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FACULTY OF ENGINEERING AND TECHNOLOGY

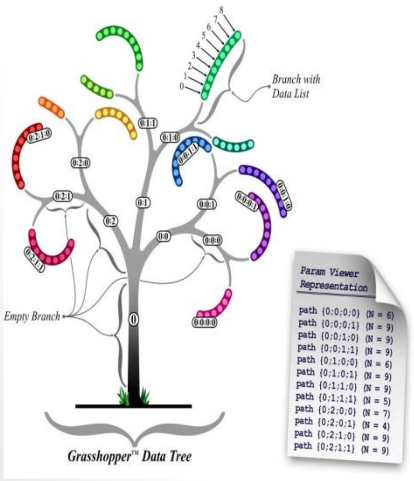
COMPUTER SCIENCE DEPARTMENT

COMP2321

Data Structures

Chapter 4 Trees

Splay Tree and B Tree

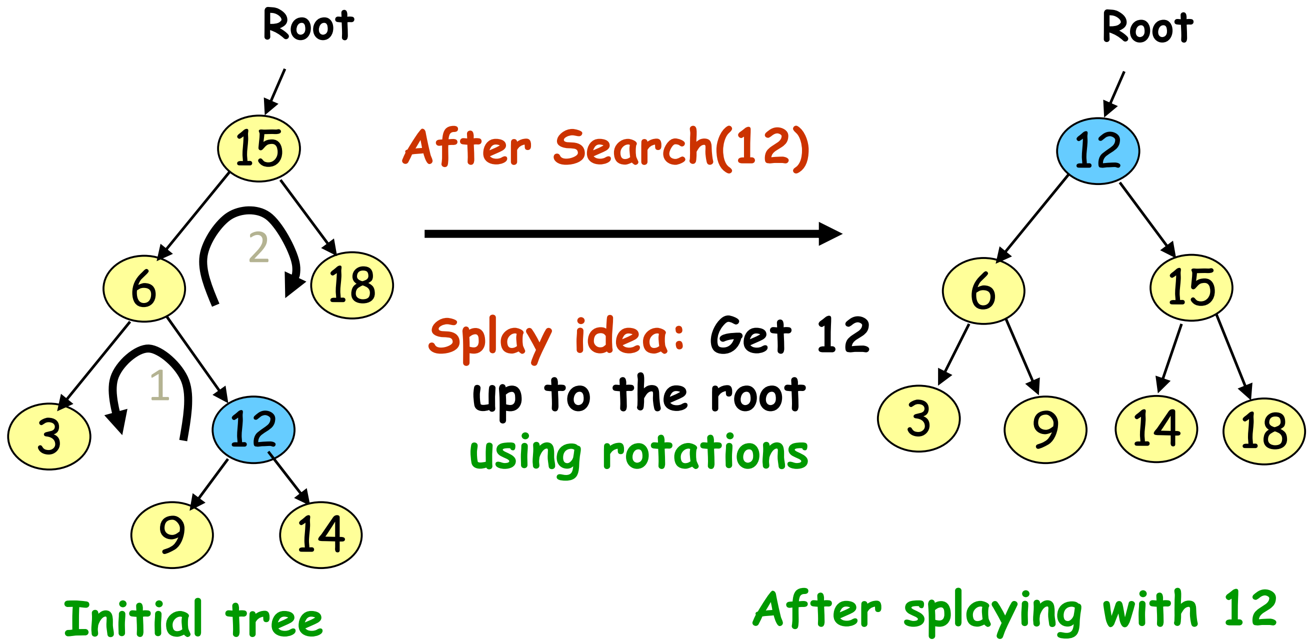


Splay Trees

- Splay Tree is binary search tree (BSTs) that:
  - Are not perfectly balanced all the time
  - It assumes that recently accessed nodes are most likely to visit them again.
  - Allow search and insertion operations to try to balance the tree so that future operations may run faster

- Based on the heuristic:
  - If X is accessed once, it is likely to be accessed again.
  - After node X is accessed, perform "splaying" operations to bring X up to the root of the tree.
  - Do this in a way that leaves the tree more or less balanced as a whole.

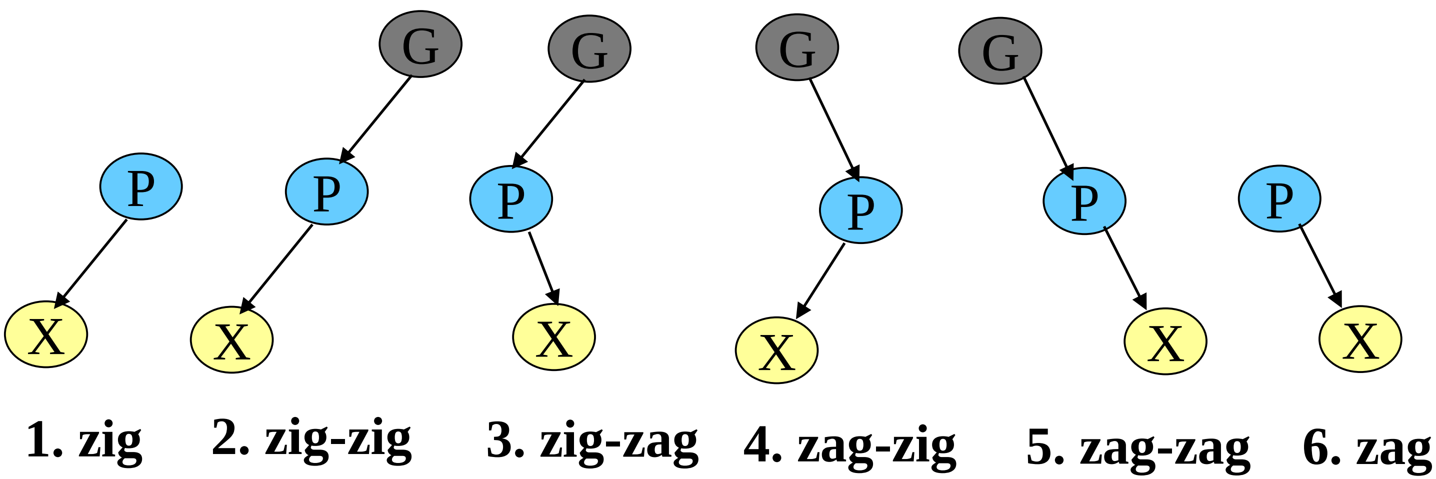
Motivating Example



- Not only splaying with 12 makes the tree **balanced**, subsequent accesses for 12 will take **O(1)** time.
- **Active (recently accessed)** nodes will move towards the root and **inactive** nodes will slowly move further from the root

Splay Tree Terminology: operations

- Let X be a **non-root** node, i.e., has at least 1 ancestor.
- Let P be its **parent** node.
- Let G be its **grandparent** node (if it exists)
- Consider a path from **G to X**:
  - Each time we go **left**, we say that we "**zig**"
  - Each time we go **right**, we say that we "**zag**"
- There are 6 possible cases:

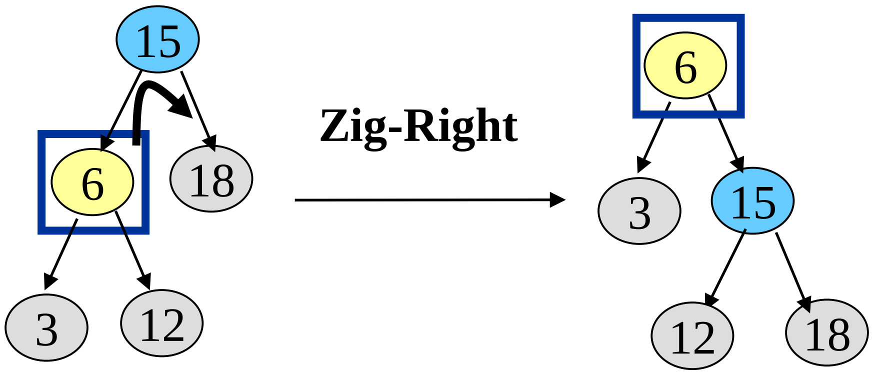


Splay Tree Operations

- When node X is accessed, apply one of six rotation operations:
  - Single Rotations (X has a P but no G)
    - zig, zag
  - Double Rotations (X has both a P and a G)
    - zig-zig, zig-zag
    - zag-zig, zag-zag

Splay Trees: Zig Operation

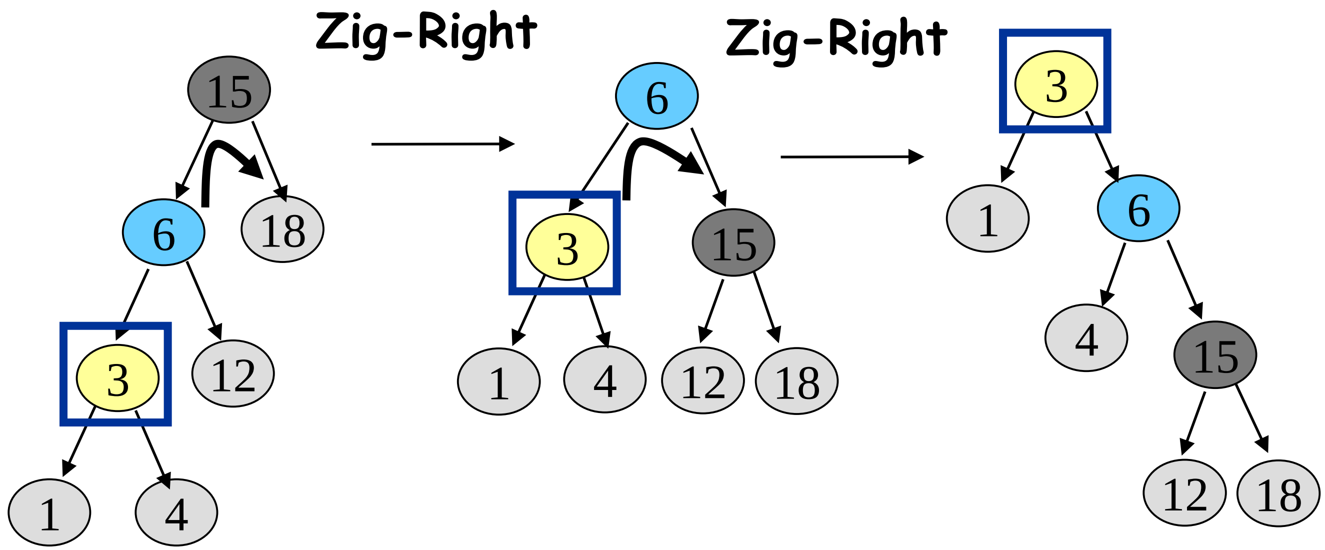
- “Zig” is just a single rotation, as in an AVL tree
- Suppose 6 was the node that was accessed (e.g. using Search)



- “Zig-Right” moves 6 to the root.
- Can access 6 faster next time: O(1)
- Notice that this is simply a right rotation in AVL tree terminology.

Splay Trees: Zig-Zig Operation

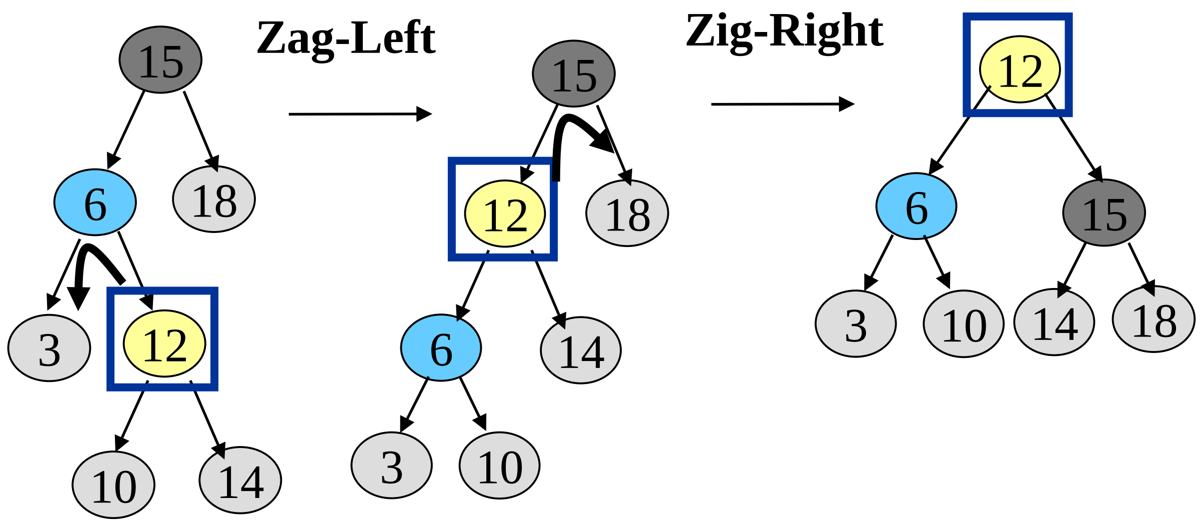
- “Zig-Zig” consists of two single rotations of the same type
- Suppose 3 was the node that was accessed (e.g., using Search)



- Due to “zig-zig” splaying, 3 has bubbled to the top!
- Note: Parent-Grandparent is rotated first.

Splay Trees: Zig-Zag Operation

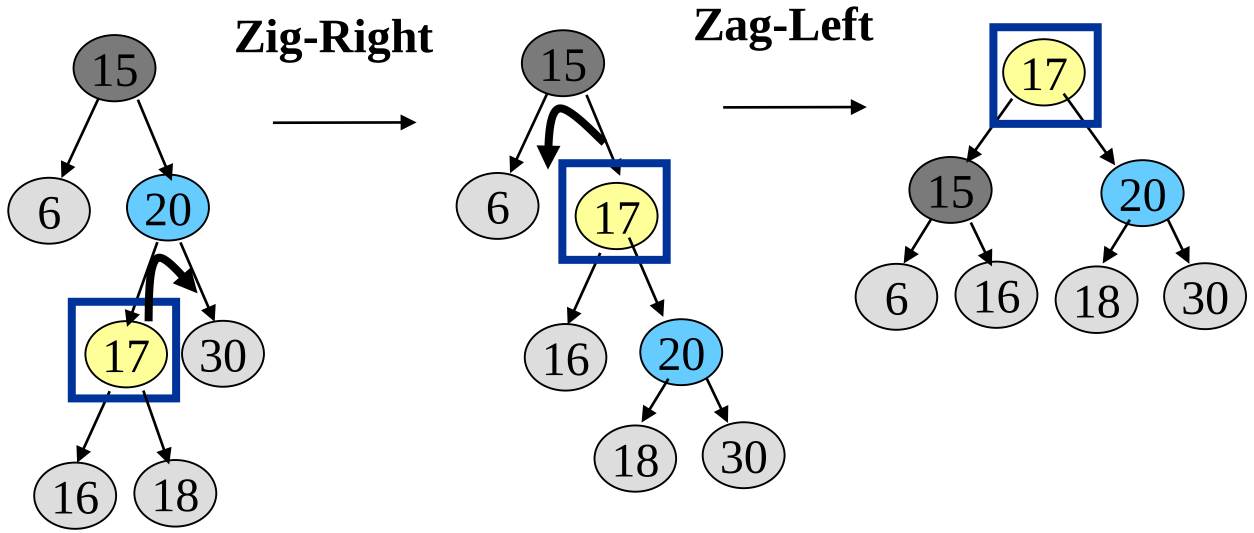
- "Zig-Zag" consists of two rotations of the opposite type
- Suppose 12 was the node that was accessed (e.g., using Search)



- Due to "zig-zag" splaying, 12 has bubbled to the top!
- Notice that this is simply an LR imbalance correction in AVL tree terminology (first a left rotation, then a right rotation)

Splay Trees: Zag-Zig Operation

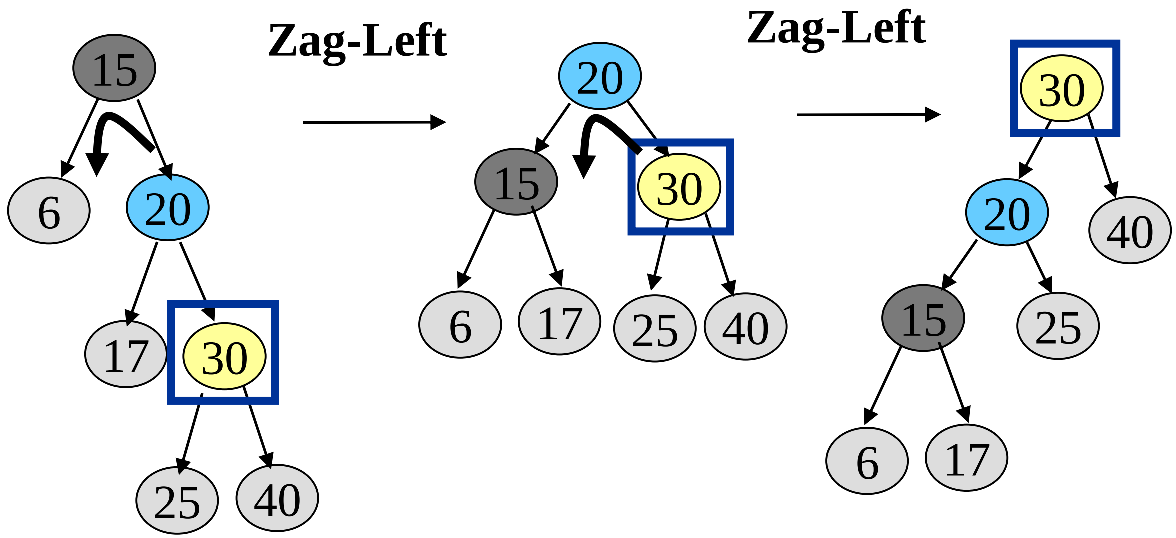
- "Zag-Zig" consists of two rotations of the opposite type
- Suppose 17 was the node that was accessed (e.g., using Search)



- Due to "zag-zig" splaying, 17 has bubbled to the top!
- Notice that this is simply an RL imbalance correction in AVL tree terminology (first a right rotation, then a left rotation)

Splay Trees: Zag-Zag Operation

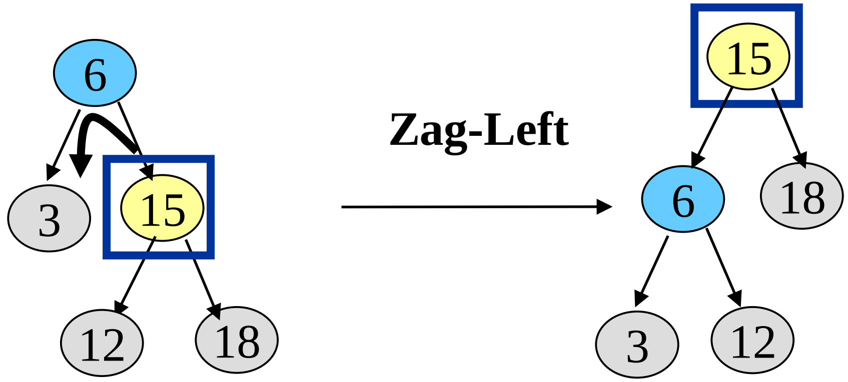
- "Zag-Zag" consists of two single rotations of the same type
- Suppose 30 was the node that was accessed (e.g., using Search)



- Due to "zag-zag" splaying, 30 has bubbled to the top!
- <u>Note: Parent-Grandparent is rotated first.</u>

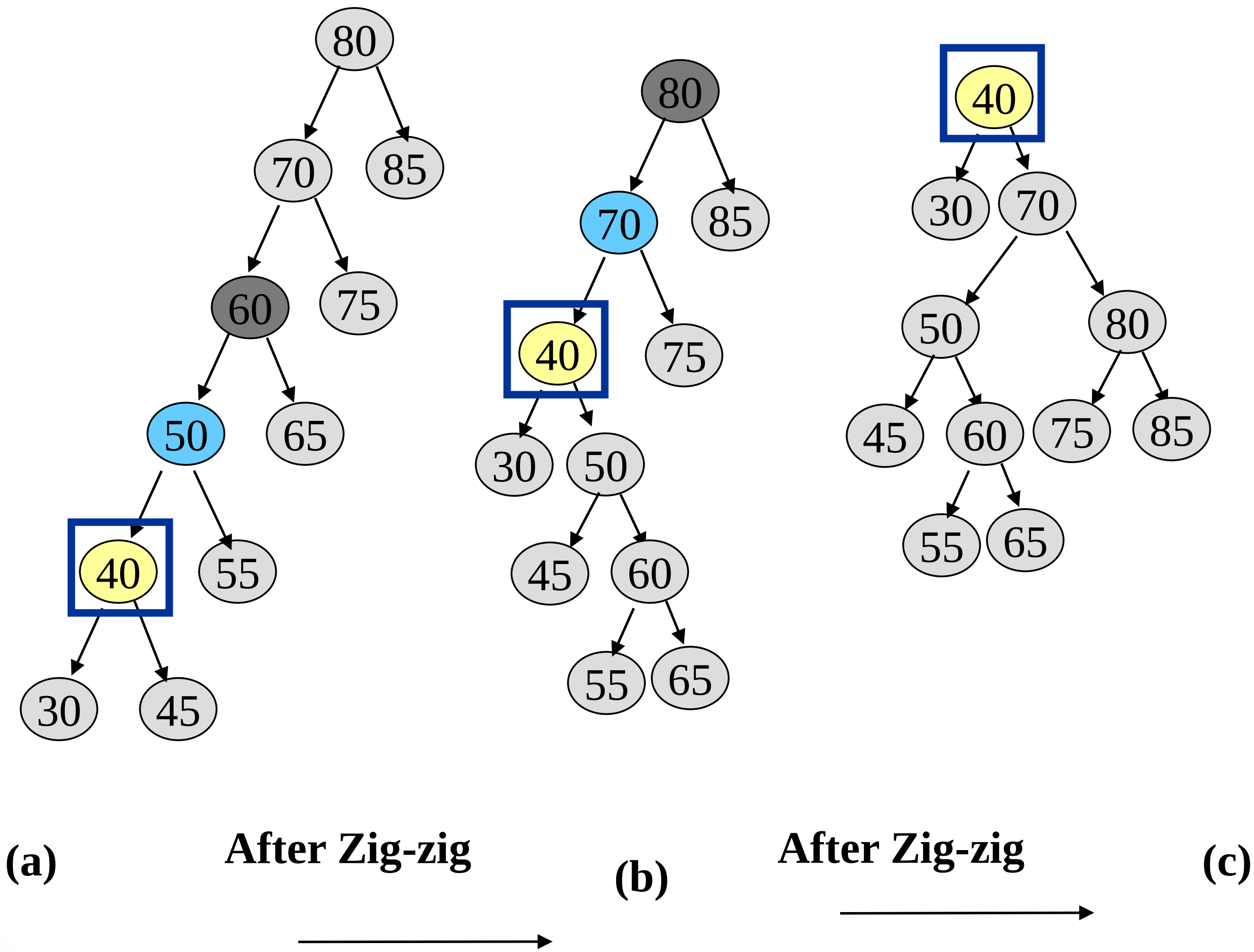
Splay Trees: Zag Operation

- "Zag" is just a single rotation, as in an AVL tree
- Suppose **15** was the node that was accessed (e.g., using Search)

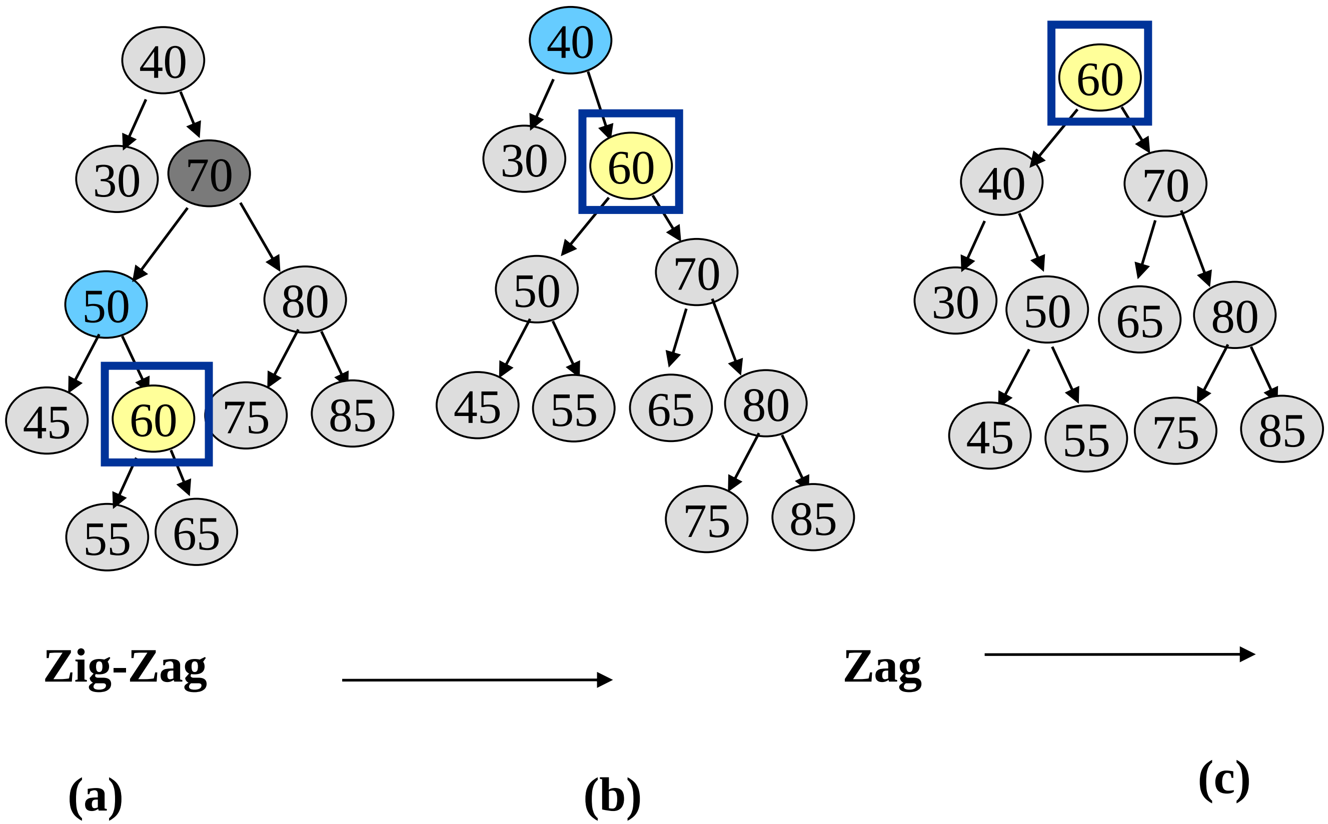


- "Zag-Left" moves 15 to the root.
- Can access 15 faster next time: O(1)
- Notice that this is simply a left rotation in AVL tree terminology

Splay Trees:
Example – 40 is accessed



Splay Trees:
Example – 60 is accessed



Splaying during other operations

- Splaying can be done not just after Search, but also after other operations such as Insert/Delete.

- **Insert X**: After **inserting X** at a leaf node (as in a regular BST), **splay X up to the root**

- **Delete X**: Do a **Search on X and get X up to the root**. Delete X at the root and move the largest item in its left sub-tree, i.e, **its predecessor, to the root using splaying.**

- **Note on Search X**: **If X was not found**, splay the leaf node that the Search **ended up with to the root**.

Any sequence of M operations on a splay tree of size N takes O(M log N) time.

Exercise: Do it by yourself

- Insert the keys 4,9,3,7,5,6 in that order into an empty splay tree.

A. Delete 9

B. Find 3

- Insert the keys 1, 2, …, 7 in that order into an empty splay tree.

What happens when you access “7”?

Hint: ensure your solution by using this website
https://www.cs.usfca.edu/~galles/visualization/SplayTree.html

B-Trees

DEF: A B-Tree of order m is an m-way tree such that

1. All leaf nodes are at the same level.
2. All non-leaf nodes (except the root) have at most m and at least m/2 children.
3. The number of keys is one less than the number of children for non-leaf nodes and at most m-1 and at least m/2 for leaf nodes.
4. The root may have as few as 2 children unless the tree is the root alone.

Example for m = 5

DEF: A B-Tree of order 5 is an 5-way tree such that

1. All leaf nodes are at the same level.
2. All non-leaf nodes (except the root) have at most 5 and at least 2 children.
3. The number of keys is one less than the number of children for non-leaf nodes and at most 4 and at least 2 for leaf nodes.
4. The root may have as few as 2 children unless the tree is the root alone.

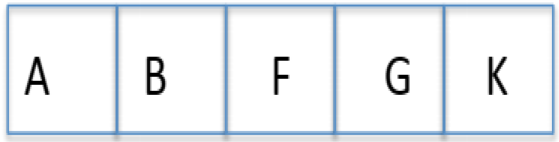
Creating a B-tree of order 5

A G F B K D H M J E S I R X C L N T U P

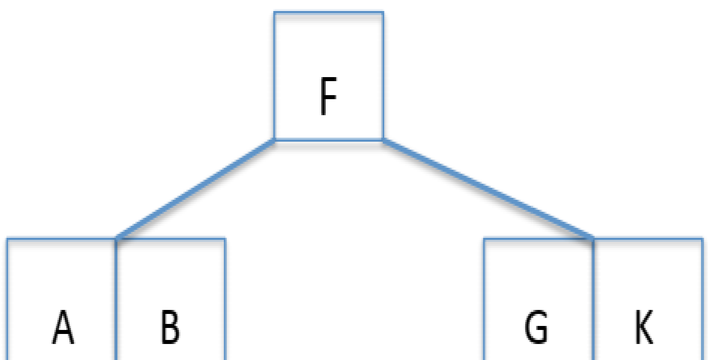
A G F B K D H M J E S I R X C L N T U P

A	B	F	G

A G F B K D H M J E S I R X C L N T U P



A G F B K D H M J E S I R X C L N T U P



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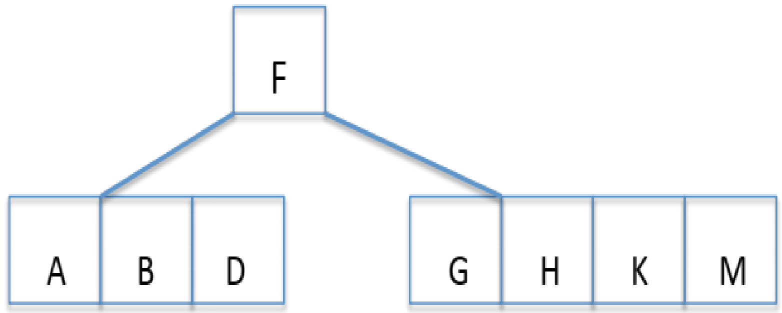

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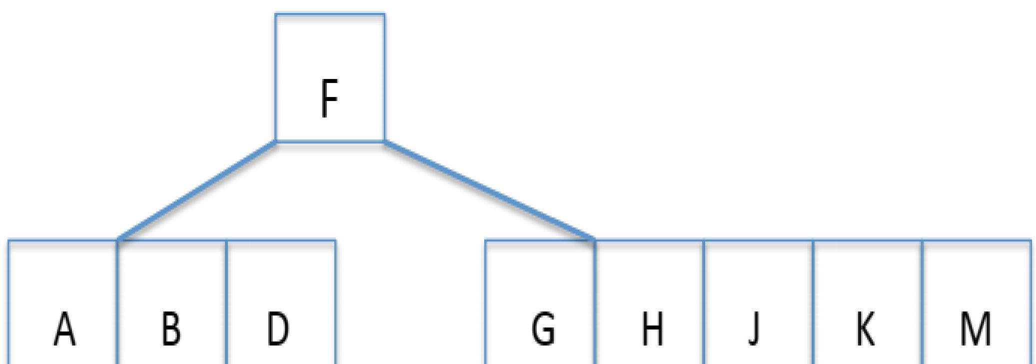

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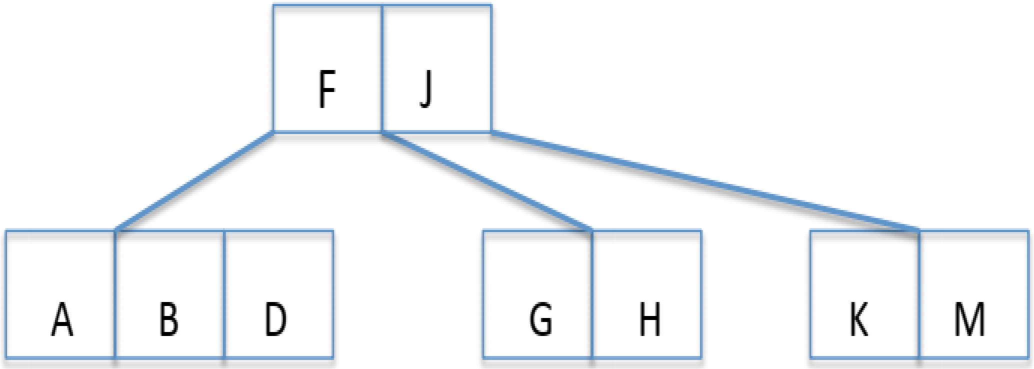

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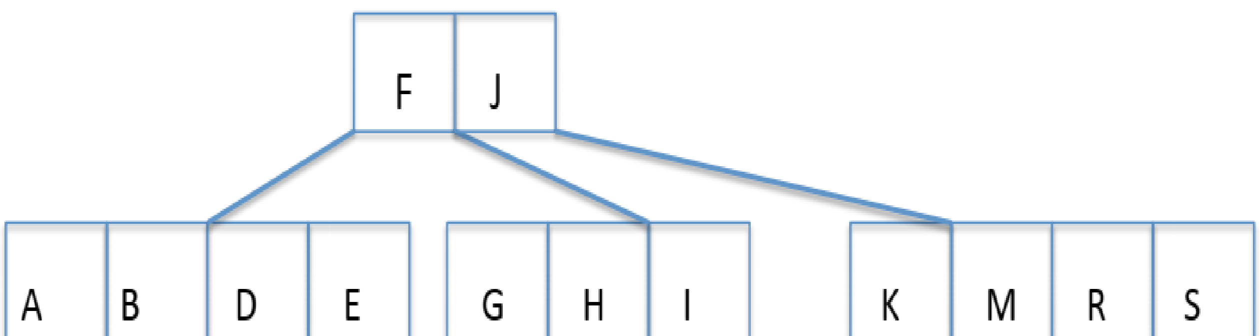

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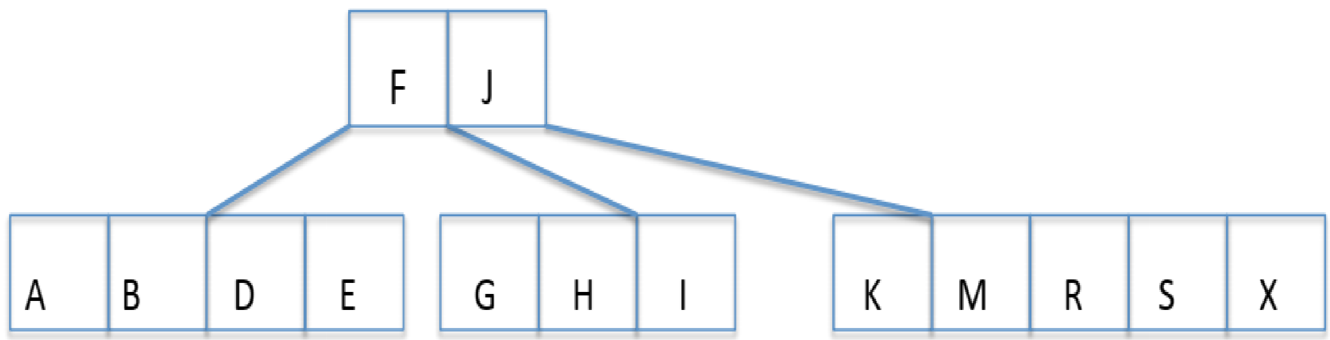

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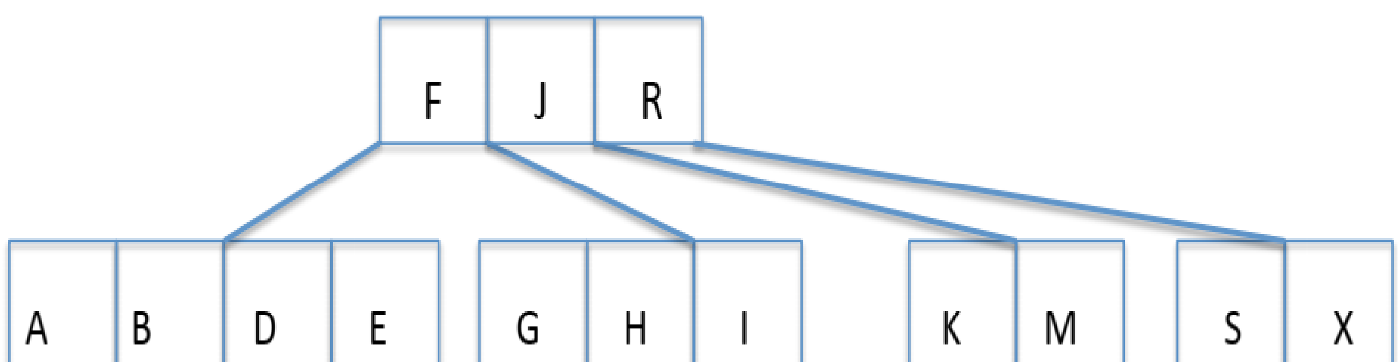

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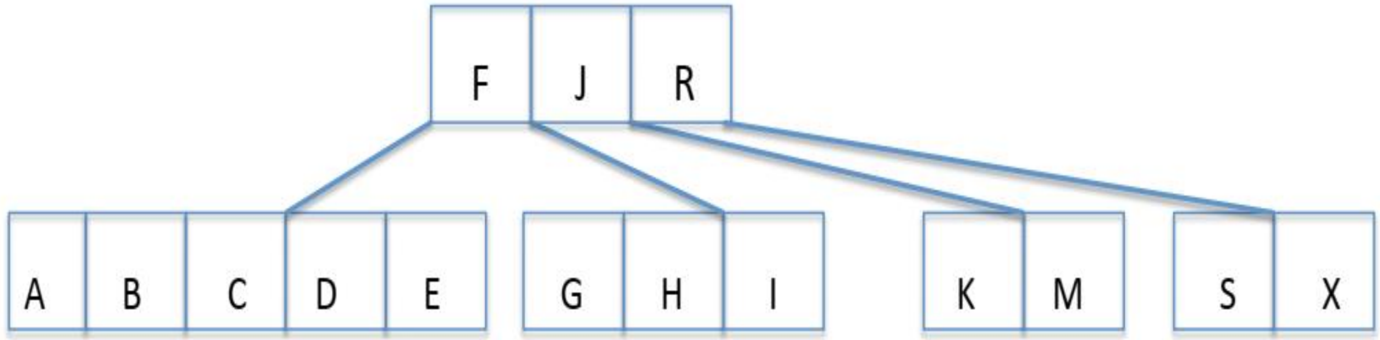

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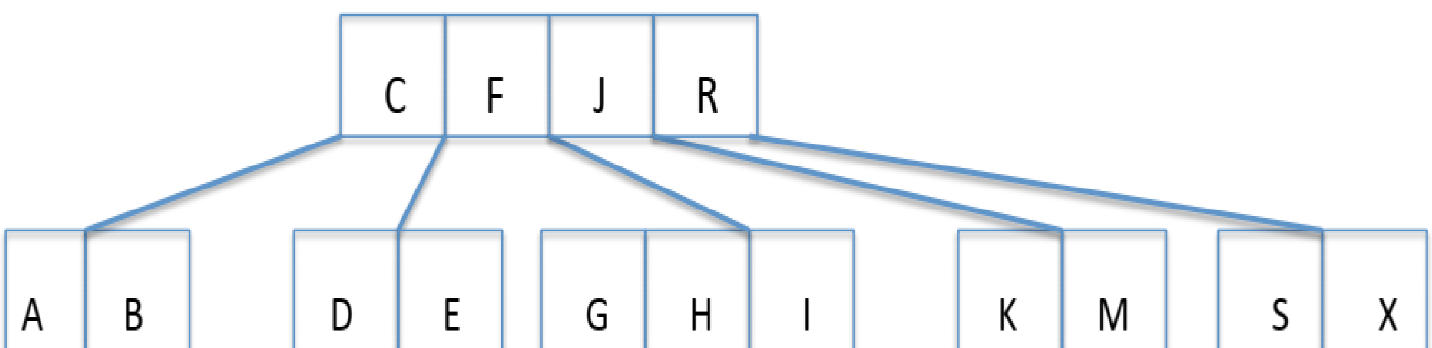

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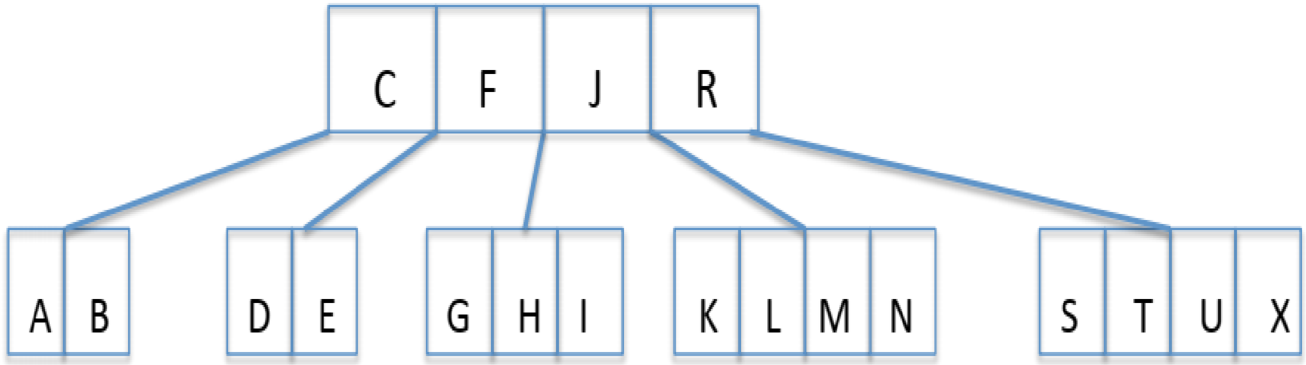

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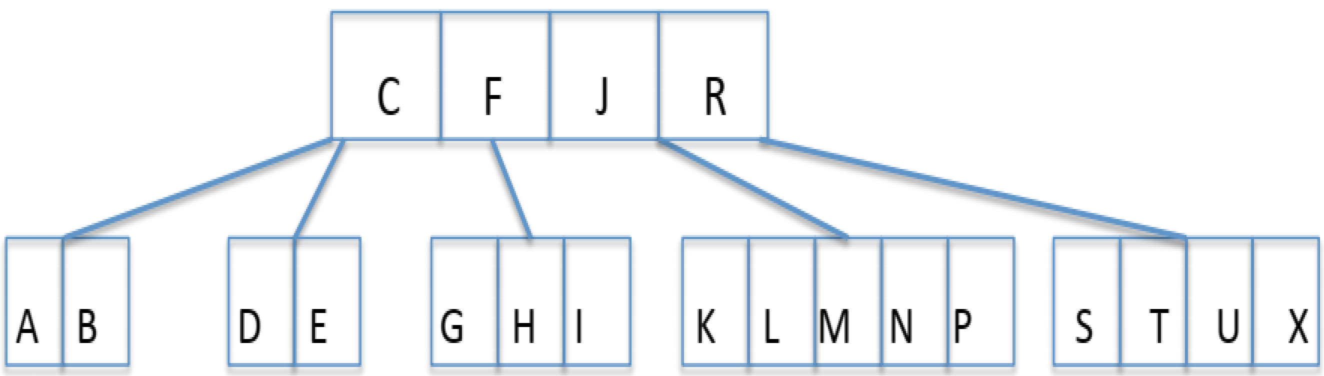

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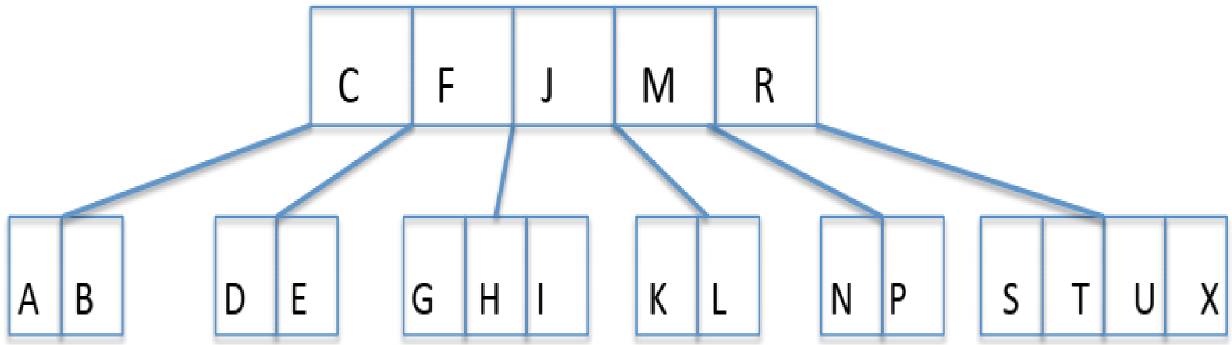

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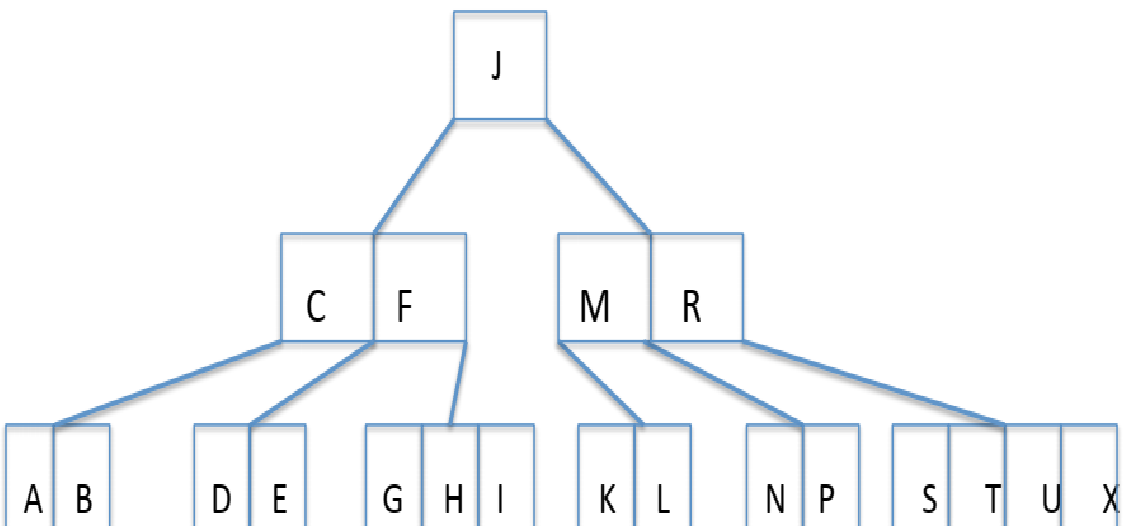
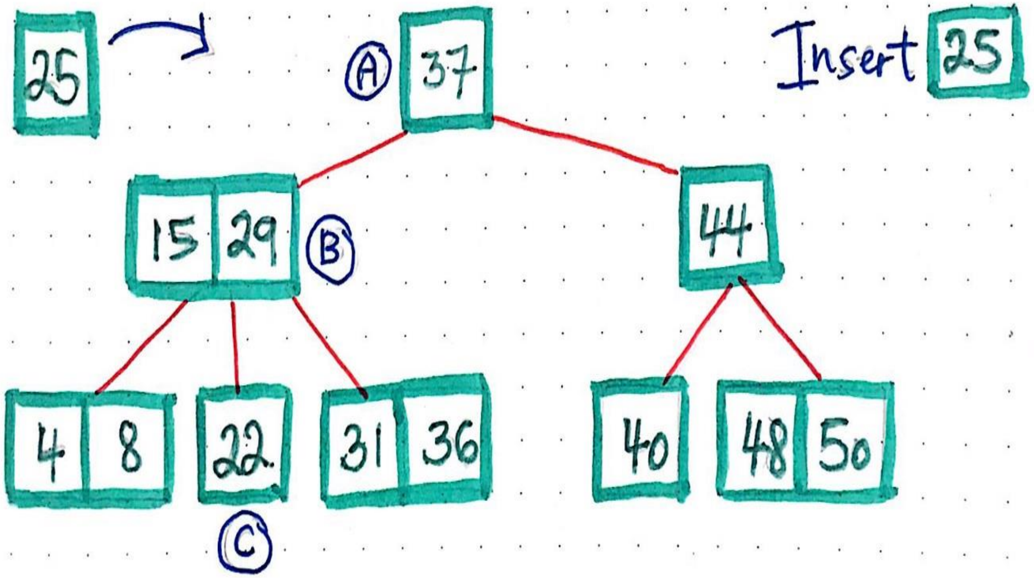
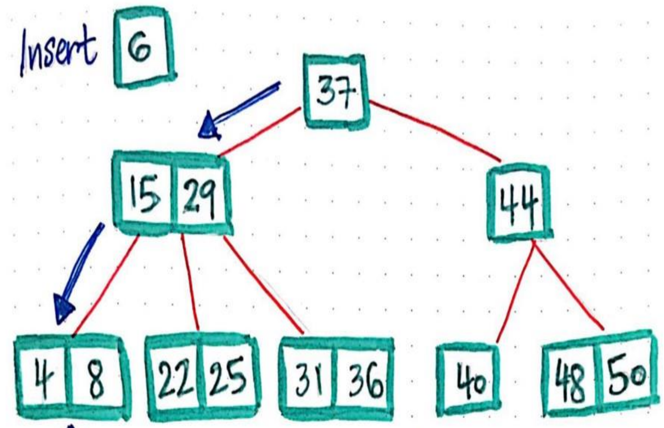
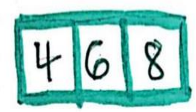




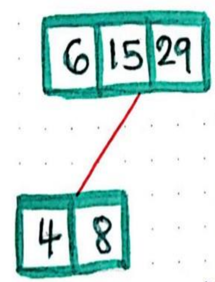
(A) Start from the root 25<37, go left.
(B) 25 is between 15 and 29, go to middle.
(C) Insert 25 into node containing Key 22.



It should be inserted here, but this node is going to overflow once we insert this Key.



We can **Split** this node to make room for the value that needs to be inserted.



We can take the middle element from the overflowed node & bring it up to the correct place in the parent node.

* If the parent overflows, we can split again, all the way up to the root node.

Homework :

Insert the following elements

10,20,40,50,60,70,80,30,35,5,15,60

in a B-Tree of order M=4

Solution using

https://www.cs.usfca.edu/~galles/visualization/BTree.html

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Homework :

Insert the following elements 9,0,8,1,7,2,6,3,5,4, in a B-Tree of order M=3,

Solution using
https://www.cs.usfca.edu/~galles/visualization/BTree.html

THANK YOU