Time comp. Recursion → Recurrence

Ex. Factorial

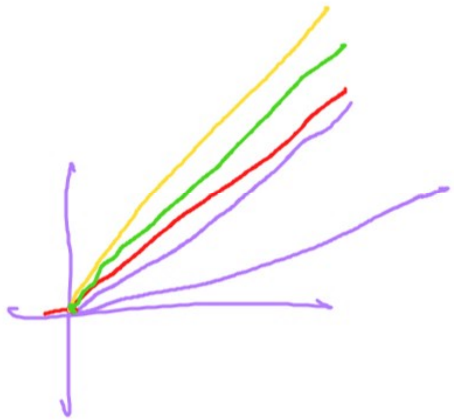
```
long Fact(int n){
  if(n==0)
    return 1;
  return n*Fact(n-1);
}
```

$\underbrace{\text{Fact}(n-1)}_{T(n-1)}$

Rec. Relation $T(n) = \begin{cases} d, & n=0 \\ T(n-1)+c, & n>0 \end{cases}$

$$T(n) = \cancel{T(n-1)} + c$$
$$\cancel{T(n-1)} = \cancel{T(n-2)} + c$$
$$\cancel{T(n-2)} = \cancel{T(n-3)} + c$$
$$\vdots$$
$$T(1) = \cancel{T(0)} + c$$
$$\cancel{T(0)} = d$$

$$T(n) = nc + d$$
$$= O(n)$$



```
MergeSort(A, L, H){
    if(L < H)
        middle = (L+H)/2

    MergeSort(A, L, middle);
    MergeSort(A, middle+1, H);

    Merge(A, L, H);

}
```

R.R.

$$T(n) = \begin{cases} d, & n=1 \\ 2T\left(\frac{n}{2}\right) + n, & n>1 \end{cases}$$

A 0 ..... n-1

(A(n, n)

$$T(n) = 2T\left(\frac{n}{2}\right) + n \quad \text{— } (1)$$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + \frac{n}{2} \quad \text{— } (2)$$

$$T\left(\frac{n}{2^2}\right) = 2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2} \quad \text{— } (3)$$

$$T\left(\frac{n}{2^3}\right) = 2T\left(\frac{n}{2^4}\right) + \frac{n}{2^3} \quad \text{— } (4)$$

...

Now (2) in (1)

$$T(n) = 2\left[2T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right] + n$$

$$= 2^2 T\left(\frac{n}{2^2}\right) + 2n \quad \text{— } (5)$$

Sub. (3) in (5)

$$T(n) = 2^2\left[2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}\right] + 2n$$

$$= 2^3 T\left(\frac{n}{2^3}\right) + 3n \quad \text{— } (6)$$

After k of steps.

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + kn$$

Let $\frac{n}{2^k} = 1$

$$n = 2^k$$

$$k = \log n$$

$$T(n) = nT(1) + n.\log n$$

$$= n.d + n\log n$$

$$= O(n \log n)$$

Wednesday, March 3, 2021 12:05 PM

$$T(n) = \begin{cases} d, & n=1 \\ 2T\left(\frac{n}{2}\right)+10, & n>1 \end{cases}$$

$$T(n) = 2T\left(\frac{n}{2}\right) + 10 \quad \text{— } 1$$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + 10 \quad \text{— } 2$$

$$T\left(\frac{n}{2^2}\right) = 2T\left(\frac{n}{2^3}\right) + 10 \quad \text{— } 3$$

Sub. 2 in 1

$$T(n) = 2\left[2T\left(\frac{n}{2^2}\right) + 10\right] + 10$$

$$= 2^2 T\left(\frac{n}{2^2}\right) + 2^1 \cdot 10 + 2^0 \cdot 10 \quad \text{— } 4$$

Sub 3 in 4

$$T(n) = 2^2\left[2T\left(\frac{n}{2^3}\right) + 10\right] + 2^1 \cdot 10 + 2^0 \cdot 10$$

$$= 2^3 T\left(\frac{n}{2^3}\right) + 2^2 \cdot 10 + 2^1 \cdot 10 + 2^0 \cdot 10$$

at K^{th} step

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + \underbrace{2^{k-1} \cdot 10 + 2^{k-2} \cdot 10 + \dots + 2 \cdot 10 + 2^0 \cdot 10}_{10 \cdot (2^{k-1} + 2^{k-2} + \dots + 2^1 + 2^0)}$$

$$\frac{n}{2^k} = 1$$

$$n = 2^K$$

$$\frac{2^k - 1}{2 - 1} = (2^k - 1) \cdot 10$$

$$\frac{r^n - 1}{r - 1}$$

$$T(n) = nT(1) + (n-1) \cdot 10$$

$$= d \cdot n + 10n - 10$$

$$= O(n)$$

Wednesday, March 3, 2021 12:23 PM

$T(n) = 2T(n-1) + c$ — 1

$T(n-1) = 2T(n-2) + c$ — 2

$T(n-2) = 2T(n-3) + c$ — 3

$T(n) = 2[2T(n-2) + c] + c$

$= 2^2 T(n-2) + 2c + c$ — 4

$T(n) = 2^2[2T(n-3) + c] + 2c + c$

$= 2^3 T(n-3) + 2^2 c + 2c + c$

after k^{th} step.

$T(n) = 2^k T(n-k) + 2^{k-1} c + 2^{k-2} c + \dots + c$

$n - k = 1$ will stop

$n = k+1 \longrightarrow k = n-1$

$T(n) = 2^{n-1} T(1) + 2^{n-2} \cdot c + 2^{n-3} \cdot c + \dots + c$

Assume n is very large

$n \approx n-1$

$T(n) = 2^n T(0) + \underbrace{2^{n-1} \cdot c + 2^{n-2} \cdot c + \dots + c}$

? $= d \cdot 2^n + (n-1) \cdot c$

$= O(2^n)$