

Data Structer

006

COMP 242

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Recursion

III Hand shake problem

$h(2) = 1, h(3) = 2, h(4) = 3$

* $h(n) = \begin{cases} 1 & \text{base, } n \leq 2 \\ h(n-1) + (n-1) & \text{Recursive Part, } n > 2 \end{cases}$

```
public static int handShake(int n) {
    if (n == 2)
        return 1;
    else
        return handShake(n-1) + (n-1);
}
```

Handshake(n) = (n-1) + handShake(n-1)

Handshake(n) = (n-1) + Handshake(n-1)

عندما يكون عدد الأشخاص 1

```
sum(int n) {
    if (n == 1)
        return 1;
    else
        return n + sum(n-1);
}
```

عندما يكون عدد الأشخاص 1
يتم استبدالها بـ 1
عندما يكون عدد الأشخاص 2
يتم استبدالها بـ 2

* بجواب رقم

100

$$n + \text{Sum}(n-1) = \sum_{i=1}^{100} i = 1 + 2 + 3 + \dots + 100$$

10

$$1 + \text{Sum}(n-1) = 1 + \text{Sum}(9)$$

$$1 + \text{Sum}(8)$$

$$1 + \text{Sum}(7)$$

⋮

31 Factorial

Public Static long Fact(int n) {

if (n == 0)

return 1;

else

return n * Fact(n-1);

}

$$F(n) = \begin{cases} 1, & n = 0 \\ n * \text{Fact}(n-1), & n > 0 \end{cases}$$

* يا إما بجيب الكود وبدء الفاتس أو الحاسن .

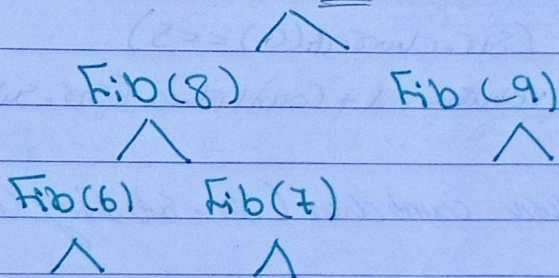
[4] Fibonacci

0 1 1 2 3 5 8 13 21 ...

$$\text{Fib}(n) = \begin{cases} 0, & n=0 \\ 1, & n=1 \\ \text{Fib}(n-1) + \text{Fib}(n-2), & n > 1 \end{cases}$$

```
public static int Fib(int n) {  
    if (n == 0)  
        return 0;  
    if (n == 1)  
        return 1;  
    else  
        Fib(n-1) + Fib(n-2);  
}
```

Fib(10) → = 55



[5] Reverse

321 ← 123

```
static int sum = 0;
```

```
public static int Reverse (int n) {
```

```
    if (n == 0)
```

```
        return sum;
```

```
    else
```

```
        sum = sum * 10 + n % 10;
```

```
        Reverse n / 10;
```

```
}
```

[6] Count Char

يوجد عدد الحروف.

```
public static int countChar (string str, char s) {
```

```
    if (str.length() == 0)
```

```
        (str.isEmpty())
```

يقدر الاستبدال.

```
        return 0;
```

```
    else if (str.charAt(0) == s)
```

```
        return 1 + countChar (str.substring(1), s);
```

```
    else
```

```
        return countChar (str.substring(1), s);
```

```
}
```

[7] Count String

يوجد كلمة سواء ألت.

```
int countString (string str, string s) {
```

```
    if (str.length() < s.length())
```

```
        return 0;
```

```
    if (str.substring(0, s.length()).equals(s))
```

```
        return 1 + countString (str.substring(1), s);
```

```
    else
```

```
        return countString (str.substring(1), s);
```

```
}
```

[8] Revers String.

```

public static String Reverse (String str) {
    if (str.is Empty ())
        return str;
    else
        return Reverse (str.substring (1)) +
            str.charAt (0);
}

```

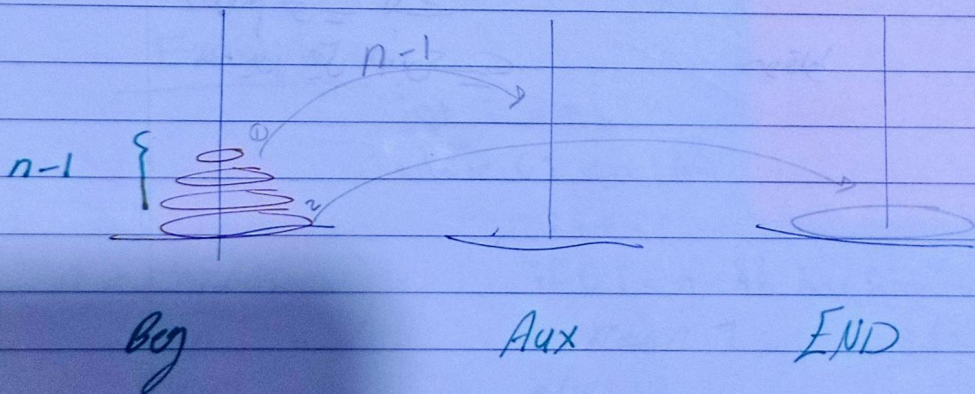
nam alle h ← Hello man

[9] Hori Tower.

```

Hori Tower (int n, char Beg, char Aux, char END) {
    if (n > 0) {
        Hori Tower (n-1, Beg, end, aux);
        System.out.printf ("Mov. disk 'd from %c to %c"
            Disk n, Beg, END);
        Hori Tower (n-1, Aux, Beg, END);
    }
}

```



Palindrome

```
public static boolean isPali (int [] arr, int i, int j) {
```

```
    if (i > j)
```

```
        return true ;
```

```
    else if (arr [i] != arr [j] )
```

```
        return false ;
```

```
    else
```

```
        return isPali (arr, ++i, --j) ;
```

```
}
```

* Done

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

لا حول ولا قوة
إلا بالله العلي
العزيز - آمين

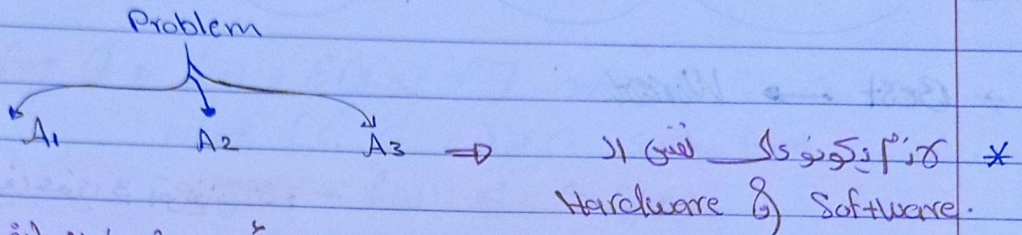
اللهم قوي بك
حين نزل هبسي

Analysis of Algorithm

Chapter 2

(OS - AOA)

Problem Solving :-



Algorithm (تستی) :-
 Speed (قدش آنقدر وقت)
 Space.

عبارت اعرف آنو الصح و الا تسنى المقبول ان أقرب

جاری Experimental → implement

timing (time)

Plotting (تستین موجود)

Asymptotic → T(n)

All possibilities

Example :-

```
int i = 0;
while (i < n && Arr[i] != key) {
    i++;
    if (i < n && Arr[i] == key)
        return 1;
    else
        return 0;
}
```

n = element
 in
 Array

Best case $\rightarrow 1$: ثابت
 Average case \rightarrow Array element (ن)
 worst case $\rightarrow N$

$$1 \leq \log n \leq n \leq n \log n \leq n^2 \leq 2^n \leq \dots$$

$$n! \leq \dots \leq n^n$$

* Best \rightarrow Worst

ثلاث تعريفات :-

Big-O \rightarrow upper bound
 Big- Ω \rightarrow lower bound
 Big- Θ \rightarrow Average bound.

$T(n) = O(g(n))$ if there exist constant C and n_0 , such that $T(n) \leq Cg(n)$ where $C > 0, n_0 > 0$

$$T(n) = \frac{n(n-1)}{2} = \frac{n^2 - n}{2} \leq \frac{1}{2}n^2 \leq n^2$$

$\therefore T(n) = O(n^2)$

* ثابت بوقت الذي
 order

Big O : $f(n) = O(g(n))$ if \exists constant C, n_0 such $\forall C \geq 0, n_0 \geq 0$ where $f(n) \leq Cg(n)$.

Big Ω : $f(n) = \Omega(g(n))$, if \exists constant C, n_0 such $\forall C \geq 0, n_0 \geq 0$ where $f(n) \geq Cg(n)$

Big Θ : $f(n) = \Theta(g(n))$ if \exists constant C_1, C_2, n_0 where $(C_1, C_2, n_0) \geq 0$ such that $C_1g(n) \leq f(n) \leq C_2g(n)$.

Ex $f(n) = 2n + 3$
 $= f(n) = O(n)$

$$f(n) \leq Cg(n)$$

$$2n + 3 \leq 2n + 3n \quad \checkmark$$

$$2n + 3 \leq 5n$$

constant 5 is least \checkmark

$\therefore f(n) = \Omega(n)$

$$f(n) \geq Cg(n)$$

$$2n + 3 \geq 1n \quad \checkmark$$

$\therefore f(n) = \Theta(n)$

$$C_1g(n) \leq f(n) \leq C_2g(n)$$

$$1n \leq 2n + 3 \leq 5n$$

Ex $n^2 \log n.$

$$f(n) = O(n^2 \log n)$$

$$f(n) \leq c g(n)$$

$$n^2 \log n + c \leq 10 n^2 \log n.$$

$$f(n) = \Omega(n^2 \log n)$$

$$f(n) \geq c g(n)$$

$$n^2 \log n + n \geq n^2 \log n.$$

$$f(n) = \Theta(n^2 \log n).$$

Ex $f(n) = n!$

$$1 \leq 1 \times 2 \times 3 \times \dots \times (n-1) \times n < n^n$$

$$f(n) = O(n^n)$$

lower

upper

$$f(n) = \Omega(1)$$

$$f(n) = \Theta \text{ unknown.}$$

$$f(n) = n^2 + n$$

$$g(n) = n \log n.$$

$$\boxed{1} \quad f(n) + g(n) = \text{Max}(f(n), g(n)) \\ (n^2 + n.)$$

∴

$$\boxed{2} \quad f(n) * g(n) = O(f(n) * g(n)) \quad (O(n^3 \log n).)$$

$$\log n! \rightarrow O(n \log n)$$

$$f(n) = n^2 \log(n)$$

$$\log n^2 \log(n)$$

$$\log n (\log n)^{10}$$

$$g(n) = n (\log n)^{10}$$

$$\frac{2 \log n + \log n \log n}{n^2} > \frac{\log n + 10 \log n \log n}{n}$$

$$f(n) = 3n^{\sqrt{n}}$$

$$3n^{\sqrt{n}}$$

$$2^{\sqrt{n} \log n}$$

$$g(n) = 2^{\sqrt{n} \log n}$$

$$3n^{\sqrt{n}}$$

$$= (n^{\sqrt{n}})^{\log 3}$$

$$a^{\log_c b}$$

$$= b^{\log_c a}$$

$$f(n) = 2^{\log n}$$

$$\log 2^{\log n}$$

$$\log n^{\sqrt{n}}$$

$$g(n) = n^{\sqrt{n}}$$

$$\log(n) (\log 2)$$

$$< \sqrt{n} \log n$$

$$f(n) = 2^n$$

$$\log 2^n$$

$$\log 2^{2n}$$

$$g(n) = 2^{2n}$$

$$n \log 2$$

$$\log 2^{2n}$$

$$\cancel{n \log 2}$$

$$< \cancel{2n \log 2}$$

$$f(n) = n^{\log n}$$

$$g(n) = 2^{n^2}$$

$$\log 2^n$$

$$\log n^{\log n}$$

$$\log n$$

$$\log (\log(n))^2$$

$$\log n$$

$$<$$

$$2 \log (\log n)$$

Multi (A, B, n) {

$n+1$ For (i=0; i<n; i++)

$n(n+1)$ For (j=0; j<n; j++)

$n \times n (n+1)$ For (k=0; k<n; k++)

$n \times n \times n$

C = C + A + B;

Time = $O(n^3)$

Space = $O(n^2)$. ?

for (int i=0; i <= n; i += 2) { i = 0, 2, 4, 6, ... k
 Statement;
 } Stop in $i > n$

$k > n$

* increment and decrement $= O(n)$.
 same time $O()$

for (int i=1; i <= n; i *= 2) { i = 2, 2', ... 2^k
 Statement;
 } Stop in $i > n$

$\log 2^k > n \log$

$= O(\log n)$. $\leftarrow k > \log n$

for (int i=n; ~~i <= n~~ i >= 1; i /= 2) { i
 Statement;
 } Stop in $i < 1$

$\log \frac{n}{2^k} < 1 \log$

$= O(\log n)$

$n \rightarrow \frac{n}{2^0}$
 $\frac{n}{2} \rightarrow \frac{n}{2^1}$
 $\frac{n}{m} \rightarrow \frac{n}{2^k}$

$O(n)$ \leftarrow جزء / جزء

$O(\log n)$. \leftarrow جزء / جزء

*


```

P = 0;
for (i = 1; i < n; i = i * 2) {
    P++;
}

```

→ log(n)

```

for (j = 1; j < P; j = j * 2) {
    Statement;
}

```

→ log(n)

∴ log(log n).

```

for (i = 0; i < n; i++) → n

```

```

for (j = 1; j < n; j = j * 2) → log n

```

Statement

∴ O(n log n).

```

P = 0;

```

```

for (int i = 1; P < n; i++) {

```

√n P += i; ⇒ P = P + i

```

for (int j = n; j > 1; j /= 2) {
    Statement;
}

```

log n

∴ O(√n log n).

while loop for statement
 ليس السجدة

```

Test (int n) {
  if (n > 0) {
    print (n);
  }
}

```

Recursion.

* Print Down.

{ Test (n-1); } *isla qazilad for qazilad *
max 11*

$$[1] \quad T(n) \begin{cases} 1 & n \leq 0 \\ T(n-1) + 1 & n > 0 \end{cases}$$

$$[2] \quad T(n) = T(n-1) + 1$$

$$T(n-1) = T(n-2) + 1$$

$$T(n-2) = T(n-3) + 1$$

*isla qazilad for qazilad **

$$\begin{aligned}
T(n) &= T(n-1) + 1 \\
&= T(n-2) + 1 + 1 \\
&\vdots \\
&\vdots
\end{aligned}$$

* Assume $n-k = 0$

$$T(n) = T(n-k) + k$$

$$n = k$$

$$T(k) = T(0) + n$$

$$T(k) = \cancel{k} + n$$

$$\therefore O(n).$$

```
void Test (int n) {
```

```
    if (n > 0)
```

```
        for (int i = 1; i <= n; i *= 2)  $\Rightarrow \log n$ 
```

```
            System.out.println(i);
```

```
        Test (n-1);
```

```
    }
```

$$T(n) = \begin{cases} 1 & n=0 \\ T(n-1) + \log n & n > 0 \end{cases}$$

$$\rightarrow T(n-1) = T(n-2) + \log(n-1) \quad \sim (1)$$

$$T(n-2) = T(n-3) + \log(n-2) \quad \sim (2)$$

$$T(n) = (T(n-2) + \log(n-1)) + \log n$$

$$T(n) = T(n-2) + \log(n-1) + \log n$$

$$T(n) = T(n-3) + \log(n-2) + \log(n-1) + \log n$$

{

⋮

$$T(n) = T(n-k) + \log(n-k-1) + \log(n-k-2) + \dots + \log n$$

Assume $k-n=0$

$k=n$

$$T(n) = T(n-n) + \log(n-n-1) + \log(n-1) + \log n$$

$$= T(0) + \log(1) + \log(2) + \dots + \log(n-1) + \log n$$

$$T(n) = 1 + \log(1 \times 2 \times 3 \times 4 \times \dots \times (n-1) \times n)$$

$$T(n) = \log n!$$

$$= O(n \log n)$$

$$T(n) = 4T(n/2) + n + 1$$

$$T(n/2) = 4T(n/2^2) + n/2 + 1 \quad \text{--- (1)}$$

$$T(n/2^2) = 4T(n/2^3) + n/2^2 + 1 \quad \text{--- (2)}$$

$$T(n) = 4T(n/2) + n + 1$$

$$= 4[4T(n/2^2) + n/2 + 1] + n + 1$$

$$= 4^2 T(n/2^2) + 2n + 4 + n + 1$$

$$= 4^3 T(n/2^3) + 4^2 n/2^2 + (4) + 2n + (4) + n + 1$$

$$= 4^3 T(n/2^3) + 7n + \cancel{1}$$

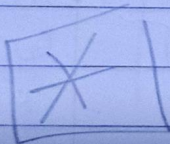
$$T(n) = 4^k T(n/2^k) + \underbrace{cn}_{\text{Constant}}$$

$$\underline{\underline{2^n (1) + cn}}$$

$$\log n = 2^k \log$$

$$\log n = k$$

Big O (2^n)



* هاد حل السؤال الآخر

في آخري صفحة

في التام كومبلكسي

$$\begin{aligned}
 T(n) &= T(n+1) + 1 \quad \rightarrow O(n) \\
 &= T(n+1) + n \quad \rightarrow O(n^2) \\
 &= T(n) + \log n \quad \rightarrow O(n \log n) \\
 &= T(n) + n^2 \quad \rightarrow O(n^3) \\
 &= T(n) + n \log n \quad \rightarrow O(n^2 \log n)
 \end{aligned}$$

constant

$$T(n) = T(n-2) + 1 \quad O(n)$$

$$T(n) + b \quad O(nb)$$

↓
 ← متن سؤا رقم مازان فالتس

alga Test (int n) {
 if (n > 0) {
 Print (n);
 Test (n-1);
 Test (n-1);
 }
 }

$$T(n) = \begin{cases} 1 & n=0 \\ 2T(n-1)+1 & n>0 \end{cases}$$

$$T(n) = 2T(n-1) + 1 \quad \rightarrow T(n-1) = 2T(n-2) + 1 \quad \text{①}$$

$$T(n-2) = 2T(n-3) + 1 \quad \text{②}$$

$$T(n) = 2(2T(n-2) + 1) + 1$$

$$= 2^2 T(n-2) + 2 + 1 \quad \sim \text{by ①}$$

$$= 2^2 (2T(n-3) + 1) + 2 + 1$$

$$= 2^3 T(n-3) + 2^2 + 2^1 + 2^0 \quad \sim \text{by ②}$$

Assume $n - k = 0 \rightarrow \boxed{n = k}$

$$T(n) = 2^k T(n-k) + 2^{k-1} + 2^{k-2} + \dots + 2^2 + 2^1 + 2^0$$

$$= 2^n + 2^{n-1} + 2^{n-2} + \dots + 2^0 \quad n=k \text{ steps} *$$

$$\sum_{k=0}^n a^k = \frac{1-a^{n+1}}{1-a} \Rightarrow \sum_{i=0}^n 2^i = \frac{1-2^{n+1}}{1-2} = \left[\frac{2^{n+1}-1}{2-1} \right]$$

$$* \left[O(2^n) \right] *$$

$$T(n) = 2T(n-1) + 1 \rightarrow O(2^n)$$

$$T(n) = 3T(n-1) + 1 \rightarrow O(3^n)$$

$$T(n) = 3T(n-1) + n \rightarrow O(n 3^n)$$

$$T(n) = 3T(n-1) + \log n \rightarrow O(3^n \log n)$$

```
Public static int Test(int n){
```

```
    if (n == 0)
```

```
        return 0;
```

```
    else
```

```
        return 2T(n/2) + n;
```

```
}
```

$$T(n) = \begin{cases} 1, & n=0 \\ 2T(n/2) + n, & n > 0 \end{cases}$$

$$T(n/2) = 2T(n/2^2) + (n/2) \quad \text{--- (1)}$$

$$T(n/2^2) = 2T(n/2^3) + (n/2^2) \quad \text{--- (2)}$$

$$T(n) = 2T(n/2) + n$$

$$= 2(2T(n/2^2) + n/2) + n$$

$$= 2^2 T(n/2^2) + 2n/2^1 + n/2^0$$

$$= 2^2 (2T(n/2^3) + n/2^2) + 2n/2^1 + n/2^0$$

$$= 2^3 T(n/2^3) + \cancel{2^2 n / 2^2} + \cancel{2n / 2^1} + n/2^0$$

$$2^3 T(n/2^3) + n$$

$$T(n) = 2^k T(n/2^k) + kn$$

$$n + n \log n$$

$$\therefore O(n \log n)$$

$$\frac{n}{2^k} = 1$$

$$\log n = 2^k \text{ (log)}$$

$$\log n = k$$

Explain this *

Test (int n) {

if (n == 1)

return 1;

else {

Print (n);

return 4T(n/2) + n;

}

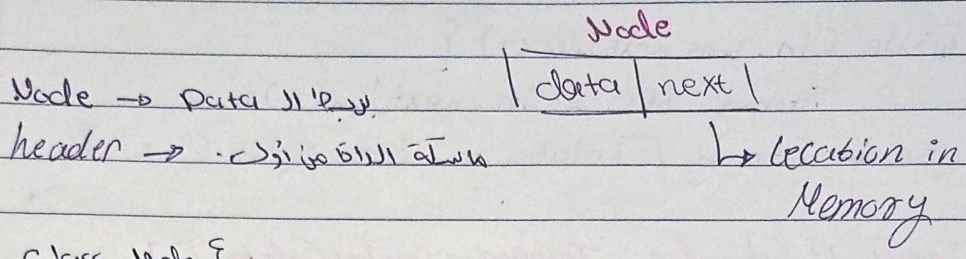
}

*

Big O
 2^n

$$T(n) = \begin{cases} 1, & n=1 \\ 4T(n/2) + n + 1, & n > 1 \end{cases}$$

Chapter 3 Linked List



```
class Node {
    private String name;
    private int id;
    Node next;
}
```

عنا في الذاكرة في الذاكرة

Operations-

- ① insert
- ② delet
- ③ update
- ④ Traversal.

```
public class Node {
    private roll;
    private int age;
    Node next;
}

Node (int roll, int age) {
    this.roll = roll;
    this.age = age;
    Next = null;
}
```

Constructor → Node () { }

getter, setter

println في الذاكرة ← to string

method في الذاكرة
بشيء اشرح


```

Node header = new Node ();
File file = new File ("data.txt");
Scanner in = new Scanner (file);
    
```

File بنا قرا

```

while (in.hasNextLine ()) {
    String str = in.nextLine ();
    String [] data = str.split (":");
    
```

قرا بنا بنا

```

insertLL (header, Integer.parseInt (data [0],
header = Integer.parseInt (data [1]);
    
```

(while)

}

insert بنا البراق *

```

public static Node insertLL (Node header, int roll, int age) {
    
```

```

Node first = header;
    
```

header

```

Node newNode = new Node (roll, age);
    
```

```

if (first == null) {
    
```

```

return newNode; header = newNode;
    
```

}

```

} else {
    
```

```

newNode.next = first;
    
```

```

first = newNode; header = newNode;
    
```

}

```

return first;
    
```

}

Big O (1)

+ constant

عن ان تاكد بعد method تصبح

نصف

```

printList (header);
    
```

print List (Node header) ?

Node ptr = header ;

while (ptr != null) ?

ptr.printInfo() ;

ptr = ptr.next ;

}

O(n).

public static Node insertAtPosition (Node header , int roll ,

int age , int key) ?

header > 1 < Node ptr = header ;

Node newNode = new Node (roll , age) ;

if (ptr == null)

ptr = newNode ; return newNode . ;

else {

while (ptr != null && ptr.getRoll() != key) {

ptr = ptr.next ;

if (ptr.getRoll() == key) {

newNode.next = ptr.next ;

ptr.next = newNode ; }

if (ptr.next == null) ?

newNode.next = ptr ;

ptr = newNode ;

}

return ptr ;

Array and linked list store collection of data.

Array \Rightarrow Store all element in one block of memory.

linkedlist \Rightarrow allocate memory for each element separately and only when necessary.

Disadvantages of Array.

I] Size is fixed. \rightarrow حجمها ثابت لا يمكن أن نعدل حجمها وإذا تغير حجمها يتغير مكانها في الذاكرة
1) Array التي حجمها ليس في الذاكرة يكون
① 90 (أو 30) من 100 من الذاكرة والباقي يكون فارغاً.
2) إذا كان حجمها 100 والباقي فارغاً.

II] Inserting and deleting elements into the middle of array is potentially expensive because existing elements need to be shifted over to make room.

الزيادة والحذف في منتصف Array
الزيادة والحذف في المنتصف يتطلب shift
بعض العناصر.

Array versus linked list :-

Array \Rightarrow insert and delete at end.

Randomly accessing any element searching the list for a particular value.

linked list \Rightarrow Insert and Delete

Application where sequential access is required.
number of element can't be predicted before hand.

```

public void insertLast (int data) {
    Node ptr = head;
    Node newNode = new Node(data);

```

```

    if (ptr == null) {
        ptr = newNode;
    }

```

```

    else { while (ptr.next != null) {
        ptr = ptr.next;
        ptr.next = newNode;
    }
}

```

```

public void deleteFirst () {

```

```

    Node ptr = head;

```

```

    if (head == null) {

```

```

        System.out.println("Empty!");
    }

```

```

    else if (head.next == null) {

```

```

        head = null;
    }

```

```

    } else {

```

```

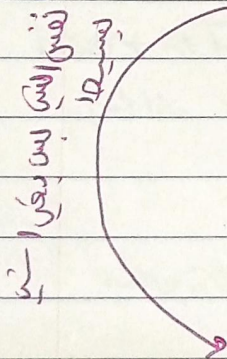
        ptr = head.next;

```

```

        head.next = ptr;
    }
}

```



delete last

```

    else if (head.next.next == null) {

```

```

        head.next = null;
    }

```

```

    else { while (ptr.next.next != null) {

```

```

        ptr = ptr.next;
    }

```

```

}

```

```

ptr.next = null;
}

```

```
public void deletePossision (int key) {
```

```
Node ptr = head; next;
```

```
Node prev = head;
```

```
if (head == null) {  
    System.out.println("Linked List Empty!");  
}
```

```
else {
```

```
while (ptr != null && ptr.getData() != key) {
```

```
    prev = ptr; next;
```

```
    ptr = head ptr.next;
```

```
}
```

```
if (ptr == null) {
```

```
    System.out.println("Not found");
```

```
}
```

```
else {
```

```
    prev.next = ptr.next;
```

```
    ptr.next = null;
```

```
}
```

```
}
```

```
}
```

```
ptr = head.next;
```

```
head = ptr;
```

double linked list

chapter 4

```
class Node {  
    private key;  
    Node next;  
    Node prev;
```

عدادة من الجداول

```
Node () {  
    }
```

```
Node (int key) {  
    this.key = key;  
    }
```

getter, setter
to string

```
class DLinkedList {
```

```
    DLinkedList () {  
        Node header, tailer prev;  
        header = null;
```

تونس

insert At First.

```
public void insertAtFirst (int key) {  
    Node newNode = new Node ();  
    { if (header == null) {  
        header = tailer = newNode ;  
    }  
    else {  
        newNode.next = header ;  
        header.prev = newNode ;  
        header = newNode ;  
    }  
}
```

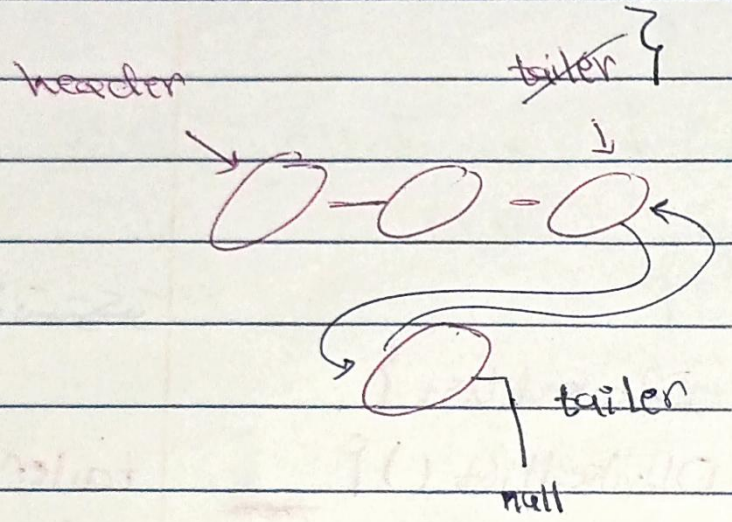
ادخال في البداية

→ insert At Last D1

```

public void insertAtLast(D1 (int key) {
    Node newNode = new Node (key);
    tailer.next = newNode;
    if (tailer == null) {
        header.prev = tailer = header = newNode;
    }
    else {
        tailer.next = newNode;
        newNode.prev = tailer;
        tailer = newNode;
    }
}

```



? ← Serqular jaisi

Delete from position:-

```
public void deleteElement (int key) {
```

```
* Node ptr = header; Node current;
```

```
if (header == null) {
```

```
    System.out.println ("out of flow");
```

```
    return;
```

```
}
```

```
else if (header.getKey() == key && header.next == null)
```

```
    header = null;
```

```
}
```

```
else {
```

```
    while (ptr != null && ptr.getKey() != key)
```

```
        current = ptr;
```

```
        if (ptr.getKey() == key)
```

```
            ptr = ptr.next;
```

```
        if (ptr.getKey() == key) {
```

```
            prev.next = ptr.next;
```

```
            ptr.next.prev = prev;
```

```
            ptr.next = null;
```

```
            ptr.prev = null;
```

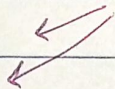
```
}
```

prev

current

next ptr

← $\frac{3}{\text{نقطه 3}}$ $\frac{2}{\text{نقطه 2}}$ $\frac{1}{\text{نقطه 1}}$



current = header

prev = current

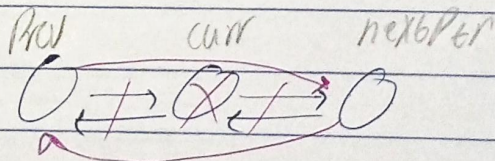
next ptr = current.next;

prev.next = next ptr;

next ptr.prev = previous;

current.next = null;

current.prev = null



~~Delete at first~~

Delete from first :-

```
public void deleteFromFirst () {
    Node ptr ;
    if (header == null) {
        ptr.print("empty");
    }
    else if (header.next == null)
        header = null;
    else {
        ptr = header.next;
        header.next = null;
        ptr.prev = null;
        header = ptr;
    }
}
```

System.gc (); | memory strip is in ~~...~~

Delete from last :-

```
public void deleteFromLast () {
    if (tailer == null) {
        ptr.print("empty");
    }
    else if (tailer.prev == null) {
        tailer = null;
    }
    else {
        ptr = tailer.prev;
        tailer.prev = null;
        ptr.next = null;
        tailer = ptr;
    }
}
```

insert At position :-

```
public void insertAtPosition (int data, int element) {
```

```
Node current = header; // data.
```

```
Node newNode = new Node ();
```

```
if (header == null) {
```

```
    print ("data not found");
```

```
    header = tailer = newNode;
```

```
}
```

```
else { while (curr.next != null && curr.getdata() != element) {
```

```
    curr = curr.next;
```

```
else if (curr.getdata() == element && curr.next != null) {
```

```
    newNode.next = curr.next;
```

```
    newNode.prev = curr;
```

```
    curr.next.prev = newNode;
```

```
    curr.next = newNode;
```

```
}
```

```
else {
```

```
    curr.next = newNode;
```

```
    newNode.prev = curr;
```

```
}
```

```
}
```

```
}
```

Radix Sort (int [] a)

int max = getMax(a);

for (int i = max; i > 0; max /= 10) {

count sort (a, p); ^{→ phase.}

element

p = p * 10;

}

$O(d * (n+k))$

number of digit

Range

Normalisation ←

←

public int getMax (int arr []) {

int max = arr [0];

for (int i = 1; i < arr.length(); i++)

if (arr [i] > max)

max = arr [i];

return max;

}

* Counting Sort :- No comparison, Sort according to keys
 counting the number having distinct
 key values.

Ex

0	2	1	1	0	2	5	4	0	2	8	7	7	9	2	0	1	9	16
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	----

Range $\rightarrow 0 \leq a[i] \leq 9^k$

Size = $k+1$

	0	1	2	3	4	5	6	7	8	9	10
count	3	3	4	0	1	1	0	2	1	2	

Update/Count:	3	4	10	10	11	12	12	14	15	17
---------------	---	---	----	----	----	----	----	----	----	----

Count Sort (a, int(p))

count { for (int i=0; i<n; i++)
 ++count[a[i] / P % 10]; } $O(n)$

update { for (int i=1; i<k; i++)
 count[i] = count[i] + count[i-1]; } $O(k)$

المسألة في
 Range ال
 قيمة كبرى

b { for (int i=n-1; i>0; i--)
 b[-count[a[i]]] = a[i]; } $O(n)$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
b	0	0	0	1	1	1	2	2	2	2	4	5	7	7	8	9	9

a { for (int i=0; i<n; i++)
 a[i] = b[i]; }

Radix Sort *
 Radix 10

$O(n+k)$

Stack

$O(1)$ insert ← Push
 $O(1)$ delete ← Pop

Stack overflow → Stack is full

Stack underflow → Stack is empty (Size = 0)

Top ← pointer to the top element
 Peek() → returns the top element without popping it

FILA

* linked list is used for queue

Application of stack

داعية بي بي سوال حلين

Main uses stack are:

1- Evaluation of Arithmetic expression

مثال * $E = A - ((B + C) * D)$

2- Execute recursive function.

depend on priority:

- | | | | |
|---|-----|----------------|------------|
| 1 | () | الاقواس | في المسائل |
| 2 | ^ | القوة | والا |
| 3 | * / | الضرب والقسمة | |
| 4 | + - | الجمع والترحيل | |

infix \rightarrow Postfix $(x) (y) \rightarrow (x) (y) (op)$

~~الاول~~ \rightarrow الاخر

الاول \rightarrow الاخر \rightarrow اللاحق \rightarrow السابق *
الاول

infix \rightarrow Prefix

~~الاول~~ \rightarrow الاخر

$(x) (op) (y) \rightarrow (op) (x) (y)$

السابق \rightarrow اللاحق \rightarrow السابق *
السابق \rightarrow اللاحق

infix \rightarrow Postfix التركيب على اليمين *

$-++A/B * CDEF/GH$

Pre \rightarrow in

الاول \rightarrow الاخر \rightarrow اللاحق \rightarrow السابق

$HG/FE DC * B - / A ++ * -$

Reverse

$(y) (op) (x)$

Stack : Expression.

C HG

(/ HG F $\xrightarrow{x} \xrightarrow{y}$

(/ HG FE DC $\xrightarrow{(x)} \xrightarrow{(y)}$

(* HG FE DC (B)

(- HG FE ~~DC~~ B * CD (A)

(- HG FE A/B * CD

(+ HG FE A/B * CD - E

(+ HG A/B * CD - E + F

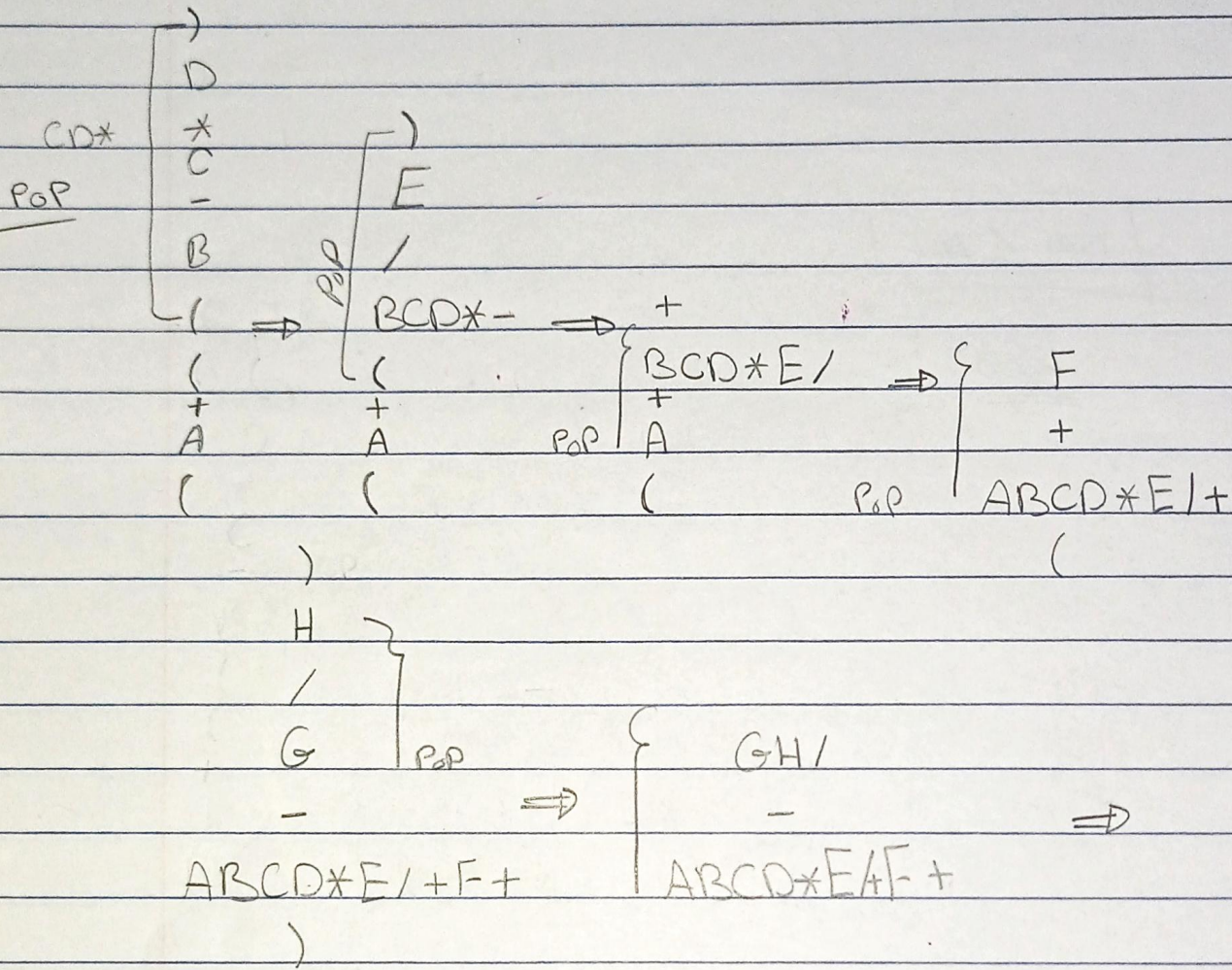
(- H A/B * CD - E + F + G

A/B * CD - E + F + G - H \rightarrow infix

Examples

II $A + ((B - C * D) / E) + F - G / H$ In \rightarrow Post

$\langle x \rangle \langle opp \rangle \langle y \rangle \rightarrow \langle x \rangle \langle y \rangle \langle opp \rangle$

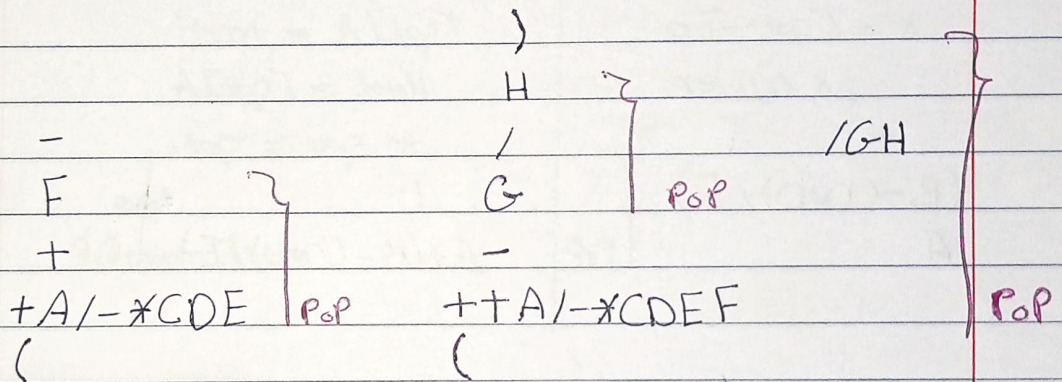
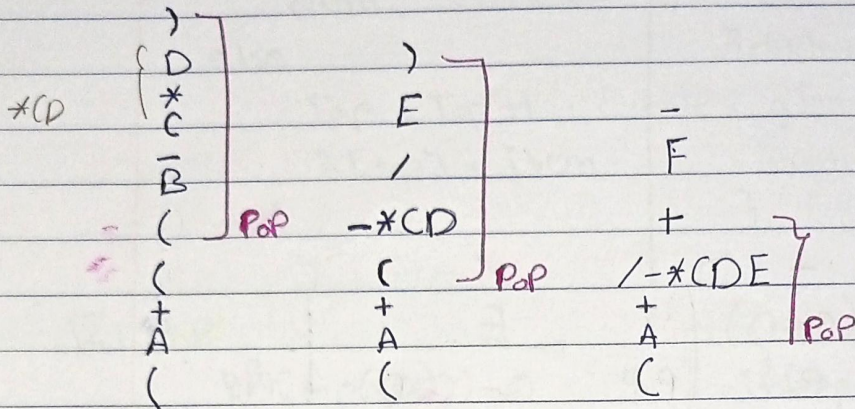


$$| \quad ABCD * E / + F + G H / - \quad |$$

Postfix

[2] $A + ((B - C * D) / E) + F - G / H$ in \rightarrow Pre

$\langle x \rangle \langle opp \rangle \langle y \rangle \rightarrow \langle opp \rangle \langle x \rangle \langle y \rangle$

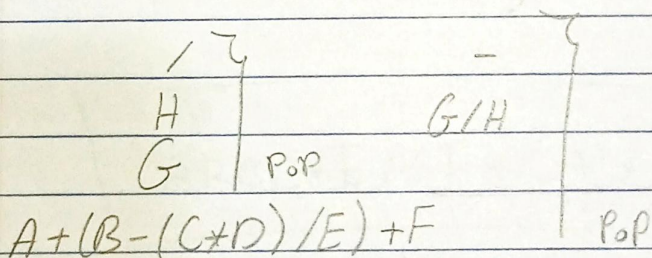
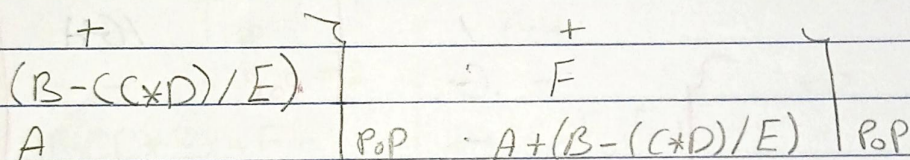
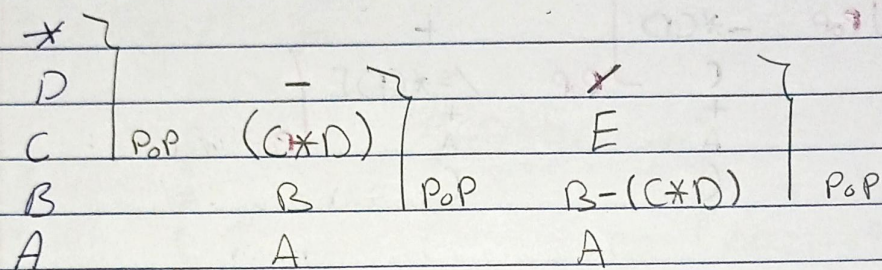


- + + A / - * C D E F / G H

Prefix

[3] ~~ABCD * - E / + F + G / H -~~ Post \rightarrow in

$\langle x \rangle \langle y \rangle \langle opp \rangle \rightarrow \langle x \rangle \langle opp \rangle \langle y \rangle$



$A + (B - (C * D) / E) + F - G / H$

infix

Stack using Array :-

1] Push.

```

if (top > size)
    print "overflow"
else
    Top = Top + 1
    A[top] = item
end
    
```

```

int Push(int x) {
    if (top == (size - 1))
        println("overflow");
    else {
        a[++top] = x;
        println(x + "Pushed");
    }
}
    
```

2] Pop

```

if (top < 0)
    print "under flow"
else
    item = A[top]
    A[top] = Null
    top = top - 1
end
return (item)
    
```

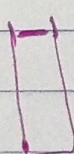
```

int Pop(int x) {
    if (top < 0)
        println("under flow");
    else {
        a[--top] = x
        return x;
    }
}
    
```

```

int Peek () {
    if (top < 0)
        print ("under flow");
    else {
        int x = a[top];
        return x;
    }
}
    
```

هاد القيس
 بوجيب احرفه
 بوجيبه في الstack
 الي في الوجه



Stack using linked List :-

1] boolean isEmpty() {

if (header == null) {

return true;

} else {

return false;

}

}

2] void push(int data) {

if (header == null) {

header = newnode;

} else {

header = newnode;

newnode->next = header;

}

}

Node temp = header;

Node newnode = new Node(data);

insert First (push) *
(new)

3] void pop() {

if (header == null) {

Print ("Stack is empty");

}

else {

header = header->next;

}

delete last *
first

```
4 | int peek () {  
    if (header == null) {  
        println("Stack empty!");  
    }  
    else {  
        return header.data;  
    }  
}
```

من القائمة التي ستأتي
التي

Queue

Queue ← زک - الطابور
مبدأ ال
الک - یخرج اولاً - یطرح اولاً
الک - یخرج آخر آخراً - یطرح آخر آخراً

Fifo
Lilo

* enqueue = List.insertFirst() ← آخر
والکرف من الاول
dequeue = List.deleteLast()

* فنیج
2 pointer
front = مؤشر ال اول
rear = مؤشر ال آخر

Stacks \Rightarrow solve problem works on recursion.

Queue \Rightarrow solve problem have sequential processing.
processing.

Queue using Linked List :-

1] void enqueue (int data) {

Node newnode = new Node (data);

if (rear == null) {

front = rear = newnode;

rear.next = front;

}

else {

rear.next = newnode;

rear = newnode;

rear.next = front; }

front }

* اذا كان ال Queue فارغاً ال front و ال rear

يساوي ال newnode ال next ال rear ال front

2] int dequeue () {

Node temp;

if (front == null) { rear = null; }

print (" Empty!");

return 0;

}

else {

Node temp = front; temp = front;

front = front.next;

rear.next = front;

temp.next = null;

}

return temp;

}

Running time

Linked List \rightarrow

- insert $O(1)$
- delete $O(1)$
- Find $O(N)$

Stack \rightarrow

- Push $O(1)$
- Pop $O(1)$

Queue \rightarrow

- enqueue $O(1)$
- dequeue $O(1)$

Stack

based in the **LIFO** or **FILO**
first in last out or
last in first out.

insertion and deleting in stacks
take place only from one end
of the lists called the **top**

insertion called **Push**
Delete called **Pop**

We need **one pointer** which
always point to the last element
called **top**

Solving problems works on
recursion

Queue

based on the **FIFO** or
LIFO, first in first out or
last in last out.

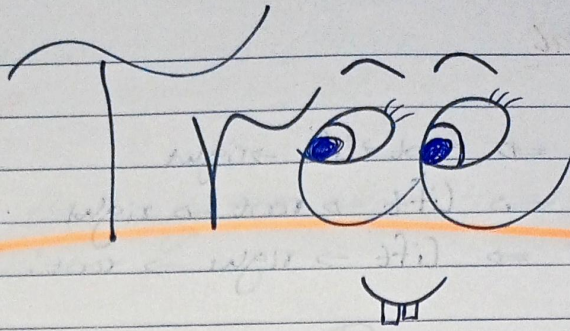
insertion ~~take~~ takes place at
the **rear** of the list and
deletion from **front**.

insertion called **enQueue**
delete called **deQueue**

We need ~~to~~ **two pointer**
front → first element inserted
in the list and is still present,
rear → always point to the
last inserted element.

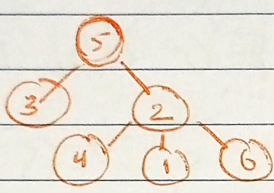
solve problems having
sequential processing

Chapter 4

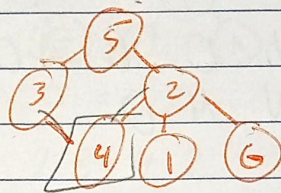


فيستويها لا Tree

- 1. ال root هو عبارة عن Node ليس عنده ابناء
- 2. كل Node له ابناء واحد
- 3. كل Node له ابناء بعض او اكثر من الاولاد
- 4. ان يوجد لا Node اكثر من يقين ابناء طرف واحد فقط

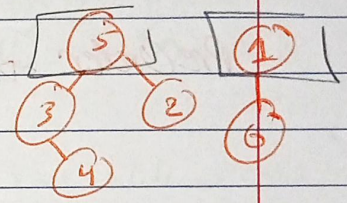


tree



Not tree

في ال (4) طرفين



Not tree

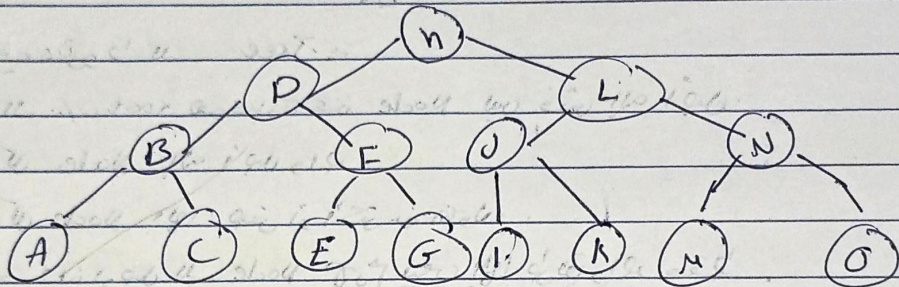
2 root

انواع ال Tree

- Ancestor
- Proper ancestor
- Descendent
- Proper descendent
- Subtree
- leaf
- internal
- Siblings
- size
- level
- Hight

Tree traversal

- ① Pre Order \Rightarrow root \rightarrow left \rightarrow right
- ② In Order \Rightarrow left \rightarrow root \rightarrow right
- ③ Post Order \Rightarrow left \rightarrow right \rightarrow root.



Pre Order \Rightarrow h D B A C F E G L J I K
N M O

InOrder \Rightarrow A B C D E F G h I J K L
M N O

Post Order \Rightarrow A C B E G F D I K J M O N
L h

Exercise

* important

Construct the binary tree whose traversal are given below.

Pre $\overset{\text{root}}{\text{H}} \text{ D B A C F E G I K } \cancel{\text{J}} \text{ M O}$ root left right
 Post A C B E $\cancel{\text{F}}$ $\cancel{\text{D}}$ I K J M $\cancel{\text{N}}$ $\cancel{\text{X}}$ $\cancel{\text{K}}$ H left right root.

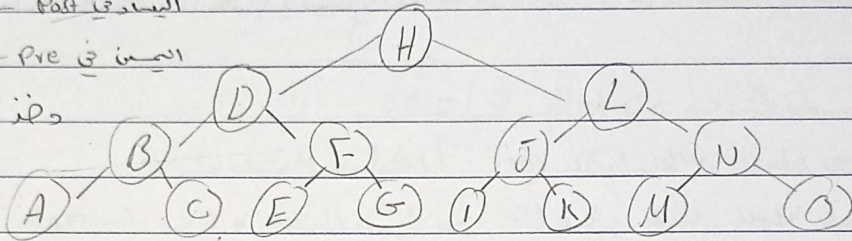
tree ال ← Post اليساري

tree ال ← Pre اليمين

وقد ال ال ال ال

وقد ال ال ال ال

وقد ال ال ال ال



root
 infix

Prefix ① - + 9 9 + * 2 3 4 2

root → left → right

infix 9 + 9 - 2 * 3 + 4 / 2

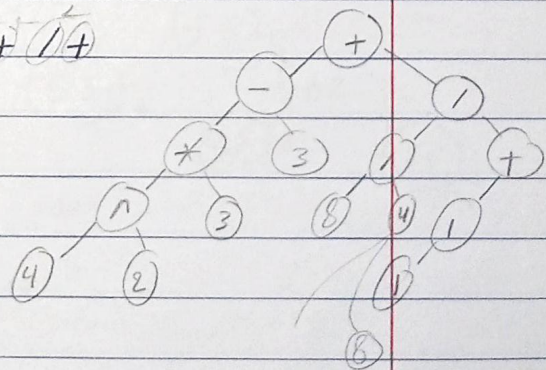
left → root → right

Postfix → ?

Pre → ⊕ ⊖ ⊗ ⊘ 4 2 3 3 1 1 8 4 ⊕ 11

infix → ?

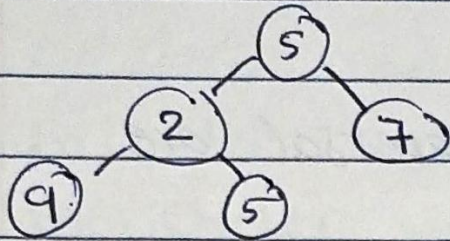
Post → 4 2 ^ 3 ⊗ 3 ⊖ 8 4 / 11 ⊕ 11



Binary tree

Node 0 درخت کی پس Tree

Node 2 درخت کی



Full Binary tree \Rightarrow

Complete Binary tree \Rightarrow درخت کی ہر سطح پر تمام جگہوں پر نود ہونے کی ضرورت نہیں ہے۔

وہاں تک کہ ہر نود کی تمام جگہوں پر نود ہونے کی ضرورت ہے۔

Binary search tree

$O(\log N)$

* كارت اللى على اليمين يكون
أكبر من ال root والى
على اليسار أصغر .

* عندها أصغر Node كبيرة بستوف
إذا دكر من ال root دوح على اليمين
وإذا كان أكبر من ال Node اللى على
يسار ال root د دوح على اليمين وهكذا
حتى اللى مكاننا نأمن .

* البنية لكارت بوي اوله 3 حالات .

1. ال Node مالها اولاد بغي leaf ← كزفها و عادي .
2. ال Node عندها ولا اولاد ← كزف ال Node و بجالي ابنا اولاد عنها .
3. ال Node عندها ولدتين ← كارت اوله ال Min أو ال Max
ال Node اللى بوي ال Node عنها و ال Node
ماتنه .

~~Min ← أصغر اللى على ال left (أصغر)~~

~~Max ← أكبر اللى على ال right (أكبر)~~

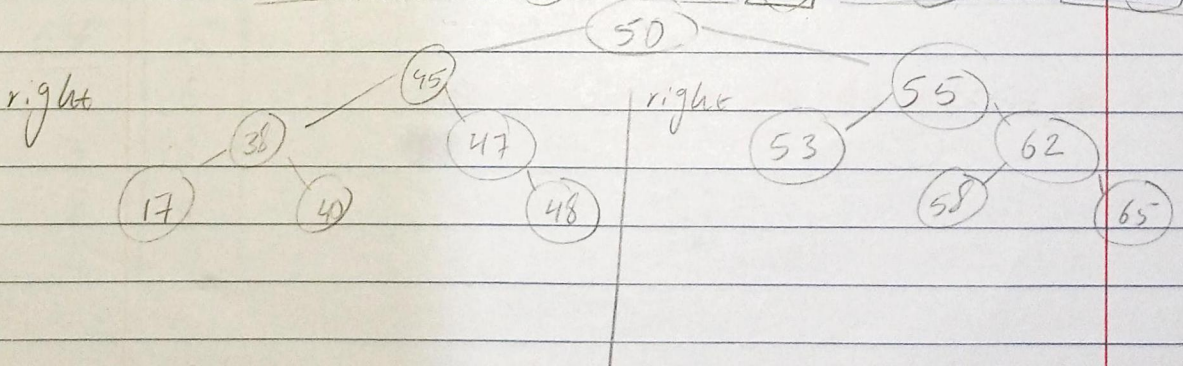
Min ⇒ اللى على ال right

* يكون فى ال left

Max ⇒ أكبر اللى على ال left

* يكون فى ال right

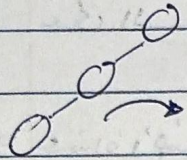
Ex Post ⇒ 17 40 38 48 47 45 53 58 65 62 55 50
in ⇒ 17 38 40 45 47 48 50 53 55 58 62 65



AVL tree

Binary search tree of x

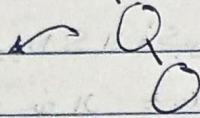
height balanced ← 'Balanced'
(-1, 0, 1)



Right Rotation ←

Rotation

Q: Left Rotation ←



Single Rotation → Right
 ↘ Left

Double Rotation → Left Right
 ↘ Right Left

Balanced if delete or insert

AVL tree

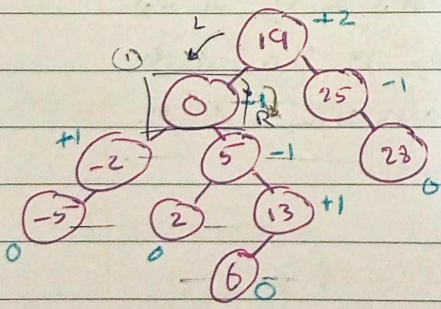
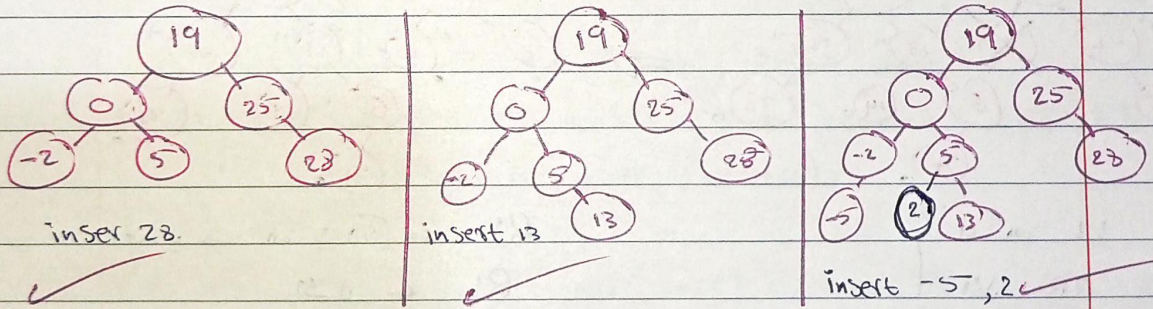
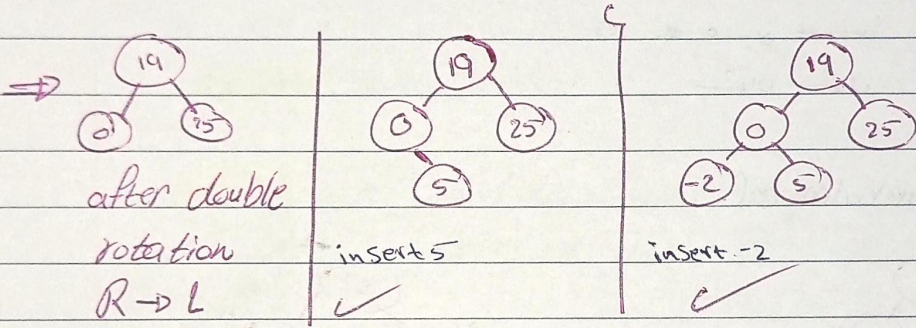
Insert an integer number consecutively in an empty AVL tree.

- 0, 25, 19, 5, -2, 28, 13, 2, 6, 14, 7
- Delete node 19 using next successors.

insert 0	insert 25	insert 19
Balanced	Balanced	imbalanced
no need to balance	no need to balance	need to balanced
		Problem: RL

* التوازن يكون اذ كان الارتفاع واحد
واليسار واليمين يعادل بعضهما
لا

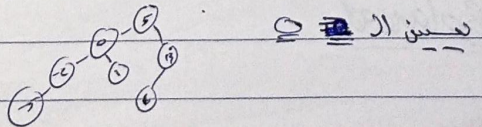
Solve: RL rotation

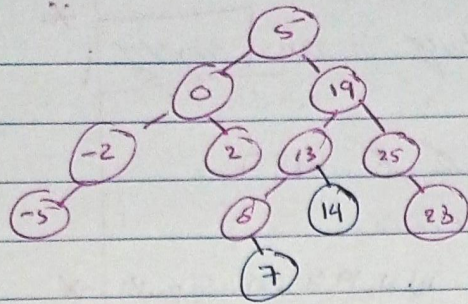


imbalanced
need to balanced
Problem ⇒ LR
Solve ⇒ LR Rotation

insert 6

* ال 5 ليصبح مكان ال 0، وال 2 ليصبح ال 5





ہیسا لیٹر پیو ال 5 مکا ان 19
 والی دک عینا ال 5 پیو ال
 لہ 19 ان ال

insert 14 → Balanced
 insert 7 → Balanced



* لیٹ آڈیو ال مکا وین

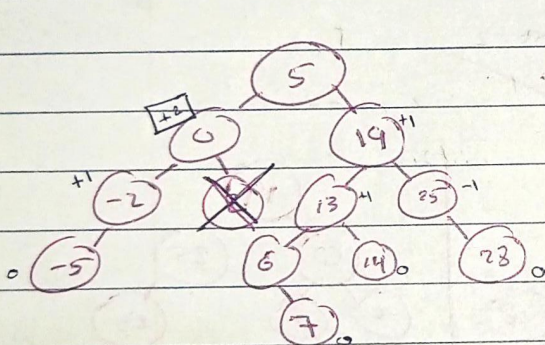
الک لہ ال ال Balanced ← + ← ال ال

ال ال ← -

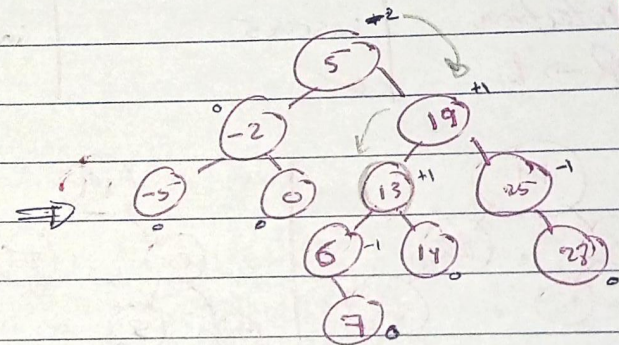
ال ال ← 0

ال ال ←

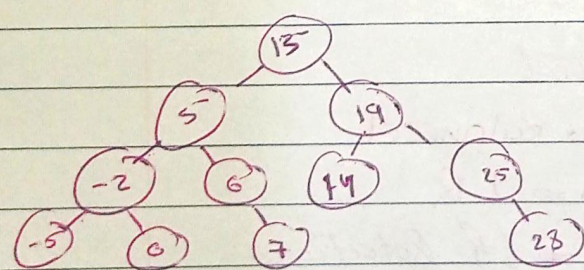
Delete 2 ⇒ imbalanced



LL ال = 1
 RR ال



RL ← ال ال ال ال ال ال ال ال
 ال ← ال ال



ال ال ال ال ال ال ال ال
 ال ال ال ال ال ال ال ال
 ال ال ال ال ال ال ال ال

Balanced

Tree

```

Node tree {
    Object data;
    Node left;
    Node right;
}
    
```

```

Node (Object data) {
    this.data = data;
}
    
```

```

class tree {
    private Node root;
    public void insert (Object data) {
        root = insert (root, data);
    }
}
    
```

Main میں جو root کی تعریف ہے وہ private آپس میں
 public void insert (Object data) { } میں method ہے
 root = insert (root, data) out میں ہے private آپس میں
 [] میں تو وہ public
 Main میں جو root کی تعریف ہے وہ private آپس میں

```

Node insert (Node root, Object data) {
    if (root == null) {
        Node newNode = new Node (data);
        return newNode (data);
    }
}
    
```

Main میں جو root کی تعریف ہے وہ private آپس میں

```

else if (root.getData() < data) {
    return root.setRight (insert (root.getRight(), data));
}
    
```

Right

```

else if (root.getData() > data) {
    return root.setLeft (insert (root.getLeft(), data));
}
    
```

Left

* recursion میں
 } وہ کارا ہے

```
Public void InOrder() {  
    InOrder (root);  
}
```

```
Private void InOrder (Node root) {  
    if (root == null)  
        return;  
    InOrder (root . getLeft ());  
    System.out.println (root . getData ());  
    InOrder (root . getRight ());  
}
```

PreOrder

PostOrder

```
public boolean delete (int data) {  
    if (root == null) {  
        return false;  
    }  
    else if ( !  
        return  
    }  
    else {  
        return delete (root , data);  
    }  
}
```

B-Trees

Database *

- 1] leaf level ke level
- 2] Node ke level root se leaf tak, m aur $\frac{m}{2}$ child
- 3] element ki khatir $m-1$ aur $\frac{m}{2}$?
- 4] root ke $\frac{m}{2}$ child aur kyon ke wo.

* اهم الترتيب :-

Example

* Create a B-tree of order 5

AGFBKDHMJESIRXCLNTUP

* لما يبي أكبر من size

بذل split و مع ليه الارتفاع

CP للي بالنسبة

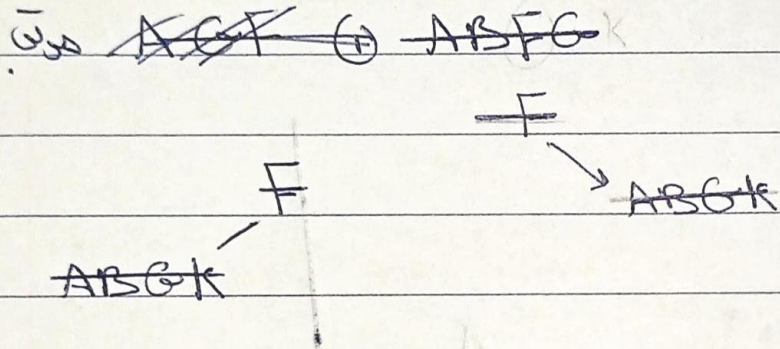
* لما يبي اقل من size

وتطلع ف حوف ←

واكبدي اليبق اجمع

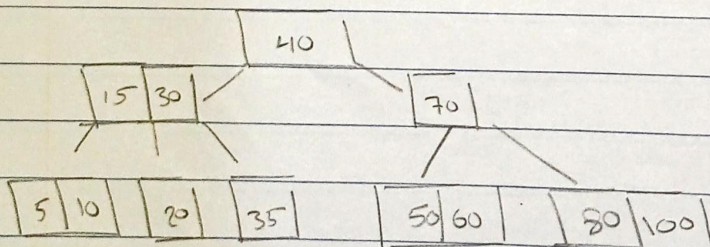
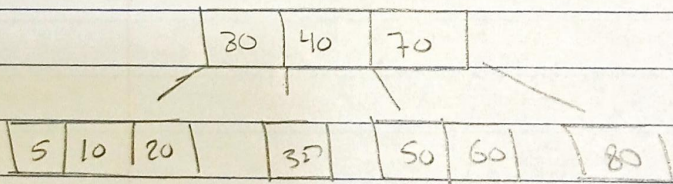
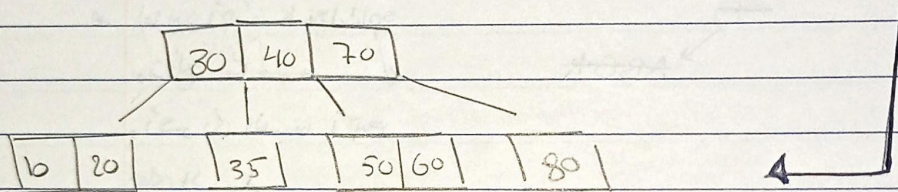
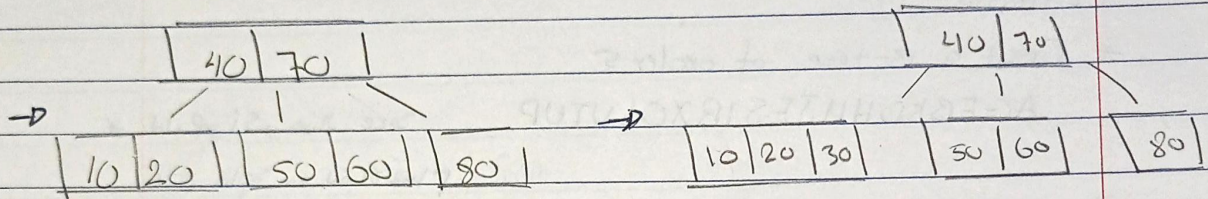
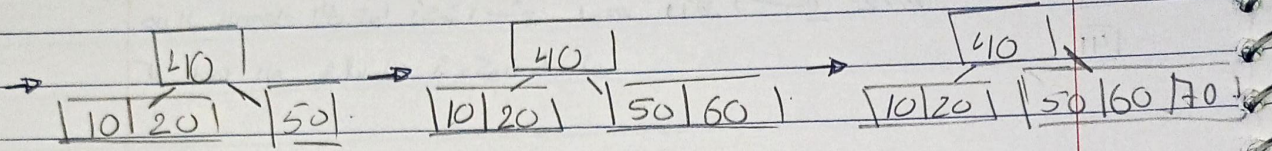
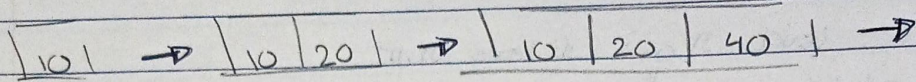
كالي اليبق

معهم الترتيب



4

~~10~~ ~~20~~ ~~40~~ ~~50~~ ~~60~~ ~~70~~ ~~80~~ ~~30~~ ~~35~~ ~~5~~ 15 60



Chapter 5

Hashing

* هون كى زيادى دى array

Fixed size ← array

- Linear search $\Rightarrow O(N)$
- Binary search $\Rightarrow O(\log N)$
- if we know index $\Rightarrow O(1)$

* Since $O(1)$ is fast, index is fast array

index! array

10 → 0, 9 → 1

... etc.

Example: Hashing $[X \% \text{table size}]$

insert 10 $\Rightarrow 10 \% 10 = 0$

insert 13 $\Rightarrow 13 \% 10 = 3$

collision

collision

0	10
1	
2	
3	3
4	4
5	5
6	6
7	13
8	8
9	

* out of array size

insert \rightarrow \leftarrow \leftarrow \leftarrow \leftarrow Hashing \rightarrow *
searching

decreation. \rightarrow increbation \rightarrow DVA \rightarrow \leftarrow \leftarrow \leftarrow \leftarrow *
*

Example:

use the hash function to load the following commodity items into a hash table of size 13 using separate chaining:

1- Onion \rightarrow $O = 111$ $n = 110$ $i = 105$

$$\text{hash}(\text{onion}) = (111 + 110 + 105 + 111 + 110) \% 13$$
$$= 1$$

2- Salt \rightarrow $s = 115$ $a = 97$ $l = 108$ $t = 116$

$$\text{has}(\text{salt}) = (115 + 97 + 108 + 116) \% 13$$
$$= 7$$

0 \rightarrow Okra \rightarrow potato

1 \rightarrow Onion \rightarrow carrot

2

3

4 \rightarrow cabbage

5

6 \rightarrow mushroom

7 \rightarrow salt

8

9 \rightarrow cucumber

10 \rightarrow tomato \rightarrow melon \rightarrow olive

11 \rightarrow banana

12 \rightarrow orange.

open addressing :-

$$* h_i(\text{key}) = [h(\text{key}) + f(i)] \% n$$

↳ linear

Quadratic

double

$$* f(0) = 0$$

value \Rightarrow between 0 and $n-1$

$$f(0) \% n, f(1) \% n, \dots, f(n-1) \% n$$

$$* \text{linear } f(i) = i$$

$$* \text{Quadratic } f(i) = i^2$$

$$* \text{double } f(i) = i * h_p(\text{key})$$

بالطبع يجب ان يكون table size الـ Prime (عدد اول)

collision

load factor λ collision

وكون بين (0.6, 0.7)

$\lambda = \text{Number of element} / \text{table size}$

undesirable

← 0.7 λ كبيرين
rehash

Type of open addressing :-

I Linear

Example

insert (18), (26), (35), (9), find (15), (48)
 delet (35), (40) find (9), insert (64), (47)
 find (35).

$f(i) = i$, table size = 13.

$h(key) = key \% \text{table size}$

$h_i(key) = (h(key) + f(i)) \% 13$

$h_0(18) = (18 \% 13 + 0) \% 13 = 5 \% 13 = 5$ 0 26

$h_0(26) = (26 \% 13 + 0) \% 13 = 13 \% 13 = 0$ 1

$h_0(35) = ((35 \% 13) + 0) \% 13 = 9 \% 13 = 9$ 2

$h_0(9) = (9 \% 13 + 0) \% 13 = 9 \% 13 = 9$ → collision 3

→ $h_1(9) = (9 \% 13 + 1) \% 13 = 10 \% 13 = 10$ 4

$h_0(15) = (15 \% 13 + 0) \% 13 = 2 \% 13 = 2$ (not found) 5 18

$h_0(48) = ((48 \% 13) + 0) \% 13 = 9 \% 13 = 9$ 6

لقد تمّ رقمّ بس 48 في خروج بدون مكان الذي يوجد

$h_1(48) = (48 \% 13 + 1) \% 13 = 10 \% 13 = 10$ 7

ما لم يتمّ 48 وبدون ايجل زيّ فوق

$h_2(48) = (48 \% 13) + 2 \% 13 = 11$ (Not found) 8 47

$h_0(35) = 9$ index of 9 has a value 35 9 35

D ← E Change status من E إلى D

$h_0(40) = (40 \% 13 + 0) \% 13 = 1 \% 13 = 1$ (Not found) 10 9

Delete من E إلى D

$h_0(9) = (9 \% 13 + 0) \% 13 = 9 \% 13 = 9$ 11 64

$h_1(9) = (9 \% 13 + 1) \% 13 = 10$ 12

D ← D in status

$h_0(64) = (64 \% 13 + 0) \% 13 = 12$

$h_0(47) = (47 \% 13 + 0) \% 13 = 8$

$h_0(35) = 9$ found!

E Empty
 D Delete
 O Oqubay

[2] Quadratic

$$f(i) = i^2$$

$$h_i(\text{key}) = (h(\text{key}) + i^2) \% \text{table size}$$

insert(13), (39), (26), (52)

* كل مرة تسمى المفتاح الى

$$h_0(13) = (13 \% 13 + 0^2) \% 13 = 0$$

$$h_0(39) = (39 \% 13 + 0^2) \% 13 = 0 \quad \text{collision}$$

$$h_1(39) = (39 \% 13 + 1^2) \% 13 = 1$$

$$h_0(26) = (26 \% 13 + 0^2) \% 13 = 0 \quad \text{collision}$$

$$h_1(26) = 1 \quad \text{collision}$$

$$h_2(26) = (26 \% 13 + 2^2) \% 13 = 4$$

* كلما زاد ال collision يزيد المثال

الحجم اكثر من 70 % مشكلة وتواجه

بناء Array جديدة وديها $\times 2$

وتوزع العناصر في مواقع جديدة.

Primary Clustering

هذا ما يسمى بـ

وهو ليس سبب آخره ال

collision.

عشان ايقن انه ال table size هو Prime

العدالة

$$* \left[f(x) = x^2 + x + 41 \right]$$

* في بالسلايدان الفرق

بين ال open و ال close

Secondary clustering

* کلاسوں میں سے بعض کلاسوں میں
انگنائے (بیکونڈی کلاسوں)

وہ آہوں سے الگ Primary cluster

31 Double Hashing :-

$$h_i(\text{key}) = [h(\text{key}) + f(i)] \% \text{table size}$$

$$* f(i) = i * h_p(\text{key})$$

is prime \Rightarrow
$$\begin{cases} h_p = 1 + \text{key} \% (\text{table size} - 1) \\ = q - (\text{key} \% q) \\ q * (\text{key} \% q) \end{cases}$$

q is prime #

Example :-

insert (26, 70, 18, 9, 47, 35, 96, 64)

* table size 13

$$* h_p = 1 + \text{key} \% 12$$

$$h_i(\text{key}) = (h(\text{key}) + i * h_p(\text{key})) \% \text{table size}$$

$$h_0(26) = (26 \% 13 + 0) \% 13 = 0$$

$$h_0(70) = (70 \% 13 + 0) \% 13 = 5$$

$$h_0(18) = ((18 \% 13) + 0) \% 13 = 5 \text{ collision}$$

$$h_1(18) = (18 \% 13 + (1 * (1 + 18 \% 12))) \% 13 =$$

$$(5 + (1 * 7)) \% 13 = 12$$

$$h_0(96) = (96 \% 13 + 0) \% 13 = 5 \text{ collision}$$

$$h_1(96) = (96 \% 13 + (1 * (1 + 96 \% 12))) \% 13 =$$

$$= (5 + 1) \% 13 = 6$$

Double

0	26
1	
2	
3	
4	
5	70
6	96
7	
8	
9	
10	
11	
12	18

Exercise 4-

if the hash table after using linear probing is as shown below?

1- insert ~~29~~ 29.

2- is there a problem in hashing table? if there is state it and give a solution.

$$h_0(29) = (29 \% 7 + 0) \% 7$$

$$= 1 \% 7 = 1 \quad \text{collision}$$

$$h_1(29) = (29 \% 7 + 1) \% 7$$

$$= 2 \% 7 = 2 \quad \text{collision}$$

$$h_2(29) = (29 \% 7 + 2) \% 7$$

$$= 3 \% 7 = 3 \quad \text{collision}$$

$$h_3(29) = (29 \% 7 + 3) \% 7$$

$$= 4 \% 7 = 4 \quad \text{inserted.}$$

0

1 15

2 37

3 25

4 29

5

6 13

~~7~~

2x array of slots - 1 collision (colision) automatic *

17 is Prime # array size = 17

Double hash of prime number insert data

table size = 17

$$hp = 1 + key \% 16$$

$$h_0(15) = (15 \% 17 + 0) \% 17 = 15$$

$$h_0(37) = (37 \% 17 + 0) \% 17 = 3$$

$$h_0(25) = (25 \% 17 + 0) \% 17 = 8$$

$$h_0(29) = (29 \% 17 + 0) \% 17 = 12$$

$$h_0(13) = (13 \% 17 + 0) \% 17 = 13$$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
			37					25				29	13		15	

$$* \text{hash} = \sum_{i=0}^{\text{keysize}-1} \text{key}[\text{keysize}-i-1] \cdot 32^i$$

String ال داتا دة *

Array

بؤبؤ ال ASCII code ال دة

Method ال دة دة دة

```
Public static int hash (String key, int table_size) {
```

```
    int hashVal = 0;
```

```
    for (int i=0; i < key.length(); i++)
```

```
        hashVal = hashVal << 5 * key.charAt(i);
```

32 دة بؤبؤ ال دة
shifting ال دة دة

```
        hashVal %= table_size;
```

```
    if (hashVal < 0)
```

```
        hashVal += table_size;
```

⇒ عسب ال دة دة
over flow.

دؤبؤ ال دة دة

positive ال دة

```
    return hashVal;
```

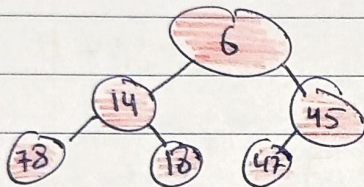
```
}
```

HEAP

Chapter 6

Binary Heap

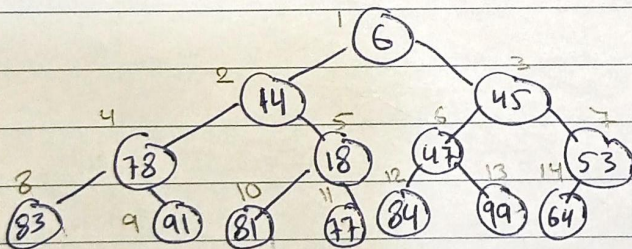
1. Complete binary Tree. \rightarrow خاصية من فوق لورا ومن اليسار لليمين
2. Min-heap ordered. \rightarrow الاكبر كانم يكونوا اكبر او يساوي الاب



\Rightarrow موقع الـ 6 اليمين اكبر
 او الـ 45 اليسار وكان من
 هم التكرار

Minimum element is in the root

Heap with N elements has height = $\frac{\log N}{2}$



\rightarrow Tree الـ 15
 ليس بـ "مجموعاً"

$$\text{Parent}(i) = i/2$$

$$\text{left}(i) = 2i$$

$$\text{Right}(i) = 2i + 1$$

Insert

insert into next available slot

Bubble up until it's heap ordered

وین 3 با 4 پیچیده
ولی این 4 > 3 پس ادا پیچ

Swap. 4 و 3 ←

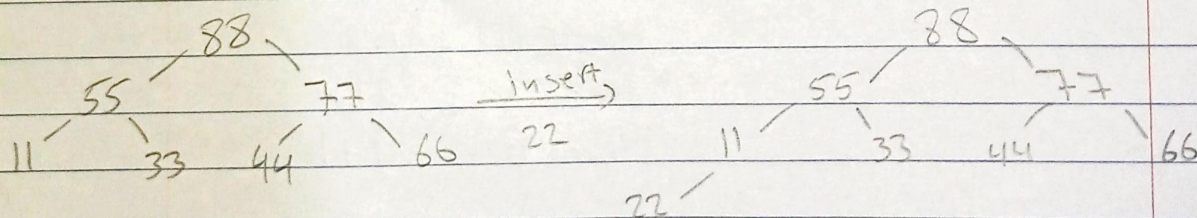
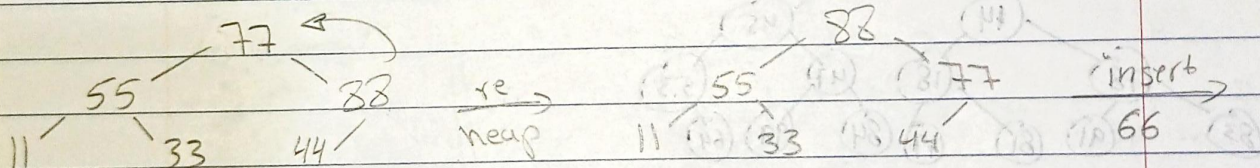
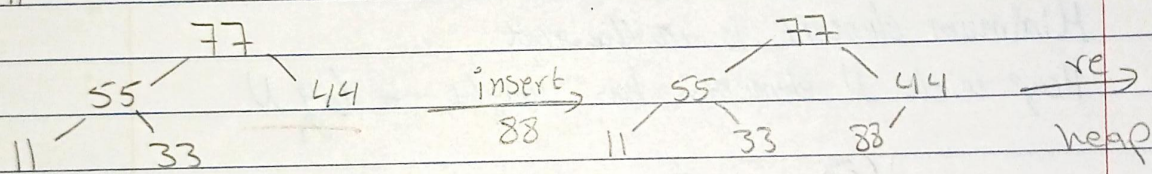
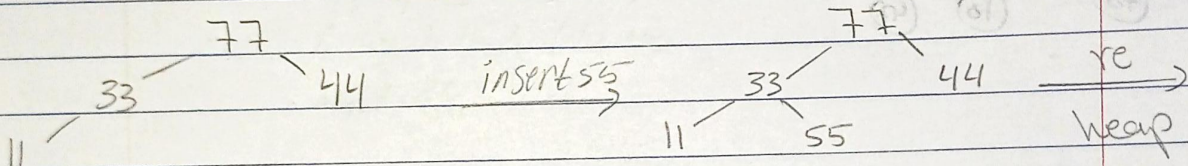
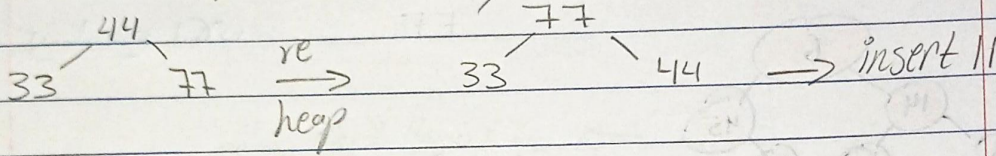
heap ordered
(4 و 3 پیچیده نیست)

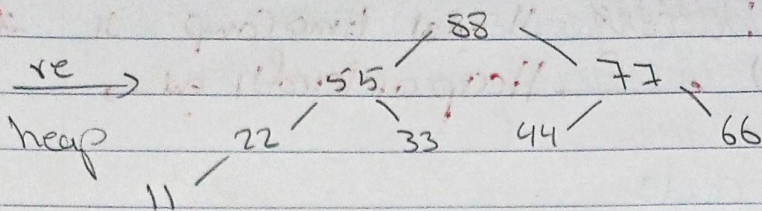
Time Complexity $O(\log N)$

Example

Build **Maximum** heap tree?

44, 33, 77, 11, 55, 88, 66, 22





Done

Delete minimum element from heap.
exchange root with rightmost leaf.

* آخر ابن اليمين

Bubble root down until it's heap ordered.

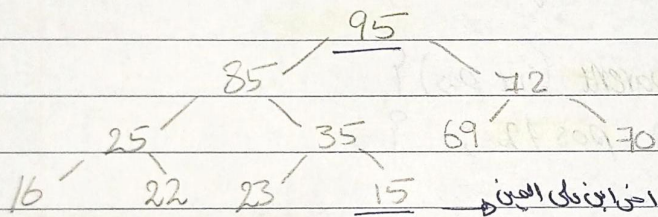
* بتدفعه اذا الابن اليمين من

الابناء

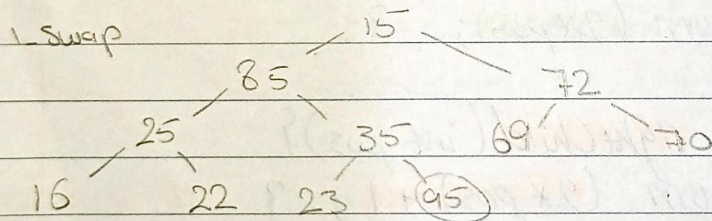
* وبتدفعه من حجم الارتفاع

Example

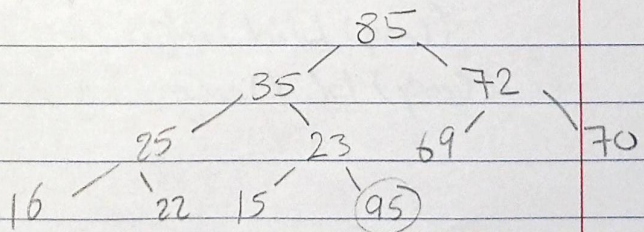
Delete root 95



maximum
Heap



2. Heap ordered



Done

$O(\log N)$ Deletion $O(\log N)$ time Comp $O(N \log N)$ \leftarrow Heap ordered $O(N \log N)$ \times

Min Heap code

```
public class MinHeap {
```

```
    private int[] Heap; size of array
```

```
    private int size; current size
```

```
    private int maxSize; max size
```

```
    private static final int FRONT = 1;
```

```
Constructor  $\rightarrow$  public MinHeap(int maxSize) {
```

```
    this.maxSize = maxSize;
```

```
    this.size = 0;
```

```
    Heap = new int[this.maxSize + 1];
```

```
    Heap[0] = Integer.MIN_VALUE; }
```

```
    private int parent(int pos) {
```

```
        return pos / 2; }
```

```
    private int leftChild(int pos) {
```

```
        return (2 * pos); }
```

```
    private int rightChild(int pos) {
```

```
        return (2 * pos) + 1; }
```

```
public void insert (int element) {
    if (size >= ele maxsize) {
        return ; } → insert فنق 3 حال نزل
```

```
Heap [++ size] = element ;
int current = size ; → مؤشر لـ size
```

```
while (Heap [current] < Heap [Parent (current)]) {
    ← نقارن الاب مع الابن
    swap (current, parent (current)); ← مباد
    current = parent (current); }
}
```

```
public void minHeap () {
    for (int pos = (size / 2); pos >= 1; pos --) {
        minHeapify (pos); }
```

```
public private void minHeapify (int pos) {
```

```
if (! isLeaf (pos)) { ← فنق الـ اولاد or
```

```
if (Heap [pos] > Heap [leftChild (pos)] ||
```

```
Heap [pos] > Heap [rightChild (pos)]) {
```

```
if (Heap [leftChild (pos)] < Heap [rightChild (pos)]) {
```

```
swap (pos, leftChild (pos)); ← اللي على اليمين اقل
    swap (pos, leftChild (pos)); ← مباد
    minHeapify (leftChild (pos)); }
}
```

```
else { ← اللي على اليمين اليه
```

```
swap (pos, rightChild (pos));
minHeapify (rightChild (pos)); }
}
```

```
} → اولاد فنق را ي اقل
```

```
private boolean isLeaf (int pos) {  
    if (pos >= (size / 2) && pos <= size) {  
        return true;  
    }  
    return false; } }
```

```
private void swap (int fpos, int spos) {  
    int temp;  
    temp = Heap [fpos];  
    Heap [fpos] = Heap [spos];  
    Heap [spos] = temp;  
}
```

```
public int remove () {  
    int popped = Heap [FRONT];  
    Heap [FRONT] = Heap [size - 1];  
    minHeapify (FRONT);  
    return popped;  
}
```

بدل اول
مع element
آخر element

Heap sort

1. Build a max heap from the input data.
2. At this point, the largest item is sorted at the root of the heap. Replace it with the last items of the heap followed by reducing the size of heap by 1. اسی 1-2
3. Repeat above steps while size of heap is greater than 1.

Time Complexity :

Heapify $\Rightarrow O(\log N)$

Build Heap $\Rightarrow O(N)$

Heap sort $\Rightarrow O(N \log N)$.

Example

A = [6, 5, 3, 1, 8, 7, 2, 4]

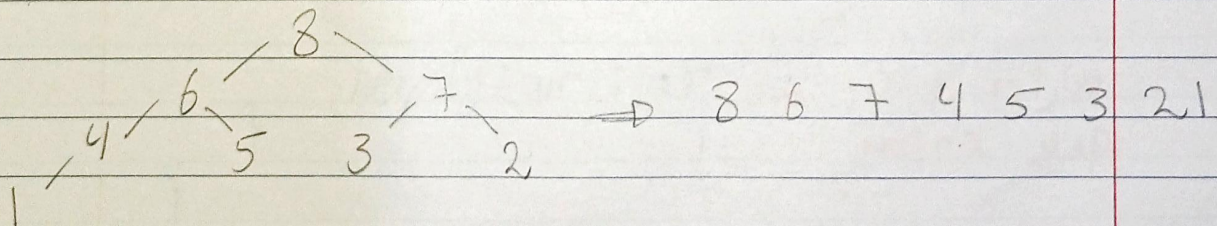
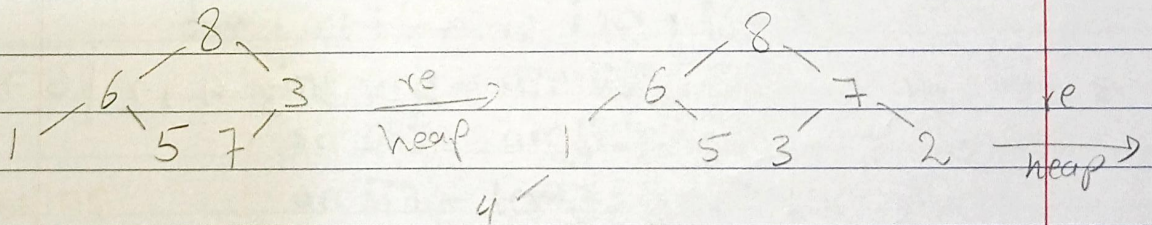
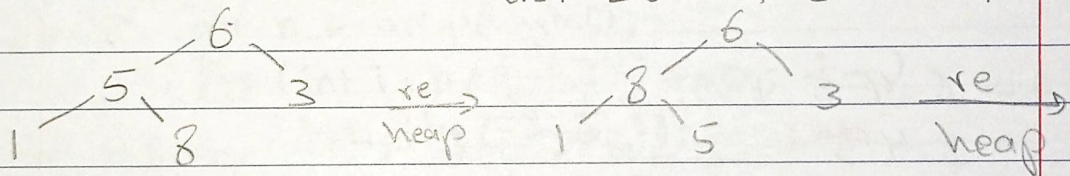
full Array.

size = 8

1. (Heapify) اول سے آخر تک

size 1, Array

دو بقیہ کے ادا کرنے کے لیے

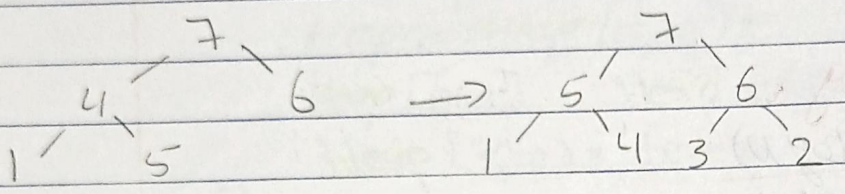
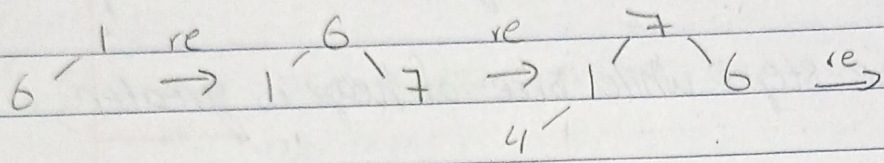


Front
 $A = [8, 6, 7, 4, 5, 3, 2, 1]$

8, 1 remove (مثلاً)

$A = [6, 7, 4, 5, 3, 2, 8]$
 size = 7

Build Heap
 وبعدين نزل

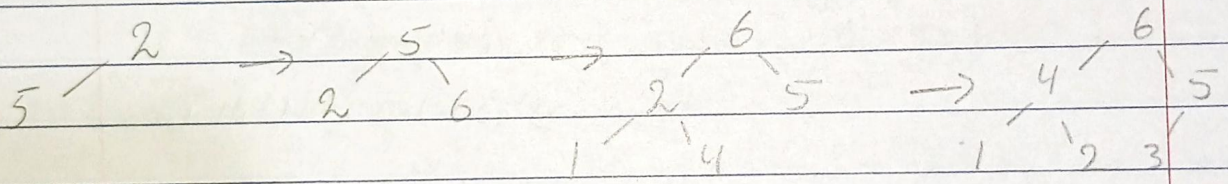


$A = [7, 5, 6, 1, 4, 3, 2, 8, 1]$

remove 7

$[2, 5, 6, 1, 4, 3, 7, 8, 1]$

size = 6

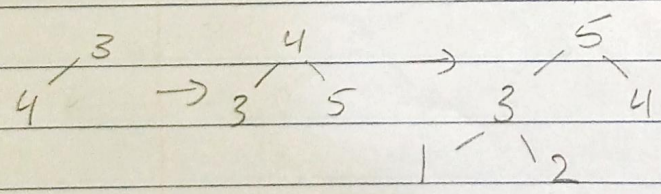


$A = 6, 4, 5, 1, 2, 3, 7, 8, 1$

$3, 4, 5, 1, 2, 6, 7, 8, 1$

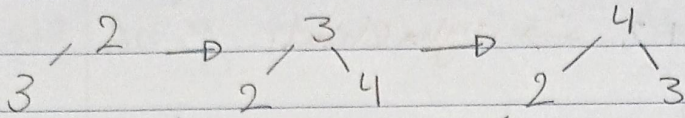
size = 5

remove 6



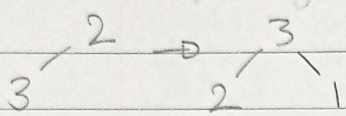
$A = 5, 3, 4, 1, 2, 6, 7, 8, 1$

A = 5 3 4 1 2 | 6 7 8 |
 2 3 4 | 5 6 7 8 |



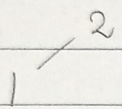
A = 4 2 3 | 5 6 7 8 |
 2 3 1 | 4 5 6 7 8 |

remove 4



A = 3 2 1 | 4 5 6 7 8 |
 2 1 | 3 4 5 6 7 8 |

remove 3



A = 2 1 | 3 4 5 6 7 8 |

remove 2

A = 1 | 2 3 4 5 6 7 8 |

remove 1

A = 1 2 3 4 5 6 7 8

is sorted. ∞

public void sort (int arr []) {

int n = arr.length;

for (int i = n/2 - 1; i >= 0; i--)

heapify (arr, n, i);

→ Data

for (int i = n - 1; i > 0; i--) {

int tmp = arr [0];

arr [0] = arr [i];

arr [i] = tmp;

swap

heapify (arr, i, 0);

→ index پر ڈیٹا

```
public void heapify(int arr[], int n, int i) {
```

```
    int largest = i → root
```

```
    int l = 2i + 1 → left child
```

```
    int r = 2i + 2 → right child.
```

```
    if (l < n && arr[l] > arr[largest])  
        largest = l;
```

الاي ج اكبر ←
أكبر من الroot

```
    if (r < n && arr[r] > arr[largest])  
        largest = r;
```

الاي ج اكبر ←
أكبر

```
    if (largest != i) {
```

```
        int swap = arr[i];
```

```
        arr[i] = arr[largest];
```

```
        arr[largest] = swap;
```

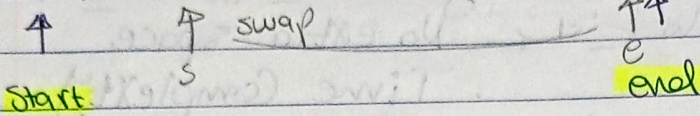
Swap
تبادل

```
        heapify(arr, n, largest); → Recursive
```

```
    }
```


divide & conquer

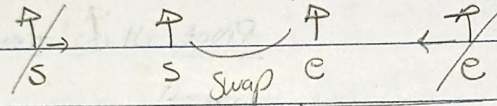
7	6	10	5	9	2	1	15	7
---	---	----	---	---	---	---	----	---



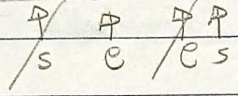
Pivot = 7

- * Move **start** to right ⇒ till find a number greater than or equal to the Pivot
- * Move **end** to left ⇒ till find a number less than the Pivot.

7	6	7	5	9	2	1	15	10
---	---	---	---	---	---	---	----	----



7	6	7	5	1	2	9	15	10
---	---	---	---	---	---	---	----	----



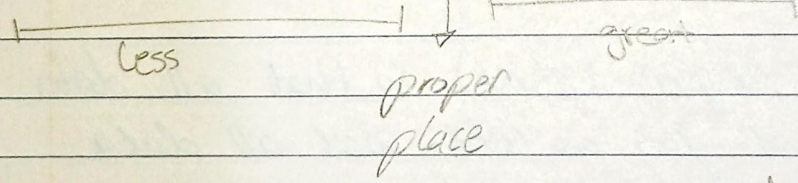
No swap because $s > e$ (cross)

Swap between element end with pivot.

Pivot ←

2	6	7	5	1	7	9	15	10
---	---	---	---	---	---	---	----	----

 Pivot = 2



...
...
...

Algorithm

Partion (a, l, h) {

int start, end, pivot;

pivot = a[l];

while (start < end) {

ما نلاقي رقم أكبر من ال
وقف Pivot while (a[start] <= pivot) {
start ++; ?

while (a[end] > pivot) {

إذا لقيت رقم أصغر وقف
end --; ?

if (start < end) {

Swap (a[start], a[end]);
}

Swap (a[l], a[end]); * اذهب الـ end
بدل بين الـ
مع الـ index
تلق الـ الـ

}

return end;

}

Qsort (a, l, h) {

int loc = Partion (a, l, h);

Qsort (a, l, loc - 1); (اليسار)

Qsort (a, loc + 1, h); (اليمين)

}

* Best Case $\Rightarrow O(N \log N)$

* Worst Case $\Rightarrow O(N^2)$

اولاً Pivot = a[L];

← Pivot اليمين

ثانياً Pivot = a[h];

Pivot = randome (a);

فكرتي

Pivot = Median3 (a, L, h);

```
int Median3 (int a[], int L, int h) {
```

```
    int center = (L+h)/2;
```

```
    if (a[L] > a[center])
```

```
        swap (a, L, center);
```

```
    if (a[L] > a[h])
```

```
        swap (a, L, h);
```

```
    if (a[center] > a[h])
```

```
        swap (a, center, h);
```

```
    swap (a, center, h-1);
```

```
    return a[h-1];
```

```
}
```

رجع ال Pivot

بدل الرقم الى اليمين
مع ال قبل ال اخر

Insertion Sort

تستعمل سجل لعبة السندرة.

Exercise

A = 5 | 4 | 10 | 1 | 6 | 2

Sort unsort

temp [4]

index 1

يقارن 4 بـ 5

و يقارن التي جوار ال temp مع sorted list.

هل التي جوار list sort اكبر من التي جوار temp

إذا أكبر اجعل shift to the right.

و يقارن باقي العناصر التي جوار ال temp

إذا بقيت فهو أصغر من ال temp.

A = ~~4~~ | 5 | ~~10~~ | 1 | 6 | 2

Sort unsort

temp [10]

و يرجع يقارن 10 مع 5 و 4
من اكبر منها 10 يتجه لليسار

A = 4 | 5 | 10 | 1 | 6 | 2

sort unsort

4 5 | 10 1 6 2

4 | 5 10 1 6 2

5 | 4 10 1 6 2

1 | 4 5 10 | 6 2

sort unsort

1 4 5 | 10 1 6 2

1 4 5 6 | 10 2

sort unsort

temp [1]

shift ← 1 5 10 لا

shift ← 1 5 لا

shift ← 1 4 لا

index 0

جوار ال 1

temp [6]

shift ← 6 5 10 لا

shift ← 6 5 لا

A = 1 4 5 6 10 / 2

sort unsort

1	4	5	6	?	10
1	4	5	?	6	10
1	4	?	5	6	10
1	?	4	5	6	10
1	2	4	5	6	10

temp [2]

shift ← 2 < 10	✓
shift ← 2 < 6	✓
shift ← 2 < 5	✓
shift ← 2 < 4	✓
lets 2 ← 2 < 1	✓

A = 1 2 4 5 6 10

sorted ✓

Algorithm

n ← 1 i ← n while i > 1
i ← n i ← i - 1 decrement

```
for (i = 1; i < n; i++) {  
    temp = A[i];  
    j = i - 1;  
    while (j >= 0 && A[j] > temp) {  
        shifting    A[j+1] = A[j];  
        j--;  
    }  
    A[j+1] = temp;  
}
```

Time Complexity ⇒ worst $O(N^2)$
Best $O(N)$

Shell Sort

سابقة في Shell Sort

تقليل الحركات التي تحدث فيها shifting *
 * شifting فيها

A = 23 29 15 19 31 7 9 5 2
 i no swap j

$$\text{gap} = \lfloor n/2 \rfloor$$

23 29 15 19 31 7 9 5 2
 i swap j

$$\text{gap} = 9/2 = 4$$

23 7 15 19 31 9 5 2
 i swap j

تقارن إذا انما الفرق هو 1 swap
 بعد

23 7 9 19 31 29 15 5 2
 i swap j

ويعبرين j++ و i++
 Backward / forward

23 7 9 5 31 29 15 19 2
 i swap j

Backward لا يتحرك في هذا الاتجاه

$$i - \text{gap} >= 0$$

A = 23 7 9 5 2 29 15 19 31
 i j

$$i - \text{gap} >= 0$$

A = 2 7 9 5 23 29 15 19 31 4 -4 = 0

phase 0 \Rightarrow جايزة

Backward

في ال phase الثانية

index 0

في ال gap القديم تقسيم 2

في تقارن ال i

$$\text{New gap} = \text{Old gap} / 2$$

Backward

$$\text{New gap} = 4/2 = 2$$

A = 2 7 9 5 23 29 15 19 31
 i j No swap

2 7 9 5 23 29 15 19 31
 i j swap and No need to look Backward

2 5 9 7 23 29 15 19 31
 i j No swap

$$2 - 2 = 0$$

need to look Backward

~~2 5 9 7~~

2 5 9 7 23 29 15 19 31

look \leftarrow \overline{i} \overline{j} No need swap

2 5 9 7 23 29 15 19 31

look \leftarrow \overline{i} \overline{j} swap \overline{i} and \overline{j}

2 5 9 7 15 29 23 19 31

look \leftarrow \overline{i} \overline{j} swap \overline{i} and \overline{j}

2 5 9 7 15 19 23 29 31

look \leftarrow \overline{i} \overline{j} No swap

A = 2 5 9 7 15 19 23 29 31

phase 2 ✓

new gap = 2/2 = 1

A = 2 5 9 7 15 19 23 29 31

\overline{i} \overline{j} No swap No look Back word

2 5 9 7 15 19 23 29 31

look \leftarrow \overline{i} \overline{j} No swap and need to look Back word

2 5 9 7 15 19 23 29 31

look \leftarrow \overline{i} \overline{j} swap

2 5 7 9 15 19 23 29 31

\leftarrow \overline{i} \overline{j} No swap

2 5 7 9 15 19 23 29 31

\leftarrow \overline{i} \overline{j} No swap

2 5 7 9 15 19 23 29 31

\leftarrow \overline{i} \overline{j} No swap

2 5 7 9 15 19 23 29 31

\leftarrow \overline{i} \overline{j} No swap

2 5 7 9 15 19 23 29 31

\leftarrow \overline{i} \overline{j} No swap

A = 2 5 7 9 15 19 23 29 31

Sorted ☺

Code

```
void shellSort (A []) {  
    int i, j, increment;  
    int temp;  
    int N = A.length;  
    for (gap = N/2; gap > 0; gap /= 2)  
        for (j = gap; j < N; j++) {  
            temp;  
            for (i = j - gap; i >= 0; i -= gap)  
                if (A[i] < A[i + gap]) {  
                    temp = A[i + gap];  
                    A[i + gap] = A[i];  
                    A[i] = temp;  
                }  
            else {  
                break;  
            }  
        }  
    }  
}
```

← هاتوا اذا كانت اوله من اوله
السر يا

Swap و بنفس الوقت يتسوف اذا
كانه يتطلع Backward

Time Complexity $\Rightarrow O(N^2)$.

Question

Suppose we have 5 GB of data using only 1 GB of Ram, what is the Best sorting algorithm could you use?

* عندي Data حجمها كبير و الرام عندي قليلة

و عندي الذاكرة التي البرنام لا يوافق لي؟

من ههنا نطلع عندي فكرة

External Sort.

نقسم ونجزل

اي نوع لسورة لي بي ياه

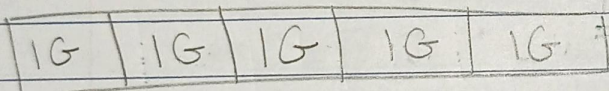
ليس اذا كان عندي اقل من 10 نجزل

وانا كانت الذاكرة كبيره استخدم

Example

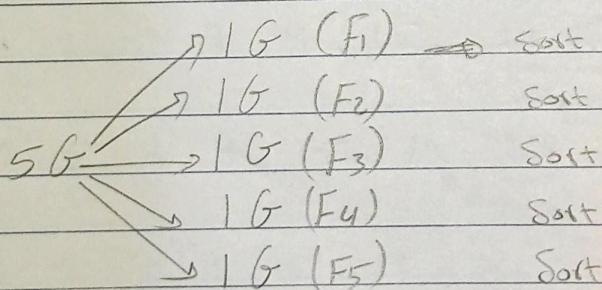
5 GB of data (file) sorted in H.D.

نجل file الى Divid



load in to Memory

Sort use quick sort

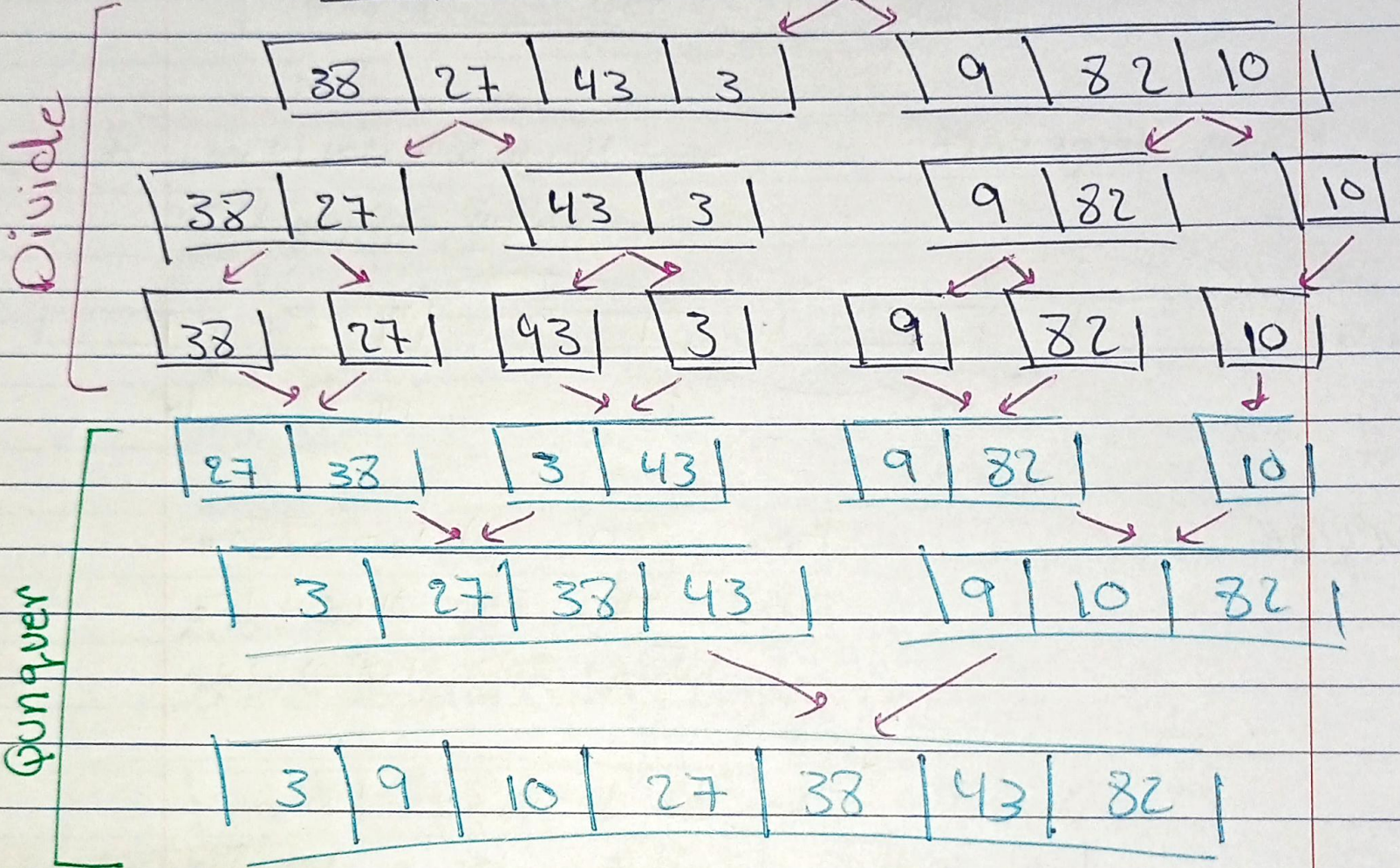


للمتاه external
لاننا انا استعملت بمساعدا
خارجية التي هي H.D
و ههنا اعمل loaded
على ال Memory .

Merge Sort

فلترية انه تقسم الذاكرة وبتحطها مرتبة
Array عند ما ال

A = | 38 | 27 | 43 | 3 | 9 | 82 | 10 |



sorted.

Algorithm

```
void merge sort (int low, int high) {  
    if (low < high) {  
        int mid = (low + high) / 2;
```

Recursion

```
    {  
        merge sort (low, mid);  
        merge sort (mid + 1, high);  
        merge sort (low, mid, high);  
    }  
}
```

Time Complexity $\Rightarrow O(n \log n)$.

$$T(n) = \begin{cases} c & , n = 1 \\ 2T(n/2) + cn & , n > 1 \end{cases}$$

$$T(n) = 2T(n/2) + cn$$

$$T(n/2) = 2T(n/4) + cn/2$$

$$\begin{aligned} T(n) &= 2(2T(n/4) + cn/2) + cn \\ &= 2^2 T(n/4) + 2cn \end{aligned}$$

$$= 2^k T(n/2^k) + kcn$$

$$T(n) = k T(1) + cn \log n$$

$$T(n) = n \log n$$

let

$$2^k = n$$

$$\log_x x^k = \log_x n$$

$$k = \log n$$