COMP242

Data Structure

Lectures Note: Stacks and Queues

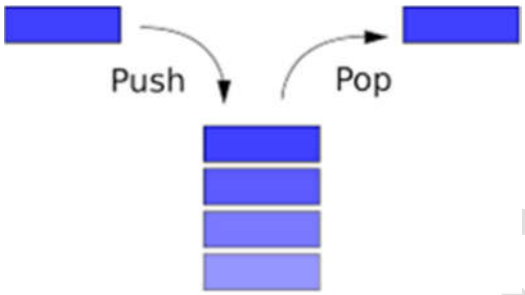
Prepared by: **Dr. Mamoun Nawahdah**

2016/2017

Stacks

Stack is an abstract data type that serves as a collection of elements, with two principal operations:

- **push** adds an element to the collection;
- **pop** removes the last element that was added.



- **Last In, First Out ➔ LIFO**

ABSTRACT DATA TYPE: STACK

DATA

- A collection of objects in reverse chronological order and having the same data type

OPERATIONS

PSEUDOCODE	UML	DESCRIPTION
`push(newEntry)`	`+push(newEntry: T): void`	Task: Adds a new entry to the top of the stack. Input: `newEntry` is the new entry. Output: None.
`pop()`	`+pop(): T`	Task: Removes and returns the stack's top entry. Input: None. Output: Returns the stack's top entry. Throws an exception if the stack is empty before the operation.
`peek()`	`+peek(): T`	Task: Retrieves the stack's top entry without changing the stack in any way. Input: None. Output: Returns the stack's top entry. Throws an exception if the stack is empty.
`isEmpty()`	`+isEmpty(): boolean`	Task: Detects whether the stack is empty. Input: None. Output: Returns true if the stack is empty.
`clear()`	`+clear(): void`	Task: Removes all entries from the stack. Input: None. Output: None.

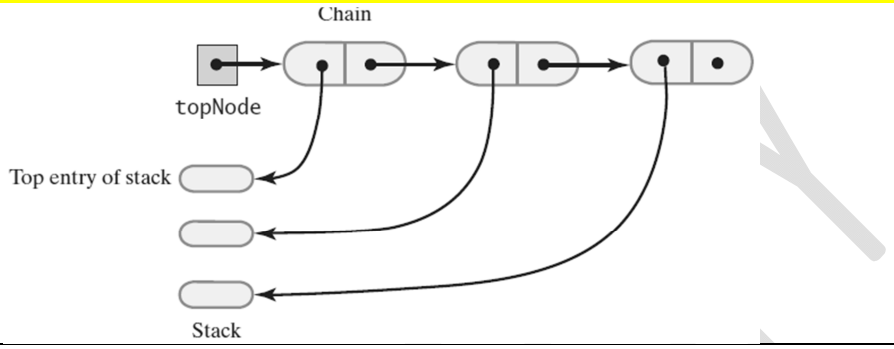
Single Linked List Implementation:

Each of the following operation involves top of stack

- **push**
- **pop**
- **peek**

Head or Tail for topNode??

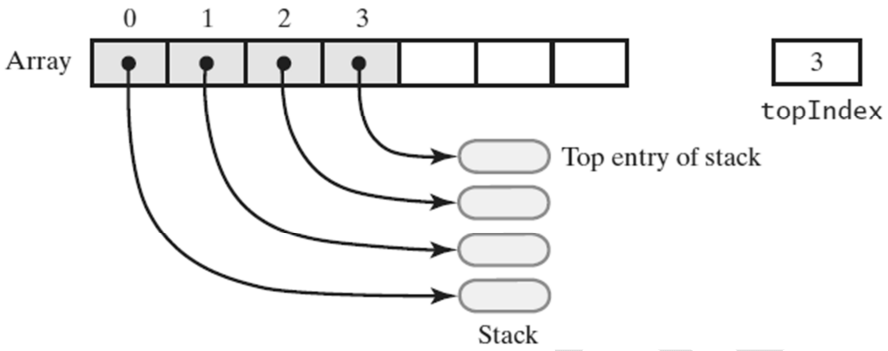
Head of linked list easiest, fastest to access ➔ Let this be the top of the stack




```
public class LinkedStack<T extends Comparable<T>> {
   private Node<T> topNode;

   public void push(T data) {
      Node<T> newNode = new Node<T>(data);
      newNode.setNext(topNode);
      topNode = newNode;
   }

   public Node<T> pop() {
      Node<T> toDel = topNode;
      if(topNode != null)
         topNode = topNode.getNext();
      return toDel;
   }

   public Node<T> peek() {   return topNode; }

   public int length() {
      int length = 0;
      Node<T> curr = topNode;
      while (curr != null) {
         length++;
         curr = curr.getNext();
      }
      return length;
   }

   public boolean isEmpty() {    return (topNode == null);    }

   public  void clear() {  topNode = null;  }
}
```

Array-Based Implementation:

- End of the array easiest to access
  - Let this be top of stack
  - Let first entry be bottom of stack




```
public class ArrayStack <T> {
   private Object[] s;
   private int n=-1;

   public ArrayStack(int capacity){
      s = new Object[capacity];
   }

   public boolean isEmpty(){ return n ==-1;}
   public int getN(){ return n;}

   public void push(T data){
      s[++n] = data;
   }

   public Object pop(){
      if(!isEmpty())
         return s[n--];
      return null;
   }

   public String toString() {
      String res = "Top-->";
      for(int i=n; i>=0;i--)
         res+="["+s[i]+"]-->";
      return res+"Null";
   }
}
```

Iteration (Optional)

Design challenge: Support iteration over stack items by client, without revealing the internal representation of the stack.

- **Java solution**. Make stack implement the **java.lang.Iterable** interface.

Q. What is an `Iterable` ?
A. Has a method that returns an `Iterator`.

Iterable interface

```
public interface Iterable<Item> {
   Iterator<Item> iterator();
}
```

Q. What is an `Iterator` ?
A. Has methods `hasNext()` and `next()`.
Q. Why make data structures `Iterable` ?
A. Java supports elegant client code.

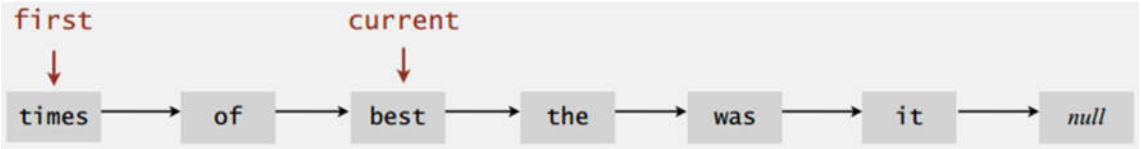
Iterator interface

```
public interface Iterator<Item> {
   boolean hasNext();
   Item next();
   void remove();   <-- optional; use at your own risk
}
```

```
import java.util.Iterator;
public class LinkedStack<T extends Comparable<T>> implements Iterable<T> {
   :
   public Iterator<T> iterator(){
      return new ListIterator();
   }

   private class ListIterator implements Iterator<T>{
      private Node<T> curr = topNode;
      public boolean hasNext(){return curr!=null;}
      public void remove(){}
      public T next(){
         T t = curr.data;
         curr = curr.next;
         return t;
      }
   }
}
```

```
Iterator<String> itt = ls.iterator();
while (itt.hasNext())
   System.out.println(itt.next());
```

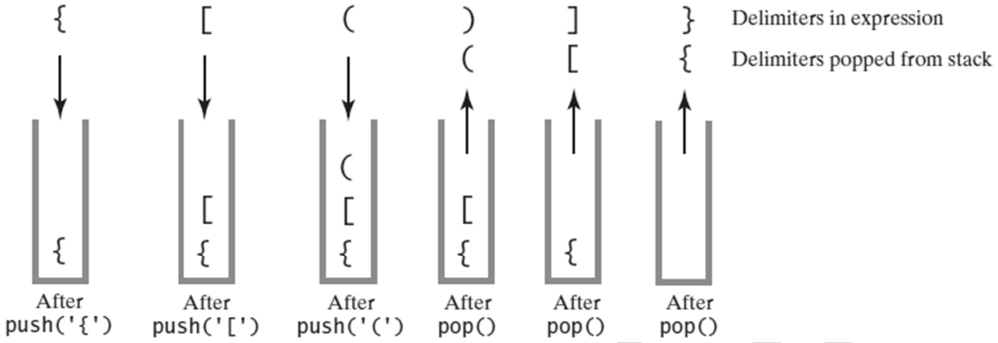
```
for(String s: ls)
   System.out.println(s);
```



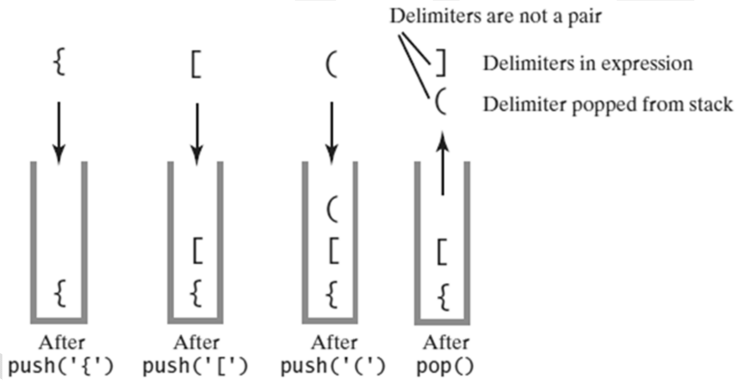
Balanced Delimiters

Problem: Find out if delimiters ("[{(]})") are paired correctly ➔ Compilers

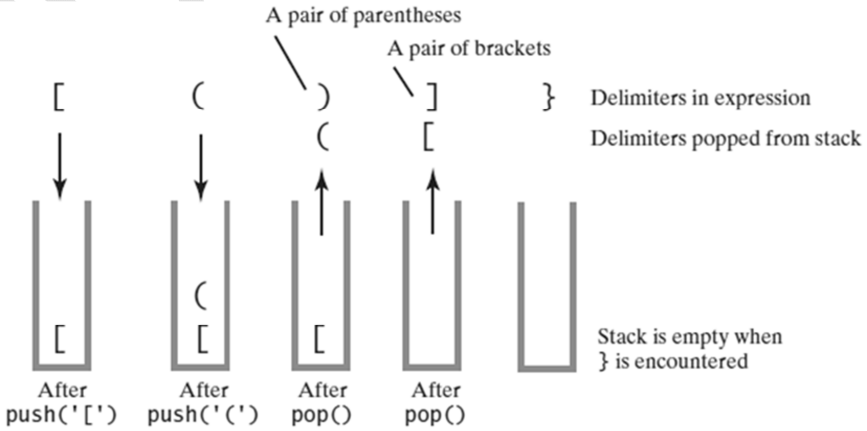
Example 1: The contents of a stack during the scan of an expression that contains the **balanced delimiters { [( )] }**



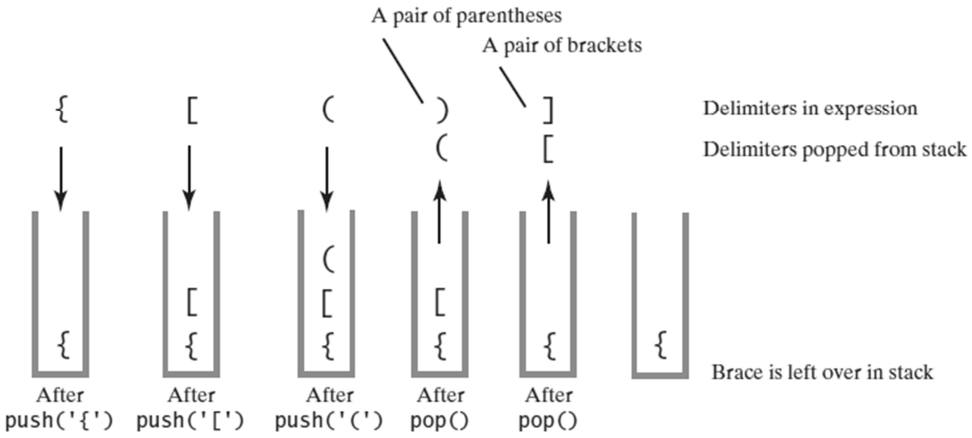
Example 2: The contents of a stack during the scan of an expression that contains the **unbalanced delimiters { [( ]) }**



Example 3: The contents of a stack during the scan of an expression that contains the **unbalanced delimiters [( )] }**



Example 4: The contents of a stack during the scan of an expression that contains the **unbalanced delimiters { [( )]**



Algorithm to process balanced expression:

```
Algorithm checkBalance(expression)
// Returns true if the parentheses, brackets,
// and braces in an expression are paired correctly.
isBalanced = true
while ((isBalanced == true) and not at end of expression) {
   nextCharacter = next character in expression
   switch (nextCharacter) {
      case '(': case '[': case '{':
         Push nextCharacter onto stack
         break

      case ')': case ']': case '}':
         if (stack is empty)
            isBalanced = false
         else {
             openDelimiter = top entry of stack
             Pop stack
             isBalanced = true or false according to whether openDelimiter
                          and nextCharacter are a pair of delimiters
          }
          break
   }
}
if (stack is not empty)  isBalanced = false
return isBalanced
```

H.W. implement check balance algorithm using linked list/array stacks

Processing Algebraic Expressions

- **Infix**: each binary operator appears between its operands *a + b*
- **Prefix**: each binary operator appears before its operands *+ a b*
- **Postfix**: each binary operator appears after its operands *a b +*

Evaluate infix expressions:

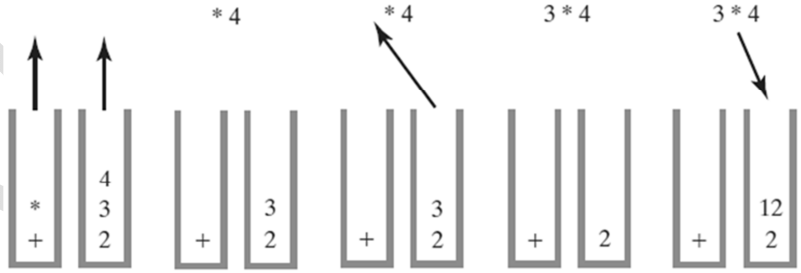
Two-stack algorithm. [E. W. Dijkstra]

- Value: push onto the value stack.
- Operator: push onto the operator stack.
- Left parenthesis: ignore.
- Right parenthesis: pop operator and two values; push the result of applying that operator to those values onto the operand stack.

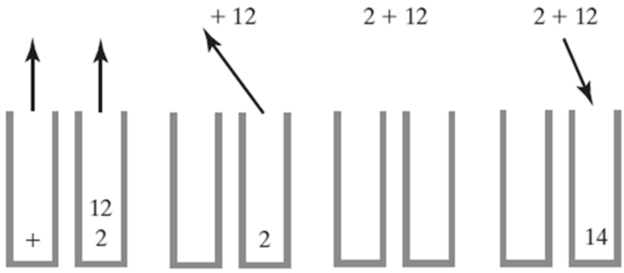
Example: evaluate a + b * c when a is 2, b is 3, and c is 4:

Step 1: Fill the two stacks until reaching the end of the expression:

Step 2: performing the multiplication:



Step 3: performing the addition:



Algorithm to evaluate infix expression:

```
Algorithm evaluateInfix(infix)
operatorStack = a new empty stack
valueStack = a new empty stack
while (infix has characters left to process) {
   nextCharacter = next nonblank character of infix
   switch (nextCharacter) {
      case variable:
         valueStack.push(value of the variable nextCharacter)
         break
      case '^' :
         operatorStack.push(nextCharacter)
         break
      case '+' : case '-' : case '*' : case '/' :
         while (!operatorStack.isEmpty() and
                precedence of nextCharacter <= precedence of operatorStack.peek()) {
            // Execute operator at top of operatorStack
            topOperator = operatorStack.pop()
            operandTwo = valueStack.pop()
            operandOne = valueStack.pop()
            result = the result of the operation in topOperator and its operands
                     operandOne and operandTwo
            valueStack.push(result)
         }
         operatorStack.push(nextCharacter)
         break
      case '(' :
         operatorStack.push(nextCharacter)
         break
      case ')' : // Stack is not empty if infix expression is valid
         topOperator = operatorStack.pop()
         while (topOperator != '(') {
            operandTwo = valueStack.pop()
            operandOne = valueStack.pop()
            result = the result of the operation in topOperator and its operands
                     operandOne and operandTwo
            valueStack.push(result)
            topOperator = operatorStack.pop()
         }
         break
      default: break // Ignore unexpected characters
   }
}
while (!operatorStack.isEmpty()) {
  topOperator = operatorStack.pop()
  operandTwo = valueStack.pop()
  operandOne = valueStack.pop()
  result = the result of the operation in topOperator and its operands
           operandOne and operandTwo
  valueStack.push(result)
}
return valueStack.peek()
```

Infix to Postfix Conversion

• Operand	Append each operand to the end of the output expression.
• Operator ^	Push ^ onto the stack.
• Operator +, -, *, or /	Pop operators from the stack, appending them to the output expression, until the stack is empty or its top entry has a lower precedence than the new operator. Then push the new operator onto the stack.
• Open parenthesis	Push (onto the stack.
• Close parenthesis	Pop operators from the stack and append them to the output expression until an open parenthesis is popped. Discard both parentheses.

Example 1: Converting the **infix** expression **a + b * c** to **postfix** form

Next Character in Infix Expression	Postfix Form	Operator Stack (bottom to top)
a	*a*	
+	*a*	+
b	*a b*	+
*	*a b*	+ *
c	*a b c*	+ *
	a b c *	+
	a b c * +	

Example 2: Successive Operators with Same Precedence: **a - b + c**

Next Character in Infix Expression	Postfix Form	Operator Stack (bottom to top)
a	*a*	
−	*a*	−
b	*a b*	−
+	*a b* −	
	a b −	+
c	*a b* − *c*	+
	a b − *c* +	

Example 3: Successive Operators with Same Precedence: **a ^ b ^ c**

Next Character in Infix Expression	Postfix Form	Operator Stack (bottom to top)
a	*a*	
^	*a*	^
b	*a b*	^
^	*a b*	^^
c	*a b c*	^^
	a b c ^	^
	a b c ^ ^	

Example 4: The steps in converting the infix expression **a / b * (c + (d – e))** to postfix form

Next Character from Infix Expression	Postfix Form	Operator Stack (bottom to top)
a	*a*	
/	*a*	/
b	*a b*	/
*	*a b* /	
	a b /	*
(	*a b* /	* (
c	*a b* / *c*	* (
+	*a b* / *c*	* (+
(	*a b* / *c*	* (+ (
d	*a b* / *c d*	* (+ (
–	*a b* / *c d*	* (+ (–
e	*a b* / *c d e*	* (+ (–
)	*a b* / *c d e* –	* (+ (
	a b / *c d e* –	* (+
)	*a b* / *c d e* – +	* (
	a b / *c d e* – +	*
	a b / *c d e* – + *	

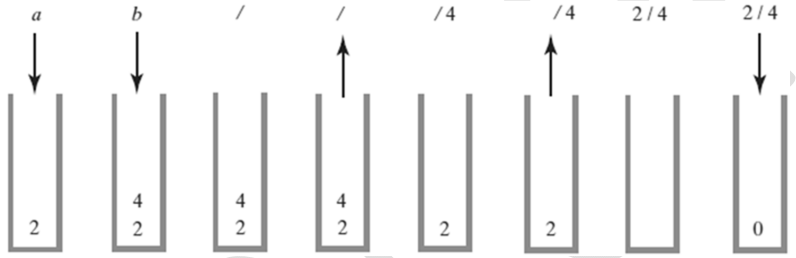
Infix-to-postfix Algorithm:

```
Algorithm convertToPostfix(infix)
operatorStack = a new empty stack
postfix = a new empty string
while (infix has characters left to parse) {
   nextCharacter = next nonblank character of infix
   switch (nextCharacter) {

      case variable:
         Append nextCharacter to postfix
         break
      case '^' :
         operatorStack.push(nextCharacter)
         break
      case '+' : case '-' : case '*' : case '/' :
         while (!operatorStack.isEmpty() and
                precedence of nextCharacter <= precedence of operatorStack.peek()){
            Append operatorStack.peek() to postfix
            operatorStack.pop()
         }
         operatorStack.push(nextCharacter)
         break
      case '( ' :
         operatorStack.push(nextCharacter)
         break
      case ')' : // Stack is not empty if infix expression is valid
         topOperator = operatorStack.pop()
         while (topOperator != '(') {
              Append topOperator to postfix
              topOperator = operatorStack.pop()
          }
          break
      default: break // Ignore unexpected characters
   }
}
while (!operatorStack.isEmpty()) {
   topOperator = operatorStack.pop()
   Append topOperator to postfix
}
return postfix
```

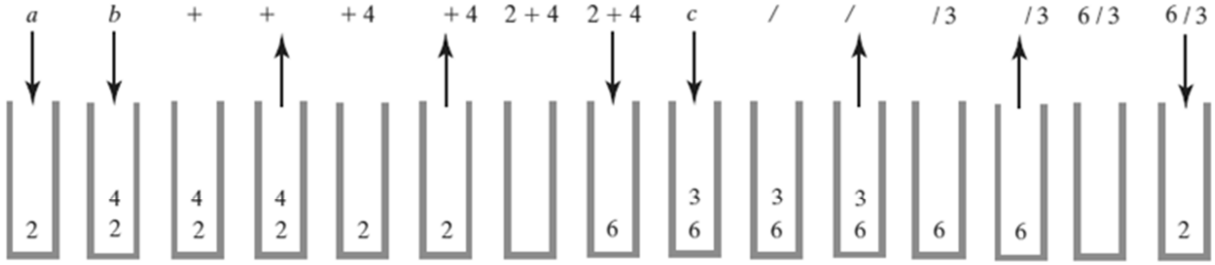
Evaluating Postfix Expressions

- When an **operand** is seen, it is **pushed** onto a stack.
- When an **operator** is seen, the appropriate numbers of **operands** are **popped** from the stack, the operator is **evaluated**, and the result is **pushed** back onto the stack.
  - Note that the **1st** item popped becomes the (right hand side) **rhs** parameter to the binary operator and that the **2nd** item popped is the (left hand side) **lhs** parameter; thus **parameters are popped in reverse order**.
  - For addition and multiplication, the order does not matter, but for subtraction and division, it does.
- When the complete postfix expression is evaluated, the result should be a single item on the stack that represents the answer.

Example 1: The stack during the evaluation of the postfix expression ***a b /*** when *a* is **2** and **b** is 4



Example 2: The stack during the evaluation of the postfix expression **a b + c /** when ***a*** is ***2***, ***b*** is ***4***, and ***c*** is ***3***



Self exercises:

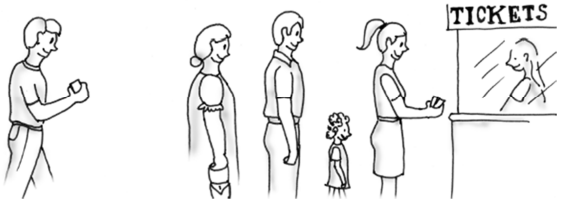
- **2 3 4 + * 6 -** ➔ **8.0**
- **2 3 + 7 9 / -** ➔ **4.222**
- **10 2 8 * + 3 -** ➔ **23.0**
- **1 2 - 4 5 ^ 3 * 6 * 7 2 2 ^ ^ / -** ➔ **-8.67**

Algorithm for evaluating postfix expressions.

```
Algorithm evaluatePostfix(postfix)
// Evaluates a postfix expression.

valueStack = a new empty stack
while (postfix has characters left to parse)
{
    nextCharacter = next nonblank character of postfix
    switch (nextCharacter)
    {
      case variable:
         valueStack.push(value of the variable nextCharacter)
         break
      case '+' : case '-' : case '*' : case '/' : case '^' :
         operandTwo = valueStack.pop()
         operandOne = valueStack.pop()
         result = the result of the operation in nextCharacter and its operands
                  operandOne and operandTwo
         valueStack.push(result)
         break
      default: break // Ignore unexpected characters
    }
}
```

Queues

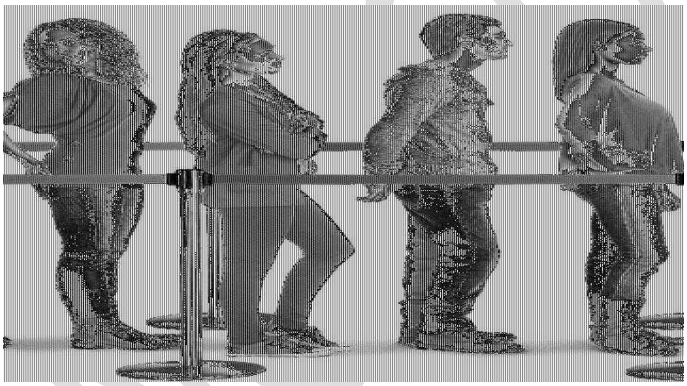
- A **queue** is another name for a **waiting line**:



- Used within operating systems and to simulate real-world events.
  - Come into play whenever processes or events must wait
- Entries organized **first-in**, **first-out**.

Terminology

- Item added first, or earliest, is at the front of the queue
- Item added most recently is at the back of the queue
- Additions to a software queue must occur at its back.
- Client can look at or remove only the entry at the front of the queue



Tail
Last
Back

FIFO: First In First Out

Head
First
Front

The ADT Queue

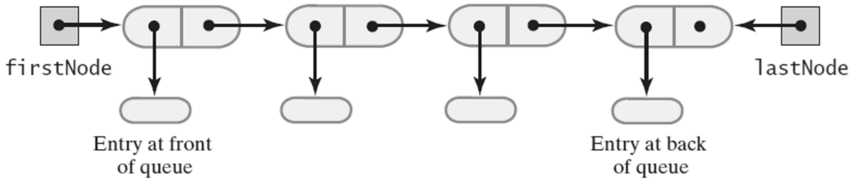
DATA

- A collection of objects in chronological order and having the same data type

OPERATIONS

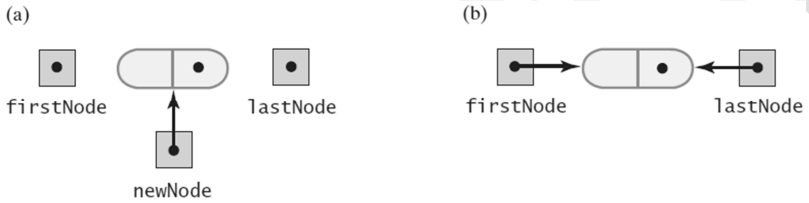
PSEUDOCODE	UML	DESCRIPTION
`enqueue(newEntry)`	`+enqueue(newEntry: integer): void`	Task: Adds a new entry to the back of the queue.
`dequeue()`	`+dequeue(): T`	Task: Removes and returns the entry at the front of the queue.
`getFront()`	`+getFront(): T`	Task: Retrieves the queue's front entry without changing the queue in any way.
`isEmpty()`	`+isEmpty(): boolean`	Task: Detects whether the queue is empty.
`clear()`	`+clear(): void`	Task: Removes all entries from the queue.

Linked-list Representation of a Queue

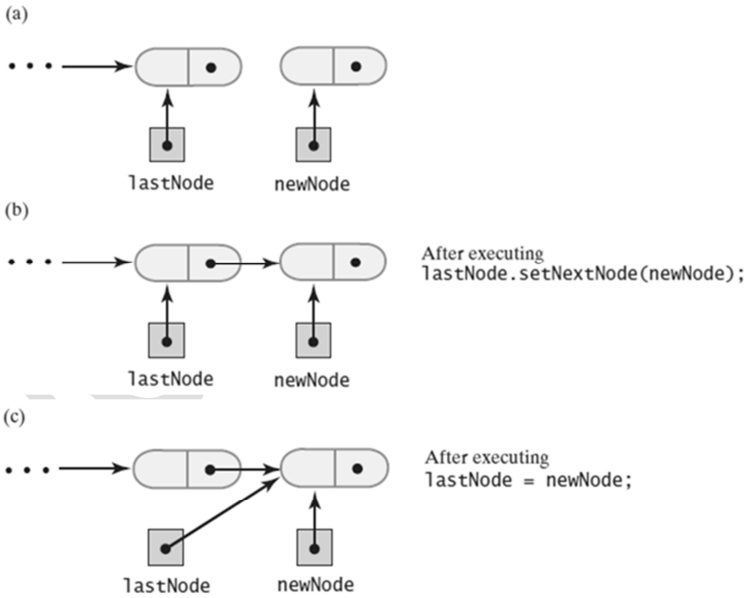



```
public class linkedQueue <T extends Comparable<T>> {
   private Node<T> first;
   private Node<T> last;

   public boolean isEmpty(){   return (first==null) && (last==null);   }
   public void clear(){
      first = null;
      last = null;
   }
}
```

- The definition of **enqueue** Performance is **O(1)**:
  - Adding a new node to an empty chain



  - Adding a new node to the end of a nonempty chain that has a **tail** reference

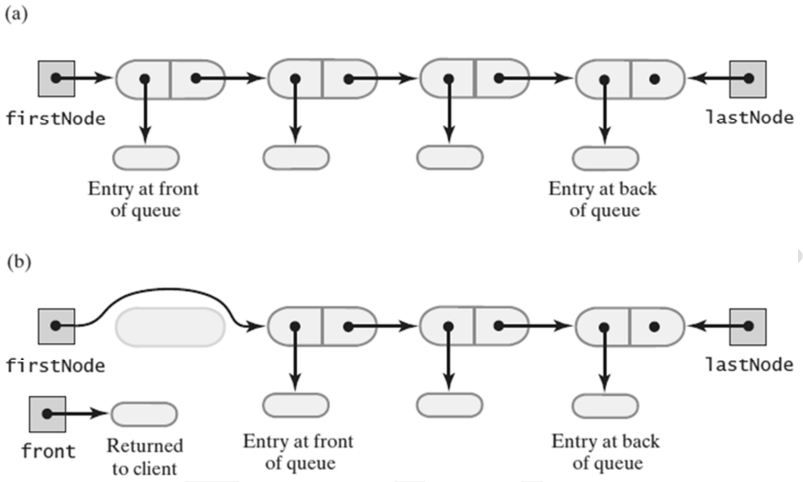



```
public void enqueue(T data){
   Node<T> newNode = new Node<T>(data);
   if(isEmpty())
      first=newNode;
   else
      last.next = newNode;
   last = newNode;
}
```

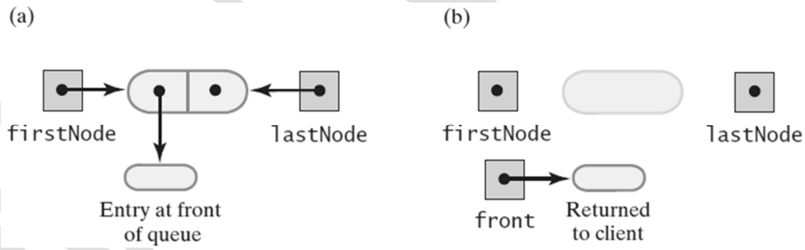
- Retrieving the front entry:

```
public T getFront(){
   if(!isEmpty())
      return first.data;
   return null;
}
```

- Removing the front entry (**dequeue**):
  - A queue of more than one entry:



  - A queue of one entry:

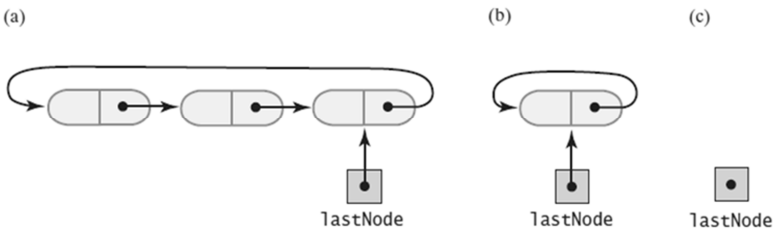



```
public T dequeue(){
   T front = getFront();
   if(!isEmpty())
      first = first.next;
   if(first==null)
      last = null;
   return front;
}
```

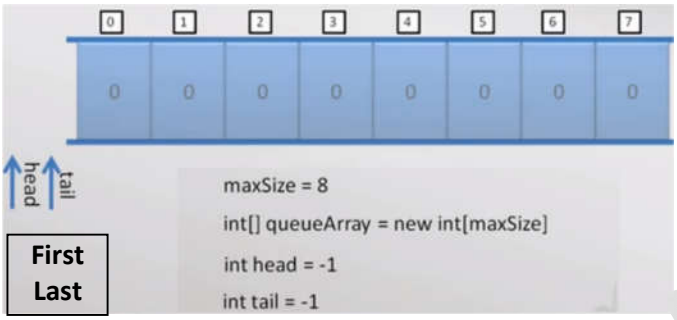
Circular Linked Implementations of a Queue

A circular linked chain with an external reference to its last node that

a) has more than one node; b) has one node; c) is empty

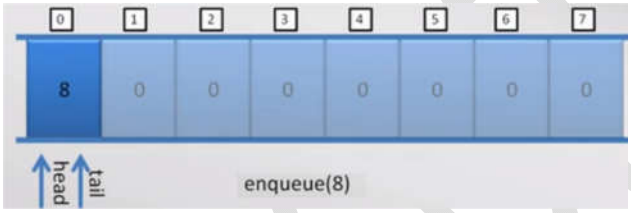


Array implementation of a Queue



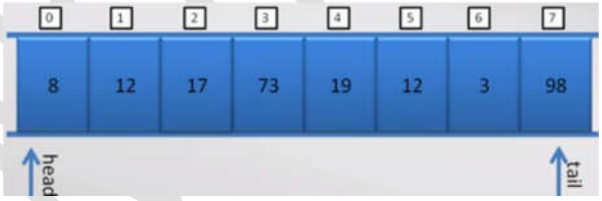
- **enqueue**(): add new item at after **last** (**tail**).
- **dequeue**(): remove item from **first** (**head**).

enqueue(8)



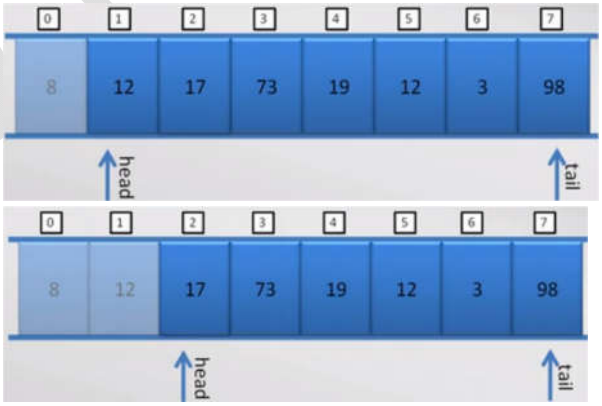
enqueue (12)

After a number of enqueues:



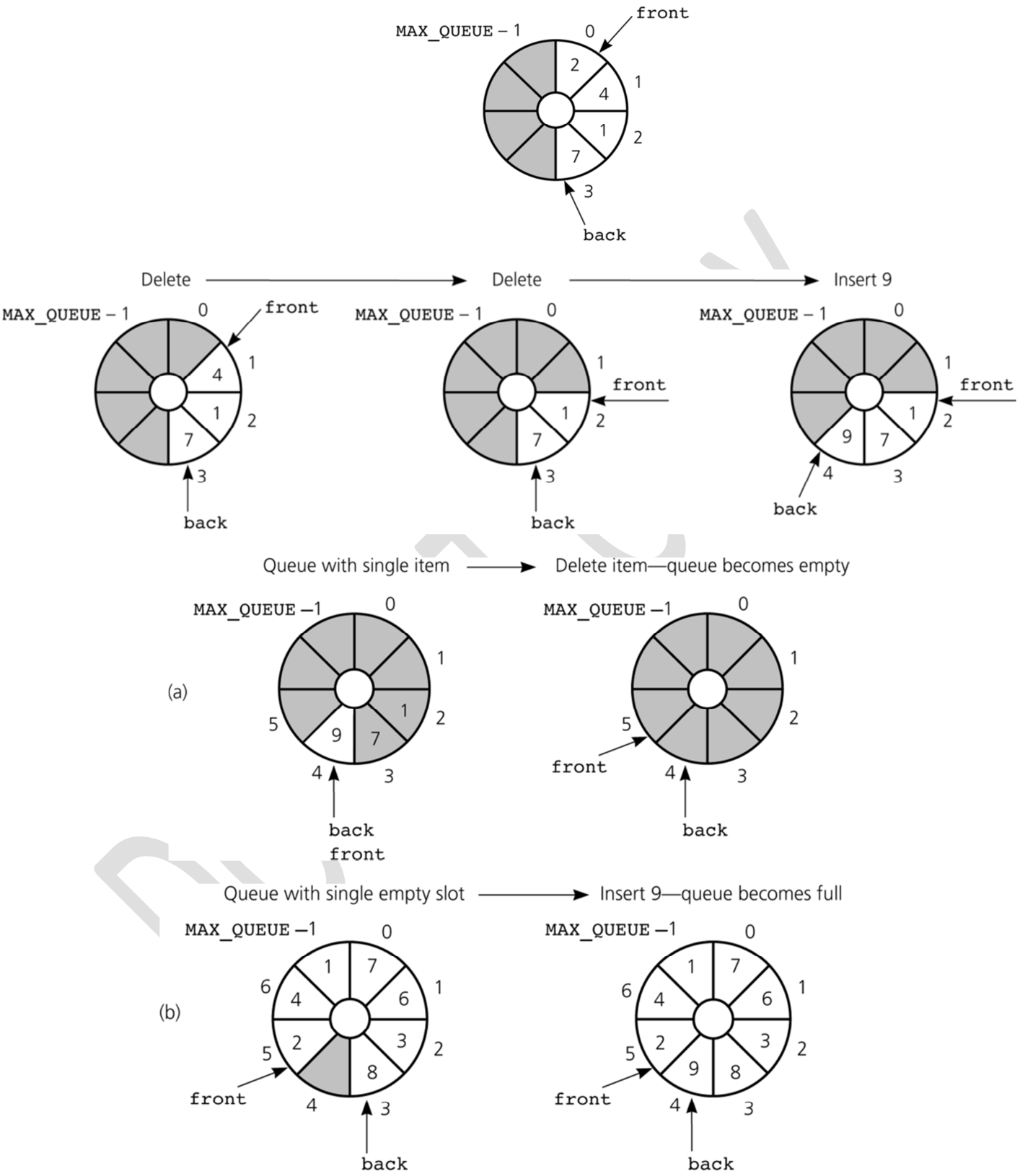
dequeue(): returns the item pointed by **head** and advances **head** pointer

dequeue()



enqueue (27) ?? How to advance **tail**?? We have space at the beginning?? Shift??

Circular Queue



- **To detect circular queue-full and queue-empty conditions**
  - Keep a **count** of the queue items

- **To initialize the circular queue, set:**
  - front to -1
  - back to -1
  - count to 0

- **Inserting into a circular queue:**

```
If(count < MAX_QUEUE)  // free
        back = (++back) % MAX_QUEUE;
        items[back] = newItem;
        ++count;
        If(count == 1) // first item
                front = back;
```

- **Deleting from a circular queue:**

```
If(count > 0) // not empty
        front = (++front) % MAX_QUEUE;
        --count;
        If(count == 0)  // empty
                front = back = -1
```

HW: Queue implementations using linked List and Arrays.

DE Queue (Double Ended Queue)

Allows add/remove elements from both head/tail.