

Recursion & stack

There are 2 conditions for recursion:

① Simple case

② Recursive case

Function stack memory
Function stack memory
Function stack memory

ex Sum of Natural Numbers:

Factorial

```
int sum(int n) {
```

```
int Fact(int n) {
```

```
if (n-1 == 0)
```

```
if (n==1 || n==0)
```

```
    return 1; or return n;
```

```
    return n;
```

```
else
```

```
else if (n==0)
```

```
    return n + sum(n-1);
```

```
    return 1;
```

```
}
```

```
else
```

```
    return n * Fact(n-1);
```

* The power of number

```
int power(int num, int power) {
```

```
if (p == 0)
```

```
    return 1;
```

```
else if (p == 1)
```

```
    return n;
```

```
else
```

```
    return n * power(n, (p-1));
```

```
}
```


* أنا كمبرمج بقدر أهد مسألة بأكثر من طريقة Algorithm ولكن لازم أختار أحسن وحدة من حيث تكون أسرع وقت وأقل مساحة

Def:- Algorithm: step by step procedure to solve a certain task.

Analysis

* The problem can be solved by more than algorithm. So the programmer should choose the best one.

less time less space

↓ ↓
Resources (CPU) Memory (RAM)

* كل برنامج يفكره أو يستعمله في الكمبيوتر على RAM

Analyze

* How to Analyze the problem:-

1) Step by step Analysis / line by line Analysis

```

void Function (int n) {
    int i, j;
    for (i=0; i<n; i++) {
        for (j=0; j<n; j++) {
            printf ("i.d j.d", i, j);
        }
    }
}

```

في يوجد (يعني بقدر يتحلل)
 البرنامج لأنه الشرح في
 C "Constant"
 n+1 ← مرة بيقدر
 n(n+1) ← كل
 n*n ←
 $T(n) = C + (n+1) + (n^2+n) + n^2$
 ← بيقدر

3 GHz means $\rightarrow 3 \times 10^9$ instruction in one second
بجاء هيرتز

Time
 $T(n) = O(n^2)$

بناج أعلى أنه وكان هو ال max للوقت التي على بإضرب البرنامج

*Q Assume the processor speed is 1 GHz,
 $n = 2000000 = 2 \times 10^6$, find the time it will take.

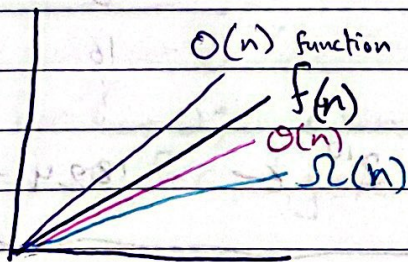
Sol Time = $\frac{n}{\text{speed}} = \frac{2 \times 10^6}{1 \times 10^6} = 2$ seconds

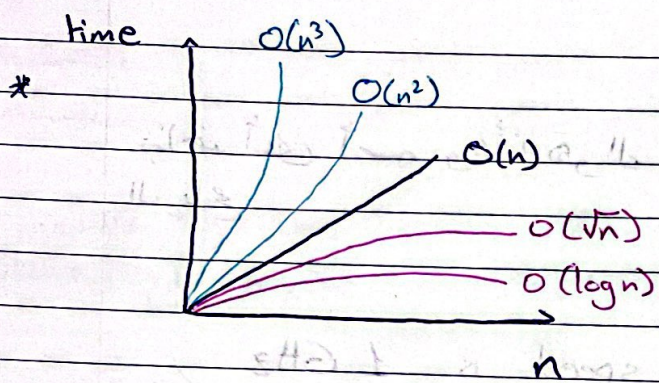
المرجع = المدة الزمنية

كل كل سرعة لو كان يوجد وبتبين بجزء من كل

* Time Functions:-

- ① Big O $O(n)$: upper bound. "maximum of time that the process will take"
- ② Omega $\Omega(n)$: lower bound "minimum ..."
- ③ Theta $\Theta(n)$: upper and lower bound.





لا تفرق بين الوقت بسبب التكرار n
 بوقت $\log n$ عن n^3

* Time (Big o) is neglected when n is small

question:- for $(i=1 ; i < n ; i \neq 2)$ {

What is the max time it will take?

لأن أول اتي بوقت قيمة ل n واول اوجر علاقة

n=1024 لو كانت

- i
 - 1
 - 2
 - 4
 - 8
 - 16
 - ...
 - 1024
- } 10 times

$$O(\log n) \leftarrow \frac{10}{2} = \log_2 1024 \leftarrow 2^{10} \leftarrow 1024$$

* for $(i=n ; i > n/2 ; --i)$

Sol

suppose

$$\frac{n}{2} = 0.5n$$

$$\frac{i}{n}$$

$$n-1$$

$$n-2$$

$$n-3$$

$$n-4$$

$$n/2 - 1$$

$$O(n)$$

*q for (i=n; i>0; i/=2)

Sol

suppose

$$n = 1024$$

$$\frac{i}{1024}$$

$$512$$

$$256$$

$$128$$

$$64$$

$$32$$

$$16$$

$$8$$

$$4$$

$$2$$

$$1$$

11 times

$$O(\log_2(n))$$

*q for (i=n; i>0; i/=3)

Sol

$$\log_3(n)$$

ما بظنہ کہ اگر ہرگز حد تک n کثیر کثیر ہے، فہلوانا مشر ح قوت عن طیبون و واحد

Q# for (i=0; i<n; ++i) {

for (j=n; j>0; j/=10) {

$$n+1$$

$$\log_{10}(n)$$

sol

$$O(n \log_{10}(n))$$

* q for (i=0; i < n; ++i) {
 for (j=i; j < n; ++j) {
 ... }
 } } $O(n^2)$

Sol $\frac{i}{1} \frac{j}{0}$
 \vdots
 n } n

$$\begin{array}{r} 1 \quad 1 \\ \vdots \\ n-1 \quad 1 \\ \hline 2 \quad n-2 \\ 3 \quad n-3 \\ 4 \quad n-4 \\ \hline \quad 2 \\ \hline n-1 \quad 1 \end{array} \Rightarrow 1+2+3+4+\dots+n-3+n-2+n-1+n$$

$$= \sum_{i=1}^n i = \frac{n(n+1)}{2} = O(n^2)$$

By using constant c_1

* q for (i=0; i < n; ++i) {
 for (j=0; j < n; ++j) {
 for (k=n; k >= 0; k -= 4) {
 } } } $O(n^2 \log(n))$

Solution $O(n^2 \log(n))$

Ex: for (i=0 ; i*i < n ; ++i) {

→ will stop → $i^2 \geq n$

⇒ $O(\sqrt{n})$

Ex: int p=0
for (i=0 ; p < n ; ++i) {
 p += i ;
}

⇒ $p = 1 + 2 + 3 + 4 + 5 + \dots + n = \sum_{i=0}^n i = \frac{n(n+1)}{2} \approx n^2$

| i | p |
|---|----|
| 0 | 0 |
| 1 | 1 |
| 2 | 3 |
| 3 | 6 |
| 4 | 10 |
| ⋮ | ⋮ |
| n | |

Ex for (i=0 ; i < n ; ++i) {
 for (j=0 ; j < i ; ++j) {
 ⋮
 }
}

⇒ $1^2 + 2^2 + 3^2 + \dots + n^2 = \sum_{i=0}^n i^2 = \frac{n(n+1)(n+2)}{6}$
 $= \frac{1}{6} (n^2+n)(2n+1)$
 $= \frac{1}{6} (2n^3 + 3n^2 + n)$

| i | j |
|---|-------|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 9 |
| ⋮ | ⋮ |
| n | n^2 |

هذا هين لأننا ما نتعامل مباشرة مع n

باللوبي التللي = $O(n^3)$


```

Ex for(i=0; i<n; ++i) {
    for(j=0; j<n; j++) {
    }
}

```

| | |
|-----|-----|
| i | j |
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| ... | ... |
| n-1 | n-1 |

→ $O(n^2)$

* Recurrences :-

- Finding time used with recursive functions:-

```

* int Factorial(int n) {
    if (n==0)
        return 1;
    else
        return n * Factorial(n-1);
}

```

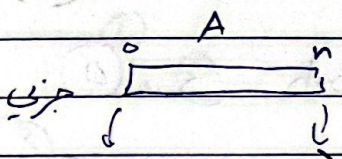
→ First, we find a Recursive Relation:-

$$T(n) = \begin{cases} d & ; n=0 \\ c + T(n-1) & ; n > 0 \end{cases}$$

$$\begin{aligned}
 \rightarrow \left. \begin{aligned}
 T(n) &= T(n-1) + c \\
 T(n-1) &= T(n-2) + c \\
 T(n-2) &= T(n-3) + c \\
 &\vdots \\
 T(1) &= T(0) + c \\
 T(0) &= d
 \end{aligned} \right\} \begin{aligned}
 \text{sum} \rightarrow T(n) &= c + c + c + \dots + d \\
 &= c \cdot n + d \\
 &= O(n)
 \end{aligned}
 \end{aligned}$$

فقطاً حل السؤال السابق راجع ليون أسرع "أقل وقت" من طريقة recursion
 لأنه كد مرة نستعمل ال Function

Ex merge sort



```

void mergeSort (int A[], int L, int H)
{
    if (L < H)
    {
        mid = (L+H)/2;
        mergeSort (A, L, mid);
        mergeSort (A, mid+1, H);
        Merge (A, L, H);
    }
}
    
```

بفضل تقسيم المصفوفة على اثنين كد مابطل ينفع ، وجيب بصير مرتبة فيهم

$$T(n) = \begin{cases} d & ; n=1 \\ 2T(\frac{n}{2}) + n & ; n > 1 \end{cases}$$

Merge Func. كد
 بصيرينه كل مرة

استعملت mergeSort
 لو استعملت 5 جزئيد
 وهذا
 كد مرة بقسم على اثنين
 عشان المعادلة بقسم
 على اثنين من طرف واحد
 المعادلة تقسم على $\frac{n}{2}$ وأما $\frac{n}{2}$

$$\begin{aligned}
 T(n) &= 2T(\frac{n}{2}) + n & \text{①} \\
 T(\frac{n}{2}) &= 2T(\frac{n}{2^2}) + \frac{n}{2} & \text{②} \\
 T(\frac{n}{2^2}) &= 2T(\frac{n}{2^3}) + \frac{n}{2^2} & \text{③}
 \end{aligned}$$

sub ② in ①

$$T(n) = 2 \left[2T\left(\frac{n}{2^2}\right) + \frac{n}{2} \right] + n$$

$$= 2^2 T\left(\frac{n}{2^2}\right) + n + n \quad \text{--- (4)}$$

Sub (3) in (4)

$$T(n) = 2^2 \left[2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2} \right] + n + n$$

$$= 2^3 T\left(\frac{n}{2^3}\right) + n + n + n$$

* after k^{th} step

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + kn$$

→ will stop $\frac{n}{2^k} = 1$

$$2^k = n$$

$$k = \log_2(n)$$

$$\rightarrow T(n) = n T(1) + \log(n) \cdot n$$

$$= \underbrace{d \cdot n}_{\text{قيمة صغرى مقارنته مع الـ (1) فقط}} + \underbrace{n \cdot \log(n)}_{\text{نسي}} = O(n \log(n))$$

$$O(n \log(n))$$

Ex: نفس قبل (5) ن

$$T(n) = \begin{cases} d & ; n=1 \\ 2T\left(\frac{n}{2}\right) + 10 & ; n > 1 \end{cases}$$

sol $T(n) = 2T\left(\frac{n}{2}\right) + 10 \dots \textcircled{1}$

$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + 10$ constant $\dots \textcircled{2}$

$T\left(\frac{n}{2^2}\right) = 2T\left(\frac{n}{2^3}\right) + 10 \dots \textcircled{3}$

$\textcircled{2} \rightarrow \textcircled{1}$

$T(n) = 2 \left[2T\left(\frac{n}{2^2}\right) + 10 \right] + 10$

$T(n) = 2^2 T\left(\frac{n}{2^2}\right) + 2 \times 10 + 10 \dots \textcircled{4}$

$\textcircled{3} \rightarrow \textcircled{4}$

$T(n) = 2^2 \left[2T\left(\frac{n}{2^3}\right) + 10 \right] + 2(10) + 10$

$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 2^3(10) + 2(10) + 10$

$+ 10(2^2 + 2 + 1)$

$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 10 \sum_{i=0}^3 2^i$

→ after K steps:

$T(n) = 2^K T\left(\frac{n}{2^K}\right) + 10 \sum_{i=0}^K 2^i$

$\frac{n}{2^K} = 1 \rightarrow n = 2^K$

$K = \log_c n$ geometric series

Note Geometric $\rightarrow \frac{r^k - 1}{r - 1} = \frac{2^k - 1}{2 - 1} = 2^k - 1$ T.S. = (n)T

$\rightarrow T(n) = n T(1) + 2^k - 1$
 $= d \cdot n + n - 1$

Final $\Rightarrow \underline{O(n)}$
 ans

Ex: $T(n) = \begin{cases} d & ; n=0 \\ 2T(n-1) + c & ; n > 0 \end{cases}$

Sol $T(n) = 2T(n-1) + c \dots (1)$
 $T(n-1) = 2T(n-2) + c \dots (2)$
 $T(n-2) = 2T(n-3) + c \dots (3)$

(2) in (1)

$T(n) = 2 [2T(n-2) + c] + c$
 $T(n) = 2^2 T(n-2) + 2c + c \dots (4)$

3 in 4

$T(n) = 2^2 [2T(n-3) + c] + 2c + c$
 $= 2^3 T(n-3) + 2^2 c + 2c + c$
 $= 2^3 T(n-3) + c \sum_{i=0}^2 2^i$

after k steps:

$T(n) = 2^k T(n-k) + c \sum_{i=0}^{k-1} 2^i$
 $\rightarrow n-k=0 \rightarrow n=k \quad i=0$

$= 2^n T(0) + c (2^k - 1)$
 $2^n d + c (2^n - 1) = O(2^n)$

24.09.2021

Linked lists

Thursday

* Array : The simplest data structure

→ fixed size, \rightarrow it leads for wasting space.

certain

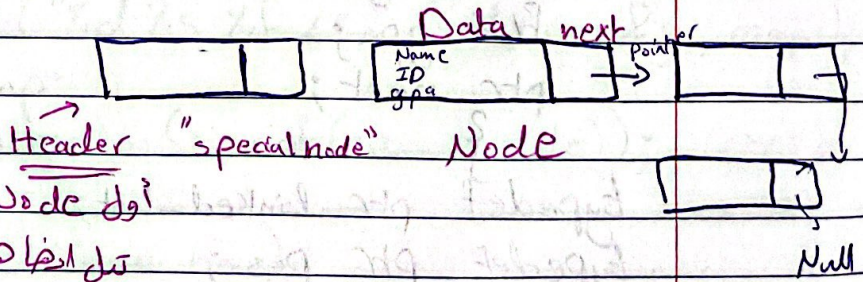
② insert new element in a position. \rightarrow takes more time

③ Delete an element. "we need to make shifts"

④ The need of sequential memory cells.

Defo- List : it is a collection of elements that are organized sequentially.

- The array follows the strategy of allocating the memory for all its elements in one block.
- The Linked List uses the strategy of allocating memory for each element separately and only when necessary.
- The linked list consists of a series of structures called nodes.



Node أول و يتكون من البيانات عن
البيانات

أخرها

L. L
* Types of Linked Lists:-

- Single L.L (Normal)
- Doubly L.L
- Circular L.L
- Doubly circular L.L.

* Operations on linked list: →

- 1) create
- 2) insert
- 3) delete
- 4) search
- 5) is Empty
- 6) get Size
- 7) sort
- 8) print List

* The creation of L.L :-

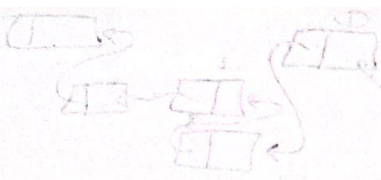
main

```
struct Node;  
typedef struct Node *ptr;
```

typedef → for creation of new datatype

```
struct Node {  
    char Name [25];  
    int ID;  
    char major [20];  
    float gpa;  
    ptr next;  
};
```

```
typedef ptr linked_list;  
typedef ptr pos;  
position
```

بترجع pointer

```

Linked_List L;
L = (Linked_List) malloc (sizeof (struct Node));
L->next = NULL;
  
```

هونا انتبات
الخير وحج
بالعموري
مساحة تميز

ptr next 2 byte
gpa 8 byte
major 20 byte
ID 4 byte
name 25 byte

دعا اول مرة بارجل NULL
الarrays اللى عندهم قبل

2) Insert in L.L :-

```

void insert (Linked_List L, pos p, ptr temp) {
    if (temp != NULL && p != NULL) {
        temp->next = p->next;
        p->next = temp;
        printf ("new node is inserted\n");
    }
    else {
        printf ("Error in insertion\n");
    }
}
  
```

هنا الخطة لانه لو انتر
الNULL اللى pointed
run-time error
Detection
قبل ما تبدأ

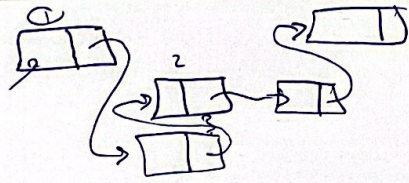
main

```

int id;
float g;
char nm [25], major [20];
while (ID >= 0) {
    printf ("Enter the student's Information");
    scanf ("%d %s %f %s", &id, nm, &g, major);
    ptr temp;
    temp = (ptr) malloc (sizeof (struct Node));
    temp->ID = id;
    strcpy (temp->Name, nm);
    temp->gpa = g;
}
  
```

while (ID >= 0)
الptr اللى
عندها

(ptr) اللى
line casting
واظروا انو الdata

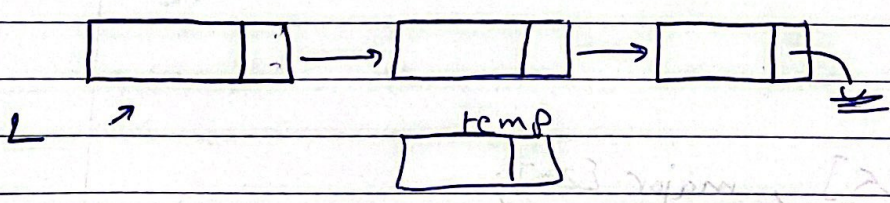


```
insert(L, L, temp);
```

هناك دهنس يبين ان node جديده و Node ال header "عاليه"
 لكن بوضو بنوع اظنه وبين بي انا ، بقدر اذ اسم ال (L) وال (temp) تانت
 ال Node الى انا صيره اظنه عليها "قبلا" او وراها "Insert(L, P2, new)"

```
Function insert_last (Linked List L, Pos P2, ptr temp) {
    // لو بي اظنه nodes ال آخر ال list
    pos p2 = L -> next;
    while (p2 -> next != NULL)
        p2 = p2 -> next;
    p2 -> next = temp;
    temp -> next = NULL;
}
```

! Null ≠ L , temp اذا if statement ← *



L: The name of the Linked List
 P: the position where the new node to be inserted
 temp: new node

Q * How to print the L.L?

```
void printList (Linked_List L) {
```

```
    ptr p;
```

```
    p = L → next;
```

```
    while (p != NULL) {
```

```
        printf ("ID: %d \t Name: %s \n",
```

```
            p → id, p → name);
```

```
        p = p → next;
```

```
    }
```

```
}
```

Function for search:-

```
Ptr search (Linked_List L, int id) {
```

```
    ptr p;
```

```
    p = L → next;
```

```
    int found = 0;
```

```
    while (p != NULL) {
```

```
        if (p → id == id) {
```

```
            return p;
```

```
        }
```

```
        p = p → next; }
```

```
    return NULL NULL;
```


* Delete:- I can delete nodes by:

- ith node
- specific property
- all nodes.

رقم "نادر" لاستبدالها
 های جستجوهای دیگر

Function void delete_ith (Linked-List, int i) {

for deleting by ith

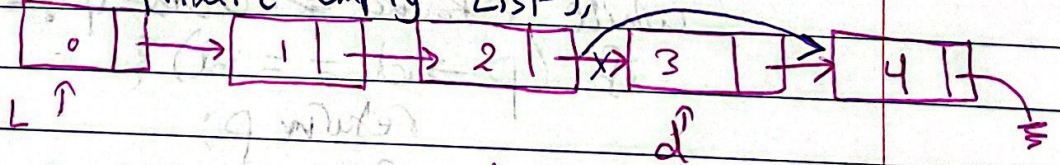
```
int a = 0;
ptr p = L; d;
```

```
if (!isEmpty(L)) {
    while (p->next != NULL && a < i-1) {
        p = p->next;
        a++;
    }
}
```

```
d = p->next;
p->next = d->next;
free(d);
```

این ptr p را به next تغییر
 دادم و به d تغییر دادم

```
else printf("Empty List-");
```



دوباره حذف اولی یعنی بجای 2 تا آخری 4 یعنی حذف 3 و خستمان ماندیم
 باقی معلوم است ال L.L

Ex. delete the node Name is Ahmad "Function"

```

void delete (Linked-List L, char Name[]) {
    pointers ptr p = L, d;
    if (!is Empty (L)) {
        while (strcmp (p->next->Name, Name) != 0)
            p = p->next;
        d = p->next;
        p->next = d->next;
        Free (d);
    }
    else
        printf ("Empty List");
}

```

لونها كان حرج غير انه احمد && && Ahmad

delete all Nodes; لازم واصل واحد كتحذفها

لانه لو حذفتي الاول بس الباقيين في ذاكرة memory حتما حرجك لال
 data wasted تمام

```

-> Void delete (Linked-List L) {

```

```

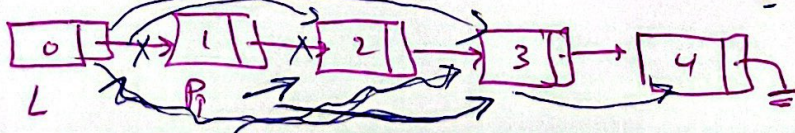
    ptr p = L->next;
    while (p != NULL) {
        L->next = p->next;
        free (p);
        p = L->next;
    }

```

```

    free (L);

```



الوقت الذي
 ياحظه

عشيرة
 ما احذف واحد
 اشارة بوجه احذف في

لا يغير اشارة على ولا اشارة احذف اول وواحدة

لو بي اعد ال Function ال recursion

```
→ void delete(Linked_List L) {  
    if (L->next == NULL) {  
        Free(L);  
    }  
    else {  
        ptr p = L->next;  
        L->next = p->next;  
        Free(p);  
        delete(L);  
    }  
}
```

لو بي اصيب Time "بفعل" $O(n)$ "بفعل"

$$\rightarrow T(n) = \begin{cases} d, & n=0 \\ T(n-1) + c, & n>0 \end{cases}$$

لأنه كل مرة كذا node

نولاً حتى لو التتبعين $O(n)$ لكن ال recursion ياخذ وقتاً أكثر
لأنه يستعمل Function كل مرة

Function getSize:

```
int getSize(Linked_List L) {  
    int counter = 0;  $O(n)$   
    ptr p = L;  
    while (p != NULL) {  
        ++counter;  
    }  
}
```


"جری" Search ہوتا ہے جو کہ size و index کے ساتھ ہے۔

Functions

① اولیٰ مرحلے میں عملیاتی creation کے لیے ptr previous ← ptr previous

```
struct Node {
```

```
    ptr next;
```

```
    ptr previous;
```

```
};
```

```
L → malloc;
```

```
L → next = NULL;
```

```
L → previous = NULL;
```

```
newNode) {
```

② void insert (Linked-List, ptr p)

```
p → next → previous = new Node;
```

```
newNode → previous = p;
```

```
newNode → next = p → next;
```

```
p → next ⇒ newNode;
```



New Node

③ void delete (Linked-List, char Name[])

```
if (!isEmpty(L)) {
```

```
    ptr p, d;
```

```
p = find-previous(L, Name);
```

```
d = p → next;
```

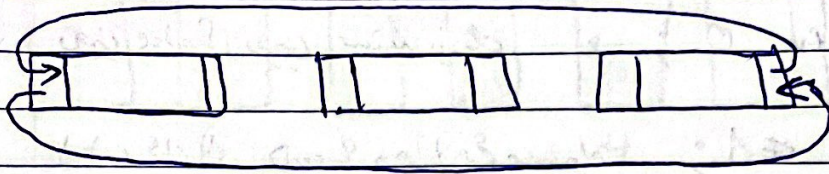
```
p → next ⇒ d → next;
```

```
p → next → previous = d → previous;
```

```
    free(d)
```

```
}
```


* Circular doubly list :-

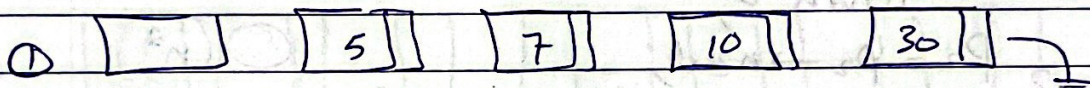


مرتبه ترتيب ايماءات و تازي

Q Assume you have 2 sorted Linked Lists, write a function that determine if these two linked lists are disjoint or not.

↳ not common elements.

فكرة الكل : انا جاد pointer تازي من ايماء رقم و يقارن



بينا تقارن هل ال -2 ايماء من حصة ولا ستاورد بها
 بروج والكيفه نفس السن ء هل يتاوي الحصة - لا وهي ايماء
 متاوي عن السجده - هل السجده ايماء من 10 - لا وهي ايماء
 فيقول بال L1 تاغها ء هل الحصة - 10 = 10 - 2 - نرج
 فبروح false

```

Function: int Disjoint (Linked-List L1, Linked-List L2) {
    ptr p1 = L1 -> next;
    ptr p2 = L2 -> next;
    while (p1 != NULL && p2 != NULL) {
        if (p1 -> data < p2 -> data)
            p1 = p1 -> next;
        else if (p2 -> data < p1 -> data)
    }
}
    
```


$P_2 = P_2 \rightarrow \text{next};$
else

} return 0; \rightarrow disjoint list False list

} return 1; \rightarrow Loop True list disjoint list

$O(n)$
↓
jante
me

nested loop $\leftarrow O(n^2) \leftarrow$ sorted list *

\rightarrow while ($P_1 \neq \text{NULL}$) {

$P_2 = L_2 \rightarrow \text{next}$

while ($P_2 \neq \text{NULL}$) {

if ($P_1 \rightarrow \text{data} == P_2 \rightarrow \text{data}$)

return 0;

} $P_2 = P_2 \rightarrow \text{next};$

$O(n^2)$

} $P_1 = P_1 \rightarrow \text{next};$

return 1;

* Radix Sort \rightarrow ترتیب دہائی

- used for integers and strings.

Examples Assume 876 253 426 913 81 51 7
64 27

steps = # digits in max num.

"أكبر من الأرقام هي من 3 منازل \leftarrow 3 خطوات"

Step # 1: حساب منزلة الأرقام

| | | | | | | | | | | |
|---|---|----|--|-----|----|--|-----|----|--|---|
| ① | | 51 | | 913 | | | 426 | 27 | | |
| | 0 | 81 | | 253 | 64 | | 876 | 7 | | 9 |

Step # 2: حساب المنزلة التالية من اليمين

| | | | | | | | | | | |
|---|---|---|-----|-----|--|-----|----|-----|---|--|
| ② | | | 27 | | | 253 | | | | |
| | 0 | 7 | 913 | 426 | | 51 | 64 | 876 | 8 | |

Step # 3: حساب المنزلة الثالثة من اليمين

| | | | | | | | | | | |
|---|---|----|-----|--|--|-----|--|--|-----|-----|
| ③ | | 81 | | | | | | | | |
| | 0 | 64 | 51 | | | | | | 876 | 913 |
| | 1 | 27 | | | | | | | | |
| | 2 | | 253 | | | 426 | | | | |
| | 3 | | | | | | | | | |
| | 4 | | | | | | | | | |
| | 5 | | | | | | | | | |
| | 6 | | | | | | | | | |
| | 7 | | | | | | | | | |
| | 8 | | | | | | | | | |
| | 9 | | | | | | | | | |

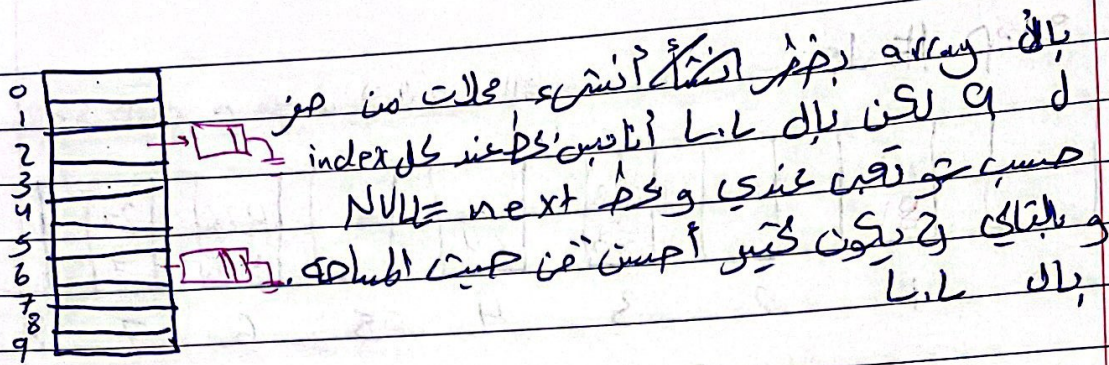
→ 7, 27, 51, 64, 81, 253, 426, 876, 913

$T(n) = n + n + n = O(n)$

$n \times n$ ~~array~~ 2-D array line Step 1 → -
 Step 2 → -
 step-1 array line Step 3 → -

⇒ 2 Arrays (at least) $n \times n$ ^{Assume $n = 10^4$} $= 2 \times 10^8 \times 4 \text{ bytes} = 8 \times 10^8 \text{ bytes}$

→ 2 Linked-List $2 \times n = 2 \times 10^4 \times 4 \text{ bytes} = 8 \times 10^4 \text{ bytes}$



سؤال: لو بي اعمل Function جيبلي خانة "منزلة" مكلوبه من رقم

```

int getDigit (int n, int i) {
    if (i == 0)
        return
    while (i > 0) {
        n = n / 10;
        --i;
    }
    return n % 10;
}

```

لو عندي اسماء بي اربطها حسب Radix ، بعد نفس الان
 هناك ، بيكون اقول اسم وطريقه Steps حسب ال ، وكل ما عندي
 اسم ملا اربط من هين للسؤال ، بحيث ملا عندي الاسم اصل الفاضي
 اما NULL او space Nulmai Sara Heba Khalid

شرح كود الورقة بال Cursor Imp.

X ملا بيكون في Pointers ← Dynamic Memory Allocation بروج

Array : fixed size

هلا بروج حاضر 100 ، لكن بيتر اعمل اكثر من لاي زي B و C
 ولح عنبر ال A

05.10.2021

Stack

Tuesday

def stack: list of elements with restriction on the insertion and deletion, whereas they are performed only on one position which called the top of stack

in stack : insert : push
delete : pop

زي صفا مويون "انا باخذ اللى من الاسفل او ديتيني من اعلى واحد"

* Applications on stack:

[1] Functions call: Functions A, B, C
مثلاً لو عندي Functions A, B, C
بسيطة B تستدعي C والـ A تستدعي B اللى هو أصلًا مستدعي C

[2] Redo/Undo

[3] expressions evaluations

[4] browsers → google → Facebook → link game

* Implementation for stack: ← 1- linked list / أيضا عالي / أي قبل "لكي" انظر من قدامه
2- array / مكاننا مفتوح

الـ stack كذا فيكون معشان هيك نستعملوا array أكثر من كذا

```
#define MAXSIZE 100;
struct stack {
    int Elements[MAXSIZE];
    int Top;
};
```



```
typedef struct stack STACK;
```

```
STACK s;
```

```
s.Top = -1;
```

هناك انشآت stack

← empty stack

Function

push

```
void push (STACK s, int e) {
```

element

```
if (s.Top < MAXSIZE-1) {
```

```
++s.Top;
```

```
s.Elements[s.Top] = e;
```

```
}
```

```
else {
```

```
printf("stack overflow \n");
```

```
}
```

```
}
```

Function

pop

int

```
pop (STACK s) {
```

```
if (s.Top > -1) {
```

```
int e;
```

```
e = s.Element[s.Top];
```

```
--s.Top;
```

```
return e;
```

```
}
```

```
else {
```

```
printf("stack is empty \n");
```

```
return -1;
```

```
}
```

```
}
```

لو كان فيها القيمة ناعنا e = -1، كين بيه جرفا اذا هي

عبر فيها -1، ويننا نحسب ولا هي واقينة؟ ← لا استعملنا Function

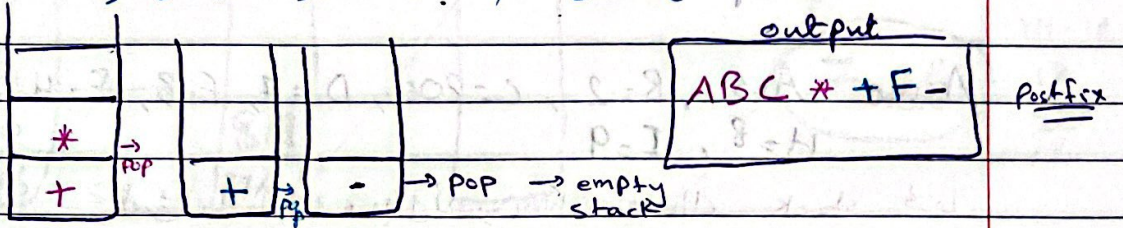
" كين لو -1 > s.Top ← استعملنا pop ← عندها ما تستعملنا "

العملية لها به يكون جواباً

① Conversion from Infix expression to Postfix

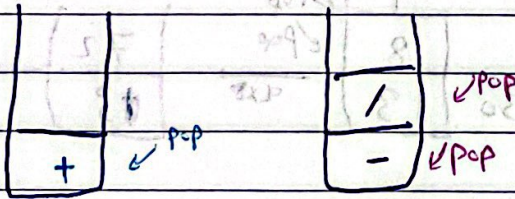
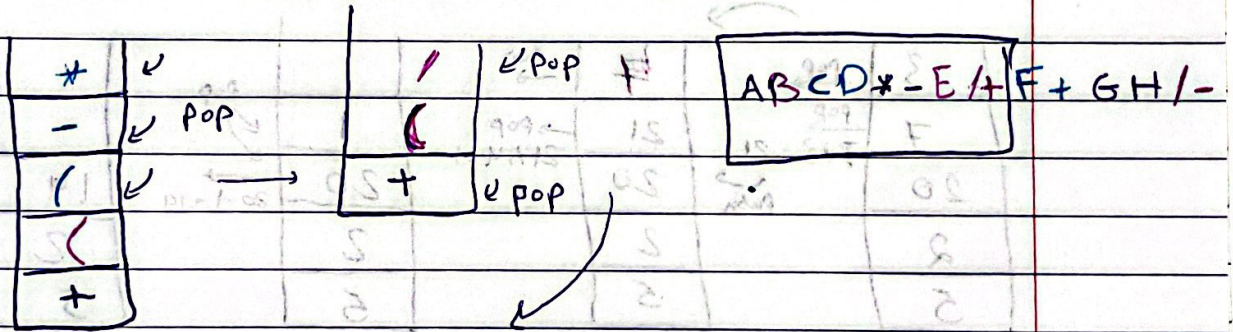
$A + B * C - F$ أولاً في جدول معين: خرج معين نطرح

الفكرة انه جدول معين العملية على stack ، وديفين فوقها الي اليها أولوية
وقت و لكن مثلاً الجواب والخرج لهم نفس الأولوية ونظراً في ستان اعتباراً



كلنا نعالج pop نزلنا على output

Ex $A + (B - C * D) / E + F - G / H$



Remarks: الأقواس يجب كلاً في ما قبلها ، وفيما كانه من موجود الي قبلها ،
فبعد pop عالي حولها ، وما أقبل نهاية القوس () ، سجل pop كما
أولاً () ، الأقواس لا تصنيفاً للناتج ، ولو يدي أضف عملية جمع
وكان موجود أصلاً عليه جمع ← تحسب الموجودة ، وديفين الجبرده
نزلنا على output

بقاولة * في اوله

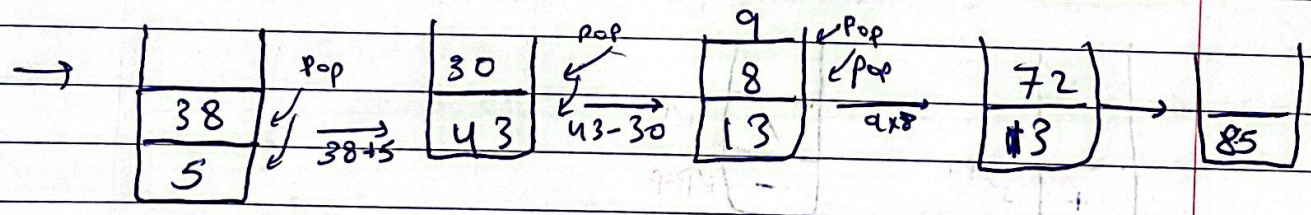
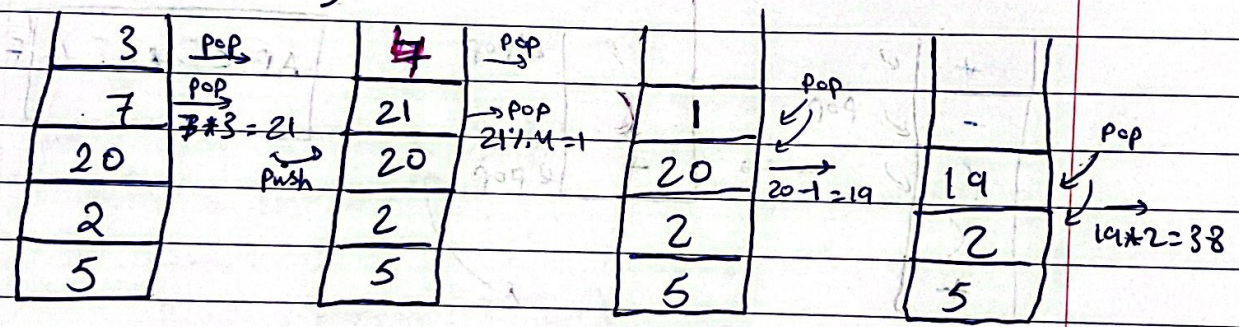
Ex Convert to postfix: $A + B * (C - D * E \% F) - G + H * I$

sol ABCDEF * % - # + G - HI * +

2 Postfix expression evaluation

Assume $A=5, B=2, C=20, D=7, E=3, F=4, G=30, H=8, I=9$

حسب ناتج السؤال السابق وبتغير اصبحت الـ variables على stack كذا
 الفأ على الـ stack السابقة وبتروح بعض اجز الـ elements وتبقى على الـ
 هاي العلية وبتبقى الـ push على الـ stack كان من الـ ناتج



عشان أعرف اذا كان صح ← لازم آفر خطوات يكون فيها جواب واحد
 ومثلا ووطا انبوا هاي اللجان لازم على الأقل أدخل قيمة متغيرين

Evaluate $ABCD * - E / + F + GH / -$
 $A=2, B=3, C=4, D=5, E=6, F=7, G=8, H=4$

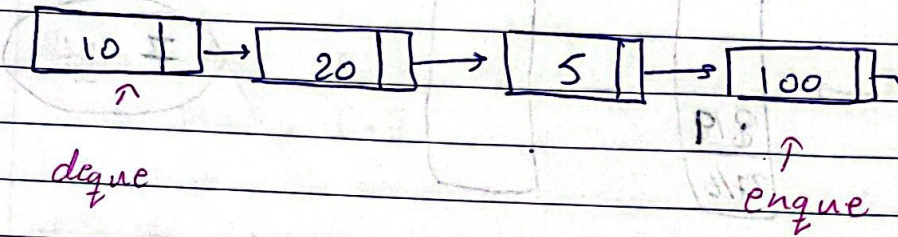
07.10.2021

Queue

Thursday

def

Queue: is a list, whereas the insertion is performed always at the end of queue, while deletion is performed at the beginning



→ Queue is FIFO "First in First out"

First In last out — stack use

- Applications :

- 1) Printers
- 2) calling centers
- 3) Registration system

* Implementation:-

1- Linked List

2- Array

← how?

```

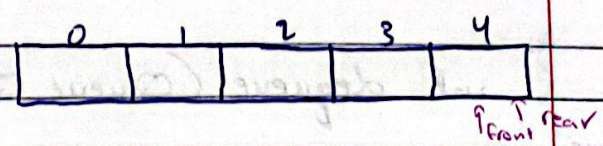
#define SIZE 5;
struct Queue {
    int Data [SIZE];
    int front;
    int rear;
} Q;
  
```

pointers

Q.front = size - 1;

Q.rear = size - 1;

ما يقبل عناصر على كيو طين ولا يقبل عناصر خارجة



Function
الأقنانه

```
void enqueue (Queue Q, int x) {
```

```
    int temp = Q.rear;
    if (Q.rear == Size-1)
        Q.rear = 0;
    else
        ++Q.rear;
```

هو بيتي (يقول) على 0 1 2 3 4
إذا كل موقع على 0
(عشان أحرك Pointer وأتمها موقع)

```
    if (Q.rear == Q.front) {
        printf("Queue is full\n");
        Q.rear = temp;
    }
```

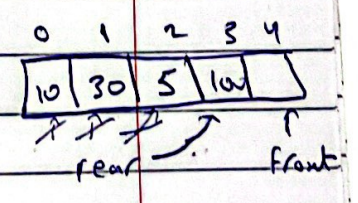
عشان بيتي (يقول) أنا
عشان وأتمها موقع

```
    else {
        Q.Data [Q.rear] = x;
    }
}
```

O(1)
constant

Assume 10, 30, 5, 100, 70, 60

| Q.front | Q.rear | x | temp |
|---------|--------|-----|------|
| 4 | 4 | 10 | 4 |
| | 0 | 30 | 0 |
| | 1 | 5 | 1 |
| | 2 | 100 | 2 |
| | 3 | 70 | 3 |
| | 4 | | |
| | 3 | | |



الفايز Header يمكن تكون أي مكان

```
Func  
dequeue  
int dequeue (Queue Q) {
```

```
if (!is Empty (Q)) {
```

```
if (Q.front == size - 1)
```

```
Q.front = 0;
```

```
else
```

```
++ Q.front;
```

```
return Q.Data [Q.front];
```

```
}
```

```
else {
```

```
printf ("Empty Queue\n");
```

```
return -1;
```

```
}
```

```
}
```

لازم "الأفضل" قبل ما استبي هاد ال Function إفتحه لئلا باله main
إذا is Empty معشان قيمة ال (-1) تابع ال Func و تحريته

```
int isEmpty (Queue Q) {
```

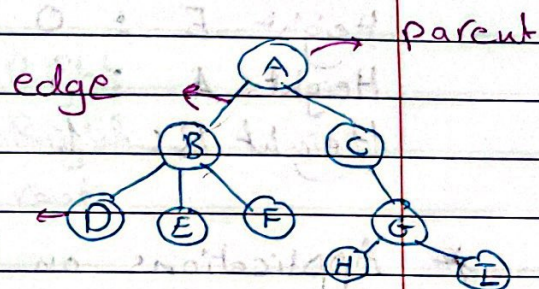
```
return Q.front == Q.rear;
```

قولاً إذا ما عندك لكن مخزن فوق القيمة المطلوب أو ال wide وهو قوس
بروح القيمة تابع ال is الي كونه 6 لأن لو في ال L غير وهو كذا قيمة
كلها من L

$O(1)$
constant

* Linear data structure:
 "كل شيء يسبقه واليه يرجع"
 "كل شيء قبله أحسن منه هو هيك"

Def Tree:- is a non-linear data structure
or is a finite set of nodes, together with
a finite set of edges, that define the
parent-child relationship.



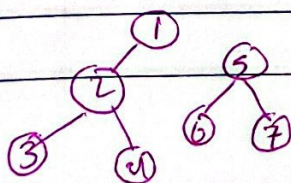
Def Path from node to node m: set of edges/
nodes from node 1 to node m, with length of m-1

Ex Path From A to H: A, C, G, H

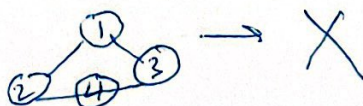
4 nodes

* Tree properties:-

- 1- It has a designated node, called root, that has no parent A
- 2- Every node, except root, has exactly one parent.
- 3- Every node has zero or more children.
- 4- There is a unique directed path from the root to each node.



Ⓛ → tree ✓
X Not a tree because there are 2 roots!



Def * Depth of node v :- the length of the path from the root to node v

ex Depth of H : 3

Depth of B : 1

Depth of A : 0

Def: * Height of node v :- The longest path from node v to the leaf node.

Height E : 0

Height A : 3

Height B : 1

* Applications on trees:-

1- Dictionary

2- Directory

3- Decision Tree

4- File directories



"Folders"

* Implementation of trees:-

```
struct Node {
```

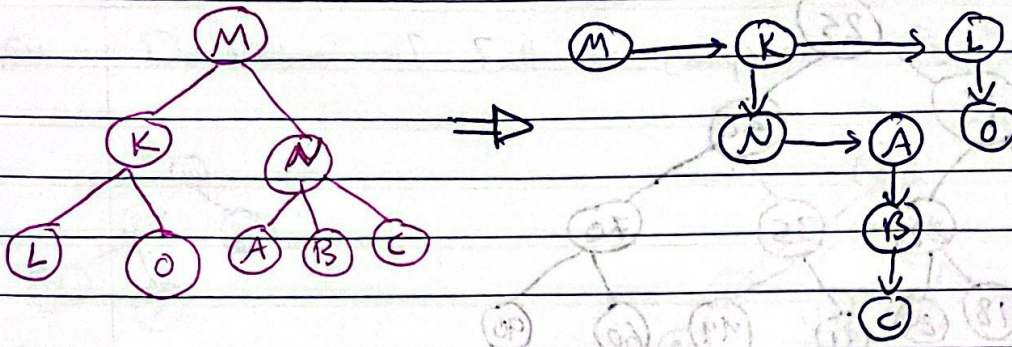
```
    ptr next;
```

```
    ptr siblings;
```

```
};
```

Note \rightarrow Siblings: nodes with same parent

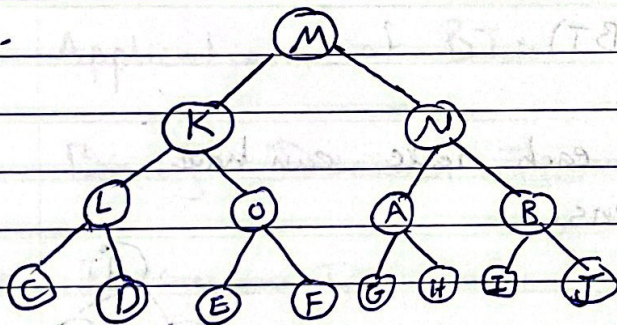
ptr next
 →
 ptr sibling ↓



Defn Tree Traversal :- visiting all nodes.

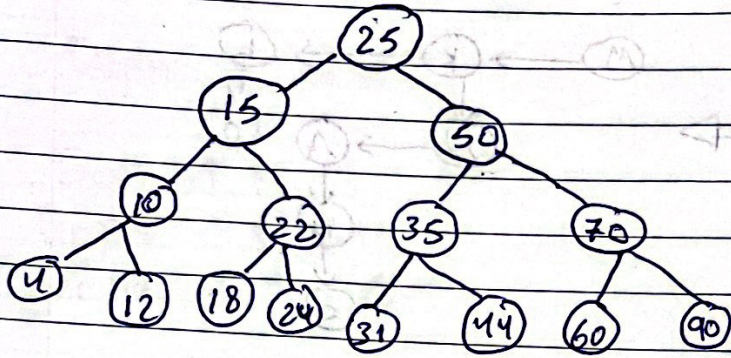
- 1- preorder: root - Left - Right
- 2- Inorder: Left - root - Right
- 3- postorder: Left - Right - root

Ex:-



1. pre-order: M, K, L, E, D, O, E, F, N, A, G, H, B, I, J
2. Inorder :- E, D, L, K, E, O, F, M, G, A, H, N, I, B, J
3. Post-order: E, D, L, E, F, O, K, G, H, A, I, J, B, N, M

Ex: Write the pre, In and post order traversal for the following tree:-

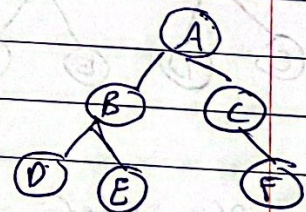


- 1- Pre-o: 25, 15, 10, 4, 12, 22, 18, 24, 50, 35, 31, 44, 70, 60, 90
- 2- In-o: 4, 10, 12, 15, 18, 22, 24, 25, 31, 35, 44, 50, 60, 70, 90
- 3- Post-o: 4, 12, 10, 18, 24, 22, 15, 31, 44, 35, 60, 90, 70, 50, 25

المدة 30 دقيقة
 من حيث 1 2 3 وبقية ارسلي

* Binary Tree:- (BT)

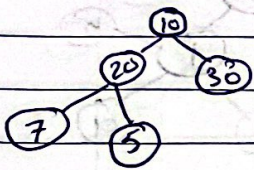
def is a tree in which each node can have maximum 2 childrens.



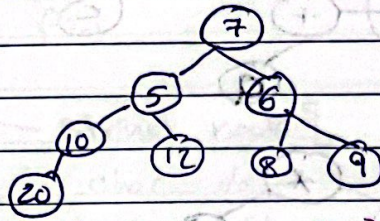
* Types of Binary Tree:

- [1] Full BT : every node has ^{zero} 0 or 2 children
- [2] Complete BT: every level, except the last, must be completely filled. The last level is filled as far left as possible.

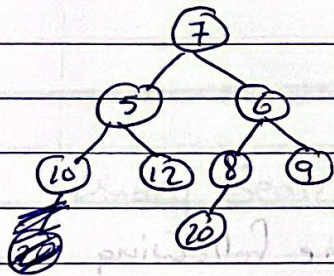
Ex Determine if Full or complete BT



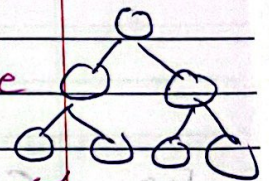
(Full BT) and (complete)



(Not FULL BT) complete BT



Not complete nor full
Just BT (filling must be from left to right)
Full + complete



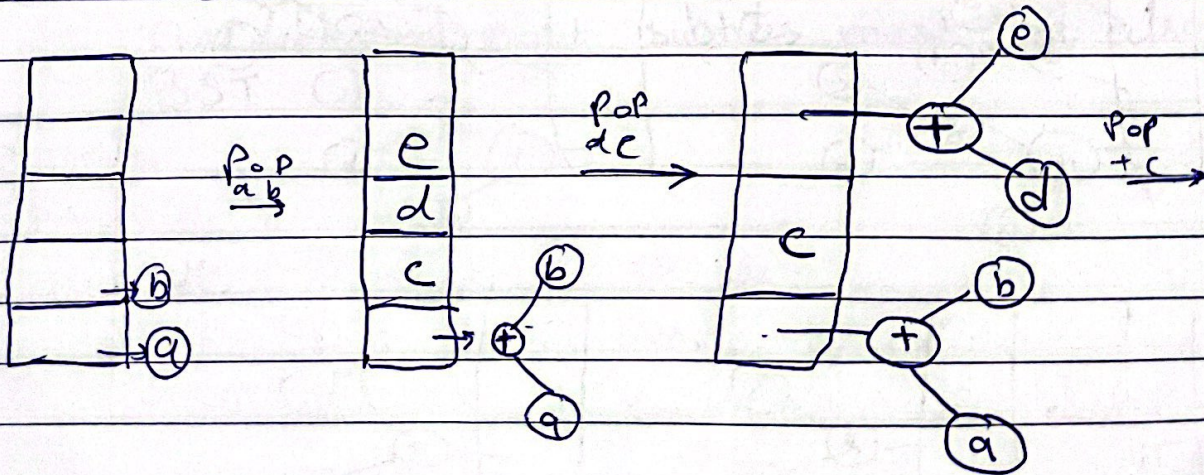
* Applications of BT:-

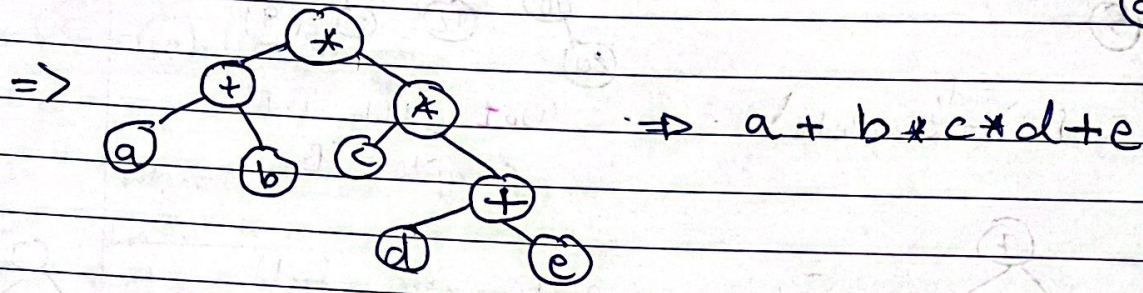
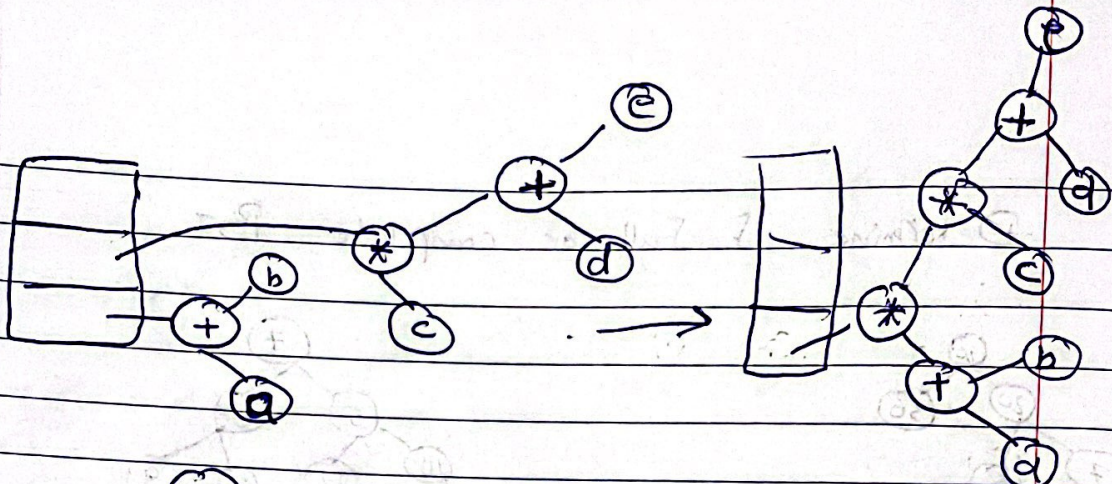
1- Binary Decision Tree

2- Expression Tree.

Ex: $ab + cde + **$

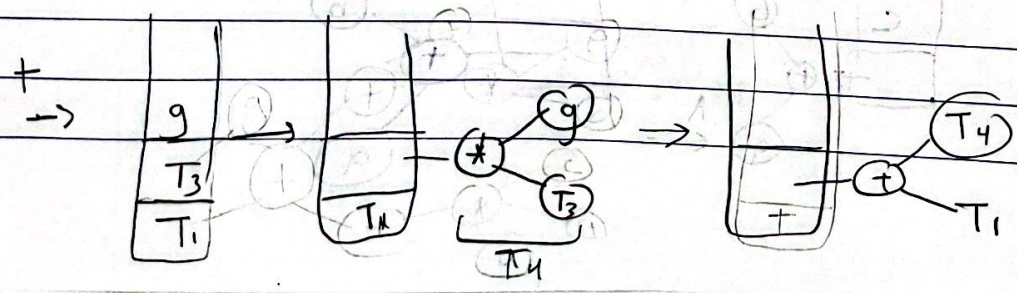
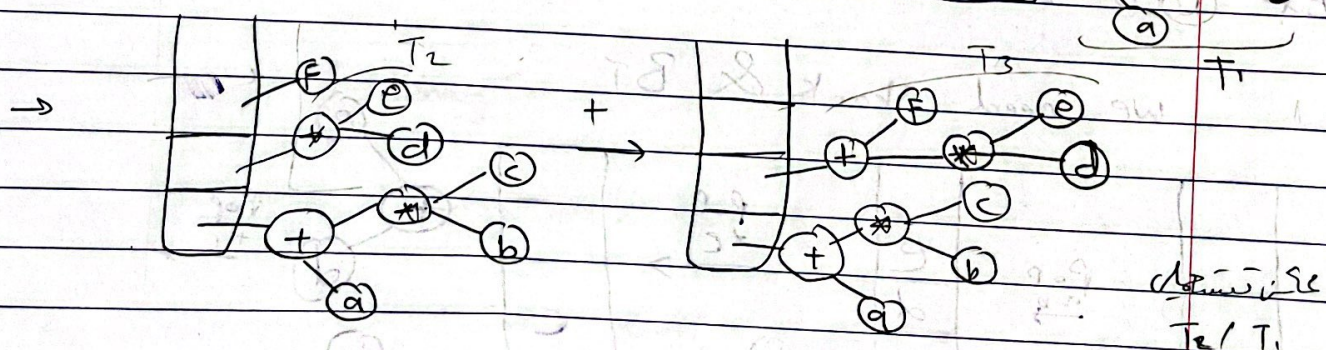
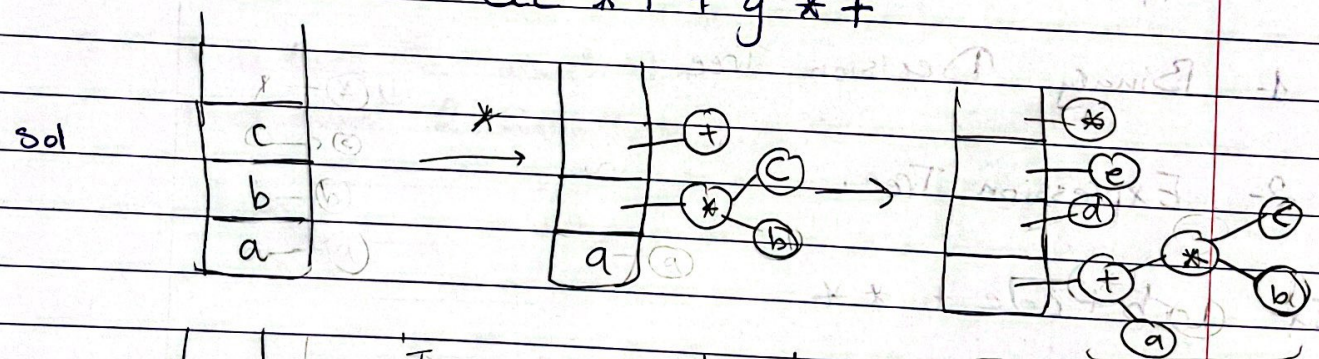
sol we need stack & BT





Ex: Construct the exp. tree for the following postfix exp.

$abc * + de * f + g * +$



node, int, int
node, int, int

Search

```
BST Search (BST T, int key) {
```

```
    BST current = T;
```

```
    int Found = 0;
```

```
    while (current != NULL && !Found) {
```

```
        if (current->ID == key)
```

```
            Found = 1;
```

```
        else if (current->ID > key) {
```

```
            current = current->left;
```

```
        else
```

```
            current = current->right;
```

```
        }
```

```
    } return current;
```

$O(\log(n))$

Q. Write search in Recursive form.

```
BST Search (BST T, int key) {
```

```
    if (T == NULL) {
```

```
        if (T->id == key)
```

```
            return 1;
```

```
        else if (T->id > key)
```

```
            return search (T->left, key);
```

```
        else
```

```
            return search (T->right, key);
```

```
    }
```

```
    return NULL;
```

```
}
```

$$T(n) = \begin{cases} d & n=0 \\ T(\frac{n}{2}) + c & n > 0 \end{cases}$$

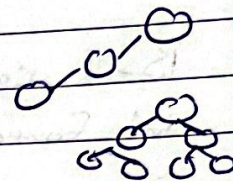
$O(\log(n))$ → if balanced BST
if the tree ideal

Time comp. for search functions:

Worst case $O(n)$

Best Case $O(1)$

Balanced $O(\log n)$



بينا في كسب دمايع كل النودز

void inorderTraversal (BST T) {

if (T != NULL) {

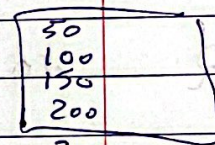
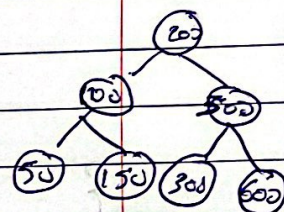
inorderTraversal (T->Left);

- printf("%d \n", T->Data);

- inorderTraversal (T->right);

}

}



→ PostOrder: بينه بين الشجرين = فقط
pre: انما يفر ان Left مع ال Right

* Insertion:-

- The idea is to do iterative traversal of the given tree,

- If we find a node whose left child is empty, we make a new key as left child of the node.

* Deletion

- Three cases:

- 1- Node to be deleted is leaf, simply Remove it.
- 2- Node to be deleted has one child, copy the child to the node and delete the child.
- 3- Node to be deleted has 2 children, either copy the minimum from right "leaf" or copy the maximum left from left

Function: find minimum

```
BST Find_Min (BST T) {  
    if (T == NULL)  
        return T;  
    else if (T->left == NULL)  
        return T;  
    else  
        return (Find_Min (T->left));  
}
```

Function BST delete (BST T, int x) {

```
delete  
    if (T == NULL)  
        return T;  
    else if (x < T->Data)  
        T->left = delete (T->left, x);  
    else if (x > T->Data)  
        T->right = delete (T->right, x);  
    else {  
        →
```


Case \neq NULL

```
if (T->left && T->right) {  
    BST temp;  
    temp = Find-Min (T->right);  
    T->Data = temp->Data;  
    T->right = delete (T->right, T->left); Copy  
    T->right = delete (T->right, T->Data);  
}  
else {  
    BST temp, c = T;  
    if (T->left == NULL)  
        temp = T->right;  
    if (T->right == NULL)  
        temp = T->left;  
    Free(c);  
    return temp;  
}  
}
```

حذف از درخت جستجوی دودویی
حذف از درخت جستجوی دودویی

میزان تعادل

* AVL Trees:-

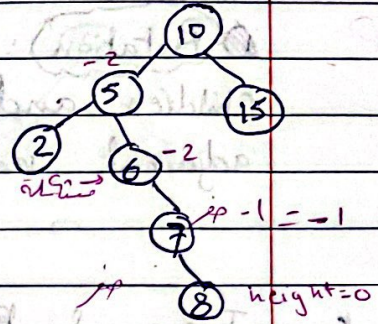
Def: AVL is a binary search tree with height balance property.

- For any node v , the height of the subtrees of v is differ by at most 1.
- Any subtree of an AVL tree is also an AVL.

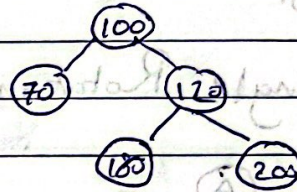
+3 خالص ارجو وعالين 3

Ex

Not AVL →



Not AVL bcoz it's not Binary tree → $180 > 120$



لكن لو عدلت الـ 180 فيستقر الـ AVL

* Why AVL tree?

- insertion or deletion in BST can cause large imbalance.

Problems of BST

- The time complexity of search, insertion or delete of any node could be $O(n)$ in worst case.

لكن الـ AVL كل ما فيها ان الـ balance يكون في الـ left و الـ right

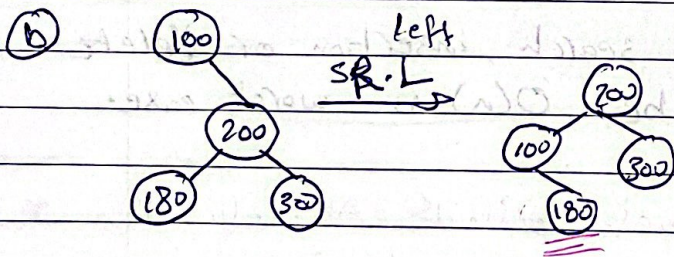
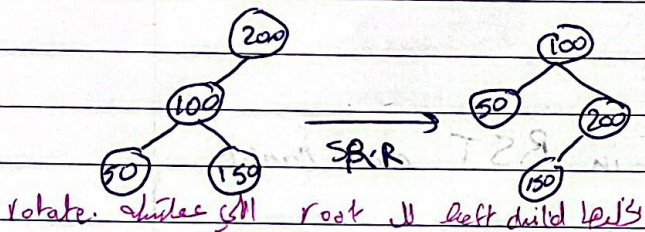
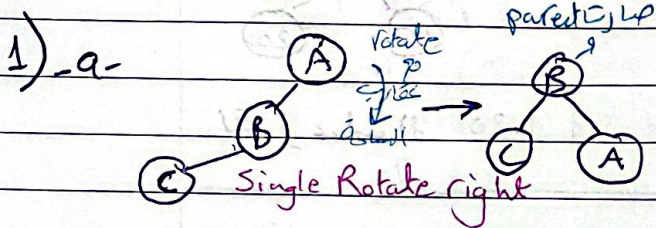
- In AVL the balance property will ensure that the height of the tree will remain $O(\log(n))$

* Balancing is done by Rotation operation.

def Rotation: is a process of switching children and parents among two or three adjacent nodes.

* Types of Rotation:-

1- Single Rotation 2- double Rotation.

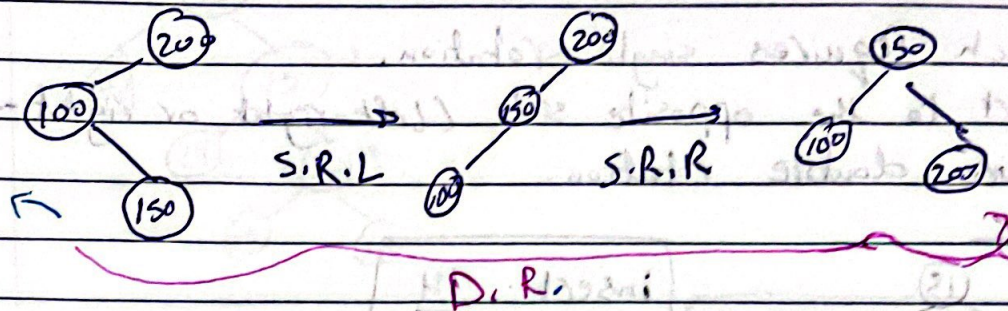


2- Double Rotation:-

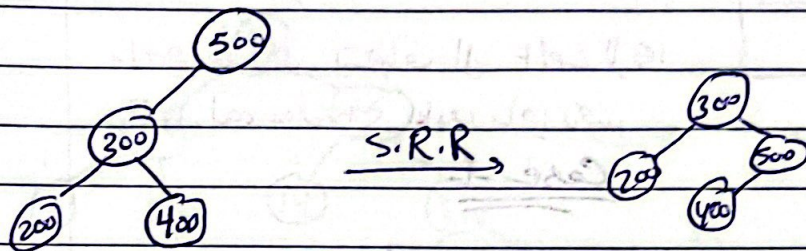
a- Double right - left

b- D. Left - Right

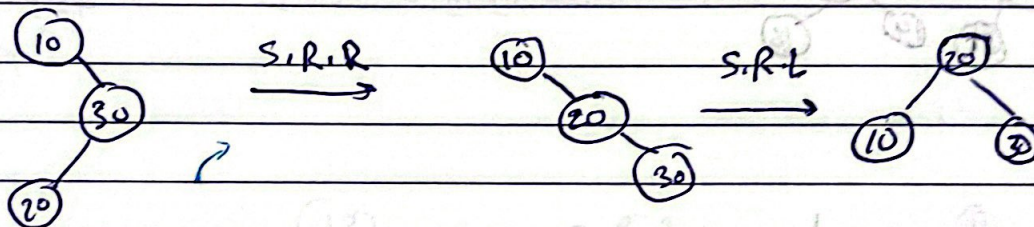
Ex



Ex



Ex



* AVL Operations on AVL:-

1. insertion
2. deletion
3. Search
4. Rotation
5. get Balance
6. Traversal

* Insertion: same algorithm used in BST, in addition to balancing the tree.

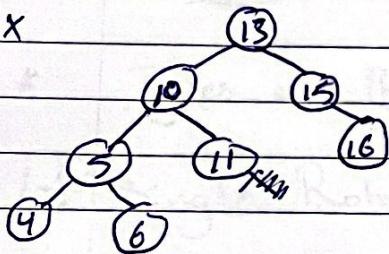
There are 3-cases in insertion

1. insert that does not cause imbalance.
2. insert to the same side (left-left or right-right)

which requires single rotation.

3- insert to the opposite side (left-right or right-left) require double rotation.

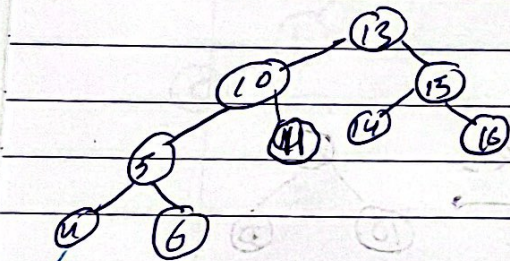
Ex



insert 14

ما قبله الی باقی الی باقی الی Left الی 15
و باقی الی باقی الی باقی الی balanced

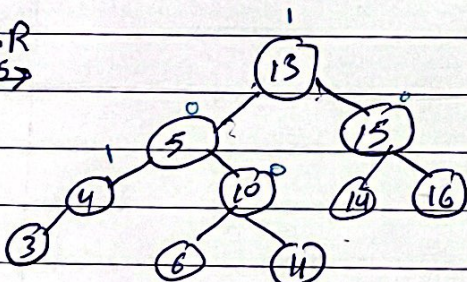
Case-1-



insert 3

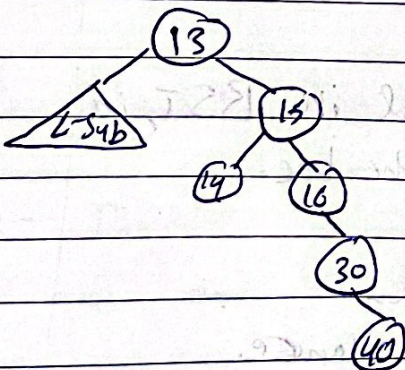
بیشتر الی Left الی 4 و باقی الی 15

S.R.R
105

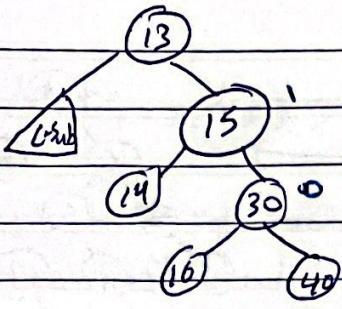


insert, 30 and 40

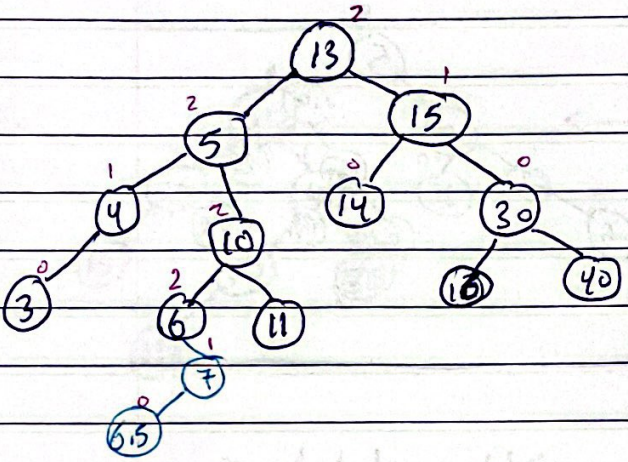
باقی الی باقی الی
Left



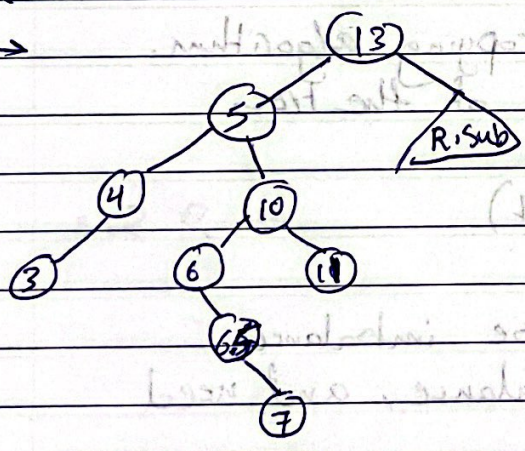
S.R.L
30-16



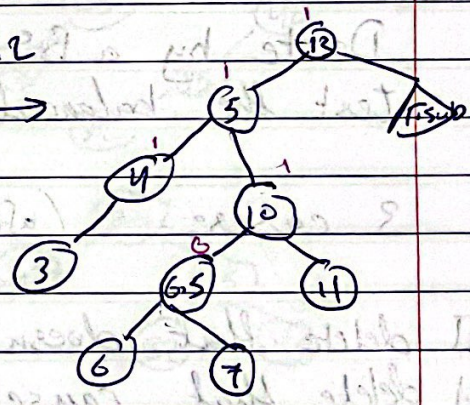
insert 7 $\hat{L} \rightarrow 6.5$



S.R.R
→

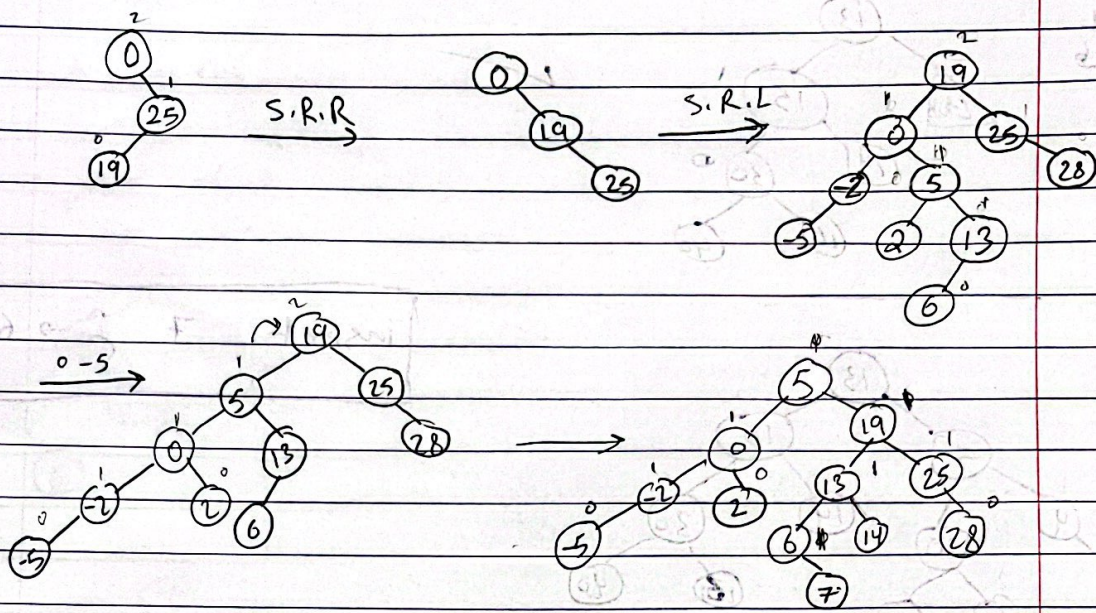


S.R.L



EX: insert the following key values into an initially empty AVL tree.

- 0, 25, 19, 5, -2, 28, 13, -5, 2, 6, 14, 7



* Deletion:-

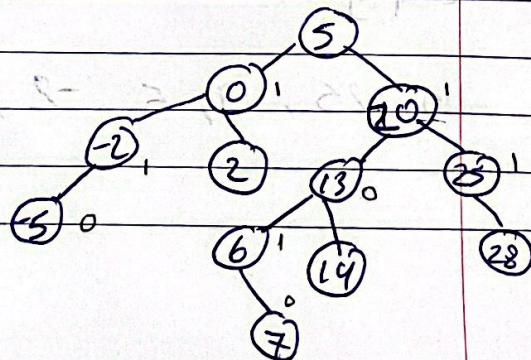
- Delete by a BST by copying algorithm.
- test the balanced factor of the tree.

→ 3 cases:- (after test)

- [1] delete that doesn't cause imbalance.
- [2] delete that cause imbalance, and need single rotation.
- [3] delete that cause imbalance, and need double rotation.

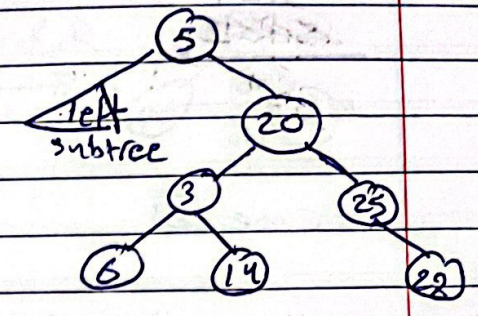
Ex

→ This AVL tree is balanced

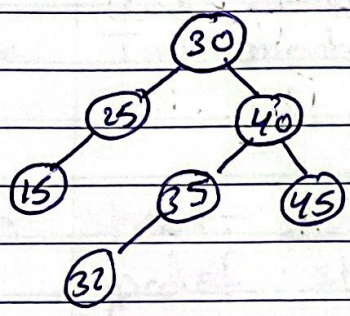


* delete 7 →

still balanced and does not
 been affected. case 1

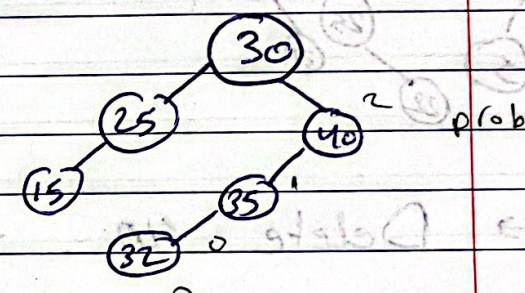


Ex



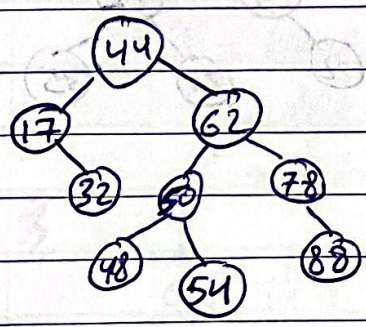
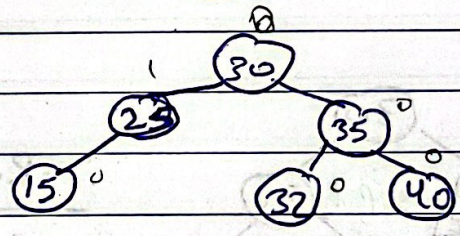
This is Balanced AVL Tree

→ delete 45 →



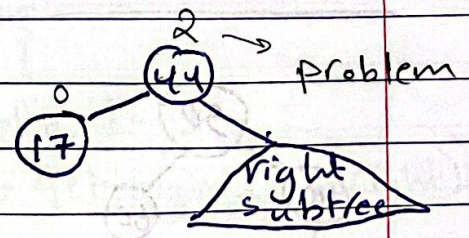
→ S.R.R →

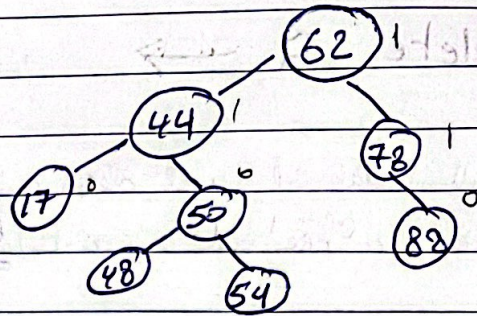
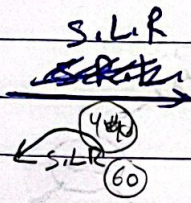
case 2



This is balanced AVL Tree

Delete 32

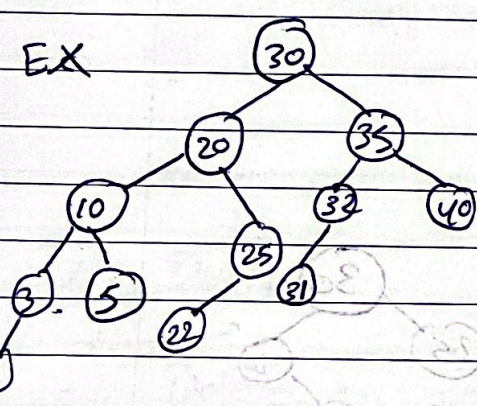




case 2

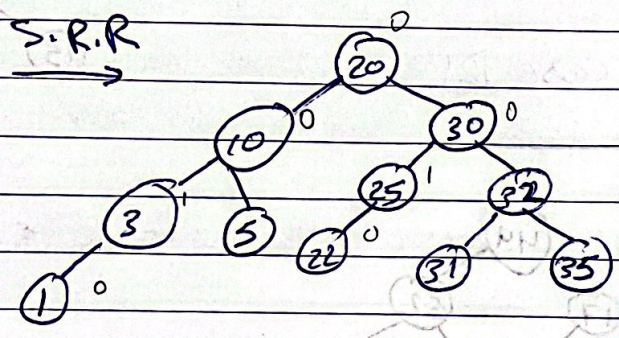
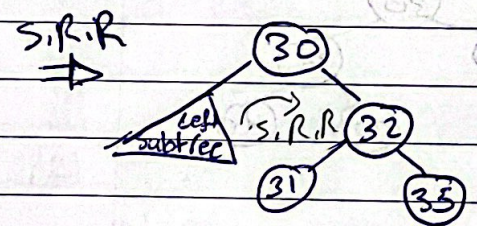
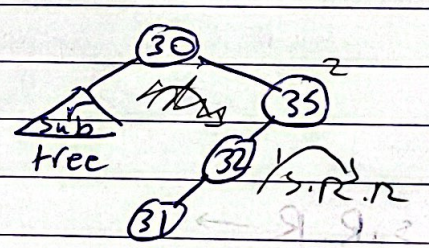
دائماً لما رخص ما في balanced وفكري قبيح نود من الجهة الأخرى فيها نودز زي للوزن بجيب من الجهة الأخرى الأهل الألف مستان يتوارثوا

Note

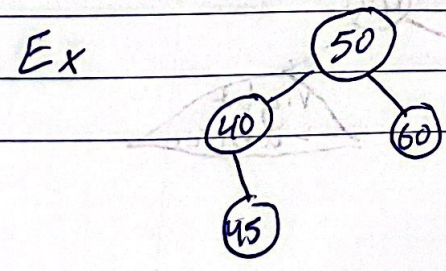


→ This is AVL T ✓

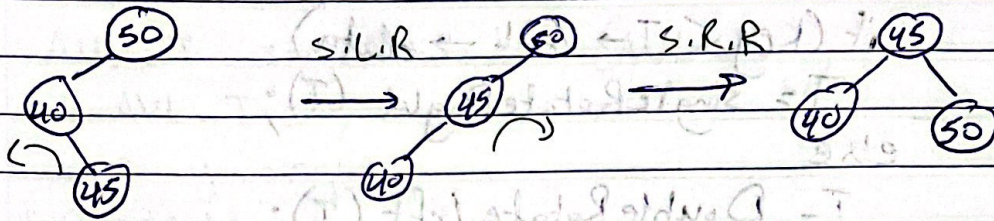
→ Delete 40



case 3



→ Delete 60



Case 3

*** Implementation of AVL Trees:-**

```

-> struct AVLnode;
typedef struct AVLnode * AVL-T;
struct AVLnode {
    double data;
    AVL-T right;
    AVL-T left;
    int Height;
}

```

*** Function insert:**

```

-> AVL-T insert (AVL-T T, double key) {
    if (T == NULL) {
        T = (AVL-T) malloc (sizeof (struct AVLnode));
        T->left = T->right = NULL;
        T->data = key;
    }
    else if (Key < T->data) {
        T->left = insert (T->left, key);
        if ((getHeight (T->left) - getHeight (T->right)) >= 2) {
            // Balance
            // AVL
            // ...
        }
    }
}

```

balance • abs
 قسمه مطلقه
 نشان عددی طرح و طرح مبالغه

موجود بالترتیب
 نکتہ بچرین


```

    if (key < T->left->data)
        T = singleRotateRight(T);
    else
        T = DoubleRotateLeft(T);
}
else if (key > T->data) {
    T->right = insert(T->right, key);
    if (abs(getHeight(T->right) - getHeight(T->left)) >= 2) {
        if (key > T->right->data)
            T = singleRotateLeft(T);
        else
            T = DoubleRotateRight(T);
    }
}
T->Height = 1 + Max(getHeight(T->left), getHeight(T->right));
return T;
}

```

```

getHeight (Function) int getHeight (AVL-T, T) {
    return T->Height;
}

```

```

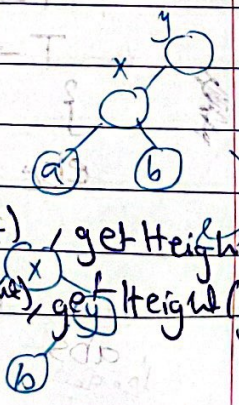
Function: AVL-T singleRotateRight (AVL-T y) {

```

```

D.R.R
    AVL-T x;
    x = y->left;
    y->left = x->right;
    x->right = y;
    x->Height = 1 + Max(getHeight(x->right), getHeight(x->left));
    y->Height = 1 + Max(getHeight(y->right), getHeight(y->left));
    return x;
}

```



Function: AVL-T singleRotateLeft (AVL-T y) {

S.R.L AVL-T x;

x = y → right;

y → right = x → left;

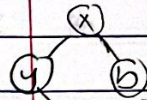
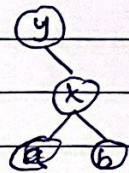
x → left = y;

x → Height = 1 + Max (getHeight(x → right), getHeight(x → left));

y → Height = 1 + Max (getHeight(y → right), getHeight(y → left));

return x;

}



Function: AVL-T doubleRotateLeft (AVL-T k3) {

Double k3 → Left = singleRotateLeft(k3 → Left);

Rotate return (singleRotateRight(k3));

Left then ?

right

* def splay Tree : is BST that :-

- Are not perfectly balanced all the time
- It assumes that recently accessed nodes are most likely accessed again
- Allow search and insertion operations to run faster.
- After node v is accessed, perform splaying operation:

def - splay : bring the node up to become the tree root.

- Active nodes (the recently accessed) will be moved towards the root, while in active nodes.

نيل من T يكون $O(n)$ ← لعيننا وبيك الحمل كمان مرة عليه (حت مثلا)
 على هذا العنصر بتغير $O(1)$

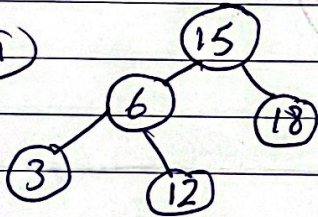
will be moved far from the root.

Splaying: - double / Right / Left ← rotate عليه

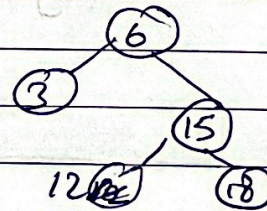
rotate

- 1- Zig to left-right
- 2- Zag to right-left
- 3- Zig-Zag
- 4- Zag-Zig
- 5- Zig-Zig
- 6- Zag-Zag

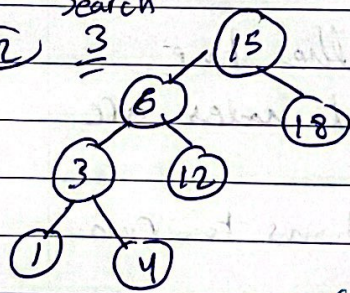
Exps: ①



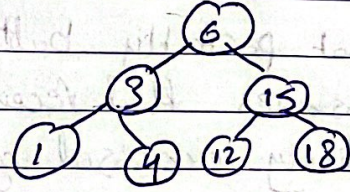
Zig search 6



② Search 3

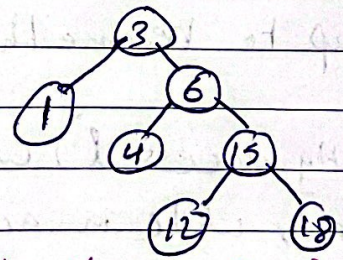


Zig Zag



3 child ال
 6 parent ال
 15 Grandparent ال
 اول مرة بعد ال
 مع ال parent ال

Zig 3

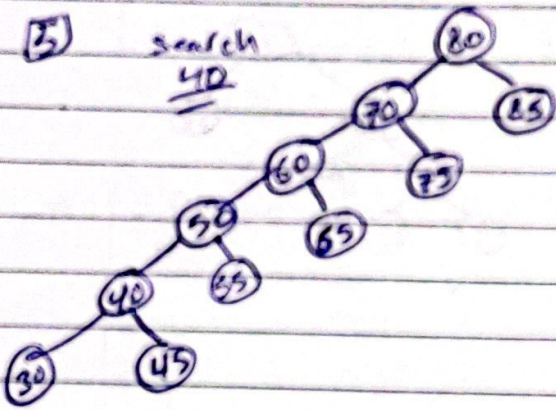


لا حرج انه هون عشان اول ال 3 بروج مرتين بنفس الاتجاه left left و يوصلها.

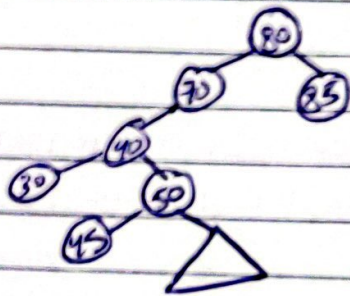
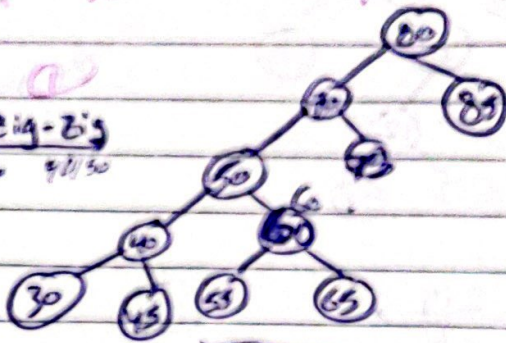
5)

Search
40

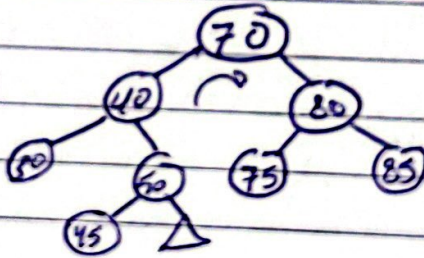
فاصله بین الیاف در وی شل 2



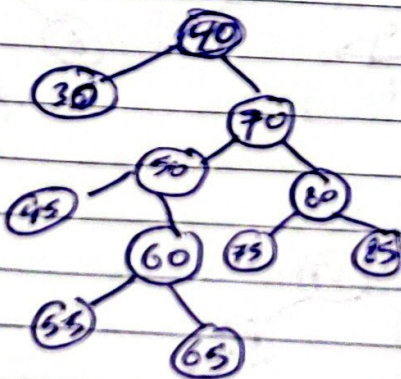
Zig-Zig
50/60 40/50



Zig
Zig

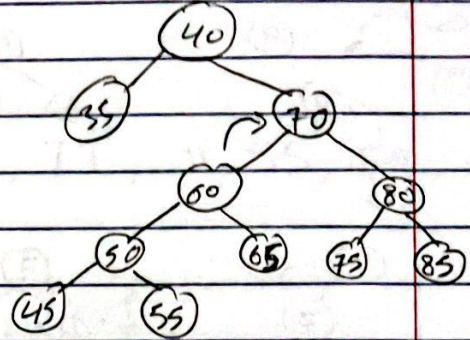
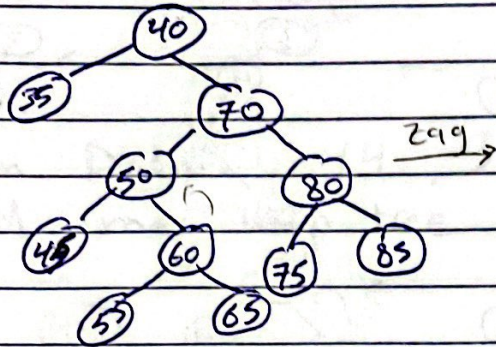


Zig

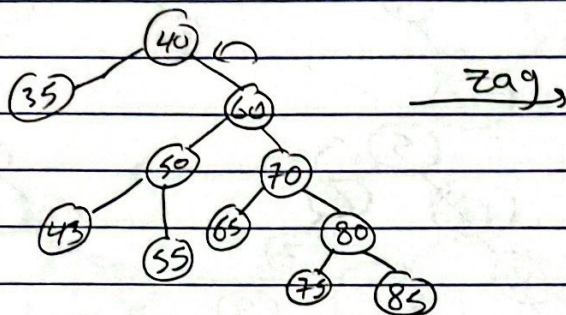


- 1) Zig-Zig
- 2) Zig-Zig

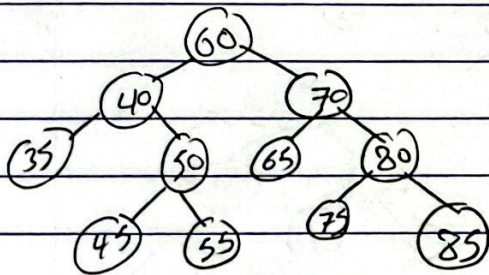
6 search
60



zig



zag



Ex. insert the following values into an empty splay tree:

4, 9, 3, 7, 5, 6, 20, 40

then delete 9

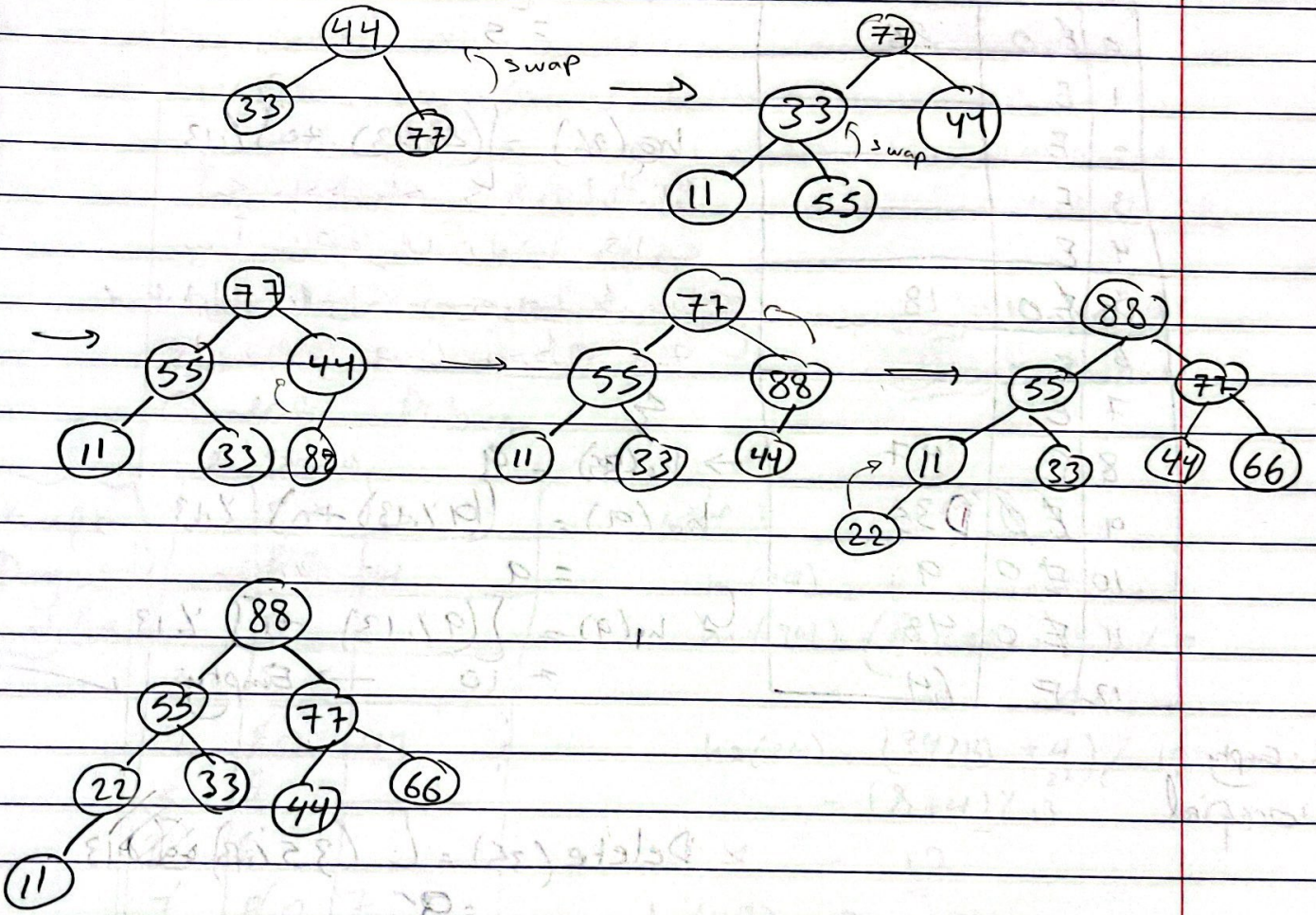
then search 3

16/11/2021

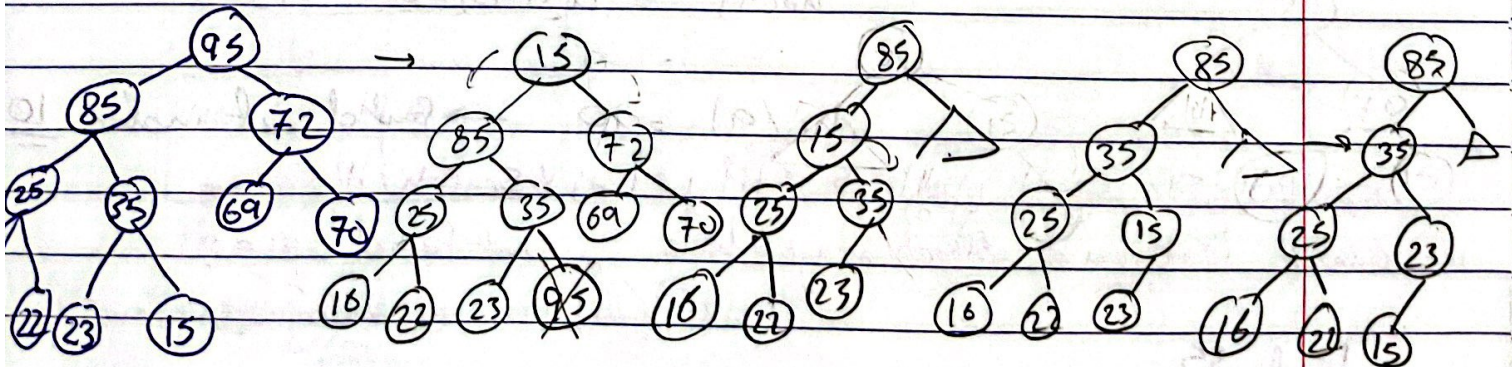
Heap

Tuesday

Q: Given Data: 44, 33, 77, 11, 55, 88, 66, 22
→ Build max heap tree.



Q: Delete root 95



Hashing

* Linear Probing (Open Addressing)

* 18 Example

المسألة

index Status

| index | Status | Value |
|-------|--------|-------|
| 0 | E | 0 |
| 1 | E | |
| 2 | E | |
| 3 | E | |
| 4 | E | |
| 5 | O | 18 |
| 6 | E | |
| 7 | E | |
| 8 | E | 47 |
| 9 | O | 35 |
| 10 | O | 9 |
| 11 | O | 48 |
| 12 | E | 64 |

$$h_0(18) = [(18 \cdot 13) + 0] \cdot 13 = 9$$

$$h_0(26) = [(26 \cdot 13) + 0] \cdot 13$$

ليه ماطح واحد لانو انا دنا
يبا من صف لكن لو صار دنا رتب
برج ن = 1 --- وهيك كمان باقى
عمل فاجني بعين رجلا فيه

$$\rightarrow h_0(35) = 9$$

$$h_0(9) = [(9 \cdot 13) + 0] \cdot 13 = 9 \quad \text{collision}$$

$$h_1(9) = [(9 \cdot 13) + 1] \cdot 13 = 10 \rightarrow \text{Empty}$$

E: Empty
O: occupied

$$* \text{Delete}(35) = [(35 \cdot 13) + 0] \cdot 13 = 9$$

$$\text{Delete}(40) \rightarrow h_0(40) = 1 \quad \text{empty}$$

$$\text{Find } 9 \rightarrow h_0(9) = [(9 \cdot 13) + 0] \cdot 13 = 9$$

$$h_1(9) = 10 \rightarrow \text{found at index } \underline{10}$$

عملية ال Search لازم اقبل انا كذا الرقم لكن لو ما لقيته
linear Probing disadvantages و ما not found

$O(n)$ و يمكن يحسب

$$\text{find } 35 \rightarrow h_0 = [(35 \cdot 13) + 0] \cdot 13 = 9 \rightarrow \text{not found} \rightarrow \text{"already deleted"}$$

* لو كان الـ index رقم زوجي الـ 35 كزي 22 مثلاً
 وطربت الـ 35 Find الـ index مشغول فإنتظر
 أدور أدور لحد ما يغير عندي شيء Empty فهو not found
 ويزيد بيهك الـ search

* إذا كانت موجودة بـ index مشغولة delete ورجعنا
 طلبنا search عليها عندنا حلين بتدي هي مني شغل كامل
 فهو خالص رجوع إذا كانت موجودة لستأ وبالتالي طلب عندي
not found

إذا كان الـ key فيه Sorting عندي عن الـ hash table
 إنه أصبح وقتك

Quadratic Probing

Example
 Page 22
 in slides.

| index | Status | Value |
|-------|--------|-------|
| 0 | E | 0 |
| 1 | E | 0 |
| 2 | E | |
| 3 | E | |
| 4 | E | 47 |
| 5 | E | |
| 6 | E | |
| 7 | E | 7 |
| 8 | E | 21 |
| 9 | E | 815 |
| 10 | E | 13 |
| 11 | E | |
| 12 | E | 34 |

صيفي :

$$\begin{aligned}
 h_0(34) &= 8 \quad \text{collision} \\
 h_1(34) &= (34 \cdot 13 + 1^2) \% 13 \\
 &= 9 \quad \text{collision} \\
 h_2(34) &= (34 \cdot 13 + 4) \% 13 \\
 &= (8 + 4) \% 13 \\
 &= 12 \\
 h_0(47) &= 8 \quad \text{collision} \\
 h_1(47) &= 9 \quad h_2 \text{ coll} \\
 h_3(47) &= (8 + 9^2) \% 13 = 17 \% 13 = 4
 \end{aligned}$$

الـ coll دهل قفزات "توزيعه"
 وبالتالي احتمال توزيع العناصر وبعدها
 أفضل

```

int HashTable[13]; parallel array
int status [13]
    
```

```

struct Node {
    int element;
    char status;
}
    
```

العنقودية بالـ Quadratic Probing
 دهلا وقت أفضل وكل ما رقتي
 de تاني رجب احتمال الـ
 Empty ألي

