{A_v}{s} = \frac{V_s}{f_{yt}d} = \frac{3.86(1000)}{(420)(310)} = 0.03mm^2/mm$$

$$\left(\frac{A_v}{s}\right)_{min} = \max of \begin{bmatrix} 0.062\sqrt{f'_c}\frac{b_w}{f_{yt}} \\ 0.35\frac{b_w}{f_{yt}} \end{bmatrix} = \frac{0.125mm^2}{mm} \quad > 0.03mm^2/mm$$

$As\ a\ minimum\ value, use\ 1\emptyset 8/150mm\ (\frac{A_v}{s} = \frac{2(50)}{150} = 0.67mm^2/mm$ $> 0.125mm^2/mm$ at ends of each rib for one quarter of clear span.

Note: For shear at exterior support, Vu=24.8kN, Vu at distance d from face of support is 22.2kN which is less than $\phi V_c = 17.4kN$, so shear reinforcement is required, same as it is at the interior support.

Design slab for flexure:

The cross section of slab is T- section. For positive moment, Mu= 20.9kN.m: Assume a < hf= 60mm, then apply formula of ρ with b= 550mm.

→ ρ = 0.00106 → As= 0.00106 x 550 x 310 = 181mm^2 (2Φ12)

$$a = \frac{A_s f_y}{0.85f'_c b} = \frac{(181)(420)}{(0.85)(24)(550)} = 7mm < 60mm \qquad ok.$$

$A_{s,\,min}$= 0.00333(150)(310)= 155mm^2 < 181mm^2 ok.

$$a_{max} = 0.375\beta_1 d = (0.375)(0.85)(310) = 99mm$$
$$> 7mm: Section\ is\ singly\ reinforced.$$

Slab shrinkage steel:

$$A_s = 0.0018bh = 0.0018(1000)(60) = 108mm^2.$$

$$Spacing\ between\ bars, s = \frac{1000}{\frac{108}{50}} = 463mm$$

$s_{max} = \min(5h, 450mm) = 300mm. Use\ \emptyset 8/300mm$ each way (E.W.).

Step 5: Analysis and design of beams:

For analysis, assume that:

Width of B1 and B2 is 600mm Width of B3 is 400mm

Wu1= 0.60(0.36)(25)(1.2)+(6.5/2)(15.3)+(21)(1.2)= 81.4kN/m

Wu2= (0.60)(0.36)(25)(1.2)+(6.5)(15.3)= 106kN/m. More accurate solution: Wu2= (0.60)(0.36)(25)(1.2)+(6.5)(15.3)(1.15)= 120.8kN/m

Wu3= (0.40)(0.36)(25)(1.4)+(21)(1.4)= 34.4kN/m

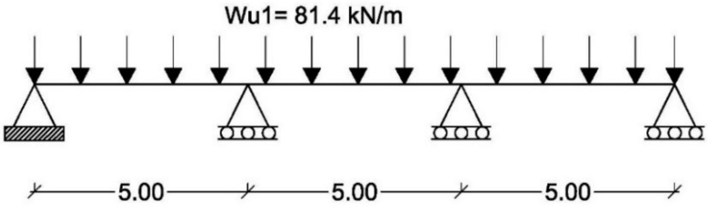


Figure 7.29: Structural model of beam B1

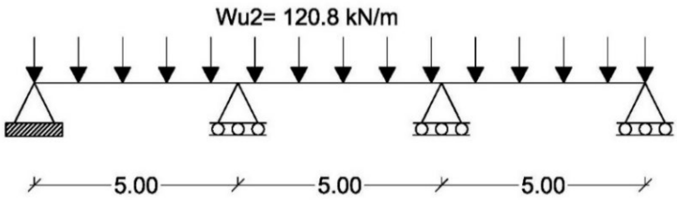


Figure 7.30: Structural model of beam B2

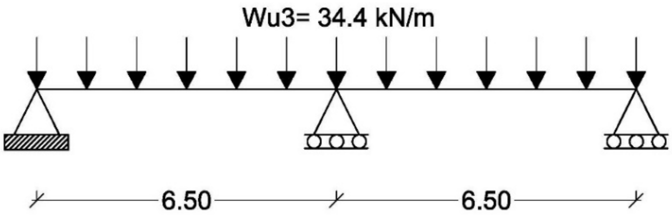


Figure 7.31: Structural model of beam B3

7.3 Beam and girder system – one-way slab (solid, ribbed)

7.3.1 Basic principles:

Beam and girder system can be used to decrease the span of slabs to minimize slab thickness.

7.3.2 Example:

Given:

Concrete compressive strength, f'c=32MPa

Steel yield strength, fy= 420MPa

Superimposed dead load, W_{SD}=3kN/m^2

Live load, W_L=5kN/m^2

Perimeter wall weight= 2kN/m^2

Floor height= 4m

No. of floors= 4

All columns are 800mm x 800mm

Determine slab thickness. Draw the structural model of the solid slab and beams

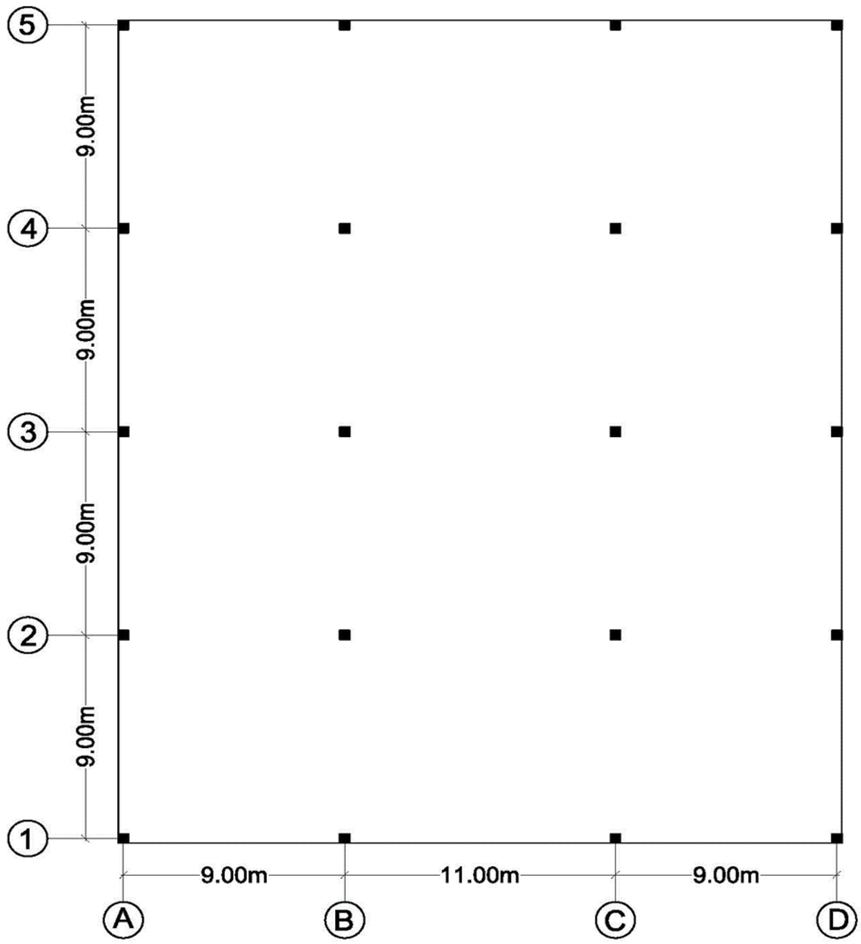


Figure 7.32: Slab layout, example 7.3.2

<u>Solution:</u>

Step 1: Slab system:

- One-way solid slab in y-direction
- Main beams are distributed in x- direction
- Girders are distributed in y-direction

Step 2: Slab thickness:

4.5/24= 0.19m

Try h= 200mm

Step 3: Loads:

WD= (0.2)(25)=5kN/m^2

Wu= 1.2(5+3)+1.6(5)=**17.6kN/m^2**

Wall weight= 2x4=8kN/m

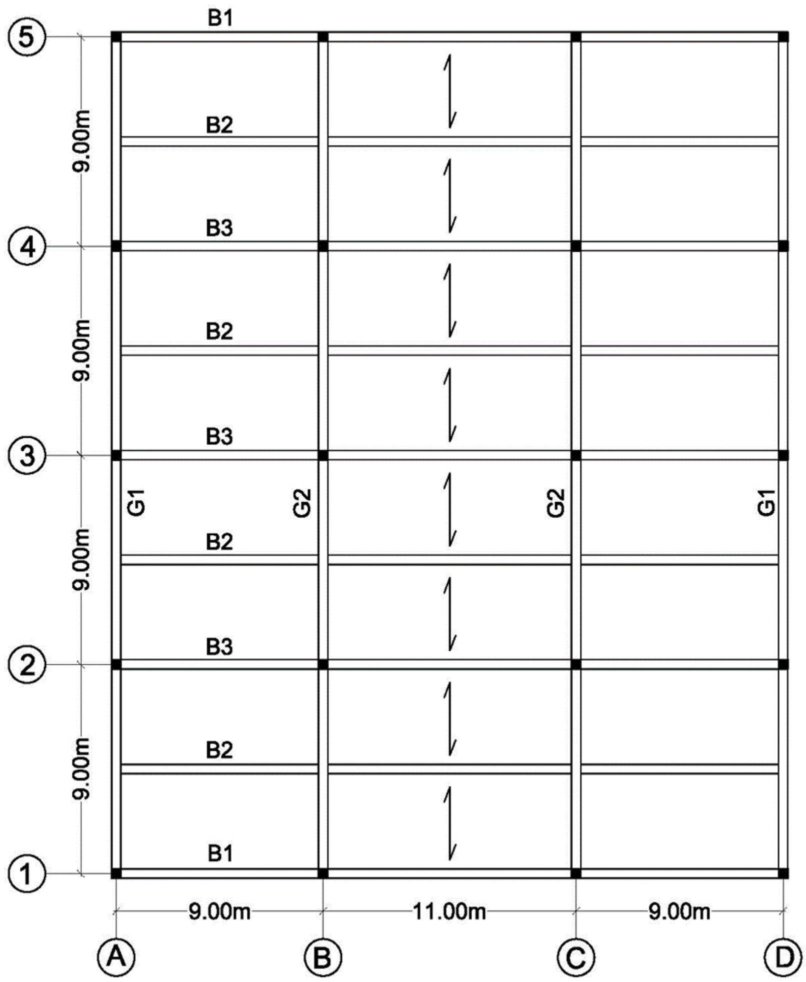


Figure 7.33: Beams layout, example 7.3.2

Step 4: Slab and beams structural models:

Beam depth, h ≥ L/18.5= 9/18.5= 0.50m

or h ≥ L/21= 11/21= 0.52m

Try h= 750mm, bw= 500mm for beams B1, B2 and B3.

Try h= 900mm, bw= 500mm for girders G1 and G2.

Interior beams are T- sections and exterior beams are L- sections.

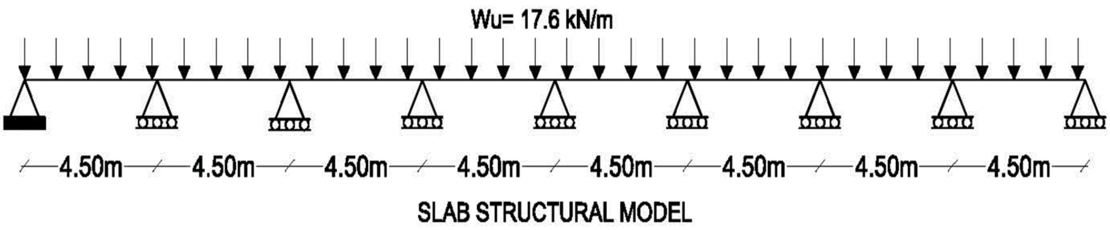


Figure 7.34: Structural model of slab

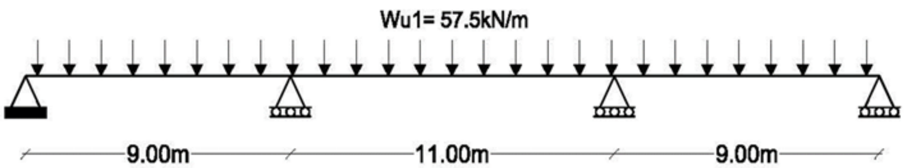


Figure 7.35: Structural model of beam B1

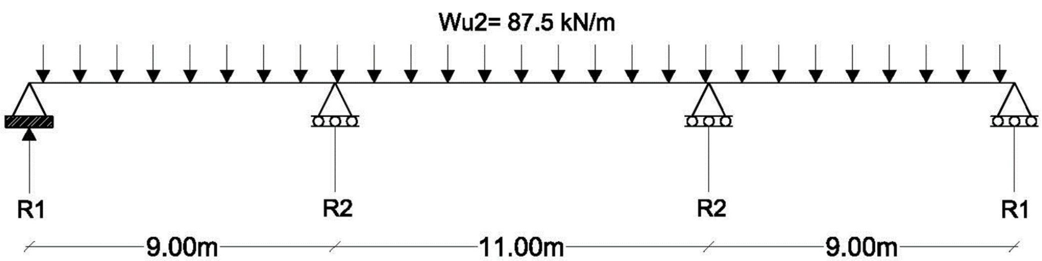


Figure 7.36: Structure model of beam B2

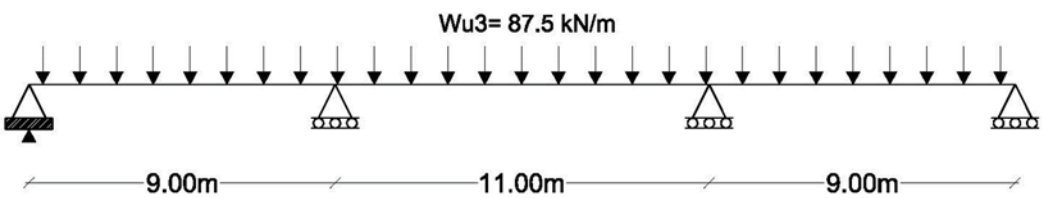


Figure 7.37: Structural model of beam B3

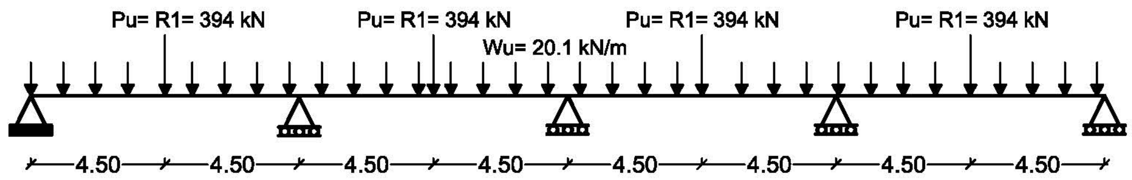


Figure 7.38: Structural model of beam (Girder) G1

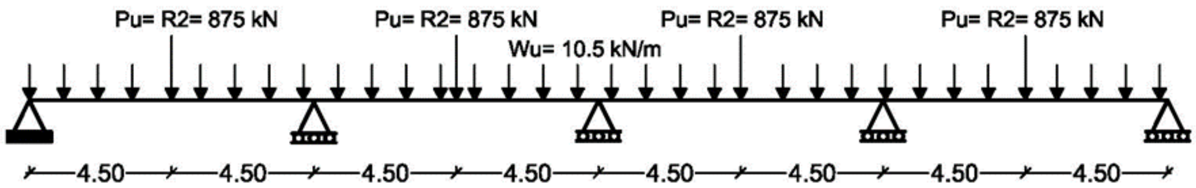


Figure 7.39: Structural model of beam (Girder) G2

Load calculations:

w_{u1}= 17.6(4.5/2)+0.50(0.75-0.20)(25)(1.2)+8(1.2)= 57.5kN/m

w_{u2}= 17.6(4.5)+0.50(0.75-0.20)(25)(1.2)= 87.5kN/m (Notice that, the factor 1.15 is not used as approximation)

w_{u3}= W_{u2}

R_1= (9/2)(87.5)= 394kN

R_2= (9/2 + 11/2)(87.5)= 875kN

Notes:

The previous calculations and models are applicable for very rigid beams and girders or for very thin slabs. When the slab-stiffness ratio is taken into account, the beam loads will differ.

The following points shall be considered in the design:

1. Torsion on beams and girders.
2. Stiffness of beams affects the internal forces in the slabs.
3. Slab strips at column lines and between column lines.
4. The supports of beams which are the girders act like springs.

Chapter 8: Two- Way Slab Systems

In two-way slab systems, the load is assumed to be transferred in two directions. Generally, two-way slabs are used to decrease the slab thickness for large spans. Also, the slab forms the diaphragm that transmits the horizontal loads to the vertical elements of the lateral forces resisting system, so, its strength is very important.

8.1 Types of two-way slabs:

1. Flat plate:

- It has uniform thickness.
- It is used for light loads; live load is less than $5kN/m^2$, as in residential and office buildings.
- There are no interior beams between columns.
- Exterior (perimeter) beams can be used.
- The punching shear capacity is achieved by slab thickness. Punching shear reinforcement can be used.
- It is economical for spans up to 7.0m

2. Flat slab:

- There are no interior beams between columns.
- Exterior (perimeter) beams can be used.
- The punching shear capacity is achieved by using drop panels and/ or column capitals.
- In drop panels: the projection below the slab is at least one-quarter of the adjacent slab thickness and the drop panel extends in each direction from centerline of a support a distance not less than one-sixth the span length measured from center- to- center of supports in that direction.
- It can be used for live loads more than $5kN/m^2$.
- It is economical for spans up to 9.0m

3. Two-way slab with beams between all columns:

- There are beams between all supports (columns).
- They are used for heavy loads.
- They are economical for spans up to 12m.

The previous types can be:

- Solid
- Voided
- Waffle
- Ribbed

The following pictures show the different slab systems.

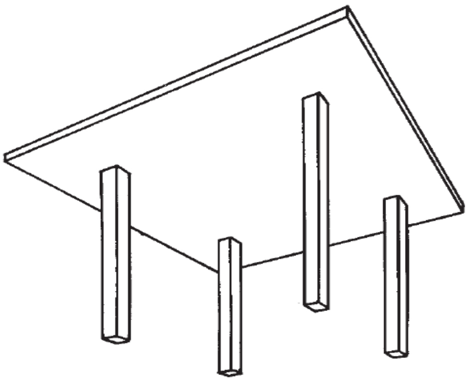


Figure 8.1: Flat plate

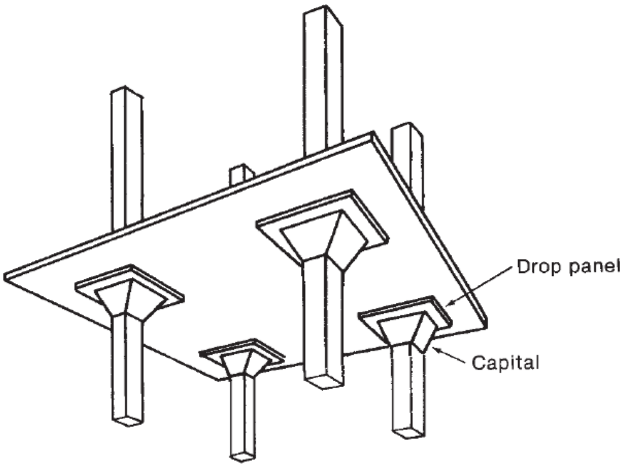


Figure 8.2: Flat slab

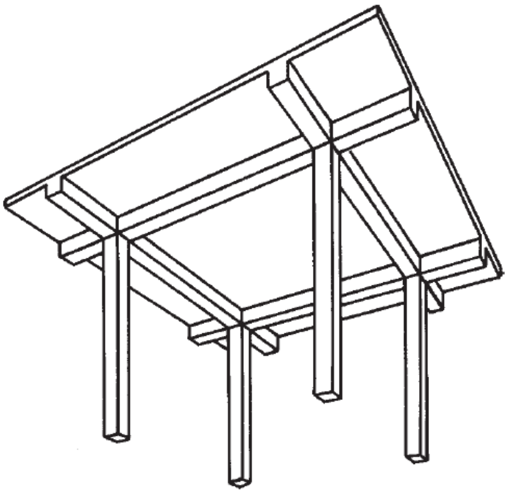


Figure 8.3: Two-way slab with beams

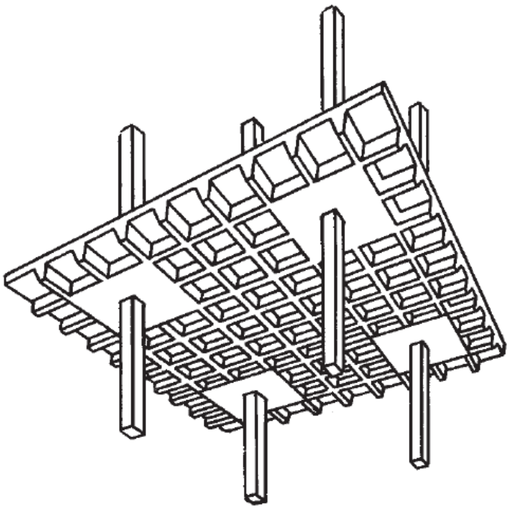


Figure 8.4: Waffle slab-1

Figure 8.5: Waffle slab-2

Figure 8.6: Voided slab – U Boot-1

Figure 8.7: Voided slab - Cobiax -1

Figure 8.8: Voided slab - Cobiax -2

Figure 8.9: Forms of waffle slab

8.2 Design methods:

There are different ways and methods for two-way slabs analysis and design. Two procedures for analysis and design of two-way floor systems are presented in detail in ACI code. These are:

1. Direct design method:

The calculation of moments is based on the total statical moment, Mo. This moment is divided between positive and negative moment in a span. These moments are further divided between middle and column strips.

2. Equivalent frame method:

The slab is divided into series of two-dimensional frames in each direction, and the positive and negative moments are computed by an elastic frame analysis. Then, these moments are divided between middle and column strips

Computer programs are available that are based on finite element method and equivalent frame method. So, the slab system is analyzed as three-dimensional structure (space frame).

8.3 Beam to slab flexural stiffness ratio, α_f:

It is the flexural stiffness, $\frac{4EI}{L}$, of the beam divided by the flexural stiffness of a width of slab bounded laterally by the center lines of the adjacent panels on each side of the beam:

$$\propto_f = \frac{4E_{cb}I_b/L}{4E_{cs}I_s/L}$$

Since, $E_{cb} = E_{cs}$ and L of the beam is the same for the slab, so

$$\propto_f = \frac{I_b}{I_s}$$

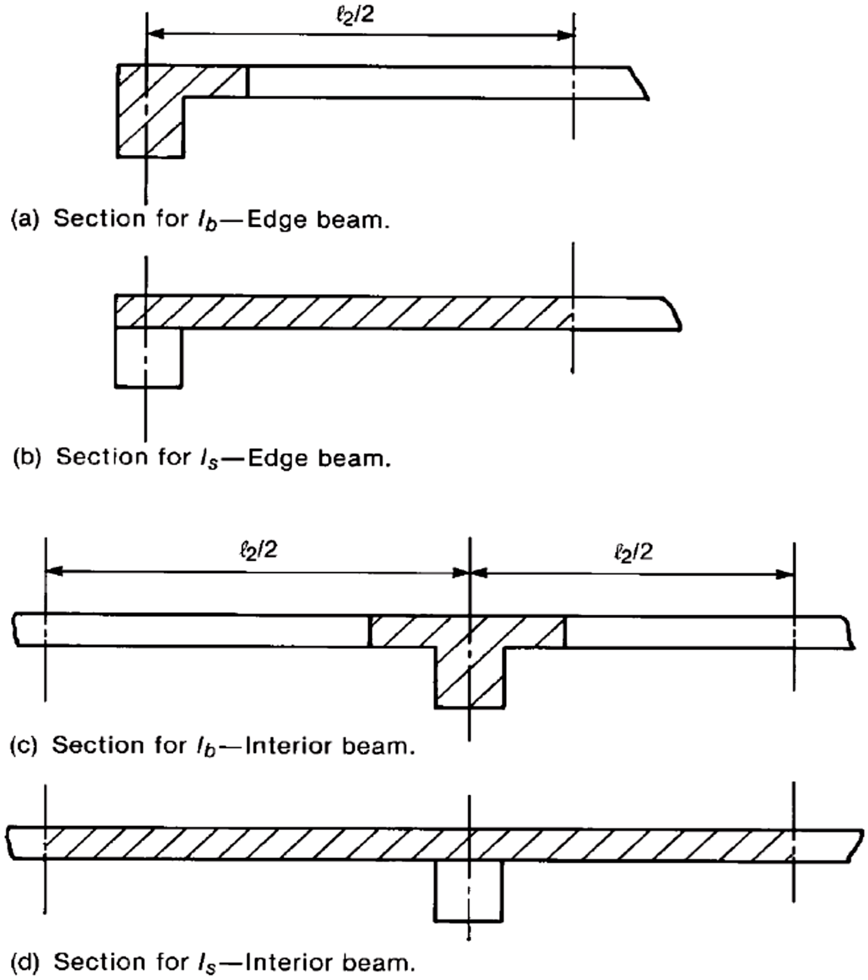


Figure 8.10: Beam and slab sections for calculations of $\propto_f$

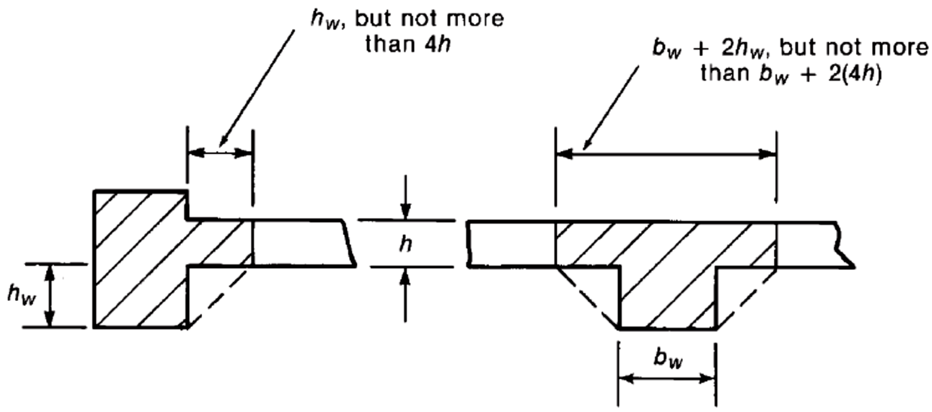


Figure 8.11: Cross section of beams in two-way slab systems

8.4 Minimum thickness of slabs:

ACI 318-19 defines minimum thicknesses that generally are sufficient to limit slab deflections to acceptable values. Thinner slabs can be used if it can be shown that the computed slab deflections will not be excessive.

The slab minimum thickness for two-way slabs can be summarized as follows:

1. For slabs without beams between interior columns and for slabs with beams and $\propto_{fm} \leq 0.2$, the slab minimum thickness can be determined using Table 8.1 (18.3.1.1 ACI 318-19).

   The slab thickness shall be not less than 125mm in slabs without drop panels or 100mm in slabs with drop panels having dimensions defined in ACI code.

   The edge beam defined in ACI 318-19 Table 8.3.1.1 has a value of $\propto_f$ not less than 0.8. In general, a beam with height of at least 2h and of gross area $4h^2$ shall have $\propto_f \geq 0.8$, h is the thickness of the slab.

   As a recommendation, one shall use a slab thickness at least 10% thicker than the ACI code minimum values to avoid excessive deflections.

Table 8.1: Minimum thickness of two-way slabs without interior beams

Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)[1]

f_y, MPa[2]	Without drop panels [3]			With drop panels [3]		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams[4]		Without edge beams	With edge beams[4]	
280	$L_n/33$	$L_n/36$	$L_n/36$	$L_n/36$	$L_n/40$	$L_n/40$
420	$L_n/30$	$L_n/33$	$L_n/33$	$L_n/33$	$L_n/36$	$L_n/36$
560	$L_n/27$	$L_n/30$	$L_n/30$	$L_n/30$	$L_n/33$	$L_n/33$

[1]l_n is the clear span in the long direction, measured face-to-face of supports (mm).
[2]For f_y between the values given in the table, minimum thickness shall be calculated by linear interpolation.
[3]Drop panels as given in 8.2.4.
[4]Slabs with beams between columns along exterior edges. Exterior panels shall be considered to be without edge beams if α_f is less than 0.8.

2. For slabs of beams with:

$$0.2 < \alpha_{fm} < 2$$

The slab thickness is given by:

$$h = \frac{L_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta(\alpha_{fm} - 0.2)} \geq 125mm$$

3. For slabs of beams with:

$$\alpha_{fm} \geq 2$$

The slab thickness is given by:

$$h = \frac{L_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta} \geq 90mm$$

Where:

h: thickness of slab

L_n: Clear span of the slab panel under consideration, measured in the longer direction

α_{fm}: The average of the values of α_f for the four sides of the panel

β: Longer clear span divided by shorter clear span of the panel

Table 8.2: ACI 318-19 Table 8.3.1.2—Minimum thickness of nonprestressed two-way slabs with beams spanning between supports on all sides

<table>
<tr><th>α_{fm}[1]</th><th colspan="2">Minimum h, mm</th><th></th></tr>
<tr><td>$\alpha_{fm} \leq 0.2$</td><td colspan="2">8.3.1.1 applies</td><td>(a)</td></tr>
<tr><td rowspan="2">$0.2 < \alpha_{fm} \leq 2.0$</td><td rowspan="2">Greater of:</td><td>$\dfrac{L_n\left(0.8 + \dfrac{f_y}{1400}\right)}{36 + 5\beta(\alpha_{fm} - 0.2)}$</td><td>(b)[1],[2]</td></tr>
<tr><td>125</td><td>(c)</td></tr>
<tr><td rowspan="2">$\alpha_{fm} > 2.0$</td><td rowspan="2">Greater of:</td><td>$\dfrac{L_n\left(0.8 + \dfrac{f_y}{1400}\right)}{36 + 9\beta}$</td><td>(d)</td></tr>
<tr><td>90</td><td>(e)</td></tr>
</table>

[1]α_{fm} is the average value of α_f for all beams on edges of a panel.

[2]L_n is the clear span in the long direction, measured face-to-face of beams (in.).

[3]β is the ratio of clear spans in long to short directions of slab.

8.5 Direct design method limitations:

1. There shall be a minimum of three continuous spans in each direction
2. Panels shall be rectangular, with a ratio of longer to shorter span center to center of supports within a panel less than 2.0
3. Successive span lengths center to center of supports in each direction shall not differ by more than one-third the longer span
4. Offset of columns by a maximum of 10% of the span in the direction of offset from either axis between centerlines of successive columns shall be permitted
5. All loads shall be due to gravity only and uniformly distributed over an entire panel.
6. The unfactored live load shall not exceed two times the unfactored dead load, $W_L/W_D \leq 2$.
7. For a panel with beams between supports on all sides, the following equation shall be satisfied for beams in the two perpendicular directions:

$$0.2 \leq \frac{\alpha_{f1} {l_2}^2}{\alpha_{f2} {l_1}^2} \leq 5$$

Where $\alpha_{f1} and\ \alpha_{f2}$ are calculated as follows:

$\alpha_{f1} = \alpha_{fA} + \alpha_{fB}$ $\alpha_{f2} = \alpha_{fC} + \alpha_{fD}$

$\alpha_{fA} and\ \alpha_{fB}$ are $\alpha_f for\ the\ beams\ in\ direction\ 1$

$\alpha_{fC} and\ \alpha_{fD}$ are $\alpha_f for\ the\ beams\ in\ direction\ 2$

L1 and L2 are spans of the panel in directions 1 and 2 respectively.

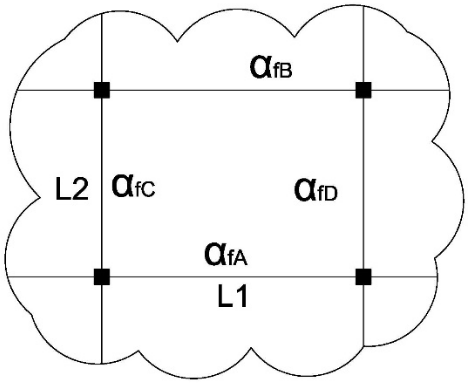


Figure 8.12: Panel with beams between all supports

8.6 Definition of column and middle strips:

The moments vary continuously across the width of the slab panel. To aid in steel placement, the design moments are averaged over the width of column strips over the columns and middle strips between the column strips.

The column strips in both directions extend one fourth of the smaller panel, Lmin, each way from the column centerline.

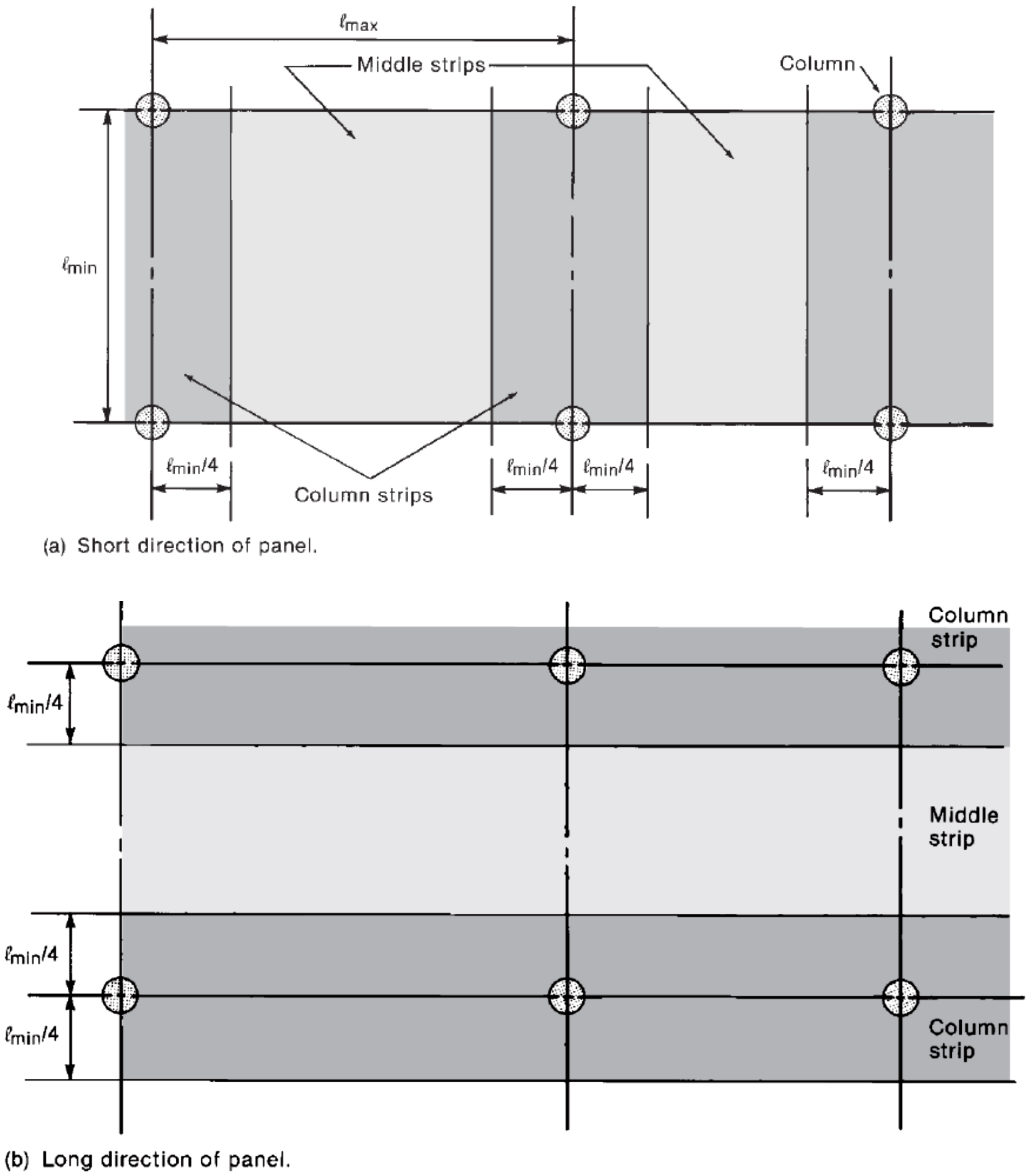


Figure 8.13: Definitions of column and middle strips

8.7 Steps of the direct design method:

The following steps are done to each frame in the slab system.

1. Calculate the total statical moment, Mo for each span. Mo is given by:

$$M_o = \frac{q_u l_2 {l_n}^2}{8}$$

Where L_n is the length of the clear span that the moments are being determined. Clear span Ln shall extend from face to face of columns, capitals, brackets, or walls. The value of Ln shall not be less than 0.65 L_1. L_1 is the length of the span center to center of supports (columns).

L_2 is the frame width and q_u is the uniform load on the slab.

Circular or regular polygon-shaped supports shall be treated as square supports with the same area.

2. Determine the negative and positive factored moments in the span.

   Negative factored moments shall be located at face of rectangular supports. Circular or regular polygon-shaped supports shall be treated as square supports with the same area.

   In an interior span, total statical moment, Mo, shall be distributed as follows:

   - Negative factored moment= 0.65 Mo
   - Positive factored moment= 0.35 Mo

   In an end span, total statical moment, Mo, shall be distributed as shown in Table below (ACI 318-14). The direct design and the equivalent frame methods are deleted from ACI 318-19.

Table 8.3: ACI 318-14 Table 8.10.4.2—Distribution coefficients for end spans

	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65

Edge of beams or edges of slab shall be proportioned to resist in torsion their share of exterior negative factored moments.

The gravity load moment to be transferred between slab and edge column shall be 0.3Mo in slabs without beams. This moment shall be used in shear- moment transfer at exterior column.

3. Determine factored moments in column strip.

   See Tables below from ACI 318-14.

Table 8.4: ACI 318-14 Table 8.10.5.1—Portion of interior negative *Mu* in column strip

α_f L2/L1	L2/L1		
	0.5	**1.0**	**2.0**
0	0.75	0.75	0.75
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

Table 8.5: ACI 318-14 Table 8.10.5.2—Portion of exterior negative *Mu* in column strip

α_f L2/L1	β*t*	*L2/L1*		
		0.5	**1.0**	**2.0**
0	0	1.0	1.0	1.0
	≥2.5	0.75	0.75	0.75
≥1.0	0	1.0	1.0	1.0
	≥2.5	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown. β*t* is calculated using Eq. (8.10.5.2a), where *C* is calculated using Eq. (8.10.5.2b).

$$\beta_t = \frac{\text{Torsional stiffness of transverse beam}}{\text{Flexural stiffness of slab}} = \frac{GC}{EI} = \frac{\left(\frac{E}{2}\right)C}{EI} = \frac{E_b C}{2E_s I_s}$$

$$G = \frac{E}{2(1+\nu)} \qquad let\ \nu = 0.0 \qquad G = \frac{E}{2}$$

$$C = \sum\left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3}$$

Table 8.6: ACI 318-14 Table 8.10.5.5—Portion of positive *Mu* in column strip

$\alpha_f\ L2/L1$	*L2/L1*		
	0.5	**1.0**	**2.0**
0	0.60	0.60	0.60
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

4. Determine the factored moments in beams:

The moments in beams shall be based on Table 8.7 (ACI 318-14 Table 8.10.5.7.1).

Table 8.7: ACI 318-14 Table 8.10.5.7.1—Portion of column strip *Mu* in beams

$\alpha_f L_2/L_1$	**Distribution Coefficient**
0.0	0.0
≥ 1.0	0.85

Note: Linear interpolation shall be made between values shown.

The beam also shall carry any direct load on it.

The rest of the moments shall be resisted by the slab in the column strip.

5. Determine the factored moments in the middle strip.

That portion of negative and positive factored moments of the frame not resisted by the column strips shall be proportionately assigned to corresponding half middle strips

Each middle strip shall be proportioned to resist the sum of the moments assigned to its two half middle strips.

A middle strip adjacent to and parallel with a wall-supported edge shall be assigned to half middle strip corresponding to the first row of interior supports.

8.8 Factored shear in slab systems with beams:

Beams between supports shall resist the portion of shear in accordance with ACI 318-14 Table 8.10.8.1 caused by factored loads on tributary areas in accordance with Figure 2.12 below.

Table 8.8: ACI 318-14 Table 8.10.8.1—Portion of shear resisted by beam

$\alpha_f L_2/L_1$	**Distribution coefficient**
0	0
≥1.0	1.0

Note: Linear interpolation shall be made between values shown.

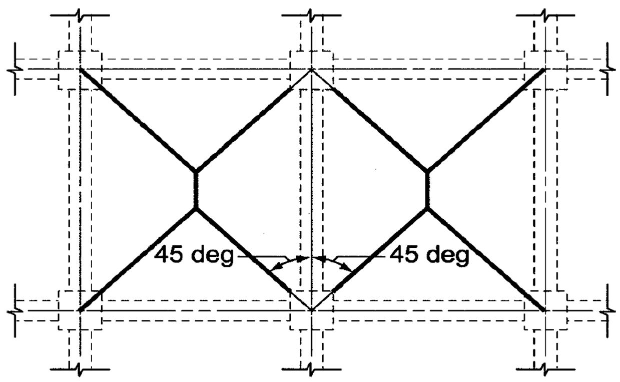


Figure 8.14: Tributary area for shear on an interior beam.

In addition to shears calculated according to 45 degrees principle, beams shall resist shears caused by factored loads applied directly to the beams, including the weight of the beam stem above and below the slab.

8.9 Factored moments in columns and walls:

Columns and walls built integrally with slab system shall resist moments caused by factored loads on the slab system.

At interior support, supporting elements above and below the slab shall resist the factored moment specified by the following equation (ACI 318-14 equation 8.10.7.2) in direct proportion to their stiffnesses unless a general analysis is made.

$$M_{sc} = 0.07\left[(q_{Du} + 0.5q_{Lu})l_2 {l_n}^2 - {q_{Du}}' {l_2}' ({l_n}')^2\right]$$

where q_{Du}', L2′, and Ln′ refer to the shorter span.

8.10 Notes on slab reinforcement:

- For nonprestressed solid slabs, maximum spacing, *s,* of deformed longitudinal reinforcement shall be the lesser of **2*h*** and 450 mm at critical sections.
- Minimum steel is $A_{s,min} = 0.0018A_g$

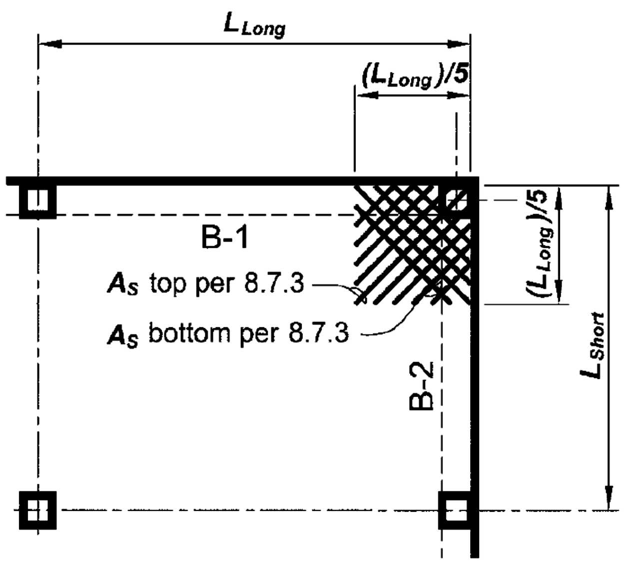
Corner reinforcement:

At exterior corners of slabs supported by edge walls or where one or more edge beams have a value of αf greater than 1.0, reinforcement at top and bottom of slab shall be designed to resist Mu per unit width due to corner effects equal to the maximum positive Mu per unit width in the slab panel.

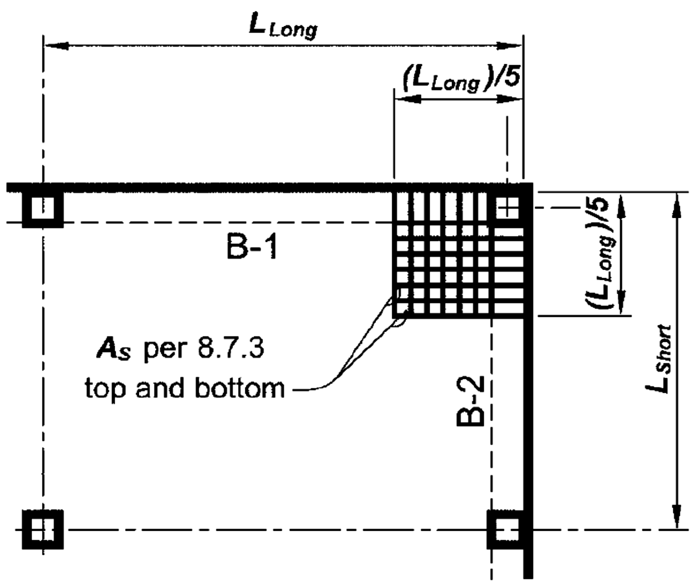
Factored moment due to corner effects, Mu, shall be assumed to be about an axis perpendicular to the diagonal from the corner in the top of the slab and about an axis parallel to the diagonal from the corner in the bottom of the slab.

Reinforcement shall be provided for a distance in each direction from the corner equal to one-fifth the longer span.

Reinforcement shall be placed parallel to the diagonal in the top of the slab and perpendicular to the diagonal in the bottom of the slab. Alternatively, reinforcement shall be placed in two layers parallel to the sides of the slab in both the top and bottom of the slab.



OPTION 1



OPTION 2

Figure 8.15: Corner reinforcement

Flexural reinforcement:

The reinforcement of two-way slab without beams is shown in Figure 8.16 ACI 318-19 Figure 8.7.4.1.3.

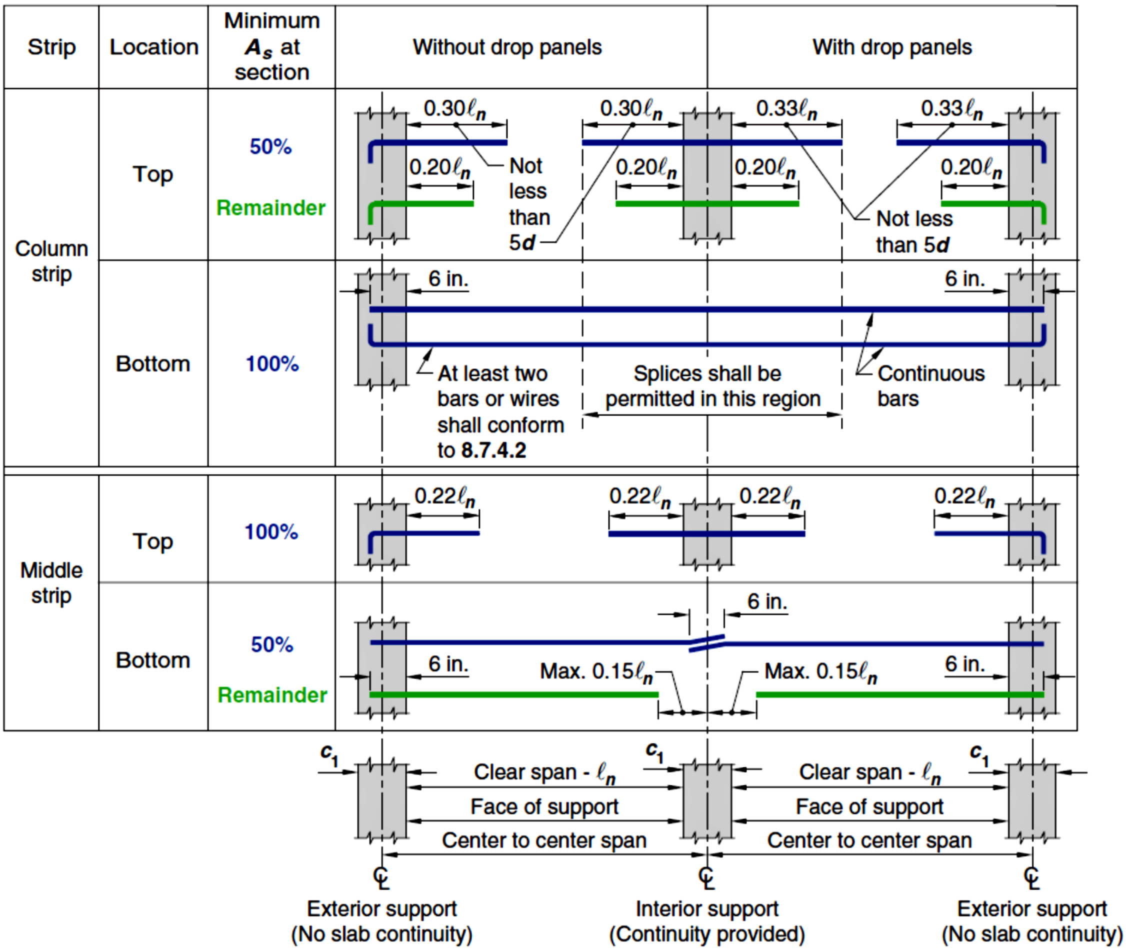


Fig. 8.7.4.1.3—Minimum extensions for deformed reinforcement in two-way slabs without beams.

Figure 8.16: Minimum extensions for deformed bars in two-way slabs without beams

Structural integrity:

1. All bottom deformed bars within the column strip, in each direction, shall be continuous or spliced with full mechanical, full welded, or Class B tension splices at or near the supports.
2. At least two of the column strip bottom bars or wires in each direction shall pass within the region bounded by the longitudinal reinforcement of the column and shall be anchored at exterior supports.

8.11 Shear – Moment transfer:

The punching shear stress resistance of the slab is given by:

$$v_c \leq 0.33\lambda_s\lambda\sqrt{f'_c}$$

$$v_c \leq 0.17\lambda_s\lambda\left(1+\frac{2}{\beta}\right)\sqrt{f'_c}$$

$$v_c \leq 0.083\lambda_s\lambda\left(2+\frac{\alpha_s d}{b_o}\right)\sqrt{f'_c}$$

Where:

b_o: The perimeter length of the critical zone

$\beta: Ratio\ of\ \dfrac{long\ side}{short\ side}\ of\ column$

α_s: Factor describes the location of the column

$\alpha_s = 40 \qquad for\ interior\ column$

$\alpha_s = 30 \qquad for\ edge\ column$

$\alpha_s = 20 \qquad for\ corner\ column$

$\lambda_s: Factor\ used\ to\ modify\ shear\ strength\ based\ on\ the\ effects\ of\ member\ depth,$

$commonly\ referred\ to\ as\ the\ size\ effect\ factor.$

$$\lambda_s = \sqrt{\frac{2}{1+0.004\,d}} \leq 1.0$$

For $d \leq 250mm, \lambda_s = 1.0$

$\lambda_s\ can\ be\ taken\ equal\ to\ 1.0\ if\ a\ or\ b\ is\ applied$:

a: $\dfrac{A_v}{S} \geq 0.17\sqrt{f'_c}\dfrac{b_o}{f_{yt}}$

b: $v_u \leq \emptyset 0.5\sqrt{f'_c}$

v_c for two-way shear with shear reinforcement (closed stirrups) is given by:

$$v_c \leq 0.17\lambda_s\lambda\sqrt{f'_c}$$

For shear capacity:

$$\emptyset V_n \geq V_u$$

$$\emptyset V_n = \emptyset(V_c + V_s)$$

Where:

V_c= shear resistance attributed to the concrete

V_s= shear resistance attributed to the steel reinforcement

V_u= factored or ultimate shear force due to the applied loads

V_n= nominal shear resistance of the slab

Two-way shear is assumed to be critical on a vertical section through the slab extending around the column. According to ACI code, this section is chosen so that it is never less than d/2 from the face of column so that its length bo, is a minimum.

For two-way members with shear reinforcement, effective depth shall be selected such that v_u calculated at critical sections does not exceed the values in ACI 318-19 Table 22.6.6.3.

Table 8.8: Table 22.6.6.3—Maximum v_u for two-way members with shear reinforcement

Type of shear reinforcement	**Maximum v_u at critical sections defined in 22.6.4.1**	
Stirrups	$\emptyset 0.50\sqrt{f'c}$	(a)
Headed shear stud reinforcement	$\emptyset 0.667\sqrt{f'c}$	(b)

Single- or multiple-leg stirrups fabricated from bars or wires shall be permitted to be used as shear reinforcement in slabs and footings satisfying (a) and (b):

(a) ***d*** is at least 150mm.

(b) ***d*** is at least **16*db***, where ***db*** is the diameter of the stirrups

For two-way members with stirrups, ***vs*** shall be calculated by:

$$v_s = \frac{A_v f_{yt}}{b_o s} \rightarrow \frac{A_v}{s} = \frac{v_s b_o}{f_{yt}}$$

where *Av* is the sum of the area of all legs of reinforcement on one peripheral line that is geometrically similar to the perimeter of the column section, and ***s*** is the spacing of the peripheral lines of shear reinforcement in the direction perpendicular to the column face.

If $v_{uv} > \emptyset\, 0.17\, \lambda_s \lambda \sqrt{f'_c}$ on the critical section for two-way shear surrounding a column, concentrated load, or reaction area, *As,min*, provided over the width b_{slab}, shall satisfy the following equation:

$$A_{s,min} = \frac{5 v_{uv} b_{slab} b_o}{\phi \alpha_s f_y}$$

$b_{slab} = 1.5h + 1.5h + \text{column width for interior column}$

$b_{slab} = 1.5h + \text{column width for edge and corner column}$

Where h is the thickness of the slab or the thickness of the drop panel if exists.

For slab with openings, refer to ACI 318-19 section 8.5.4.

"Tests on interior column-to-slab connections with lightly reinforced slabs with and without shear reinforcement have shown that yielding of the slab flexural tension reinforcement in the vicinity of the column or loaded area leads to increased local rotations and opening of any inclined crack existing within the slab. In such cases, sliding along the inclined crack can cause a flexure-driven punching failure at a shear force less than the strength calculated by the two-way shear equations of Table 22.6.5.2 for slabs without shear reinforcement and less than the strength calculated in accordance with 22.6.6.3 for slabs with shear reinforcement.

Tests of slabs with flexural reinforcement less than $A_{s,min}$ have shown that shear reinforcement does not increase the punching shear strength. However, shear reinforcement may increase plastic rotations prior to the flexure-driven punching failure.

Inclined cracking develops within the depth of the slab at a shear stress of approximately $0.17\lambda\lambda_s\sqrt{f_c'}$. At higher shear stresses, the possibility of a flexure-driven punching failure increases if $A_{s,min}$ is not satisfied. $A_{s,min}$ was developed for an interior column, such that the factored shear force on the critical section for shear equals the shear force associated with local yielding at the column faces.
To derive Eq. (8.6.1.2) the shear force associated with local yielding was taken as $8A_{s,min}\, f_y d/b_{slab}$ for an interior column connection (Hawkins and Ospina 2017) and generalized as $(\alpha_s/5)A_{s,min}\, f_y d/b_{slab}$ to account for edge and corner conditions. $A_{s,min}$ also needs to be provided at the periphery of drop panels and shear caps." ACI 318-19 section 8.6.1.2.

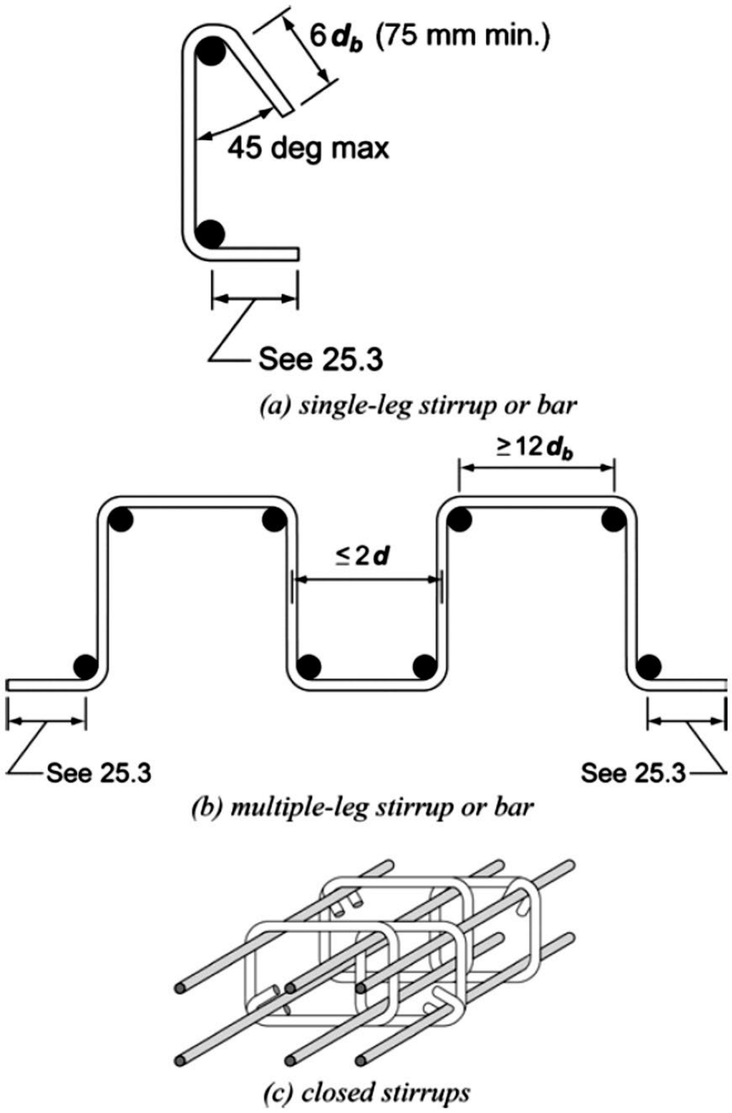


***Fig. R8.7.6(a)-(c)**—Single- or multiple-leg stirrup-type slab shear reinforcement.*

Figure 8.17: Slab shear reinforcement- stirrups (ACI 318-14)

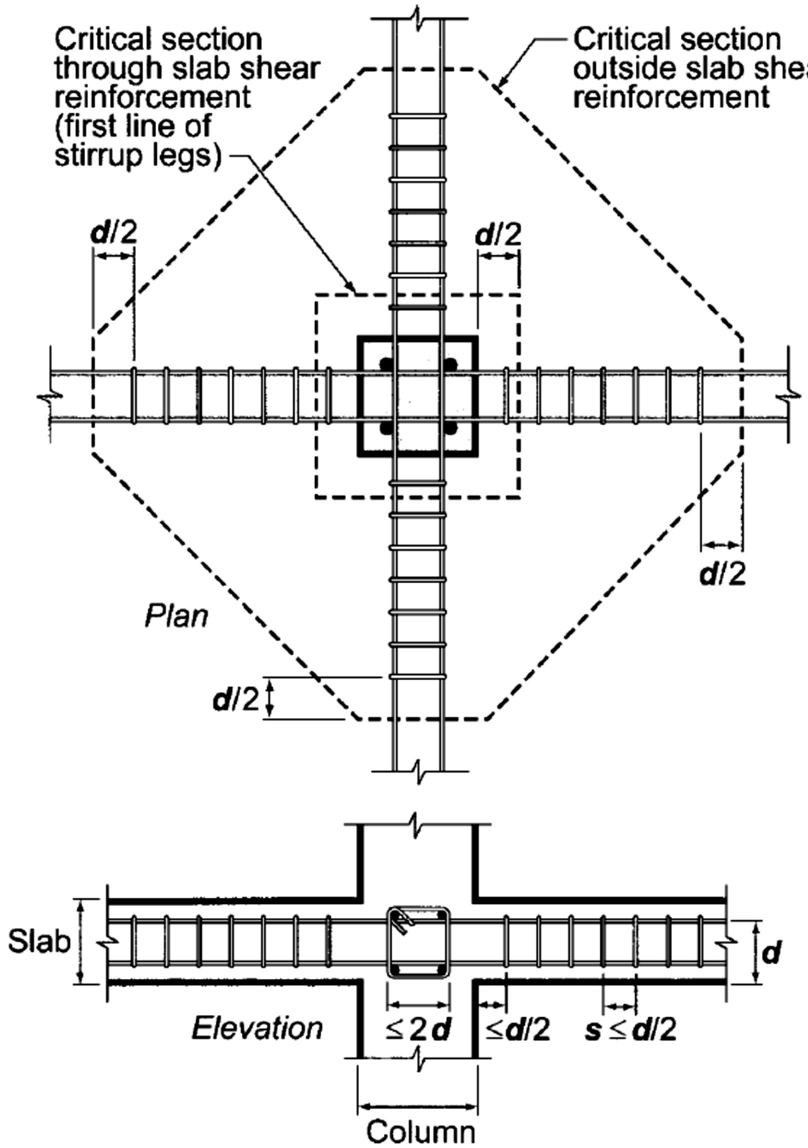


Figure 8.18: Shear reinforcement at interior column (ACI 318-14)

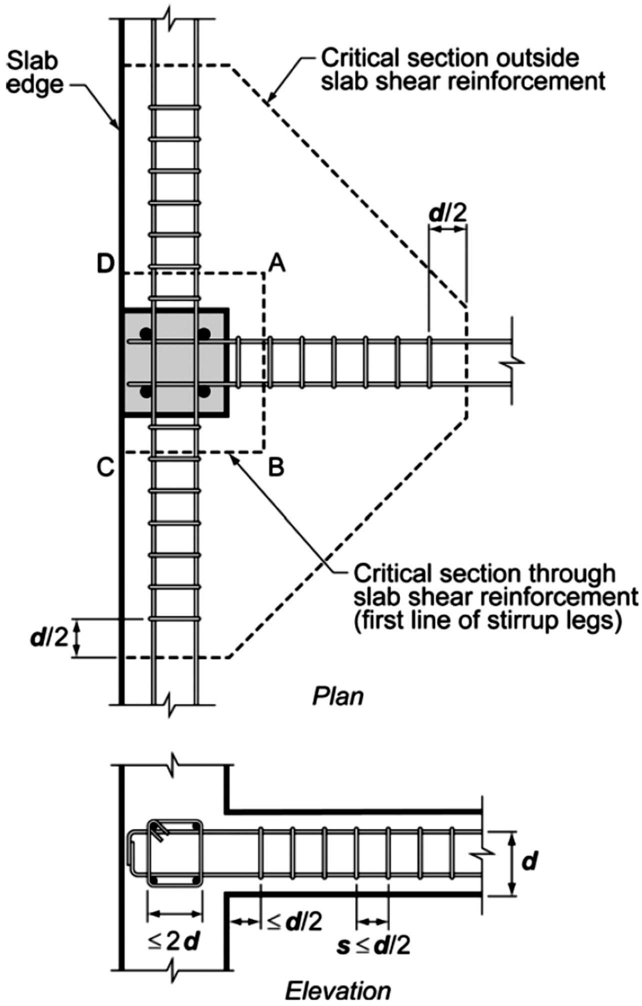
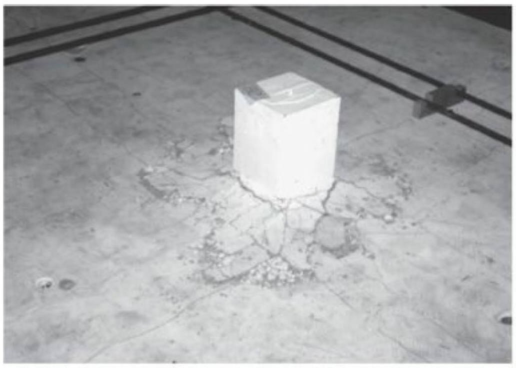
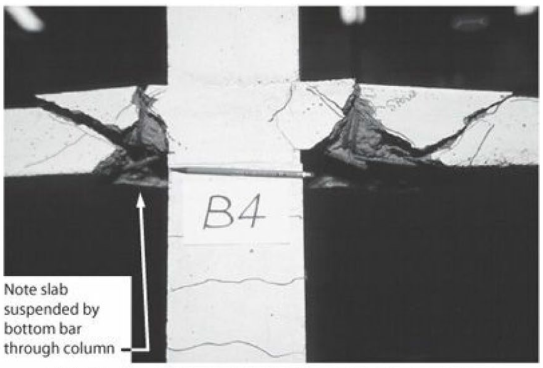


Figure 8.19: Shear reinforcement at edge column (ACI 318-14)

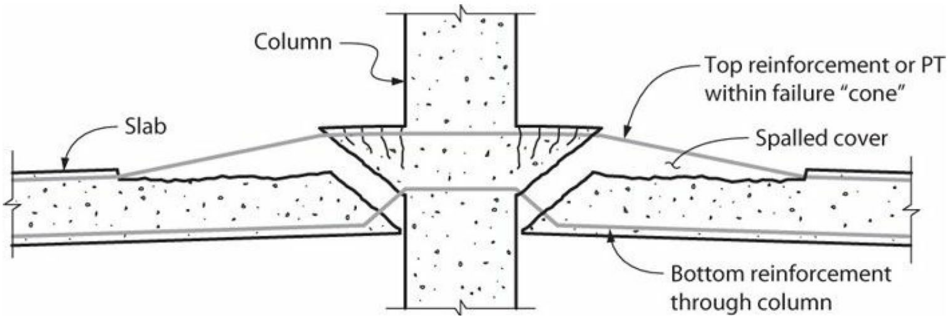


(a) Interior connection viewed from top at angle



(b) Exterior connection viewed from exterior

Picture 8.1: Punching shear failure



Picture 8.1- continued: Punching shear failure

Picture 1 shows the punching shear failure in a flat plate slab. On the basis of these observations, ACI 352.1 recommends structural integrity reinforcement in the form of continuous bottom reinforcement passing through the column core at every slab-column connection. The amount of reinforcement is calculated from equilibrium considerations. The total load to be resisted at an interior connection is taken equal to $w_u A_t$, where At is the column tributary area. Defining the area of reinforcement along each principal direction as $A_{s,min}$, the total available steel area at an interior connection is $4A_{s,min}$ (area $A_{s,min}$ enters each of four faces of the column). Assuming the catenary effective at an angle of 30° with respect to horizontal, the total resistance at yield stress is $2A_{s,min}f_y$. Equating demand and capacity, and using a strength reduction factor of $\emptyset = 0.9$, the design recommendation is to provide continuous bottom slab reinforcement passing within the column core in each principal direction (These bottom bars shall have at least development length into the slab in all directions from face of column) satisfying:

$$A_{s,min} = \frac{w_u A_t}{2\emptyset f_y}$$

Where $A_{s,min}$ is the area of steel that passes in the column core at each face of column ($4A_{s,min}$ for an interior column).

For an edge column:

$$A_{s,min} = \frac{w_u A_t}{1.5\emptyset f_y}$$

And for a corner column:

$$A_{s,min} = \frac{w_u A_t}{1.0\emptyset f_y}$$

ACI 318-19 section 8.7.5.6.3.1 states equations for the bottom bars through column core for prestressed concrete slabs if tendons are not provided through the column core to achieve structural integrity.

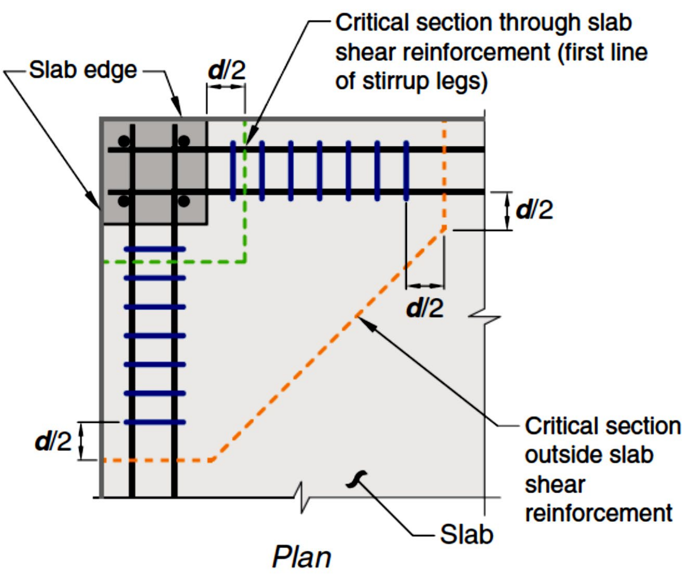


Fig. R22.6.4.2c—Critical sections for two-way shear in slab with shear reinforcement at corner column.

Figure 8.20: Shear reinforcement at corner column (ACI 318-19)

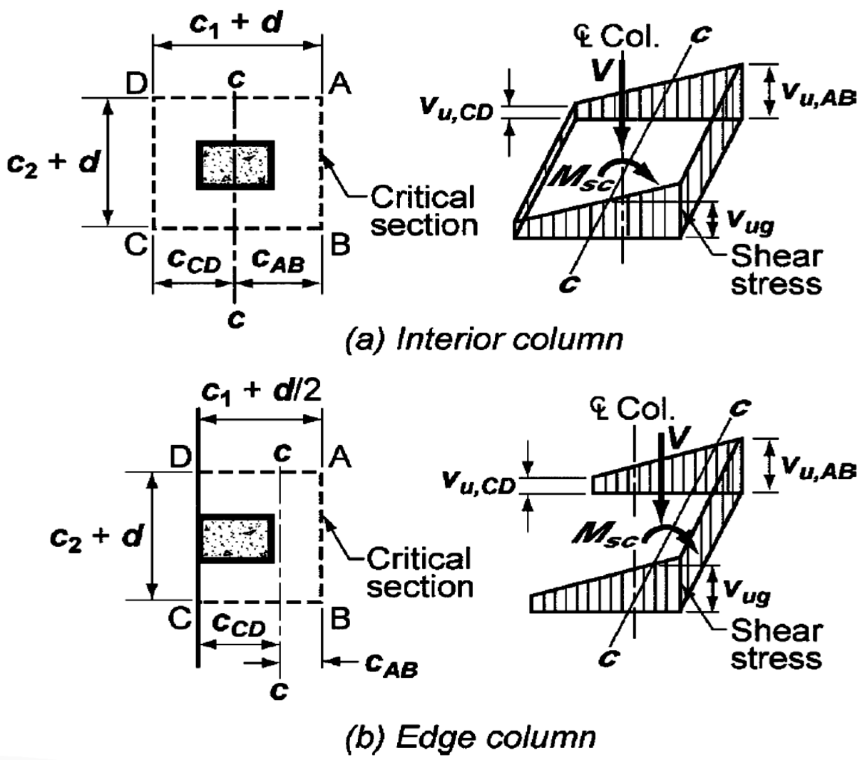


Fig. R8.4.4.2.3—Assumed distribution of shear stress.

Figure 8.21: Distribution of shear stress for shear-moment transfer (ACI 318-14)

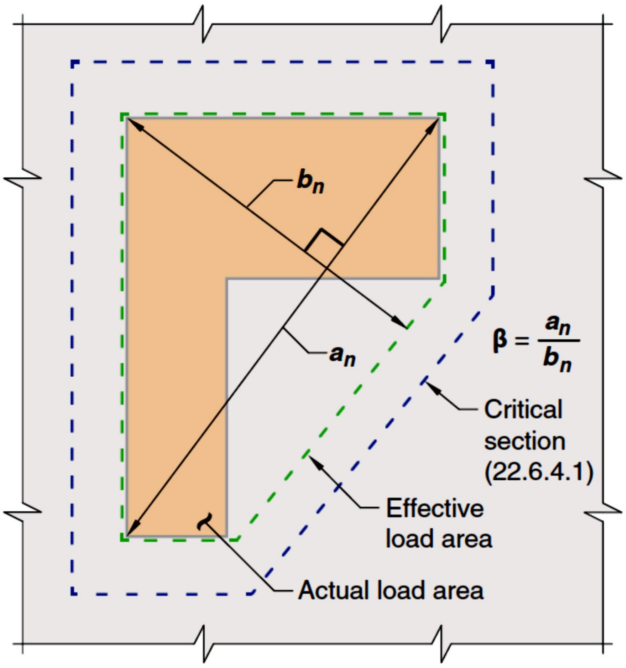


Figure 8.22: Value of β for a nonrectangular loaded area (ACI 318-19)

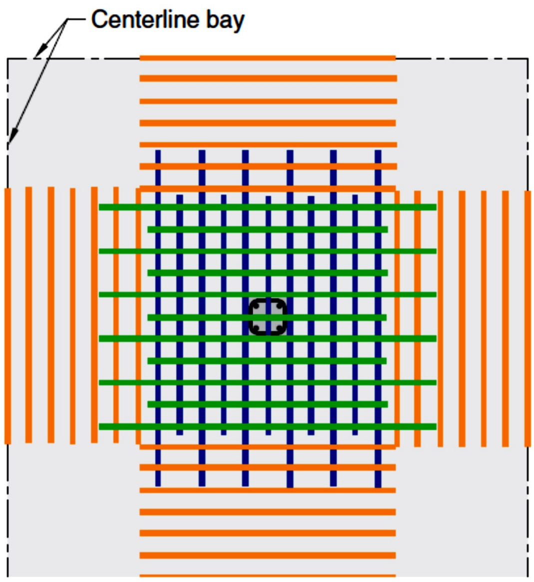


Fig. R8.6.1.1—Arrangement of minimum reinforcement near the top of a two-way slab.

Figure 8.23: Arrangement of minimum reinforcement near the top of a two-way slab (ACI 318-19)

The maximum shear stress on the critical section surrounding the column for slab- column connections transferring shear and moment is given by:

$$v_u = \frac{V_u}{b_o d} + \frac{\gamma_{v1} M_{u1} c_1{}'}{J_{c1}} + \frac{\gamma_{v2} M_{u2} c_2{}'}{J_{c2}}$$

Where:

Vu= the factored shear being transferred from the slab to the column, and it is assumed to act through the centroid of the critical section for shear. It equals to column load considering loads outside the critical zone for punching.

Mu= the factored moment being transferred at the connection (unbalanced moment).

b_o= the length of the critical shear perimeter.

J_c= property of assumed critical section analogous to polar moment of inertia.

c'= the measurement from the centroid of the critical shear perimeter to the edge of the perimeter where the stress, vu, is being calculated.

γ_v= the fraction of the moment that is transferred by shear stresses on the critical section and is defined as:

$$\gamma_v = 1 - \gamma_f$$

$$\gamma_f = \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}}$$

γ_f= the fraction of the moment that is transferred by direct flexure. Reinforcement already designed for flexure in this region can be used to satisfy all or part of this strength requirement.

b_1= the total width of the critical section in direction of frame, or perpendicular to the axis about which the moment acts.

b_2= the total width of the critical section perpendicular to the frame direction.

The value of $\gamma_f M_u$ shall be resisted by the slab at a section of width equal to the column side length c2 + 1.5 h at each side for a column in an interior frame or to a section width of c2 + 1.5h at one side for column in an exterior frame.

For interior column:

The centroid of the critical section is located at the center of the columns and at the center distances of b1 and b2. Jc is given by:

$$J_c = 2\left(\frac{{b_1}^3 d}{12} + \frac{b_1 d^3}{12} + b_2 d\left(\frac{b_1}{2}\right)^2\right)$$

$$b_1 = c_1 + d \qquad b_2 = c_2 + d$$

For edge column of frame in x- direction or direction 1:

In direction 1, Jc is given by:

$$J_{c1} = 2\left(\frac{{b_1}^3 d}{12} + \frac{b_1 d^3}{12} + b_1 d\left(x' - \frac{b_1}{2}\right)^2\right) + b_2 d x'^2$$

$$b_1 = c_1 + d/2 \qquad b_2 = c_2 + d$$

The value x' which is the distance from the centroid of the critical section to the right edge is given by:

$$x' = \frac{2b_1\left(\frac{b_1}{2}\right)}{2b_1 + b_2}$$

In direction 2, Jc is given by:

$$J_{c2} = \left(\frac{{b_1}^3 d}{12} + \frac{b_1 d^3}{12}\right) + 2b_2 d\left(\frac{b_1}{2}\right)^2$$

The unbalanced moment, Mu, in direction 1, can be determined by:

$$Mu1 = (M_{uA} - M_{uB}) + (\frac{V_{uA}c_1}{2} - \frac{V_{uB}c_1}{2})$$

Where:

M_{uA}= moment at column left face

M_{uB}= moment at column right face

V_{uA}= shear at column left face

V_{uB}= shear at column right face

C1= length of column side in direction 1

One can neglect

$$(\frac{V_{uA}c_1}{2} - \frac{V_{uB}c_1}{2})$$

8.12 Notes on ribbed, waffle and voided slabs:

1. Slab thickness: The slab thickness is determined based on α_{fm}. The calculated value is for a solid slab. A ribbed, waffle or voided slab can be proposed which has a moment of inertia greater than or equals that for solid slab.
2. Slab self-weight: The slab self-weight shall account for the voids and blocks in the slab.
3. Slab shear capacity: The shear strength is provided by the width of web in addition to web shear reinforcement. It is not recommended (Shear reinforcement shall be used if required) to use shear reinforcement in U-Boot and Cobiax voided slab systems. Shear reinforcement can be used for large thickness ribbed and waffle slabs. The shear capacity of the rib can be increased by 10%.
4. Slab- beam stiffness, α_f: The beam can be considered as rectangle, L-shape or T-shape. The moment of inertia of the slab must take the voids into account
5. Slab flexural design: The U-Boot and Cobiax slabs can be designed for flexure as solid slab since the depth of the compression zone is within the flange. The waffle and the ribbed slabs are designed as T- sections. The moment of a rib can be determined by:

$$M_{rib} = \frac{M_{strip}}{b_{strip}} b_f$$

Where:

M_{rib}: bending moment in a strip; column strip or middle strip

b_{strip}: width of strip; column strip or middle strip

b_f: width of flange of the rib

Example 1: Two-way solid slab with beams

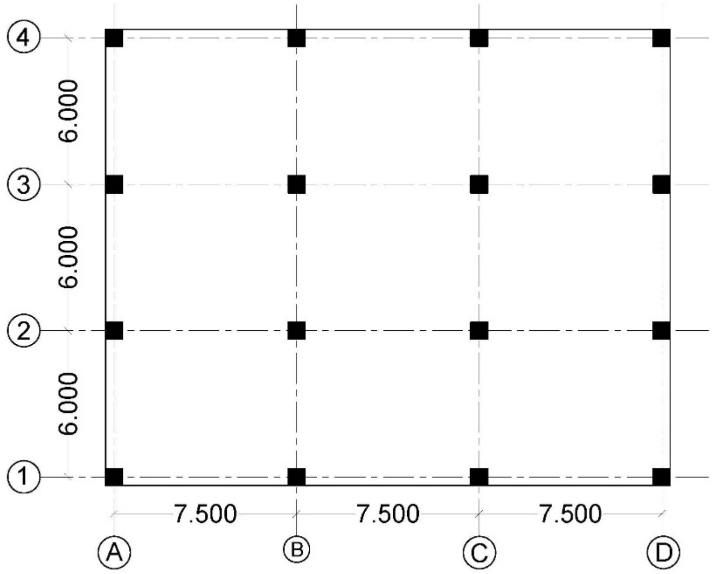


Figure 8.24: Plan for example 1

Given:

- Concrete, f'c= 24MPa
- Steel, fy= 420MPa
- Superimposed dead load, W_{SD}= 4kN/m^2
- Live load, WL= 5kN/m^2
- Perimeter wall weight, W_{WALL}= 21kN/m
- All columns are 0.50m x 0.50m
- Column height, h= 3.50m
- All beams are 400mm width and 600mm thickness

Determine slab thickness and design frame (strip) 2.

Solution:

Slab thickness:

Assume that $\alpha_{fm} \geq 2$, so:

$$h = \frac{l_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta} = \frac{7.1\left(0.8 + \frac{420}{1400}\right)}{36 + 9(1.273)} = 0.165m \geq 0.09m \quad ok$$

where: Ln= 7.50-0.40=7.10m

$$\beta = \frac{long\ clear\ span}{short\ clear\ span} = \frac{7.5-0.4}{6-0.4} = 1.27$$

Try slab thickness, h= 200mm

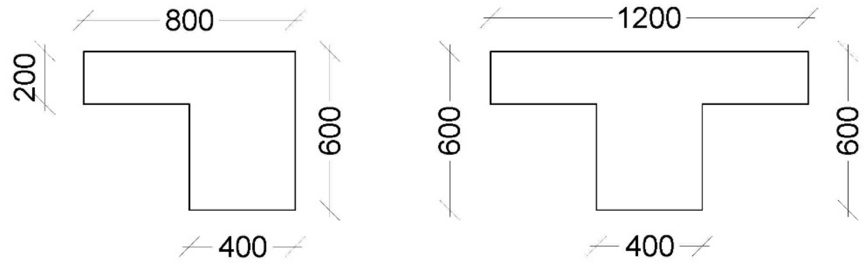


Figure 8.25: Exterior and interior beams in the slab

The moments of inertias are calculated for the two sections, they are:

Edge (Exterior) beam: I= 9.867x10^{-3}m^{4}

Interior beam: I= 11.573x10^{-3}m^{4}

The distribution of α_f is shown in Figure 8.26.

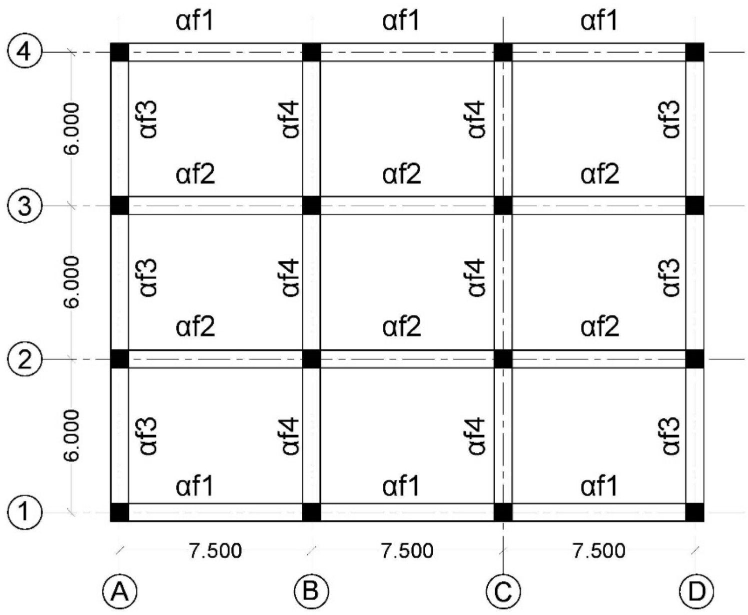


Figure 8.26: Distribution of α_f for the beams

$$\alpha_{f1} = \frac{I - edge\ beam}{I - slab\ of\ 3m\ width} = \frac{9.867x10^{-3}}{\frac{1}{12}x3x0.2^3} = 4.93$$

$$\alpha_{f2} = \frac{I - interior\ beam}{I - slab\ of\ 6m\ width} = \frac{11.573x10^{-3}}{\frac{1}{12}x6x0.2^3} = 2.89$$

$$\alpha_{f3} = \frac{I - edge\ beam}{I - slab\ of\ 3.75m\ width} = \frac{9.867x10^{-3}}{\frac{1}{12}x3.75x0.2^3} = 3.95$$

$$\alpha_{f4} = \frac{I - interior\ beam}{I - slab\ of\ 7.5m\ width} = \frac{11.573x10^{-3}}{\frac{1}{12}x7.5x0.2^3} = 2.31$$

Since all values of α_f are greater than 2.0, the average of any four values shall be not less than 2.0. So, $\alpha_{fm}$for each panel is greater than 2.0.

Slab self-weight, w_D= 0.2(25)=5kN/m^2

Slab ultimate load, w_u= 1.2(5+4)+1.6(5)= 18.8kN/m^2

Check wide beam shear (one-way shear): for stiff beams, the shear can be calculated considering the short direction of the largest panel. Here, the short span, L= 6.0m.

Shear can be calculated at distance d from face of beam, so:

$$V_u = W_u\left(\frac{L}{2} - \frac{b_1}{2} - d\right) = 18.8\left(\frac{6}{2} - \frac{0.4}{2} - 0.16\right) = 49.6kN$$

The shear strength capacity of the slab is given by:

$$\boldsymbol{ACI\ 318-14}: \emptyset V_c = \emptyset\frac{1}{6}\lambda\sqrt{f'_c}b_w d = \frac{0.75\left(\frac{1}{6}\right)(1)\sqrt{24}(1000)(160)}{1000} = 98kN > 49.6kN \quad ok$$

$\boldsymbol{ACI\ 318-19}$:

$$Let\ \rho_w = 0.0018\left(\frac{h}{d}\right) = 0.0018\left(\frac{200}{160}\right) = 0.00225 \qquad \lambda_s = 1.0$$

$$\emptyset V_c = \emptyset\left(0.66\lambda_s\lambda(\rho_w)^{\frac{1}{3}}\sqrt{f'_c} + \frac{N_u}{6A_g}\right)b_w d$$

$$= \frac{0.75\left(0.66(1)(1)(0.00225)^{\frac{1}{3}}\sqrt{24} + 0.0\right)(1000)(160)}{1000} = 50.85kN > 49.6kN \qquad OK$$

Analysis and design of frame 2:

1. Compute the total statical moment, M_o for each span. Here the three spans are equal.

$$M_o = \frac{q_u l_2 {l_n}^2}{8} = \frac{18.8(6)(7)^2}{8} + \frac{0.4(0.4)(25)(1.2)(7)^2}{8} = 720.3kN$$

2. Compute positive and negative moments in the span:

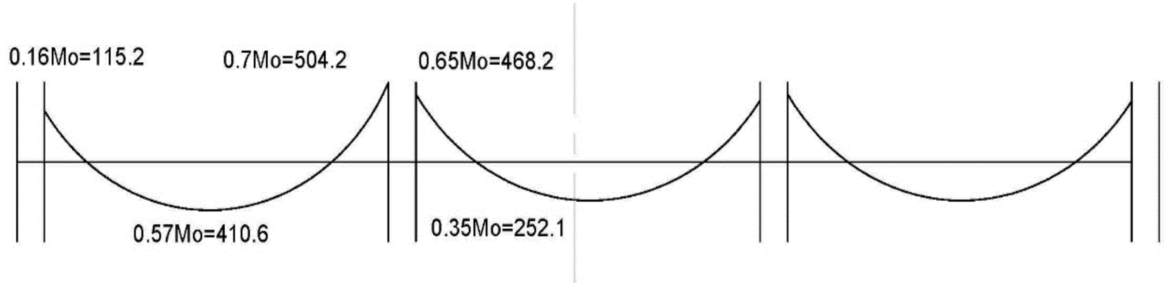


Figure 8.27: Frame bending moment diagram- kN.m

3. Compute the moments in the column strip (slab + beam)

Frame width= 6.0m

Column strip width= 3.0m

Middle strip width= 3.0m

$$\frac{\alpha_f L_2}{L_1} = \frac{2.89(6)}{7.5} = 2.3 > 1$$

$$\frac{L_2}{L_1} = \frac{6}{7.5} = 0.8$$

From ACI 318-14 Table 8.10.5.5, for positive moment, the ratio = 81%

From ACI 318-14 Table 8.10.5.1, for interior negative moment, the ratio = 81%

Calculations for β_t:

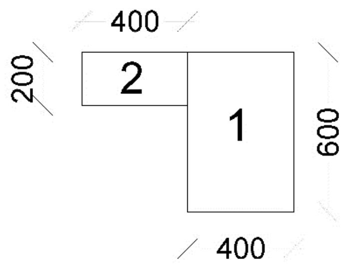


Figure 8.28: Edge beam parts for β_t computations

$$C = \sum\left(1 - 0.63\frac{x}{y}\right)\frac{x^3y}{3} = \left(1 - 0.63\frac{0.4}{0.6}\right)\frac{0.4^3(0.6)}{3} + \left(1 - 0.63\frac{0.2}{0.4}\right)\frac{0.2^3(0.4)}{3}$$
$$= 8.155x10^{-3}m^4$$

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s} = \frac{C}{2I_s} = \frac{8.155(10)^{-3}}{2(\frac{1}{12})(6)(0.2)^3} = 1.02$$

$$\frac{\alpha_f L_2}{L_1} = \frac{2.89(6)}{7.5} = 2.3 > 1$$

$$\frac{L_2}{L_1} = \frac{6}{7.5} = 0.8$$

From ACI 318-14 Table 8.10.5.2, for exterior negative moment, the ratio = 92%

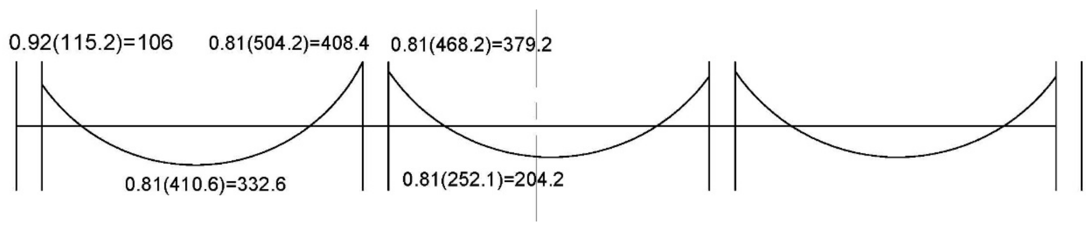


Figure 8.29: Column strip bending moment diagram- kN.m

4. Compute moments in beam:

Since $\frac{\alpha_f L_2}{L_1} > 1.0$, 0.85 of column strip moments are transferred to the beams.

Figure 8.30 below shows the bending moment diagram of the beam in frame 2.

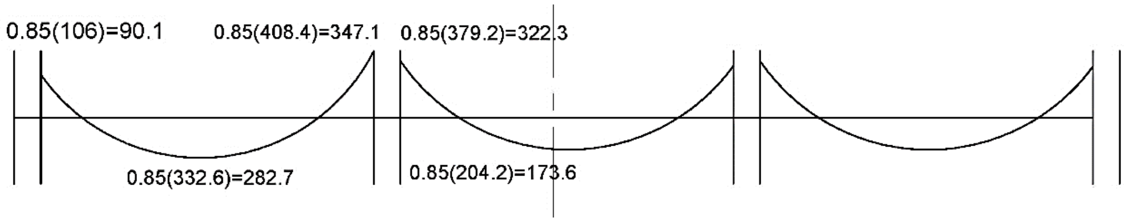


Figure 8.30: Beam bending moment diagram- kN.m

The beam section is shown above, so the steel area can be computed.

Beam section effective depth, d= 600-60=540mm

5. Compute moments in slab column strip:

The slab column strip moment = the column strip moment – the beam moment

Figure 8.31 shows the moments in the slab column strip.

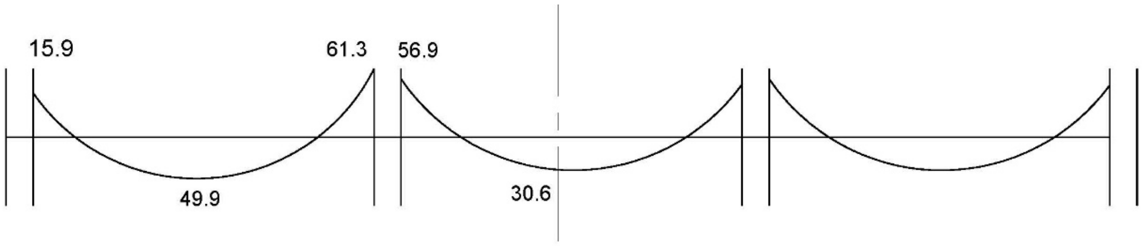


Figure 8.31: Slab column strip bending moment diagram- kN.m

Slab column strip width = 3000mm – 400mm = 2600mm

Effective depth, d= 160mm

Then the steel area can be computed.

6. Compute moments in slab middle strip:

The slab middle strip moment = the frame moment – the column strip moment.

Figure 8.32 shows the moments in the slab middle strip.

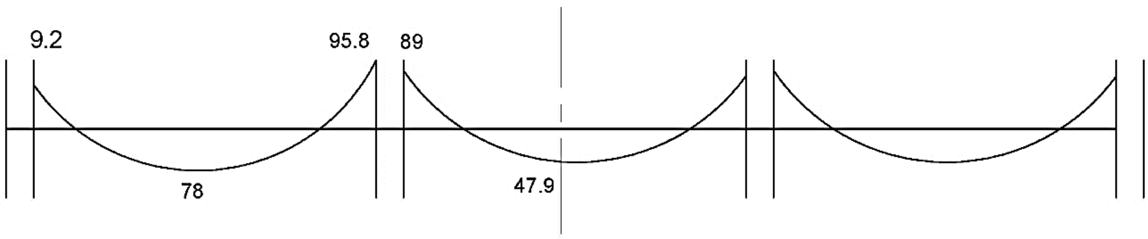


Figure 8.32: Slab middle strip bending moment diagram- kN.m

Slab middle strip width = 3000mm

Effective depth, d= 160mm

Then the steel area can be computed.

7. Compute ultimate shear in the beam:

Beam weight = 0.4(0.4)(25)(1.2)= 4.8kN/m

Exterior span end moments: M_L= 90.1kN.m and M_R= 347kN.m

Load on the beam from slab = 6m x 18.8 = 112.8kN/m

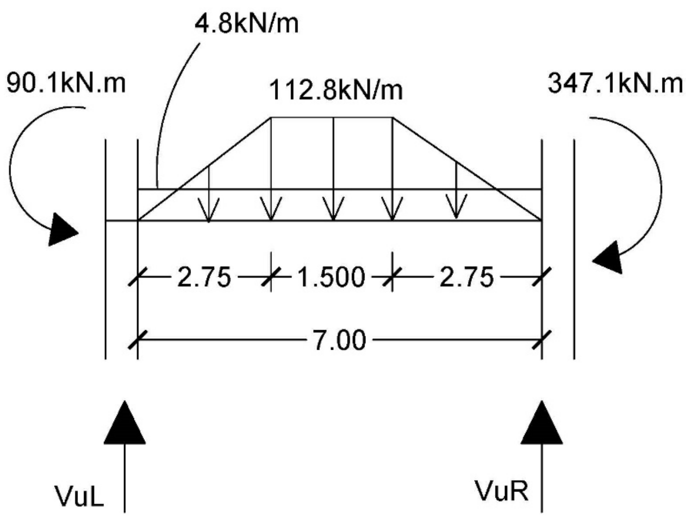


Figure 8.33: Free body diagram for the exterior span in beam in frame 2

The shear values at left and right ends of span are computed as follows:

$$V_{uL} = 112.8(2.75)(0.50) + \left(\frac{1.5}{2}\right)112.8 + \left(\frac{7}{2}\right)(4.8) - \frac{(347.1 - 90.1)}{7} = 220kN$$

$$V_{uR} = 112.8(2.75)(0.50) + \left(\frac{1.5}{2}\right)112.8 + \left(\frac{7}{2}\right)(4.8) + \frac{(347.1 - 90.1)}{7} = 293.2kN$$

For the interior span, the end moments are equal, so they can be cancelled, then:

$$V_{uL} = V_{uR} = 112.8(2.75)(0.50) + \left(\frac{1.5}{2}\right)112.8 + \left(\frac{7}{2}\right)(4.8) = 256.5kN$$

So, the shear reinforcement can be computed.

Example 2: Flat plate

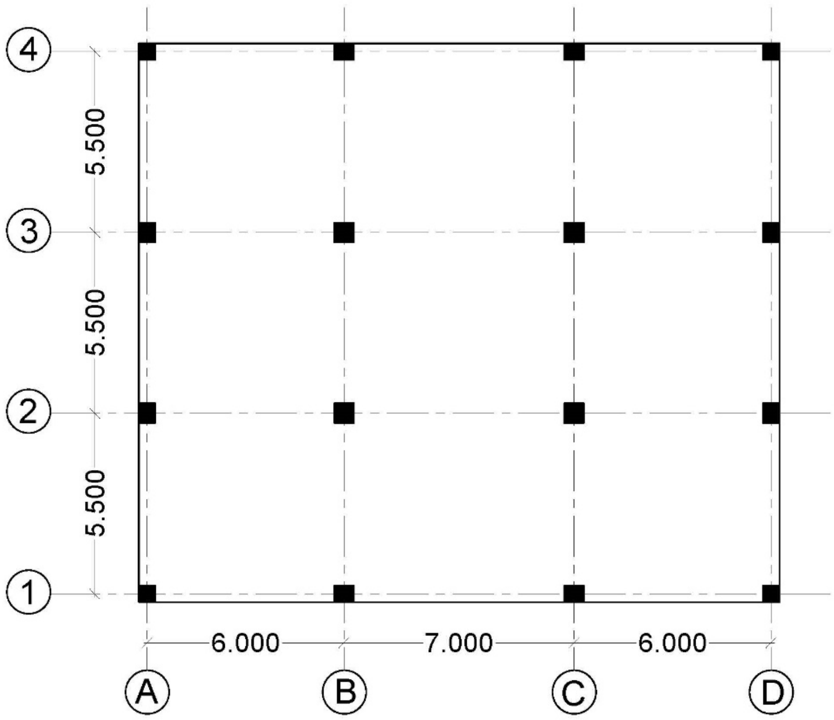


Figure 8.34: Slab layout for Example 2

Given:

- Slab system: flat plate- no beams
- Concrete, f'c= 28MPa
- Steel, fy= 420MPa
- Superimposed dead load, W_{SD}= 3.5kN/m^2
- Live load, WL= 3kN/m^2
- Perimeter wall weight, W_{WALL}= 5kN/m
- Interior columns are 0.60m x 0.60m
- Corner columns are 0.50m x 0.50m

- Edge columns are 0.50m x 0.60m
- Column height, h= 3.00m
- Slab thickness, h= 230mm, d=180mm

Check shear-moment transfer at column A-3.

Solution:

Slab loads:

Slab self-weight, WD= 0.23(25)=5.75kN/m^2

Slab ultimate load, Wu= 1.2(5.75+3.5)+1.6(3)= 15.9kN/m^2

Load on column, Vu= 15.9[(1.15x5.5/2 + 5.5/2)(6/2 + 0.25) – 0.59 x 0.78] + 1.2(5)(1.15x5.5/2 + 5.5/2 -0.78)= 329kN

Critical section properties- Frame 3- Direction 1:

$$M_o = \frac{W_u L_2 {L_n}^2}{8} = \frac{15.9(5.5)(5.45)^2}{8} = 325kN.m$$

$$M_{u1} = 0.3M_o = 0.3(325) = 97.5kN.m$$

$$b_1 = c_1 + \frac{d}{2} = 0.59m \qquad b_2 = c_2 + d = 0.78m$$

$$x' = \frac{2b_1\left(\frac{b_1}{2}\right)}{2b_1 + b_2} = \frac{2(0.59)(\frac{0.59}{2})}{2(0.59) + (0.78)} = 0.178m$$

$$J_{c1} = 2\left(\frac{{b_1}^3 d}{12} + \frac{b_1 d^3}{12} + b_1 d\left(x' - \frac{b_1}{2}\right)^2\right) + b_2 d x'^2 = 14.091x10^{-3}m^4$$

$$\gamma_{f1} = \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}} = 0.633$$

$$\gamma_{v1} = 1 - \gamma_{f1} = 1 - 0.633 = 0.367$$

$$b_o = 2b_1 + b_2 = 2(0.59) + 0.78 = 1.96m \quad (1960mm)$$

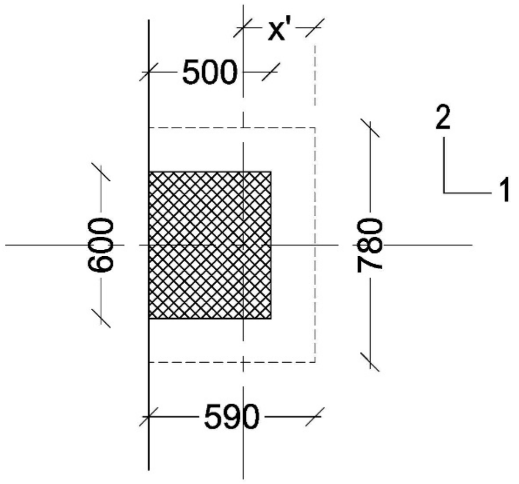


Figure 8.35: Critical section for the column

Critical section properties- Frame A- Direction 2:

$$b_1 = c_2 + d = 0.78m \qquad b_2 = c_1 + d/2 = 0.59m$$

$$J_{c2} = \left(\frac{{b_1}^3 d}{12} + \frac{b_1 d^3}{12}\right) + 2b_2 d\left(\frac{b_1}{2}\right)^2 = 39.8x10^{-3}m^4$$

$$\gamma_{f2} = \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}} = 0.566$$

$$\gamma_{v2} = 1 - \gamma_{f2} = 1 - 0.566 = 0.434$$

$$M_{u2} = 0.07\left((q_{Du} + 0.5q_{Lu})L_2{L_n}^2 - q_{Du}'L_2'{L_{n'}}^2\right) = 14.6kN.m$$

Where:

$$q_{Du} = q_{Du}' = 1.2(5.75 + 3.5) = \frac{11.1kN}{m^2}$$

$$q_{Lu} = 1.6(3) = \frac{4.8kN}{m^2}$$

$$L_2 = 3.25m \qquad L_n = 4.95m \qquad L_2' = 3.25m \quad L_{n'} = 4.90m$$

Check stress:

$$c_1' = 0.178m \quad c_2' = 0.39m$$

$$v_u = \frac{V_u}{b_o d} + \frac{\gamma_{v1} M_{u1} c_1'}{J_{c1}} + \frac{\gamma_{v2} M_{u2} c_2'}{J_{c2}} = 1.447MPa$$

$$\frac{v_u}{\emptyset} = \frac{1.447}{0.75} = 1.93MPa$$

$$v_c \leq 0.33\lambda_s \lambda \sqrt{f'_c} = 1.75MPa$$

$$v_c \leq 0.17\lambda_s \lambda \left(1 + \frac{2}{\beta}\right) \sqrt{f'_c} = 2.4MPa$$

$$v_c \leq 0.083\lambda_s \lambda \left(2 + \frac{\alpha_s d}{b_o}\right) \sqrt{f'_c} = 2.1MPa$$

1.75MPa < 1.93MPa so, shear reinforcement is required.

$Check\ that\ v_u \leq \emptyset 0.50\sqrt{f'c}$: $1.447MPa \leq (0.75)(0.5)\sqrt{28} = 1.984MPa$ $ok.$

$d = 180mm > 150mm\ and > 16d_b = 16(10) = 160mm$

$$v_c = 0.17\lambda_s \lambda \sqrt{f'_c} = 0.88MPa$$

$$v_s = \frac{v_u}{\emptyset} - v_c = 1.93 - 0.88 = 1.05MPa$$

$$V_s = v_s b_o d = \frac{1.05(1960)(180)}{1000} = 370.44kN$$

$$\frac{A_v}{s} = \frac{V_s}{f_{yt} d} = \frac{370440}{420(180)} = 4.9mm^2/mm$$

$$spacing\ of\ stirrups, s = \frac{6(78.5)}{4.9} = 96mm$$

$$\frac{d}{2} = \frac{180}{2} = 90mm < 96mm\ use\ stirrups\ \ 3\phi 10/90mm$$

Flexural stresses:

Direction 1:

$\gamma_{f1} M_{u1} = 0.633(97.5) = 61.7kN.m$

In X-direction, the moment shall be resisted by a cross section of thickness, h= 230mm with d= 180mm and a width equals to column width plus 1.5h from each side, so, width of section is given by:

b= 600 + 2(1.5 x 230) = 1300mm

This moment requires:

$$steel\ ratio, \rho = 0.00401, steel\ area, As = 0.00401(1300)(180) = 938mm^2$$

This area of steel should be available in 1.30m width at column centerline.

Check:

$$If\ v_{uv} > \emptyset\ 0.17\ \lambda_s \lambda \sqrt{f'_c}, \quad A_{s,min} = \frac{5v_{uv}b_{slab}b_o}{\phi \alpha_s f_y}$$

$$1.447MPa \ > \quad (0.75)(0.17)(1)(1)\sqrt{28} = 0.675MPa$$

$$A_{s,min} = \frac{5v_{uv}b_{slab}b_o}{\phi \alpha_s f_y} = \frac{5(1.447)(1300)(1960)}{(0.75)(30)(420)} = 1951mm^2 > 938mm^2\ .$$

$$Use\ A_{s,min} = 1951mm^2$$

This reinforcement shall be checked if it is provided in a width of 1.30m in the column strip. If not, additional steel shall be provided.

Direction 2:

The moment can be neglected because it is very small and the minimum steel shall be used.

$\gamma_{f2} M_{u2} = 0.566(14.6) = 8.3kN.m$

Width of strip, b_{slab}= 500+1.5(230)=845mm.

$$A_{s,min} = \frac{5v_{uv}b_{slab}b_o}{\phi \alpha_s f_y} = \frac{5(1.447)(845)(1960)}{(0.75)(30)(420)} = 1268mm^2$$

This reinforcement shall be checked if it is provided in a width of 0.845m in the column strip. If not, additional steel shall be provided.

__Notes:__

- Based on ACI 318-19, the shear stress can be calculated for each direction alone. In direction 1, V_u and M_{u1} are used and in direction 2, V_u and M_{u2} are used.
- If Vu, M_{u1} and M_{u2} are used in shear stress calculations, the punching shear capacity, v_c can be increased by a ratio like 20%.

Example 3: U- Boot voided slab:

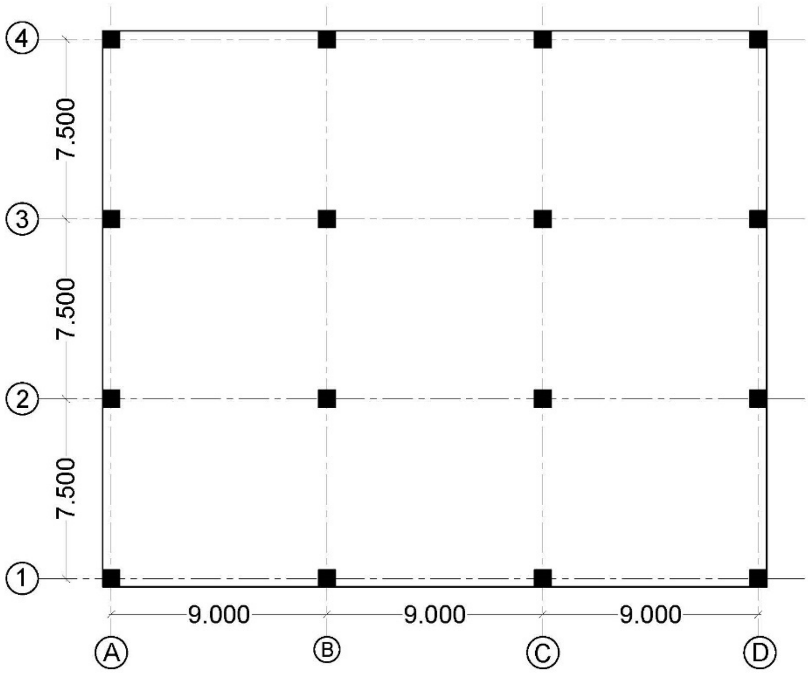


Figure 8.36: Slab layout for Example 3

Given:

- Slab system: voided- U Boot
- Concrete, f'c= 32MPa
- Steel, fy= 420MPa
- Superimposed dead load, W_{SD}= 4kN/m^2
- Live load, W_L= 2.5kN/m^2
- Perimeter wall weight, W_{WALL}= 15kN/m
- All columns are 0.60m x 0.60m
- Column height, h= 3.80m
- Design frame (strip) 2.

Solution:

Dimensions of beams:

Width, b= L/20 = 9/20= 0.45m Try, b= 0.40m

Thickness, h= L/18.5=9/18.5= 0.49m Try, h= 0.70m

Slab thickness:

Assume that $\alpha_{fm} \geq 2$, so:

$$h = \frac{l_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta} = \frac{8.6\left(0.8 + \frac{420}{1400}\right)}{36 + 9(1.21)} = 0.2m \geq 0.09m \quad ok$$

Where: Ln= 9.0-0.40=8.40m

$$\beta = \frac{long\ clear\ span}{short\ clear\ span} = \frac{9 - 0.4}{7.5 - 0.4} = 1.21$$

Try voided slab thickness, h= 320mm:

Check α_m:

The distribution of α_f is shown in Figure 8.37 below.

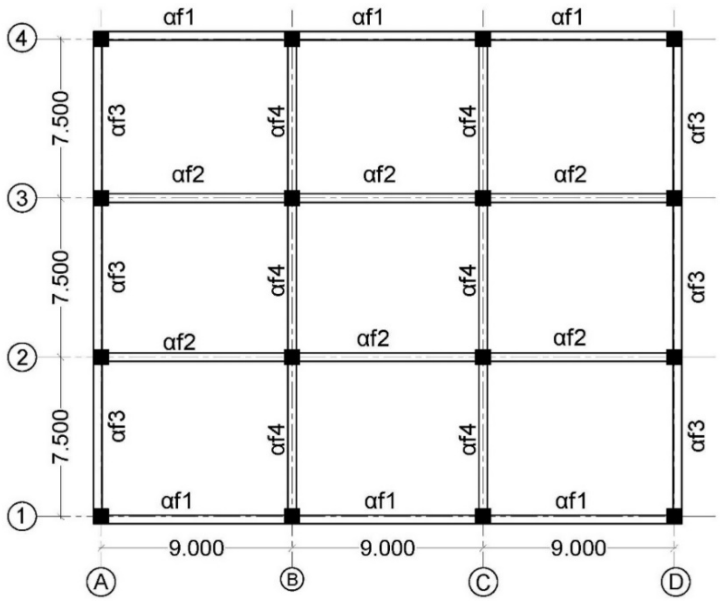


Figure 8.37: Distribution of α_f for the beams

The moment of inertia of the beam, Ib is given by:

$$I_{beam} = \frac{(0.4)(0.7)^3}{12} = 0.011433m^4$$

The flange width of a voided slab unit (I- section), bf= 520mm + 150mm = 670mm

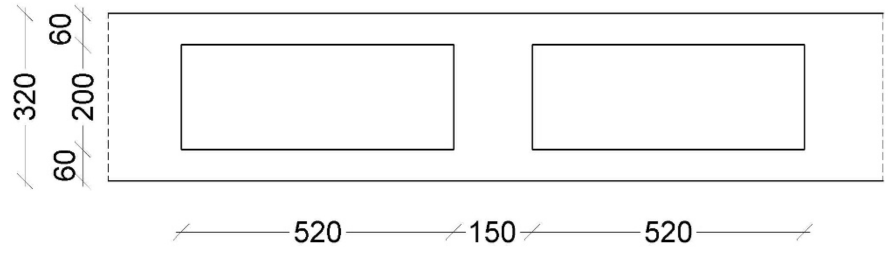


Figure 8.38: Cross section in slab

The moment of inertia of one unit of width 670mm is given by:

$$I = \frac{0.67(0.32)^3}{12} - \frac{0.52(0.2)^3}{12} = 1.4828x10^{-3}m^4$$

$$\alpha_{f1} = \frac{I - edge\ beam}{I - slab\ of\ 3.75m\ width} = \frac{0.011433}{\frac{3.75}{0.67}x1.4828x10^{-3}} = 1.38$$

$$\alpha_{f2} = \frac{I - interior\ beam}{I - slab\ of\ 7.5m\ width} = \frac{0.011433}{\frac{7.5}{0.67}x1.4828x10^{-3}} = 0.69$$

$$\alpha_{f3} = \frac{I - edge\ beam}{I - slab\ of\ 4.5m\ width} = \frac{0.011433}{\frac{4.5}{0.67}x1.4828x10^{-3}} = 1.15$$

$$\alpha_{f4} = \frac{I - interior\ beam}{I - slab\ of\ 9m\ width} = \frac{0.011433}{\frac{9.0}{0.67}x1.4828x10^{-3}} = 0.57$$

Value of $\alpha_{fm}$for interior panel is given by:

$$\alpha_{fm} = \frac{2\alpha_{f2} + 2\alpha_{f4}}{4} = 0.63 < 2$$

So, slab thickness is given by:

$$h = \frac{l_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta(\alpha_{fm} - 0.2)} = \frac{8.4\left(0.8 + \frac{420}{1400}\right)}{36 + 5(1.22)(0.63 - 0.2)} = 0.24m$$

the moment of inertia of the solid slab of 0.24m thickness and 0.67m width is given by:

$$I = \frac{0.67(0.24)^3}{12} = 0.77x10^{-3}m^4 < 1.4828x10^{-3}m^4 \qquad ok$$

Slab self-weight, W_D= [0.67x0.67x0.32-0.52x0.52x0.2]x25 / (0.67x0.67)=5kN/m^2

Slab ultimate load, Wu= 1.2(5+4)+1.6(2.5)= 14.8kN/m^2

Check wide beam shear (one-way shear):

Here, the short span, L= 7.50m

Load on one unit of slab= 14.8(0.67)=9.92kN/m

Shear can be calculated at distance d from face of beam, so:

$$V_u = W_u\left(\frac{L}{2} - \frac{b_1}{2} - d\right) = 9.92\left(\frac{7.6}{2} - \frac{0.4}{2} - 0.28\right) = 32.9kN$$

The shear strength capacity of the slab is given by:

$$ACI\ 318-14: \emptyset V_c = \emptyset\frac{1}{6}\lambda\sqrt{f'_c}b_w d = \frac{0.75\left(\frac{1}{6}\right)(1)\sqrt{32}(150)(280)(1.1)}{1000} = 32.7kN \approx 32.9kN \quad ok$$

$$Size\ factor, \qquad \lambda_s = \sqrt{\frac{2}{1+0.004\ d}} \le 1.0 \rightarrow \lambda_s = \sqrt{\frac{2}{1+0.004\ (280)}} = 0.97$$

$$Let\ \rho_w = 0.0018\left(\frac{h}{d}\right) = 0.0018\left(\frac{320}{280}\right) = 0.002$$

So,

$$ACI\ 318-19: \emptyset V_c = \emptyset\left(0.66\lambda_s\lambda(\rho_w)^{\frac{1}{3}}\sqrt{f'_c} + \frac{N_u}{6A_g}\right)b_w d$$

$$= \frac{(1.1)(0.75)\left(0.66(0.97)(1)(0.002)^{\frac{1}{3}}\sqrt{32} + 0.0\right)(150)(280)}{1000}$$

$$= 15.8kN < 32.9kN \qquad N.G.$$

So, use minimum shear reinforcement in the ribs at least for ¼ the clear span at each end.

$$Use\ 1\emptyset 8/140mm, \qquad A_v/s = (2(50))/140 = 0.71mm^2/mm$$

$$\left(\frac{A_v}{s}\right)_{min} = max\left[\frac{0.062\sqrt{f'_c}b_w}{f_{yt}}, \frac{0.35b_w}{f_{yt}}\right] = max[0.13, 0.125] = 0.13mm^2/mm$$

$< 0.71mm^2/mm$ ok.

Analysis and design of frame 2:

1. Compute the total statical moment, Mo for the span. Here the three spans are equal.

$$M_o = \frac{q_u l_2 {l_n}^2}{8} = \frac{14.8(7.5)(8.4)^2}{8} + \frac{0.4(0.7)(25)(1.2)(8.4)^2}{8} = 1053.108kN.m$$

2. Compute positive and negative moments in the span:

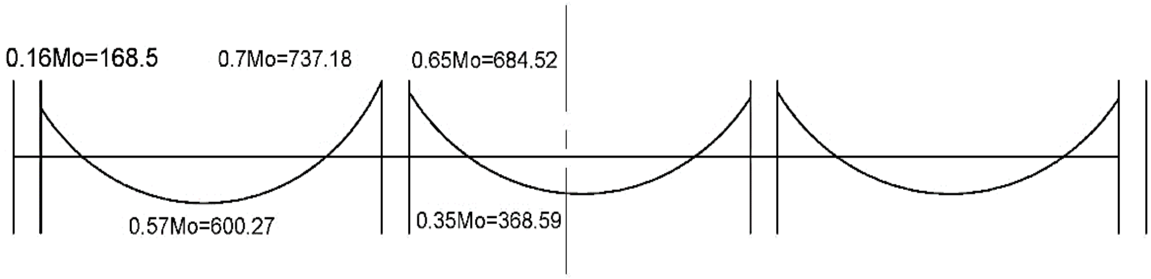


Figure 8.39: Frame bending moment diagram- kN.m

3. Compute the moments in the column strip (slab + beam)

Frame width= 7.5m

Column strip width= 3.75m

Middle strip width= 3.75m

$$\frac{\alpha_f L_2}{L_1} = \frac{0.69(7.5)}{9} = 0.575 < 1$$

$$\frac{L_2}{L_1} = \frac{7.5}{9} = 0.83$$

From ACI 318-14 Table 8.10.5.5, for positive moment, the ratio = 71.6%

From ACI 318-14 Table 8.10.5.1, for interior negative moment, the ratio = 77.9%

Calculations for β_t:

Rectangular section: b= 0.4m h=0.7m

$$C = \sum \left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3} = \left(1 - 0.63\frac{0.4}{0.7}\right)\frac{0.4^3(0.7)}{3} = 0.00955733 m^4$$

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s} = \frac{C}{2I_s} = \frac{0.00955733}{2\frac{7.5}{0.67}(1.4828x10^{-3})} = 0.29$$

$$\frac{\alpha_f L_2}{L_1} = 0.575$$

$$\frac{L_2}{L_1} = 0.83$$

From ACI 318-14 Table 8.10.5.2, for exterior negative moment, the ratio = 97.4%

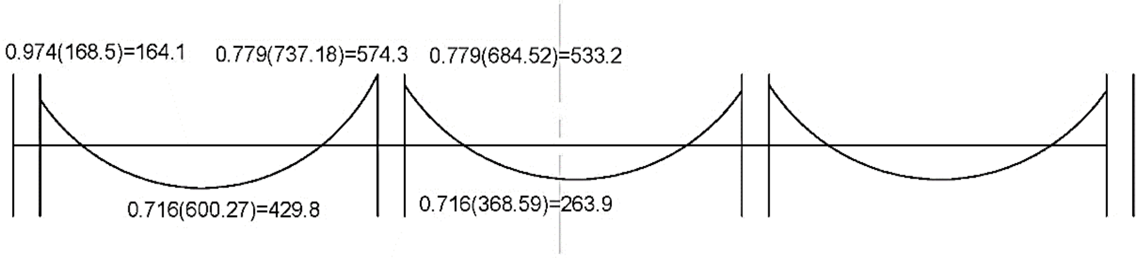


Figure 8.40: Column strip bending moment diagram- kN.m

4. Compute moments in beam:

Since $\frac{\alpha_f L_2}{L_1} = 0.575$, the ratio of moments that are transferred to beams is:

0.575 (0.85)=0.49

Figure 8.41 shows the bending moment diagram of the beam in frame 2.

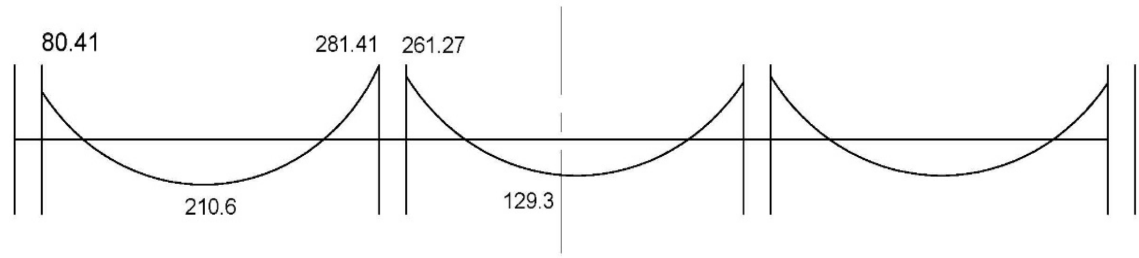


Figure 8.41: Beam bending moment diagram- kN.m

5. Compute moments in slab column strip:

The slab column strip moment = the column strip moment – the beam moment

Figure 8.42 shows the moments in the slab column strip.

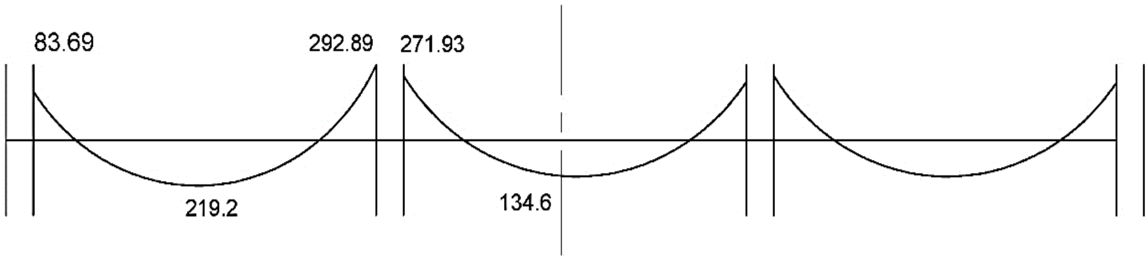


Figure 8.42: Slab- column strip bending moment diagram- kN.m

Slab column strip width = 3750mm

Effective depth, d= 280mm

Then the steel area can be computed.

6. Compute moments in slab middle strip:

The slab middle strip moment = the frame moment – the column strip moment

Figure 8.43 shows the moments in the slab column strip.

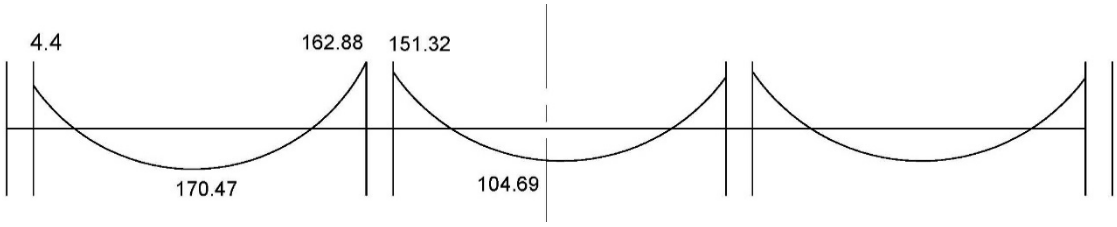


Figure 8.43: Slab middle strip bending moment diagram- kN.m

Slab middle strip width = 3750mm

Effective depth, d= 280mm

Then the steel area can be computed.

The flexural steel can be determined assuming a solid slab.

For example: the maximum bending moment in the column strip, Mu= 292.89kN.m

Section width, b= 3750mm

Section thickness, h= 320mm

Section effective depth, d= 280mm

Then,

$$\rho = \frac{0.85f'_c}{f_y}\left(1-\sqrt{1-\frac{2.61M_u}{bd^2f'_c}}\right) = \frac{0.85(32)}{420}\left(1-\sqrt{1-\frac{2.61(292.89x10^6}{3750(280)^2(32)}}\right) = 0.00269$$

$$A_s = 0.00269(3750)(280) = 2825mm^2 \quad A_s = \frac{2825}{3.75} = \frac{753mm^2}{m}$$

Minimum area of steel is given by:

$$A_{s,min} = 0.0018(1000)(320) = 576mm^2 < 753mm^2$$

Use $1\emptyset 14/200mm$

Example 4: Waffle slab:

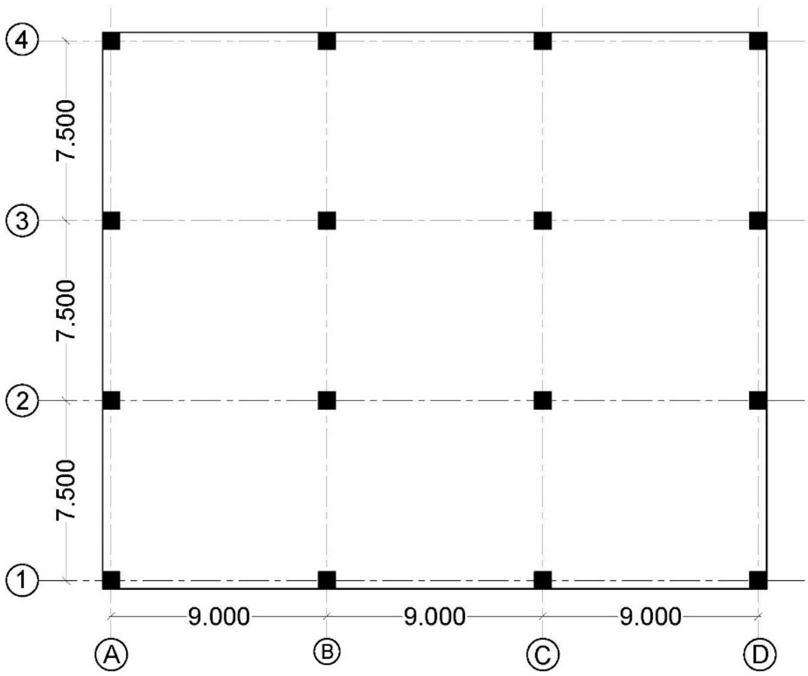


Figure 8.44: Slab layout for Example 4

Given:

- Slab system: Waffle
- Concrete, f'c= 32MPa
- Steel, fy= 420MPa
- Superimposed dead load, W_{SD}= 4kN/m^2
- Live load, W_L= 2.5kN/m^2
- Perimeter wall weight, W_{WALL}= 15kN/m
- All columns are 0.60m x 0.60m
- Column height, h= 3.80m
- Determine Mo for spans in frame (strip) 2.

Solution:

Dimensions of beams:

Width, b= L/20 = 9/20= 0.45m Try, b= 0.40m

Thickness, h= L/18.5=9/18.5= 0.49m Try, h= 0.70m

Slab thickness:

Assume that $\alpha_{fm} \geq 2$, so:

$$h = \frac{l_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta} = \frac{8.6\left(0.8 + \frac{420}{1400}\right)}{36 + 9(1.21)} = 0.2m \geq 0.09m \quad ok$$

where: Ln= 9.0-0.60=8.40m

$$\beta = \frac{long\ clear\ span}{short\ clear\ span} = \frac{9 - 0.4}{7.5 - 0.4} = 1.21$$

Try waffle slab thickness, h= (4/3)(0.20)= 0.27m:

Check α_m:

The distribution of α_f is shown in the Figure 8.45 below.

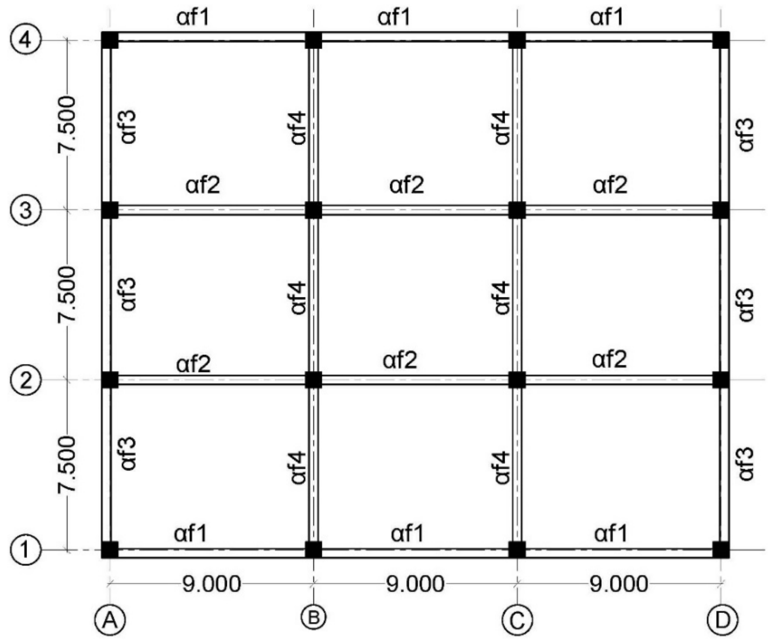


Figure 8.45: Distribution of α_f for the beams

The moment of inertia of the beam is given by:

$$I_{beam} = \frac{(0.4)(0.7)^3}{12} = 0.011433m^4$$

The flange width of the waffle slab unit (T- section), bf= 600mm + 150mm = 750mm

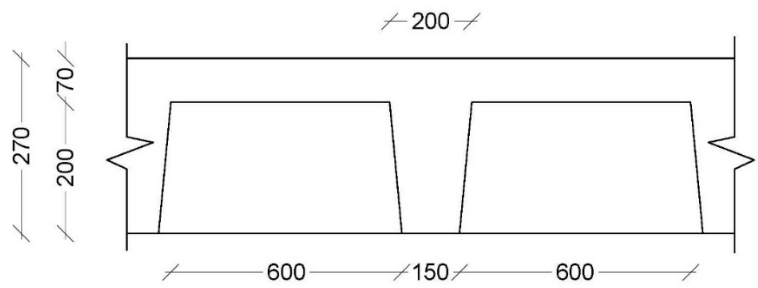


Figure 8.46: Cross section in slab

The moment of inertia of one unit of width 750mm is equal to 4.935x10^{-4} mm^4.

$$\alpha_{f1} = \frac{I - edge\ beam}{I - slab\ of\ 3.75m\ width} = \frac{0.011433}{\frac{3.75}{0.75}x4.935x10^{-4}} = 4.6$$

$$\alpha_{f2} = \frac{I - interior\ beam}{I - slab\ of\ 7.5m\ width} = \frac{0.011433}{\frac{7.5}{0.75}x4.935x10^{-4}} = 2.3$$

$$\alpha_{f3} = \frac{I - edge\ beam}{I - slab\ of\ 4.5m\ width} = \frac{0.011433}{\frac{4.5}{0.75}x4.935x10^{-4}} = 3.8$$

$$\alpha_{f4} = \frac{I - interior\ beam}{I - slab\ of\ 9m\ width} = \frac{0.011433}{\frac{9.0}{0.75}x4.935x10^{-4}} = 1.93$$

Minimum value of α_{fm} for interior panel is given by:

$$\alpha_{fm} = \frac{2\alpha_{f2} + 2\alpha_{f4}}{4} = 2.1 > 2 \quad OK$$

Slab self-weight, w_D= {[0.75x0.75x0.27-0.575x0.575x0.2]x25}/(0.75x0.75)= 3.8kN/m^2

Slab ultimate load, w_u= 1.2(3.8+4)+1.6(2.5)= 13.4kN/m^2

Check wide beam shear (one-way shear):

Here, the short span, L= 7.50m

Load on one unit of slab= 13.4(0.75)=10.05kN/m

Shear can be calculated at distance d from face of beam, so:

$$V_u = W_u\left(\frac{L}{2} - \frac{b_1}{2} - d\right) = 10.05\left(\frac{7.6}{2} - \frac{0.4}{2} - 0.22\right) = 34.0kN$$

The shear strength capacity of the slab is given by:

$$ACI\ 318 - 14: ØV_c = Ø\frac{1}{6}\lambda\sqrt{f'_c}b_wd = \frac{0.75\left(\frac{1}{6}\right)(1)\sqrt{32}(150)(220)(1.1)}{1000} = 23.3kN < 33.5kN \quad Use\ stirrups.$$

$$V_c = 31.1kN.m$$

$$V_s = \left(\frac{34.0}{0.75}\right) - 31.1 = 14.2kN.m$$

$$\frac{A_v}{s} = \frac{14.2x1000}{420x220} = 0.154mm^2/mm$$

$$\left(\frac{A_v}{s}\right)_{min} = max\left[\frac{0.062\sqrt{f'_c}b_w}{f_{yt}}, \frac{0.35b_w}{f_{yt}}\right] = \frac{0.13mm^2}{mm} < 0.154mm^2/mm \quad OK$$

$$Use\ \frac{A_v}{s} = 0.154mm^2/mm$$

For Ø$8mm$ stirrups:

$$s = \frac{100}{0.15} = 667mm > \ s_{max} = \frac{d}{2} = 100mm$$

So, use closed stirrups at ends of ribs for one quarter the clear span (The accurate distance can be specified).

This shear design is applicable for ACI 318-19.

Analysis of frame 2:

Compute the total statical moment, Mo for the span. Here the three spans are equal.

$$M_o = \frac{q_u l_2 {l_n}^2}{8} = \frac{13.4(7.5)(8.4)^2}{8} + \frac{0.4(0.7)(25)(1.2)(8.4)^2}{8} = 960.5kN.m$$

Note:

The moments in the frame, beam, slab column strip and slab middle strip can be found using the same procedure in the previous examples.

The moment in the rib is given by:

$$M_{rib,column\ strip} = \frac{M_{column\ strip}}{width\ of\ column\ strip}\ x\ flange\ width; 0.75m$$

$$M_{rib,middle\ strip} = \frac{M_{middle\ strip}}{width\ of\ middle\ strip}\ x\ flange\ width; 0.75m$$

And the cross section is T for the rib.

Chapter 9: Design for Shear and Torsion

9.1 Introduction:

A torque, a twisting moment or a torsional moment is a moment that acts about the longitudinal axis of a member.

In a circular member, the shearing stresses are zero at the axis of the member and increase linearly to a maximum stress at the outside of the member as shown in **Figure 9.1.**

In a rectangular member, the shearing stresses vary from zero at the center to a maximum at the centers of the long sides. Around the perimeter of a square member, the shearing stresses vary from zero at the corners to a maximum at the center of each side, as shown in **Figure 9.1**

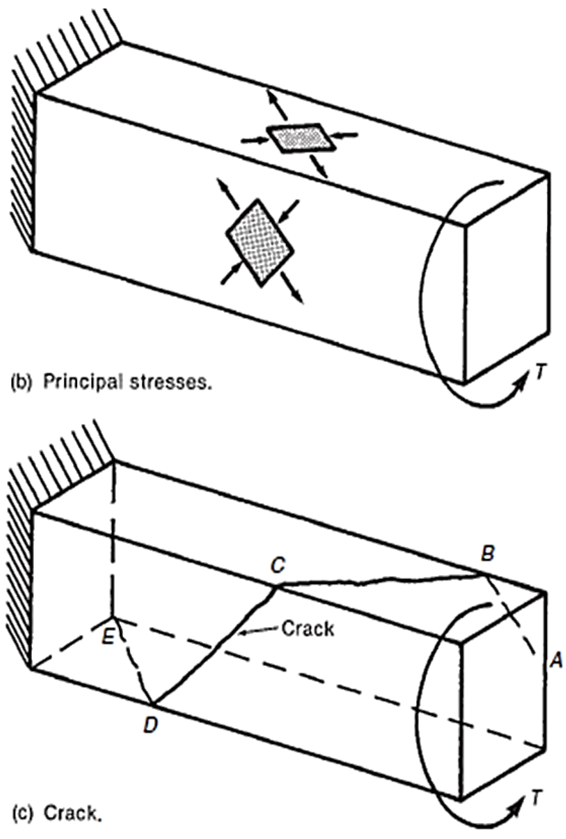


Figure 9.1: Distribution of torsional shear stresses

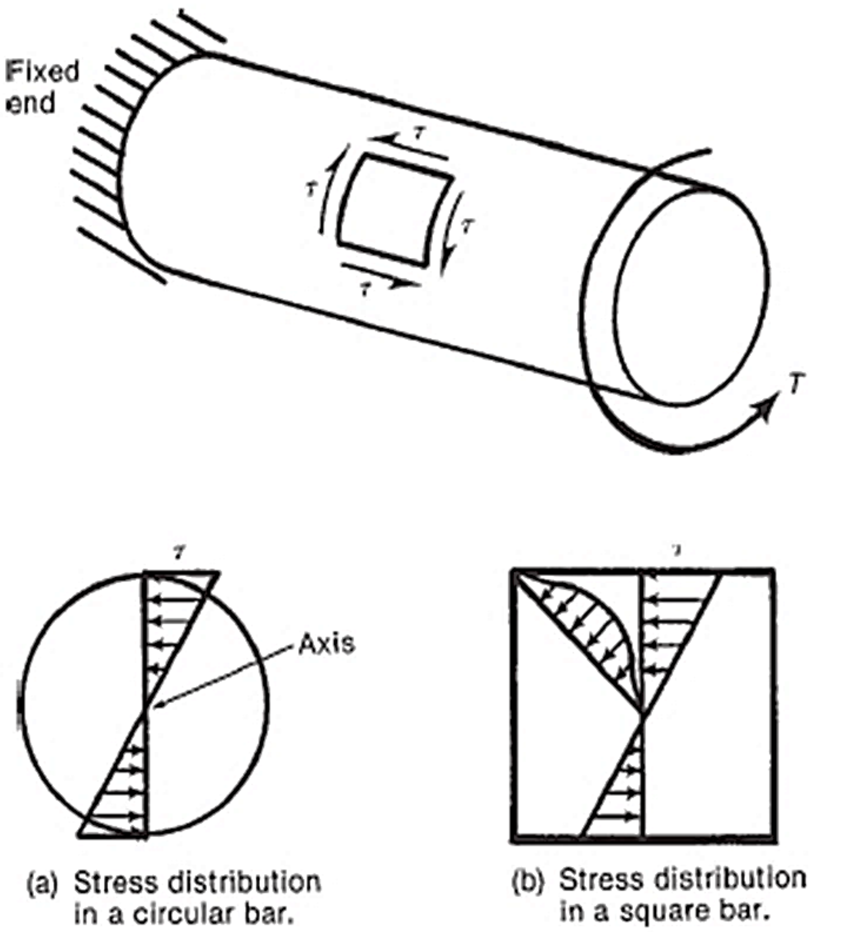
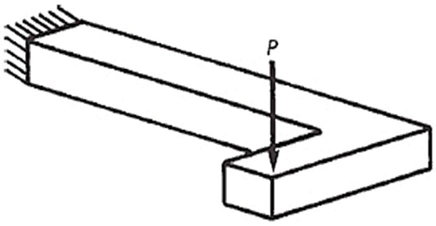


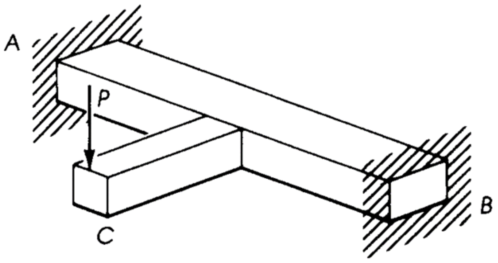
Figure 9.1- continued: Distribution of torsional shear stresses

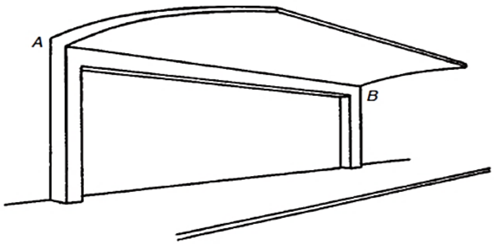
In structures, torsion results from:

1. Eccentric loading of beams.
2. Deformations resulting from continuity of beams or similar members that join at an angle to each other.



(a) Cantilever beam with eccentrically applied load.





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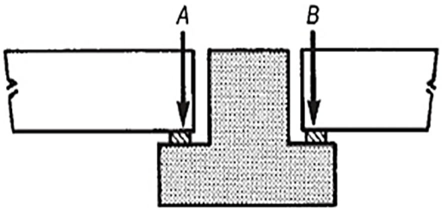


Section through a beam supporting precast floor slabs.

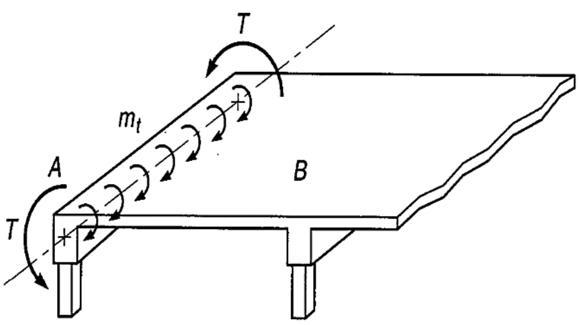


Figure 9.2: Torsion in structures

Torsion can be classified into two types:

1. Equilibrium torsion: affects equilibrium.
2. Compatibility torsion: does not affect equilibrium.

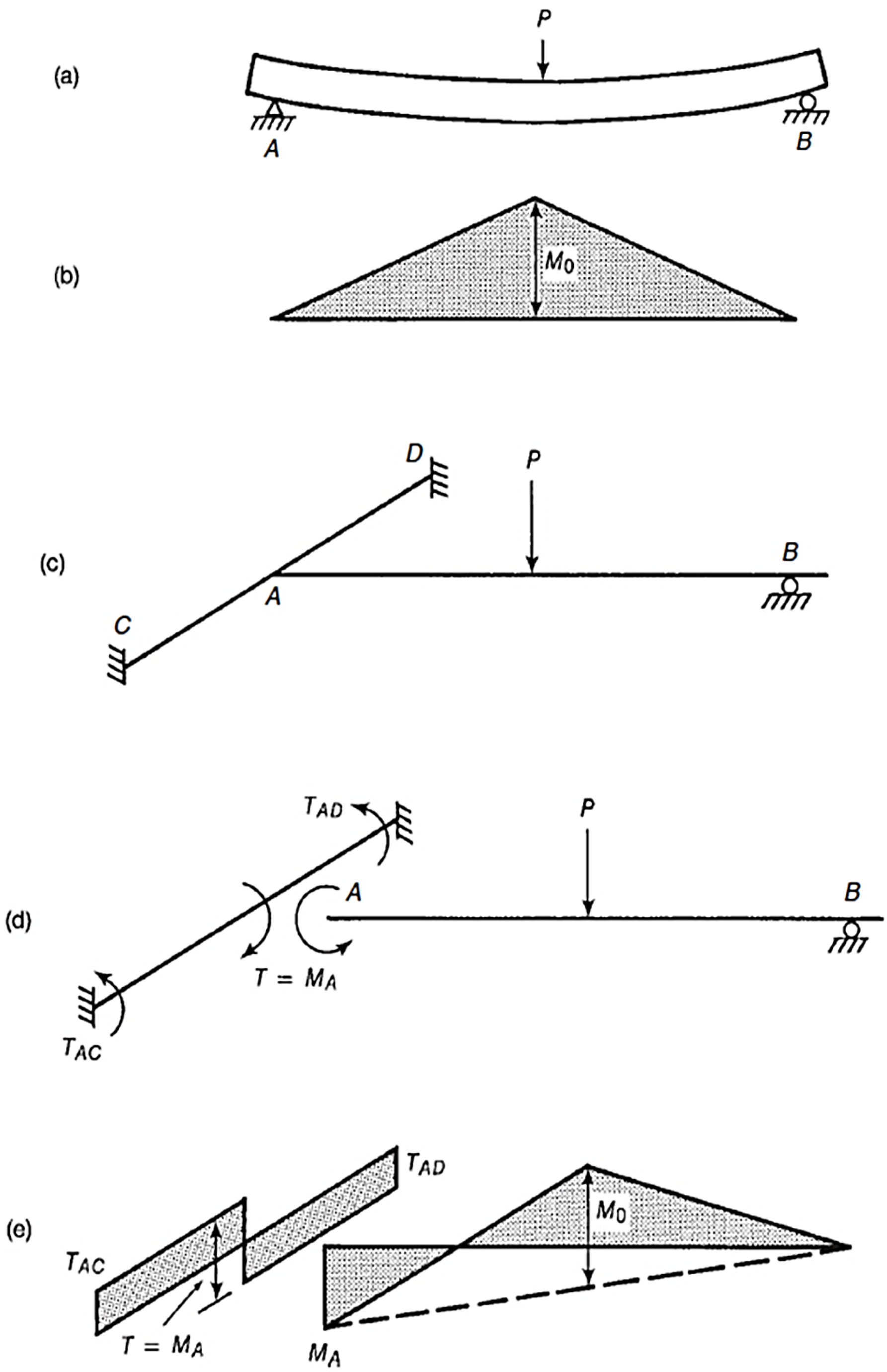


Figure 9.3: Compatibility torsion

9.2 Behavior of reinforced concrete members subjected to torsion:

When a concrete member is loaded in pure torsion, shearing stresses develop. One or more cracks (inclined) develop when the maximum principal tensile stress reaches the tensile strength of the concrete. The onset of cracking failure of unreinforced concrete.

Furthermore, the addition of longitudinal steel without stirrups has little effect on the strength of the beam loaded in pure torsion because it is effective only in increasing the longitudinal component of the diagonal tension forces.

A rectangular beam with longitudinal bars in the corners and closed stirrups can resist increased load after cracking.

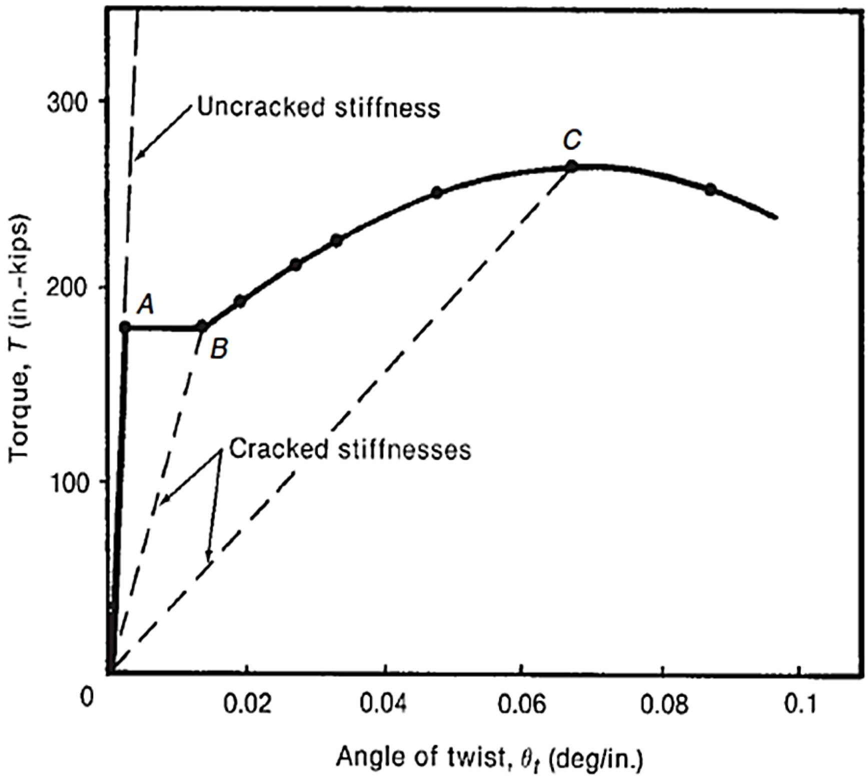


Figure 9.4: Torque twist curve for a rectangular beam

At the cracking load, point A, the angle of twist increases without an increase in torque as some of the forces formerly in the uncracked concrete are distributed to the reinforcement.

After the cracking of a reinforced concrete beam, failure may occur in several ways. The stirrups, or longitudinal reinforcement, or both, may yield, or, for beams that are over-reinforced in torsion, the concrete between the inclined cracks may be crushed by the principal compression stresses prior to yield of the steel. The more ductile behavior results when both reinforcements yield prior to crushing of the concrete.

9.3 Combined shear and torsion:

In combined shear and torsion, the cracking load follows a circular interaction diagram as in **Figure 9.6.**

In **Figure 9.6:**

V_{cu}= the cracking shear in the absence of torque.

T_{cu}= the cracking torque in the absence of shear.

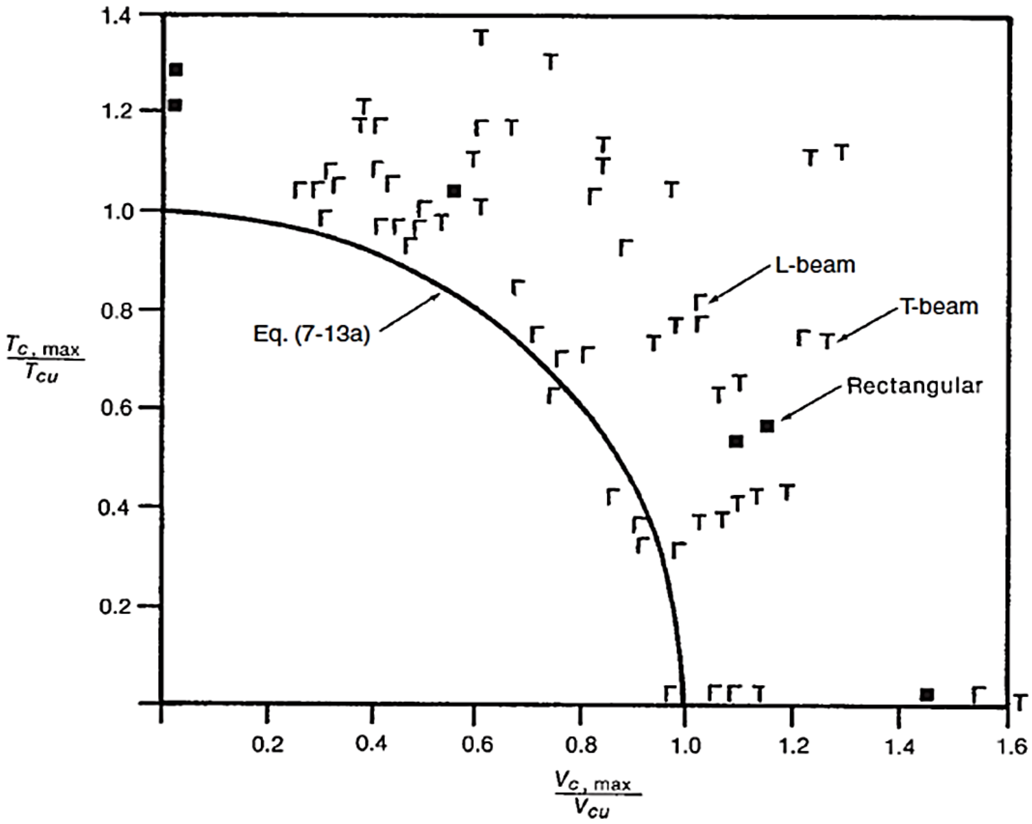


Figure 9.5 Interaction of torsion and shear

$$\left(\frac{T_c}{T_{cu}}\right)^2 + \left(\frac{V_c}{V_{cu}}\right)^2 = 1$$

9.4 Design methods for torsion:

* Skew bending theory: (1971- 1981) ACI CODES. It assumes that the shear and torsion are resisted by concrete (Vc and Tc) and the reinforcing steel (Vs and Ts).

* Thin-walled tube/ plastic space truss model: in European codes and in ACI code since 1995.

Assumptions of thin-walled tube/ plastic space truss:

1. Both solid and hollow members are considered as tubes: tests for solid and hollow beams suggest that, once the torsional cracking has occurred, the concrete in the center of the member has little effect on the torsional strength of the cross section and hence can be ignored. This, in effect, produce an equivalent tubular member.

2. After cracking the tube is idealized as a hollow truss consisting of closed stirrups, longitudinal bars in the corners, and compression diagonals approximately centered on the stirrups. The diagonals are idealized as being between the cracks that are at angle θ, generally taken as 45 degrees for reinforced concrete.

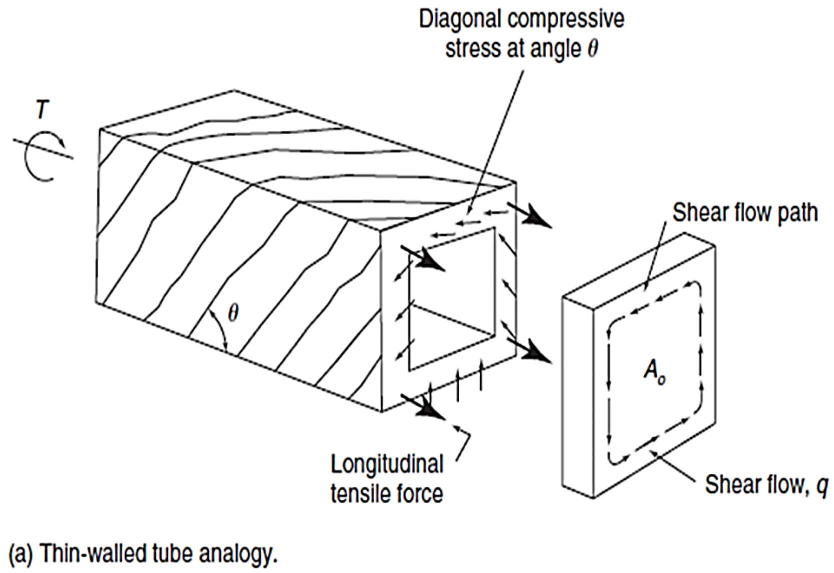


Figure 9.6: Thin-walled tube analogy and space truss analogy.

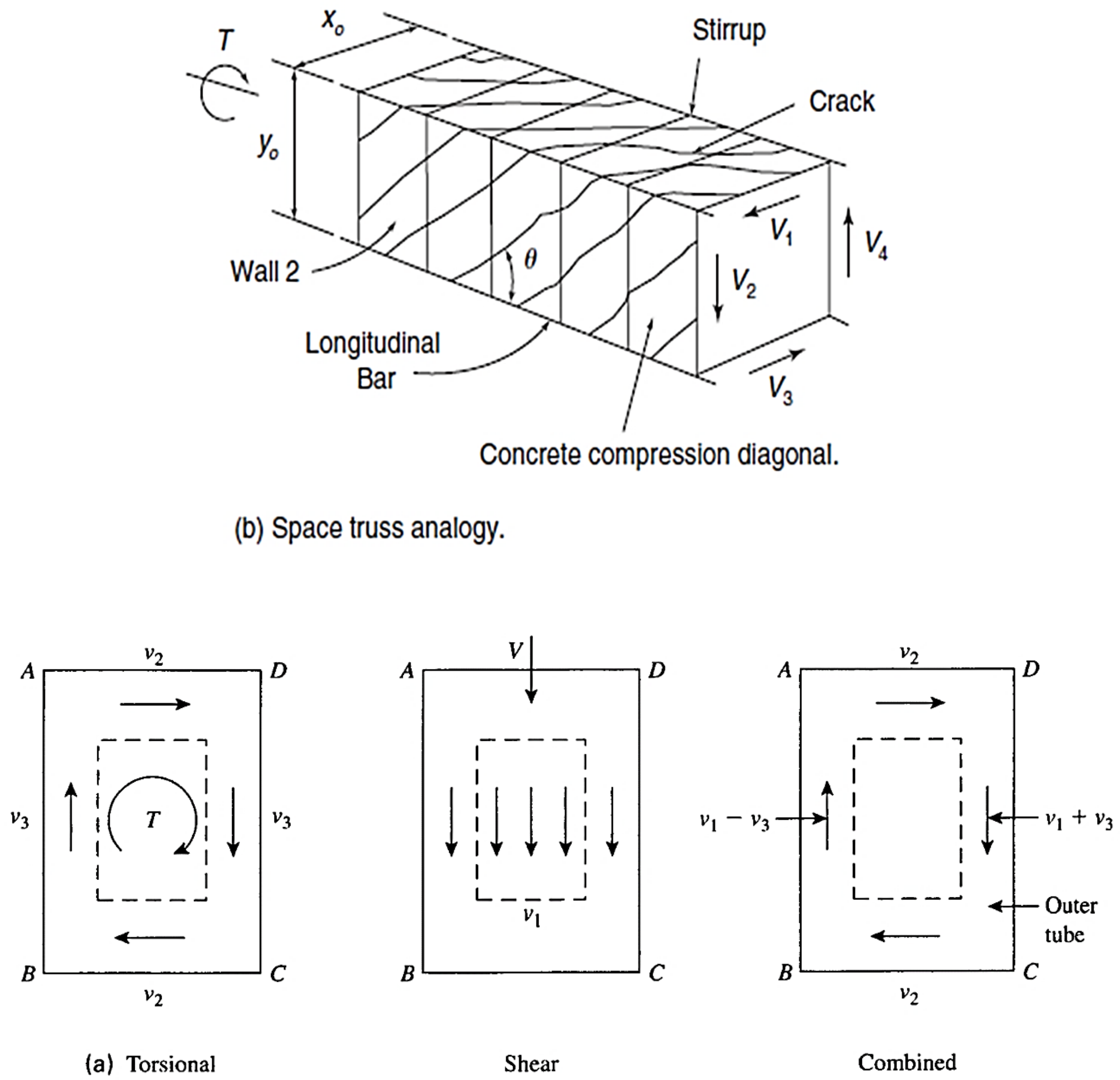


Figure 9.7: Combined shear and torsion

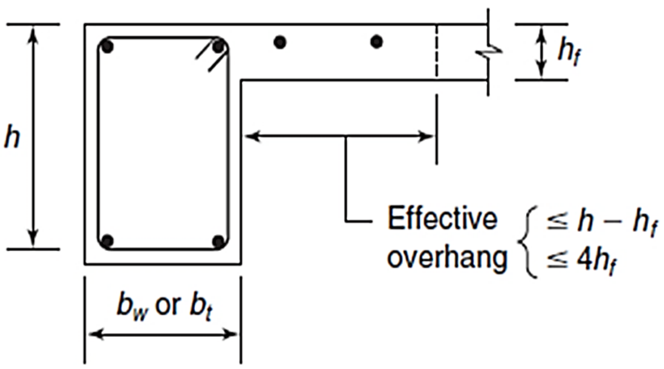


Figure 9.8: Part of overhanging flange effective for torsion

Note:

The cracking torque, the threshold torsion, area of stirrups for torsion and the area of the longitudinal steel needed for torsion resistance can be derived. For details refer to textbook.

The cracking torsion:

From mechanics of materials principles and from ACI assumptions, the cracking torsion, Tcr, is given by:

$$T_{cr} = \frac{1}{3}\sqrt{f'_c}\frac{{A_{cp}}^2}{P_{cp}}$$

This formula is derived based on that the torsional cracking is assumed to occur when the principal tensile stress reaches the tensile strength of concrete in biaxial tension- compression which is:

$$\sigma_t = \frac{1}{3}\sqrt{f'_c}$$

Also,

$$\left(\frac{T_c}{T_{cu}}\right)^2 + \left(\frac{V_c}{V_{cu}}\right)^2 = 1$$

$If\ T_c = 0.25\ T_{cu}, then$:

$$\frac{V_c}{V_{cu}} = \sqrt{1 - \left(\frac{0.25T_{cu}}{T_{cu}}\right)} \rightarrow V_c = 0.97V_{cu}$$

So, the existence of a torque equal to 0.25 of the cracking torque will reduce the cracking shear by only 3%. This is deemed to be negligible. **In ACI code, the threshold torsion, $\emptyset T_{th}$, below which torsion can be neglected in a solid section is given by:**

$$\emptyset T_{th} = \emptyset\lambda\frac{1}{12}\sqrt{f'_c}\frac{{A_{cp}}^2}{P_{cp}}$$

Tables 9.1 and 9.2 show T_{th} equations.

Table 9.3 shows T_{cr} equations.

Table 9.1: ACI 318-19 Table 22.7.4.1(a)—Threshold torsion for solid cross sections

Type of member	T_{th}	
Nonprestressed member	$\frac{1}{12}\lambda\sqrt{f'_c}\frac{A_{cp}^2}{P_{cp}}$	(a)
Prestressed member	$\frac{1}{12}\lambda\sqrt{f'_c}\frac{A_{cp}^2}{P_{cp}}\sqrt{1+\frac{f_{pc}}{0.33\lambda\sqrt{f'_c}}}$	(b)
Nonprestressed member subjected to axial force	$\frac{1}{12}\lambda\sqrt{f'_c}\frac{A_{cp}^2}{P_{cp}}\sqrt{1+\frac{N_u}{0.33A_g\lambda\sqrt{f'_c}}}$	(c)

Table 9.2: ACI 318-19 Table 22.7.4.1(b)—Threshold torsion for hollow cross sections

Type of member	T_{th}	
Nonprestressed member	$\frac{1}{12}\lambda\sqrt{f'_c}\frac{A_g^2}{P_{cp}}$	(a)
Prestressed member	$\frac{1}{12}\lambda\sqrt{f'_c}\frac{A_g^2}{P_{cp}}\sqrt{1+\frac{f_{pc}}{0.33\lambda\sqrt{f'_c}}}$	(b)
Nonprestressed member subjected to axial force	$\frac{1}{12}\lambda\sqrt{f'_c}\frac{A_g^2}{P_{cp}}\sqrt{1+\frac{N_u}{0.33A_g\lambda\sqrt{f'_c}}}$	(c)

Table 9.3: ACI 318-19 Table 22.7.5.1—Cracking torsion

Type of member	T_{cr}	
Nonprestressed member	$\frac{1}{3}\lambda\sqrt{f'_c}\frac{A_{cp}^2}{P_{cp}}$	(a)
Prestressed member	$\frac{1}{3}\lambda\sqrt{f'_c}\frac{A_{cp}^2}{P_{cp}}\sqrt{1+\frac{f_{pc}}{0.33\lambda\sqrt{f'_c}}}$	(b)
Nonprestressed member subjected to axial force	$\frac{1}{3}\lambda\sqrt{f'_c}\frac{A_{cp}^2}{P_{cp}}\sqrt{1+\frac{N_u}{0.33A_g\lambda\sqrt{f'_c}}}$	(c)

Where:

A_g: gross area of concrete section, mm^2 . For a hollow section, A_g is the area of the concrete only and does not include the area of the void(s).

A_{cp}: area enclosed by outside perimeter of concrete cross section, mm^2.

P_{cp}: outside perimeter of concrete cross section, in.

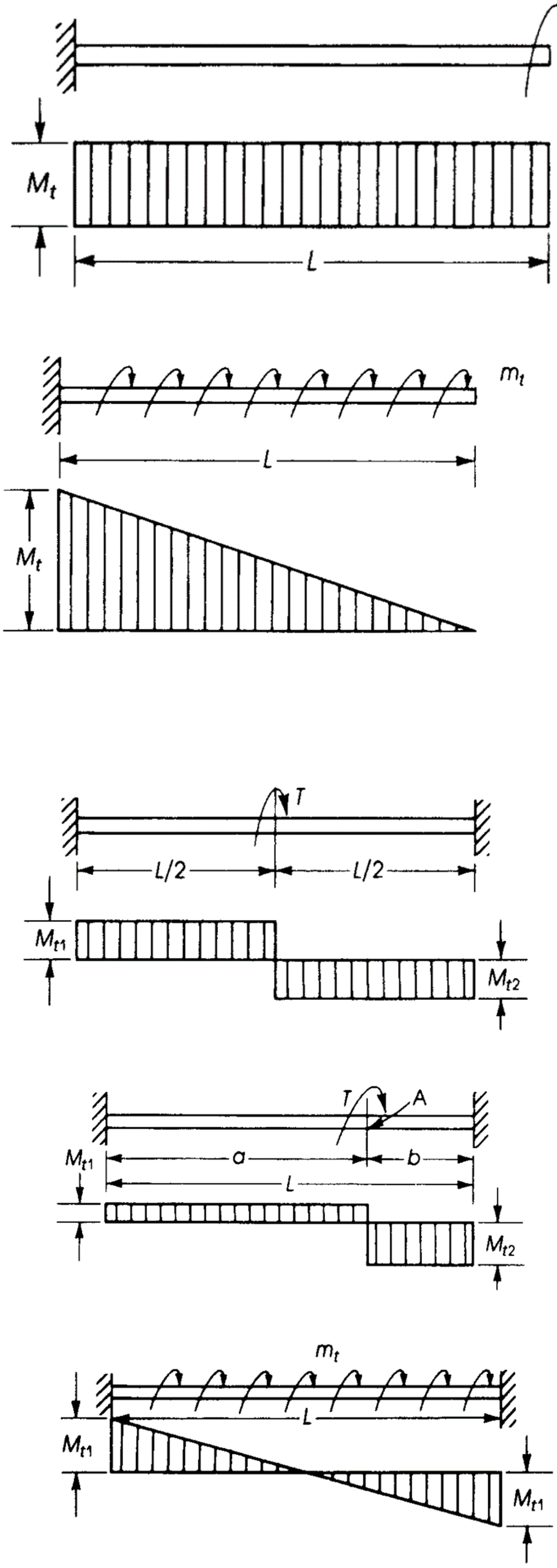
f_{pc}: compressive stress in concrete, after allowance for all prestress losses, at centroid of cross section, MPa.

N_u: factored axial force normal to cross section occurring simultaneously with *Vu* or *Tu*; to be taken as positive for compression and negative for tension, N.

Maximum shear and torsion:

A serviceability failure may occur if the inclined cracks are too wide at service loads. The limit on combined shear and torsion in ACI code was derived to limit the service load crack width.

9.5 Torsion diagrams:



At support

$M_t = T$

At support

$M_t = m_t L$

m_t = uniform torque

$$M_{t1} = M_{t2} = \frac{T}{2}$$

$$M_{t1} = \frac{Tb}{L}$$

$$M_{t2} = \frac{Ta}{L}$$

$$M_{t1} = \frac{m_t L}{2}$$

Figure 9.9: Torsion diagrams

9.6 ACI design method for shear and torsion:

1. Calculate Vu and Tu at a section. Usually, the critical section is located at distance d from face of support.
2. Determine whether torsion is compatibility or equilibrium. For compatibility torsion, the calculated torsion can be reduced to the cracking torsion $\emptyset T_{cr}$.

   If T_u is reduced to $\emptyset T_{cr}$, moment redistribution shall be applied.

3. Design for torsion if $T_u > \emptyset T_{th}$.

4. Check whether section is large enough for torsion design (check section adequacy).

For solid sections:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u P_h}{1.7\, A_{oh}{}^2}\right)^2} \leq \emptyset \frac{5}{6}\sqrt{f'_c}$$

Where:

V_u= ultimate shear force, N.

V_u= ultimate torsion, N.mm.

P_h= perimeter of centerline of outermost closed transverse torsional reinforcement, mm.

A_{oh}= area enclosed by centerline of the outermost closed transverse torsional reinforcement, mm^2.

b_w= width of web.

d= effective depth.

For hollow sections:

$$\frac{V_u}{b_w d} + \frac{T_u P_h}{1.7\, A_{oh}{}^2} \leq \emptyset \frac{5}{6}\sqrt{f'_c}$$

For hollow sections where the wall thickness is less than A_{oh}/P_h, the term $(T_u P_h/1.7A_{oh}{}^2)$ shall be taken as $(T_u/1.7A_{oh}t)$, where t is the thickness of the wall of the hollow section at the location where the stresses are being checked.

5. Compute the area of stirrups required for shear, Av/S, mm^2/mm.

6. Compute the area of stirrups required for torsion, At/S , mm²/mm, using the following equation:

$$\frac{A_t}{s} = \frac{T_n}{2A_o f_{yt}} \qquad T_n = \frac{T_u}{\phi} \qquad A_o = 0.85A_{oh}$$

Where A_o= gross area enclosed by shear flow path, mm².

7. Add the required stirrup amounts together:

$$\frac{A_{v+t}}{s} = \frac{A_v}{s} + 2\frac{A_t}{s} \qquad for\ 2\ legs\ closed\ stirrup.$$

$$\frac{A_{v+t}}{s} = \frac{A_v}{s} + 4\frac{A_t}{s} \qquad for\ 4\ legs\ closed\ stirrup.$$

$$\frac{A_{v+t}}{s} \geq max\left[0.062\sqrt{f'_c}\frac{b_w}{f_{yt}}, \frac{0.35b_w}{f_{yt}}\right]$$

The maximum spacing between the closed stirrups is the smaller of $\frac{P_h}{8}$ and 300mm.

8. Determine the longitudinal reinforcement for torsion:

$$A_l = \frac{T_n P_h}{2A_o f_y} = \left(\frac{A_t}{s}\right) P_h \left(\frac{f_{yt}}{f_y}\right)$$

$$A_{l,min} = \frac{5\sqrt{f'_c}}{12f_y}A_{cp} - \left(\frac{A_t}{s}\right) P_h \left(\frac{f_{yt}}{f_y}\right) \qquad and\ \frac{A_t}{s} \geq \frac{0.175b_w}{f_{yt}}$$

9.7 Notes:

1. Torsional reinforcement shall continue a distance $(b_t + d)$ past the point where the torque is less than the threshold torsion. Where b_t is the width of that part of cross section containing the closed stirrups resisting torsion, mm.
2. The stirrups must be closed.
3. Longitudinal torsion reinforcement shall be developed at both ends.
4. The longitudinal reinforcement shall be distributed around the perimeter of the closed stirrups with a maximum spacing of 300mm.
5. The longitudinal bars shall be inside the closed stirrups.
6. There shall be at least one longitudinal bar in each corner of the stirrups.
7. Longitudinal bars shall have a diameter at least 1/24 (0.042) times the stirrup spacing, but not less than 10mm.

Example 1:

Given:

f'c= 28MPa

fy= 420MPa

Rectangular section: bw= 350mm h=600mm

Clear cover to stirrup= 40mm

Assume $\Phi 12mm$ stirrups and $\Phi 25mm$ longitudinal bars

Mu (negative moment) = 310kN.m

Vu= 260kN

Tu= 38kN.m

Solution

Step 1: Determine the flexural reinforcement:

For Mu= 310kN.m, bw=350mm and h=600mm:

d= 600- (40+12+25/2)= 600-65=535mm

→ $\rho = 0.00887$

$\rho_{min} = 0.00333$ $\rho_{max,singly} = 0.375\beta_1 \dfrac{0.85f'_c}{f_y} = 0.01806$

$\rho_{min} < \rho < \rho_{max,singly}$ ok

$A_s = 0.00887(350)(535) = 1661mm^2$

Step 2: Check torsion: $T_u = 38kN.m$

$$\emptyset T_{th} = \emptyset \frac{1}{12}\sqrt{f'_c}\frac{{A_{cp}}^2}{P_{cp}}$$

$$A_{cp} = 350(600) = 210\ 000mm^2$$

$$P_{cp} = 2(350 + 600) = 1900mm$$

$$\emptyset T_{th} = \emptyset\lambda\frac{1}{12}\sqrt{f'_c}\frac{{A_{cp}}^2}{P_{cp}} = \frac{(0.75)(1)\frac{1}{12}\sqrt{28}\frac{(210000)^2}{1900}}{10^6} = 7.68kN.m$$

So, consider torsion.

Step 3: Check section adequacy: check section dimensions:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u P_h}{1.7\,{A_{oh}}^2}\right)^2} \leq \emptyset\frac{5}{6}\sqrt{f'_c}$$

$V_u = 260 \ x \ 10^3 \ N \quad T_u = 38 \ x \ 10^6 \ N.mm$

$b_w = 350mm \quad d = 535mm$

$x_1 = 350 - (40 + 12/2)(2) = 350 - 92 = 258mm$

$y_1 = 600 - 92 = 508mm$

$A_{oh} = x_1 y_1 = 258(508) = 131064mm^2$

$P_h = 2(258 + 508) = 1532mm$

Applied stress (left side of the equation) = 2.43MPa

Allowed stress (right side of the equation) = 3.31MPa

→ Section dimensions are ok.

Step 4: Compute shear reinforcement: Applicable for ACI 318-14 and ACI 318-19:

$V_u = 260kN \qquad \frac{V_u}{\Phi} = 346.7kN$

$$V_c = \frac{1}{6}\lambda\sqrt{f'_c}b_w d = \frac{\frac{1}{6}(1)\sqrt{28}(350)(535)}{1000} = 165kN$$

$V_s = 346.7 - 165 = 181.7kN$

$$\frac{A_v}{s} = \frac{V_s}{f_{yt} d} = \frac{181700}{(420)(535)} = 0.81mm^2/mm$$

Step 5: Compute torsion transverse reinforcement:

$$\frac{A_t}{s} = \frac{T_u/\emptyset}{2A_o f_{yt}} = \frac{(38x10^6)/0.75}{(2)(0.85)(131064)(420)} = \frac{0.541mm^2}{mm}$$

Step 6: Add shear and torsion transverse reinforcement:

$$\frac{A_{v+t}}{s} = \frac{A_v}{s} + 2\frac{A_t}{s} = 0.81 + 2(0.541) = 1.892mm^2/mm$$

$$\frac{A_{v+t}}{s} \geq max\left[0.062\sqrt{f'_c}\frac{b_w}{f_{yt}}, \frac{0.35b_w}{f_{yt}}\right] = \frac{0.29mm^2}{mm} < \frac{1.892mm^2}{mm} \quad ok.$$

Stirrups spacing, S= 113x2/1.892= 119mm

$$S_{max} = \frac{P_h}{8} = \frac{1532}{8} = 192mm < 300mm$$

Use S= 100mm

Step 6: compute torsion longitudinal reinforcement:

$$A_l = \frac{T_n P_h}{2A_o f_y} = \left(\frac{A_t}{s}\right) P_h \left(\frac{f_{yt}}{f_y}\right)$$

$$A_{l,min} = \frac{5\sqrt{f'_c}}{12f_y} A_{cp} - \left(\frac{A_t}{s}\right) P_h \left(\frac{f_{yt}}{f_y}\right) \quad and \quad \frac{A_t}{s} \geq \frac{0.175b_w}{f_{yt}} = \frac{0.175(350)}{420} = \frac{0.15mm^2}{mm}.$$

$$\frac{A_t}{s} = \frac{0.541mm^2}{mm} > \frac{0.15mm^2}{mm} \qquad P_h = 1532mm$$

$$A_{cp} = 210\,000mm^2$$

$$\rightarrow A_l = 829mm^2 \qquad Al, min = 126mm^2$$

$$Use\ A_l = 829mm^2$$

Step 7: Bars distribution:

Torsion longitudinal bars: they can be divided into three layers to have maximum spacing between bars not larger than 300mm; bottom, middle and top.

So,

$$A_s = \frac{829}{3} = 276mm^2$$

Top bars, $A_s = 1661 + 276 = 1937mm^2 \quad 4\phi25$

Middle bars, $A_s = 276mm^2 \quad (2\phi14)$

Bottom bars, $A_s = 276mm^2 \ (2\phi14)$

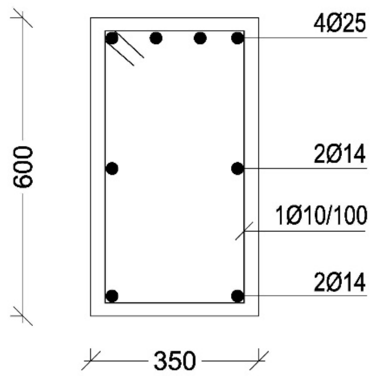


Figure 9.10: Beam cross section

Chapter 10: Deflection of Beams and One-Way Slabs

10.1 Structures should be designed for:

- Adequate strength at ultimate loads.
- Limited and accepted deflections at service loads.
- Limited crack widths.
- Ductility provisions: the deflection at ultimate loads should be large enough to give warning of failure so that the total collapse could be prevented

10.2 Behavior of beams:

When a beam is subjected to load, it is subjected to bending moment, Ma. The beam will be subjected to the following stages:

Stage 1: Pre-cracking stage

$M_a < M_{cr}$

$M_u \leq \emptyset M_n$

$$M_{cr} = \frac{f_r I_g}{y_t}$$

$f_r = 0.62\lambda\sqrt{f'_c}$

Where:

M_a: Service bending moment.

M_{cr}: Cracked moment.

M_u: Ultimate moment.

$\emptyset M_n$: Design strength for flexure.

I_g: Gross moment of inertia.

f_r: Modulus of rupture, MPa.

λ: Modification factor reflecting the reduced mechanical properties of lightweight concrete,

all relative to normal weight concrete of the same compressive strength.

y_t: Distance from centroidal axis of gross section, neglecting reinforcement, to tension face, mm.

Stage 2: Post-cracking stage

$M_a \geq M_{cr}$

$M_u \leq \emptyset M_n$

Stage 3: Post-serviceability

Where the stress in the tension reinforcement reaches the limit state of yielding, then a failure will develop.

$M_u > \emptyset M_n$

10.3 Effective moment of inertia:

<u>**ACI 318-14:**</u>

For nonprestressed members, effective moment of inertia, I_e, shall be calculated by Eq. (24.2.3.5a) unless obtained by a more comprehensive analysis, but I_e shall not be greater than I_g.

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) I_{cr} \qquad ACI318 - 14\ eq. (24.2.3.5a)$$

Where I_{cr} is the section cracked moment of inertia.

$$If\ M_a < M_{cr} \rightarrow I_e = I_g$$

$$If\ 3M_{cr} \geq M_a \geq M_{cr} \rightarrow I_e \quad ACI318 - 14\ eq. (24.2.3.5a)$$

$$If\ M_a > 3M_{cr} \rightarrow I_e = I_{cr}$$

<u>**ACI 318-19:**</u>

For nonprestressed members, unless obtained by a more comprehensive analysis, effective moment of inertia, I_e shall be calculated in accordance with Table 24.2.3.5.

Table 10.1: ACI 318-19 Table 24.2.3.5—Effective moment of inertia, I_e

Service moment	Effective moment of inertia, I_e mm^4	
$M_a \le \frac{2}{3} M_{cr}$	I_g	(a)
$M_a > \frac{2}{3} M_{cr}$	$I_e = \dfrac{I_{cr}}{1 - \left(\dfrac{\left(\frac{2}{3}\right) M_{cr}}{M_a}\right)^2 \left(1 - \dfrac{I_{cr}}{I_g}\right)}$	(b)

10.4 Deflection computations:

The deflection value depends on:

- Span
- Loads
- Supports
- Modulus of elasticity
- Moment of inertia

The deflection can be divided into:

- Immediate (instantaneous) deflection
- Long term deflection (due to creep and shrinkage)

The total long-term deflection, Δ_{LT} is given by:

$$\Delta_{LT} = \Delta_L + \lambda_\infty \Delta_D + \lambda_t \Delta_{LS}$$

where:

Δ_L: immediate live load deflection.

Δ_D : immediate dead load deflection.

Δ_{LS} : immediate sustained live load deflection.

λ_Δ: multiplier for additional deflection due to long-term effects

$$\lambda_\Delta = \frac{\xi}{1 + 50\rho'}$$

where ρ' (compression steel ratio) shall be the value at midspan for simple and continuous spans, and at support for cantilevers.

It shall be permitted to assume ξ, the time-dependent factor for sustained loads, to be equal to:

5 years or more 2.0

12 months.............................. 1.4

6 months............................... 1.2

3 months................................ 1.0

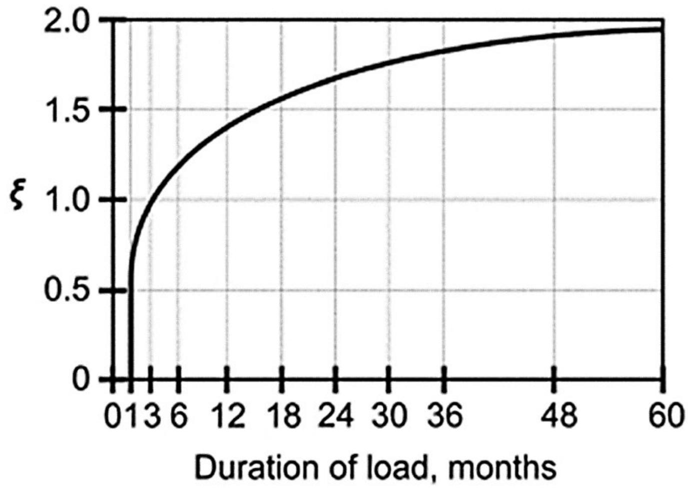


Figure 10.1: Multipliers for long-term deflections

10.5 Member effective moment of inertia

Computations of I_e (option 1):

For continuous members, I_e shall be permitted to be taken as the average of values for the critical positive and negative moment sections.

Computations of I_e (option 2):

For prismatic members, I_e shall be permitted to be taken as the value at midspan for simple and continuous spans, and at support for cantilevers.

Computations of I_e (option 3):

Or, one can use the following formulas to compute I_e in beams:

$$For\ simple\ span: I_e = I_{e,midspan}$$

$$For\ cantilever: I_e = I_{e,fixed\ end}$$

$$For\ one\ end\ continuous\ span: I_e = 0.85 I_{e,midspan} + 0.15 I_{e,continuous\ end}$$

$$For\ two\ ends\ continuous\ span:$$

$$I_e = 0.7 I_{e,midspan} + 0.15 I_{e,continuous\ end-1} + 0.15 I_{e,continuous\ end-2}$$

10.6 Allowable deflections:

The minimum thickness stipulated in ACI 318-19 code Tables 7.3.1.1, 9.3.1.1 shall apply for one-way construction not supporting or attached to partitions or other construction likely to be damaged by large deflections, unless computation of deflection indicates a lesser thickness can be used without adverse effects.

Table 10.2: ACI 318-19 Table 24.2.2—Maximum permissible calculated deflections

Member	Condition		Deflection to be considered	Deflection limitation
Flat roofs	Not supporting or attached to nonstructural elements likely to be damaged by large deflections Immediate deflection due to L		Immediate deflection due to maximum of L_r, S, and R	$L/180$[1]
Floors			$L/360$	
Roof or floors	Supporting or attached to nonstructural elements	Likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements, which is the sum of the time- dependent deflection due to all sustained loads and the immediate deflection due to any additional live load [2]	$L/480$[3]
		Not likely to be damaged by large deflections		$L/240$[4]

[1] Limit not intended to safeguard against ponding. Ponding shall be checked by calculations of deflection, including added deflections due to ponded water, and considering time- dependent effects of sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

[2] Time-dependent deflection shall be calculated in accordance with 24.2.4, but shall be permitted to be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be calculated on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

[3] Limit shall be permitted to be exceeded if measures are taken to prevent damage to supported or attached elements.

[4] Limit shall not exceed tolerance provided for nonstructural elements.

10.7 Note:

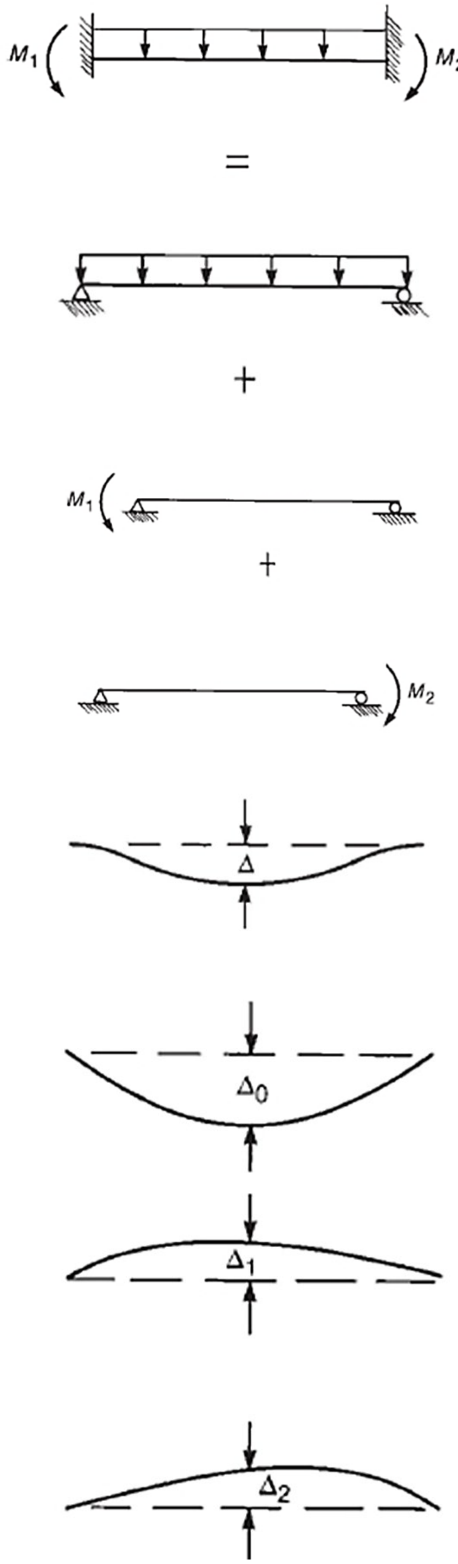


Figure 10.2: Deflection at mid span

The midspan deflection for a continuous beam with uniform loads and unequal end moments can be computed with the use of superposition as follows (notice the signs of deflections):

$$\Delta = \Delta_o - \Delta_1 - \Delta_2$$

$$\Delta = \frac{5M_oL^2}{48EI} - \frac{3M_1L^2}{48EI} - \frac{3M_2L^2}{48EI}$$

Where:

M_o: The moment at midspan due to uniform loads on a simple span.

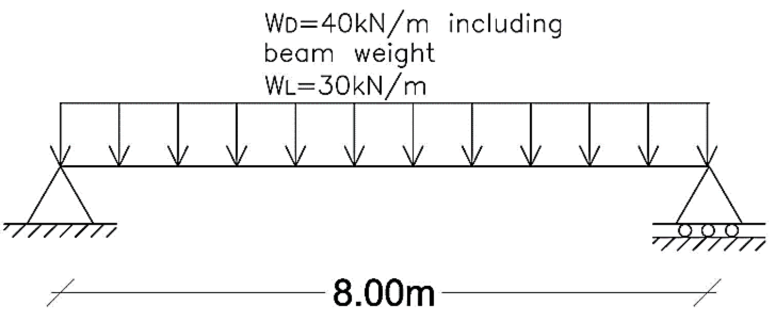
M_1 and M_2: The span end moments.

Example 1:

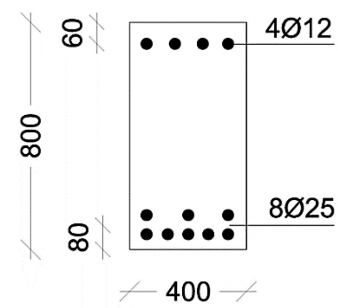
Calculate the immediate and long-term deflections for the beam shown in Figure 10.3 below assuming half the live load is sustained forever and compare with code allowable values.

Given: f'c= 24MPa

fy= 420MPa



BEAM STRUCTURAL MODEL



CROSS SECTION

Figure 10.3: Beam model and section for example 1

Solution:

Immediate deflection due to dead load:

$$M_a = M_D = \frac{w_D L^2}{8} = \frac{(40)(8)^2}{8} = 320kN.m$$

$$I_g = \frac{bh^3}{12} = \frac{(400)(800)^3}{12} = 1.707x10^{10}mm^4$$

$$f_r = 0.62\lambda\sqrt{f'_c} = 0.62(1)\sqrt{24} = 3.037MPa$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{(3.037)(1.707x10^{10})}{800/2}/10^6 = 129.6kN.m$$

Since $M_a > M_{cr}, the\ moment\ of\ inertia\ shall\ be\ reduced.$

Calculations of I_{cr}:

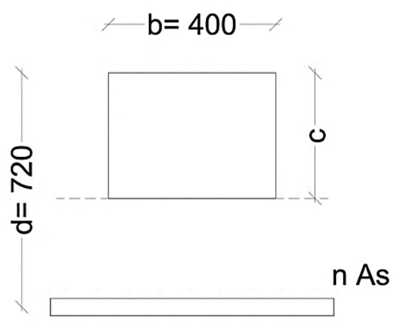


Figure 10.4: Cracked section for the beam

Since As' is very small comparing with As, so it will be neglected.

The location of the neutral axis can be calculated as follows:

$$bc\frac{c}{2} = nA_s(d-c)$$

$$E_c = 4700\sqrt{f'_c} = 4700\sqrt{24} = 23000MPa$$

$$n = \frac{E_s}{E_c} = \frac{200000}{23000} = 8.7$$

$$A_s = 8(491) = 3928mm^2$$

$$bc\frac{c}{2} = nA_s(d-c) = (400)(c)\left(\frac{c}{2}\right) = (8.7)(3928)(720-c)$$

$$c^2 + 171c - 123000 = 0 \rightarrow c = 275mm$$

$$I_{cr} = \frac{bc^3}{3} + nA_s(d-c)^2 = 9.54x10^9 mm^4$$

ACI 318-14:

$$3M_{cr} \geq M_a \geq M_{cr}$$

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) I_{cr} = 1x10^{10} mm^4$$

$$\Delta = \frac{5w_D L^4}{384E_c I_e} = \frac{(5)(40)(8000)^4}{(384)(23000)(1x10^{10})} = 9.3mm$$

ACI 318-19:

$$I_e = \frac{I_{cr}}{1 - \left(\frac{\left(\frac{2}{3}\right) M_{cr}}{M_a}\right)^2 \left(1 - \frac{I_{cr}}{I_g}\right)} = \frac{9.54x10^9}{1 - \left(\frac{\frac{2}{3}(129.6)}{320}\right)^2 \left(1 - \frac{9.54x10^9}{1.707x10^{10}}\right)} = 9.86x10^9 mm^4$$

$$\Delta = \frac{5w_D L^4}{384E_c I_e} = \frac{(5)(40)(8000)^4}{(384)(23000)(9.86x10^9)} = 9.4mm$$

Immediate deflection due to dead + live load:

$$M_a = M_{D+L} = \frac{w_{D+L} L^2}{8} = \frac{(70)(8)^2}{8} = 560kN.m$$

ACI 318-14:

$$M_a \geq 3M_{cr} \rightarrow I_e = I_{cr} = 9.54x10^9 mm^4$$

$$\Delta = \frac{5w_{D+L} L^4}{384E_c I_e} = \frac{(5)(70)(8000)^4}{(384)(23000)(9.54x10^9)} = 17mm$$

ACI 318-19:

$$I_e = \frac{I_{cr}}{1-\left(\frac{\left(\frac{2}{3}\right)M_{cr}}{M_a}\right)^2\left(1-\frac{I_{cr}}{I_g}\right)} = \frac{9.54x10^9}{1-\left(\frac{\frac{2}{3}(129.6)}{560}\right)^2\left(1-\frac{9.54x10^9}{1.707x10^{10}}\right)} = 9.64x10^9 mm^4$$

$$or \qquad M_a \geq 3M_{cr} \rightarrow I_e = I_{cr} = 9.54x10^9 mm^4$$

$$\Delta = \frac{5w_{D+L}L^4}{384E_cI_e} = \frac{(5)(70)(8000)^4}{(384)(23000)(9.54x10^9)} = 17mm$$

$$ACI\ 318-19: \qquad \Delta_L = \Delta_{D+L} - \Delta_D = 17 - 9.4 = 7.6mm$$

Immediate deflection due to dead + sustained live load:

$$M_a = M_{D+Ls} = \frac{w_{D+Ls}L^2}{8} = \frac{(55)(8)^2}{8} = 440kN.m > 3M_{cr}$$

$$I_e = I_{cr} = 9.54x10^9 mm^4$$

$$\Delta = \frac{5w_{D+Ls}L^4}{384E_cI_e} = \frac{(5)(55)(8000)^4}{(384)(23000)(9.54x10^9)} = 13.4mm$$

$$ACI\ 318-19: \qquad \Delta_{Ls} = \Delta_{D+Ls} - \Delta_D = 13.4 - 9.4 = 4.0mm$$

$$ACI\ 318-19: \qquad \Delta_{L,unsustained} = \Delta_{D+L} - \Delta_{D+Ls} = 17 - 13.4 = 3.6mm$$

Total long-term deflection:

$$\Delta_{LT} = \Delta_L + \lambda_\infty \Delta_D + \lambda_t \Delta_{LS}$$

$$\rho' = \frac{A_s'}{b_w d} = \frac{(4)(113)}{(400)(740)} = 0.00153$$

$$\lambda_\Delta = \frac{\xi}{1+50\rho'} = \frac{2}{1+(50)(0.00153)} = 1.86$$

$$\Delta_{LT} = 3.6 + (1.86)(9.4) + (1.86)(4.0) = 28.5mm$$

Compare deflections with ACI allowable values:

L/180= 44mm > Δ_L= 7.6mm OK

L/360= 22mm > Δ_L= 7.6mm OK

L/480= 16.7mm < Δ_{LT}= 28.5mm N.G

L/240= 33.3mm > Δ_{LT}= 28.5mm OK

So, this beam is not applicable for:

Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections.

Example 2:

Calculate I_g and I_{cr} for the section shown in Figure 10.5 below.

Given: n= 8.72

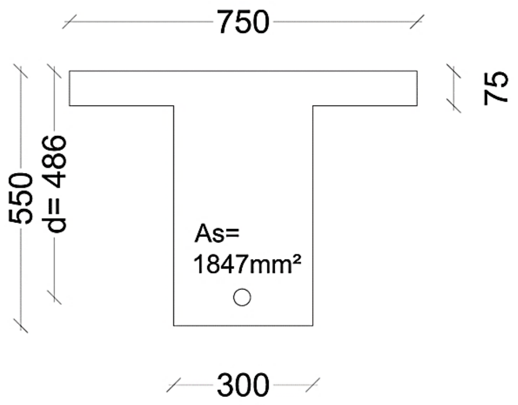


Figure 10.5: Beam section for example 2

Solution:

Gross moment of inertia:

Calculate location of centroid (take the reference line at top edge of section):

(750-300)(75)(75/2)+ (300)(550)(550/2)= y' ((750-300)(75)+ (300)(550))

→ y'= 234.7mm

Ig= $(1/12)(750-300)(75)^3 + (750-300)(75)(234.7-75/2)^2 + (1/12)(300)(550)^3 + (300)(550)(234.7-55/2)^2 = 5.76 \times 10^9\ mm^4$

Cracked moment of inertia:

The centroid is not clear, it is within the flange or within the web. So, assume that the centroid is located at the web- flange junction, c= 75mm, so:

(750)(75)(75/2) < ? (8.72)(1847)(486-75)

210 9375 < 661 9500 → c > 75mm

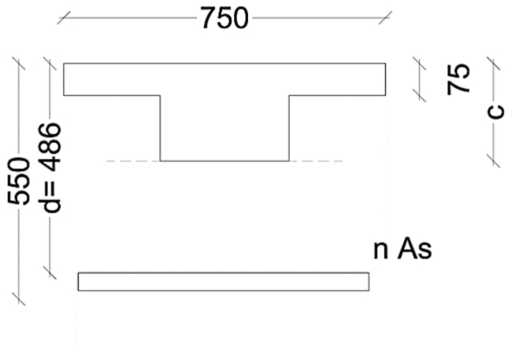


Figure 10.6: Cracked section

Calculate c:

(750-300)(75)(c-75/2) + (300)(c) (c/2) = (8.72) (1847)(486- c) → c= 126 mm

I_{cr} = (1/12)(750-300)$(75)^3$ + (750-300)(75)$(126-75/2)^2$ + (1/3)(300)$(126)^3$ + (8.72)(1847)$(486-126)^2$ = 2.4 x 10^9 mm^4

$$\frac{I_{cr}}{I_g} = 0.42$$

Chapter 11: Slender Columns

11.1 Introduction:

If the column fails due to initial material failure, it is classified as a short column. As the length of the column increases, the probability that failure will occur by buckling also increases. Therefore, the transition from the short column to the long column is defined using the slenderness ratio which is given by:

KL_u/r

Where:

K= effective length factor that depends on end or support conditions of the column

L_u= unsupported length of the column

r= radius of gyration of the section

$$r = \sqrt{\frac{I}{A}}$$

I= column moment of inertia in the direction of buckling

A= column section area

r= 0.3 h   for rectangular column

r= 0.25 D for circular column

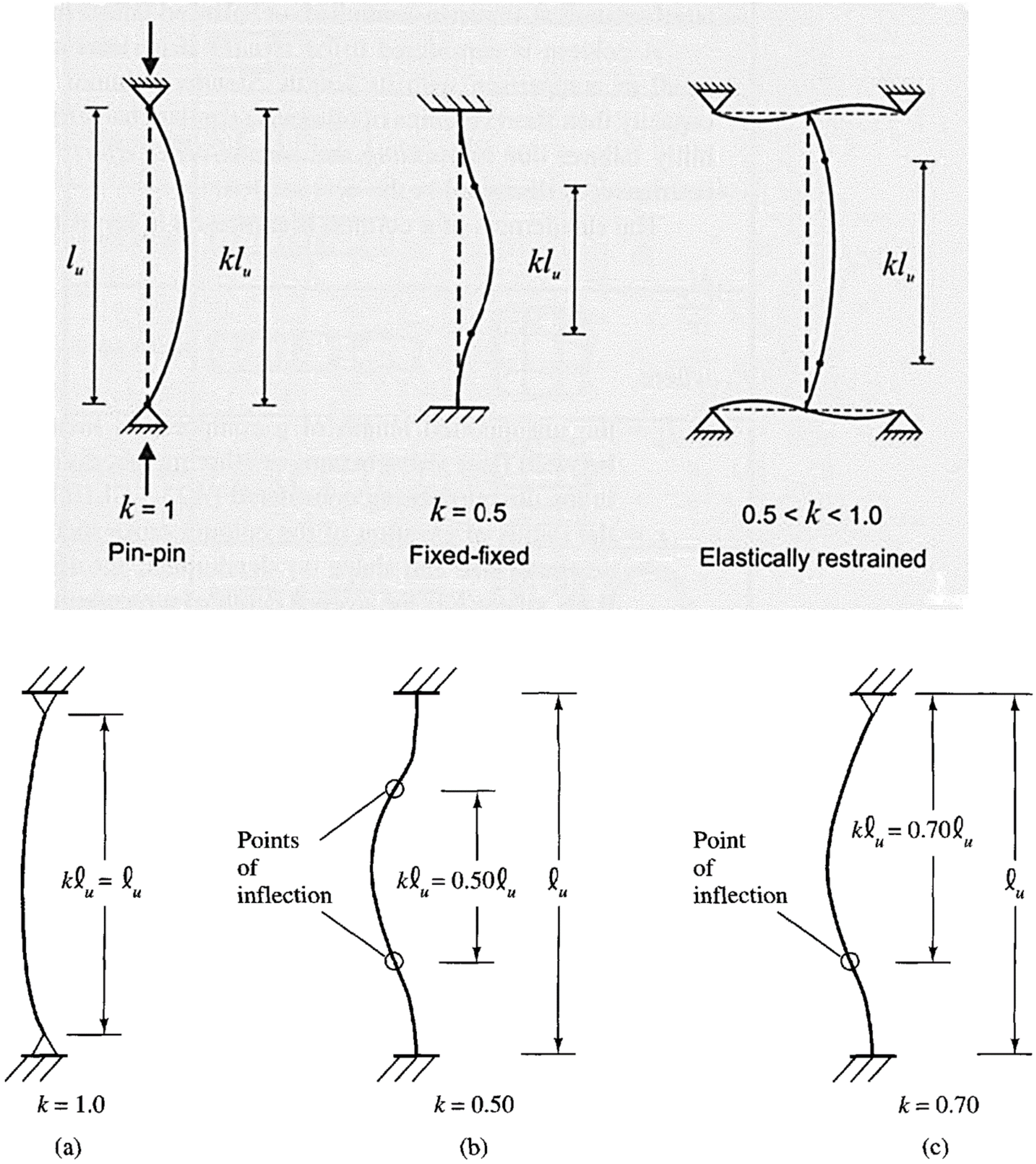
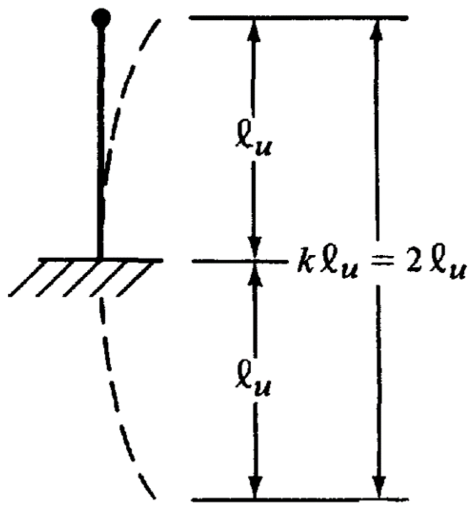


Figure 11.1: Unbraced length factors for columns



(a) Upper end free to rotate and translate, lower end fixed

Figure 11.1 - continued: Unbraced length factors for columns

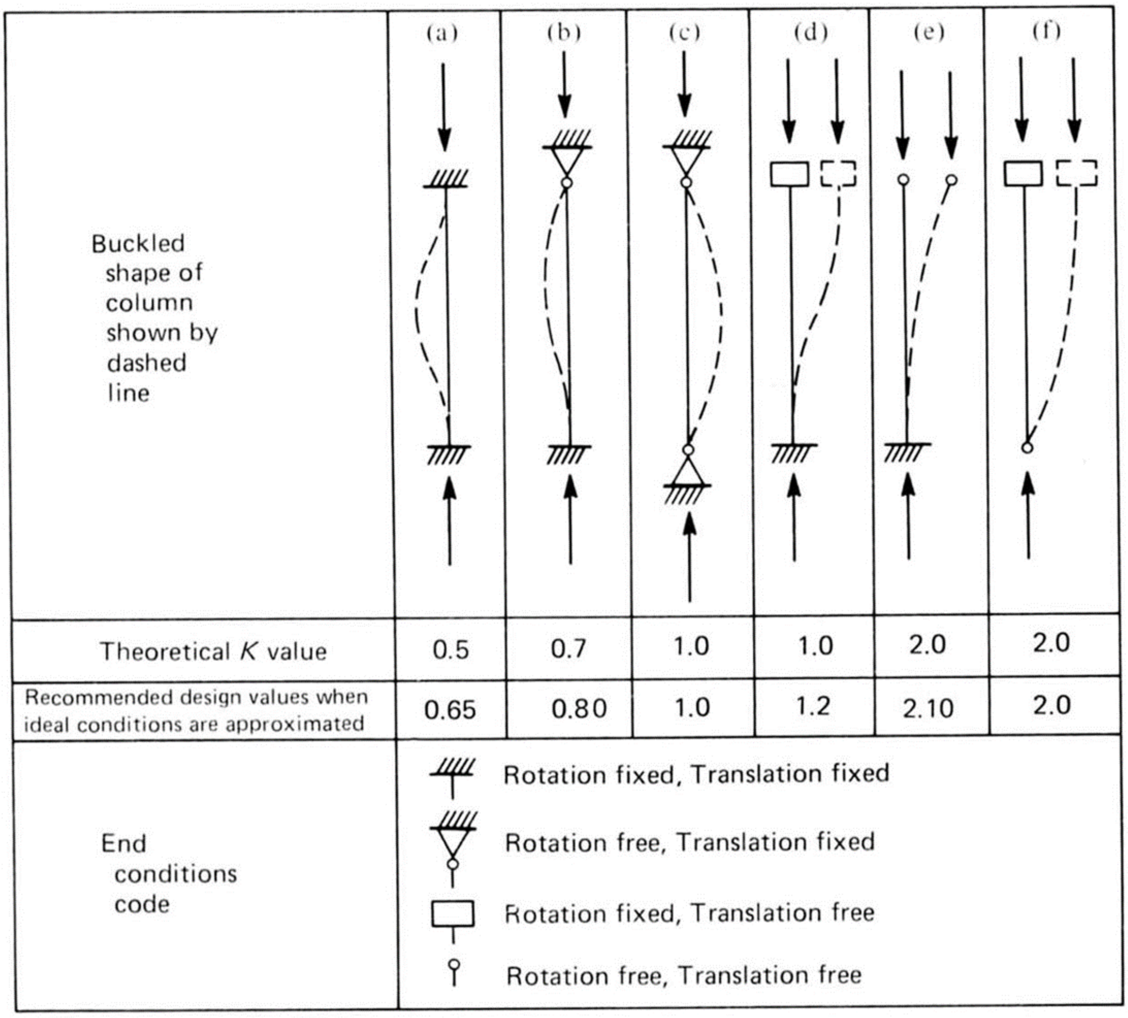


Figure 11.1- continued: Unbraced length factors for columns

Calculations of K:

K can be calculated from monographs for effective length factors using the factor Ψ as follows:

$$\psi = \frac{\sum \frac{EI}{L} \ of\ columns}{\sum \frac{EI}{L} \ of\ beams}$$

L= length of member center to center of the joints.

E= modulus of elasticity of concrete.

I= section moment of inertia.

Ψ= 0 if the column is fully fixed at that end.

Ψ= infinity if the column end is perfect hinge.

In practical structures, there is no such thing as truly fixed end or truly hinged end

Reasonable upper and lower limits on Ψ are 20 and 0.2.

Note: For columns in nonsway frames, k should never be taken less than 0.6

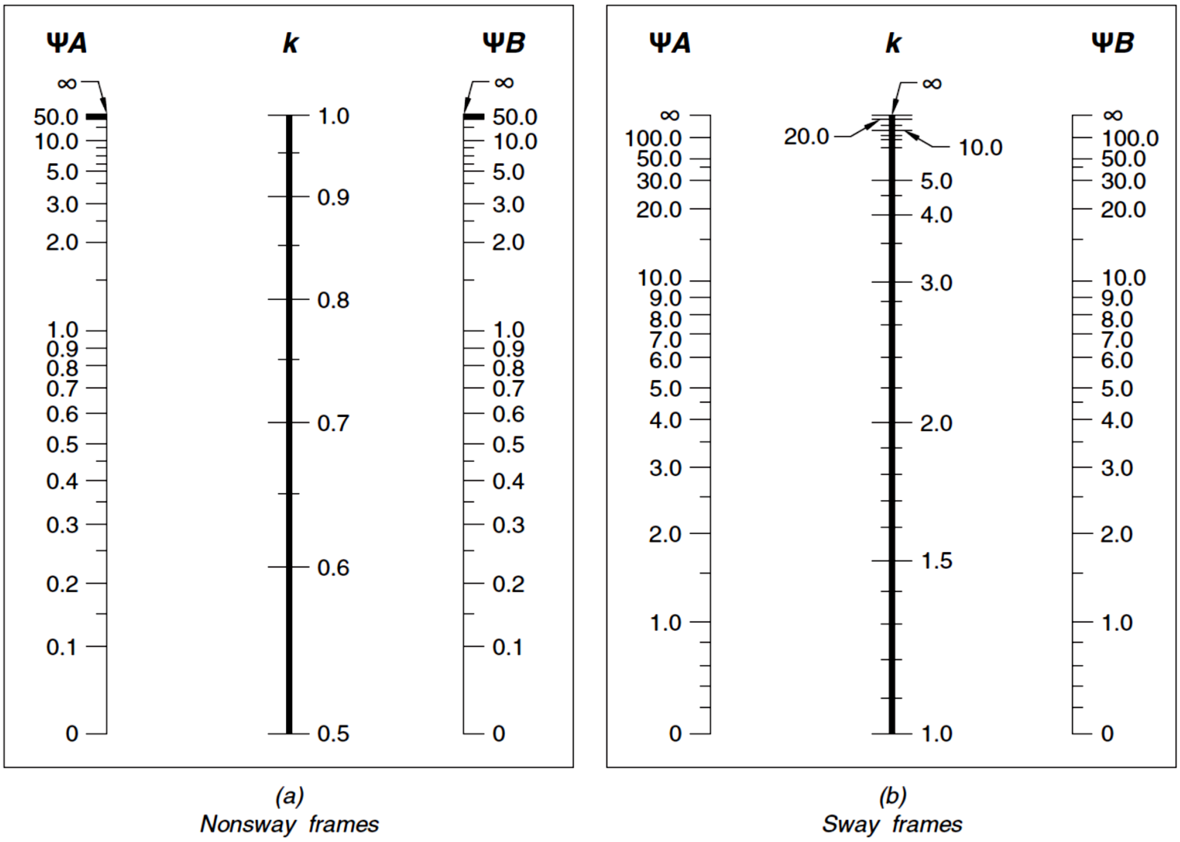


Figure 11.2: ACI 318-19 Fig. R6.2.5.1 Effective length factor k

11.2 P – delta moments:

P-δ moments (member P- delta): these moments result from deflections of the axis of bent column away from the chord joining the ends of column. The slenderness effects in pin- ended columns and in nonsway frames result from **P-δ** effects.

P-Δ moments (structure P- delta): these moments result from lateral deflections of the beam- column joints from their original undeflected locations. The slenderness effects in sway frames result from **P-Δ** effects.

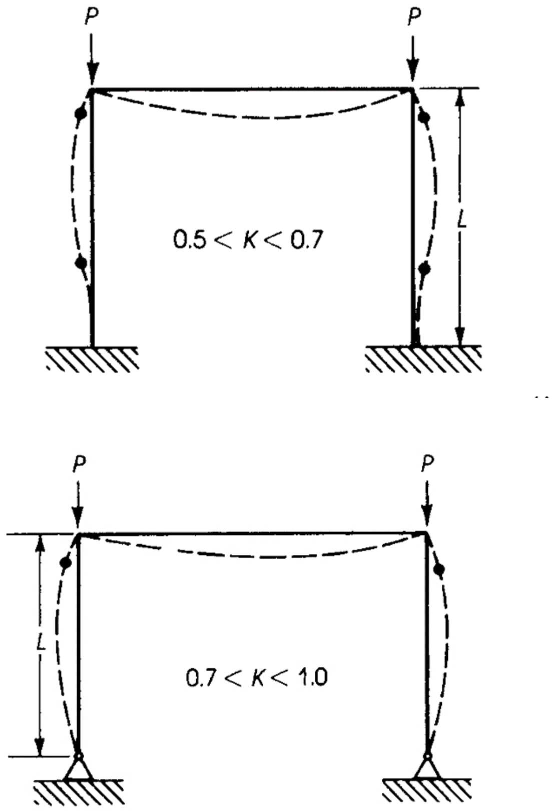


Figure 11.3: Unbraced length factors in braced frame

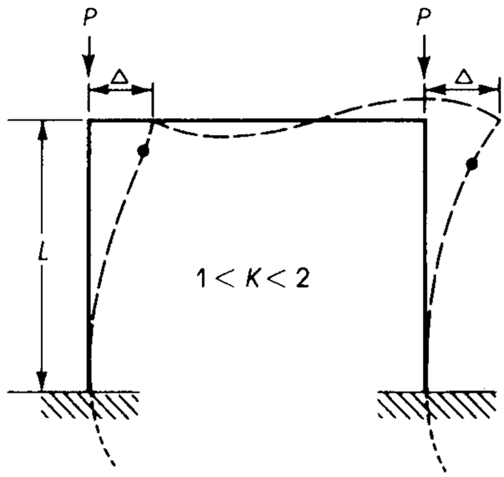


Figure 11.4: Unbraced length factors in unbraced frame with fixed supports

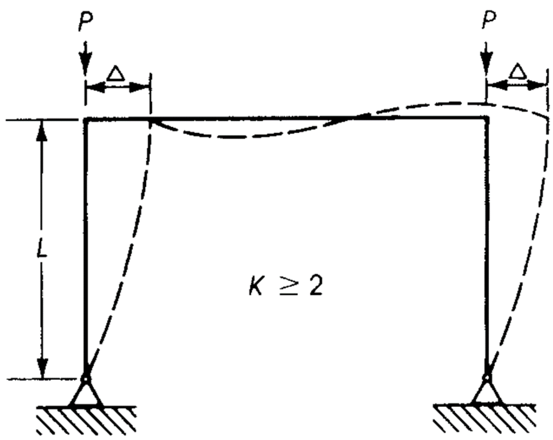


Figure 11.5: Unbraced length factors in unbraced frame with pin supports

11.3 First and second order analyses:

A. In a first order analysis, the equations of equilibrium are derived by assuming that deflections have a negligible effect on the internal forces in the members
B. In a second order analysis, the equations of equilibrium considered the deformed shape of the structure

C. Instability can be investigated only via a second order analysis, because it is the loss of equilibrium of the deformed structure that causes instability

D. However, because many engineers' calculations and computer programs are based on first order analysis, methods have been derived to modify the results of a first order analysis to approximate the second order effects.

11.4 Braced frames (columns):

It shall be permitted to analyze columns and stories in structures as nonsway frames if a, b or c is satisfied:

(a) If bracing elements resisting lateral movement of a story have a total stiffness of at least 12 times the gross lateral stiffness of the columns in the direction considered.
(b) The increase in column end moments due to second- order effects does not exceed 5% of the first – order end moments.
(c) Q (Stability index) in accordance with ACI 318-19 section 6.6.4.4.1 does not exceed 0.05.

$$Q = \frac{\sum P_u \Delta_o}{V_{us} l_c}$$

where $\sum P_u$ and V_{us} are the total factored vertical load and horizontal story shear, respectively, in the story being evaluated, and Δ_o is the first order relative lateral deflection between the top and bottom of that story due to V_{us}. l_c is the length of compression member, measured center to- center of the joints.

11.5 Moment computations in slender columns:

1. Second order analysis
2. Moment magnification for first order analysis for K $L_u/r \leq 100$

11.6 Moment magnification in nonsway (braced) frames:

For columns braced against sidesway, slenderness effects shall be permitted to be neglected if:

$$\frac{kl_u}{r} \leq 34 + 12\frac{M_1}{M_2} \leq 40$$

where M_1/M_2 is negative if the column is bent in single curvature, and positive for double curvature. M_1 and M_2 are the smaller and larger end column moments respectively.

The magnified moment is given as:

$$M_c = \delta_{ns} M_2$$

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \geq 1.0$$

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2}$$

$$(EI)_{eff} = \frac{0.4E_c I_g}{1 + \beta_{dns}} \qquad (1)$$

$$(EI)_{eff} = \frac{\left(0.2E_c I_g + E_s I_{se}\right)}{1 + \beta_{dns}} \qquad (2)$$

$$(EI)_{eff} = \frac{E_c I}{1 + \beta_{dns}} \qquad (3)$$

$$C_m = 0.6 - 0.4\frac{M_1}{M_2}$$

where β_{dns} shall be the ratio of maximum factored sustained axial load to maximum factored axial load associated with the same load combination and I is calculated according to ACI 318-19 Table 6.6.3.1.1(b) for columns and walls. For equation 3, table 11.1 shall be used.

For columns with transverse loads applied between supports, $C_m = 1.0$.

Table 11.1: ACI 318-19 Table 6.6.3.1.1(b) Alternative moments of inertia for elastic analysis at factored loads

Member	Alternative value of I for elastic analysis		
	Minimum	I	Maximum
Columns and walls	$0.35I_g$	$\left(0.80+25\frac{A_{st}}{A_g}\right)\left(1-\frac{M_u}{P_u h}-0.5\frac{P_u}{P_o}\right)I_g$	$0.875I_g$
Beams, flat plates, and flat slabs	$0.25I_g$	$(0.10+25\rho)\left(1.2-0.2\frac{b_w}{d}\right)I_g$	$0.5I_g$

Notes: For continuous flexural members, I shall be permitted to be taken as the average of values obtained for the critical positive and negative moment sections. P_u and M_u shall be calculated from the load combination under consideration, or the combination of P_u and M_u that produces the least value of I.

The factor *Cm* is a correction factor relating the actual moment diagram to an equivalent uniform moment diagram. The derivation of the moment magnifier assumes that the maximum moment is at or near midheight of the column. If the maximum moment occurs at one end of the column, design should be based on an equivalent uniform moment $C_m M_2$ that leads to the same maximum moment at or near midheight of the column when magnified (MacGregor et al. 1970).

Where:

δ_{ns}= nonsway moment magnification factor.

P_u= ultimate axial compression on the column.

P_c= Euler buckling load for pin- ended column.

I_g= gross moment of inertia of the concrete section about its centroidal axis ignoring reinforcement.

I_{se}= moment of inertia of reinforcement about the centroidal axis of concrete section.

M_2 must be greater than M_{min}.

$$M_{min} = P_u e_{min}$$

$$e_{min} = 0.015 + 0.03h$$

Where h is the side length of section in meters and e_{min} is the minimum eccentricity.

If M_{min} exceeds M_2, C_m shall be taken equal to 1.0 or calculated based on the ratio of the calculated end moments M_1/M_2, using the considered equation.

Note: The cross-sectional dimensions of each member used in an analysis shall be within 10 percent of the specified member dimensions in construction documents or the analysis shall be repeated. If the stiffnesses of Table 6.6.3.1.1(b) are used in an analysis, the assumed member reinforcement ratio shall also be within 10 percent of the specified member reinforcement in construction documents.

Unless slenderness effects are neglected, the design of columns, restraining beams, and other supporting beams shall be based on the factored forces and moments considering second-order effects. M_u including second-order effects shall not exceed $1.4M_u$ due to first- order effects. So, if $\boldsymbol{\delta_{ns}}$ is greater than 1.4 enlarge section.

11.7 Moment magnification in sway (unbraced) frames:

Slenderness effects can be ignored if:

$$\frac{kl_u}{r} \leq 22$$

The end moments will be:

$$M_1 = M_{1ns} + \delta_s M_{1s}$$

$$M_2 = M_{2ns} + \delta_s M_{2s}$$

Where:

δ_s= sway moment magnification factor.

The magnified moment is given as:

$$M_c = \delta_{ns} M_2$$

The sway moment magnification factor δs, can be computed using the following procedures:

A. Second order analysis: ACI code allows the use of second order analysis to compute δ_s M_s.
B. Moment magnification procedure.

Elastic second-order analysis shall consider section properties determined taking into account the influence of axial loads, the presence of cracked regions along the length of the member, and the effects of load duration.

In elastic analysis, it shall be permitted to use the following properties for the members in the structure as shown in Table 11.2 below.

Table 11.2: Table 6.6.3.1.1(a)—Moments of inertia and cross- sectional areas permitted for elastic analysis at factored load level

<table>
<tr><th colspan="2">Member and condition</th><th>Moment of inertia</th><th>Cross- sectional area for axial deformations</th><th>Cross- sectional area for shear deformations</th></tr>
<tr><td colspan="2">Columns</td><td>$0.70I_g$</td><td rowspan="5">$1.0A_g$</td><td rowspan="5">b_wh</td></tr>
<tr><td rowspan="2">Walls</td><td>Uncracked</td><td>$0.70I_g$</td></tr>
<tr><td>Cracked</td><td>$0.35I_g$</td></tr>
<tr><td colspan="2">Beams</td><td>$0.35I_g$</td></tr>
<tr><td colspan="2">Flat plates and flat slabs</td><td>$0.25I_g$</td></tr>
</table>

The moment magnification factor, δ_s is given by:

$$(a) \qquad \delta_s = \frac{1}{1-Q} \geq 1$$

Or

$$(b) \qquad \delta_s = \frac{1.0}{1 - \frac{\sum P_u}{0.75 \sum P_c}} \geq 1.0$$

If δ_s exceeds 1.5, δs shall be calculated using second order elastic analysis or from equation (b).

Where:

ΣP_u= The summation for all the factored vertical loads in a story.

ΣP_c= the summation of Pc for all columns in the story.

When sustained lateral loads are present, *I for compression members shall be divided* by (1 + β*ds). The term* βds *shall be taken as the ratio of* maximum factored sustained shear within a story to the maximum factored shear in that story associated with the same load combination, but shall not be taken greater than 1.0.

Notes: ACI 318-19

6.6.3.1.2 For factored lateral load analysis, it shall be permitted to assume ***I* = 0.5*Ig*** for all members or to calculate I by more detailed analysis, considering the effective stiffness of all members under the loading conditions.

6.6.3.2.2 It shall be permitted to calculate immediate lateral deflections using a moment of inertia of 1.4 times I defined in 6.6.3.1, or using a more detailed analysis, but the value shall not exceed ***Ig***.

11.8 Moment – axial force interaction diagrams:

Column design for axial force and a moment can be achieved by:

1. Try a section with a specific steel ratio, draw P-M interaction diagram then check the location of P and M in the diagram
2. Try a section and use P-M interaction diagram sheets

In this chapter, P-M interaction diagram sheets are used.

P-M interaction diagram sheets:

- Section shape (rectangular or circular)
- Concrete strength, f'c
- Steel strength, fy
- Factor γ

$$\gamma = \frac{h - 2\ covers\ to\ bars\ centroid}{h}$$

- Bars distribution (at two sides/ distributed at 4 sides)
- $Determine\ \ \emptyset P_n/bh\ \ and\ \ \emptyset M_n/bh^2$

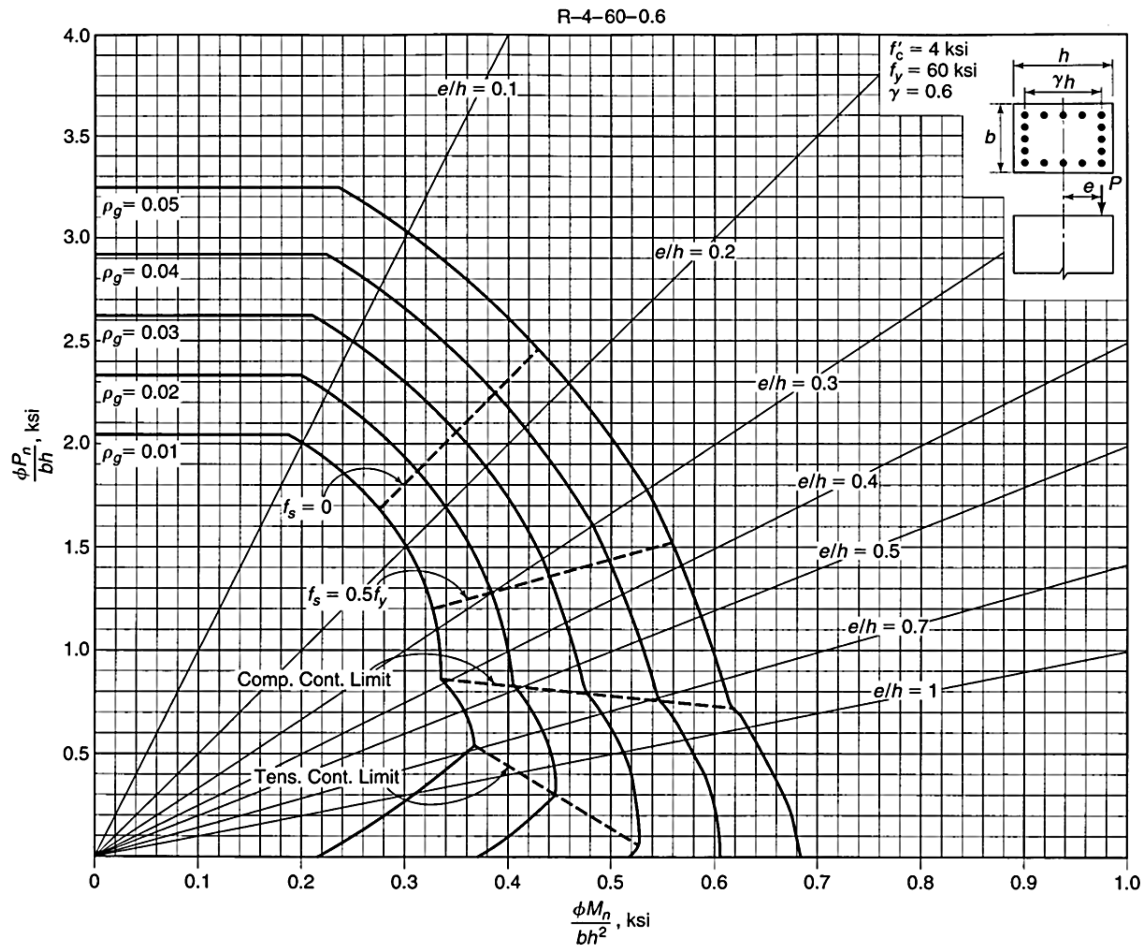


Figure 11.6: Moment- Axial force interaction diagram for rectangular column with γ=0.6

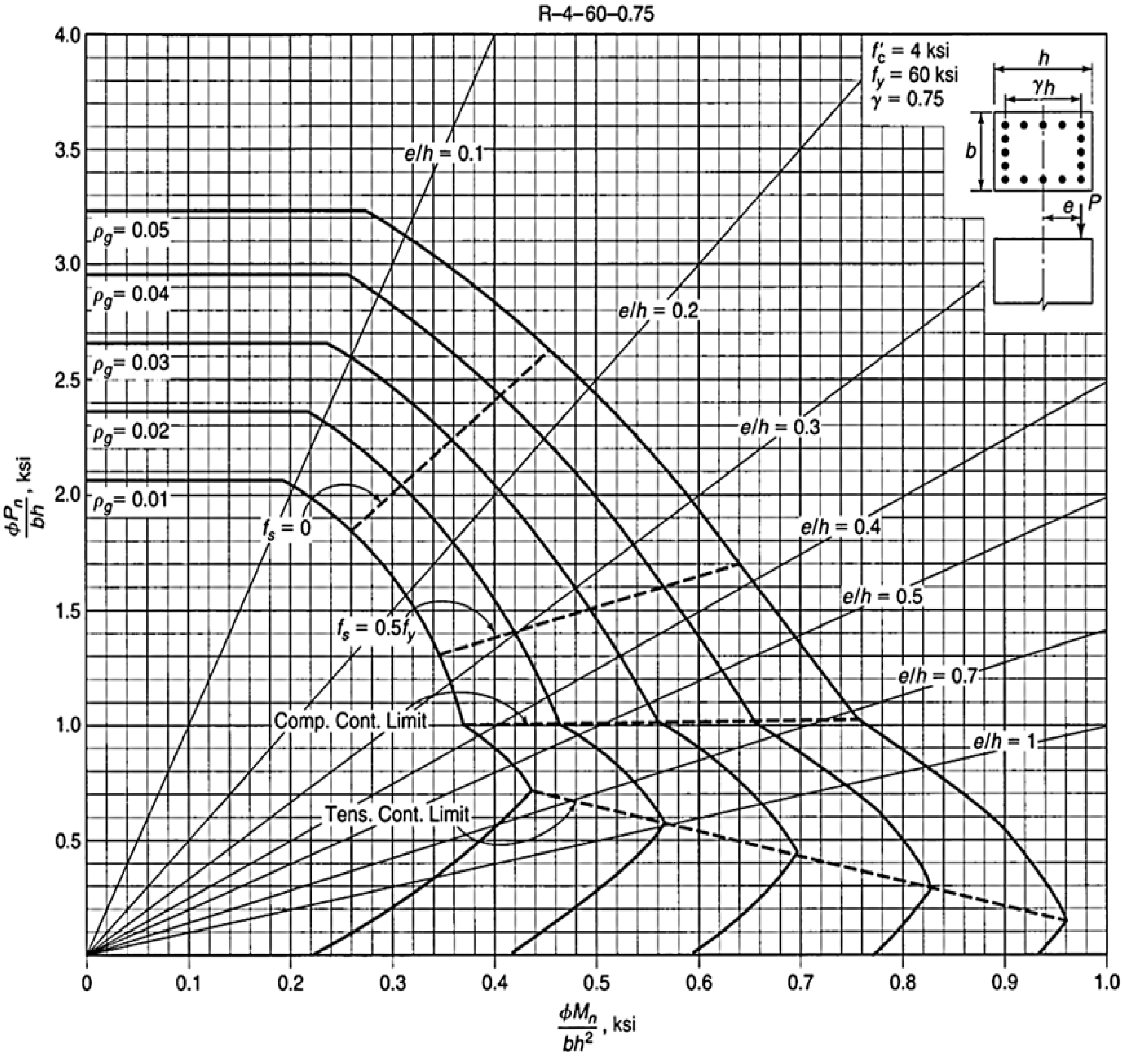


Figure 11.7: Moment- Axial force interaction diagram for rectangular column with γ=0.75

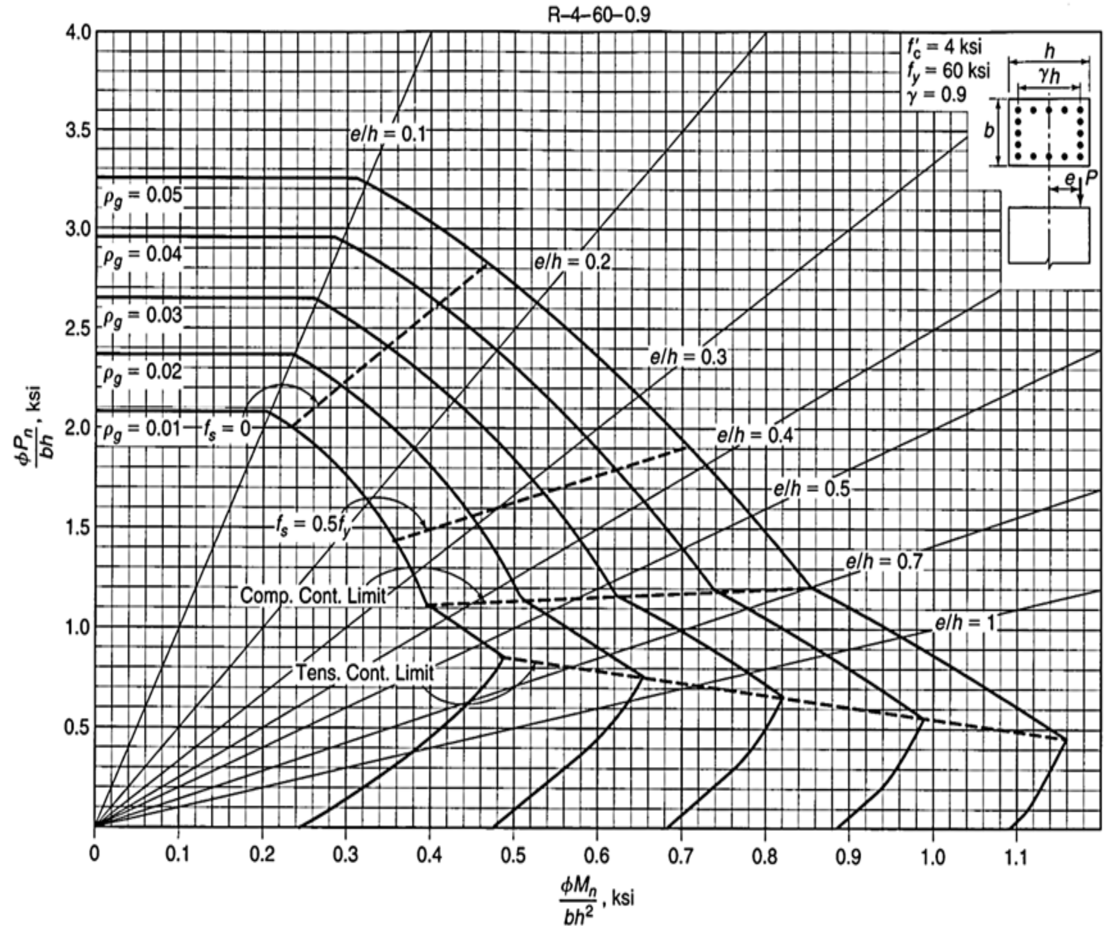


Figure 11.8: Moment- Axial force interaction diagram for rectangular column with γ=0.9

Example 1:

Design a 6m tall column to support an unfactored dead load of 410kN and unfactored live load of 340kN.

F'c= 28MPa

Fy= 420MPa

Assume that the column braced and K=1.0.

The bending moment diagram for the column is shown in Figure 11.9.

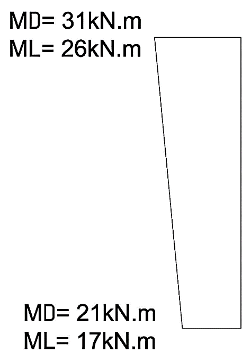


Figure 11.9: Bending moment diagram for the column in example 1

Solution:

Step1 : Compute ultimate loads:

P_u= 1.2 P_D + 1.6 P_L

P_u= 1.2 (410) + 1.6 (340)= 1036 kN

M_{u1}= 1.2 M_{D1} + 1.6 M_{L1}

M_{u1}= 1.2 (21) + 1.6 (17)= 52.4 kN.m

M_{u2}= 1.2 M_{D2}+ 1.6 M_{L2}

M_{u2}= 1.2 (31) + 1.6 (26)= 78.8 kN.m

Step2 : Estimate column size:

Assume steel ratio, ρ= 0.01. Assume M_u= 0.0

$$\emptyset P_n = \emptyset \lambda \left(0.85 f'_c \left(A_g - A_s\right) + f_y A_s\right)$$

Φ= 0.65

λ= 0.8

f'c= 28MPa

fy= 420MPa

$A_s = \rho A_g = 0.01 A_g$

Substitute in above equation → Ag= 71764mm^2

(270mm x 270mm) **try: 400mm x 400mm**

Step 3: Check slenderness

$$\frac{kl_u}{r} \leq 34 + 12\frac{M_1}{M_2} \leq 40$$

K= 1 Lu= 6m

r= 0.3 h = 0.3 (0.4)= 0.12

kLu/r= 50

M1= 52.4 kN.m

M2= 78.8 kN.m

M1/M2 Negative single curvature

34-12(M1/M2)= 26 < 50 ***consider slenderness***

Step 4: Compute moment magnification factor

$$M_c = \delta_{ns} M_2$$

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75 P_c}} \geq 1.0$$

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2}$$

$$(EI)_{eff} = \frac{0.4 E_c I_g}{1 + \beta_{dns}}$$

$$C_m = 0.6 - 0.4 \frac{M_1}{M_2}$$

C_m= 0.866

P_u= 1036 kN

K=1

L_u= 6000mm

$E_c = 4700\sqrt{28} = 24870 MPa$

I_g= (1/12)(400)^4= 2.133 x 10^9 mm^4

$$\beta_{dns} = \frac{1.2 P_D}{1.2 P_D + 1.6 P_L}$$

P_D= 410kN

P_L= 340kN

P_u= 1036kN

→ $\beta_{dns} = 0.475$

EI= 1.439 x 10^{13} N.mm^2

Pc= 3945kN

→ δns= 1.333

M_{u2}= 78.8kN.m

$M_{min} = P_u e_{min}$

$e_{min} = 0.015 + 0.03h$

h= 0.4m → e_{min}= 0.027m

M_{min}= 1036 (0.027)= 28kN.m < M_{u2}= 78.8kN.m

OK

M_c= (1.333)(78.8)= 105kN.m

Step 5: Section design

Column dimensions: 400 x 400 mm

b= 400mm h= 400mm

Pu= 1036kN Mu= 105kN.m

Concrete cover= 60mm

$$\gamma = (400 - 2\,x\,60)/400 = 0.7$$

$$\frac{\phi P_n}{bh} = \frac{(1036)(1000)}{(400)(400)(7)} = 0.93ksi$$

$$\frac{\phi M_n}{bh^2} = \frac{(105)(1000\,000)}{(400)(400)^2(7)} = 0.23ksi$$

Using column design aids: P-M interaction diagrams

→ Steel ratio, ρ= 0.01 A_s= 0.01(400)(400)= 1600mm^2 Use 8Φ16 reinforcing bars

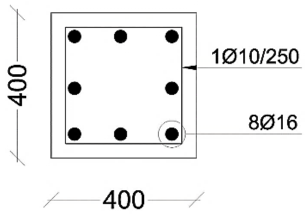


Figure 11.10: Section in column

Ties:

$$s \le 48d_s = 48(10) = 480mm$$

$$s \le 16d_b = 16(16) = 256mm \qquad\qquad controls$$

$$s \le least\ column\ dimension = 400mm$$

Example 2:

Design the column shown in the frame in Figure 11.11.

The axial force and the bending moment diagrams are shown in Figure 11.12.

Given:

f'c= 28MPa

fy= 420MPa

Load combination: U= 1.2D + 1.0L + 1.6W

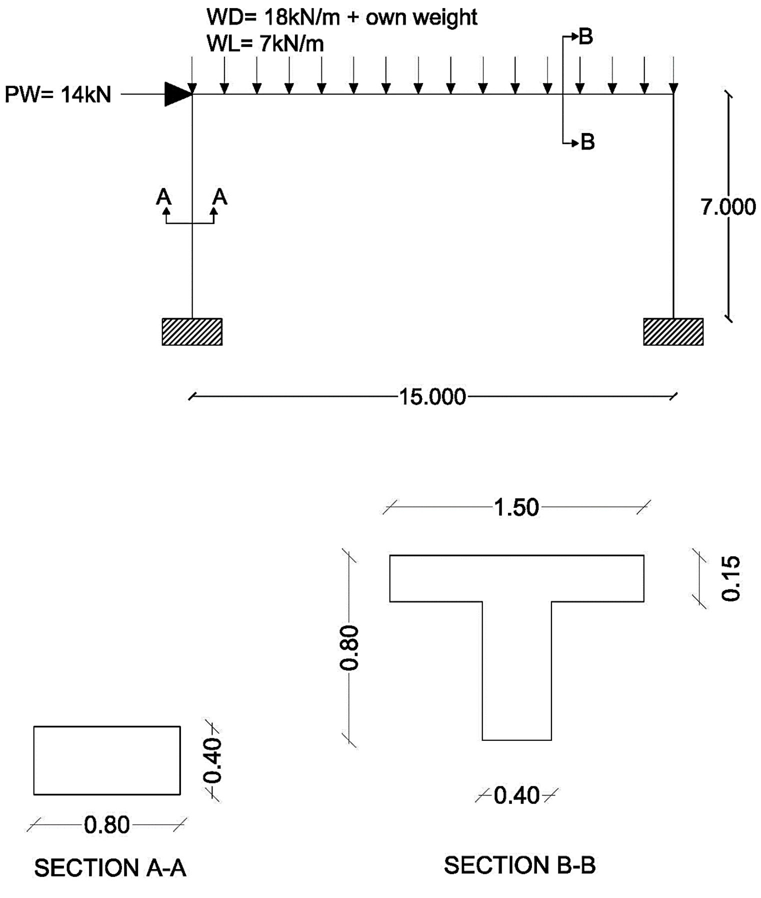


Figure 11.11: Frame and sections for example 2

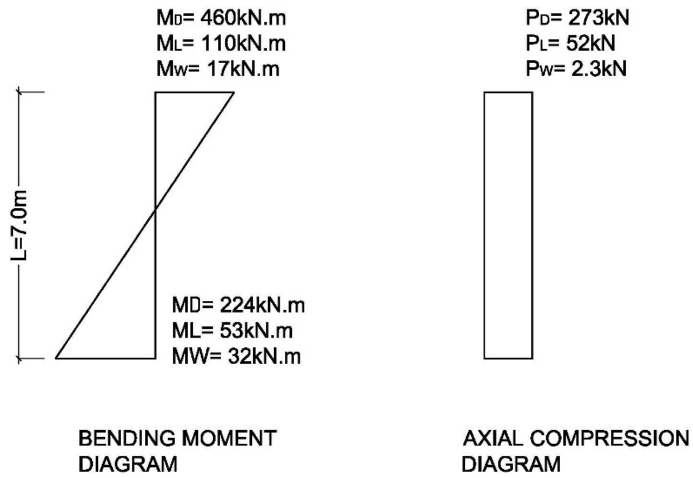


Figure 11.12: Bending moment and axial force diagram for the column for example 2

Solution:

Step1: Compute ultimate axial loads:

P_u= 1.2 P_D + 1.0 P_L + 1.6W

P_u= 1.2 (273) + 1.0 (52) + 1.6 (2.3)= 383.3 kN

Step 2: Check slenderness

$$\frac{kl_u}{r} \leq 22$$

$$\psi = \frac{\sum \frac{EI}{L}\ of\ columns}{\sum \frac{EI}{L}\ \ of\ beams}$$

I column= 1.707 x 10^{10} mm^4 (0.0171 m^4)

I beam= 2.89 x 10^{10} mm^4 (0.0289 m^4)

Modified I:

I column= 0.7 I = 0.01197 m^4

I beam= 0.35 I = 0.01012 m^4

ψ_A = 0 (fixed end)

ψ_B= (0.01197/7)/(0.01012/15) = 2.54

Using monograph for effective length factor of sway frames:

K= 1.35

Radius of gyration, r= o.3h = 0.3 (0.8) = 0.24m

$$\frac{kl_u}{r} = \frac{(1.35)(7-0.4)}{0.24} = 37.1 \ > \ 22$$

→ **consider slenderness**

Step 3: Calculate moment magnification factor, δs:

$$\delta_s = \frac{1.0}{1 - \frac{\sum P_u}{0.75 \sum P_c}} \geq 1.0$$

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2}$$

$$(EI)_{eff} = \frac{0.4 E_c I_g}{1 + \beta_{ds}}$$

$$\beta_{ds} = \ 0.0$$

K= 1.35

L_u= 6600mm

EI= 1.7 x 10^{14} N.mm^2

Pc= 21111kN

ΣP_u= 2(1.2 x 273 + 1 x 52) = 759.2 kN

(note ΣP_W=0)

δs= 1.025

$$M_1 = M_{1ns} + \delta_s M_{1s}$$

$$M_2 = M_{2ns} + \delta_s M_{2s}$$

M_2 is larger than M_1, so

M_{2ns}= 1.2 M_D + 1.0 M_L

M_{2ns}= 1.2 (460) + 1.0 (110) = 662 kN.m

M_{2s}= 1.6 M_W

M_{2s}= 1.6 (17) = 27.2 kN.m

M_2= 662 + 1.025 (27.2) = 690 kN.m

M_{1ns}= 1.2 M_D + 1.0 M_L

M_{1ns}= 1.2 (224) + 1.0 (53) = 321.8 kN.m

M_{1s}= 1.6 M_W

M_{1s}= 1.6 (32) = 51.2 kN.m

M_1= 321.8 + 1.025 (51.2) = 374.28 kN.m

Step 4: Calculate moment magnification factor, δns:

$$M_c = \delta_{ns} M_2$$

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \geq 1.0$$

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2}$$

$$(EI)_{eff} = \frac{0.4E_c I_g}{1 + \beta_{dns}}$$

$$C_m = 0.6 - 0.4\frac{M_1}{M_2}$$

M_1= 374.28kN.m

M_2= 690kN.m

Double curvature. $\frac{M_1}{M_2}$ is positive.

K=1

→ C_m= 0.38

$$\beta_{dns} = \frac{1.2P_D}{1.2P_D + 1.0P_L + 1.6P_w} = \frac{1.2P_D}{P_u} = \frac{(1.2)(273)}{383.3} = 0.85$$

EI= 9.18 x 10^{13} mm^4

P_c= 20778 kN

P_u= 383.3 kN

δ_{ns}= 0.39 < 1.0 use **δ_{ns}= 1.0**

So, the design moment will be,

Mc= 1 x 690= 690 kN.m

Step 5: Section design:

b= 400mm

h= 800mm

P_u= 383.3 k.N

M_u= 690 kN.m

$$\gamma = (800 - 2\,x\,60)/800 = 0.85$$

$$\frac{\phi P_n}{bh} = \frac{(383.3)(1000)}{(400)(800)(7)} = 0.17ksi$$

$$\frac{\phi M_n}{bh^2} = \frac{(690)(1000\,000)}{(400)(800)^2(7)} = 0.39ksi$$

Using column design aids: P-M interaction diagrams →

For γ = 0.75 → ρ = 0.017

For γ = 0.9 → ρ = 0.014

→ ρ = 0.015

A_s= 0.015(400)(800)= 4800mm^2

Use 16Φ20 reinforcing bars

Ties:

S ≤ 16 (20) = 320 mm

S ≤ 48 (10) = 480 mm

S ≤ 400 mm

So,

Use s= 300mm

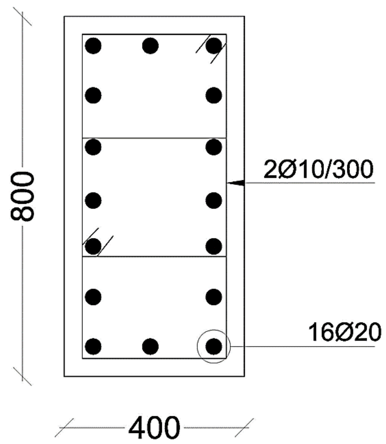


Figure 11.13: Section in column

Chapter 12: Introduction to Seismic Design

12.1 Introduction:

Earthquakes result from the sudden movement of tectonic plates in earth's crust. The movement takes place at fault lines, and the energy released is transmitted through the earth in the form of waves that cause ground motion many kilometers from the epicenter.

The mapped values, expressed as a percent of gravity, represent the expected peak acceleration of a single-degree-of-freedom system with 0.2 seconds period and 5% of critical damping, known as the 0.2 sec. spectral response acceleration Ss (Subscript s for short period), it is used along with 1.0 second spectral response acceleration S1, to establish the loading criteria for seismic design based on IBC 2012/ ASCE 7-10.

Accelerations Ss and S1 are based on historical records and local geology. They represent earthquake ground motion with a likelihood of exceedance of 2% in 50 years, a value that is equivalent to a return period of about 2500 years. Refer to Figure 12.1 for single degree of freedom system.

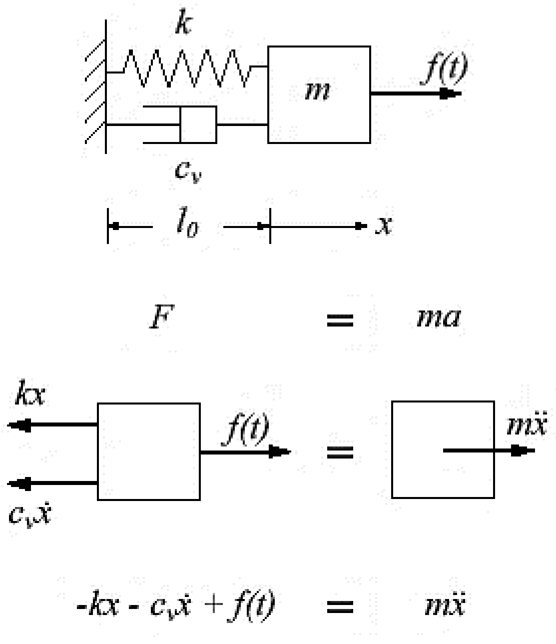


Figure 12.1: Single degree of freedom system

$$-kx - c\dot{x} + f(t) = m\ddot{x}$$

$$kx + c\dot{x} + m\ddot{x} = f(t)$$

(Second order nonhomogeneous ordinary differential equation)

Note:

The return period 1/r can be computed from the mathematical following equation:

$$1 - P_o = e^{-yr}$$

Where:

Po= Probability of exceedance, 2%

y: number of years, 50 years

$$1 - 0.02 = e^{-50r} = 0.98$$

$$\mathrm{Ln}(0.98) = -50r = -0.0202 \rightarrow r = \frac{1}{2475} \rightarrow \frac{1}{r} = 2475 \text{ years}$$

Or:

$$1 - P_o = e^{-y/r}$$

Here, the return period is r.

As experienced by structures, earthquakes consist of random horizontal and vertical movements of the earth's surface. As the ground moves, inertia tends to keep structures in place, resulting in the imposition of displacements and forces that can have catastrophic results. The purpose of the seismic design is to proportion structures so that they withstand the displacements and the forces induced by the ground motion.

12.2 Structure response:

Design of earthquakes differ from design for gravity and wind loads in the relatively greater sensitivity of earthquake-induced forces to the geometry of the structures.

A. Structural considerations:

- The closer the frequency of the ground motion to one of the natural frequencies of a structure, the greater the likelihood of the structure experiencing resonance, resulting in an increase in displacement and damage.
- Earthquake response depends on the geometric properties of a structure, especially height. The building has many mode shapes. The relative contribution of each mode to the lateral displacement of the structure depends on the frequency characteristics of the ground motion.
- The configuration of the structure has major effect on its response to an earthquake. Structures with a discontinuity in stiffness or geometry can be subjected to high displacements or forces.

- Stiffer members tend to pick up a greater portion of the load like the existence of shear walls. However, when the effects of higher stiffness members, such as masonry infill walls, are not considered in the design, unexpected and often undesirable results can occur.
- There is a need to provide an adequate separation between structures. Spacing requirements to ensure that adjacent structures do not come into contact as the result of earthquake induced motion are specified in codes.

B. Member considerations:

- Members must perform in a ductile fashion and dissipate energy.
- The principal method of ensuring ductility in members subject to shear and bending is to provide confinement for the concrete by using closed stirrups (hoops) in beams and columns. So, beams and columns can undergo nonlinear cyclic bending while maintaining their flexural strength and without deteriorating due to diagonal tension cracking. The formation of ductile (plastic) hinges allow reinforced concrete frames to dissipate energy. Hinges will form in the beams rather than in columns, minimizing the portion of the structure affected by nonlinear behavior and maintaining the overall vertical load capacity. So, the “weak beam-strong column” approach is used to design reinforced concrete frames subjected to seismic loading. The ends of the beam must be designed to resist maximum shear that can be developed. Refer to Figures 12.2 and 12.3.

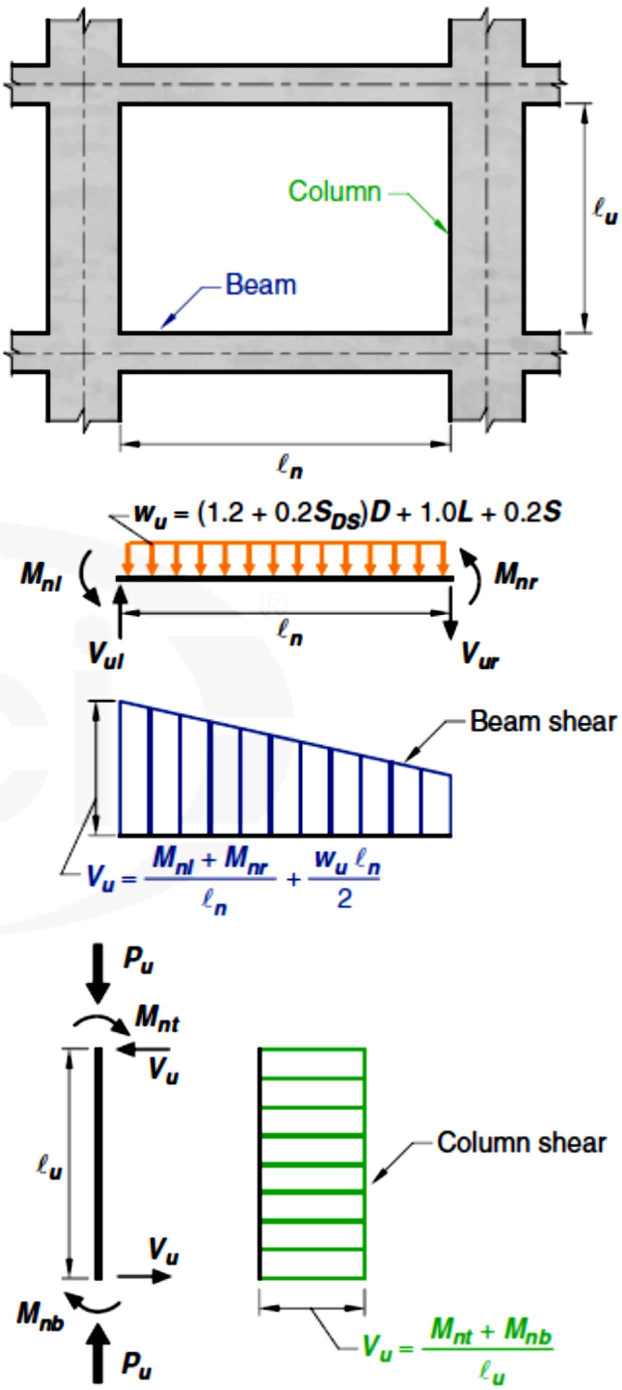


Fig. R18.4.2—Design shears for intermediate moment frames.

Figure 12.2: Design shears for intermediate moment

Frames

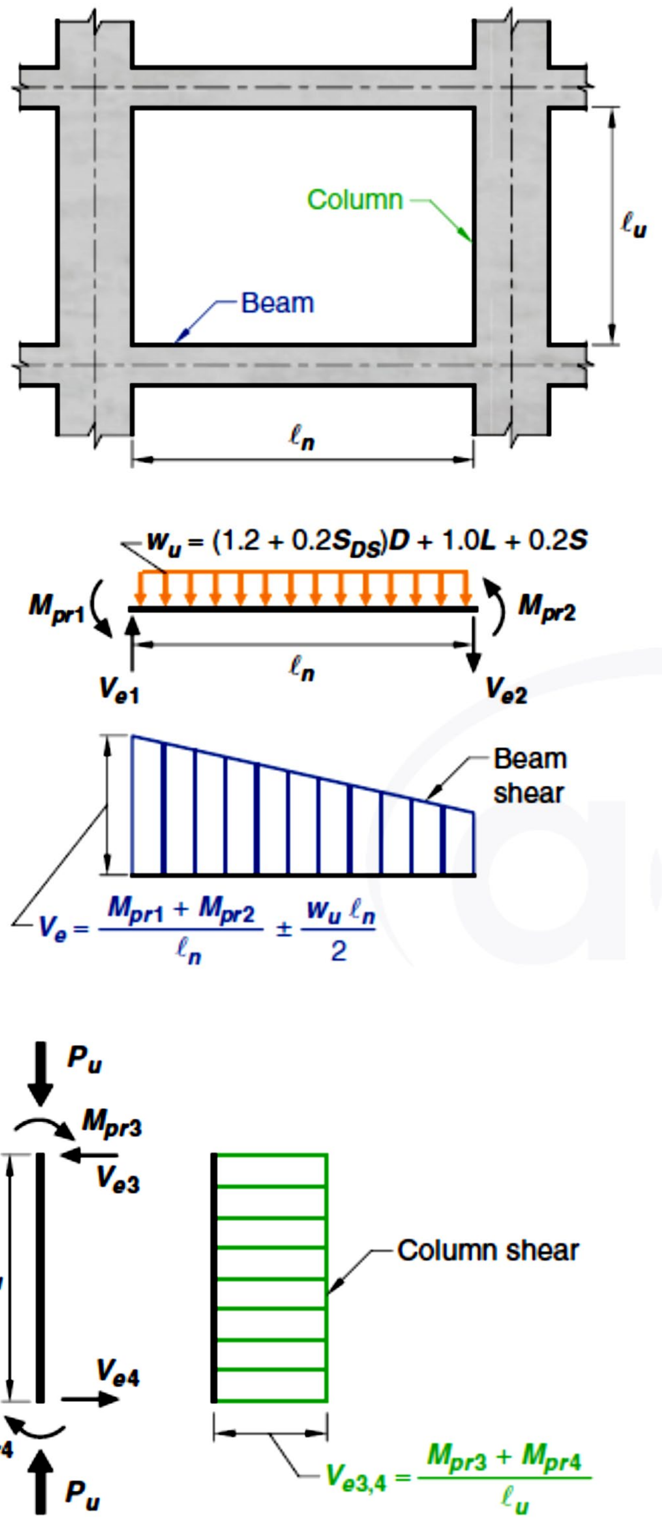


Figure 12.3: Design shears for special moment

Frames

Where:

M_n = Nominal flexural strength at section.

M_{pr} = Probable flexural strength of members, with or without axial load, determined using the properties of the member at joint faces assuming a tensile stress in the longitudinal bars of at least 1.25fy and a strength reduction factor Ø of 1.0.

D= Dead load

L= Live load

S= Snow load

Ln= length of clear span measured face-to-face of supports

Lu= unsupported length of column or wall

- Two-way systems without beams are especially vulnerable because of low ductility at the slab-column intersection.

12.3 Load combinations:

Table 12.1: ACI 318-19 Table R5.2.2—Correlation between seismic-related terminology in model codes

Code, standard, or resource document and edition	**Level of seismic risk or assigned seismic performance or design categories as defined in the Code**		
ACI 318-08, ACI 318-11, ACI 318-14, ACI 318-19; IBC of 2000, 2003, 2006, 2009, 2012, 2015, 2018; NFPA 5000 of 2003, 2006, 2009, 2012, 2015, 2018; ASCE 7-98, 7-02, 7-05, 7-10, 7-16; NEHRP 1997, 2000, 2003, 2009, 2015	SDC[1] A, B	SDC C	SDC D, E, F
ACI 318-05 and previous editions	Low seismic risk	Moderate/intermediate seismic risk	High seismic risk
BOCA National Building Code 1993, 1996, 1999; Standard Building Code 1994, 1997, 1999; ASCE 7-93, 7-95; NEHRP 1991, 1994	SPC[2] A, B	SPC C	SPC D, E
Uniform Building Code 1991, 1994, 1997	Seismic Zone 0, 1	Seismic Zone 2	Seismic Zone 3, 4

[1] SDC = seismic design category as defined in code, standard, or resource document.

[2] SPC = seismic performance category as defined in code, standard, or resource document.

The seismic design category in IBC 2012/ ASCE 7-10 depends on the values of S_{D1} and S_{DS} and the risk category. (Tables 11.6-1 and 11.6-2 in ASCE 7-10)

Based on seismic design category, the lateral forces resisting system is selected and the reinforcement details are specified based on ACI code.

Refer to Table 12.2 for load combinations in ACI 318-19.

Table 12.2: ACI 318-19 Table 5.3.1 Load combinations

Load combination	Equation	Primary load
U = 1.4D	(5.3.1a)	D
U = 1.2D + 1.6L + 0.5(L_r or S or R)	(5.3.1b)	L
U = 1.2D + 1.6(L_r or S or R) + (1.0L or 0.5W)	(5.3.1c)	L_r or S or R
U = 1.2D + 1.0W + 1.0L + 0.5(L_r or S or R)	(5.3.1d)	W
U = 1.2D + 1.0E + 1.0L + 0.2S	(5.3.1e)	E
U = 0.9D + 1.0W	(5.3.1f)	W
U = 0.9D + 1.0E	(5.3.1g)	E

Where:

D= Dead loads

L= Live loads

Lr= Roof live loads

S= Snow loads

R= Rain loads

W= wind loads

E= Earthquake loads

Load combinations in ASCE 7-10 section 2.3.2:

1. 1.4D

2. 1.2D + 1.6L + 0.5(Lr or S or R)

3. 1.2D + 1.6(Lr or S or R) + (L or 0.5W)

4. 1.2D + 1.0W + L + 0.5(Lr or S or R)

5. 1.2D + 1.0E + L + 0.2S

6. 0.9D + 1.0W

7. 0.9D + 1.0E

Where fluid loads F are present, they shall be included with the same load factor as dead load D in combinations 1 through 5 and 7.

Where load H (Soil pressure) are present, they shall be included as follows:

1. where the effect of H adds to the primary variable load effect, include H with a load factor of 1.6;

2. where the effect of H resists the primary variable load effect, include H with a load factor of 0.9 where the load is permanent or a load factor of 0 for all other conditions.

Seismic Load Effect: ASCE 7-10

The seismic load effect, E, shall be determined in accordance with the following:

- For use in load combination 5, E shall be determined in accordance with:

$$E = E_h + E_v$$

- For use in load combination 7, E shall be determined in accordance with:

$$E = E_h - E_v$$

Where:

E = seismic load effect

Eh = effect of horizontal seismic forces

Ev = effect of vertical seismic forces

Horizontal Seismic Load Effect:

The horizontal seismic load effect, Eh, shall be determined in accordance with:

$$Eh = \rho Q_E$$

Where:

Q_E = effects of horizontal seismic forces

ρ = redundancy factor. In general, it equals to 1.0 for seismic design categories B and C and it equals 1.30 for seismic design categories D, E and F.

Vertical Seismic Load Effect:

The vertical seismic load effect, Ev, shall be determined in accordance with:

$$E_v = 0.2S_{DS}D$$

Where:

S_{DS} = design spectral response acceleration parameter at short periods

D = effect of dead load

Seismic Load Effect Including Overstrength Factor:

Where specifically required, conditions requiring overstrength factor applications shall be determined in accordance with the following:

- For use in load combination 5, E shall be taken equal to Em as determined in accordance with:

$$E_m = E_{mh} + E_v$$

- For use in load combination, E shall be taken equal to Em as determined in accordance with:

$$E_m = E_{mh} - E_V$$

Where:

E_m = seismic load effect including overstrength factor

E_{mh} = effect of horizontal seismic forces including overstrength factor

E_v = vertical seismic load effect

Horizontal Seismic Load Effect with Overstrength Factor:

The horizontal seismic load effect with overstrength factor, E_{mh}, shall be determined in accordance with:

$$E_{mh} = \Omega_o Q_E$$

Where:

Q_E = effects of horizontal seismic forces

Ω_o = overstrength factor

EXCEPTION: The value of E_{mh} need not exceed the maximum force that can develop in the element as determined by a rational, plastic mechanism analysis or nonlinear response analysis utilizing realistic expected values of material strengths.

12.4 Equivalent lateral forces procedure:

The seismic base shear, V, in a given direction shall be determined in accordance with the following equation:

$$V = C_s W$$

Where:

C_s = the seismic response coefficient

W = the effective seismic weight

The seismic response coefficient, C_s, shall be determined in accordance with:

$$C_s = \frac{S_{DS}}{R/I_e}$$

Where:

S_{DS} = the design spectral response acceleration parameter in the short period

R = the response modification factor in Table 12.2-1 ASCE 7-10

I_e = the importance factor determined in accordance with Section 11.5.1 ASCE 7-10

The value of C_s need not exceed the following:

$$C_{s,max} = \frac{S_{D1}}{T\left(\frac{R}{I_e}\right)} \text{ for } T \leq T_L$$

$$C_{s,max} = \frac{S_{D1}T_L}{T^2\left(\frac{R}{I_e}\right)} \text{ for } T > T_L$$

C_s shall not be less than:

$$C_{s,min} = 0.044 S_{DS} I_e \geq 0.01$$

$$C_{s,min} = \frac{0.5 S_1}{R/I_e} \text{ if } S_1 \geq 0.6g$$

S_{D1} = the design spectral response acceleration parameter at a period of 1.0 s.

T = the fundamental period of the structure in seconds.

T_L= long-period transition period in seconds.

S_S = mapped MCER (Maximum Considered Earthquake), 5 percent damped, spectral response acceleration parameter at short period of 0.2 second.

S_1 = mapped MCER, 5 percent damped, spectral response acceleration parameter at a period of 1.0 second.

Effective Seismic Weight:

The effective seismic weight, *W*, of a structure shall include the dead load above the base and other loads above the base as listed below:

1. In areas used for storage, a minimum of 25 percent of the floor live load shall be included.

2. Where provision for partitions is required in the floor load design, the actual partition weight or a minimum weight of 0.48 kN/m^2 of floor area, whichever is greater.

3. Total operating weight of permanent equipment.

4. Where the flat roof snow load, Pf, exceeds 1.44 kN/m^2, 20 percent of the uniform design snow load, regardless of actual roof slope.

5. Weight of landscaping and other materials at roof gardens and similar areas.

Site Class:

Based on the site soil properties, the site shall be classified as Site Class A, B, C, D, E, or F in accordance with Chapter 20 in ASCE 7-10. Where the soil properties are not known in sufficient detail to determine the site class, Site Class D shall be used unless the authority having jurisdiction or geotechnical data determines Site Class E or F soils are present at the site.

Site Coefficients and Risk-Targeted Maximum Considered Earthquake (MCER) Spectral Response Acceleration Parameters:

The MCER spectral response acceleration parameter for short periods (*SMS*) and at 1 s (*SM*1), adjusted for Site Class effects, shall be determined by:

$$S_{MS} = F_a S_S$$

$$S_{M1} = F_v S_1$$

Where: site coefficients *Fa* and *Fv* are defined in Tables 11.4-1 and 11.4-2 in ASCE 7-10, respectively.

Design Spectral Acceleration Parameters:

Design earthquake spectral response acceleration parameter at short period, S_{DS}, and at 1 s period, S_{D1}, shall be determined from:

$$S_{DS} = \frac{2}{3} S_{MS}$$

$$S_{D1} = \frac{2}{3} S_{M1}$$

Risk category:

Buildings and other structures shall be classified, based on the risk to human life, health, and welfare associated with their damage or failure by nature of their occupancy or use, according to Table 1.5-1 in ASCE 7-10 for the purposes of applying flood, wind, snow, earthquake, and ice provisions. Each building or other structure shall be assigned to the highest applicable risk category or categories. Minimum design loads for structures shall incorporate the applicable importance factors given in Table 1.5-2 in ASCE 7-10, as required by other sections of this Standard (ASCE 7-10). Assignment of a building or other structure to multiple risk categories based on the type of load condition being evaluated (e.g., snow or seismic) shall be permitted. When the building code or other referenced standard specifies an Occupancy Category, the Risk Category shall not be taken as lower than the Occupancy Category specified therein.

Approximate Fundamental Period:

The approximate fundamental period (T_a), in seconds, shall be determined from the following equation:

$$T_a = C_t {h_n}^x$$

Where: h_n is the structural height as defined in Section 11.2 (ASCE 7-10: STRUCTURAL HEIGHT: The vertical distance from the base to the highest level of the seismic force-resisting system of the structure. For pitched or sloped roofs, the structural height is from the base to the average height of the roof.) and the coefficients Ct and x are determined from Table 12.8-2.

The general equation for the period T (Frequency= 1/Period) is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Where:

m= mass

k= stiffness

f= frequency

12.5Distribution of base shear to floors:

The lateral seismic force, *Fx* induced at any level shall be determined from the following equations:

$$F_x = C_{vx}V$$

$$C_{vx} = \frac{w_x {h_x}^k}{\sum_{i=1}^{n} w_i {h_i}^k}$$

$$k = 1 \; for \; T \leq 0.5 \sec \qquad k = 2 \; for \; T \geq 2.5 sec$$

Where:

C_{vx} = vertical distribution factor

V = total design lateral force or shear at the base of the structure (kN)

w_i and w_x = the portion of the total effective seismic weight of the structure (W) located or assigned to Level i or x

h_i and h_x = the height from the base to Level i or x.

12.6 UBC 97 code provisions:

$$V = \frac{C_v I}{RT} W$$

$$V_{max} = \frac{2.5 C_a I}{R} W$$

$$V_{min} = 0.11 C_a I W$$

$$T = C_t h_n^{\frac{3}{4}}$$

$C_t = 0.0853\ for\ steel\ moment\ resisting\ frames$

$C_t = 0.0731\ for\ concrete\ moment\ resisting\ frames$

$C_t = 0.0488\ for\ all\ other\ buildings$

$E_v = 0.5C_aID \qquad E = \rho Q_E + E_v$

$V = F_t + \sum_{i=1}^{n} F_i$

$F_t = 0.07TV \ \leq 0.25V \quad for\ T > 0.7\sec \ \ and\ F_t = 0.0 \quad for\ T \leq 0.7sec$

$$F_x = \frac{(V - F_t)w_x h_x}{\sum_{i=1}^{n} w_i h_i}$$

$UBC\ sesimic\ zones\ 0{,}1 \quad use \quad SDC: A, B$

$UBC\ sesimic\ zone\ 2\ \ use \quad SDC: C$

$UBC\ sesimic\ zones\ 3, 4 \quad use \quad SDC: D, E, F$

Where:

W: seismic effective weight

I: importance factor, Table 16-K

R: response factor, Table 16-N

C_a: seismic factor, Table 16-Q

C_v: seismic factor, Table 16-R

Soil profile as stated in Table 16-J

T: approximate period in seconds.

The calculated structural period from structural analysis shall be less than 1.3 T from method A (The above equation) if the structure exists in seismic zone 4 and 1.4 T from method A if the structure exists in seismic zones 1,2 and 3.

The redundancy factor, ρ is equal to 1.0 for seismic zones 0, 1 and 2 and it equals 1.5 for seismic zone 3. In general, it can be calculated based on code provisions.

E_v: vertical seismic force component

w_i and w_x : The portion of the total effective seismic weight of the structure (W) located or assigned to Level i or x

h_i and h_x = the height (ft or m) from the base to Level i or x.

F_t: the concentrated force at the top

Moment resisting frames:

For seismic zones 3 and 4, the lateral forces resisting system shall be special moment resisting frames and for seismic zone 2, it shall be intermediate moment resisting frames.

Example 1:

Given:

- Concrete strength, f'c= 28MPa
- Steel strength, fy= 420MPa
- Office building
- Five floors
- Floor height= 4m
- Live load, WL= 3kN/m^2
- Superimposed dead load, WSD= 4kN/m^2
- Perimeter wall weight= 6kN/m
- Slab system: Two way solid. Slab thickness, t= 0.17m.
- All columns are 0.60m x 0.60m
- All beams are 0.30m (Width) x 0.55m (Depth)
- Location: Ss= 1.12g, S1=0.53g, z=0.3
- Soil class= C or Sc

Determine the base shear and its distribution to the floors.

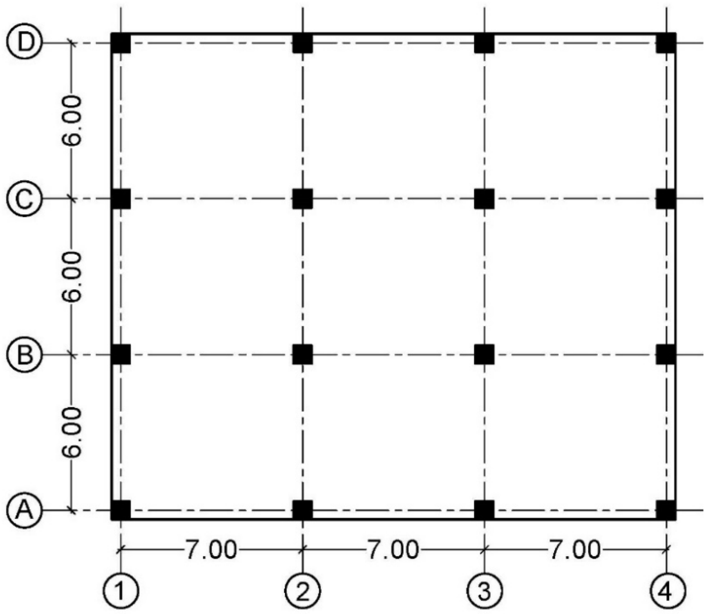


Figure 12.4: Building plan for the example

Solution:

ASCE 7-10 / IBC 2012 codes:

From ASCE Table 11.4.1, F_a= 1.0

From ASCE Table 11.4-2, F_v=1.30

$S_{MS} = F_a S_S = (1)(1.12) = 1.12$

$S_{M1} = F_v S_1 = (1.30)(0.53) = 0.689$

$S_{DS} = \frac{2}{3} S_{MS} = (2/3)(1.12) = 0.75$

$S_{D1} = \frac{2}{3} S_{M1} = (2/3)(0.689) = 0.46$

Risk category: III ASCE 7-10 Table 1.5-1

I_e= 1.25 ASCE Table 1.5-2

Seismic design category: D Table 11.6-1 ASCE 7-10

Seismic design category: D Table 11.6-2 ASCE 7-10

So, the seismic design category is D.

Seismic force resisting system: Special reinforced concrete moment frames.

From ASCE 7-10 Table 12.2-1:

$$R = 8$$

$$\Omega_o = 3$$

$$C_d = 5.5$$

Period:

$T_a = C_t h_n{}^x = (0.0466)(5x4)^{0.9} = 0.69\ seconds$

If structural analysis is done, the period is limited to Ta C_u= (0.69)(1.4)=0.97 seconds.

Building effective weight:

Slab: (21)(18)(0.17)(25)=1606.5kN

Beams: {(21)(4)+(18)(4)}(0.3)(0.55)(25)=643.5kN

Columns: (16)(0.6)(0.6)(4)(25)=576kN

Walls: {(21)(2)+(18)(2)}(6)=468kN

Superimposed dead load: (21)(18)(4)=1512kN

0.25 of Live loads: (0.25)(21)(18)(3)=283.5kN

Weight of one floor= 5089.5kN

Weight of five floors= (5)(5089.5)=25447.5kN

$$C_s = \frac{S_{DS}}{R/I_e} = \frac{0.75}{(8/1.25)} = 0.117$$

$$C_{s,max} = \frac{S_{D1}}{T\left(\frac{R}{I_e}\right)} = \frac{0.46}{(0.69)(8/1.25)} = 0.104 \qquad controlds$$

$$C_{s,min} = 0.044 S_{DS} I_e = 0.044(0.75)(1.25) = 0.041 \geq 0.01$$

So, use $C_s = 0.104$

$$V = C_s W = (0.104)(25447.5) = 2647kN$$

Distribution of base shear to floors:

K=1.095

$$C_{vx} = \frac{w_x {h_x}^k}{\sum_{i=1}^{n} w_i {h_i}^k}$$

Table 12.3: Floor forces – ASCE 7-10/ IBC 2012

Floor	Height (m) hi	Weight (kN) Wi	Wi hi ^{k}	Cvx	Force to floor Fx (kN)
5	20	5089.5	135301.6	0.346	916
4	16	5089.5	105970.9	0.271	717
3	12	5089.5	77335.4	0.198	524
2	8	5089.5	49608.8	0.127	336
1	4	5089.5	23223.7	0.059	156
			391440.4	1.001	2649

UBC 97 code:

Effective weight, W= 25447.5kN

I=1.0 Table 16-K

Seismic zone: 3 Z=0.3

Special moment resisting frame.

$$R = 8.5 \qquad \Omega_o = 2.8$$

Soil profile: Sc: given

C_a= 0.33 Table 16-Q C_v= 0.45 Table 16-R

$$T = C_t h_n^{\frac{3}{4}} = (0.0731)(20)^{3/4} = 0.69\ seconds$$

$$V = \frac{C_v I}{RT} W = \frac{(0.45)(1)}{(8.5)(0.69)}(25447.5) = 1953kN$$

$$V_{max} = \frac{2.5 C_a I}{R} W = \frac{(2.5)(0.33)(1)}{8.5}(25447.5) = 2470kN\ > 1953kN$$

$$V_{min} = 0.11 C_a I W = (0.11)(0.33)(1)(25447.5) = 924kN$$

Use V= 1953kN

Distribution of base shear to floors:

Since T< 0.7 seconds, F_t=0.0 kN

$$F_x = \frac{(V - F_t) w_x h_x}{\sum_{i=1}^{n} w_i h_i}$$

Table 6.4: Floor forces – UBC 97

Floor	Height (m) hi	Weight (kN) Wi	Wi hi ^{k}	Cvx	Force to floor Fx (kN)
5	20	5089.5	101790	0.333	650
4	16	5089.5	81432	0.267	521
3	12	5089.5	61074	0.200	391
2	8	5089.5	40716	0.133	260
1	4	5089.5	20358	0.067	131
			305370	1.000	1953

12.7 Seismic response spectrum:

The effect of the size and type of vibration waves released during a given earthquake can be organized so as to be more useful in design in terms of a *response spectrum* for a given earthquake or family of earthquakes. Figure 5a shows a family of inverted, damped pendulums, each of which has a different period of vibration, *T*. To derive a point on a response spectrum, one of these hypothetical pendulum structures is analytically subjected to the vibrations recorded during a particular earthquake. The largest acceleration of this pendulum structure during the entire record of a particular earthquake can be plotted as shown in Figure 6.5b. Repeating this for each of the other pendulum structures shown in Figure 6.5a and plotting the peak values for each of the pendulum structures produces an *acceleration response spectrum*.

Generally, the vertical axis of the spectrum is normalized by expressing the computed accelerations in terms of the acceleration due to gravity. If, for example, the ordinate of a point on the response spectrum is 2 for a given period *T*, it means that the peak acceleration of the pendulum structure for that value of *T* and for that earthquake was twice that due to gravity. The random wave content of an earthquake causes the derived acceleration response spectrum to plot as a jagged line, as shown in Figure 6.6c. The spectra in Figure 6.5b has been smoothed.

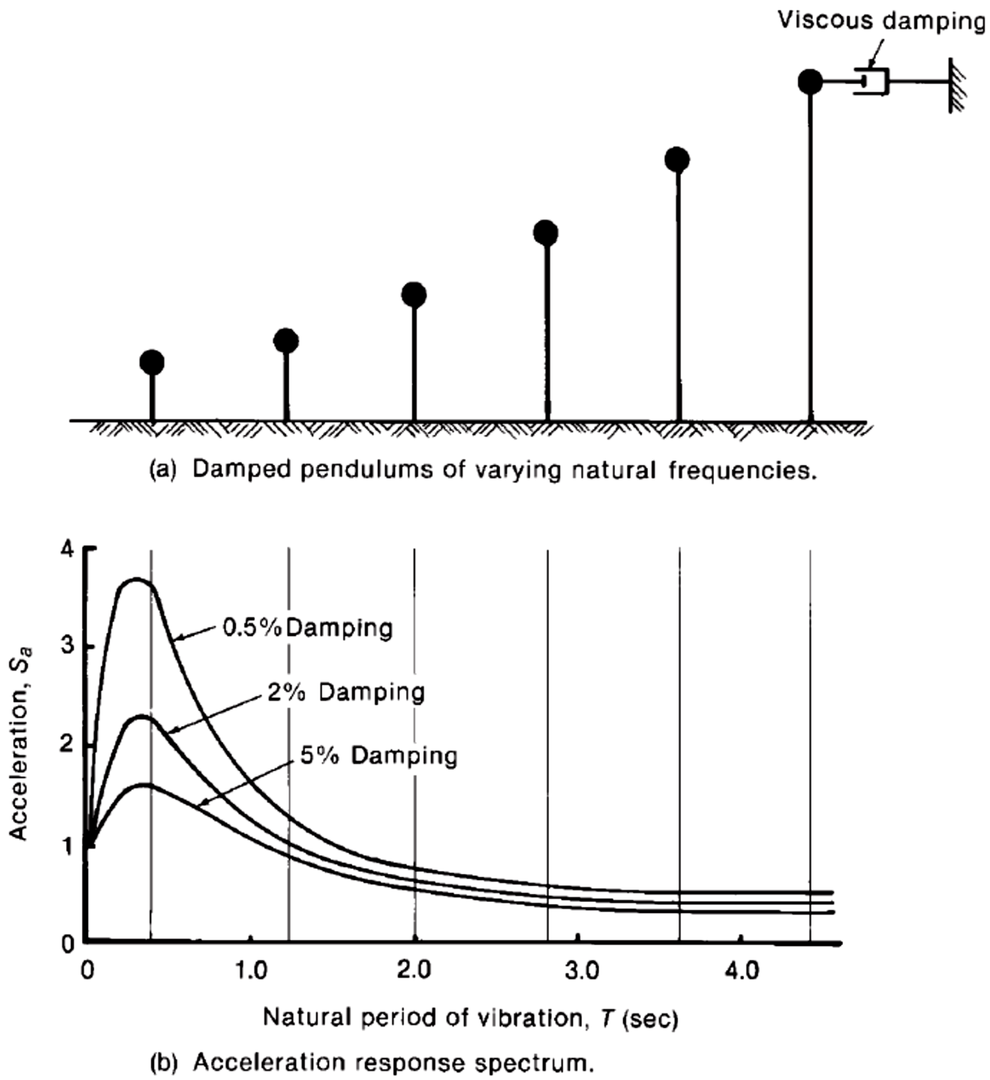


Figure 12.5: Earthquake response spectrum

Velocity and Displacement Spectra:

Following the procedure used to obtain an acceleration spectrum, but plotting the peak velocity relative to the ground during the entire earthquake against the periods of the family of pendulum structures, gives a velocity response spectrum. A plot of the maximum displacements of the structure relative to the ground during the entire earthquake is called a displacement response spectrum. These three spectra for a particular earthquake measured on rock or firm soil sites are shown in Figure 6.

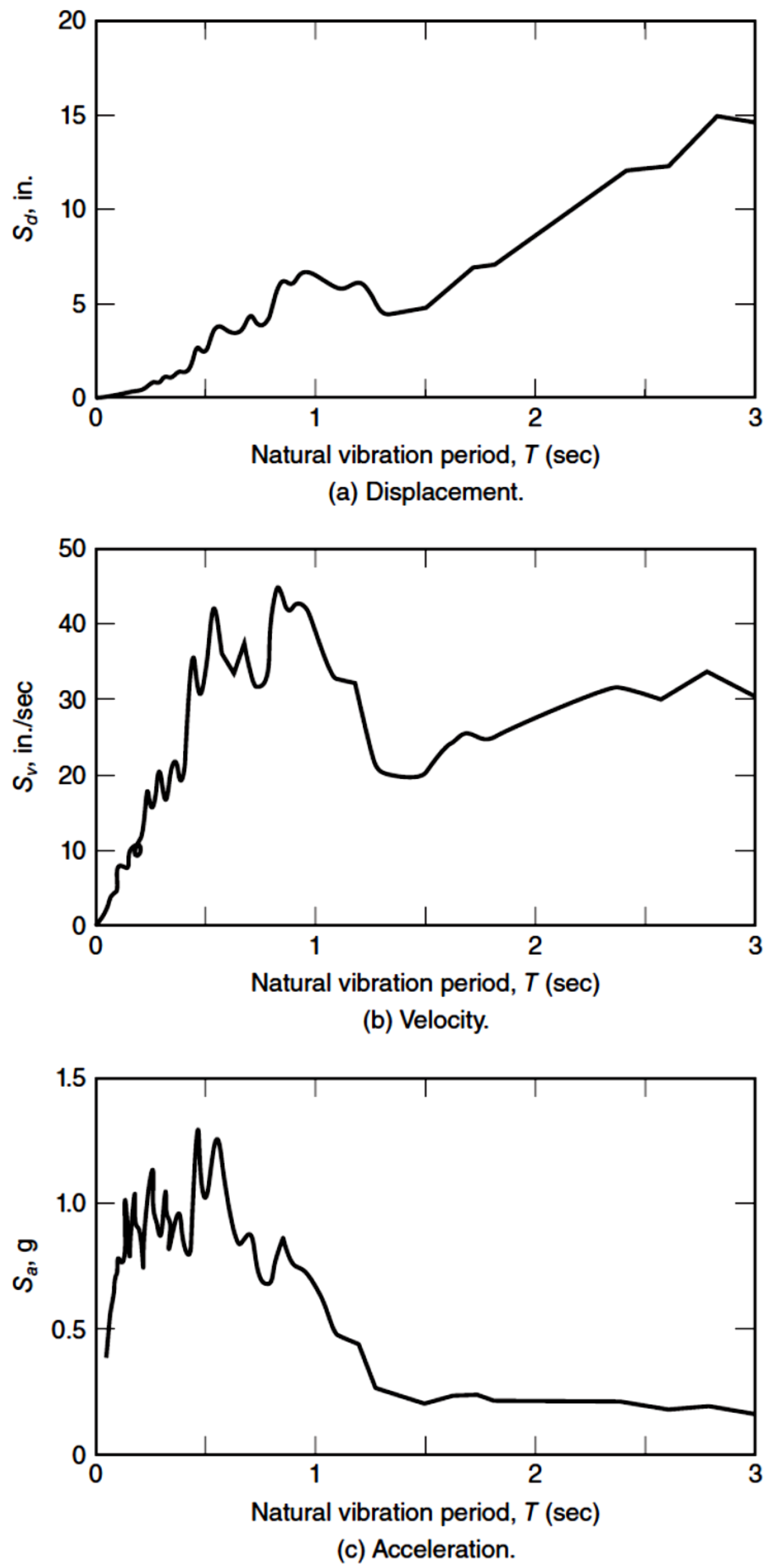


Figure 12.6: Displacement, velocity, and acceleration spectra for a given earthquake

Response spectrum analysis:

An elastic dynamic analysis of a structure utilizing the peak dynamic response of all modes having a significant contribution to total structural response. Peak modal responses are calculated using the ordinates of the appropriate response spectrum curve which correspond to the modal periods. Maximum modal contributions are combined in a statistical manner to obtain an approximate total structural response.

Time-history analysis:

An analysis of the dynamic response of a structure at each increment of time when the base is subjected to a specific ground motion time history.

Table 1.5-1 Risk Category of Buildings and Other Structures for Flood, Wind, Snow, Earthquake, and Ice Loads

Use or Occupancy of Buildings and Structures	Risk Category
Buildings and other structures that represent a low risk to human life in the event of failure	I
All buildings and other structures except those listed in Risk Categories I, III, and IV	II
Buildings and other structures, the failure of which could pose a substantial risk to human life.	III
Buildings and other structures, not included in Risk Category IV, with potential to cause a substantial economic impact and/or mass disruption of day-to-day civilian life in the event of failure.	
Buildings and other structures not included in Risk Category IV (including, but not limited to, facilities that manufacture, process, handle, store, use, or dispose of such substances as hazardous fuels, hazardous chemicals, hazardous waste, or explosives) containing toxic or explosive substances where their quantity exceeds a threshold quantity established by the authority having jurisdiction and is sufficient to pose a threat to the public if released.	
Buildings and other structures designated as essential facilities.	IV
Buildings and other structures, the failure of which could pose a substantial hazard to the community.	
Buildings and other structures (including, but not limited to, facilities that manufacture, process, handle, store, use, or dispose of such substances as hazardous fuels, hazardous chemicals, or hazardous waste) containing sufficient quantities of highly toxic substances where the quantity exceeds a threshold quantity established by the authority having jurisdiction to be dangerous to the public if released and is sufficient to pose a threat to the public if released.[a]	
Buildings and other structures required to maintain the functionality of other Risk Category IV structures.	

[a]Buildings and other structures containing toxic, highly toxic, or explosive substances shall be eligible for classification to a lower Risk Category if it can be demonstrated to the satisfaction of the authority having jurisdiction by a hazard assessment as described in Section 1.5.2 that a release of the substances is commensurate with the risk associated with that Risk Category.

Table 1.5-2 Importance Factors by Risk Category of Buildings and Other Structures for Snow, Ice, and Earthquake Loads[a]

Risk Category from Table 1.5-1	Snow Importance Factor, I_s	Ice Importance Factor—Thickness, I_i	Ice Importance Factor—Wind, I_w	Seismic Importance Factor, I_e
I	0.80	0.80	1.00	1.00
II	1.00	1.00	1.00	1.00
III	1.10	1.25	1.00	1.25
IV	1.20	1.25	1.00	1.50

[a]The component importance factor, I_p, applicable to earthquake loads, is not included in this table because it is dependent on the importance of the individual component rather than that of the building as a whole, or its occupancy. Refer to Section 13.1.3.

Table 11.4-1 Site Coefficient, F_a

	Mapped Risk-Targeted Maximum Considered Earthquake (MCE_R) Spectral Response Acceleration Parameter at Short Period				
Site Class	$S_S \le 0.25$	$S_S = 0.5$	$S_S = 0.75$	$S_S = 1.0$	$S_S \ge 1.25$
A	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0
C	1.2	1.2	1.1	1.0	1.0
D	1.6	1.4	1.2	1.1	1.0
E	2.5	1.7	1.2	0.9	0.9
F	See Section 11.4.7				

Note: Use straight-line interpolation for intermediate values of S_S.

Table 11.4-2 Site Coefficient, F_v

	Mapped Risk-Targeted Maximum Considered Earthquake (MCE_R) Spectral Response Acceleration Parameter at 1-s Period				
Site Class	$S_1 \leq 0.1$	$S_1 = 0.2$	$S_1 = 0.3$	$S_1 = 0.4$	$S_1 \geq 0.5$
A	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0
C	1.7	1.6	1.5	1.4	1.3
D	2.4	2.0	1.8	1.6	1.5
E	3.5	3.2	2.8	2.4	2.4
F	See Section 11.4.7				

Note: Use straight-line interpolation for intermediate values of S_1.

Table 11.6-1 Seismic Design Category Based on Short Period Response Acceleration Parameter

	Risk Category	
Value of S_{DS}	I or II or III	IV
$S_{DS} < 0.167$	A	A
$0.167 \leq S_{DS} < 0.33$	B	C
$0.33 \leq S_{DS} < 0.50$	C	D
$0.50 \leq S_{DS}$	D	D

Table 11.6-2 Seismic Design Category Based on 1-S Period Response Acceleration Parameter

	Risk Category	
Value of S_{D1}	I or II or III	IV
$S_{D1} < 0.067$	A	A
$0.067 \leq S_{D1} < 0.133$	B	C
$0.133 \leq S_{D1} < 0.20$	C	D
$0.20 \leq S_{D1}$	D	D

Table 12.2-1 Design Coefficients and Factors for Seismic Force-Resisting Systems

Seismic Force-Resisting System	ASCE 7 Section Where Detailing Requirements Are Specified	Response Modification Coefficient, R[a]	Overstrength Factor, Ω_0[g]	Deflection Amplification Factor, C_d[b]	Structural System Limitations Including Structural Height, h_n (ft) Limits[c] — Seismic Design Category				
					B	C	D[d]	E[d]	F[e]
A. BEARING WALL SYSTEMS									
1. Special reinforced concrete shear walls[l, m]	14.2	5	2½	5	NL	NL	160	160	100
2. Ordinary reinforced concrete shear walls[l]	14.2	4	2½	4	NL	NL	NP	NP	NP
3. Detailed plain concrete shear walls[l]	14.2	2	2½	2	NL	NP	NP	NP	NP
4. Ordinary plain concrete shear walls[l]	14.2	1½	2½	1½	NL	NP	NP	NP	NP
5. Intermediate precast shear walls[l]	14.2	4	2½	4	NL	NL	40[k]	40[k]	40[k]
6. Ordinary precast shear walls[l]	14.2	3	2½	3	NL	NP	NP	NP	NP
7. Special reinforced masonry shear walls	14.4	5	2½	3½	NL	NL	160	160	100
8. Intermediate reinforced masonry shear walls	14.4	3½	2½	2¼	NL	NL	NP	NP	NP
9. Ordinary reinforced masonry shear walls	14.4	2	2½	1¾	NL	160	NP	NP	NP
10. Detailed plain masonry shear walls	14.4	2	2½	1¾	NL	NP	NP	NP	NP
11. Ordinary plain masonry shear walls	14.4	1½	2½	1¼	NL	NP	NP	NP	NP
12. Prestressed masonry shear walls	14.4	1½	2½	1¾	NL	NP	NP	NP	NP
13. Ordinary reinforced AAC masonry shear walls	14.4	2	2½	2	NL	35	NP	NP	NP
14. Ordinary plain AAC masonry shear walls	14.4	1½	2½	1½	NL	NP	NP	NP	NP
15. Light-frame (wood) walls sheathed with wood structural panels rated for shear resistance or steel sheets	14.1 and 14.5	6½	3	4	NL	NL	65	65	65
16. Light-frame (cold-formed steel) walls sheathed with wood structural panels rated for shear resistance or steel sheets	14.1	6½	3	4	NL	NL	65	65	65
17. Light-frame walls with shear panels of all other materials	14.1 and 14.5	2	2½	2	NL	NL	35	NP	NP
18. Light-frame (cold-formed steel) wall systems using flat strap bracing	14.1	4	2	3½	NL	NL	65	65	65
B. BUILDING FRAME SYSTEMS									
1. Steel eccentrically braced frames	14.1	8	2	4	NL	NL	160	160	100
2. Steel special concentrically braced frames	14.1	6	2	5	NL	NL	160	160	100
3. Steel ordinary concentrically braced frames	14.1	3¼	2	3¼	NL	NL	35[j]	35[j]	NP[j]

Continued

Table 12.2-1 (*Continued*)

Seismic Force-Resisting System	ASCE 7 Section Where Detailing Requirements Are Specified	Response Modification Coefficient, R^a	Overstrength Factor, Ω_0^g	Deflection Amplification Factor, C_d^b	Structural System Limitations Including Structural Height, h_n (ft) Limits[c] — Seismic Design Category B	C	D^d	E^d	F^e
4. Special reinforced concrete shear walls[l,m]	14.2	6	2½	5	NL	NL	160	160	100
5. Ordinary reinforced concrete shear walls[l]	14.2	5	2½	4½	NL	NL	NP	NP	NP
6. Detailed plain concrete shear walls[l]	14.2 and 14.2.2.8	2	2½	2	NL	NP	NP	NP	NP
7. Ordinary plain concrete shear walls[l]	14.2	1½	2½	1½	NL	NP	NP	NP	NP
8. Intermediate precast shear walls[l]	14.2	5	2½	4½	NL	NL	40[k]	40[k]	40[k]
9. Ordinary precast shear walls[l]	14.2	4	2½	4	NL	NP	NP	NP	NP
10. Steel and concrete composite eccentrically braced frames	14.3	8	2 ½	4	NL	NL	160	160	100
11. Steel and concrete composite special concentrically braced frames	14.3	5	2	4½	NL	NL	160	160	100
12. Steel and concrete composite ordinary braced frames	14.3	3	2	3	NL	NL	NP	NP	NP
13. Steel and concrete composite plate shear walls	14.3	6½	2½	5½	NL	NL	160	160	100
14. Steel and concrete composite special shear walls	14.3	6	2½	5	NL	NL	160	160	100
15. Steel and concrete composite ordinary shear walls	14.3	5	2½	4½	NL	NL	NP	NP	NP
16. Special reinforced masonry shear walls	14.4	5½	2½	4	NL	NL	160	160	100
17. Intermediate reinforced masonry shear walls	14.4	4	2½	4	NL	NL	NP	NP	NP
18. Ordinary reinforced masonry shear walls	14.4	2	2½	2	NL	160	NP	NP	NP
19. Detailed plain masonry shear walls	14.4	2	2½	2	NL	NP	NP	NP	NP
20. Ordinary plain masonry shear walls	14.4	1½	2½	1¼	NL	NP	NP	NP	NP
21. Prestressed masonry shear walls	14.4	1½	2½	1¾	NL	NP	NP	NP	NP
22. Light-frame (wood) walls sheathed with wood structural panels rated for shear resistance	14.5	7	2½	4½	NL	NL	65	65	65
23. Light-frame (cold-formed steel) walls sheathed with wood structural panels rated for shear resistance or steel sheets	14.1	7	2½	4½	NL	NL	65	65	65
24. Light-frame walls with shear panels of all other materials	14.1and 14.5	2½	2½	2½	NL	NL	35	NP	NP
25. Steel buckling-restrained braced frames	14.1	8	2½	5	NL	NL	160	160	100
26. Steel special plate shear walls	14.1	7	2	6	NL	NL	160	160	100

Table 12.2-1 *(Continued)*

Seismic Force-Resisting System	ASCE 7 Section Where Detailing Requirements Are Specified	Response Modification Coefficient, R^a	Overstrength Factor, Ω_0^g	Deflection Amplification Factor, C_d^b	Structural System Limitations Including Structural Height, h_n (ft) Limits[c] — Seismic Design Category: B	C	D[d]	E[d]	F[e]
C. MOMENT-RESISTING FRAME SYSTEMS									
1. Steel special moment frames	14.1 and 12.2.5.5	8	3	5½	NL	NL	NL	NL	NL
2. Steel special truss moment frames	14.1	7	3	5½	NL	NL	160	100	NP
3. Steel intermediate moment frames	12.2.5.7 and 14.1	4½	3	4	NL	NL	35[h]	NP[h]	NP[h]
4. Steel ordinary moment frames	12.2.5.6 and 14.1	3½	3	3	NL	NL	NP[i]	NP[i]	NP[i]
5. Special reinforced concrete moment frames[n]	12.2.5.5 and 14.2	8	3	5½	NL	NL	NL	NL	NL
6. Intermediate reinforced concrete moment frames	14.2	5	3	4½	NL	NL	NP	NP	NP
7. Ordinary reinforced concrete moment frames	14.2	3	3	2½	NL	NP	NP	NP	NP
8. Steel and concrete composite special moment frames	12.2.5.5 and 14.3	8	3	5½	NL	NL	NL	NL	NL
9. Steel and concrete composite intermediate moment frames	14.3	5	3	4½	NL	NL	NP	NP	NP
10. Steel and concrete composite partially restrained moment frames	14.3	6	3	5½	160	160	100	NP	NP
11. Steel and concrete composite ordinary moment frames	14.3	3	3	2½	NL	NP	NP	NP	NP
12. Cold-formed steel—special bolted moment frame[p]	14.1	3½	3[o]	3½	35	35	35	35	35
D. DUAL SYSTEMS WITH SPECIAL MOMENT FRAMES CAPABLE OF RESISTING AT LEAST 25% OF PRESCRIBED SEISMIC FORCES	12.2.5.1								
1. Steel eccentrically braced frames	14.1	8	2½	4	NL	NL	NL	NL	NL
2. Steel special concentrically braced frames	14.1	7	2½	5½	NL	NL	NL	NL	NL
3. Special reinforced concrete shear walls[l]	14.2	7	2½	5½	NL	NL	NL	NL	NL
4. Ordinary reinforced concrete shear walls[l]	14.2	6	2½	5	NL	NL	NP	NP	NP
5. Steel and concrete composite eccentrically braced frames	14.3	8	2½	4	NL	NL	NL	NL	NL
6. Steel and concrete composite special concentrically braced frames	14.3	6	2½	5	NL	NL	NL	NL	NL

Table 12.2-1 (*Continued*)

Seismic Force-Resisting System	ASCE 7 Section Where Detailing Requirements Are Specified	Response Modification Coefficient, R^a	Overstrength Factor, Ω_0^g	Deflection Amplification Factor, C_d^b	Structural System Limitations Including Structural Height, h_n (ft) Limits[c] — Seismic Design Category: B	C	D[d]	E[d]	F[e]
7. Steel and concrete composite plate shear walls	14.3	7½	2½	6	NL	NL	NL	NL	NL
8. Steel and concrete composite special shear walls	14.3	7	2½	6	NL	NL	NL	NL	NL
9. Steel and concrete composite ordinary shear walls	14.3	6	2½	5	NL	NL	NP	NP	NP
10. Special reinforced masonry shear walls	14.4	5½	3	5	NL	NL	NL	NL	NL
11. Intermediate reinforced masonry shear walls	14.4	4	3	3½	NL	NL	NP	NP	NP
12. Steel buckling-restrained braced frames	14.1	8	2½	5	NL	NL	NL	NL	NL
13. Steel special plate shear walls	14.1	8	2½	6½	NL	NL	NL	NL	NL
E. DUAL SYSTEMS WITH INTERMEDIATE MOMENT FRAMES CAPABLE OF RESISTING AT LEAST 25% OF PRESCRIBED SEISMIC FORCES	12.2.5.1								
1. Steel special concentrically braced frames[f]	14.1	6	2½	5	NL	NL	35	NP	NP
2. Special reinforced concrete shear walls[l]	14.2	6½	2½	5	NL	NL	160	100	100
3. Ordinary reinforced masonry shear walls	14.4	3	3	2½	NL	160	NP	NP	NP
4. Intermediate reinforced masonry shear walls	14.4	3½	3	3	NL	NL	NP	NP	NP
5. Steel and concrete composite special concentrically braced frames	14.3	5½	2½	4½	NL	NL	160	100	NP
6. Steel and concrete composite ordinary braced frames	14.3	3½	2½	3	NL	NL	NP	NP	NP
7. Steel and concrete composite ordinary shear walls	14.3	5	3	4½	NL	NL	NP	NP	NP
8. Ordinary reinforced concrete shear walls[l]	14.2	5½	2½	4½	NL	NL	NP	NP	NP
F. SHEAR WALL-FRAME INTERACTIVE SYSTEM WITH ORDINARY REINFORCED CONCRETE MOMENT FRAMES AND ORDINARY REINFORCED CONCRETE SHEAR WALLS[l]	12.2.5.8 and 14.2	4½	2½	4	NL	NP	NP	NP	NP

Table 12.2-1 *(Continued)*

Seismic Force-Resisting System	ASCE 7 Section Where Detailing Requirements Are Specified	Response Modification Coefficient, R^a	Overstrength Factor, Ω_0^g	Deflection Amplification Factor, C_d^b	Structural System Limitations Including Structural Height, h_n (ft) Limits[c] — Seismic Design Category B	C	D^d	E^d	F^e
G. CANTILEVERED COLUMN SYSTEMS DETAILED TO CONFORM TO THE REQUIREMENTS FOR:	12.2.5.2								
1. Steel special cantilever column systems	14.1	2½	1¼	2½	35	35	35	35	35
2. Steel ordinary cantilever column systems	14.1	1¼	1¼	1¼	35	35	NP[i]	NP[i]	NP[i]
3. Special reinforced concrete moment frames[n]	12.2.5.5 and 14.2	2½	1¼	2½	35	35	35	35	35
4. Intermediate reinforced concrete moment frames	14.2	1½	1¼	1½	35	35	NP	NP	NP
5. Ordinary reinforced concrete moment frames	14.2	1	1¼	1	35	NP	NP	NP	NP
6. Timber frames	14.5	1½	1½	1½	35	35	35	NP	NP
H. STEEL SYSTEMS NOT SPECIFICALLY DETAILED FOR SEISMIC RESISTANCE, EXCLUDING CANTILEVER COLUMN SYSTEMS	14.1	3	3	3	NL	NL	NP	NP	NP

[a]Response modification coefficient, R, for use throughout the standard. Note R reduces forces to a strength level, not an allowable stress level.
[b]Deflection amplification factor, C_d, for use in Sections 12.8.6, 12.8.7, and 12.9.2.
[c]NL = Not Limited and NP = Not Permitted. For metric units use 30.5 m for 100 ft and use 48.8 m for 160 ft.
[d]See Section 12.2.5.4 for a description of seismic force-resisting systems limited to buildings with a structural height, h_n, of 240 ft (73.2 m) or less.
[e]See Section 12.2.5.4 for seismic force-resisting systems limited to buildings with a structural height, h_n, of 160 ft (48.8 m) or less.
[f]Ordinary moment frame is permitted to be used in lieu of intermediate moment frame for Seismic Design Categories B or C.
[g]Where the tabulated value of the overstrength factor, Ω_0, is greater than or equal to 2½, Ω_0 is permitted to be reduced by subtracting the value of 1/2 for structures with flexible diaphragms.
[h]See Section 12.2.5.7 for limitations in structures assigned to Seismic Design Categories D, E, or F.
[i]See Section 12.2.5.6 for limitations in structures assigned to Seismic Design Categories D, E, or F.
[j]Steel ordinary concentrically braced frames are permitted in single-story buildings up to a structural height, h_n, of 60 ft (18.3 m) where the dead load of the roof does not exceed 20 psf
(0.96 kN/m²) and in penthouse structures.
[k]An increase in structural height, h_n, to 45 ft (13.7 m) is permitted for single story storage warehouse facilities.
[l]In Section 2.2 of ACI 318. A shear wall is defined as a structural wall.
[m]In Section 2.2 of ACI 318. The definition of "special structural wall" includes precast and cast-in-place construction.
[n]In Section 2.2 of ACI 318. The definition of "special moment frame" includes precast and cast-in-place construction.
[o]Alternately, the seismic load effect with overstrength, E_{mh}, is permitted to be based on the expected strength determined in accordance with AISI S110.
[p]Cold-formed steel – special bolted moment frames shall be limited to one-story in height in accordance with AISI S110.

Table 12.6-1 Permitted Analytical Procedures

Seismic Design Category	Structural Characteristics	Equivalent Lateral Force Analysis, Section 12.8[a]	Modal Response Spectrum Analysis, Section 12.9[a]	Seismic Response History Procedures, Chapter 16[a]
B, C	All structures	P	P	P
D, E, F	Risk Category I or II buildings not exceeding 2 stories above the base	P	P	P
	Structures of light frame construction	P	P	P
	Structures with no structural irregularities and not exceeding 160 ft in structural height	P	P	P
	Structures exceeding 160 ft in structural height with no structural irregularities and with $T < 3.5T_s$	P	P	P
	Structures not exceeding 160 ft in structural height and having only horizontal irregularities of Type 2, 3, 4, or 5 in Table 12.3-1 or vertical irregularities of Type 4, 5a, or 5b in Table 12.3-2	P	P	P
	All other structures	NP	P	P

[a]P: Permitted; NP: Not Permitted; $T_s = S_{D1}/S_{DS}$.

Table 12.8-1 Coefficient for Upper Limit on Calculated Period

Design Spectral Response Acceleration Parameter at 1 s, S_{D1}	Coefficient C_u
≥ 0.4	1.4
0.3	1.4
0.2	1.5
0.15	1.6
≤ 0.1	1.7

Table 12.8-2 Values of Approximate Period Parameters C_t and x

Structure Type	C_t	x
Moment-resisting frame systems in which the frames resist 100% of the required seismic force and are not enclosed or adjoined by components that are more rigid and will prevent the frames from deflecting where subjected to seismic forces:		
Steel moment-resisting frames	0.028 (0.0724)[a]	0.8
Concrete moment-resisting frames	0.016 (0.0466)[a]	0.9
Steel eccentrically braced frames in accordance with Table 12.2-1 lines B1 or D1	0.03 (0.0731)[a]	0.75
Steel buckling-restrained braced frames	0.03 (0.0731)[a]	0.75
All other structural systems	0.02 (0.0488)[a]	0.75

[a]Metric equivalents are shown in parentheses.

Table 12.12-1 Allowable Story Drift, Δ_a[a,b]

Structure	Risk Category		
	I or II	III	IV
Structures, other than masonry shear wall structures, 4 stories or less above the base as defined in Section 11.2, with interior walls, partitions, ceilings, and exterior wall systems that have been designed to accommodate the story drifts.	$0.025h_{sx}$[c]	$0.020h_{sx}$	$0.015h_{sx}$
Masonry cantilever shear wall structures[d]	$0.010h_{sx}$	$0.010h_{sx}$	$0.010h_{sx}$
Other masonry shear wall structures	$0.007h_{sx}$	$0.007h_{sx}$	$0.007h_{sx}$
All other structures	$0.020h_{sx}$	$0.015h_{sx}$	$0.010h_{sx}$

[a] h_{sx} is the story height below Level x.
[b] For seismic force-resisting systems comprised solely of moment frames in Seismic Design Categories D, E, and F, the allowable story drift shall comply with the requirements of Section 12.12.1.1.
[c] There shall be no drift limit for single-story structures with interior walls, partitions, ceilings, and exterior wall systems that have been designed to accommodate the story drifts. The structure separation requirement of Section 12.12.3 is not waived.
[d] Structures in which the basic structural system consists of masonry shear walls designed as vertical elements cantilevered from their base or foundation support which are so constructed that moment transfer between shear walls (coupling) is negligible.

Table 20.3-1 Site Classification

Site Class	$\bar{v}_s$	$\bar{N}$ or $\bar{N}_{ch}$	$\bar{s}_u$
A. Hard rock	>5,000 ft/s	NA	NA
B. Rock	2,500 to 5,000 ft/s	NA	NA
C. Very dense soil and soft rock	1,200 to 2,500 ft/s	>50	>2,000 psf
D. Stiff soil	600 to 1,200 ft/s	15 to 50	1,000 to 2,000 psf
E. Soft clay soil	<600 ft/s	<15	<1,000 psf
	Any profile with more than 10 ft of soil having the following characteristics: —Plasticity index $PI > 20$, —Moisture content $w \geq 40\%$, —Undrained shear strength $\bar{s}_u < 500$ psf		
F. Soils requiring site response analysis in accordance with Section 21.1	See Section 20.3.1		

For SI: 1 ft/s = 0.3048 m/s; 1 lb/ft^2 = 0.0479 kN/m^2.

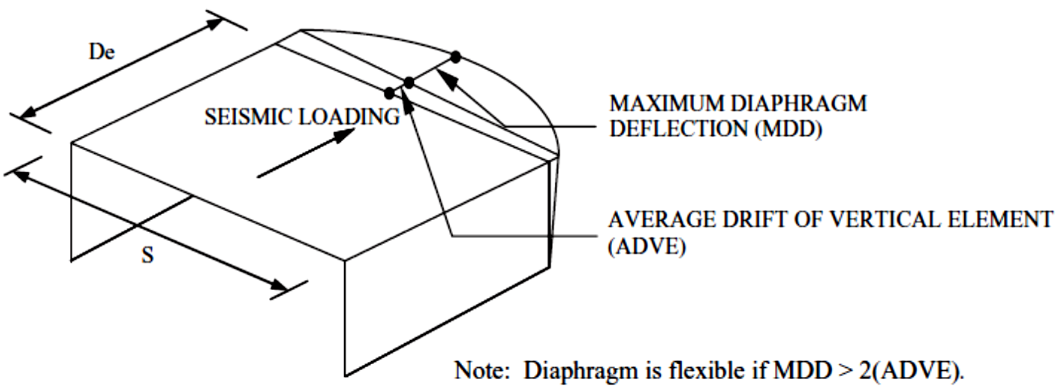


FIGURE 12.3-1 Flexible Diaphragm

Table 12.3-1 Horizontal Structural Irregularities

Type	Description	Reference Section	Seismic Design Category Application
1a.	**Torsional Irregularity:** Torsional irregularity is defined to exist where the maximum story drift, computed including accidental torsion with $A_x = 1.0$, at one end of the structure transverse to an axis is more than 1.2 times the average of the story drifts at the two ends of the structure. Torsional irregularity requirements in the reference sections apply only to structures in which the diaphragms are rigid or semirigid.	12.3.3.4 12.7.3 12.8.4.3 12.12.1 Table 12.6-1 Section 16.2.2	D, E, and F B, C, D, E, and F C, D, E, and F C, D, E, and F D, E, and F B, C, D, E, and F
1b.	**Extreme Torsional Irregularity:** Extreme torsional irregularity is defined to exist where the maximum story drift, computed including accidental torsion with $A_x = 1.0$, at one end of the structure transverse to an axis is more than 1.4 times the average of the story drifts at the two ends of the structure. Extreme torsional irregularity requirements in the reference sections apply only to structures in which the diaphragms are rigid or semirigid.	12.3.3.1 12.3.3.4 12.7.3 12.8.4.3 12.12.1 Table 12.6-1 Section 16.2.2	E and F D B, C, and D C and D C and D D B, C, and D
2.	**Reentrant Corner Irregularity:** Reentrant corner irregularity is defined to exist where both plan projections of the structure beyond a reentrant corner are greater than 15% of the plan dimension of the structure in the given direction.	12.3.3.4 Table 12.6-1	D, E, and F D, E, and F
3.	**Diaphragm Discontinuity Irregularity:** Diaphragm discontinuity irregularity is defined to exist where there is a diaphragm with an abrupt discontinuity or variation in stiffness, including one having a cutout or open area greater than 50% of the gross enclosed diaphragm area, or a change in effective diaphragm stiffness of more than 50% from one story to the next.	12.3.3.4 Table 12.6-1	D, E, and F D, E, and F
4.	**Out-of-Plane Offset Irregularity:** Out-of-plane offset irregularity is defined to exist where there is a discontinuity in a lateral force-resistance path, such as an out-of-plane offset of at least one of the vertical elements.	12.3.3.3 12.3.3.4 12.7.3 Table 12.6-1 Section 16.2.2	B, C, D, E, and F D, E, and F B, C, D, E, and F D, E, and F B, C, D, E, and F
5.	**Nonparallel System Irregularity:** Nonparallel system irregularity is defined to exist where vertical lateral force-resisting elements are not parallel to the major orthogonal axes of the seismic force-resisting system.	12.5.3 12.7.3 Table 12.6-1 Section 16.2.2	C, D, E, and F B, C, D, E, and F D, E, and F B, C, D, E, and F

Table 12.3-2 Vertical Structural Irregularities

Type	Description	Reference Section	Seismic Design Category Application
1a.	**Stiffness-Soft Story Irregularity:** Stiffness-soft story irregularity is defined to exist where there is a story in which the lateral stiffness is less than 70% of that in the story above or less than 80% of the average stiffness of the three stories above.	Table 12.6-1	D, E, and F
1b.	**Stiffness-Extreme Soft Story Irregularity:** Stiffness-extreme soft story irregularity is defined to exist where there is a story in which the lateral stiffness is less than 60% of that in the story above or less than 70% of the average stiffness of the three stories above.	12.3.3.1 Table 12.6-1	E and F D, E, and F
2.	**Weight (Mass) Irregularity:** Weight (mass) irregularity is defined to exist where the effective mass of any story is more than 150% of the effective mass of an adjacent story. A roof that is lighter than the floor below need not be considered.	Table 12.6-1	D, E, and F
3.	**Vertical Geometric Irregularity:** Vertical geometric irregularity is defined to exist where the horizontal dimension of the seismic force-resisting system in any story is more than 130% of that in an adjacent story.	Table 12.6-1	D, E, and F
4.	**In-Plane Discontinuity in Vertical Lateral Force-Resisting Element Irregularity:** In-plane discontinuity in vertical lateral force-resisting elements irregularity is defined to exist where there is an in-plane offset of a vertical seismic force-resisting element resulting in overturning demands on a supporting beam, column, truss, or slab.	12.3.3.3 12.3.3.4 Table 12.6-1	B, C, D, E, and F D, E, and F D, E, and F
5a.	**Discontinuity in Lateral Strength–Weak Story Irregularity:** Discontinuity in lateral strength–weak story irregularity is defined to exist where the story lateral strength is less than 80% of that in the story above. The story lateral strength is the total lateral strength of all seismic-resisting elements sharing the story shear for the direction under consideration.	12.3.3.1 Table 12.6-1	E and F D, E, and F
5b.	**Discontinuity in Lateral Strength–Extreme Weak Story Irregularity:** Discontinuity in lateral strength–extreme weak story irregularity is defined to exist where the story lateral strength is less than 65% of that in the story above. The story strength is the total strength of all seismic-resisting elements sharing the story shear for the direction under consideration.	12.3.3.1 12.3.3.2 Table 12.6-1	D, E, and F B and C D, E, and F

2.4 COMBINING NOMINAL LOADS USING ALLOWABLE STRESS DESIGN

2.4.1 Basic Combinations

Loads listed herein shall be considered to act in the following combinations; whichever produces the most unfavorable effect in the building, foundation, or structural member being considered. Effects of one or more loads not acting shall be considered.

1. D
2. $D + L$
3. $D + (L_r \text{ or } S \text{ or } R)$

4. $D + 0.75L + 0.75(L_r \text{ or } S \text{ or } R)$
5. $D + (0.6W \text{ or } 0.7E)$

6a. $D + 0.75L + 0.75(0.6W) + 0.75(L_r \text{ or } S \text{ or } R)$

6b. $D + 0.75L + 0.75(0.7E) + 0.75S$

7. $0.6D + 0.6W$
8. $0.6D + 0.7E$

Basic Combinations for Strength Design (see Sections 2.3.2 and 2.2 for notation).

5. $(1.2 + 0.2S_{DS})D + \rho Q_E + L + 0.2S$
6. $(0.9 - 0.2S_{DS})D + \rho Q_E + 1.6H$

Basic Combinations for Allowable Stress Design (see Sections 2.4.1 and 2.2 for notation).

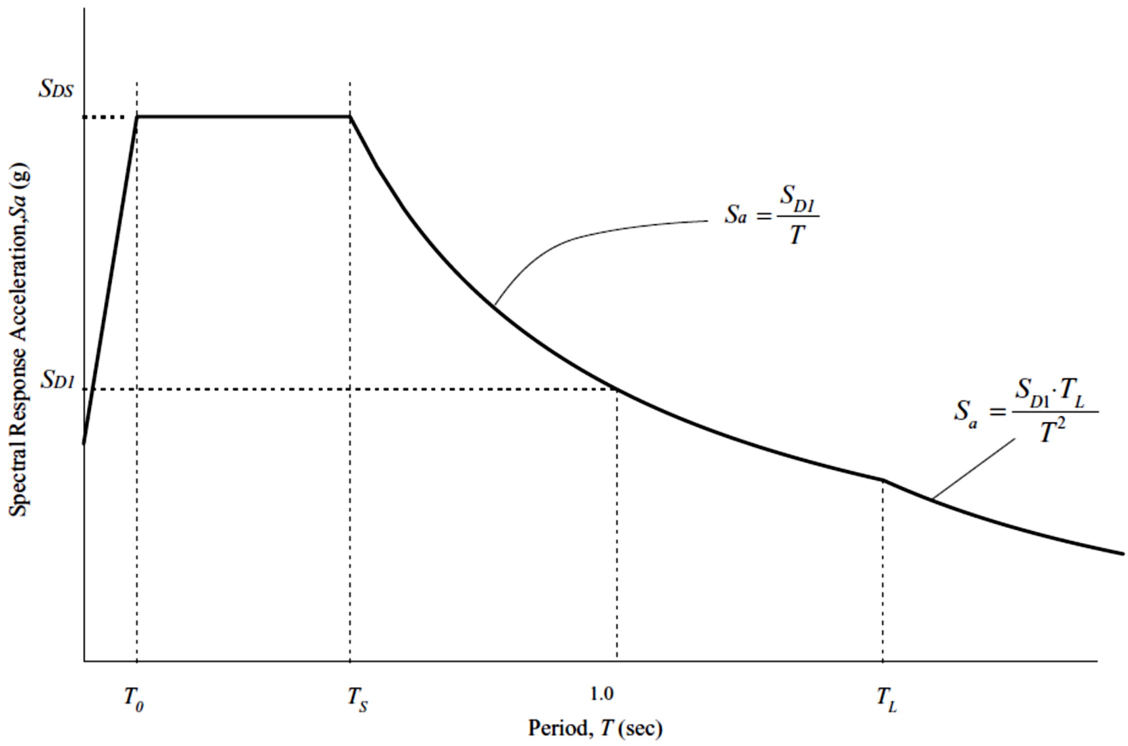
5. $(1.0 + 0.14S_{DS})D + H + F + 0.7\rho Q_E$
6. $(1.0 + 0.10S_{DS})D + H + F + 0.525\rho Q_E + 0.75L + 0.75(L_r \text{ or } S \text{ or } R)$
8. $(0.6 - 0.14S_{DS})D + 0.7\rho Q_E + H$

Basic Combinations for Strength Design with Overstrength Factor (see Sections 2.3.2 and 2.2 for notation).

5. $(1.2 + 0.2S_{DS})D + \Omega_o Q_E + L + 0.2S$
7. $(0.9 - 0.2S_{DS})D + \Omega_o Q_E + 1.6H$

Basic Combinations for Allowable Stress Design with Overstrength Factor (see Sections 2.4.1 and 2.2 for notation).

5. $(1.0 + 0.14S_{DS})D + H + F + 0.7\Omega_o Q_E$
6. $(1.0 + 0.105S_{DS})D + H + F + 0.525\Omega_o Q_E + 0.75L + 0.75(L_r \text{ or } S \text{ or } R)$
8. $(0.6 - 0.14S_{DS})D + 0.7\Omega_o Q_E + H$



1. For periods less than T_0, the design spectral response acceleration, S_a, shall be taken as given by Eq. 11.4-5:

$$S_a = S_{DS}\left(0.4 + 0.6\frac{T}{T_0}\right) \qquad (11.4\text{-}5)$$

2. For periods greater than or equal to T_0 and less than or equal to T_S, the design spectral response acceleration, S_a, shall be taken equal to S_{DS}.
3. For periods greater than T_S, and less than or equal to T_L, the design spectral response acceleration, S_a, shall be taken as given by Eq. 11.4-6:

$$S_a = \frac{S_{D1}}{T} \qquad (11.4\text{-}6)$$

4. For periods greater than T_L, S_a shall be taken as given by Eq. 11.4-7:

$$S_a = \frac{S_{D1}T_L}{T^2} \qquad (11.4\text{-}7)$$

$T_0 = 0.2S_{D1}/S_{DS}$
$T_S = S_{D1}/S_{DS}$

TABLE 16-I—SEISMIC ZONE FACTOR *Z*

ZONE	1	2A	2B	3	4
Z	0.075	0.15	0.20	0.30	0.40

NOTE: The zone shall be determined from the seismic zone map in Figure 16-2.

TABLE 16-J—SOIL PROFILE TYPES

SOIL PROFILE TYPE	SOIL PROFILE NAME/GENERIC DESCRIPTION	AVERAGE SOIL PROPERTIES FOR TOP 100 FEET (30 480 mm) OF SOIL PROFILE		
		Shear Wave Velocity, $\bar{v}_s$ feet/second (m/s)	Standard Penetration Test, $\bar{N}$ [or $\bar{N}_{CH}$ for cohesionless soil layers] (blows/foot)	Undrained Shear Strength, $\bar{s}_u$ psf (kPa)
S_A	Hard Rock	> 5,000 (1,500)	—	—
S_B	Rock	2,500 to 5,000 (760 to 1,500)		
S_C	Very Dense Soil and Soft Rock	1,200 to 2,500 (360 to 760)	> 50	> 2,000 (100)
S_D	Stiff Soil Profile	600 to 1,200 (180 to 360)	15 to 50	1,000 to 2,000 (50 to 100)
S_E[1]	Soft Soil Profile	< 600 (180)	< 15	< 1,000 (50)
S_F	Soil Requiring Site-specific Evaluation. See Section 1629.3.1.			

[1]Soil Profile Type S_E also includes any soil profile with more than 10 feet (3048 mm) of soft clay defined as a soil with a plasticity index, $PI > 20$, $w_{mc} \geq 40$ percent and $s_u < 500$ psf (24 kPa). The Plasticity Index, PI, and the moisture content, w_{mc}, shall be determined in accordance with approved national standards.

TABLE 16-K—OCCUPANCY CATEGORY

OCCUPANCY CATEGORY	OCCUPANCY OR FUNCTIONS OF STRUCTURE	SEISMIC IMPORTANCE FACTOR, I	SEISMIC IMPORTANCE[1] FACTOR, I_p	WIND IMPORTANCE FACTOR, I_w
1. Essential facilities[2]	Group I, Division 1 Occupancies having surgery and emergency treatment areas Fire and police stations Garages and shelters for emergency vehicles and emergency aircraft Structures and shelters in emergency-preparedness centers Aviation control towers Structures and equipment in government communication centers and other facilities required for emergency response Standby power-generating equipment for Category 1 facilities Tanks or other structures containing housing or supporting water or other fire-suppression material or equipment required for the protection of Category 1, 2 or 3 structures	1.25	1.50	1.15
2. Hazardous facilities	Group H, Divisions 1, 2, 6 and 7 Occupancies and structures therein housing or supporting toxic or explosive chemicals or substances Nonbuilding structures housing, supporting or containing quantities of toxic or explosive substances that, if contained within a building, would cause that building to be classified as a Group H, Division 1, 2 or 7 Occupancy	1.25	1.50	1.15
3. Special occupancy structures[3]	Group A, Divisions 1, 2 and 2.1 Occupancies Buildings housing Group E, Divisions 1 and 3 Occupancies with a capacity greater than 300 students Buildings housing Group B Occupancies used for college or adult education with a capacity greater than 500 students Group I, Divisions 1 and 2 Occupancies with 50 or more resident incapacitated patients, but not included in Category 1 Group I, Division 3 Occupancies All structures with an occupancy greater than 5,000 persons Structures and equipment in power-generating stations, and other public utility facilities not included in Category 1 or Category 2 above, and required for continued operation	1.00	1.00	1.00
4. Standard occupancy structures[3]	All structures housing occupancies or having functions not listed in Category 1, 2 or 3 and Group U Occupancy towers	1.00	1.00	1.00
5. Miscellaneous structures	Group U Occupancies except for towers	1.00	1.00	1.00

[1]The limitation of I_p for panel connections in Section 1633.2.4 shall be 1.0 for the entire connector.
[2]Structural observation requirements are given in Section 1702.
[3]For anchorage of machinery and equipment required for life-safety systems, the value of I_p shall be taken as 1.5.

TABLE 16-N—STRUCTURAL SYSTEMS[1]

BASIC STRUCTURAL SYSTEM[2]	LATERAL-FORCE-RESISTING SYSTEM DESCRIPTION	R	Ω_o	HEIGHT LIMIT FOR SEISMIC ZONES 3 AND 4 (feet) × 304.8 for mm
1. Bearing wall system	1. Light-framed walls with shear panels			
	a. Wood structural panel walls for structures three stories or less	5.5	2.8	65
	b. All other light-framed walls	4.5	2.8	65
	2. Shear walls			
	a. Concrete	4.5	2.8	160
	b. Masonry	4.5	2.8	160
	3. Light steel-framed bearing walls with tension-only bracing	2.8	2.2	65
	4. Braced frames where bracing carries gravity load			
	a. Steel	4.4	2.2	160
	b. Concrete[3]	2.8	2.2	—
	c. Heavy timber	2.8	2.2	65
2. Building frame system	1. Steel eccentrically braced frame (EBF)	7.0	2.8	240
	2. Light-framed walls with shear panels			
	a. Wood structural panel walls for structures three stories or less	6.5	2.8	65
	b. All other light-framed walls	5.0	2.8	65
	3. Shear walls			
	a. Concrete	5.5	2.8	240
	b. Masonry	5.5	2.8	160
	4. Ordinary braced frames			
	a. Steel	5.6	2.2	160
	b. Concrete[3]	5.6	2.2	—
	c. Heavy timber	5.6	2.2	65
	5. Special concentrically braced frames			
	a. Steel	6.4	2.2	240
3. Moment-resisting frame system	1. Special moment-resisting frame (SMRF)			
	a. Steel	8.5	2.8	N.L.
	b. Concrete[4]	8.5	2.8	N.L.
	2. Masonry moment-resisting wall frame (MMRWF)	6.5	2.8	160
	3. Concrete intermediate moment-resisting frame (IMRF)[5]	5.5	2.8	—
	4. Ordinary moment-resisting frame (OMRF)			
	a. Steel[6]	4.5	2.8	160
	b. Concrete[7]	3.5	2.8	—
	5. Special truss moment frames of steel (STMF)	6.5	2.8	240
4. Dual systems	1. Shear walls			
	a. Concrete with SMRF	8.5	2.8	N.L.
	b. Concrete with steel OMRF	4.2	2.8	160
	c. Concrete with concrete IMRF[5]	6.5	2.8	160
	d. Masonry with SMRF	5.5	2.8	160
	e. Masonry with steel OMRF	4.2	2.8	160
	f. Masonry with concrete IMRF[3]	4.2	2.8	—
	g. Masonry with masonry MMRWF	6.0	2.8	160
	2. Steel EBF			
	a. With steel SMRF	8.5	2.8	N.L.
	b. With steel OMRF	4.2	2.8	160
	3. Ordinary braced frames			
	a. Steel with steel SMRF	6.5	2.8	N.L.
	b. Steel with steel OMRF	4.2	2.8	160
	c. Concrete with concrete SMRF[3]	6.5	2.8	—
	d. Concrete with concrete IMRF[3]	4.2	2.8	—
	4. Special concentrically braced frames			
	a. Steel with steel SMRF	7.5	2.8	N.L.
	b. Steel with steel OMRF	4.2	2.8	160
5. Cantilevered column building systems	1. Cantilevered column elements	2.2	2.0	35[7]
6. Shear wall-frame interaction systems	1. Concrete[8]	5.5	2.8	160
7. Undefined systems	See Sections 1629.6.7 and 1629.9.2	—	—	—

N.L.—no limit
[1]See Section 1630.4 for combination of structural systems.
[2]Basic structural systems are defined in Section 1629.6.
[3]Prohibited in Seismic Zones 3 and 4.
[4]Includes precast concrete conforming to Section 1921.2.7.
[5]Prohibited in Seismic Zones 3 and 4, except as permitted in Section 1634.2.
[6]Ordinary moment-resisting frames in Seismic Zone 1 meeting the requirements of Section 2211.6 may use a R value of 8.
[7] Total height of the building including cantilevered columns.
[8]Prohibited in Seismic Zones 2A, 2B, 3 and 4. See Section 1633.2.7.

TABLE 16-Q—SEISMIC COEFFICIENT C_a

SOIL PROFILE TYPE	SEISMIC ZONE FACTOR, Z				
	Z = 0.075	Z = 0.15	Z = 0.2	Z = 0.3	Z = 0.4
S_A	0.06	0.12	0.16	0.24	$0.32N_a$
S_B	0.08	0.15	0.20	0.30	$0.40N_a$
S_C	0.09	0.18	0.24	0.33	$0.40N_a$
S_D	0.12	0.22	0.28	0.36	$0.44N_a$
S_E	0.19	0.30	0.34	0.36	$0.36N_a$
S_F	See Footnote 1				

[1]Site-specific geotechnical investigation and dynamic site response analysis shall be performed to determine seismic coefficients for Soil Profile Type S_F.

TABLE 16-R—SEISMIC COEFFICIENT C_v

SOIL PROFILE TYPE	SEISMIC ZONE FACTOR, Z				
	Z = 0.075	Z = 0.15	Z = 0.2	Z = 0.3	Z = 0.4
S_A	0.06	0.12	0.16	0.24	$0.32N_v$
S_B	0.08	0.15	0.20	0.30	$0.40N_v$
S_C	0.13	0.25	0.32	0.45	$0.56N_v$
S_D	0.18	0.32	0.40	0.54	$0.64N_v$
S_E	0.26	0.50	0.64	0.84	$0.96N_v$
S_F	See Footnote 1				

[1]Site-specific geotechnical investigation and dynamic site response analysis shall be performed to determine seismic coefficients for Soil Profile Type S_F.

TABLE 16-P—*R* AND Ω_o FACTORS FOR NONBUILDING STRUCTURES

STRUCTURE TYPE	R	Ω_o
1. Vessels, including tanks and pressurized spheres, on braced or unbraced legs.	2.2	2.0
2. Cast-in-place concrete silos and chimneys having walls continuous to the foundations.	3.6	2.0
3. Distributed mass cantilever structures such as stacks, chimneys, silos and skirt-supported vertical vessels.	2.9	2.0
4. Trussed towers (freestanding or guyed), guyed stacks and chimneys.	2.9	2.0
5. Cantilevered column-type structures.	2.2	2.0
6. Cooling towers.	3.6	2.0
7. Bins and hoppers on braced or unbraced legs.	2.9	2.0
8. Storage racks.	3.6	2.0
9. Signs and billboards.	3.6	2.0
10. Amusement structures and monuments.	2.2	2.0
11. All other self-supporting structures not otherwise covered.	2.9	2.0

TABLE 16-L—VERTICAL STRUCTURAL IRREGULARITIES

IRREGULARITY TYPE AND DEFINITION	REFERENCE SECTION
1. **Stiffness irregularity—soft story** A soft story is one in which the lateral stiffness is less than 70 percent of that in the story above or less than 80 percent of the average stiffness of the three stories above.	1629.8.4, Item 2
2. **Weight (mass) irregularity** Mass irregularity shall be considered to exist where the effective mass of any story is more than 150 percent of the effective mass of an adjacent story. A roof that is lighter than the floor below need not be considered.	1629.8.4, Item 2
3. **Vertical geometric irregularity** Vertical geometric irregularity shall be considered to exist where the horizontal dimension of the lateral-force-resisting system in any story is more than 130 percent of that in an adjacent story. One-story penthouses need not be considered.	1629.8.4, Item 2
4. **In-plane discontinuity in vertical lateral-force-resisting element** An in-plane offset of the lateral-load-resisting elements greater than the length of those elements.	1630.8.2
5. **Discontinuity in capacity—weak story** A weak story is one in which the story strength is less than 80 percent of that in the story above. The story strength is the total strength of all seismic-resisting elements sharing the story shear for the direction under consideration.	1629.9.1

TABLE 16-M—PLAN STRUCTURAL IRREGULARITIES

IRREGULARITY TYPE AND DEFINITION	REFERENCE SECTION
1. **Torsional irregularity—to be considered when diaphragms are not flexible** Torsional irregularity shall be considered to exist when the maximum story drift, computed including accidental torsion, at one end of the structure transverse to an axis is more than 1.2 times the average of the story drifts of the two ends of the structure.	1633.1, 1633.2.9, Item 6
2. **Re-entrant corners** Plan configurations of a structure and its lateral-force-resisting system contain re-entrant corners, where both projections of the structure beyond a re-entrant corner are greater than 15 percent of the plan dimension of the structure in the given direction.	1633.2.9, Items 6 and 7
3. **Diaphragm discontinuity** Diaphragms with abrupt discontinuities or variations in stiffness, including those having cutout or open areas greater than 50 percent of the gross enclosed area of the diaphragm, or changes in effective diaphragm stiffness of more than 50 percent from one story to the next.	1633.2.9, Item 6
4. **Out-of-plane offsets** Discontinuities in a lateral force path, such as out-of-plane offsets of the vertical elements.	1630.8.2; 1633.2.9, Item 6; 2213.9.1
5. **Nonparallel systems** The vertical lateral-load-resisting elements are not parallel to or symmetric about the major orthogonal axes of the lateral-force-resisting system.	1633.1

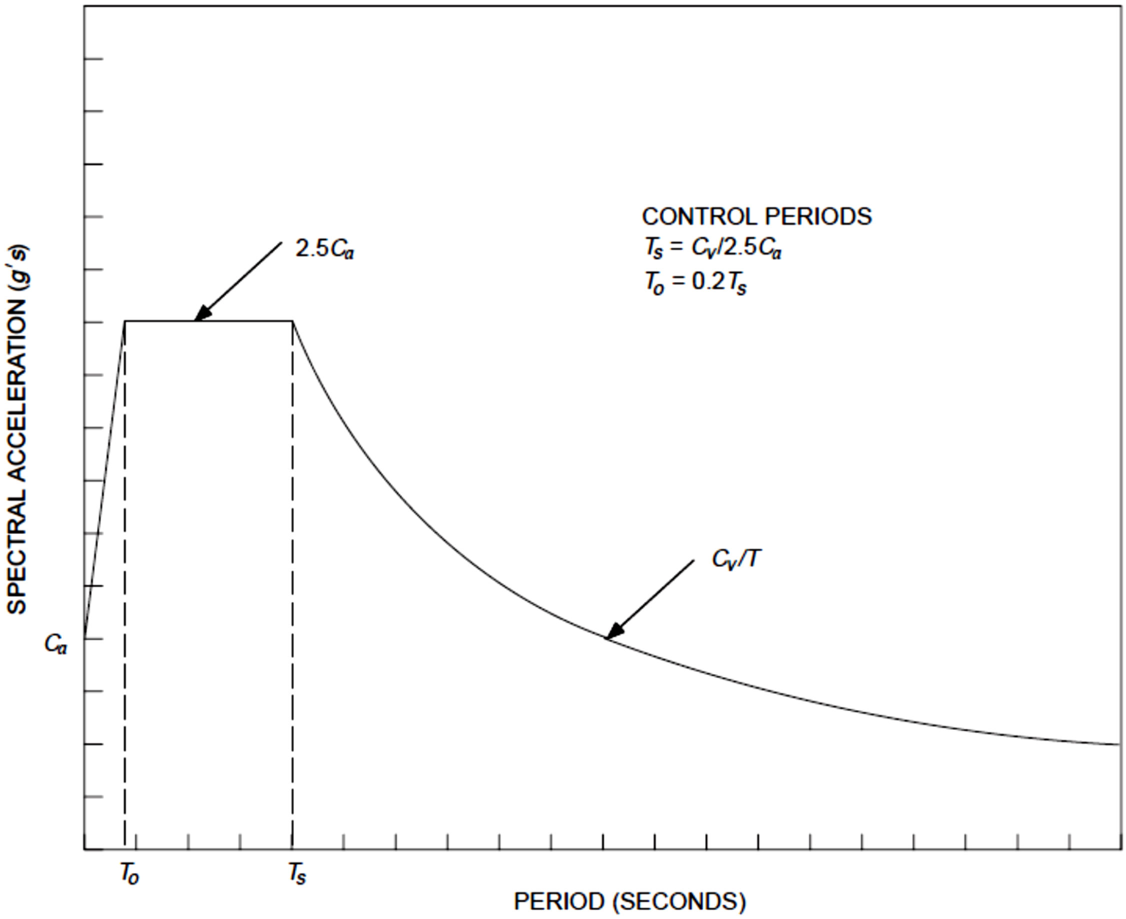


FIGURE 16-3—DESIGN RESPONSE SPECTRA

Chapter 13: Design of Footings

This chapter illustrates the design of the following types of footings:

- Wall footing
- Single footing
- Combined footing
- Strap footing
- Mat foundation
- Pile foundation

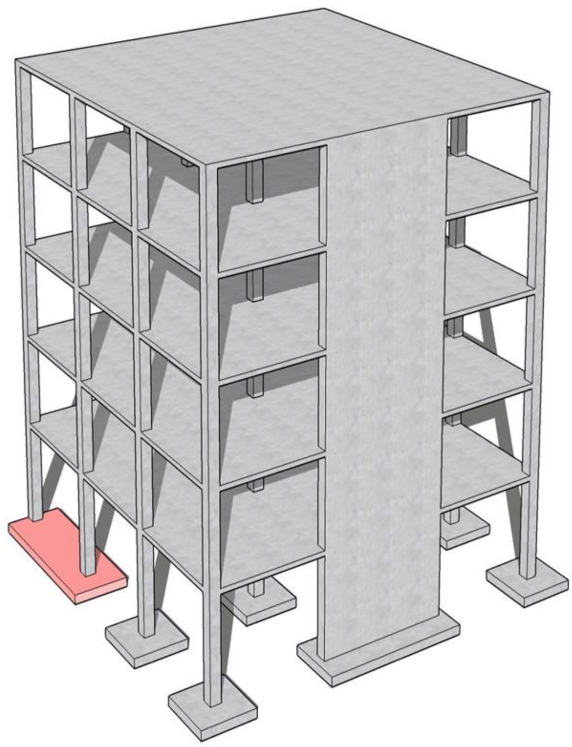


Figure 13.1: Wall, single and combined footings

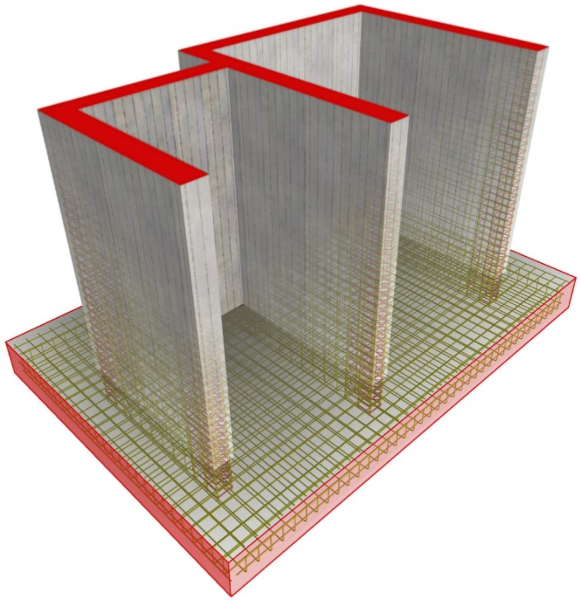


Figure 13.2: Mat foundation

13.1 Design of Wall Footing:

Design steps:

1. Determine footing width, B:

$$A_f = \frac{P_{service}}{q_{all}}$$

Where:

A_f= area of footing= B x 1m

B= width of footing, m

1 m= unit length of wall footing

$P_{service}$= summation of service compression axial force, kN/m

q_{all}= allowable soil bearing capacity, kN/m^2

So,

$$B = \frac{P_{service}}{q_{all}}$$

2. Determine footing thickness, h:

Footing thickness, h= d + cov

d= footing effective depth, mm

cov= concrete cover to flexural bars centroid = clear concrete cover + half bar diameter, mm

Clear cover= 75mm if the footing is casted directly on soil.

Clear cover= 40mm if the footing is casted on plain concrete.

d>= 150mm for footing on soil.

d>= 300mm for footing on piles.

It is recommended to use the thickness of the footing not less than 300mm to take into account environmental conditions to protect concrete.

Footing effective depth, d can be determined from wide beam shear strength (one-way shear), as follows:

The ultimate shear force, V_u should be less than or equals the section shear strength, $\emptyset V_c$.

$$V_u \leq \emptyset V_c$$

$$\text{ACI } 318-14\text{: } \emptyset V_c = \emptyset \frac{1}{6} \lambda \sqrt{f'_c} b_w d$$

ACI 318-19:

V_c can be calculated by:

$$For\ A_v \geq A_{v.min}\ (or\ \frac{A_v}{s} \geq \left(\frac{A_v}{s}\right)_{min})\ use\ either\ of:$$

$$V_c = \left(0.17\lambda\sqrt{f'_c} + \frac{N_u}{6A_g}\right) b_w d \quad and \quad V_c = \left(0.66\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right) b_w d$$

$$For\ A_v < A_{v.min}\ \left(or\ \frac{A_v}{s} < \left(\frac{A_v}{s}\right)_{min}\right) use:$$

$$V_c = \left(0.66\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right) b_w d$$

Where A_v is the area of shear reinforcement within spacing s, mm^2.

And, V_c shall not be taken greater than:

$$V_c \leq 0.42\lambda\sqrt{f'_c}b_w d$$

$$Size\ factor, \lambda_s = \sqrt{\frac{2}{1 + 0.004\ d}} \leq 1.0$$

For $d \leq 250mm, \lambda_s = 1.0$

$$\frac{N_u}{6A_g} \leq 0.05f'_c$$

Axial load, *Nu*, is positive for compression and negative for tension.

$$\rho_w = \frac{A_s}{b_w d}$$

The value of A_s to be used in the calculation of ρ_w may be taken as the sum of the areas of longitudinal bars located more than two thirds of the overall member depth away from the extreme compression fiber.

The value of $\sqrt{f'c}$ used to calculate ***Vc*** for one-way shear shall not exceed 100 psi (8.3MPa), unless allowed in 22.5.3.2 ($A_v \geq A_{v,min}$).

When no shear reinforcement is used, the ACI 318-19 equation of V_c will give the same value as in ACI 318-14 if the steel ratio, $\rho_w = 0.017$. In footings, this ratio is high and will not be obtained, so, the controlling equation will be that of ACI 318-19.

$V_u = q_u(l_1 - d)$

$b_w = 1000mm$

q_u = ultimate stress under the footing, kN/m^2

$q_u = \frac{P_u}{B}$

P_u= ultimate axial downward load on the footing in kN/m

L1= clear distance from face of wall (support) to edge of footing

3. Determine flexural steel:

The moment at face of the wall, M_u is given by:

$M_u = \frac{q_u l1^2}{2}$: cantilever of span L1

The steel ratio, ρ is given by:

$$\rho = \frac{0.85f'_c}{f_y}\left(1 - \sqrt{1 - \frac{2.61M_u}{bd^2 f'_c}}\right)$$

Where:

f'c= concrete compressive strength, Mpa

fy= steel yield strength, Mpa

Mu= ultimate moment at face of wall, N.mm

b= width of section= 1000mm

d= effective depth of section, mm

The flexural steel area, As= ρbd $\geq A_{s,min}$

$A_{s,min}$ = $\rho_{shrinkage}$ x b x h b=1000mm

$\rho_{shrinkage} = 0.0018$

Shrinkage steel should be used in the longitudinal direction of footing.

The flexural steel is used in the transverse direction of footing.

It is recommended to use top reinforcement equals to half the shrinkage steel if the thickness of footing is large (may be larger than 500mm).

Check development of flexural bars:

The length of bar from face of wall to the end of footing should be larger than or equal to the bar development length, L_{dt} or L_{dh}.

The development length in tension, L_{dt} is given by:

$$L_{dt} \geq \frac{0.48f_y}{\lambda\sqrt{f'_c}}d_b \geq 300mm \;\; for\; d_b < 20mm$$

$$L_{dt} \geq \frac{0.59f_y}{\lambda\sqrt{f'_c}}d_b \geq 300mm \;\; for\; d_b \geq 20mm$$

For top bars and d>= 300mm, increase these values by 30%.

The development length for hooked bars, L_{dh} is given by:

$$ACI\ 318-14: L_{dh} \geq \frac{0.24 f_y}{\lambda \sqrt{f'_c}} d_b$$

$$ACI\ 318-19: l_{dh} \geq \frac{0.087 f_y}{\lambda \sqrt{f'_c}} d_b{}^{1.5}$$

$$\geq 8d_b$$

$$\geq 150mm$$

Example (wall footing):

Given:

Concrete f'c= 24MPa

Steel yield strength, fy= 420MPa

Soil allowable bearing capacity, qall= 300kN/m^2

Wall thickness= 0.25m

Dead load, P_D= 300kN/m

Live load, P_L= 200kN/m

Design the required wall footing?

Solution:

1. **Footing width, B:**

$P_{service}$= P_D+ P_L= 300 + 200= 500kN

$P_{ultimate}$= 1.2 P_D + 1.6 P_L= 1.2(300) + 1.6(200) = 680kN

$$B = \frac{P_{service}}{q_{all}} = \frac{500}{300} = 1.7m$$

2. Footing thickness, h:

$$q_u = \frac{P_{ultimate}}{B} = \frac{680}{1.7} = 400kN/m^2$$

The distance from face of wall to the edge of footing, L1 is:

$$l1 = \frac{1.7 - 0.25}{2} = 0.725m$$

The ultimate shear at distance d from face of wall is given by:

$$V_u = q_u(l1 - d) = 400(0.725 - d) = 290 - 400\text{d}$$

ACI 318-14:

The concrete shear capacity, $\emptyset V_c$ is:

$$\emptyset V_c = \emptyset \frac{1}{6}\lambda\sqrt{f'_c}b_w d = \frac{(0.75)\left(\frac{1}{6}\right)(1)\sqrt{24}(1000)(dx1000)}{1000}$$

$$V_u = \emptyset V_C \rightarrow d = 0.29m$$

Footing thickness, h = d + cover to bars centroid= 0.29+0.05= 0.34m Use h=0.35 for practical purposes.

ACI 318-19:

The concrete shear capacity, $\emptyset V_c$ is:

$$\emptyset V_c = \emptyset\left(0.66\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right)b_w d$$

$$Let\ \rho_w = 0.0018\left(\frac{h}{d}\right) = 0.0018(1.1) = 0.00198$$

Based on ACI 318-19 section 13.2.6.2, the size factor in footings can be neglected.

$$\frac{0.75\left(0.66(1)(1)(0.00198)^{\frac{1}{3}}\sqrt{24} + 0.0\right)(1000)(d\,X1000)}{1000} = 290 - 400d$$

$$305d = 290 - 400d \rightarrow d = \frac{290}{705} = 0.41m \rightarrow h = 0.41 + 0.05 = 0.46m \rightarrow use\ h = 0.50m.$$

3. Design for flexure:

The ultimate bending moment at face of wall, Mu is given by:

$$M_u = \frac{q_u\, l1^2}{2} = \frac{(400)(0.725)^2}{2} = 105kN.m$$

ACI 318-14: Footing thickness, h= 350mm, d= 300mm.

Steel ratio, ρ is:

$$\rho = \frac{0.85(24)}{420}\left(1 - \sqrt{1 - \frac{2.61(105x10^6)}{(1000)(300)^2(24)}}\right) = 0.0032$$

Steel area (bottom), As= ρbd= 0.0032(1000) (300) = 960mm^2

Minimum area of steel (bottom), As,min= As,shrinkage = 0.0018bh= 0.0018(1000)(350)= 630mm^2 < 960mm^2.

Use As= 960mm^2/m. Use $\emptyset 16/200mm$

Longitudinal bars: As, shrinkage = 630mm^2/m. Use $\emptyset 16/300mm$

ACI 318-19: Footing thickness, h= 500mm, d= 450mm.

Steel ratio, ρ is:

$$\rho = \frac{0.85(24)}{420}\left(1 - \sqrt{1 - \frac{2.61(105x10^6)}{(1000)(450)^2(24)}}\right) = 0.0014$$

Steel area (bottom), As= ρbd= 0.0014(1000) (450) = 630mm^2

Minimum area of steel (bottom), As,min= As,shrinkage = 0.0018bh= 0.0018(1000)(500)= 900mm^2 > 630mm^2.

Use As= 900mm^2/m. Use $\emptyset 16/200mm$

Longitudinal bars: As, shrinkage = 900mm^2/m. Use $\emptyset 16/200mm$

As the footing has large thickness, it is recommended to use top bars not less than half the shrinkage steel.

$$A_s = 0.5(900) = 450mm^2 \qquad use\ 1\emptyset 12/200mm$$

Check development of flexural bars:

The bars are extended to the end of the cantilever (footing), so, the length of bar after the critical section extends a distance of:

L= 0.725 – 0.05 = 0.675m

The development length in tension, Ldt is:

$$L_{dt} \geq \frac{0.48 f_y}{\lambda \sqrt{f'_c}} d_b \geq 300mm$$

$$L_{dt} \geq \frac{0.48(420)}{(1)\sqrt{24}}(16) = 658m \geq 300mm$$

So, 675mm > 658mm, there is no need for standard hook.

As a common practice and for bars fixation, hooks can be used.

$$ACI\ 318-14: L_{\mathrm{dh}} \geq \frac{0.24 f_y}{\lambda \sqrt{f'_c}} d_b = \frac{(0.24)(420)}{(1)\sqrt{24}}(16) = 330mm$$

$$ACI\ 318-19: l_{dh} \geq \frac{0.087 f_y}{\lambda \sqrt{f'_c}} {d_b}^{1.5} = \frac{0.087(420)}{(1)\sqrt{24}}(16)^{1.5} = 477mm$$

$$\geq 8d_b = (8)(16) = 128mm$$

$$\geq 150mm$$

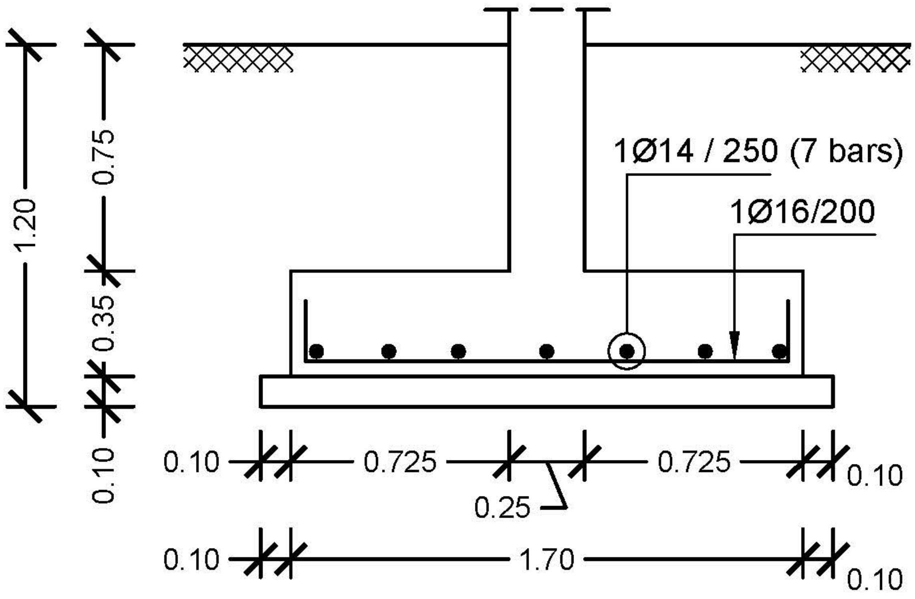


Figure 13.3: Section in wall footing – ACI 318-14

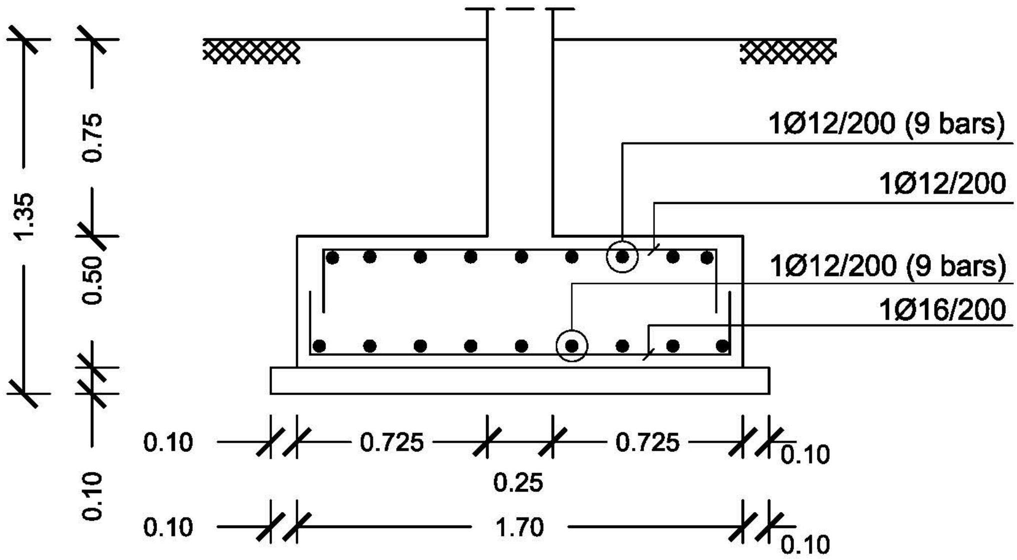


Figure 13.4: Section in wall footing – ACI 318-19

13.2 Design of single footing

Design steps:

1. Determine footing area:

$$A_f = \frac{P_{service}}{q_{all}}$$

Where:

A_f= area of footing= B x L

B= width of footing

L= length of footing

It is preferred to have the distance from the column edge to the four footing edges constant, so the shear and moment have the same values.

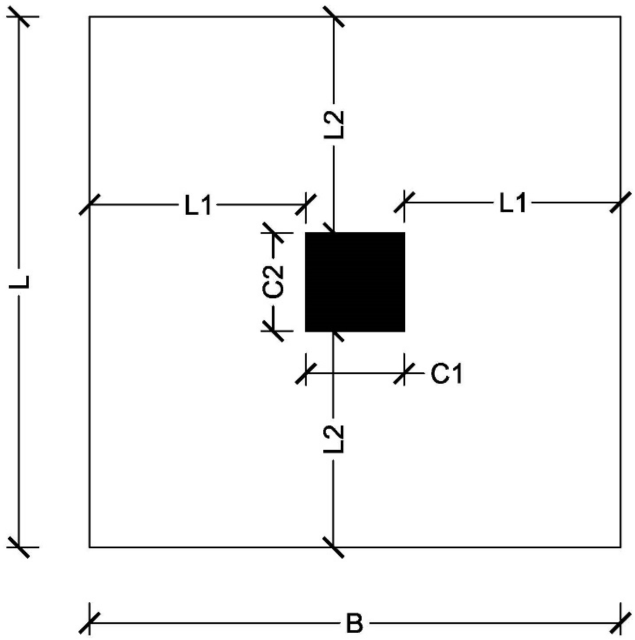


Figure 13.5: Footing layout

2. Determine footing thickness, h:

Footing thickness, h= d + cov

d= footing effective depth, mm

cov= concrete cover to flexural bars centroid = clear concrete cover + half bar diameter, mm

Clear cover= 75mm if the footing is casted directly on soil

Clear cover= 40mm if the footing is casted on plain concrete

d>= 150mm for footing on soil

d>= 300mm for footing on piles

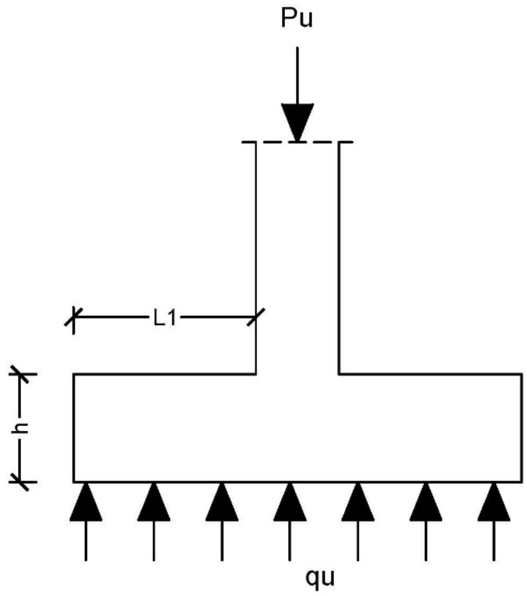


Figure 13.6: Section in footing

Wide beam shear (or one-way shear):

Footing effective depth, d can be determined from wide beam shear strength.

$$V_u \leq \emptyset V_c$$

$$\text{ACI 318} - 14\text{: } \emptyset V_c = \emptyset \frac{1}{6} \lambda \sqrt{f'_c} b_w d$$

$$ACI\ 318 - 19\text{: } \emptyset V_c = \emptyset \left(0.66 \lambda_s \lambda (\rho_w)^{1/3} \sqrt{f'_c} + \frac{N_u}{6A_g} \right) b_w d$$

$$V_u = q_u (L' - d)$$

$$b_w = 1000mm$$

$$q_u = \frac{P_u}{BL}$$

Pu= ultimate axial downward load on the footing in kN/m

L1= the larger distance from face of column to edge of footing

Punching shear (or two-way shear):

$$\emptyset V_c \leq \emptyset 0.33 \lambda_s \lambda \sqrt{f'_c} b_o d$$

$$\emptyset V_c \leq \emptyset 0.17 \lambda_s \lambda \left(1 + \frac{2}{\beta}\right) \sqrt{f'_c} b_o d$$

$$\emptyset V_c \leq \emptyset 0.083 \lambda_s \lambda \left(2 + \frac{\alpha_s d}{b_o}\right) \sqrt{f'_c} b_o d$$

Where:

b_o: The perimeter length of the critical zone

$\beta: Ratio\ of\ \dfrac{long\ side}{short\ side}\ of\ column$

α_s: Factor describes the location of the column

$\alpha_s = 40 \qquad for\ interior\ column$

$\alpha_s = 30 \qquad for\ edge\ column$

$\alpha_s = 20 \qquad for\ corner\ column$

d: effective depth of section

The punching shear force, V_{up}= P_u- q_u A_1

Pu= ultimate load on column, kN

q_u= ultimate pressure at footing, kN/m^2

A1= area inside the critical section, m^2

The critical section is located at minimum of d/2 from face of column and maximum of d. Use d/2 to be conservative.

3. Determine flexural steel:

The bending moments in the two directions are computed based on a cantilever span of L1 and L2.

$$M_{u1} = \frac{q_u \, {L_1}^2}{2}$$

$$M_{u2} = \frac{q_u \, {L_2}^2}{2}$$

Then area of steel is computed and compared with As,min which is As,shrinkage as discussed in design of wall footings.

Note:

In rectangular footings of dimensions L and B, where L is larger than B, determine the portion of As1 of the total steel area As for the short direction to be uniformly distributed over the central band. The central band has a width B.

$$A_{S1} = \frac{2}{\beta + 1} A_S$$

Where: β is the long side divided by the short side of footing

And the remainder of steel As-As1, will be distributed out the central band.

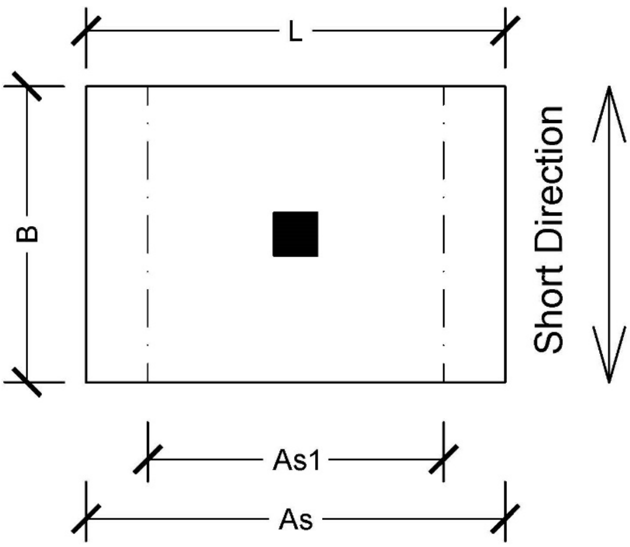


Figure 13.7: Band reinforcement

Check development of flexural bars:

The length of bar from face of wall to the end of footing should be larger than or equal to the bar development length, $L_{dt}\ or\ L_{dh}$.

Example (single footing):

Given:

- Column: 0.50 x 0.50m
- Square footing
- Concrete f'c= 24MPa
- Steel yield strength, fy= 420MPa
- Soil allowable bearing capacity, qall= 350kN/m^2
- Dead load, PD= 1500kN
- Live load, PL= 1000kN

Design the square footing.

Solution:

1. Footing area:

Total service axial compression force, $P_{service}$= P_D+ P_L

$P_{service}$ = 1500 + 1000= 2500kN

$P_{ultimate}$= 1.2 P_D + 1.6 P_L = 1.2(1500) +1.6(1000) =3400kN

A_f = 2500/350 = 7.14 m^2

Footing side length = $\sqrt{7.14} = 2.70m$

B=L= 2.70m

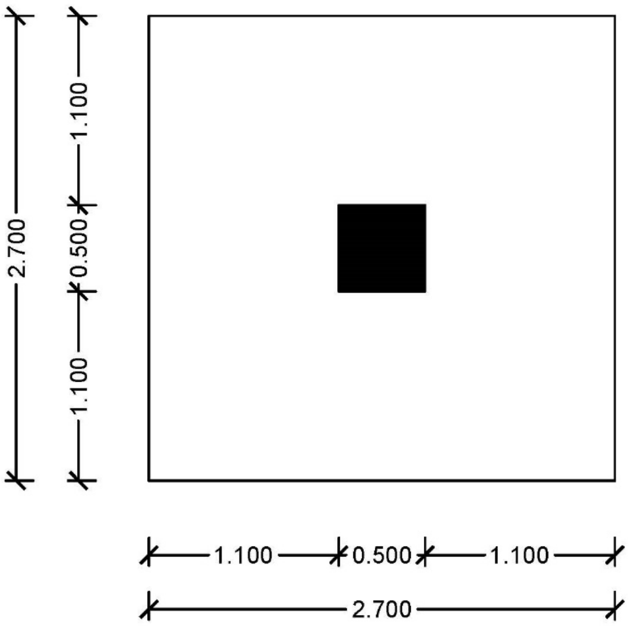


Figure 13.8: Footing plan

2. Footing thickness:

Wide beam shear:

$$q_u = \frac{P_u}{Af} = \frac{3400}{2.70x2.70} = 466.4kN/m^2$$

$L_1 = L_2 = (2.70-0.5)/2 = 1.10m$

$$V_u = q_u(L' - d) = 466.4(1.10 - d)$$

The concrete shear capacity, $\emptyset V_c$ is:

ACI 318-14:

Wide beam shear:

$$\emptyset V_c = \emptyset \frac{1}{6} \lambda \sqrt{f'_c} b_w d = \frac{(0.75)\left(\frac{1}{6}\right)(1)\sqrt{24}(1000)(dx1000)}{1000}$$

From Vu = ΦVc → d=0.48m

Check punching shear:

d= 0.48m (480mm)

b_o= 4(column side + d/2) = 4(500+480) = 3920mm^2

$\beta = 1$

$\alpha_s = 40$

$\gamma = 1$ normal weight concrete

P_u= 3400kN

A_1= (0.5+0.48)^2= 0.9604m^2

Then Vup= 2952kN $\emptyset V_{c,p} = 2281kN \quad N.G$

Try d= 0.58m

So, V_{up}= 2856kN $\emptyset V_{c,p} = 3038kN \quad O.K.$

Footing thickness, h= 0.58+0.06=0.64 m use h=0.65m.

ACI 318-19:

$$V_u = q_u(L' - d) = 466.4(1.10 - d) = 513.04 - 466.4\text{d}$$

$$Let\ \rho_w = 0.0018\left(\frac{h}{d}\right) = 0.0018(1.1) = 0.00198$$

Based on ACI 318-19 section 13.2.6.2, the size factor in footings can be neglected.

$$\frac{0.75\left(0.66(1)(1)(0.00198)^{\frac{1}{3}}\sqrt{24} + 0.0\right)(1000)(d\ X1000)}{1000} = 513.04 - 466.4d$$

$$305d = 513.04 - 466.4d \rightarrow d = \frac{513.04}{771.4} = 0.67m \rightarrow h = 0.67 + 0.06 = 0.73m \rightarrow use\ h = 0.75m.$$

Check punching shear:

d= 750-60=690mm.

b_o= 4(column side + d/2) = 4(500+690) = 4760mm^2

$\beta = 1$

$\alpha_s = 40$

$\gamma = 1$ normal weight concrete

P_u= 3400kN

A_1= $(0.5+0.69)^2$= 1.416m^2

Then Vup= 3400-(466.4)(1.416)=2740kN $\emptyset V_{c,p} = 4023kN \quad ok.$

3. Flexural steel:

$$M_{u1} = \frac{q_u L'^2}{2} = \frac{466.4(1.1)^2}{2} = 282.2kN.m/m$$

ACI 318-14: h= 650mm, d= 580mm

Steel ratio = 0.00227 As= 1317mm^2

$A_{s,min}$= 0.0018(1000)(650)= 1170mm^2 < 1317mm^2

Use As= 1317mm^2 $\emptyset 16mm/150mm$ in each direction bottom bars.

Top bars for shrinkage can be used with As= 1170/2= 585mm^2 $\emptyset 12/150mm\ or\ \emptyset 16/300mm$

ACI 318-19: h= 750mm, d= 690mm

Steel ratio = 0.0016 As= 1104mm^2

$A_{s,min}$= 0.0018(1000)(750)= 1350mm^2 > 1104mm^2

Use As= 1350mm^2 $\emptyset 16mm/150mm$ in each direction bottom bars.

Top bars for shrinkage can be used with As= 1350/2= 675mm^2 $\emptyset 12/150mm\ or\ \emptyset 16/300mm$

Check development of bars:

L= 1.1 – 0.05 = 1.05m

The development length in tension, L_{dt} is:

$$L_{dt} \geq \frac{0.48f_y}{\lambda\sqrt{f'_c}}d_b \geq 300mm$$

$$L_{dt} \geq \frac{0.48(420)}{(1)\sqrt{24}}(16) = 658m \geq 300mm$$

So, 1050mm > 658mm.

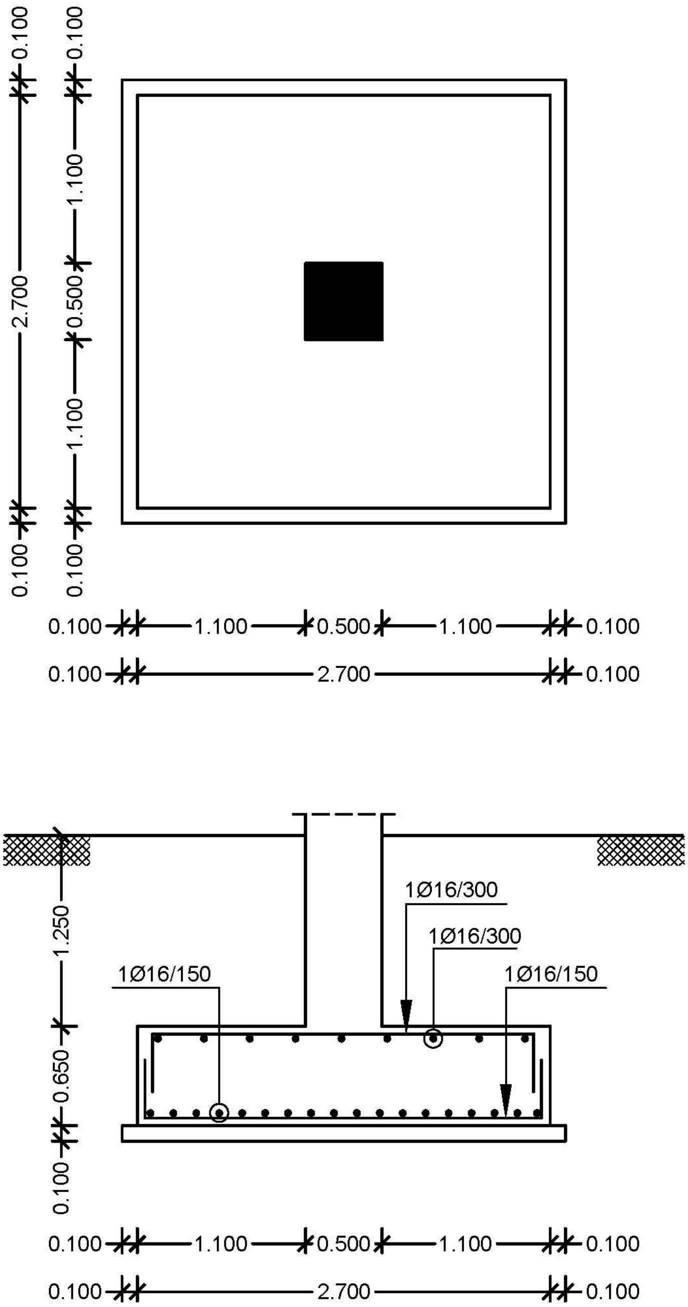


Figure 13.9: Footing details – ACI318-14

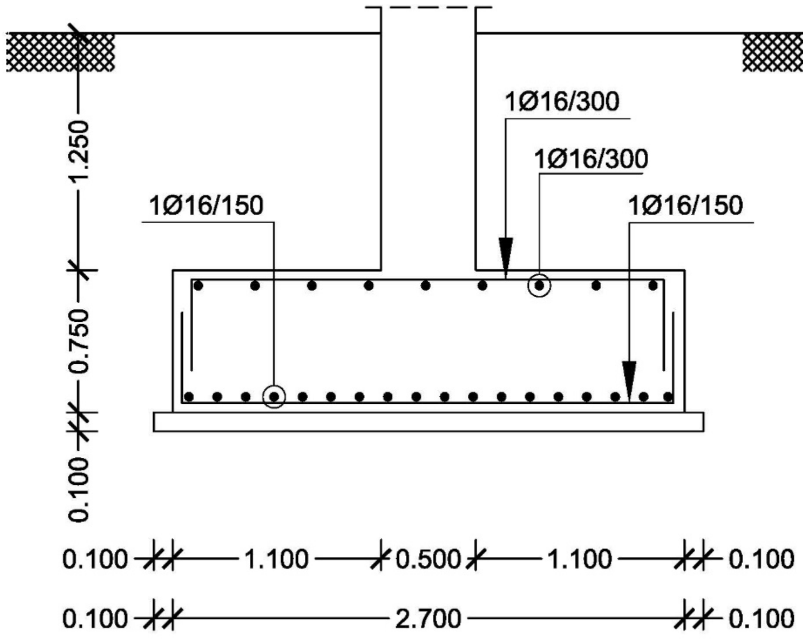


Figure 13.10: Footing details – ACI318-19

13.3 Design of single footing with combined compression and bending moments:

The stress at the footing is determined from the known formula:

$$\sigma = \frac{P}{A} \pm \left(\frac{My}{I}\right)_1 \pm \left(\frac{My}{I}\right)_2$$

Where:

P= compression force on footing

A= area of footing

M= applied moment in a direction

I = moment of inertia in a direction

y= distance from centroidal axis to a point where pressure will be computed

The pressure under the footing is variable.

For simplicity, the maximum pressure can be used for flexural design, wide beam shear and punching shear computations. The punching shear force can be computed by multiplying the maximum stress by the footing area to approximately taking into account shear- moment transfer.

So,

$$V_{up} = q_{u,max} A_f$$

$$M_{u,max} = \frac{q_{u,max}\, \mathrm{l_1}^2}{2}$$

$$V_{u,max} = q_{u,max}(l_1 - d)$$

Af= area of footing

L1= distance from face of column to edge of footing

Note:

The known equation of stress calculation is used when there is no tension at soil when the eccentricity, e, is less than or equals to footing length divided by 6.

But, when tension exists; eccentricity larger than L/6, the minimum stress is zero. And the maximum stress is computed by:

$$q_{max} = \frac{(4/3)P}{B(L - 2e)}$$

Where:

P= axial compression force

L= footing side in direction of moment

B= footing transverse direction

e= eccentricity= M/P

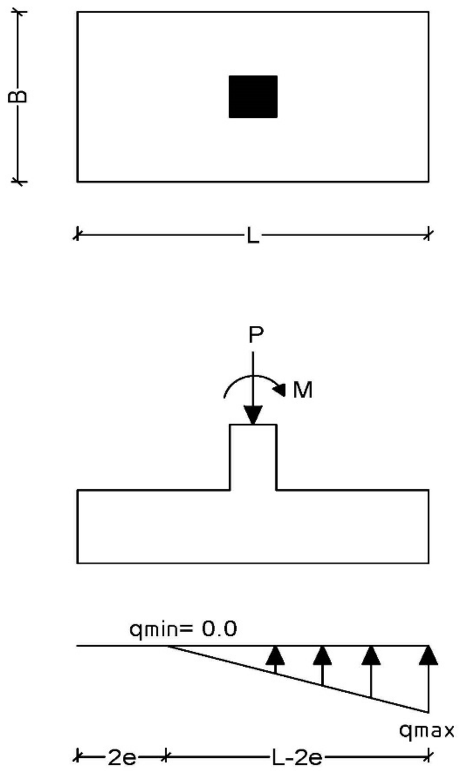
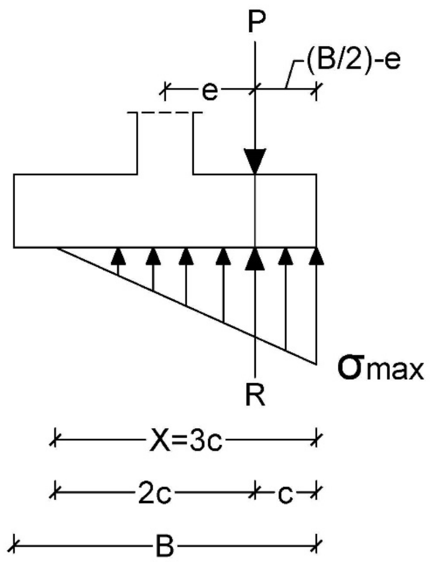


Figure 13.11: Stresses under footing with e >L/6

Derivation:



$$R = P = \sigma_{max}\frac{X}{2}$$

$$X = 3c = 3\left(\frac{B}{2} - e\right)$$

$$\sigma_{max} = \frac{2P}{X} = \frac{2}{3}\frac{P}{\left(\frac{B}{2} - e\right)} = \frac{4P}{3(B - 2e)} = \frac{(4/3)P}{(B - 2e)}$$

Note that when a footing is subjected to an axial compression force and a bending moment on a wall or a column, the footing can be shifted with an eccentricity equals to the bending moment divided by the axial compression force to have a uniform pressure under the footing.

Example: (single footing with combined compression and moment)

Given:

- Column: 0.40 x 0.80m
- Rectangular footing.
- Concrete f'c= 28MPa
- Steel yield strength, fy= 420MPa
- Soil allowable bearing capacity, qall= 400kN/m^2
- Dead load, PD= 1700kN
- Live load, PL= 1300kN
- Moments: MD= 255kN.m ML= 195kN.m in long direction of column
- Assume that weight of footing, backfill and surcharge equal to 10% of total compression load on footing.

Design the required rectangular footing?

Solution:

1. **Footing area:**

Total service load, P= 1.1(1700+1300) = 3300kN

Total service moment, M= 255+ 195= 450kN

Pu= 1.1(1.2x1700+1.6x1300) = 4532kN

Mu= 618kN

If the total service moment, M=0.0, then, the area of footing will be:

$$Area\ of\ footing, A_f = \frac{P_{service}}{q_{aa}} = \frac{3300}{400} = 8.25m^2$$

$$Footing\ side, L = B = \sqrt{8.25} = 2.90m$$

So, the footing will be larger than 2.90m x 2.90m.

Assume length of footing = 3.5m in direction of column long side, so:

$$\sigma = \frac{P}{A} \pm \left(\frac{My}{I}\right) = 400 = \frac{3300}{3.5B} + \frac{450(\frac{3.5}{2})}{\frac{1}{12}3.5^3B}$$

B= 3.0m

So, use footing 3m x 3.5m

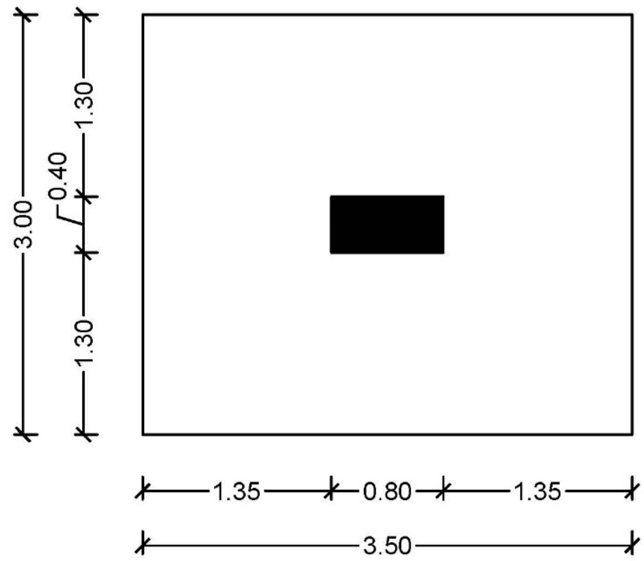


Figure 13.12: Footing layout

The stresses at the footing should be checked to check the existence of tension stress.

So,

Minimum stress= 240.82kN/m^2 compression

Maximum stress= 387.76kN/m^2 compression

Or the eccentricity, e can be computed as follows:

$$e = \frac{M}{P} = \frac{450}{3300} = 0.136m < \frac{L}{6} = \frac{3.50}{6} = 0.583m, so\ no\ tension\ exists\ at\ footing.$$

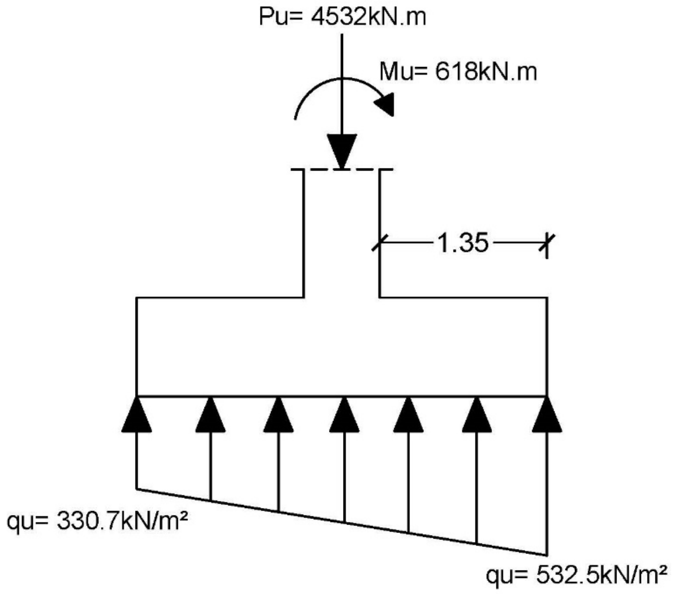


Figure 13.13: ultimate stresses under the footing

2. Footing thickness:

Apply the equation of stress computation using ultimate loads:

$$\sigma = \frac{P_u}{A} \pm \frac{Mu\ y}{I}$$

Maximum ultimate stress, $q_{u,max}$= 532.5kN/m^2

Minimum ultimate stress, $q_{u,min}$= 330.7kN/m^2

Assume d= 0.62m and h= 0.70m

The length of cantilevers in the two directions are:

$$l_1 = \frac{3.5 - 0.8}{2} = 1.35m$$

$$l_2 = \frac{3.0 - 0.4}{2} = 1.30m$$

So, the maximum cantilever distance is L1= 1.35m.

Ultimate shear at distance d from face of column, Vu:

$$V_u = q_u(l1 - d) = 532.5(1.35 - 0.62) = 388.7kN$$

ACI 318-14: Concrete shear capacity, $\emptyset Vc$:

$$\emptyset V_c = \frac{0.75\left(\frac{1}{6}\right)\sqrt{28}(1000)(620)}{1000} = 410kN > 388.7kN$$

Check punching shear:

$$V_{u,p} = (3)(3.5)(532.5) = 5591.25kN$$

$$\emptyset Vc,p = 3962\ kN < 5591.25kN \quad N.G$$

Assume h= 0.90m and d= 0.82m:

$$\emptyset Vc,p = 6161kN > 5591.25kN \quad O.K$$

ACI 318-19: Concrete shear capacity, $\emptyset Vc$:

$$\frac{0.75\left(0.66(1)(1)(0.00198)^{\frac{1}{3}}\sqrt{28} + 0.0\right)(1000)(820\ X1000)}{1000} = 270kN$$

$$V_u = q_u(l1 - d) = 532.5(1.35 - 0.82) = 282.2kN > 270kN \quad N.G$$

Increase footing thickness, h= 950mm, d= 870mm, so:

$$\phi V_c = 286.5kN$$

$$V_u = 255.6kN < 286.5kN \quad ok.$$

3. Design for flexure:

h=950mm, d=870mm.

The ultimate moment, M_u= 532.2(1.35)^2/2= 485. 2kN.m

This moment is the maximum in the two directions of footing.

Steel ratio, $\rho = 0.00172$ $\quad A_s = 1496mm^2$

This moment requires minimum area of steel:

$$As, min = 0.0018(1000)(950) = 1710mm^2$$

Use $1\emptyset 20/150mm$ bottom bars in each direction.

Use $1\emptyset 14/150mm$ top bars shrinkage in each direction or use $1\emptyset 20/300mm$ bars.

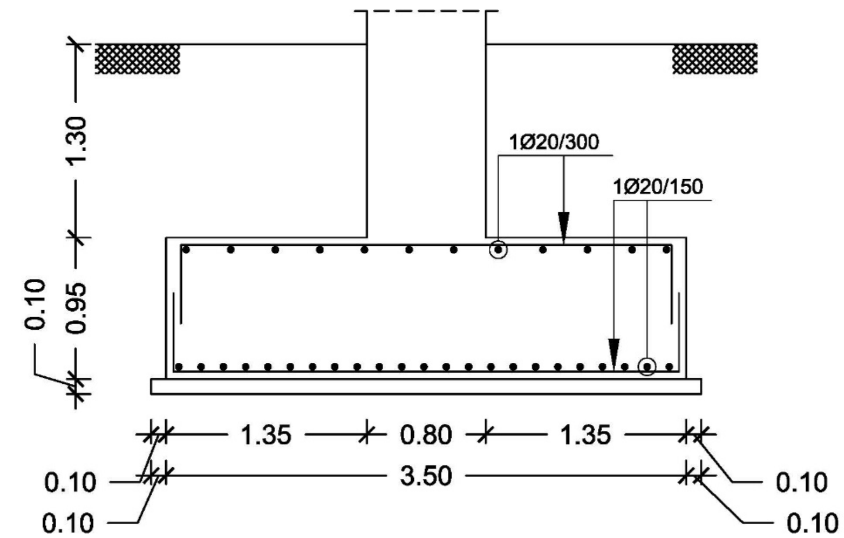


Figure 13.14: Footing reinforcement

13.4 Design of combined footing:

- Combined footing is a footing that is used to support more than one column; usually two.
- In general, the combined footing has a uniform width or it has a trapezoidal area.
- It is recommended to have the centroid of forces coincides with the centroid of footing area especially when considering the footing rigid.
- Combined footing is recommended to be used to have a uniform pressure under the footings when one column is located at edge of a single footing, so, this column is connected with another column in a combined footing with uniform pressure.
- Also, combined footing is used to support two columns when the areas of single footings for these columns are large and the clear distance between these footings is small.

Steps:

1. Determine footing area and its dimensions: it is recommended to have the centre of area to coincide with the centroid of loads to have uniform pressure as the footing is considered rigid.

2. Determine footing thickness based on wide beam shear and punching shear. Shear reinforcement can be used. The combined footing that supports two columns can be modeled as a beam element. The column loads are downward point loads and the line soil pressure is the uniformly distributed line load. The downward point loads are equal to the upward line load multiplied by the beam element length. The shear force diagram can be constructed to this model.

Also, the footing thickness must be determined or checked based on punching shear or moment- transfer strength.

3. Determine flexural reinforcement in longitudinal direction. Analyze and design the footing as a beam element. The bending moment diagram can be constructed to the beam (footing) structural model.

4. Determine flexural reinforcement in the transverse direction. It is considered that there is a strip in the transverse direction under the column of width equals column side plus d/2 at each side of column, so strip width equals: c2 + d/2 for exterior column and c2 + d for interior column where c2 is the transverse dimension of column.

Example (combined footing):

<u>Given:</u>

- Refer to **Figure 13.15** below.
- Concrete strength, f'c= 21MPa
- Steel strength, fy= 420MPa
- Soil allowable bearing capacity, qall= 180kN/m^2
- Column C1: 0.35 m x 0.35m P_{D1}= 580kN P_{L1}= 312kN
- Column C2: 0.40m x 0.40m P_{D2}= 670kN PL2= 423kN
- Distance between the two columns, L= 4.85m

Design a combined footing to support the two columns.

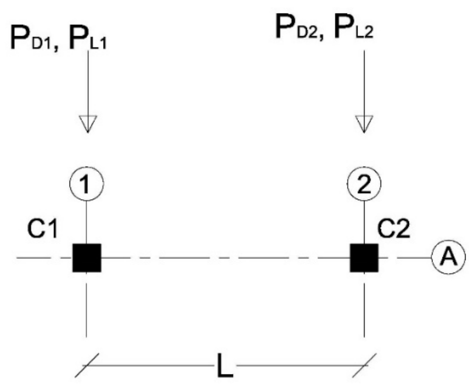


Figure 13.15: Columns layout- combined footing

Solution:

1. Footing area:

The centroid of the footing area shall coincide with the centre of the loads.

To locate the center of the loads, the moments of loads about point A at the centroid of left column is calculated as follows: refer to **Figure 13.16**:

$(P_{D1}+P_{L1})\ (0.0) + (P_{D2}+P_{L2})\ (L) = R\ X$ x=2.67m.

Where:

R= resultant of vertical loads

X= distance from the resultant force R to the centroid of the left column, C1

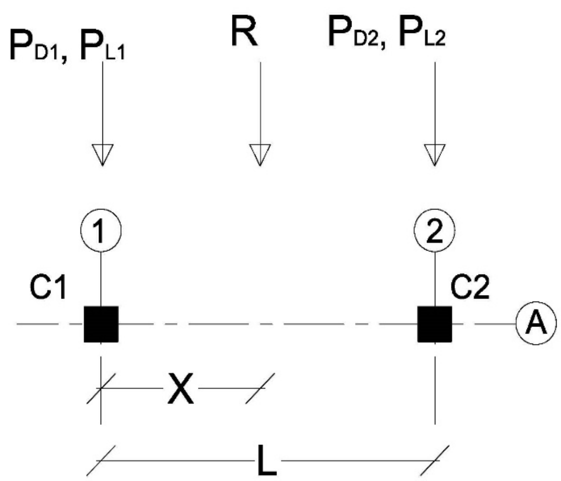


Figure 13.16: Resultant force

Distance from the resultant force to the edge of column C1, L1= 0.35/2 + 2.67 = 2.845m

Length of footing, L= 2(L1) = 5.70m

Total force= resultant= R= P_{D1}+ P_{L1}+ P_{D2}+ P_{L2}= 1985kN

Area of footing, Af= 1985/180= 11.03m^2

Width of footing, B= 11.03/5.70= 1.93 m (2.00m)

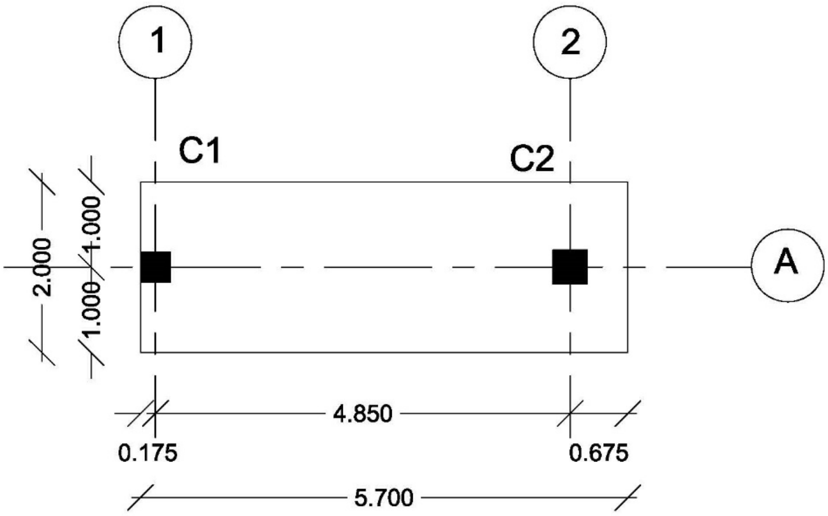


Figure 13.17: Footing layout

2. Footing thickness

Average load factor =

$$\frac{\sum ultimate\ loads}{\sum service\ loads} = \frac{1.2(580+670)+1.6(312+423)}{(580+670+312+423)} = 1.348$$

P_{u1}= 1.348(580+312) = 1202.4kN

P_{u2}= 1.348(670+423) = 1473.4kN

Ultimate load at footing = (P_{u1}+ P_{u2})/5.7= 469.4kN/m

Which equals to 234.7kN/m^2

Figure 13.18 shows the structural model, shear force diagram and bending moment diagram for the combined footing.

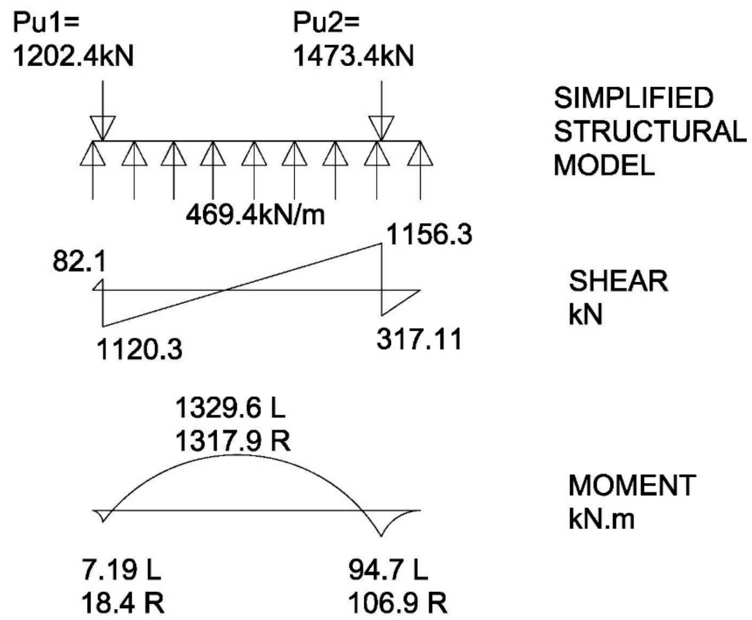


Figure 13.18: Footing shear and moment diagrams

The maximum shear force at support, Vu= 1156.3kN

The maximum shear force at distance d from face of column, Vu1= 1156.3-469.4(0.4/2+d)

ACI 318-14:

$$\emptyset V_c = \frac{0.75\left(\frac{1}{6}\right)\sqrt{21}(2000)(dX1000)}{1000}$$

$$Vu1 = \emptyset Vc \rightarrow d = 0.66m$$

ACI 318-19:

$$\frac{0.75\left(0.66(1)(1)(0.00198)^{\frac{1}{3}}\sqrt{21} + 0.0\right)(2000)(d\,X1000)}{1000} = 1156.3 - 469.4(0.4/2 + d)$$

$$569.8d = 1156.3 - 93.88 - 469.4d \rightarrow 569.8d = 1062.42 - 469.4d \rightarrow d = \frac{1062.42}{1039.2} = 1.02m$$

$$If\ \rho_w = 0.00333 \rightarrow 676.7d = 1062.42 - 469.4d \rightarrow d = \frac{1062.42}{1146.1} = 0.93m$$

ACI 318-14: Check punching shear – column C1: d=660mm:

The critical section is located at distance d/2 from face of column.

Vu,p= 1202.4-234.7{(0.35+0.66)(0.35+0.66/2)}= 1041.2kN

$$\emptyset V_{c,p} = \frac{0.75(0.33)\sqrt{21}\left(350 + 660 + 2\left(350 + \frac{660}{2}\right)\right)(660)}{1000} = 1774kN > 1041.2kN \quad ok$$

ACI 318-14: Check punching shear – column c2: d=660mm:

The critical section is located at distance d/2 from face of column.

Vu,p= 1473.4-234.7{0.4+0.66}2= 1209.7kN

$$\emptyset V_{c,p} = \frac{0.75(0.33)\sqrt{21}(400 + 660)(4)(660)}{1000} = 3174kN > 1209.7kN \quad ok$$

ACI 318-19: Check punching shear – column C1: d=1020mm:

The critical section is located at distance d/2 from face of column.

Vu,p= 1202.4-234.7{(0.35+1.02)(0.35+1.02/2)}= 926kN

$$\emptyset V_{c,p} = \frac{0.75(0.33)\sqrt{21}\left(350 + 1020 + 2\left(350 + \frac{1020}{2}\right)\right)(1020)}{1000} = 3607kN > 926kN \quad ok$$

ACI 318-19: Check punching shear – column C2: d=1020mm:

The critical section is located at distance d/2 from face of column.

Vu,p= 1473.4-234.7{0.4+1.02}2= 1000kN

$$\emptyset V_{c,p} = \frac{0.75(0.33)\sqrt{21}(400 + 1020)(4)(1020)}{1000} = 6571kN > 1000kN \quad ok$$

If it is suggested to use $\left(\frac{A_v}{s}\right)_{min}$, then the thickness will be reduced as follows:

$$\left(\frac{A_v}{s}\right)_{min} = max\,of \begin{bmatrix} 0.062\sqrt{f'_c}\,\frac{b_w}{f_{yt}} \\ 0.35\,\frac{b_w}{f_{yt}} \end{bmatrix} = max[1.35,1.67] = 1.67mm^2/mm$$

Spacing of stirrups, s=d/2 in the long direction and it is d across the section.

The maximum shear force at distance d from face of column, Vu1= 1156.3-469.4(0.4/2+d)

$$\emptyset V_c = \frac{0.75\left(\frac{1}{6}\right)\sqrt{21}(2000)(dX1000)}{1000}$$

$$Vu1 = \emptyset Vc \rightarrow d = 0.66m$$

Try h=600mm, d= 520mm.

Check punching shear – column C1: d=520mm:

The critical section is located at distance d/2 from face of column.

Vu,p= 1202.4-234.7{(0.35+0.52)(0.35+0.52/2)}= 1078kN

$$\emptyset V_{c,p} = \frac{0.75(0.33)\sqrt{21}\left(350 + 520 + 2\left(350 + \frac{520}{2}\right)\right)(520)}{1000} = 1233kN > 1078kN \quad ok$$

Check punching shear – column C2: d=520mm:

The critical section is located at distance d/2 from face of column.

Vu,p= 1473.4-234.7$\{0.4+0.52\}^2$= 1275kN

$$\emptyset V_{c,p} = \frac{0.75(0.33)\sqrt{21}(400 + 520)(4)(520)}{1000} = 2170kN > 1275kN \quad ok$$

$$V_c = \frac{\left(\frac{1}{6}\right)\sqrt{21}(2000)(520X1000)}{1000} = 794.3kN$$

$$V_u = 1156.3 - 469.4\left(\frac{0.4}{2} + 0.52\right) = 818.3kN > V_c$$

$$V_s = \frac{V_u}{\emptyset} - V_c = \frac{818.3}{0.75} - 794.3 = 296.8kN < \frac{\frac{1}{3}\sqrt{21}(2000)(420)}{1000} = 1282kN$$

$$So, S_{max} = min\left(\frac{d}{2}, 600mm\right) = 210mm$$

$$\frac{A_v}{s} = \frac{V_s}{f_{yt}d} = \frac{296800}{(420)(420)} = \frac{1.68mm^2}{mm} \approx \left(\frac{A_v}{s}\right)_{min} \qquad ok.$$

$$d = 520mm, \frac{d}{2} = 260mm$$

Section width, b=2000mm. Section effective width=2000-150=1850mm.

Number of =1850/520=3.6 4 spacings; 5 legs.

So,

$$For\ \emptyset12\ closed\ stirrups: A_v = 113(5) = 565mm^2 > 1.68mm^2/mm \qquad \frac{A_v}{s} = \frac{565}{250} = 2.825mm^2$$

$$For\ \emptyset10\ closed\ stirrups: A_v = 78.5(5) = 392.5mm^2 < 1.68mm^2/mm \qquad \frac{A_v}{s} = \frac{392.5}{250} = 1.57mm^2$$

$$Use\ \emptyset\frac{12}{250mm} closed\ stirrups\ 5legs.$$

3. Flexural reinforcement in longitudinal direction:

ACI 318-14: d=660mm, h=750mm

Section width, b= 2000mm

Section thickness, h= 750mm

Section effective depth, d= 660mm

For Mu= 1329.6 KN.m:

Steel ratio, ρ= 0.00424 As= 0.00424(2000) (660) = 5600mm^2. Use 12Ø25

For the other moment values, use minimum steel area.

$$A_{s,min} = A_{s,shrinkage} = 0.0018(2000)(750) = 2700mm^2 \ \ (12\emptyset18)$$

ACI 318-19: d=520mm, h=600mm

Section width, b= 2000mm

Section thickness, h= 600mm

Section effective depth, d= 520mm

For Mu= 1329.6 KN.m:

Steel ratio, ρ= 0.0071 As= 0.0071(2000) (520) = 7384mm^2. Use 15Ø25

For the other moment values, use minimum steel area.

Shrinkage steel, $A_{s,shrinkage} = 0.0018(2000)(600) = 2160mm^2$ (12Ø16)

4. Flexural reinforcement in transverse direction:

$\boldsymbol{d = 660mm}$:

Left column, C1:

$$b_{effective} = column\ width + d/2 = 0.35 + (0.66/2) = 0.68m$$

$$q_u = \frac{1202.4}{0.68x2} = 884kN/m^2$$

$$l_1 = \frac{2 - 0.35}{2} = 0.825$$

$$M_u = 884(0.825)^2/2 = 301kN.m/m$$

For strip of 1000mm width, steel ratio, ρ = 0.00186, As= 0.00186(1000) (660) =1227.6mm^2/m

For width of 680mm, As= 0.68(1227.6) = 835mm^2

$$A_{s,min} = 0.0018(680)(750) = 918mm^2 \qquad 3Ø20 \qquad \textbf{controls}$$

Right column, C2:

$$b_{effective} = column\ width + d = 0.40 + 0.66 = 1.06m$$

$$q_u = \frac{1473.4}{1.06x2} = 695kN/m^2$$

$$M_u = 695\frac{(2-0.40)^2}{2} = 222.4kN.m/m$$

For strip of 1000mm width, steel ratio, $\rho = 0.00137$, As= 0.00137(1000) (660) =904mm^2/m

For width of 1060mm, As= 1.06(904) = 958mm^2

$$A_{s,min} = 0.0018(1060)(750) = 1431mm^2 \quad 5\emptyset 20 \quad \boldsymbol{controls}$$

The steel area in the transverse direction top and bottom in other zones will be half the shrinkage steel.

$$A_s = 0.0018(1000)(750)(0.5) = 675mm^2 \quad 1\emptyset 16/300$$

Figure 13.18 shows the reinforcement details for the combined footing.

$\boldsymbol{d = 520mm}$:

Left column, C1:

$$b_{effective} = column\,width + d/2 = 0.35 + (0.52/2) = 0.61m$$

$$q_u = \frac{1202.4}{0.61x2} = 986kN/m^2$$

$$l_1 = \frac{2-0.35}{2} = 0.825$$

$$M_u = 986(0.825)^2/2 = 335.5kN.\frac{m}{m}$$

For strip of 1000mm width, steel ratio, $\rho = 0.00342$, As= 0.00342(1000) (520) =1778mm^2/m

For width of 610mm, As= 0.61(1778) = 1085mm^2

$A_{s,min} = 0.0018(610)(600) = 659mm^2 < 1085mm^2 \quad 4\emptyset 20$

Right column, C2:

$$b_{effective} = column\,width + d = 0.40 + 0.61 = 1.01m$$

$$q_u = \frac{1473.4}{1.01x2} = 729kN/m^2$$

$$l_1 = \frac{2-0.4}{2} = 0.8$$

$$M_u = \frac{729(0.8)^2}{2} = 233.3kN.\frac{m}{m}$$

For strip of 1000mm width, steel ratio, $\rho = 0.00234$, As= 0.00234(1000) (520) =1217mm^2/m

For width of 1010mm, As= 1.01(1217) = 1219mm^2

$$A_{s,min} = 0.0018(1010)(600) = 1091mm^2$$

$use\ \ 4\emptyset 20$

The steel area in the transverse direction top and bottom in other zones will be half the shrinkage steel.

$$A_s = 0.0018(1000)(600)(0.5) = 540mm^2 \qquad 1\emptyset 12/200$$

$The\ used\ stirrups\ are\ \emptyset 12/250$ which are less than Ø12/200mm, so use Ø12/200mm to serve as shear reinforcement and shrinkage steel.

Figure 13.19 shows the reinforcement details for the combined footing using h=750mm. And Figure 13.20 shows the reinforcement details using h=600mm.

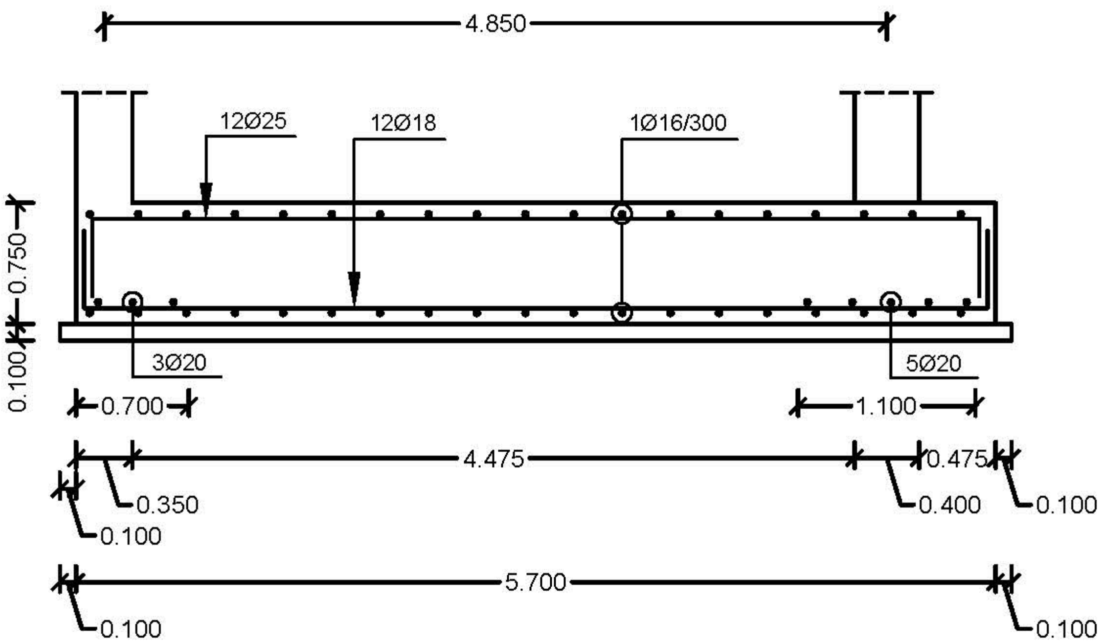


Figure 13.19: Combined footing details, h=750mm

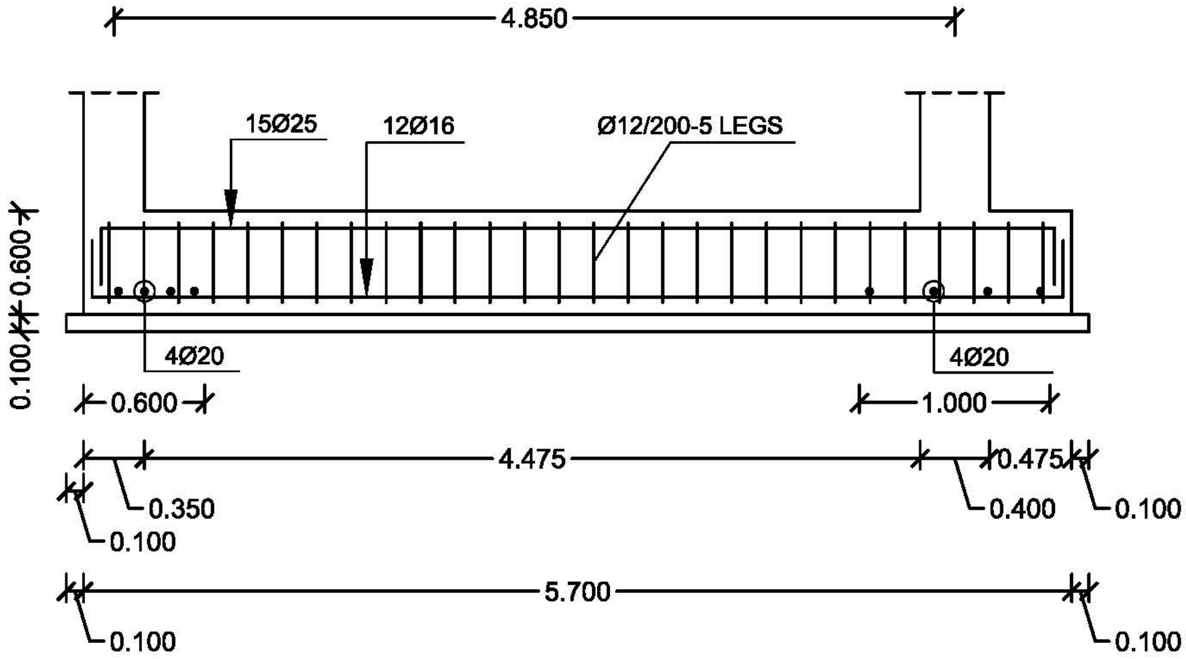


Figure 13.20: Combined footing details, h=600mm

13.5 Design of cantilever or strap footing:

The strap footing is a footing that combines two columns in one footing. It is composed of two single footings for the two columns and a connecting beam between them. One of the single footings is eccentric. The connecting beam (strap beam) is not supported on soil. The major purpose of using this type of footing is to have uniform pressure under the footing system. So, the two single footings and the connecting beam form one structure.

Example (strap footing):

Given:

- Refer to **Figure 13.21**.
- Left column, C1: exterior: 0.30m x0.30m. PD1= 320kN, PL1= 250kN
- Right column, C2: interior: 0.35m x 0.35m. PD2= 600kN, PL2= 360kN
- Soil allowable bearing capacity, qall= 160Kn/M2
- Concrete strength, f'c= 21MPa
- Steel strength, fy= 420MPa
- Distance between the centerlines of the two columns, L= 5.50m

Design a strap footing to carry the columns C1 and C2.

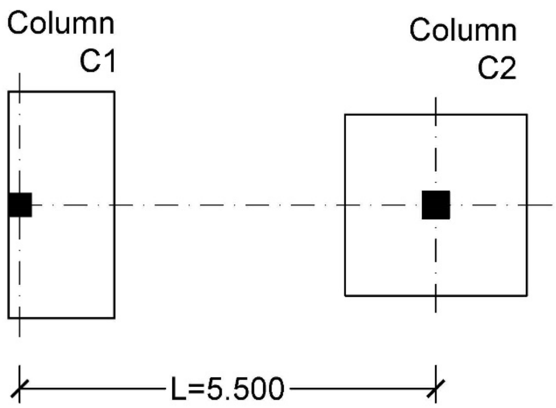


Figure 13.21: Strap footing

Solution:

1. Determine area of the two footings:

Service load on left column, C1, P1= 320 + 250 = 570kN

Service load on right column, C2, P2= 600 + 360 = 960kN

Total service loads, P= P1 + P2 = 1530kN

Assume that width of left footing, F1, b = 1.40m.

Take summation of moments about a point located at center of column C2:

$$\sum M_A = 0 \qquad P_1(5.5) = R_1(4.95) \qquad so, \qquad R_1 = 633.3kN$$

$$R_1 + R_2 = P \qquad so, \qquad R_2 = P - R_1 = 1530 - 633.3 = 896.7kN$$

Area of footing F1 is:

$$A_1 = \frac{633.3}{160} = 3.96m^2$$

Width of footing, B= 1.40m

Length of footing = 3.96/1.40 = 2.83m (use 3.00m)

Area of footing F2 is:

$$A_2 = \frac{896.7}{160} = 5.60m^2$$

$$B = L = \sqrt{5.60} = 2.40m$$

2. Ultimate loads and structural model:

$$Average\ load\ factor, F = \frac{summation\ of\ ultimate\ loads}{summation\ of\ service\ loads}$$

$$Average\ load\ factor, F = \frac{1.2(320 + 600) + 1.6(250 + 360)}{(320 + 600 + 250 + 360)} = 1.36$$

So,

Ultimate load on column C1, Pu1 = 1.36(320+250) = 775.2kN

Ultimate load on column C2, Pu2= 1.36(600+360) = 1305.6kN

Ultimate load (reaction) on footing F1, Ru1= 1.36(633.3) = 861.3kN

Ultimate load (reaction) on footing F2, Ru2= 1.36(896.7) = 1219.5kN

$$Ultimate\ pressure\ at\ footing\ F1, q_{u1} = \frac{861.3}{1.4x3} = 205.1kN/m^2$$

$$Ultimate\ pressure\ at\ footing\ F2, q_{u2} = \frac{1219.5}{2.4x2.4} = 211.7kN/m^2$$

Note that, the pressures q_{u1} and q_{u2} must be equal. Here, these two numbers are not equal due to numerical approximations.

$$ultimate\ linear\ load\ at\ footing\ F1, q_{u1'} = \frac{861.3}{1.4} = 615.2kN/m$$

$$ultimate\ linear\ load\ at\ footing\ F2, q_{u2'} = \frac{1219.5}{2.4} = 508.1kN/m$$

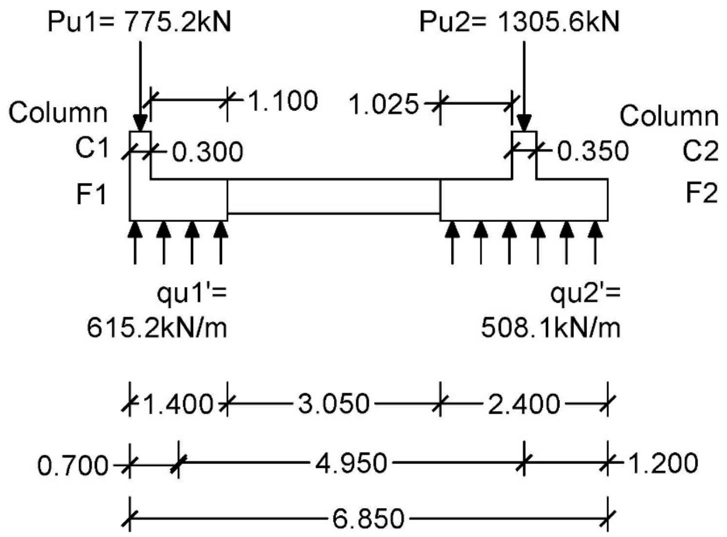


Figure 13.22: Footing model and loads

3. Design of strap beam:

Assume that the strap beam has a width, b= 600mm and thickness, h= 500mm. effective depth, d= 430mm.

Length of strap beam= 3.05m.

$$Left\ shear, V_{uL} = 615.2(1.4) - 775.2 = 86.1kN$$

$$Right\ shear, V_{uR} = 86.1kN$$

$$Left\ moment, M_{uL} = 775.2(1.4 - 0.15) - 615.2(1.4)^2(0.5) = 366.1kN.m$$

$$Right\ moment, M_{uR} = 775.2(3.05 + 1.4 - 0.15) - 615.2(1.4)(3.05 + 0.7) = 103.6kN.m$$

$$Shear\ strength, V_c = 0.75(1/6)\sqrt{21}(600)(430)/1000 = 197kN$$

$$\frac{V_u}{\emptyset} = \frac{86.1}{0.75} = 114.8kN$$

$$\frac{V_u}{\emptyset} > \frac{V_c}{2}\ use\ \left(\frac{A_v}{S}\right)_{min} = \frac{0.35(600)}{420} = 0.5mm^2/mm$$

$$For\ \emptyset 10mm\ stirrups, s = \frac{157}{0.5} = 314mm \qquad use\ stirrups\ at\ d/2\ =\ 200mm$$

For left moment, Mu = 366.1KN.m:

$$\rho = 0.00986, A_s = 0.00986(600)(430) = 2544mm^2 \quad 8\emptyset 20$$

For right moment, Mu = 103.6KN.m:

$$\rho = 0.0025,\ use\ \rho_{min} = 0.00333, A_s = 0.00333(600)(430) = 859mm^2 \quad 5\emptyset 16$$

4. Design of left footing, F1:

Let the footing thickness is 0.60m which is 0.10m larger than the thickness of the strap beam. Note that this beam should not be supported on soil. h= 0.60m. d= 0.53m.

Check punching:

$$q_u = 205.1kN/m^2$$

$$V_{u,p} = 205.1(1.4x3 - 0.565x0.830 = 765.2kN$$

$$\emptyset V_c = 0.75(0.33)\sqrt{f_c{}'}b_o d = 0.75(0.33)\sqrt{21}(565x2 + 830)(530)/1000 = 1178kN > 765.2kN \quad OK$$

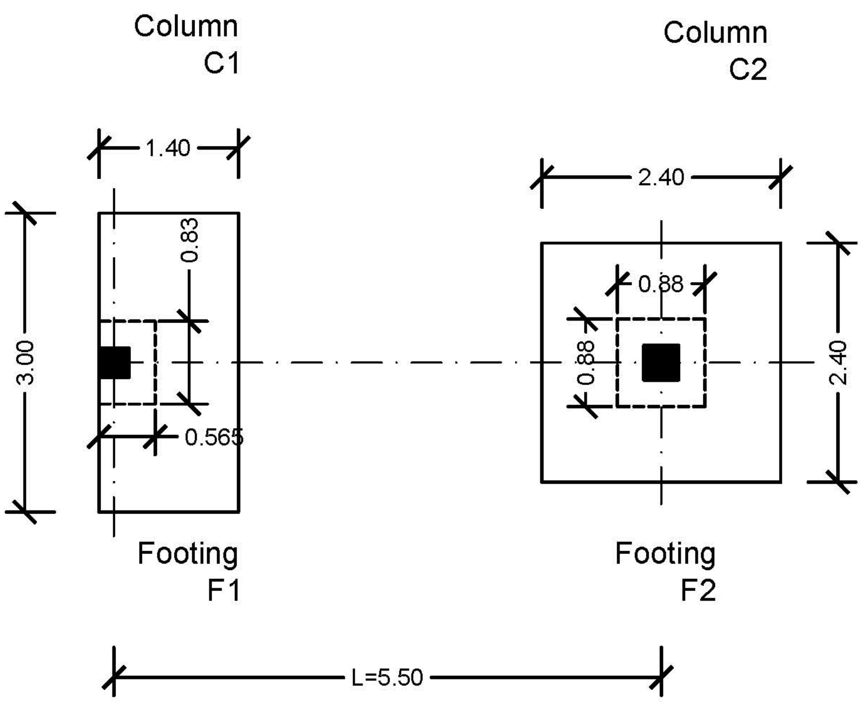


Figure 13.23: Footings dimensions

Check wide beam shear:

In longitudinal direction:

$$V_u = 615.2(0.3 + 0.53) - 775.2 = -264.6kN$$

And

$$V_u = \frac{264.6}{3} = 88.2kN/m$$

In transverse direction:

$$V_u = 205.1(\frac{3 - 0.3}{2} - 0.53) = 168.2kN/m$$

$$\emptyset V_c = 0.75(1/6)\sqrt{21}(1000)(530)/1000 = 303.6kN\ >\ 168.2kN$$

Design for flexure:

In longitudinal direction: at face of column:

$$M_u = 615.2(0.3)^2/2 - 775.2(0.15) = -88.6kN.m \quad Tension\ at\ top$$

Or:

$$M_u = 88.6/3 = 30kN.m/m$$

In transverse direction:

$$M_u = 205.1\left(\frac{3 - 0.3}{2}\right)^2/2 = 187kN.m/m$$

Mu in the longitudinal direction at the end of the footing is:

$$M_u = 615.2(1.4)^2/2 - 775.2(1.4 - 0.15) = -366.1kN.m \quad Tension\ at\ top\ face.$$

Or:

$$M_u = 366.1/3 = 122kN.m/m$$

$$A_{s,min} = 0.0018(1000)(600) = 1080mm^2 \qquad 1\emptyset 16/180mm$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1080(420)}{0.85(21)(1000)} = 25.4mm$$

$$\emptyset M_n = \emptyset A_s f_y(d - \frac{a}{2}) = 0.9(1080)(420)(530 - \frac{25.4}{2})/10^6 = 211kN.m$$
$$> M_{u,max}\ in\ each\ direction$$

Use As,min top and bottom in the longitudinal direction and bottom in the transverse direction.

5. Design of right footing, F2:

Let the thickness of the footing is 0.60m as for the left footing, F1.

Check punching:

$$q_u = 211.7kN/m^2$$

$$V_{up} = 211.7(2.4^2 - 0.88^2) = 1055.5kN$$

$$\emptyset V_{cp} = 0.75(0.33)\sqrt{21}(880x4)(530)/1000 = 2116kN$$

Check wide beam shear:

In longitudinal direction:

$$V_{u,R} = 508.1\left(\frac{2.4 - 0.35}{2} - 0.53\right) = 251.5kN$$

Or:

$$V_{u,R} = \frac{251.5}{2.4} = 104.8kN/m$$

$$V_{u,L} = 508.1(1.025 + 0.35 + 0.53) - 1305.6 = 338kN$$

Or:

$$V_{u,L} = \frac{338}{2.4} = 161.7kN/m$$

In transverse direction:

$$V_u = 211.7\left(\frac{2.4 - 0.35}{2} - 0.53\right) = 104.8kN$$

$$\emptyset V_c = 303.6kN > 161.7kN \quad OK$$

Design for flexure:

In longitudinal direction:

$$M_{u,R} = 508.1(1.025)^2/2 = 267kN.m$$

Or:

$$M_{u,R} = \frac{267}{2.4} = 111.25kN/m$$

$$M_{u,L} = 508.1(1.025+0.35)^2 - 1305.6\left(\frac{0.35}{2}\right) = 251.8kN.m$$

Or:

$$M_{u,R} = \frac{251.8}{2.4} = 104.9kN/m$$

In transverse direction:

$$M_u = 211.7\left(\frac{2.4-0.35}{2}\right)^2/2 = 111.2kN.m/m$$

$$\emptyset M_{n,As,min} = 211kN.m/m > 111.2kN.m \quad OK$$

ACI 318-19:

$$\emptyset V_c = \frac{0.75\left(0.66(1)(1)(0.00198)^{\frac{1}{3}}\sqrt{21}+0.0\right)(1000)(530)}{1000} = 151.3kN < V_{u,max} = 168.2kN$$

So, the footings shall be increased by a small amount like 100mm.

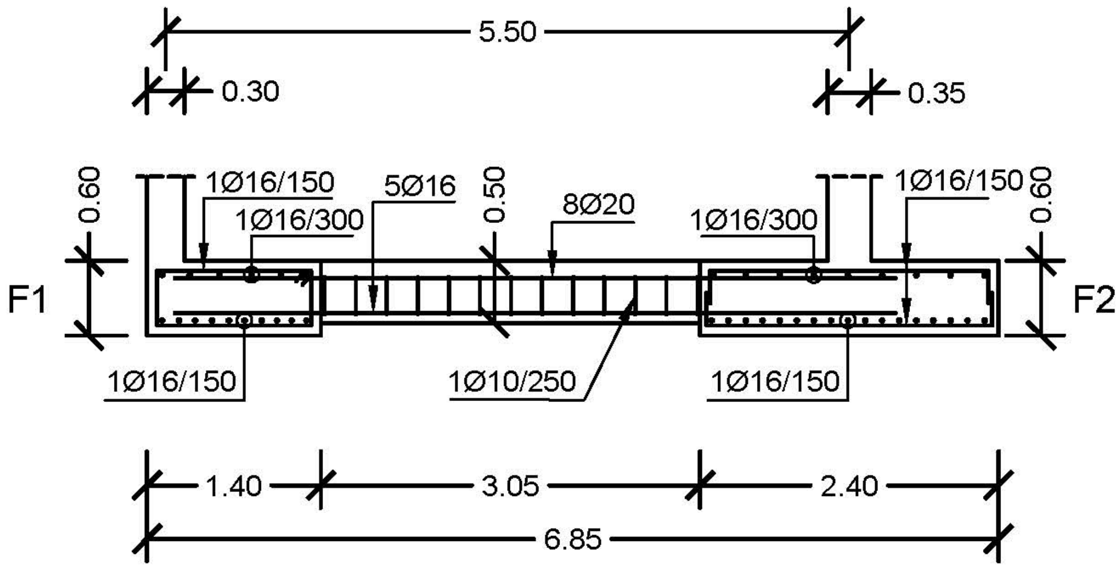


Figure 13.24: Strap footing reinforcement

Cross sections in the two footing and in the strap beam can be constructed.

13.6 Design of mat foundation:

A mat foundation, which sometimes referred to as a raft foundation, is a combined footing that may cover the whole area under a structure supporting several columns and walls.

In some conditions where spread footings may cover more than half the building area, mat foundations may prove to be more economical.

Some of the common types of mat foundations are:

- Flat plate: uniform thickness mat
- Flat slab: mat with drop panels to resist punching shear upward or downward
- Mat with beams: one way or two way
- Slab (mat) with basement walls as part of the mat
- Voided mat
- Mat on piles

Mats are sometimes supported on piles. The piles help in reducing the settlement of the structure located over highly compressive soil. where the ground water table is high, mats are often placed over piles to control buoyancy.

The structural design of mat foundation can be carried by two conventional methods: the conventional rigid method and the approximate flexible method. Finite difference and finite element methods can be used; however, this section will cover the basic concepts of the conventional rigid method.

Steps of conventional rigid method:

1. Determine area of mat: check stresses under the mat:

The maximum compression stress under the mat should be less than the soil allowable bearing capacity and there is no tension under the mat, this is the typical case. If tension exists, this should be taken into account and part of the area is excluded. Nonlinear analysis is recommended for the case of tension stresses under the mat.

The procedure here assumes that the mat is rigid. In general, finite element analysis is recommended for analysis of mat foundations. Also, soil – structure interaction is recommended to be used if it is required to know the effect of soil settlements on the superstructure.

The stress under the mat is given by:

$$q = -\frac{P}{A} \pm \frac{M_y X}{I_y} \pm \frac{M_x Y}{I_x}$$

Where:

P= summation of column (walls) loads, kN

A= area of mat, m^2

M_y= bending moment about Y axis, kN.m

M_x= bending moment about X axis, kN.m

I_y= moment of inertia about Y axis, m^4

I_x= moment of inertia about X axis, m^4

X= distance from the point at which stress will be computed to the Y axis, m

Y= distance from the point at which stress will be computed to the X axis, m

X and Y are the axes that pass through the centroid of the mat area.

2. Determine thickness of mat:

The thickness of mat is controlled by wide beam shear and punching shear (shear- moment transfer).

3. Design the mat for flexure:

The mat should be analyzed using structural analysis principles. For a two-way mat like flat plate, the mat should be divided into strips (frames) in the two directions.

The following procedure is usually used for analysis of a strip in flat plate mat foundation:

- The pressure (stress) on soil shall be computed for at least two points; at the start and at the end of the strip q_1 and q_2.
- The average soil pressure in the strip will be:

$$q_{av} = \frac{q_1 + q_2}{2}$$

- The total soil reaction in the strip will be:

$$R = q_{av}\ x\ strip\ width\ x\ strip\ length$$

- Usually for a strip, the sum of the column loads are not equal to the soil reaction, R. This will be a problem in drawing shear and bending moment diagrams, so this problem must be solved by modifying column loads and pressure on soil to be equal. So:

$$Q_{av} = \frac{R + sum\ of\ column\ loads}{2}$$

- The soil pressure under the strip shall be modified to:

$$q_{avm} = q_{av}\left(\frac{Q_{av}}{R}\right)$$

- The column loads shall be multiplied by load modification factor, F, which is:

$$F = \frac{Q_{av}}{summation\ of\ column\ loads}$$

- A structural model shall be constructed for the strip with modified column loads downward and with line upward load which is equal to q_{avm} multiplied by width of strip. Then the shear and bending moment diagrams can be constructed.

Example (Mat foundation):

Given:

- Mat foundation plan is shown in Figure 13.25
- All given loads are service (unfactored)
- All columns are 0.50 m x0.50 m
- Concrete strength, f'c= 20MPa
- Steel strength, fy= 420MPa
- Soil allowable bearing capacity, qall= 60kN/m^2
- Ultimate load factor= 1.4
- Check mat area and design the interior strip in Y direction.

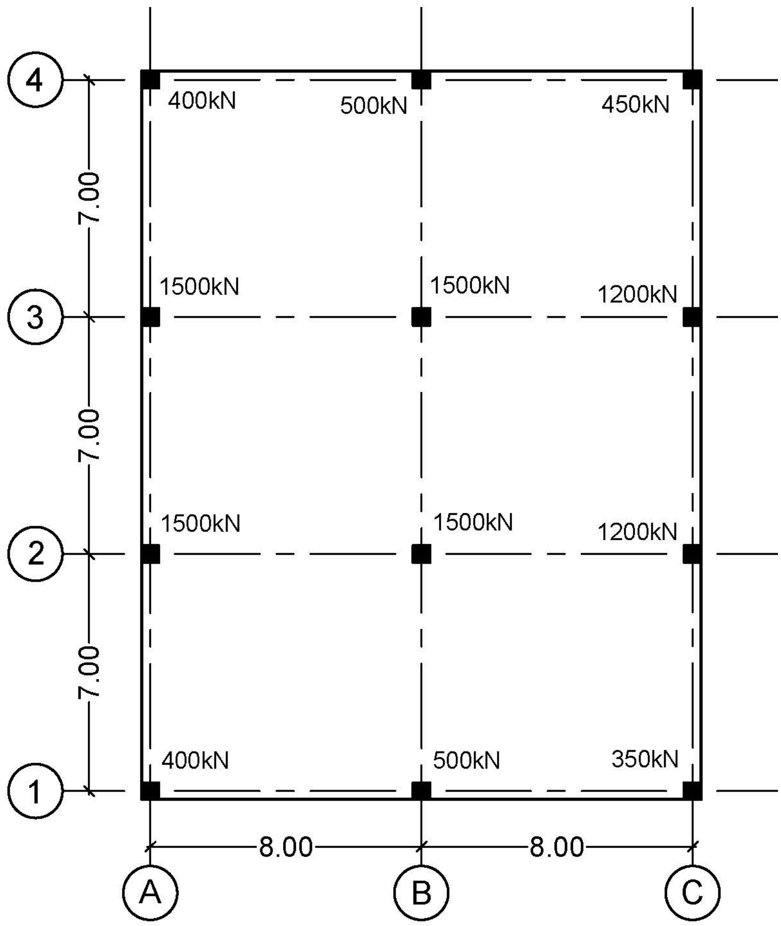


Figure 13.25: Mat layout

Solution:

1. Footing (Mat) area and stresses:

The stress at a point is given by:

$$q = -\frac{P}{A} \pm \frac{M_y X}{I_y} \pm \frac{M_x Y}{I_x}$$

P= summation of column loads = 11000kN

A= area of mat = BL= (16.5) (21.5) =354.75kN

I_x= moment of inertia about X axis which passes through centroid of mat area, it is given by:

$$I_x = \frac{BL^3}{12} = \frac{(16.5)(21.5)^3}{12} = 13655m^4$$

I_y= moment of inertia about Y axis which passes through centroid of mat area, it is given by:

$$I_y = \frac{LB^3}{12} = \frac{(21.5)(16.5)^3}{12} = 8048m^4$$

M_x= bending moment about X axis= P e_y

M_y= bending moment about Y axis= P e_x

The eccentricity of loads in X direction is e_x and it is given by:

$$e_x = \frac{\sum Q_i x_i}{\sum Q_i} - \frac{B}{2}$$

$$e_x = \{[(400 + 1500 + 1500 + 400)(0.25) + (500 + 1500 + 1500 + 500)(8.25) + (450 + 1200 + 1200 + 350)(16.25)]/11000\} - (16.5/2) = -0.436m$$

The eccentricity of loads in Y direction is e_y and it is given by:

$$e_y = \frac{\sum Q_i y_i}{\sum Q_i} - \frac{L}{2}$$

$$e_y = \{[(400 + 500 + 350)(0.25) + (1500 + 1500 + 1200)(7.25)(1500 + 1500 + 1200)(14.25) + (400 + 500 + 450)(21.25)]/11000\} - (21.5/2) = 0.095m$$

So,

M_x= (11000) (0.095) =1045kN.m

M_y= (11000) (0.436) =4800kN.m

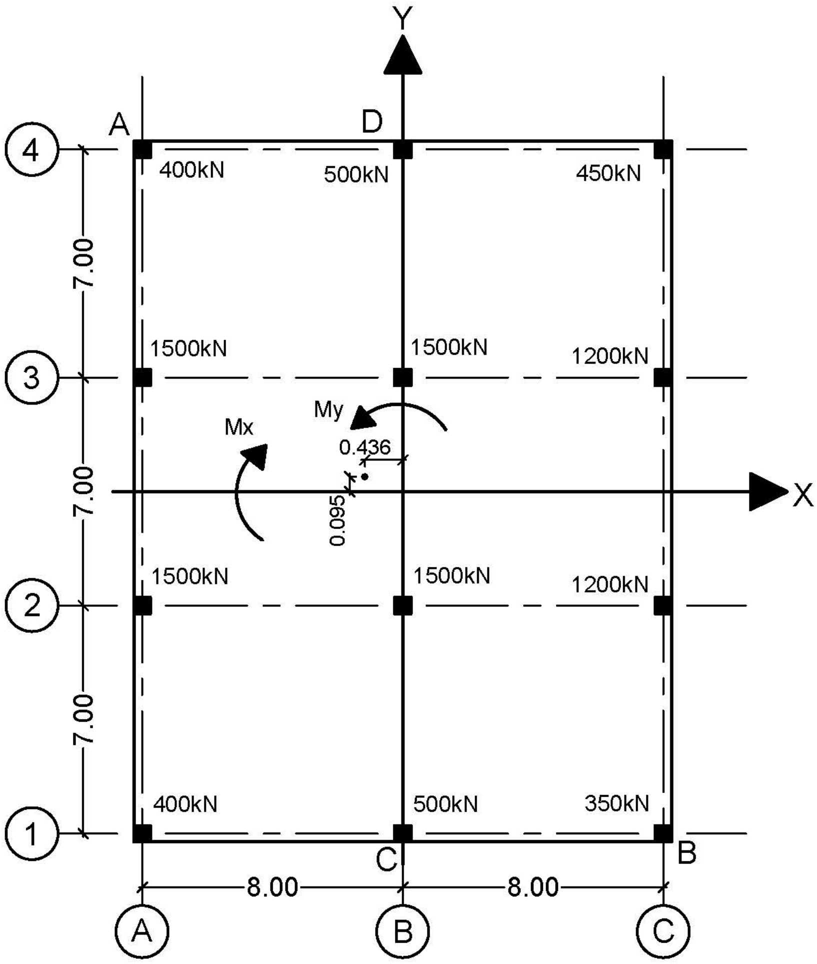


Figure 13.26: Eccentricity and moments in mat foundation

The maximum pressure under the area of mat is located at point A and the minimum pressure under the area of mat is located at point B.

$$q_A = -\frac{11000}{354.75} - \frac{4800(8.25)}{8048} - \frac{1045(10.75)}{13655} = -36.8kN/m^2$$

$$q_B = -\frac{11000}{354.75} + \frac{4800(8.25)}{8048} + \frac{1045(10.75)}{13655} = -25.26kN/m^2$$

These stresses are less than q_{all} and there is no tension under the mat.

2. Mat thickness:

Try thickness of mat, h= 800mm and d= 700mm.

$$V_{u,p} = 1500(1.4) = 2100kN$$

$$\emptyset V_{c,p} = 0.75(0.333)\sqrt{20}((500+350)(2)+(500+700))(700)/1000 = 2247kN > 2100kN.$$

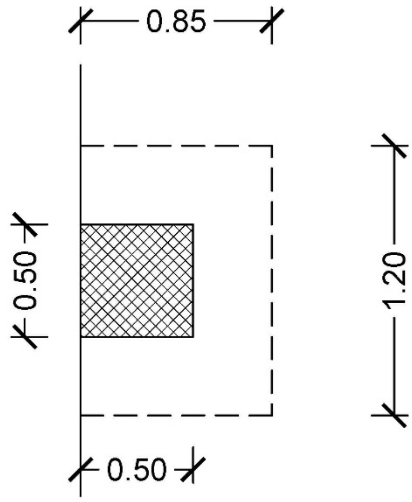


Figure 13.27: Critical section for punching shear

Note: Shear – moment transfer should be done for columns for axial force and moments.

3. Flexural design of the interior strip in Y direction:

Strip width, L2= 8.0m

Strip length, L= 21.5m

$$q_C = -\frac{11000}{354.75} + \frac{1045(10.75)}{13655} = -30.2kN/m^2$$

$$q_D = -\frac{11000}{354.75} - \frac{1045(10.75)}{13655} = -31.8kN/m^2$$

$$q_{av} = \frac{q_C + q_D}{2} = 31kN/m^2$$

Total soil reaction, R= $q_{av}(8)(21.5) = 5332kN$

Total column loads in the strip, Q1= 4000kN

Average load, Qav:

$$Q_{av} = \frac{R + Q_1}{2} = \frac{5332 + 4000}{2} = 4666kN$$

Average pressure in the strip is given by:

$$q_{avm} = q_{av}\left(\frac{Q_{av}}{R}\right) = 31\left(\frac{4666}{5332}\right) = 27.13kN/m^2$$

Column loads modification factor, F1 is given by:

$$F_1 = \frac{Q_{av}}{Q_1} = \frac{4666}{4000} = 1.1665$$

So, the line load on the strip will be:

$$q = (8)(27.13) = 217kN/m$$

Figure 13.28 shows the structural model, the shear force and the bending moment diagrams for the strip.

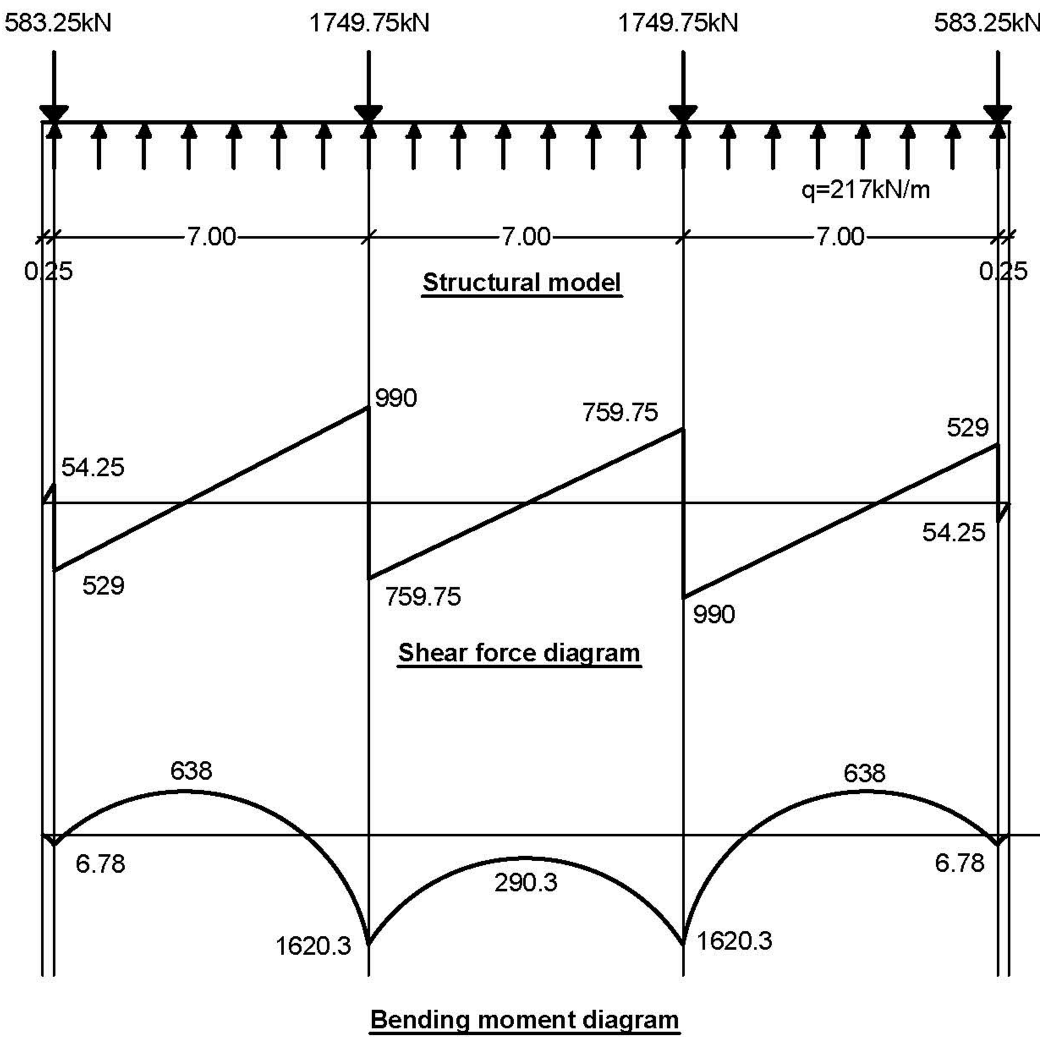


Figure 13.28: Structural model, shear force diagram and bending moment diagram for the interior strip in Y direction in the mat

The strip (frame) can be divided into column and middle strips based on ACI code specifications. The moments in the column strip are about 2/3 the moments of the frame.

As an example: for Mu= 1620.3 kN.m x 1.4 = 2268 kN.m:

Moment in column strip = 0.6667 (2268) = 1512kN.m. b= 3500mm, d= 700mm, $\rho = 0.0024, A_s = 0.0024(3500)(700) = 5880mm^2 \; or \; A_s = 5880/3.5 = 1680mm^2/m$

Moment in middle strip = 2268-1512 = 756kN.m. b= 4500mm, d= 700mm, $\rho = 0.00092, A_s = 0.00092(4500)(700) = 2898mm^2 \; or \; A_s = 2898/4.5 = \; 644mm^2/m$

Minimum steel area= shrinkage steel area= 0.0018(1000) (800) = 1440mm^2/m

So, for column strip, use A_s= 1680mm^2/m $1\emptyset 22/200mm$

For middle strip, use A_s= 1440mm^2/m $1\emptyset 20/200mm$

13.7 Pile foundations

Pile foundation is used to transmit structure loads to deeper soil stratum. They are used if the soil is weak and having low bearing capacity and so spread foundation is not practical. They are effective to minimize structure settlement especially when water is found in the site.

Piles can be divided into two types:

- Bearing piles
- Friction piles

The bearing pile develops its capacity by the bearing end of the pile. These piles usually supported on rock.

The friction pile develops its capacity by the friction between its surface and the surrounding soil or rock.

There are equations to compute the pile strength whether it is bearing or friction pile.

In general, pile construction is fast in construction and practical and provide stability for the structure more than spread or shallow foundation.

The minimum distance between piles centerlines is three times the pile diameter, d.

When the distance between piles centerlines is less than 3d, the pile efficiency shall be determined, as there is overlap between the soil area affected by pile load. More detailed are found in Foundation Design references.

The column can be supported on one pile or on group of piles which depends on the load value and the pile capacity. The group of piles shall be connected by a pile cap. The pile cap thickness shall be determined from punching and one-way shear. Also, the cap shall be designed for flexure in each direction. In general, the pile cap can be considered rigid.

The minimum steel in the cap is similar to than in a footing which is the value specified for shrinkage.

Group of piles supported by a cap can be subjected to a vertical eccentric or concentric force in addition to moments about the two axes.

Example 1 (Pile foundation):

Design a pile system to carry a column load of P_D= 500kN and P_L= 300kN. The pile can carry a load of 450kN. The pile is 800mm diameter. Concrete strength, f'c= 24MPa and steel strength, fy= 420MPa. The column is 350mm x 350mm. Assume the pile cap weight is 5% of the total applied loads.

Solution:

Load, P= (1.05) (500+300) =840kN

Number of piles, N= 840/450=1.87 (Two piles)

Figure 13.29 shows the pile foundation.

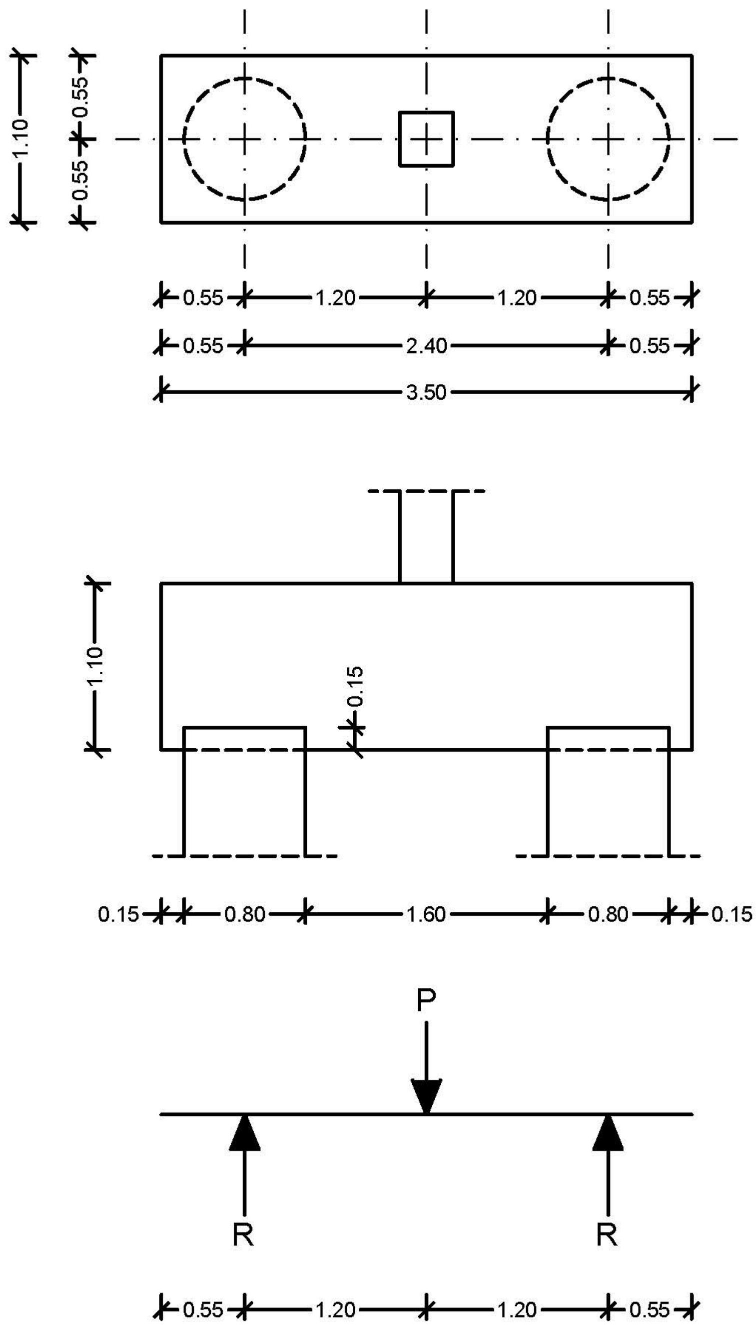


Figure 13.29: Pile foundation

Ultimate column load, Pu= 1.05(1.2(500) +1.6(300)) =1134kN

$$M_u = \frac{P_u L}{4} = \frac{1134(2.4)}{4} = 680 kN.m$$

$$V_u = \frac{P_u}{2} = \frac{1134}{2} = 567 kN$$

At least a minimum transverse reinforcement shall be used that resists shear and shrinkage.so,

$$\emptyset V_c = (0.75)(1/6)\sqrt{24}(1100)(d)/1000$$

$$V_u = \emptyset V_c \qquad d = 842mm$$

h= d+ cover = 842 + 200 = 1042mm

Try h= 900mm, d= 700mm.

$$V_c = \frac{(0.17)(1)\sqrt{24}(1100)(700)}{1000} = 641.3kN$$

$$V_s = \frac{V_u}{\emptyset} - V_c = \frac{567}{0.75} - 641.3 = 114.7kN < \frac{1}{3}\sqrt{24}(1100)(700) = 1257.4kN$$

$$S_{max} = min\left[600mm, \frac{d}{2}\right] = \min\left(600mm, \frac{700}{2} = 350mm\right) = 350mm$$

$$\frac{A_v}{s} = \frac{V_s}{f_{yt} d} = \frac{114700}{(420)(700)} = \frac{0.39mm^2}{mm}.$$

$$\left(\frac{A_v}{s}\right)_{min} = max\ of \begin{bmatrix} 0.062\sqrt{f'_c}\frac{b_w}{f_{yt}} \\ 0.35\frac{b_w}{f_{yt}} \end{bmatrix} = max[0.74, 0.92] = 0.92mm^2/mm$$

$$use\ \frac{A_v}{s} = \frac{0.92mm^2}{mm}$$

$$Use\ \emptyset 12\ closed\ stirrups, 4\ legs: A_v = (4)(113) = 452mm^2$$

$$Spacing, s = \frac{452}{0.92} = 491mm > 350mm, use\ s = 350mm$$

Shrinkage steel in the transverse direction = 0.0018(1000)(900)(0.5)=810mm^2/m. So,

$$Use\ \frac{810}{113} = 7.2\frac{bars}{m}.$$

$Use\ \emptyset 12/120mm\ \ large\ stirrup\ and\ \emptyset 12/360mm\ interior\ small\ stirrup.$

d= 900mm b= 700mm Mu= 680kN.m steel ratio, ρ=0.0035>0.00333

As= 0.0035(1100) (700) =2690mm^2 $(6\emptyset 25)$

Top bars, As= 0.0018(1100) (900)/2= 891mm^2 $(6\emptyset 14)$

Note: See Figure 13.30.

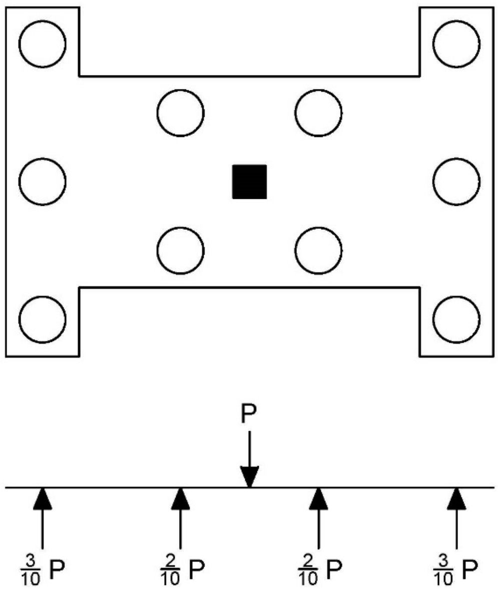


Figure 13.30: Piles group

Example 2 (Pile foundation):

Calculate the maximum and the minimum pile load for the pile foundation shown in Figure 13.31 if the column load, P= 2500kN.

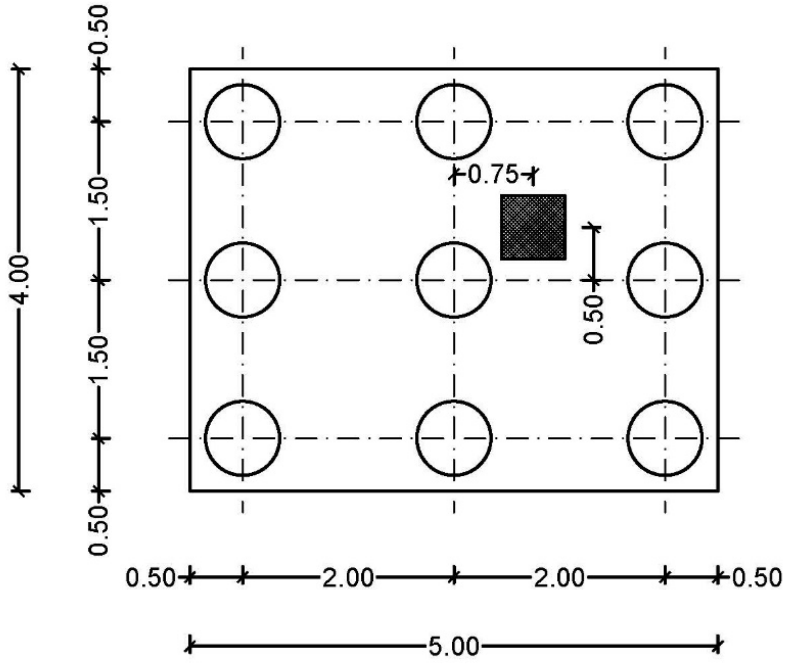


Figure 13.31: Pile group with cap

Solution:

P= 2500kN

Mx= 2500(0.5) =1250kN.m

My= 2500(0.75) =1875kN.m

Ix= 6(1.5)2= 13.5m^4

Iy= 6(2)2= 24m^4

$$Pile\ load, P\ = \frac{P}{N} \pm \frac{M_x y}{I_x} \pm \frac{M_y x}{I_y}$$

$$Pile\ load, P\ = \frac{-2500}{9} \pm \frac{1250y}{13.5} \pm \frac{1875x}{24}$$

$$Maximum\ pile\ load,\ P_{max}\ = \frac{-2500}{9} - \frac{1250(1.5)}{13.5} - \frac{1875(2)}{24}$$
$$= -573kN \quad (Compression)$$

$$Minimum\ pile\ load,\ P_{min}\ = \frac{-2500}{9} + \frac{1250(1.5)}{13.5} + \frac{1875(2)}{24} = +17.3kN \quad (Tension)$$

Chapter 14: Design of Retaining Walls

- Gravity retaining wall
- Cantilever retaining wall
- Counterfort retaining wall
- Basement wall
- Special retaining walls

14.1 Introduction

- The coefficient of lateral pressure is based on soil type and compaction. It is determined from backfill soil properties. In general, there are three types: static, active and passive. Static: if the retaining wall is stiff and difficult to move away from soil. Active: if the retaining wall can deflect and move away from the soil. Passive: if the retaining wall is inclined toward or moves toward the soil.
- In this chapter, the backfill soil has uniform properties (one type of soil). More details are found in soil mechanics and foundations textbooks and references for backfill composed of soil layers with different properties and soil with water.
- The design of a retaining wall is composed of two stages:

    - Design for serviceability: Preliminary dimensions: Check stability of retaining wall structure.
    - Design for strength: Shear, moment, axial, ……….. . The stem and the base thicknesses are controlled by shear.

- The retaining wall stability includes: Overturning, sliding and bearing.
- The factor of safety against overturning should be not less than 2.0
- The factor of safety against sliding should be not less than 1.5
- The pressure under the base of the retaining wall (footing) should be less than the soil allowable bearing capacity. Tension pressure under the footing shall be considered. The footing of the retaining wall can be supported on piles.
- The retaining wall should be designed for the most critical cases. Construction phases control the design of the retaining walls.
- Retaining walls shall be designed for soil lateral pressure, self-weight, surcharge on soil surfaces, wind loads and seismic forces.
- Refer to geotechnical references for computations of seismic forces due to soil effects.
- For a **cantilever retaining wall**, the preliminary dimensions are as follows:

    - Stem thickness at top is not less than 200mm. Minimum of 250mm and 300mm are recommended.
    - Stem thickness at bottom (at base) is about 0.10 the height of the wall from top surface of soil fill to bottom of footing, h.

- Base thickness is about 0.10 the height, h.
- Base length is about 0.33 to 0.75 the height, h.
- The length of the heel is not less than 1.5 times the length of the toe.
- A key can be used to increase sliding resistance.
- Refer to Figure 14.1.

- The surcharge on backfill soil for vehicle movement is not less than 12kN/m^2. Refer to AASHTO for more details.

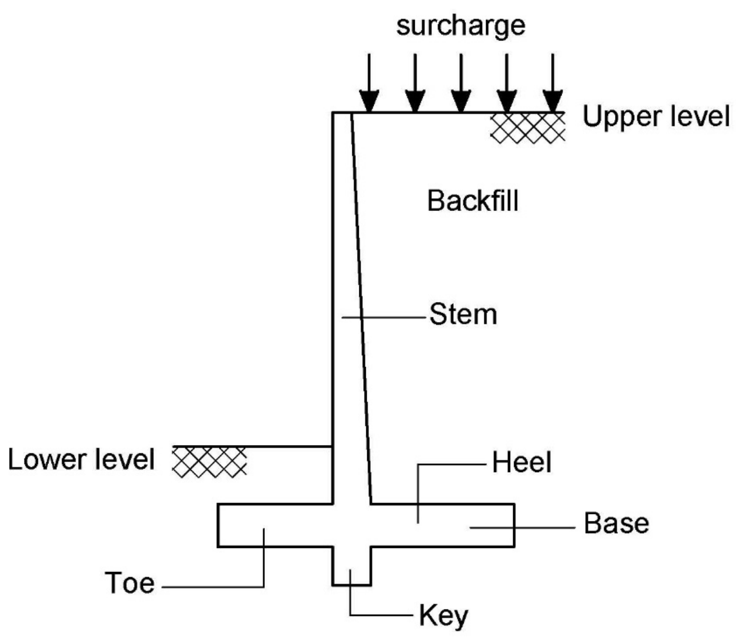


Figure 14.1: Cantilever retaining wall

Example: Cantilever retaining wall:

Given:

- Refer to Figure 14.2.
- Concrete strength, f'c= 28MPa
- Steel yield strength, fy= 420MPa
- Soil allowable bearing capacity, qall= 400kN/m^2
- Soil unit weight, γ_s= 19.2kN/m^3
- Soil friction internal angle, Ø= 30 degrees
- Base friction coefficient, F1= 0.5

Check retaining wall dimensions and design it.

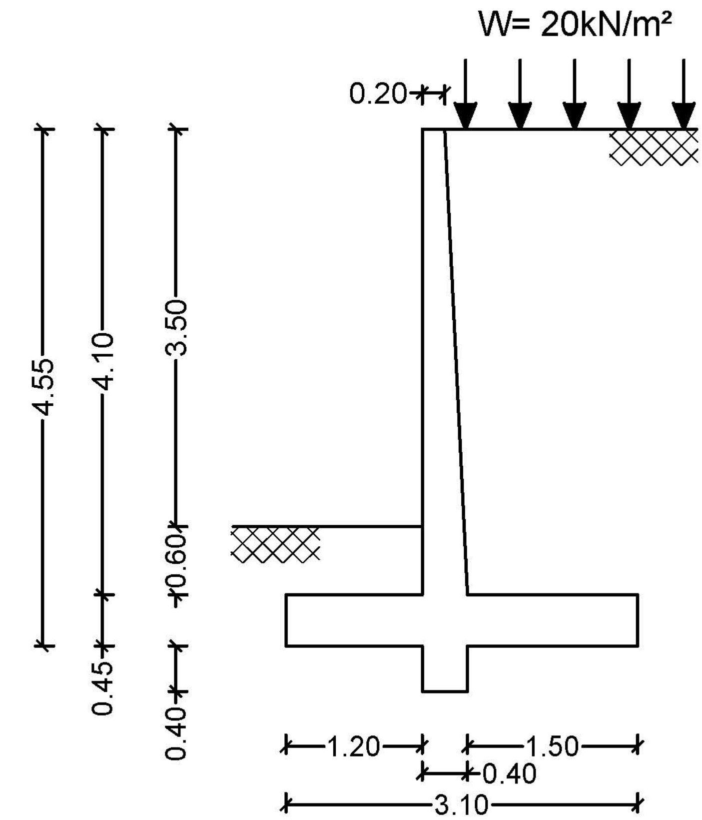


Figure 14.2: Cantilever retaining wall for the example

Solution:

Step 1: Check stability

- Check overturning of the retaining wall: refer to Figure 14.3.

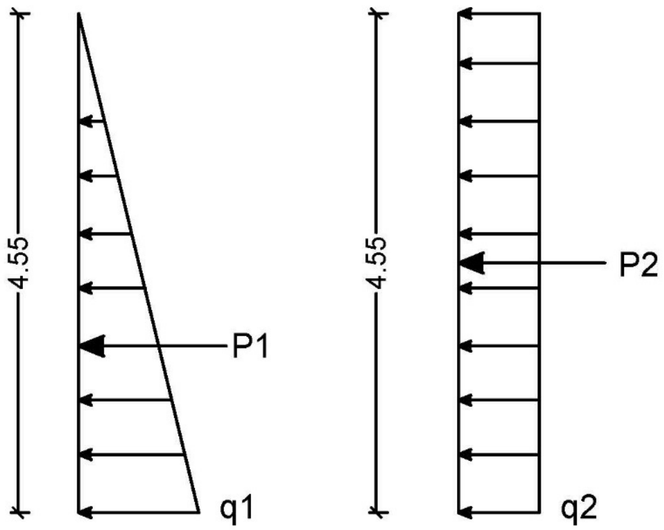


Figure 14.3: Soil pressure on wall

$$q_1 = \gamma hk$$

$$K_a = \frac{1 - sin\emptyset}{1 + sin\emptyset} = 0.333$$

$$q_1 = (19.2)(4.55)(0.333) = 29.1kN/m^2$$

$$q_2 = wK_a = (20)(0.333) = 6.66kN/m^2$$

$$P_1 = \frac{1}{2}q_1 h = \frac{1}{2}(29.1)(4.55) = 66.2kN$$

$$P_2 = q_2 h = (6.66)(4.55) = 30.3kN$$

$$P = P_1 + P_2 = 66.2 + 30.3 = 96.5kN$$

$$M_{ov} = P_1(\frac{4.55}{3}) + P_2(\frac{4.55}{2}) = 169.3kN.m$$

Figure 14.4 shows the weight zones for the retaining wall to compute the resisting moment. **Table 14.1** shows calculations for weight and moments of the zones.

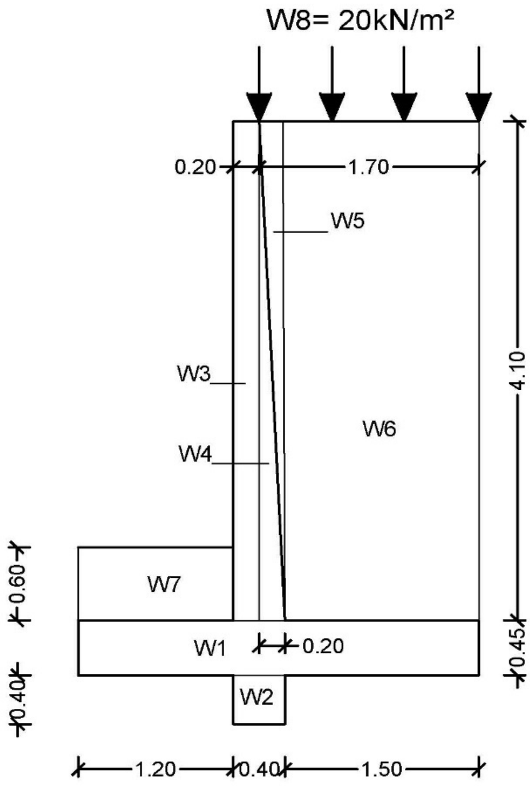


Figure 14.4: Weight zones for the retaining wall

Table 14.1: Weight and resisting moments in the cantilever retaining wall

Part	Weight (kN)	Moment arm (m)	Moment (kN.m)
W1	(0.45)(3.1)(25)=34.88	1.55	54.06
W2	(0.4)(0.4)(25)=4.0	1.4	5.6
W3	(0.2)(4.10)(25)=20.5	1.3	26.65
W4	(0.5)(0.2)(4.1)(25)=10.25	1.47	15.07
W5	(0.5)(0.2)(4.1)(19.2)=7.87	1.53	12.04
W6	(1.5)(4.1)(19.2)=118.08	2.35	277.49
W7	(1.2)(0.6)(19.2)=13.82	0.6	8.29
W8	(20)(1.7)=34	2.25	76.5
	W=243.4kN		**MR=475.7kN.m**

$$Factor\ of\ safety\ against\ overturning, F.S. = \frac{M_R}{M_{ov}} = \frac{475.7}{169.3} = 2.8 > 2 \quad OK.$$

- Check sliding of the retaining wall: Refer to **Figure 14.5**.

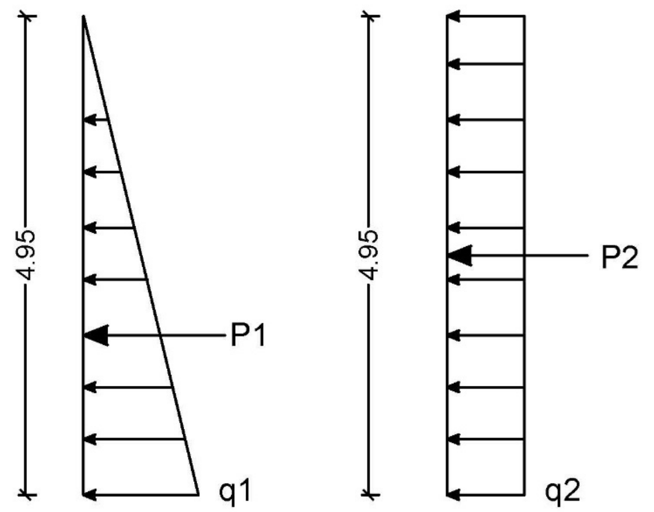


Figure 14.5: Soil lateral pressure for computing sliding force

$$P_{sliding} = P_1 + P_2 = (0.5)(4.95)(19.2x4.95x0.333) + (20x0.333)(4.95)$$
$$= 78.33 + 32.97 = 111.3kN$$

$$P_{resisting} = Wf + \frac{1}{2}\gamma_s h^2 K_p = (243.4)(0.5) + \frac{1}{2}(19.2)(1.45)^2(3) = 121.7 + 60.552$$
$$= 182.25kN$$

$$Factor\ of\ safety\ against\ sliding, F.S. = \frac{F_{Resisting}}{F_{Sliding}} = \frac{182.25}{111.3} = 1.64 > 1.5 \quad OK.$$

- Check bearing: check pressure under the base of the retaining wall: Refer to **Figure 14.6**.

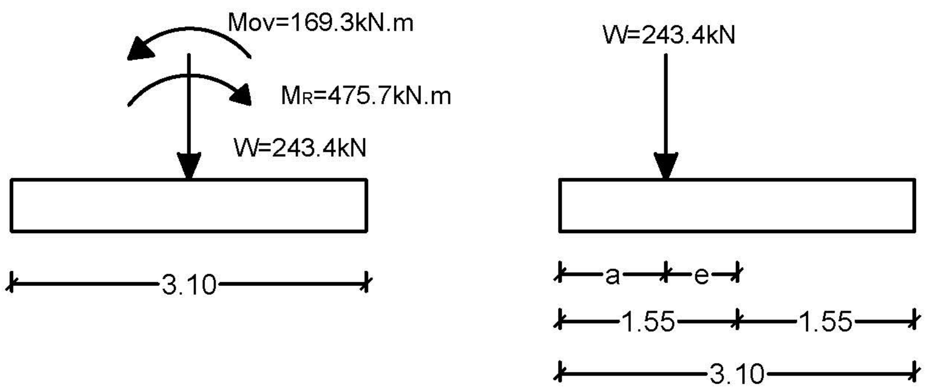


Figure 14.6: Forces at the base of the retaining wall

$$a = \frac{\Delta M}{W} = \frac{475.7 - 169.3}{243.4} = 1.26m$$

$$M_{about\ center\ of\ footing} = \left(\frac{3.1}{2} - 1.26\right)(243.4) = 0.29(243.4) = 70.6kN,m$$

$$q_1 = \frac{-P}{A} - \frac{Mc}{I} = \frac{-243.4}{1(3.1)} - \frac{70.6(1.55)}{\frac{1}{12}(1)(3.1)^3} = -122.62kN/m^2$$

$$q_2 = \frac{-P}{A} + \frac{Mc}{I} = \frac{-243.4}{1(3.1)} + \frac{70.6(1.55)}{\frac{1}{12}(1)(3.1)^3} = -34.42kN/m^2$$

See **Figure 14.7**.

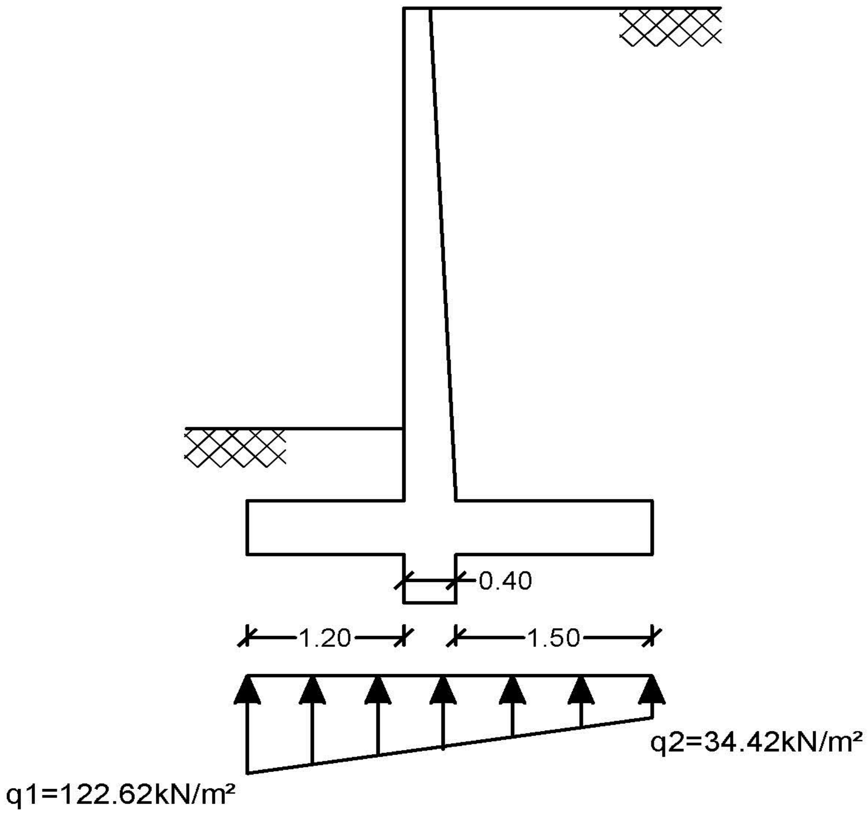


Figure 14.7: Pressure under the base of retaining wall

Note that q1 and q2 are compression.

The eccentricity is located in the middle third of the base length. The eccentricity is given by:

$$e = \frac{M}{W} = \frac{70.6}{243.4} = 0.29m < \frac{3.1}{6} = 0.517m$$

Step 2: Strength design:

- Design of the stem:

Height, h= 4.10m

Thickness at base, t= 0.40m d= 0.33m section width= one unit = 1.0m

See **Figure 14.8**.

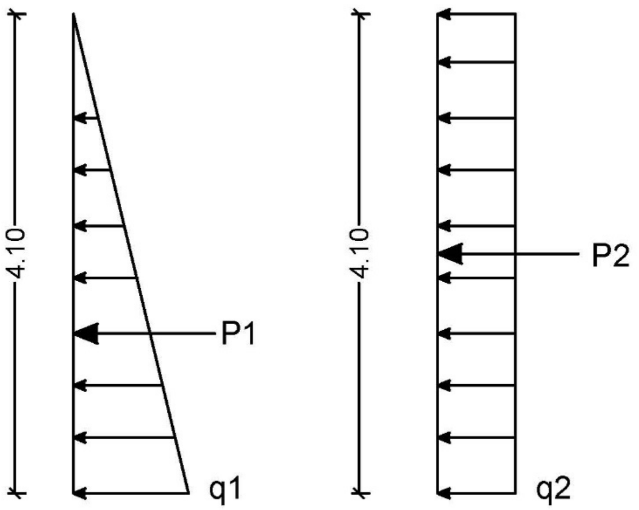


Figure 14.8: Soil lateral pressure for computing internal forces in the stem of the cantilever retaining wall

$$q_1 = \gamma h k_a = (19.2)(4.1)(0.333) = 26.21 kN/m^2$$

$$q_2 = w k_a = (20)(0.333) = 6.66 kN/m^2$$

$$P_1 = \frac{1}{2} q_1 h = \frac{1}{2}(26.21)(4.1) = 53.73 kN$$

$$P_2 = q_2 h = (6.66)(4.1) = 27.31 kN$$

$$P = V = P_1 + P_2 = (53.73) + (27.31) = 81.04 kN$$

$$M = P_1\left(\frac{h}{3}\right) + P_2\left(\frac{h}{2}\right) = (53.73)\left(\frac{4.1}{3}\right) + (27.31)\left(\frac{4.1}{2}\right) = 129.43 kN$$

$$V_u = (1.6)(81.04) = 129.7 kN$$

$$M_u = (1.6)(129.43) = 207.1 kN.m$$

$ACI\ 318-14: \emptyset V_c = (0.75)(1/6)\sqrt{28}(1000)(330)/1000 = 218.3 kN \quad > \ 129.7 kN \quad OK$

$From\ Mu = 207.1 kN.m\,, \quad steel\ ratio, \rho = 0.0053, As = (0.0053)(1000)(330) = 1749 mm^2 \qquad use\ 1\emptyset 20/150 mm$

ACI 318-19: shear design

V_c can be calculated by:

$$For\ A_v \geq A_{v.min}\ (or\ \frac{A_v}{s} \geq \left(\frac{A_v}{s}\right)_{min})\ use\ either\ of:$$

$$V_c = \left(0.17\lambda\sqrt{f'_c} + \frac{N_u}{6A_g}\right)b_w d \quad and \quad V_c = \left(0.66\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right)b_w d$$

$$For\ A_v < A_{v.min}\ \left(or\ \frac{A_v}{s} < \left(\frac{A_v}{s}\right)_{min}\right) use:$$

$$V_c = \left(0.66\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right)b_w d$$

Where A_v is the area of shear reinforcement within spacing *s*, mm^2.

And, V_c shall not be taken greater than:

$$V_c \leq 0.42\lambda\sqrt{f'_c}b_w d$$

$$\lambda_s = \sqrt{\frac{2}{1 + 0.004\,d}} \leq 1.0$$

For $d \leq 250mm, \lambda_s = 1.0$

$$\frac{N_u}{6A_g} \leq 0.05f'_c$$

Axial load, *Nu*, is positive for compression and negative for tension.

$$\rho_w = \frac{A_s}{b_w d}$$

The value of *As* to be used in the calculation of ρ_w may be taken as the sum of the areas of longitudinal bars located more than two thirds of the overall member depth away from the extreme Compression fiber.

The value of $\sqrt{f'c}$ used to calculate ***Vc*** for one-way shear shall not exceed 100 psi (8.3MPa), unless allowed in 22.5.3.2 ($A_v \geq A_{v,min}$).

$$\lambda_s = \sqrt{\frac{2}{1 + 0.004\, d}} \leq 1.0 \rightarrow \lambda_s = \sqrt{\frac{2}{1 + 0.004\,(330)}} = 0.928$$

$$\rho_w = \frac{(6.67\, bars)\,(314)}{(1000)(330)} = 0.0063$$

$$\emptyset\, V_c = 0.75 \frac{\left(0.66(0.928)(1)(0.0063)^{\frac{1}{3}}\sqrt{28} + 0.0\right)(1000)(330)}{1000} = 148.4kN$$

$$> 129.7kN \quad ok.$$

At exterior face of wall, one can use steel ratio= 0.0025/2 if the wall is interior and is not subjected to environmental hazards.

If the wall can be subjected to environmental hazards, one can use steel ratio based on ACI 350, and steel ratio= 0.003/2.

Here, use steel ratio= 0.003/2

$$A_s = (0.003/2)(1000)(400) = \; 600mm^2 \qquad use\; 1\emptyset 12/150$$

For horizontal steel, one can use steel ratio= 0.0025/2 if the wall is interior and is not subjected to environmental hazards.

If the wall can be subjected to environmental hazards, one can use steel ratio based on ACI 350, and steel ratio= 0.003/2 if expansion joints are used with spacing not larger than 9.0m.

Here, use steel ratio= 0.003/2

$$A_s = (0.003/2)(1000)(400) = \; 600mm^2 \qquad use\; 1\emptyset 12/150\; or\;\; 1\emptyset 14/250$$

Check shear key: Refer to Figure 14.9.

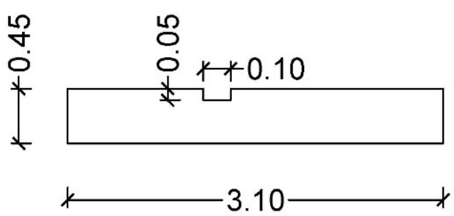


Figure 14.9: Shear key at base of the cantilever retaining wall

Check that:

$$\frac{V_u}{A_{key}} \leq \emptyset(0.2f'_c) \leq 5.5MPa$$

$$\frac{129.7(1000)}{1000(100)} = 1.3MPa < 0.75(0.20)(28) = 4.2MPa$$

- **Design of the toe:** Refer to Figure 14.10.

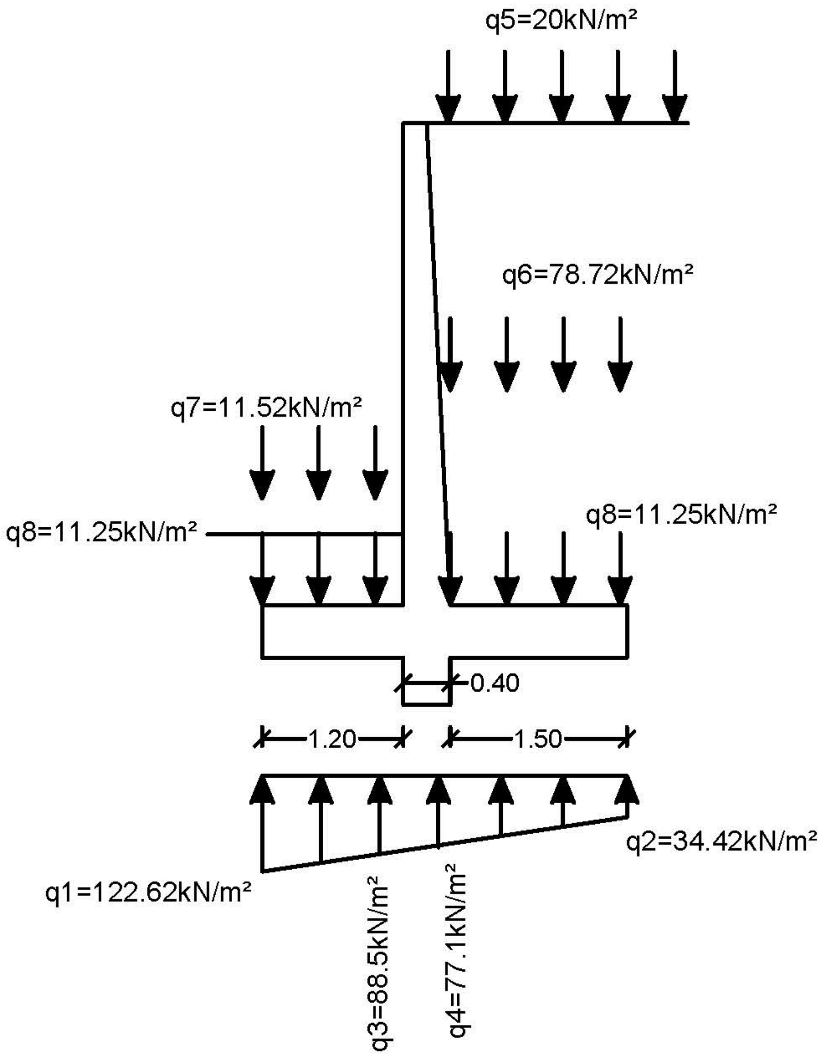


Figure 14.10: Loads at the base of the cantilever retaining wall

$$q_6 = \gamma_s h = (19.2)(4.1) = 78.72kN/m^2$$

$$q_7 = \gamma_s h = (19.2)(0.6) = 11.52kN/m^2$$

$$q_8 = \gamma_c h = (25)(0.45) = 11.25kN/m^2$$

Use an average load factor of 1.4 for shear and moment computations.

$$V = \left(\frac{(122.62 + 88.5)}{2}\right)(1.2) - (11.52)(1.2) - (11.25)(1.2) = 99.35kN$$

$$V_u = (1.4)(99.35) = 139.1kN$$

$$M = (88.5)(1.2)(0.6) + (122.62 - 88.5)(0.5)(1.2)(0.667)(1.2) - (11.52)(0.5)(1.2)^2 - (11.25)(0.5)(1.2)^2 = 63.71kN.m$$

$$M_u = (1.4)(63.71) = 89.2kN.m$$

$$ACI\ 318-14: \emptyset V_c = (0.75)(1/6)\sqrt{21}(1000)(380)/1000 = 218kN \quad > \ 139.1kN \quad OK$$

Flexure:

$$\rho = 0.001664 \qquad A_s = 0.001664(1000)(380) = 632mm^2$$

$$A_{s,min} = 0.0018(1000)(450) = 810mm^2 \quad use\ 1\emptyset 20/300mm$$

ACI 318-19: shear:

$$\lambda_s = \sqrt{\frac{2}{1 + 0.004\,d}} \leq 1.0 \rightarrow \lambda_s = \sqrt{\frac{2}{1 + 0.004\,(380)}} = 0.89$$

$$\rho_w = \frac{(3.3\ bars)\ (314)}{(1000)(380)} = 0.0027$$

$$\emptyset\, V_c = 0.75\frac{\left(0.66(0.89)(1)(0.0027)^{\frac{1}{3}}\sqrt{28} + 0.0\right)(1000)(380)}{1000} = 138.6kN \approx 139.1kN \quad ok.$$

- **Design of heel:** Refer to Figure 14.10 above:

Use an average load factor of 1.4 for shear and moment computations.

$$V = (20)(1.5) + (78.72)(1.5) + (11.25)(1.5) - (34.42 + 77.1)(0.5)(1.5) = 81.32kN$$

$$V_u = (1.4)(81.32) = 113.85kN$$

$$M = \frac{(20)(1.5)^2}{2} + \frac{(78.72)(1.5)^2}{2} + \frac{11.25(1.5)^2}{2} - \frac{(34.42)(1.5)^2}{2} - (77.1 - 34.42)(0.5)(1.5)(1.5/3) = 69kN.m$$

$$M_u = (1.4)(69) = 96.6kN$$

Flexure:

$$\rho = 0.001805 \qquad A_s = 0.001664(1000)(380) = 686mm^2$$

$$A_{s,min} = 0.0018(1000)(450) = 810mm^2 \quad use\ 1\emptyset 20/300mm$$

$$ACI\ 318-14{:}\ \emptyset V_c = 218kN > V_u = 113.85kN \quad ok.$$

$$ACI\ 318-19{:}\ \emptyset V_c = 138.6kN > V_u = 113.85kN \quad ok.$$

Shrinkage steel in the base: $use\ 810/2 = 405mm^2 \qquad use\ 1\emptyset 12/250mm$

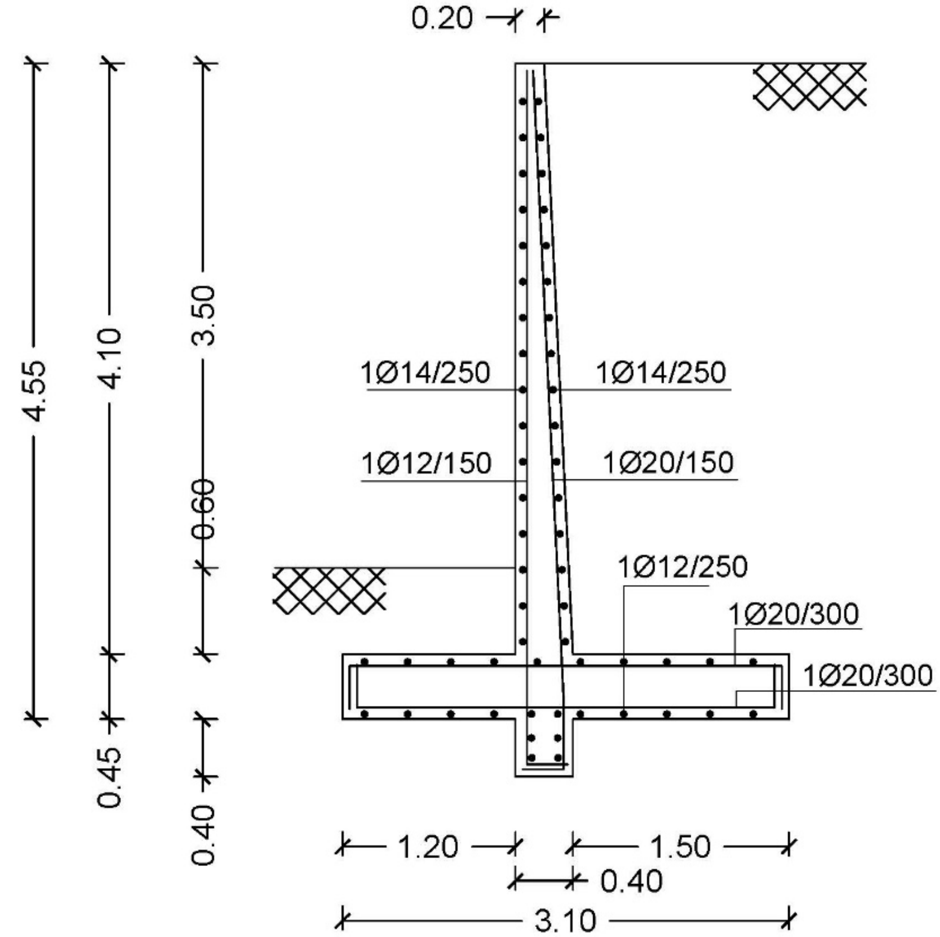


Figure 14.11: Cantilever wall reinforcement details

14.2 Design of counterfort retaining wall:

The counterfort retaining walls are composed of slabs and counterforts.

The toe slab represents a cantilever built in a long the front face of the wall, loaded upward by the bearing pressure, exactly as the cantilever walls.

The panel of the vertical wall between two counterforts is a slab acted upon by the horizontal earth pressure and supported along three sides; the two counterforts and the base slab, while the fourth side; the top edge, is not supported. Slab moments are determined for strips one meter wide spanning horizontally, usually for the strip at the bottom of the wall and for other strips at higher elevations.

The heel slab is supported as in the wall slab; by counterforts and at the wall (stem). It is loaded downward by the weight of the fill resting on it, its own weight, and surcharge as there may be. This load is partially counteracted by the bearing pressure. Horizontal strips are taken with the corresponding loads; the top and downward loads.

The counterforts are wedge-shaped cantilevers built in the base slab. They support the wall slab (stem) and therefore are loaded by the total soil pressure over a length equal to the distance center to center between counterforts.

Example: (counterfort wall):

Given:

- Refer to **Figure 14.12**
- Concrete strength, f’c= 32MPa
- Steel yield strength, fy= 420MPa
- Soil unit weight, γ_s=20kN/m^3
- Soil internal friction angle, Ø=30 degrees
- Clear distance between counterforts is 2.50m
- Thickness of counterfort wall is 0.30m

Design the stem and the counterfort wall.

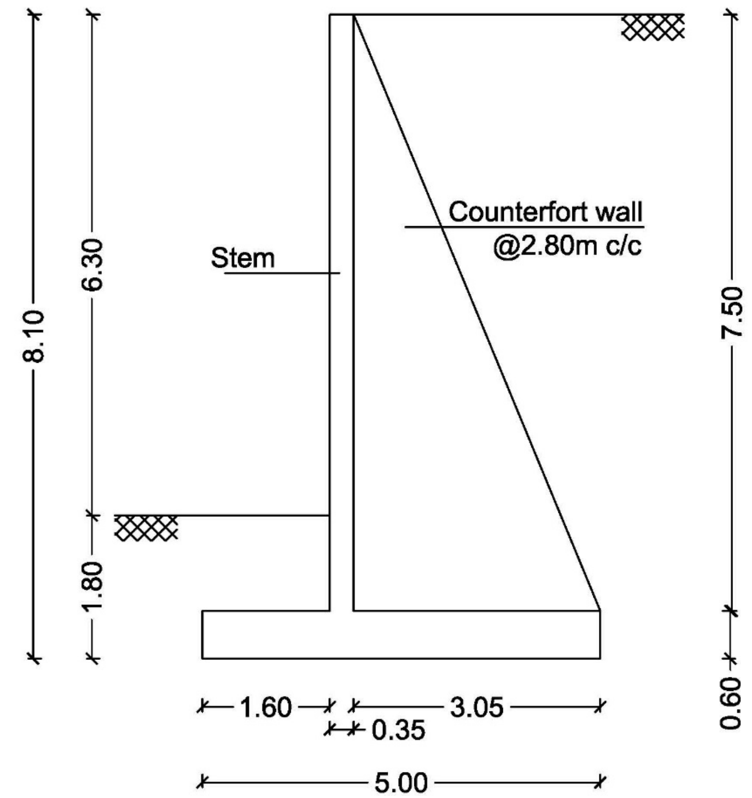


Figure 14.12: Counterfort retaining wall

Solution:

- Design of wall (stem):

$q = \gamma_s h k_a = (20)(7.5)(0.333) = 50kN/m^2$

$q_u = (1.6)(50) = 80kN/m^2$

Clear span length, L_n= 2.50m

Load, q_u= 80kN/m on a horizontal strip of 1.0m width at the base.

$$V_u = 1.15\frac{q_u L_n}{2} = 1.15\frac{(80)(2.5)}{2} = 115kN$$

$$M_{u,max} = \frac{q_u {L_n}^2}{10} = \frac{(80)(2.5)^2}{10} = 50kN.m$$

Effective depth, d= 350-60=290mm.

$ACI\ 318-14{:}\ \emptyset V_c = (0.75)(1/6)\sqrt{32}(1000)(290)/1000 = 205kN \quad > \ 115kN \quad OK$

$Steel\ ratio, \rho = 0.0016 < 0.00333 \qquad A_s = (0.00333)(1000)(290) = 966mm^2 \quad use\ 1\emptyset 16/200mm$

ACI 318-19: shear:

$$\lambda_s = \sqrt{\frac{2}{1 + 0.004\, d}} \leq 1.0 \rightarrow \lambda_s = \sqrt{\frac{2}{1 + 0.004\,(290)}} = 0.96$$

$$\rho_w = \frac{(5\, bars)\,(201)}{(1000)(290)} = 0.0035$$

$$\emptyset\, V_c = 0.75 \frac{\left(0.66(0.96)(1)(0.0035)^{\frac{1}{3}}\sqrt{32} + 0.0\right)(1000)(290)}{1000} = 118.6kN > 115kN \quad ok.$$

Vertical steel in the wall = (0.003/2)(1000)(350)=525mm^2. Use $1\emptyset 12/200mm$.

- Design of the counterfort wall:

$q_u = (80)(2.8) = 224kN/m$. Triangular load.

$V_u = (0.5)(224)(7.5) = 840kN$

$M_u = (840)(7.5/3) = 2100kN.m$

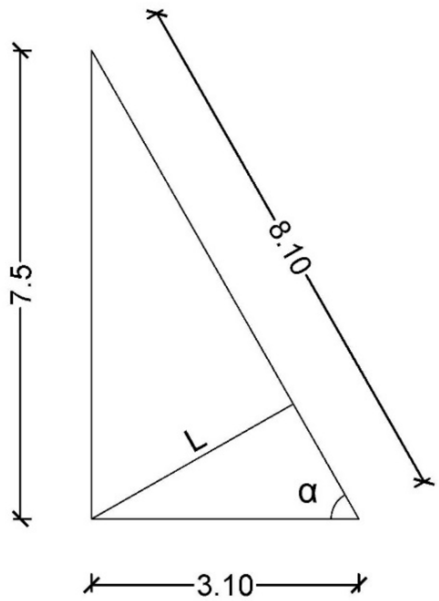


Figure 14.13: Counterfort wall

$$sin\,\alpha = \frac{L}{3.1} = \frac{7.5}{8.1} \rightarrow L = 2.87m$$

$\emptyset M_n = \emptyset A_s f_y jd$

$2100(10)^6 = (0.9)(A_s)(420)(0.9)(0.8x2870) \rightarrow A_s = 2689mm^2 \ use\ 9\emptyset 20$

Shear:

$V_c = (1/6)\sqrt{32}(300)(0.8x2870)/1000 = 649kN$

$\frac{V_u}{\emptyset} = \frac{840}{0.75} = 1120kN$

$V_s = 1120 - 649 = 471kN$

$\frac{A_v}{s} = \frac{V_s}{f_{yt}d} = \frac{471000}{(420)(0.8x2870)} = 0.49mm^2/m$

$\left(\frac{A_v}{s}\right)_m = max\left(\frac{0.35b_w}{f_{yt}}, \frac{0.062\sqrt{f'_c}b_w}{f_{yt}}\right) = 0.25mm^2/m$

$use\ \frac{A_v}{s} = \frac{0.49mm^2}{m} \rightarrow s = \frac{113x2}{0.49} = 461mm \qquad \emptyset 12/450mm$

$Shrinkage\ steel = \frac{0.003}{2}(1000)(300) = 450mm^2/m$

$Use\ 1\emptyset 12/250mm\ in\ horizontal\ and\ vertical\ directions$

Chapter 15: Design of Water Tanks

15.1 Design of rectangular water tanks:

- Tanks can be divided to:
    - On-ground tanks
    - Underground tanks
    - Elevated tanks
- Tanks can be of different shapes:
    - Rectangular
    - Circular
    - Irregular
- Based on load transfer, water tanks can be divided to:
    - Shallow tanks
    - Medium tanks
    - Deep tanks

Shallow water tanks:

In shallow tanks, mainly, water loads are transferred in walls in the vertical direction. So, walls can be designed for one meter strip in the central zone of wall.

- For tanks without roof: $L/H \geq 4$
- For tanks with roof: $L/H \geq 2$

Where L is the side length and H is the side height.

Figure 15.1 shows the shear and moments for a wall.

Here, w= unit weight of water= 10kN/m^3.

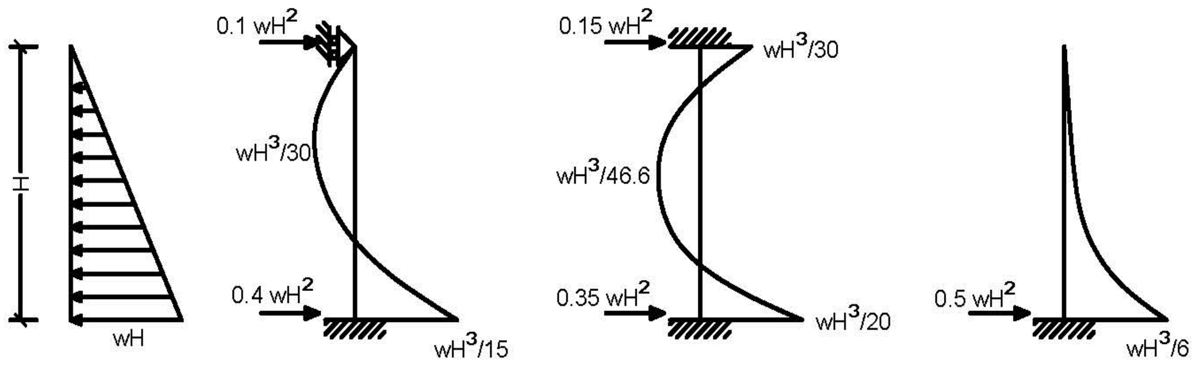


Figure 15.1: Shear and moments in a wall of shallow tank in the vertical direction

Horizontal forces in walls of shallow tanks (At corners of tank):

Loads are transferred to walls at corners by the 45 degrees load distribution principle.

Refer to **Figure 15.2**.

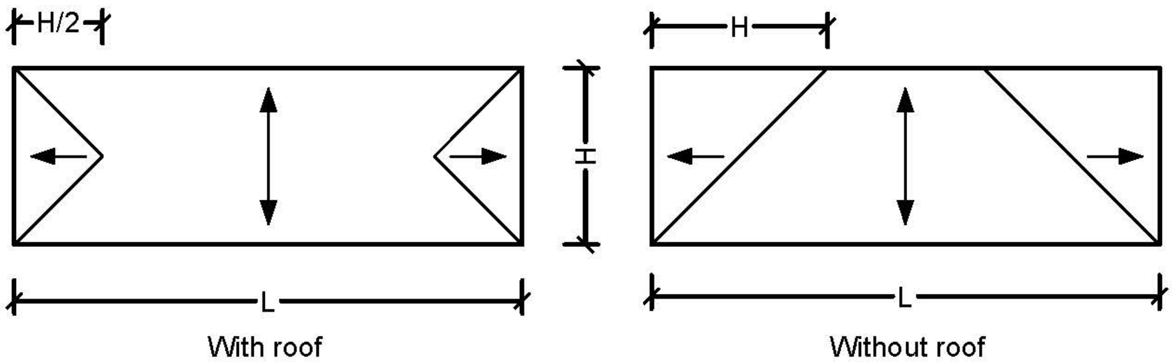


Figure 15.2: Water loads distribution in walls of shallow water tank

Tank with roof:

$$F = \frac{1}{2}H\frac{H}{2}\frac{wH}{2} = \frac{wH^3}{8}$$

$$M = F * \ Distance = \frac{wH^3}{8}\left(\frac{1}{2}\frac{H}{2}\right) = \frac{wH^4}{32}$$

Force (tension force on the perpendicular wall= shear force in this wall) per meter height is given by:

$$F = \frac{wH^3}{8} / H = \frac{wH^2}{8}$$

Moment per meter height is given by:

$$M = \frac{wH^4}{32} / H = \frac{wH^3}{32}$$

Tank without roof:

$$F = \frac{1}{2} H H \frac{wH}{2} = \frac{wH^3}{4}$$

$$M = F * \ Distance = \frac{wH^3}{4} \left(\frac{H}{2}\right) = \frac{wH^4}{8}$$

Force (tension force on the perpendicular wall= shear force in this wall) per meter height is given by:

$$F = \frac{wH^3}{4} / H = \frac{wH^2}{4}$$

Moment per meter height is given by:

$$M = \frac{wH^4}{8} / H = \frac{wH^3}{8}$$

Medium water tanks:

In medium tanks, water loads are transferred in walls in the vertical and horizontal directions. Here L/H or H/L is between 1 and 2. In general, the wall shall be analyzed as two-way slab subjected to triangular surface load.

Deep water tanks:

In deep water tanks, mainly, water loads are transferred in walls in the horizontal direction. Here, $H/L \geq 2$.

Refer to **Figure 15.3**.

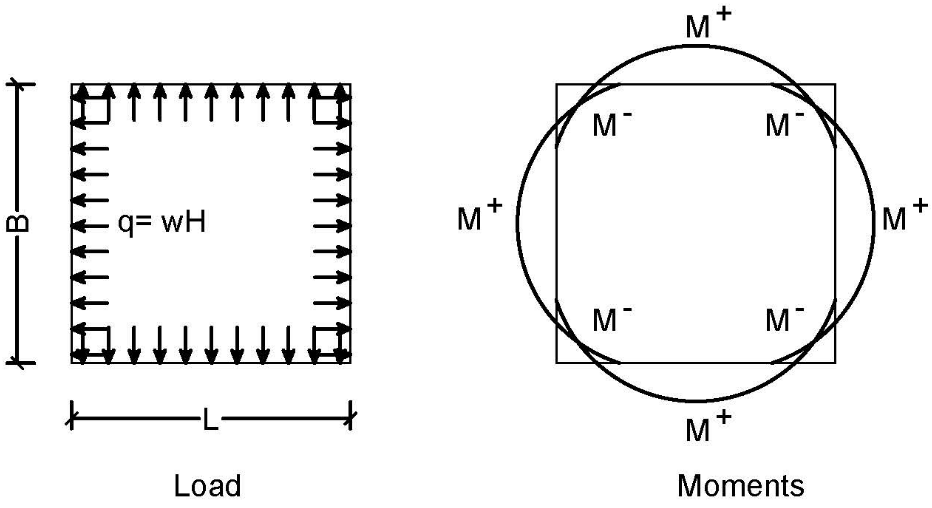


Figure 15.3: Load and moments in horizontal direction of deep water tank

$$Tension\ in\ side\ B = \frac{qL}{2}$$

$$Tension\ in\ side\ L = \frac{qB}{2}$$

$$The\ negative\ moment\ at\ corner\ is\ given\ by: M^- = \frac{q(L^3 + B^3)}{12(L + B)}$$

And the span moments will be:

$$At\ side\ L, M^+ = \frac{qL^2}{8} - (M^-)$$

$$At\ side\ B, M^+ = \frac{qB^2}{8} - (M^-)$$

Design methods:

- Working design method: Allowable stress method.
- Ultimate design method with serviceability check.

In this chapter, the ultimate design method is used.

The environmental durability factor, S_d will be used in the design for moment, tension, shear and torsion. It is given by:

$$S_d = \frac{\emptyset f_y}{\gamma f_s}$$

$$\gamma = \frac{factored\ load}{unfactored\ load}$$

Note that S_d is equal to 1.0 for seismic forces and for compression.

The allowable stress in steel, fs is discussed below.

Flexure:

For normal exposure:

$$f_s = \frac{2200MPa}{\beta\sqrt{S^2 + 4\left(50 + \frac{d_b}{2}\right)^2}/25.4} = \frac{55880}{\beta\sqrt{S^2 + 4\left(50 + \frac{d_b}{2}\right)^2}}MPa$$

$$f_s \le 250MPa$$

$$f_s \ge 138MPa\ for\ one\ way\ members.$$

$$f_s \ge 165MPa\ for\ two\ way\ members.$$

For severe exposure:

$$f_s = \frac{1794MPa}{\beta\sqrt{S^2 + 4\left(50 + \frac{d_b}{2}\right)^2}/25.4} = \frac{45560}{\beta\sqrt{S^2 + 4\left(50 + \frac{d_b}{2}\right)^2}}MPa$$

$$f_s \le 250MPa$$

$$f_s \ge 117MPa\ for\ one\ way\ members.$$

$$f_s \ge 138MPa\ for\ two\ way\ members.$$

Where:

S= spacing between bars, mm

d_b= diameter of bar, mm

$$\beta = \frac{h - c}{d - c}$$

h= overall thickness of member

d= effective depth of section

C= distance from extreme compression fiber to neutral axis. It is calculated at service loads for cracked section.

For simplicity:

$\beta = 1.2\ for\ h \geq 400mm$

$\beta = 1.35\ for\ h < 400mm$

Tension:

$f_s = 138MPa\ for\ normal\ exposure$

$f_s = 117MPa\ for\ severe\ exposure$

Shear:

$f_s = 165MPa\ for\ normal\ exposure$

$f_s = 138MPa\ for\ severe\ exposure$

It is not recommended to use shear reinforcement in walls and base of tank, so:

$\emptyset V_c \geq V_u$

The environmental durability factor, S_d will be multiplied by Vs if shear reinforcement is used in the member like for beams.

In walls and base of water tank, the minimum steel ratio is similar to beams.

The shrinkage steel area in the vertical direction of walls shall be not less than 0.003 multiplied by the gross sectional area.

The minimum shrinkage steel ratio in the horizontal direction is given in Table 3.1 based on ACI 350-06.

Table 15.1: Minimum shrinkage and temperature reinforcement in environmental structures

Length between movement joints, L, m	Steel ratio for fy= 280MPa	Steel ratio for fy= 420MPa
$L < 6$	0.003	0.003
$6 \leq L < 9$	0.004	0.003
$9 \leq L < 12$	0.005	0.004
$12 \leq L$	0.006	0.005

Notes:

- For underground water tanks, major two loading cases shall be taken into account:
    - The tank is full of water and no backfill: this case happens mainly during water tightness test.
    - The tank is empty and there is backfill.
- The minimum concrete strength is 28MPa.
- The minimum thickness of buried wall or wall subjected to water and its height is more than 3.0m, is 300mm.

In addition to that, construction method shall be considered in the design stage.

The design of a water tank for seismic loads can be done with the reference to ACI 350.3-06; Seismic Design of Liquid-Containing Concrete Structures and Commentary.

Example: Rectangular on-ground water tank:

Given:

- Concrete strength, f'c= 28MPa
- Steel yield strength, fy= 420MPa
- Rectangular on-ground tank.
- No roof.
- Tank area dimensions: 25m x 25m
- Tank height= 5.0m
- Soil allowable bearing capacity, qall= 340kN/m^2
- Assume severe exposure.

Solution:

$$\frac{L}{H} = \frac{25}{5} > 5$$

So, the tank is shallow.

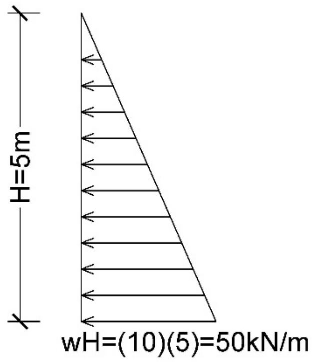


Figure 15.4: Load on wall of tank - example

Vertical direction:

Shear= reaction = area under the load diagram = (0.5)(5)(50)=125kN

Moment = shear x (h/3)=125(5/3)=208.33kN.m

Load combination: U=1.4D+1.4F

$$V_u = (1.4)(125) = 175kN$$

$$M_u = (1.4)(208.33) = 291.66kN.m$$

Try wall thickness at top= 300mm

Try wall thickness at bottom= 500mm d= 430mm

Check shear in walls:

$$\emptyset V_c = (0.75)(1/6)\sqrt{28}(1000)(430)/1000 = 284kN > 175kN \quad OK$$

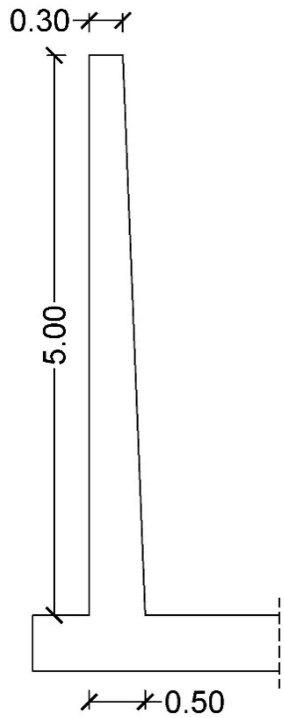


Figure 15.5: Section in wall of the rectangular tank - example

Design for flexure:

Let S=150mm

Let db=25mm

$\beta = 1.2 \; since \; 500mm \; > \; 400mm$

$$f_s = \frac{45560}{\beta\sqrt{S^2 + 4\left(50 + \frac{d_b}{2}\right)^2}} MPa = \frac{45560}{(1.2)\sqrt{(150)^2 + 4\left(50 + \frac{25}{2}\right)^2}} = 194MPa < 250MPa$$

And > 117MPa for one-way member – sever exposure

$$S_d = \frac{\emptyset f_y}{\gamma f_s} = \frac{(0.9)(420)}{(1.4)(194)} = 1.4$$

$$M_u{}' = (1.4)(291.66) = 408.3kN.m$$

$\rho = 0.0062 \; > \; 0.00333 \; OK \; , A_s = (0.0062)(1000)(430) = 2666mm^2 \;\; 1\emptyset 25/150mm)$

Check stress in reinforcing steel at service load:

$A_s = 3273mm^2 \qquad for\ 1\emptyset 25/150mm$

$$n = \frac{E_s}{E_c} = \frac{200000}{4700\sqrt{28}} = 8$$

$$\rho = \frac{3273}{(1000)(430)} = 0.00761$$

$$k = -n\rho + \sqrt{(n\rho)^2 + 2n\rho} = 0.2934$$

$$j = 1 - \frac{k}{3} = 0.902$$

$$f_s = \frac{M}{A_s jd} = \frac{208.33(10)^6}{(3273)(0.902)(430)} = 164MPa\ < 194MPa \quad OK$$

ACI 318-19: shear:

$$\lambda_s = \sqrt{\frac{2}{1 + 0.004\ d}} \leq 1.0 \rightarrow \lambda_s = \sqrt{\frac{2}{1 + 0.004\ (430)}} = 0.86$$

$$\rho_w = 0.0076$$

$$\emptyset V_c = \emptyset\left(0.66\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right)b_w d$$

$$\emptyset\ V_c = 0.75\frac{\left(0.66(0.86)(1)(0.0076)^{\frac{1}{3}}\sqrt{28} + 0.0\right)(1000)(430)}{1000} = 191kN > 175kN \quad ok.$$

Horizontal direction:

Total force = (0.5)(5)(5)(10x5/2)=312.5kN

Total moment= Total shear x Distance = 312.5(0.5x5)=781.25kN.m

Force/m= 312.5/5= 62.5kN

Moment/m= 781.25/5=156.25kN.m

$M_u = (1.4)(156.25) = 218.75kN.m$

$P_u = (1.4)(62.5) = 87.5kN$

Design for flexure:

$M_u{}' = (218.75)(1.4) = 306.3kN.m$

b= 1000mm

h= 400mm

d= 330mm

$\rho = 0.008 \quad > \; 0.00333\; OK \qquad , A_s = (0.008)(1000)(330) = 2640mm^2$

Use the above procedure to check fs at service loads.

Design for tension:

$$S_d = \frac{\emptyset f_y}{\gamma f_s} = \frac{(0.9)(420)}{(1.4)(117)} = 2.31$$

$$P_u{}' = (2.31)(87.5) = 202.125kN$$

$$A_s = \frac{P_u}{\emptyset f_y} = \frac{P}{f_s} = \frac{202.125(1000)}{(0.9)(420)} = 535mm^2$$

Total horizontal steel at side of water= 2665 + (535/2) = 2933mm^2 $(1\emptyset 25/150mm)$

Minimum horizontal steel at a side = (0.005/2)(1000)(400)=1000mm^2 < 2933mm^2 OK

Horizontal steel at outer side of wall = 1000mm^2 $(1\emptyset 14/150mm)$

Vertical steel at outer side of wall = (0.003/2)(1000)(500)= 750mm^2 $(1\emptyset 14/200mm)$

See Figure 15.6.

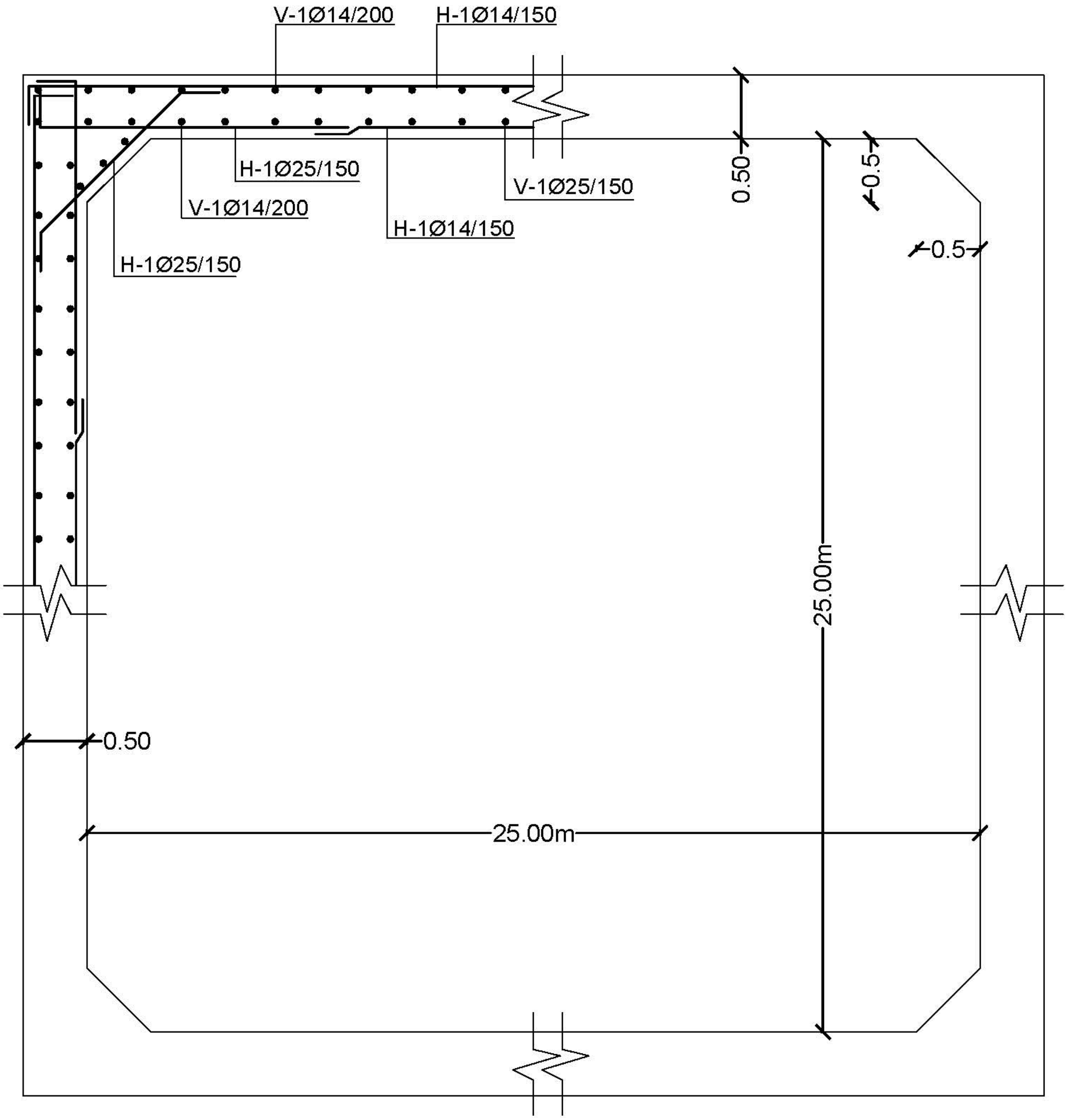


Figure 15.6: Horizontal cross section in tank

15.2 Design of circular water tanks:

The loads are transferred in the horizontal direction as hoop tension in the cylindrical walls. When there are no restraints at the wall ends, the total load is transferred to the walls as tension. But when there are restraints at top and or at bottom ends, bending moments developed and the tension in the walls decreased at these regions.

Cylindrical tanks can be divided into:

- Free base: tension only in the walls
- Fixed base: tension, moment and shear in the walls

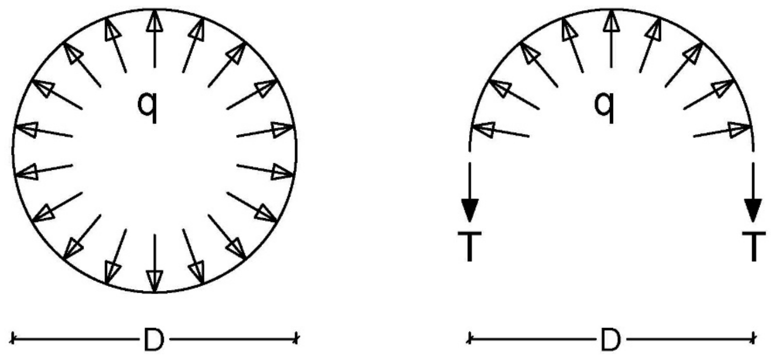


Figure 15.7: Loads at ring (horizontal section in cylindrical wall)

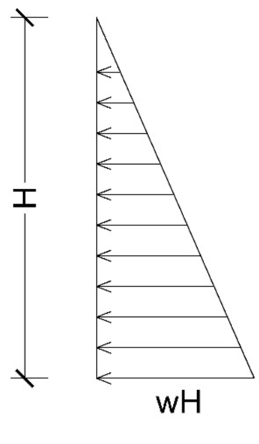


Figure 15.8: Loads at cylindrical walls – vertical distribution

$$\sum F_Y = 0 \ \rightarrow \ 2T = qD \rightarrow \ T = \frac{1}{2}qD = qr = wHr$$

The hoop tension in concrete must be checked to be less than ft.

$$f_t = 0.333\lambda\sqrt{f'_c}$$

The stress in concrete is given by:

$$f_c = \frac{CE_sA_s + T}{A_g + nA_s}$$

Where:

C= coefficient of shrinkage of reinforced concrete. It is in the range of 0.0002 and 0.0004. It can be taken equal to 0.0003. ($3x10^{-4}$).

When the bottom of walls is fixed, moments and shears are developed and the maximum tensile force in the walls will be less than that for free base.

The following tables show the tension, moments and shear in walls of cylindrical tank with fixed base.

CYLINDRICAL TANKS WITH FIXED BASE, FREE TOP

Coefficients for tension in circular ring **Triangular Load**

T= Coefficient x (wHR)

Positive values indicate tension

w= water unit weight H= depth of water R= radius of tank
D= diameter of tank t= thickness of wall

H^2/Dt	Coefficient at a point									
	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	0.149	0.134	0.12	0.101	0.082	0.066	0.049	0.029	0.014	0.004
0.8	0.263	0.239	0.215	0.19	0.16	0.13	0.096	0.063	0.034	0.01
1.2	0.283	0.271	0.254	0.234	0.209	0.18	0.142	0.099	0.054	0.016
1.6	0.265	0.268	0.268	0.266	0.25	0.226	0.185	0.134	0.075	0.023
2	0.234	0.251	0.273	0.285	0.285	0.274	0.232	0.172	0.104	0.031
3	0.134	0.203	0.267	0.322	0.357	0.362	0.303	0.262	0.157	0.052
4	0.067	0.164	0.256	0.339	0.403	0.429	0.409	0.334	0.21	0.073
5	0.025	0.137	0.245	0.346	0.428	0.477	0.469	0.398	0.259	0.092
6	0.018	0.119	0.234	0.344	0.441	0.504	0.514	0.447	0.301	0.112
8	-0.011	0.104	0.218	0.335	0.443	0.534	0.575	0.53	0.381	0.151
10	-0.011	0.098	0.208	0.323	0.437	0.542	0.608	0.589	0.44	0.179
12	-0.005	0.097	0.202	0.312	0.429	0.543	0.628	0.633	0.494	0.211
14	-0.002	0.098	0.2	0.306	0.42	0.539	0.639	0.666	0.541	0.241
16	0	0.099	0.199	0.301	0.413	0.531	0.641	0.687	0.582	0.265

H^2/Dt	Coefficient at a point				
	0.75H	0.80H	0.85H	0.90H	0.95H
20	0.716	0.654	0.52	0.325	0.115
24	0.746	0.702	0.577	0.372	0.137
32	0.782	0.768	0.663	0.459	0.182
40	0.8	0.805	0.731	0.53	0.217
48	0.791	0.828	0.785	0.593	0.254
56	0.763	0.838	0.824	0.536	0.285

CYLINDRICAL TANKS WITH FIXED BASE, FREE TOP

Coefficients for moments in cylindrical walls **Triangular Load**

M= Coefficient x (wH^3)

Positive values indicate tension in the outside face of wall

w= water unit weight H= depth of water R= radius of tank

D= diameter of tank t= thickness of wall

H^2/Dt	Coefficient at a point									
	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
0.4	0.0005	0.0014	0.0021	0.0007	-0.0042	-0.015	-0.0302	-0.0529	-0.0816	-0.1205
0.8	0.0011	0.0037	0.0063	0.008	0.007	0.0023	-0.0068	-0.0224	-0.0465	-0.0705
1.2	0.0012	0.0042	0.0077	0.0103	0.0112	0.009	0.0022	-0.0108	-0.0311	-0.0602
1.6	0.0011	0.0041	0.0075	0.0107	0.0121	0.0111	0.0058	-0.0051	-0.0232	-0.0505
2	0.001	0.0035	0.0068	0.0099	0.012	0.0115	0.0075	-0.0021	-0.0185	-0.0436
3	0.0006	0.0024	0.0047	0.0071	0.009	0.0097	0.0077	0.0012	-0.0119	-0.0333
4	0.0003	0.0015	0.0028	0.0047	0.0066	0.0077	0.0069	0.0023	-0.008	-0.0268
5	0.0002	0.0008	0.0016	0.0029	0.0046	0.0059	0.0059	0.0028	-0.0058	-0.0222
6	0.0001	0.0003	0.0008	0.0019	0.0032	0.0046	0.0051	0.0029	-0.0041	-0.0187
8	0	0.0001	0.0002	0.0008	0.0016	0.0028	0.0038	0.0029	-0.0022	-0.0146
10	0	0	0.0001	0.0004	0.0007	0.0019	0.0022	0.0028	-0.0012	-0.0122
12	0	-0.0001	0.0001	0.0002	0.0003	0.0013	0.0023	0.0026	-0.0005	-0.0104
14	0	0	0	0	0.0001	0.0008	0.0019	0.0023	-0.0001	-0.009
16	0	0	-0.0001	-0.0001	-0.0001	0.0004	0.0013	0.0019	-0.0001	-0.0079

H^2/Dt	Coefficient at a point				
	0.80H	0.85H	0.90H	0.95H	1.0H
20	0.0015	0.0014	0.0005	-0.0018	-0.0063
24	0.0012	0.0012	0.0007	-0.0013	-0.0053
32	0.0007	0.0009	0.0007	-0.0008	-0.004
40	0.0002	0.0005	0.0006	-0.0005	-0.0032
48	0	0.0001	0.0006	-0.0003	-0.0026
56	0	0	0.0004	-0.0001	-0.0023

Coefficients for shear at base **shear, V= coefficient x (wH^2)**

H^2/Dt	0.4	0.8	1.2	1.6	2	3	4	5	6	8
Coeff.	0.436	0.374	0.339	0.317	0.299	0.262	0.236	0.213	0.197	0.174
H^2/Dt	10	12	14	16	20	24	32	40	48	56
Coeff.	0.158	0.145	0.135	0.127	0.114	0.102	0.089	0.08	0.072	0.067

Example 1:

Design the walls of a circular water tank with free base for capacity of 1000m^3 and height of 6.0m with free board of 0.25m. Concrete strength, f'c= 28MPa and steel strength, fy= 420MPa. Assume sever exposure.

Solution:

- Determine diameter of tank:

$$Tank\ volume, V = \frac{\pi}{4} D^2 H = 1000 = \frac{\pi}{4} D^2 (6 - 0.25) \rightarrow D = 15m$$

- Forces in the wall:

$$Tension, T = qr = (6)(10)(15/2) = 450kN$$

- Design the wall:

$$T_u{}' = S_d T_u$$

$$T_u = (1.4)(450) = 630kN$$

$$S_d = \frac{\emptyset f_y}{\gamma f_s} = \frac{(0.9)(420)}{(1.4)(117)} = 2.31$$

$$T_u{}' = (2.31)(630) = 1455.3kN$$

$$A_s = \frac{T_u{}'}{\emptyset f_y} = \frac{(1455.3)(1000)}{(0.90)(420)} = 3850mm^2/m$$

Or:

$$A_s = \frac{T}{f_s} = \frac{(450)(1000)}{117} = 3850mm^2/m$$

Use $1\emptyset 18/125mm$. As= 4064mm^2.

This reinforcement is for the maximum tension in the walls at bottom of wall. The tensile force can be calculated at different heights and then the required reinforcing steel can be calculated. This can be done for economical purposes.

The minimum horizontal reinforcing steel is 0.005Ag and the minimum vertical reinforcing steel is 0.003Ag.

The stress in concrete is given by:

$$f_c = \frac{CE_sA_s + T}{A_g + nA_s} = \frac{(0.0003)(200000)(4064) + 450(1000)}{(1000)(t) + \frac{200000}{4700\sqrt{28}}(4064)} = 0.333(1)\sqrt{28} \rightarrow t$$
$$= 400mm$$

$$A_{s,minimum} = (0.005)(1000)(400) = 2000mm^2 > 3850mm^2 \quad OK$$

Vertical reinforcement:

$$A_s = 0.003(1000)(400) = 1200mm^2. \quad (600mm^2 \ at\ each\ face: 1\emptyset14/250mm)$$

Example 2:

Resolve the previous example assuming fixed base.

Solution:

Assume wall thickness, t= 0.30m

$$\frac{H^2}{Dt} = \frac{(6)^2}{(15)(0.3)} = 8$$

Tension:

Maximum hoop tension occurs at 0.6H from top.

Maximum hoop tension is:

$$T = 0.575wHR = (0.575)(10)(6)(7.5) = 259kN$$

$$T_u' = S_dT_u$$

$$T_u = (1.4)(259) = 362.6kN$$

$$S_d = \frac{\emptyset f_y}{\gamma f_s} = \frac{(0.9)(420)}{(1.4)(117)} = 2.31$$

$$T_u' = (2.31)(362.6) = 838kN$$

$$A_s = \frac{T_u'}{\emptyset f_y} = \frac{(838)(1000)}{(0.90)(420)} = 2217mm^2/m \quad use\ 1\emptyset14/125mm, A_s = 2464mm^2$$
$$Or\ use\ 1\emptyset16/150mm, A\_s = 2680mm^2$$

$$A_{s,minimum} = 0.005(1000)(300) = 1500mm^2 < 2217mm^2 \quad OK$$

Wall thickness:

The stress in concrete is given by:

$$f_c = \frac{CE_sA_s + T}{A_g + nA_s} = \frac{(0.0003)(200000)(2464) + (259)(1000)}{(1000)(300) + \frac{200000}{4700\sqrt{28}}(2464)} = 1.27MPa$$
$$< 0.333(1)\sqrt{28} = 1.75MPa \qquad OK$$

Shear:

Maximum shear is located at bottom of wall:

$$V = 0.174wH^2 = (0.174)(10)(6)^2 = 62.64kN$$

$$V_u = (1.4)(62.64) = 88kN$$

$$\emptyset V_c = (0.75)\left(\frac{1}{6}\right)\sqrt{28}(1000)(230)/1000 = 152kN \ > 88kN \qquad OK$$

$$ACI\ 318-19: \emptyset V_c = \emptyset\left(0.66\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right)b_wd$$

$$\emptyset V_c = \frac{0.75\left(0.66(1)(1)(0.00333)^{\frac{1}{3}}\sqrt{28} + 0.0\right)(1000)(230)}{1000} = 90.1kN > 88kN \quad ok$$

Flexure:

Maximum positive moment is located at 0.7H:

$$M^+ = 0.0038wH^3 = (0.0038)(10)(6)^3 = 8.21kN.m$$

Maximum negative moment is located at bottom of wall:

$$M^- = 0.0146wH^3 = (0.0146)(10)(6)^3 = 31.5kN.m$$

For negative moment:

$$M_u' = S_dM_u$$

$$f_s = \frac{45560}{\beta\sqrt{S^2 + 4\left(50 + \frac{d_b}{2}\right)^2}}MPa = \frac{45560}{(1.35)\sqrt{(150)^2 + 4\left(50 + \frac{14}{2}\right)^2}} = 179MPa$$
$$< 250MPa$$

$$> 138MPa \quad for\ two\ way\ member.$$

$$S_d = \frac{\emptyset f_y}{\gamma f_s} = \frac{(0.9)(420)}{(1.4)(179)} = 1.51$$

$$M_u{}' = S_d M_u = (1.51)(1.4)(31.5) = 68kN.m$$

$$\rho = 0.0035 > 0.00333, A_s = (0.0035)(1000)(230) = 805mm^2 \quad use\ 1\emptyset 14/150mm$$

The stress in this steel can be checked using the service design method as shown in the example of rectangular tank.

For positive moment of 8. 21kN.m, one can use minimum steel of $\rho = 0.00333$.

$$A_s = 0.00333(1000)(230) = 766mm^2 \quad use\ 1\emptyset 14/200mm$$

Chapter 16: Design of Shell Structures: Spherical domes and Conical shells

16.1 Design of spherical domes:

The internal forces, the dome geometry and the normal distributed force are related together by the following equation:

$$\frac{N_{\emptyset}}{R_1} + \frac{N_{\theta}}{R_2} = p_r$$

Where:

R1: the radius of curvature of the meridian, m

R2: the radius of curvature of the second principal curve (horizontal direction), m

$N_{\emptyset}$: the meridian force, kN/m

$N_{\theta:}$ the hoop (horizontal) force, kN/m

P_r: the normal force (distributed) on the shell surface toward the center, kN/m^2

Surface loads:

The surface load on the dome is defined as g in kN/m^2 downward (-Z direction).

The internal force $N_{\emptyset}$ can be determined as follows:

The load on surface area is given by:

$$W_o = \int_0^{\emptyset} g 2\pi r R d\emptyset_1$$

$$W_o = \int_0^{\emptyset} 2\pi g R^2 sin\emptyset_1 d\emptyset_1$$

$$= -2\pi g R^2 cos\emptyset_1 {}_0^{\emptyset}$$

$$= -2\pi g R^2 (\cos\emptyset - 1)$$

$$= 2\pi g R^2 (1 - cos\emptyset)$$

$$Vertical\ reaction = 2\pi r N_{\emptyset} sin\emptyset = 2\pi(R\ sin\emptyset)N_{\emptyset} sin\emptyset = 2\pi R\ sin^2\emptyset N_{\emptyset}$$

Surface load = reaction

$$2\pi g R^2(1 - cos\emptyset) = 2\pi R\ sin^2\emptyset N_{\emptyset}$$

$$gR(1 - cos\emptyset) = sin^2\emptyset N_{\emptyset} \qquad (Divided\ by\ 2\pi R)$$

$$N_{\emptyset} = \frac{gR(1 - cos\emptyset)}{sin^2\emptyset} = \frac{gR(1 - cos\emptyset)}{(1 - cos^2\emptyset)} = \frac{gR(1 - cos\emptyset)}{(1 - cos\emptyset)(1 + cos\emptyset)} = \frac{gR}{1 + cos\emptyset}$$

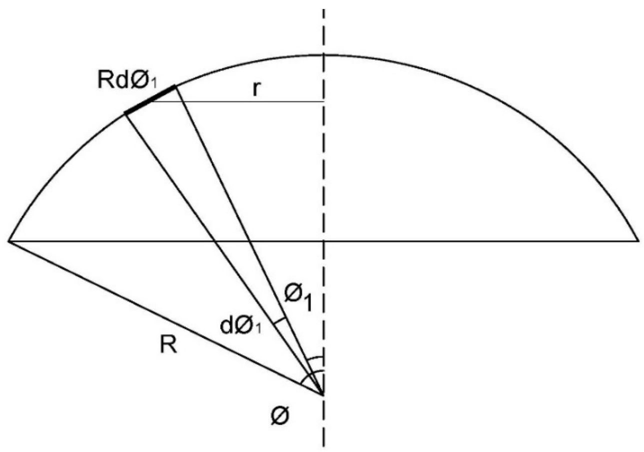


Figure 16.1: Spherical dome – derivation for $N_{\emptyset}$

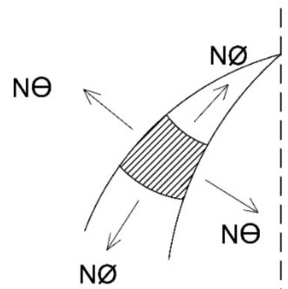


Figure 16.2: Internal forces in spherical dome

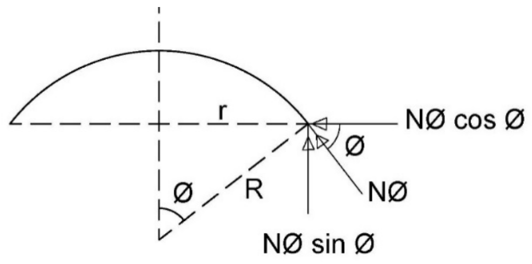


Figure 16.3: Components of $N_{\emptyset}$ in spherical dome

The normal component of g is **g cos**∅ as shown in Figure 16.4.

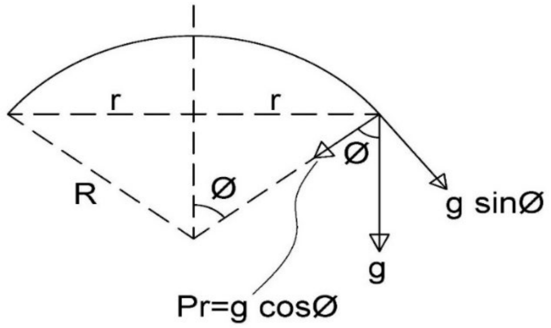


Figure 16.4: Pr in spherical dome due to load g

In spherical domes, R1=R2=R

$$\frac{N_{\emptyset}}{R}+\frac{N_{\theta}}{R}=P_r$$

$$\frac{gR}{(1+cos\emptyset)R}+\frac{N_{\theta}}{R}=gcos\emptyset$$

$$\frac{N_{\theta}}{R}=g\ cos\emptyset-\frac{g}{1+cos\emptyset}$$

$$N_{\theta}=gR(cos\emptyset-\frac{1}{1+cos\emptyset})$$

$$At\ \emptyset=51^{o}50',N_{\theta}=0$$

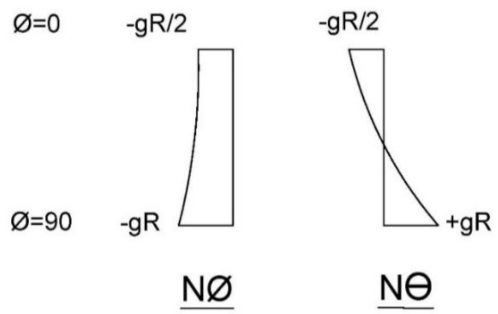


Figure 16.5: Variation of $N_\emptyset$ and N_θ due to $surface$ $load$ in $spherical$ $dome$

Projected loads:

Total live load on the section= $q\pi r^2$

Reaction= $2\pi r N_\emptyset sin\emptyset$

Total live load = Reaction

$$q\pi r^2 = 2\pi r N_\emptyset sin\emptyset$$

$$N_\emptyset = \frac{qr}{2\ sin\emptyset} = \frac{q\ R\ sin\emptyset}{2\ sin\emptyset} = \frac{qR}{2}$$

$$\frac{N_\emptyset}{R} + \frac{N_\theta}{R} = P_r$$

$$\frac{qR}{2R} + \frac{N_\theta}{R} = q\ cos^2\emptyset$$

Where: $P_r = surface\ laod\ x\ cos\emptyset = \ projected\ load\ x\ cos\emptyset\ x\ cos\emptyset$

$$\frac{N_\theta}{R} = q\ cos^2\emptyset - \frac{q}{2}$$

$$N_\theta = qR(cos^2\emptyset - \frac{1}{2}) = \frac{qR}{2}(2\ cos^2\emptyset - 1) = \frac{qR}{2} cos\ 2\emptyset$$

Figure 16.6 shows the distributions of $N_\emptyset$ and N_θ for the angles from zero to 90 degrees.

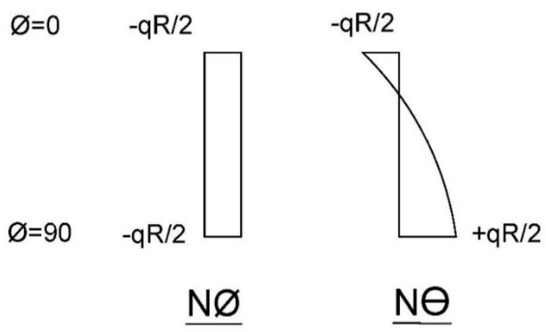


Figure 16.6: Variation of $N_{\emptyset}$ $and\ N_{\theta}\ due\ to\ live\ load\ in\ spherical\ dome$

$$N_{\theta} = 0\ at\ angle, \emptyset = 45\ degrees$$

Load on the ring beam:

The dome is attached to a ring beam in many structures. There are two cases:

Case 1: the ring beam is attached to walls. Here, there are no spans for this beam, and so no shear or moment is developed. The main internal force in the ring beam in this case is tension. Torsion may be developed.

Case 2: the ring beam is supported on columns. Here, there are spans to the beam, and so, internal forces of shear, moments and torsion are developed in addition to the tensile force.

The vertical load on the ring beam is: $N_{\emptyset} sin\emptyset$

The horizontal load on the ring beam is: $N_{\emptyset} cos\emptyset$

The tension force in the ring beam is given by:

$$Tension, T = N_{\emptyset} cos\emptyset\ r$$

Where: r is the radius of the ring beam.

For the beam in case 2, the vertical load causes the internal forces of shear, moment and torsion.

Table 16.1 shows the internal forces in the ring beam that is supported by columns. Here, the vertical load on the ring beam is denoted by q and the radius of the ring beam is denoted by r.

Table 16.1: Internal forces in ring beam supported by columns

No. of columns	***Load on column***	***shear***	***Moment at midspan***	***Moment at support***	***Torsion***
4	$2\pi rq/4$	$2\pi rq/8$	$0.11qr^2$	$0.22qr^2$	$0.033qr^2$
6	$2\pi rq/6$	$2\pi rq/12$	$0.047qr^2$	$0.1qr^2$	$0.0088qr^2$
8	$2\pi rq/8$	$2\pi rq/16$	$0.026qr^2$	$0.052qr^2$	$0.0038qr^2$
12	$2\pi rq/12$	$2\pi rq/24$	$0.012qr^2$	$0.023qr^2$	$0.0013qr^2$

Example:

Given:

- Spherical dome
- It is a roof for cylindrical tank. Tank diameter= 25m
- Dome height= 4m
- Dome thickness, t= 0.12m
- Concrete strength, f'c=28MPa
- Steel strength, fy= 420MPa
- Live load (projected)= 1.5kN/m^2

Design the dome and the ring beam.

Solution:

The radius, R and the inclination angle, Ø are determined as follows:

Refer to Figure 16.7.

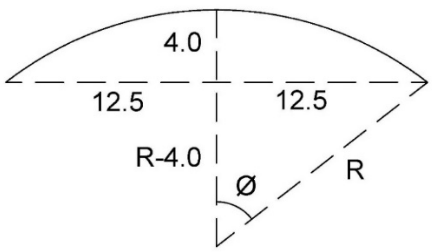


Figure 16.7: Dome properties

$$(12.5)^2 + (R-4)^2 = R^2 = 156.25 + R^2 + 16 - 8R$$

$$172.25 = 8R \rightarrow R = 21.531m$$

$$sin\ \emptyset = \frac{12.5}{21.531} = 0.5806 \rightarrow \emptyset = 35.5^o$$

Loads:

Weight of dome, WD=0.12(25)=3.0kN/m^2

Live load, WL= 1.5kN/m^2

$N_\emptyset$ *and* N_θ from dead load:

$$N_\emptyset = \frac{gR}{1+cos\emptyset}$$

$$N_\theta = gR(cos\emptyset - \frac{1}{1+cos\emptyset})$$

$$N_{\emptyset,\emptyset=0.0} = -32.3kN/m$$

$$N_{\emptyset,\emptyset=35.5} = -35.6kN/m$$

$$N_{\theta,\emptyset=0.0} = -32.3kN/m$$

$$N_{\theta,\emptyset=35.5} = -16.98kN/m$$

$N_\emptyset$ *and* N_θ from live load:

$$N_\emptyset = \frac{qR}{2}$$

$$N_\theta = \frac{qR}{2}cos\ 2\emptyset$$

$$N_{\emptyset,\emptyset=0.0} = -16.1kN/m$$

$$N_{\emptyset,\emptyset=35.5} = -16.1kN/m$$

$$N_{\theta,\emptyset=0.0} = -16.1kN/m$$

$$N_{\theta,\emptyset=35.5} = -5.3kN/m$$

Note that $N_{\emptyset}$ and N_{θ} are compression.

$$N_{\emptyset,max} = N_{\emptyset,dead} + N_{\emptyset,live}$$

$$N_{\emptyset,max} = 35.6 + 16.1 = 51.7kN/m$$

$$N_{\emptyset,max,ultimate} = 1.2(35.6) + 1.6(16.1) = 68.48kN/m \qquad at\ \emptyset = 35.5\ degrees$$

$$N_{\theta,max} = N_{\theta,dead} + N_{\theta,live}$$

$$N_{\theta,max} = 32.3 + 16.1 = 48.4kN/m$$

$$N_{\theta,max,ultimate} = 1.2(32.3) + 1.6(16.1) = 64.52kN/m \qquad at\ \emptyset = 0.0\ degrees$$

The compressive capacity of section is given by:

$$\emptyset P_n = (0.65)(0.80)(0.85f'_c(A_g - A_s) + f_y A_s)$$

$$\emptyset P_n = (0.65)(0.80)(0.85)f'_c\, A_g = (0.442)(28)(1000)(120)/1000 = 1485kN \gg 68.48kN \quad OK.$$

Use minimum steel area in the dome, As= 0.003(1000)(120)=360mm^2. Use $1\emptyset 10/200mm$.

Ring beam:

The ring beam is supported on the walls of tank.

Horizontal force on the ring beam, P=$N_{\emptyset}\ cos\ \emptyset = (51.7)(cos\ 35.5) = 42.09kN/m$

Tension in the ring beam, T= P.r= (42.09)(12.5)=526kN

The required reinforcement is given by:

$$A_s = \frac{T}{f_s} = \frac{526000}{117} = 4496mm^2$$

The tensile stress in the ring beam is given by:

$$f_c = \frac{CE_sA_s + T}{A_g + nA_s} = \frac{(0.0003)(200000)(4496) + (526)(1000)}{(A_g) + \frac{200000}{4700\sqrt{28}}(4496)} = 0.333(1)\sqrt{28} = 1.75MPa \quad \rightarrow A_g = 416194mm^2$$

Use square section with side length= 650mm.

Use 15Ø20

16.2 Design of Conical Shells:

The following paragraphs illustrate the calculations of the membrane forces for a conical shell that is supported at its bottom end.

The surface downward load is P in kN/m^2.

If the load P is the self-weight, then it is equal to $\gamma_c\ t$, where γ_c is the unit weight of concrete and t is the thickness of the shell.

Figure 16.8 shows a section in a conical shell.

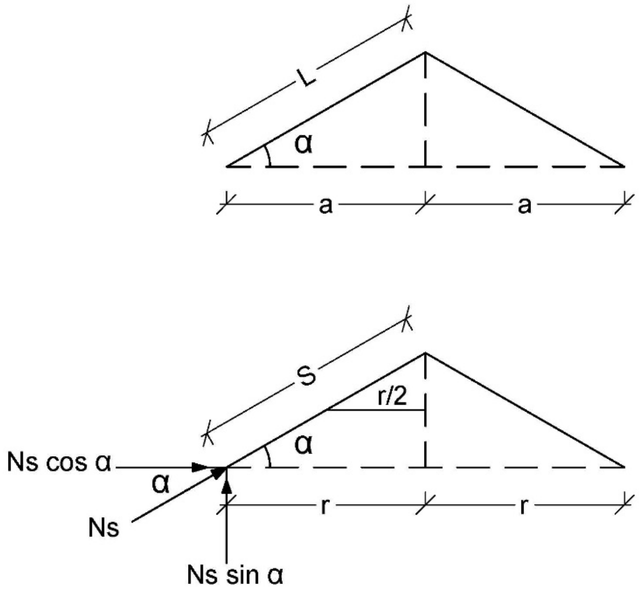


Figure 16.8: Conical shell

Weight of the conical shell is given by:

$$W = 2\pi \frac{r}{2} SP$$

The reaction or the internal force is given by:

$$R = 2\pi r\ N_s \sin \alpha$$

$$W = R = 2\pi \frac{r}{2} SP = 2\pi r\ N_s \sin \alpha \rightarrow N_s = \frac{-SP}{2 \sin\alpha}$$

The hoop force N_θ can be determined as follows:

$$N_\theta = P_r\, r_2$$

Refer to Figure 16.9.

$$tan\ \alpha = \frac{S}{r_2} \rightarrow r_2 = \frac{S}{tan\ \alpha} = S\ cot\ \alpha$$

And,

$$P_r = P\ cos\ \alpha$$

$$N_\theta = P_r\, r_2 = P\ cos\ \alpha\ S\ cot\ \alpha$$

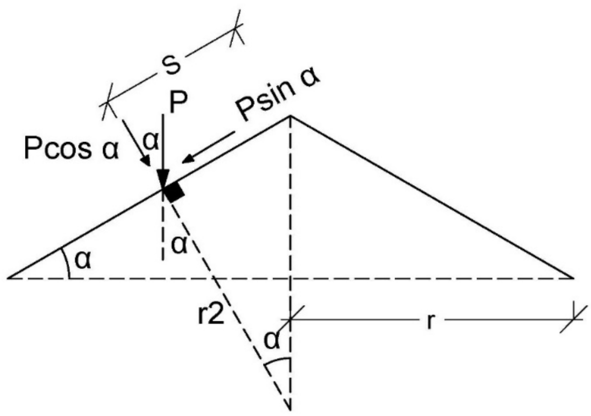


Figure 16.9: Conical shell, determination of N_θ

Example 1:

Given:

- Conical shell
- It is a roof for a circular tank. Diameter of tank= 25m.
- Height of shell=4m
- Shell thickness, t= 0.12m
- Live load on surface= 1.5kN/m^2
- Concrete strength, f’c= 28MPa
- Steel strength, fy= 420MPa

Design the shell and the ring beam.

Solution:

$$P_D = (0.12)(25) = 3kN/m^2$$

$$P_L = 1.5kN/m^2$$

$$P = P_D + P_L = 3 + 1.5 = 4.5kN/m^2$$

$$P_u = 1.2P_D + 1.6P_L = 1.2(3) + 1.6(1.5) = 6kN/m^2$$

$$N_\theta = P_r\, r_2 = P\, cos\, \alpha\, S\, cot\, \alpha$$

$$N_s = \frac{SP}{2sin\alpha}$$

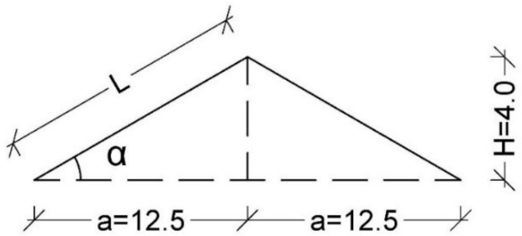


Figure 16.10: Conical shell example

$$tan\, \alpha = \frac{4}{12.5} \rightarrow \alpha = 17.74\, degrees$$

$$S = L = \sqrt{4^2 + 12.5^2} = 13.12m$$

Substitute in the previous equations:

$$N_\theta = P_r\, r_2 = P\, cos\, \alpha\, S\, cot\, \alpha = (4.5)(13.12)(cos\, 17.74)(cot\, 17.74)$$
$$= 175.8kN/m \quad compression$$

$$N_{\theta,u} = P_r\, r_2 = P\, cos\, \alpha\, S\, cot\, \alpha = (6)(13.12)(cos\, 17.74)(cot\, 17.74)$$
$$= 234.4kN/m \quad compression$$

$$N_s = \frac{SP}{2sin\alpha} = \frac{(13.12)(4.5)}{2\, sin\, 17.74} = 97kN/m \;\; compression$$

$$N_{s,u} = \frac{SP}{2sin\alpha} = \frac{(13.12)(6)}{2\, sin\, 17.74} = 129.2kN/m \;\; compression$$

The tension in the ring beam is given by:

$$T = N_s cos\alpha\ a = (97)(cos\ 17.74)(12.5) = 1155kN$$

The required reinforcement is given by:

$$A_s = \frac{T}{f_s} = \frac{1155000}{117} = 9872mm^2$$

The tensile stress in the ring beam is given by:

$$f_c = \frac{CE_sA_s + T}{A_g + nA_s} = \frac{(0.0003)(200000)(9872) + (1155)(1000)}{(A_g) + \frac{200000}{4700\sqrt{28}}(9872)} = 0.333(1)\sqrt{28}$$
$$= 1.75MPa \quad \rightarrow A_g = 912258mm^2$$

Use square section with side length= 1000mm.

Use 26Ø22

Example 2:

Determine the membrane forces in the concrete umbrella shown in Figure 16.11 at point A.

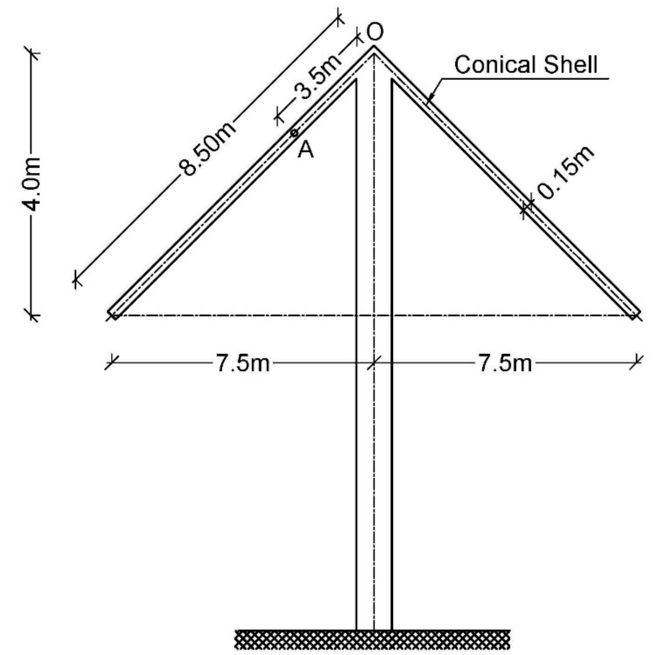


Figure 16.11: Reinforced concrete umbrella

Solution:

$$P = (0.15)(25) = 3.75kN/m^2$$

$$tan\ \alpha = \frac{4}{7.5} \rightarrow \alpha = 28.07\ degrees$$

$$\frac{x_1}{5} = cos\ \propto \rightarrow x_1 = 4.41m \rightarrow x_2 = 7.5 - 4.41 = 3.09m$$

$$(3.75)(5)(2\pi)(\frac{7.5 + 3.09}{2}) = N_s\ sin\ \alpha\ (2\pi)(3.09) \rightarrow N_s = 68.3kN/m\ \ tension$$

$$N_\theta = P_r r_2 = P\ cos\ \alpha\ r_2 \qquad tan\ 28.07\ = \frac{3.5}{r_2} \rightarrow r_2 = 6.56m$$

$$N_\theta = P\ cos\ \alpha\ r_2\ =\ (3.75)(cos\ 28.07)(6.56) = 21.7\ kN/m \quad compression$$

Example 3:

For the reinforced concrete tank shown in Figure 16.12, determine the membrane forces at point A due to:

- Dome loads
- Tank self-weight
- Water loads

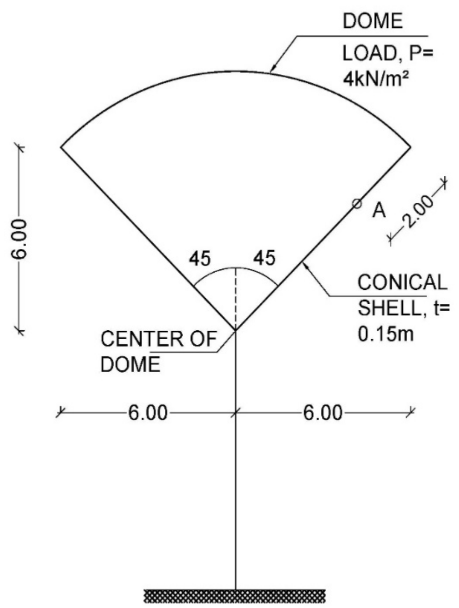


Figure 16.12: Reinforced concrete conical tank

Solution:

Membrane forces due to dome loads:

Refer to Figure 16.13 below.

The horizontal force $N_\emptyset\ cos\emptyset$ will be taken by a ring beam.

Dome radius, R= 8.49m

$$N_\emptyset = \frac{gR}{1 + cos\ \emptyset} = \frac{(4)(8.49)}{1 + cos\ 45} = 20kN/m$$

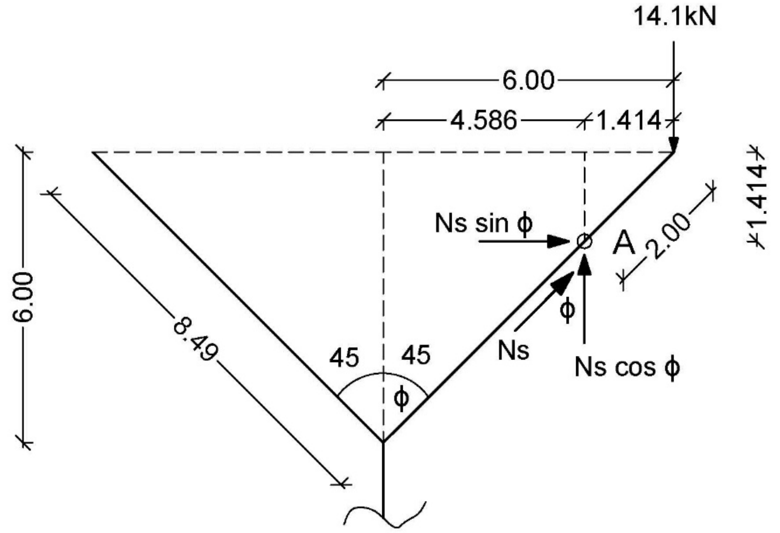


Figure 16.13: Dome loads on the conical shell

Vertical reaction on the conical shell= $N_\emptyset\ sin\ \emptyset = (20)(sin\ 45) = 14.1kN$

$$(14.1)(2\pi)(6) = N_s\ cos\emptyset\ (2\pi)(4.586) \rightarrow N_s = 26.1kN/m$$

Membrane forces due to tank self-weight:

Refer to Figure 16.14.

Dead load (self-weight) = (0.15)(25)=3.75kN/m^2

$$(3.75)(2)(2\pi)(6 - \frac{1.414}{2}) = N_s\ cos\emptyset\ (2\pi)(6 - 1.414) \rightarrow N_s = 12.25kN/m$$

$$N_\theta = P_r . r_2 = P\ sin\emptyset\ r_2 = (3.75)(sin\ 45)(6.485) = 17.2kN/m$$

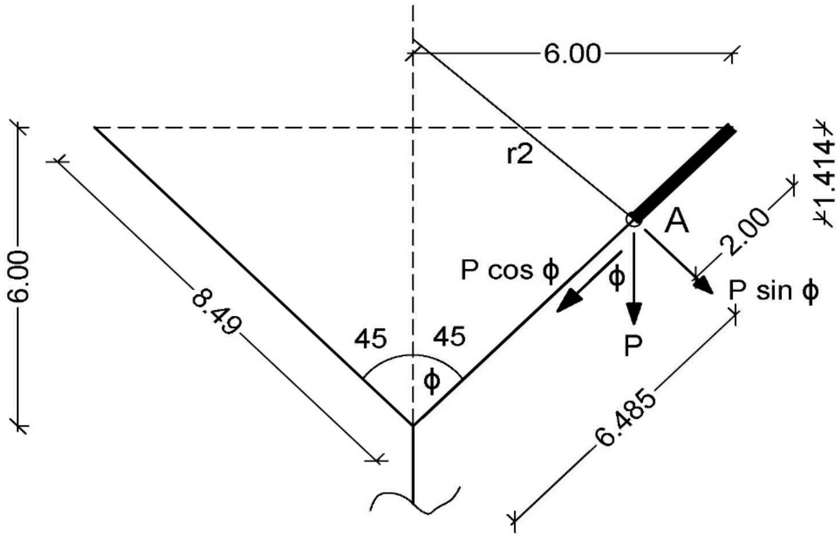


Figure 16.14: Effect of self-weight of tank

Membrane forces due to water loads:

Refer to Figure 16.15 below.

$$(\frac{1}{2})(1.414)(1.414)(10)(2\pi)(6-\frac{2}{3}(1.414)) = N_s\, cos\emptyset\; (2\pi)(4.586) \rightarrow N_s = 15.6kN/m \;\; compression$$

$$N_\theta = P_r.r_2 = \gamma_w h\, r_2 = (10)(1.414)(6.485) = 91.7kN/m \;\; tension$$

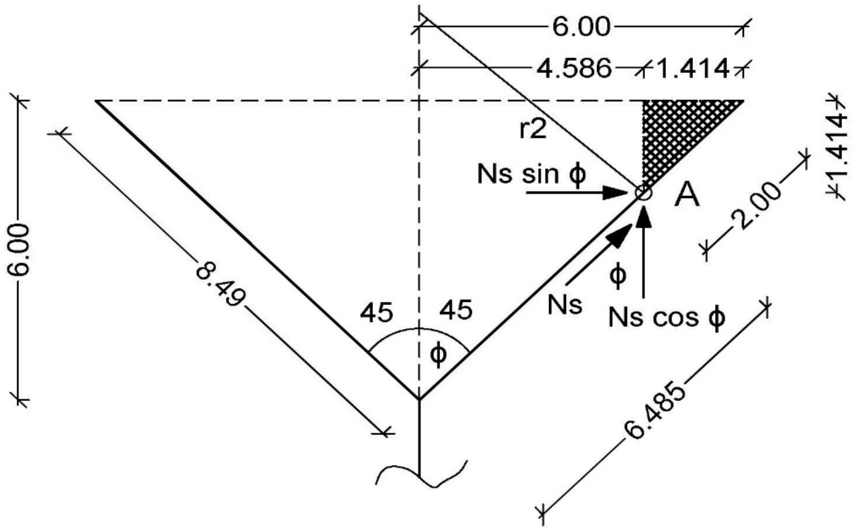


Figure 16.15: Water loads on the conical shell

Chapter 17: Design of Shell Roofs Using Beam Theory

It is known that cylindrical shells with large (L/a) values, spanning between diaphragm supports, tend to behave as simply supported beams. Folded plates, which are composed of interconnected plates, can be considered to span as beams between diaphragm supports.

This leads to consider that the longitudinal stresses in a folded plate or cylindrical shell could be obtained by using simple formula of beam theory; $\sigma_x = \frac{MZ}{I}$, where M is the bending moment and I is the moment of inertia of the cross section about the neutral axis. For symmetrical sections, the neutral axis is horizontal.

In a folded plate or a shell, there are also transverse bending moments, shears and in plane forces ($M_\emptyset, Q_\emptyset\ and\ N_\emptyset$) which are generally not considered for simple beams. These have to be found by a secondary analysis called arch analysis, in the case of shells and folded plates. The analysis of shells by this simple combined procedure is called beam theory.

Deformations of the cross section are not considered when deriving formulas for beams in strength of materials. For shells or folded plates, such deformations of the cross section in the transverse direction do take place, and they can change the resultants $M_\emptyset$ and $N_\emptyset$ obtained from simple beam theory.

Transverse deformations and forces can be neglected for:

Single shells: $\frac{L}{a} \geq 5$

Single shells with edge beams: $\frac{L}{a} \geq 3$

Internal shell of multi-barrel shell structures: $\frac{L}{a} \geq 1.67$

<u>Selection of shell configuration and dimensions:</u>

Although smaller column spacings generally result in economical structure, the requirement for spacing of columns; in both the longitudinal and transverse directions; is usually directed by the client, depending on what size of column – free bays he can work with.

The loads from the shell are transmitted to the diaphragm beams, which span on columns in the transverse direction. Hence the width of each bay of multi-bay shell need not have any relation to the column spacing in the transvers direction, since each span of the diaphragm beam can carry loads from one or more shell bays, or from any fractions thereof.

The depth of the shell can be:

$$\frac{L}{8} \ to \ \frac{L}{12}$$

The thickness of the cylindrical shell can be:

$$\frac{L}{160} \ to \ \frac{L}{200}$$

The thickness of the folded plate can be:

$$\frac{L}{80} \ to \ \frac{L}{120}$$

Thus, the choice of using a folded plate or a barrel shell cross section for a given span would thus depend on the relative saving in cost of concrete and steel compared to the added cost of preparing formwork for the curved barrels. These costs vary from country to country, as well as within a country.

For barrel shells, the angle $\emptyset_k$ is kept between 20 degrees and 45 degrees. Angles between 30 degrees and 40 degrees are preferred. Angles greater than 45 degrees are not used, so as to avoid double formwork when pouring concrete, and also because a larger perimeter is required to cover the same bay width. If the depth of the curved cross section of the shell itself is less than L/8 to L/12, the balance should be made up by the provision of sufficiently deep edge beams.

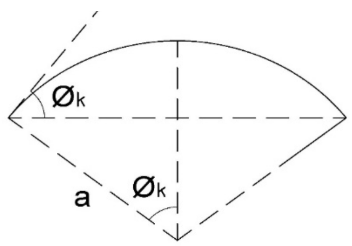


Figure 17.1: Cross section in a cylindrical shell

17.1 Application of Beam Theory for cylindrical shell:

Refer to Figure 17.2 below.

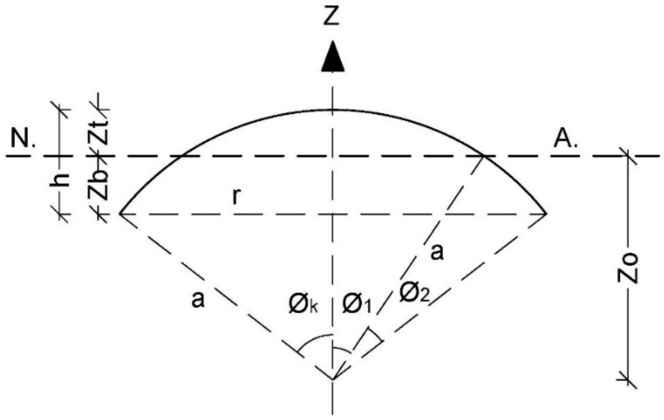


Figure 17.2: Cylindrical shell parameters

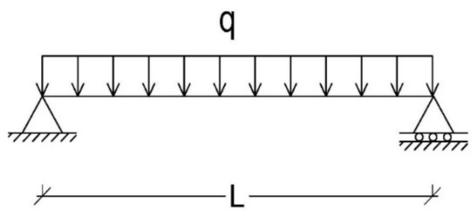


Figure 17.3: Structural model of a shell unit

1. Calculate the load per unit length, q, along the span, for the given distributed loading P per unit area:

$$q = 2aP\emptyset_k$$

2. Find the location of the neutral axis and the moment of inertia about the neutral axis for the cross section:

$$Z_o = \frac{a\ sin\emptyset_k}{\emptyset_k}$$

$$I = a^3t\left(\emptyset_k + sin\ \emptyset_k\left(cos\ \emptyset_k - \frac{2\ sin\ \emptyset_k}{\emptyset_k}\right)\right)$$

3. Find the maximum stresses in the cross section:

$$\sigma_{top} = \frac{MZ_{top}}{I} \quad and \qquad \sigma_{bottom} = \frac{MZ_{bottom}}{I}$$

4. Compare the maximum compression stress to the allowable concrete strength which is 0.45 f'c.
5. Find the tensile force from the tensile stress multiplied by the area of section in tension. Then, the area of steel can be calculated.

$$cos\ \emptyset_1 = \frac{Z_o}{a} \rightarrow find\ \emptyset_1$$

$$\emptyset_2 = \emptyset_k - \emptyset_1$$

$$Tension\ force, T = tensile\ area\ \ x\ average\ tensile\ stress = \emptyset_2\ a\ t\ \frac{\sigma_{tension}}{2}$$

$$Reinforcing\ steel\ area, A_s = \frac{T}{f_s}$$

$$A_{s,min} = 0.003A_g = 0.003(b)(h)$$

17.2 Application of Beam Theory for folded plate:

Use the same procedure as for cylindrical shells. The neutral axis and the moment of inertia can be determined using simple principles in statics.

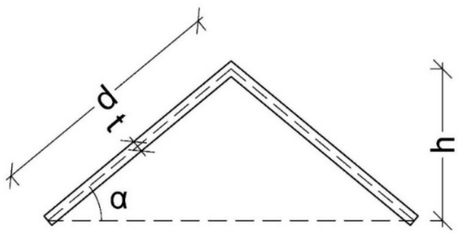


Figure 17.4: Folded plate unit

For the folded plate in Figure 17.4, the moment of inertia is given by:

$$I = 2\left(\frac{td^3}{12}\right)sin^2\alpha$$

$$Z_{top} = Z_{bottom} = \frac{h}{2}$$

Example 1:

Given:

- Refer to Figure 17.5
- Cylindrical shell
- Shell thickness, t= 0.12m
- Span, L=20m
- Concrete strength, f'c= 28MPa
- Steel strength, fy= 420MPa
- Surface live load, WL= 0.8kN/m^2

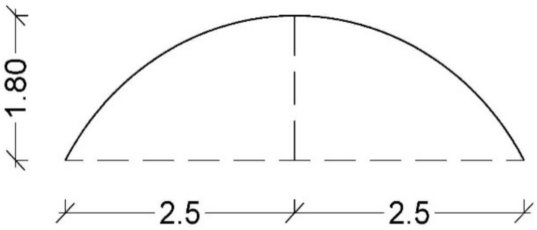


Figure 17.5: Cylindrical shell -1

Design the cylindrical shell.

Solution:

$q = 2aP\emptyset_k$ $P = (0.12)(25) + 0.8 = 3.8kN/m^2$

Refer to Figure 17.6 below.

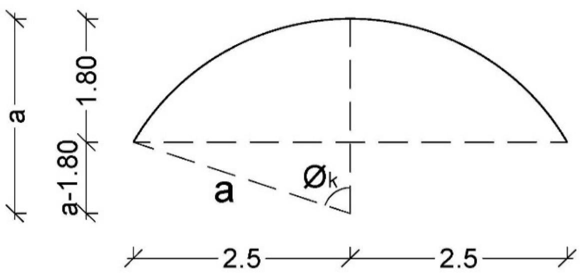


Figure 17.6: Cylindrical shell-2

$$a^2 = 2.5^2 + (a - 1.80)^2 = 6.25 + a^2 - 3.6\,a + 3.24 \rightarrow a = 2.64m$$

$$sin\,\emptyset_k = \frac{2.5}{a} \rightarrow \emptyset_k = 71.26^o \quad (\frac{71.26(\pi)}{180} = 1.243\,radians)$$

$$q = (2)(1.243)(2.64)(3.8) = 24.94kN/m$$

$$M = \frac{qL^2}{8} = \frac{(24.94)(20)^2}{8} = 1247kN.m$$

$$Z_o = \frac{a\,sin\emptyset_k}{\emptyset_k} = \frac{(2.64)(sin\,71.26)}{1.243} = 2.01m$$

$$I = a^3t\left(\emptyset_k + sin\,\emptyset_k\left(cos\,\emptyset_k - \frac{2\,sin\,\emptyset_k}{\emptyset_k}\right)\right) = 0.23m^4$$

Refer to Figure 17.7 below.

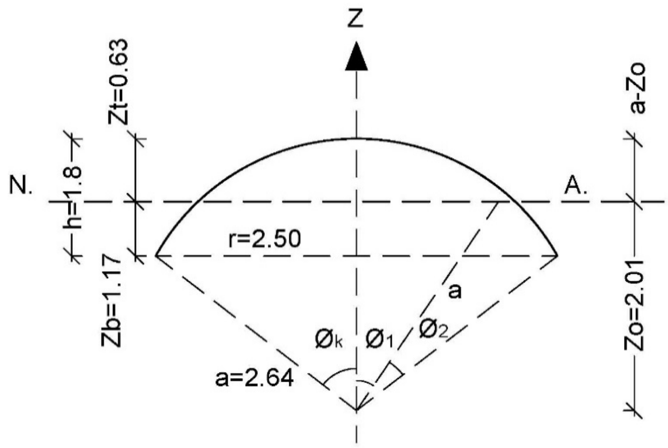


Figure 17.7: Cylindrical shell-3

$$\sigma_{top} = \frac{MZ_{top}}{I} = \frac{(1247)(0.63)}{0.23} = 3416kN/m^2 = 3.416MPa \quad < 0.45(28) = 12.6MPa$$

$$\sigma_{bottom} = \frac{MZ_{bottom}}{I} = \frac{(1247)(1.17)}{0.23} = 6343kN/m^2$$

$$cos\,\emptyset_1 = \frac{2.01}{2.64} \rightarrow \emptyset_1 = 40.4\,degrees$$

$$\emptyset_2 = \emptyset_k - \emptyset_1 = 71.26 - 40.4 = 30.86\,degrees \quad (0.538\,radians)$$

$$Tension\ force, T = \ tensile\ area \ \ \ x\ average\ tensile\ stress = \ \emptyset_2\ a\ t\ \frac{\sigma_{tension}}{2}$$
$$= (0.538)(2.64)(0.12)(\frac{6343}{2}) = 540kN$$

$$Reinforcing\ steel\ area, A_s = \frac{T}{f_s} = \frac{540000}{0.4(420)} = 3214mm^2 \qquad (11\emptyset 20mm)$$

$$A_{s,min} = 0.003A_g = 0.003(b)(h) = (0.003)(1000)(120)$$
$$= 360mm^2/m\ (1\emptyset 10/200mm)$$

Example 2:

Given:

- Refer to Figure 17.8
- Folded plate
- Plate thickness, t= 0.12m
- Span, L=20m
- Concrete strength, f'c= 28MPa
- Steel strength, fy= 420MPa
- Surface live load, WL= 0.8kN/m^2

Design the folded plate unit.

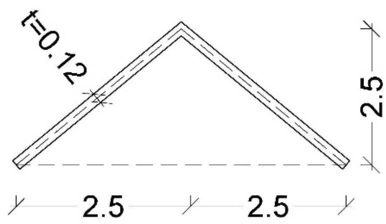


Figure 17.8: Folded plate

Solution:

$$P = (0.12)(25) + 0.8 = 3.8kN/m^2$$

$$Length\ of\ inclined\ line = \sqrt{2.5^2 + 2.5^2} = 3.536m$$

$$q = P(2)(3.536) = (3.8)(2)(3.536) = 26.87kN/m$$

$$M = \frac{qL^2}{8} = \frac{(26.87)(20)^2}{8} = 1343.5kN.m$$

$$I = 2\left(\frac{td^3}{12}\right)sin^2\alpha = (2)\left(\frac{(0.12)(3.536)^3}{12}\right)sin^2 45 = 0.442m^4$$

$$Z_{top} = Z_{bottom} = \frac{h}{2} = \frac{2.5}{2} = 1.25m$$

$$\sigma_{top} = \frac{MZ_{top}}{I} = \frac{(1343.5)(1.25)}{0.442} = 3799.5kN/m^2 = 3.8MPa \quad < 0.45(28) = 12.6MPa \quad Compression \quad OK$$

$$\sigma_{bottom} = \frac{MZ_{bottom}}{I} = 3799.5kN/m^2 \quad Tension$$

$$Tension, T = (0.12)(0.5)(3.536)(\frac{3799.5}{2}) = 403kN$$

$$Reinforcing\ steel\ area, A_s = \frac{T}{f_s} = \frac{403000}{0.4(420)} = 2399mm^2 \quad (8\emptyset 20mm)$$

$$A_{s,min} = 0.003A_g = 0.003(b)(h) = (0.003)(1000)(120) = 360mm^2/m\ (1\emptyset 10/200mm)$$

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This Third Edition of *Design of Reinforced Concrete Structures: A Practical Approach* book covers the analysis and design principles of reinforced concrete sections, members and systems in a simplified way. It introduces the design of beams, slabs, columns and footings based on ACI 318-19. In addition, it introduces the design of special structures like retaining walls, water tanks and shell structures.

This book presents the basic mechanics of structural concrete in a practical approach. It presents many practical examples in the design of reinforced concrete elements and systems.