

Splay Trees

CSE 326
Data Structures
Lecture 8

Readings and References

- Reading
 - › Sections 4.5-4.7

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Self adjustment for better living

- Ordinary binary search trees have no balance conditions
 - › what you get from insertion order is it
- Balanced trees like AVL trees enforce a balance condition when nodes change
 - › tree is always balanced after an insert or delete
- Self-adjusting trees get reorganized over time as nodes are accessed

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Splay Trees

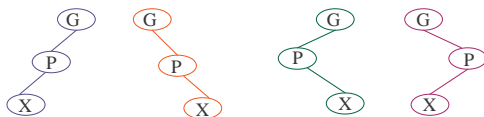
- Splay trees are tree structures that:
 - › Are not perfectly balanced all the time
 - › Data most recently accessed is near the root.
- The procedure:
 - › After node X is accessed, perform "splaying" operations to bring X to the root of the tree.
 - › Do this in a way that leaves the tree more balanced as a whole

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Splay Tree Terminology

- Let X be a non-root node with ≥ 2 ancestors.
 - P is its parent node.
 - G is its grandparent node.



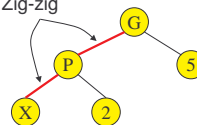
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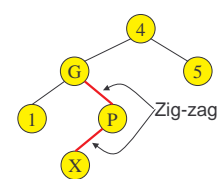
Zig-Zig and Zig-Zag

Parent and grandparent in same direction.

Zig-zig



Parent and grandparent in different directions.



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Splay Tree Operations

1. Helpful if nodes contain a parent pointer.



2. When X is accessed, apply one of six rotation routines.

- Single Rotations (X has a P (the root) but no G)
ZigFromLeft, ZigFromRight
- Double Rotations (X has both a P and a G)
ZigZigFromLeft, ZigZigFromRight
ZigZagFromLeft, ZigZagFromRight

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Zig at depth 1

- “Zig” is just a **single rotation**, as in an AVL tree
- Let R be the node that was accessed (e.g. using Find)



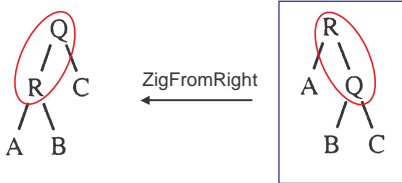
- ZigFromLeft moves R to the top → faster access next time

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Zig at depth 1

- Suppose Q is now accessed using Find



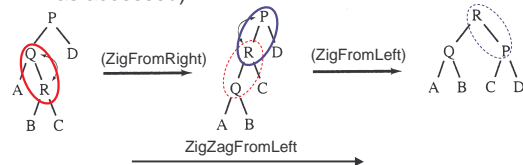
- ZigFromRight moves Q back to the top

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Zig-Zag operation

- “Zig-Zag” consists of **two rotations of the opposite direction** (assume R is the node that was accessed)

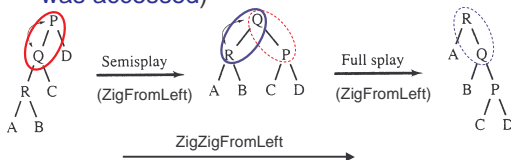


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Zig-Zig operation

- “Zig-Zig” consists of **two single rotations of the same direction** (R is the node that was accessed)



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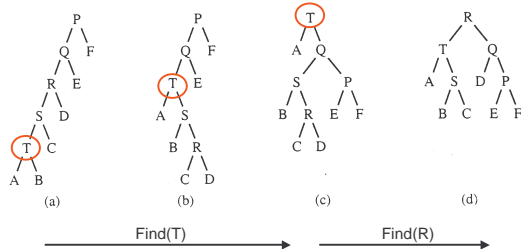
Find Operation

- Find operation
 - › Do a normal find in the binary search tree
 - › Splay the the node found to the root by a series of zig-zig and zig-zag operations with an additional zig at the end if the length of the path to the node is odd.
 - › If nothing found splay the last node visited to the root.

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Decreasing depth - "autobalance"



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Details of SplayFind

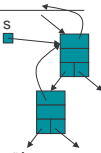
```
SplayFind(p: node pointer, x: key): node pointer {
  r, s : node pointer;
  r := Find(p, x); //if x is not in the tree then
                  //the last node visited is returned
  while r.parent ≠ null do {
    s := r.parent.parent;
    case {
      s = nil:
        if r.parent.right = r then ZigFromRight(r.parent) ;
        else ZigFromLeft(r.parent);
      s.right.right = r: ZigZigFromRight(s);
      s.left.left = r: ZigZigFromLeft(s);
      s.right.left = r: ZigZagFromRight(s);
      s.left.right = r: ZigZagFromLeft(s);
    }
  }
  return r //r contains x if it is in the tree
}
```

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ZigFromLeft

```
ZigFromLeft(s: node pointer): {
  c: node pointer;
  c := s.left;
  s.left := c.right;
  if s.left ≠ null then s.left.parent := s;
  c.parent := s.parent;
  if c.parent ≠ null then
    if c.parent.right = s then c.parent.right := c;
    else c.parent.left := c;
  s.parent := c;
  c.right := s;
}
```



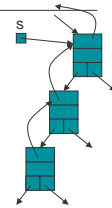
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Try ZigZigFromLeft

- Design ZigZigFromLeft

```
ZigZigFromLeft(s: node pointer) {
  ???
}
```



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Splay Tree Insert

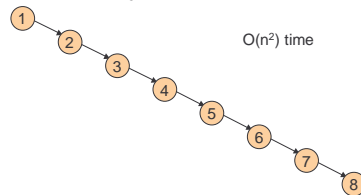
- Insert x
 - › Insert x as normal then splay x to root.

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Example Insert

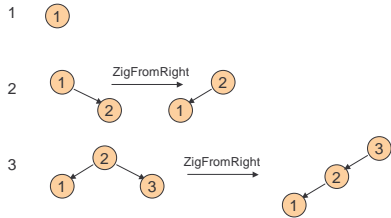
- Inserting in order 1,2,3,...,8
- Without self-adjustment



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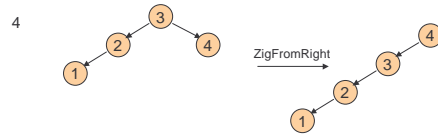
With Self-Adjustment



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With Self-Adjustment



$O(n)$ time!!

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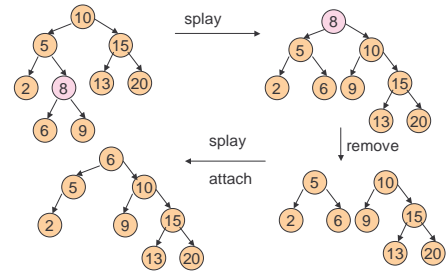
Splay Tree Deletion

- Delete
 - › Splay x to root and remove it. Two trees remain, right subtree and left subtree.
 - › Splay the max in the left subtree to the root
 - › Attach its right subtree to the new root of the left subtree and return it. The predecessor of x becomes the root.

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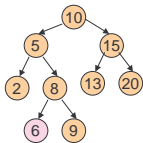
Example Deletion



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Practice Delete



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Analysis of Splay Trees

- Splay trees tend to be balanced
 - › M operations takes time $O(M \log N)$ for $M \geq N$ operations on N items.
 - › Amortized $O(\log n)$ time.
- Splay trees have good "locality" properties
 - › Recently accessed items are near the root of the tree.
 - › Items near an accessed node are pulled toward the root.

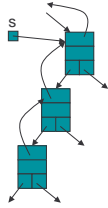
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Solution to First Exercise

```

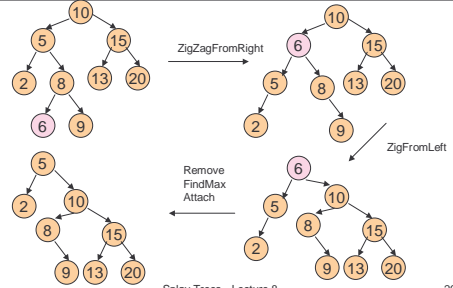
ZigZigFromLeft(s: node pointer) {
  c: node pointer;
  c := s.left;
  ZigFromLeft(s);
  ZigFromLeft(c);
}
    
```



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Solution to Second Exercise



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