Faculty of Engineering and Tecnology

Computer Science Department

Trees_3

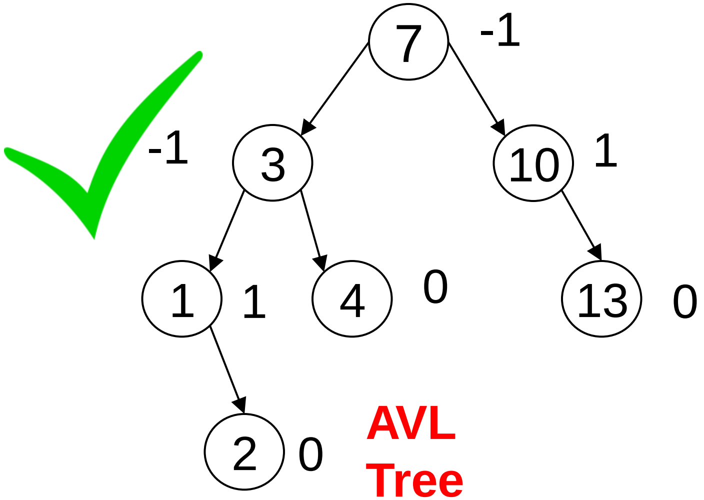
AVL Trees

AVL Trees

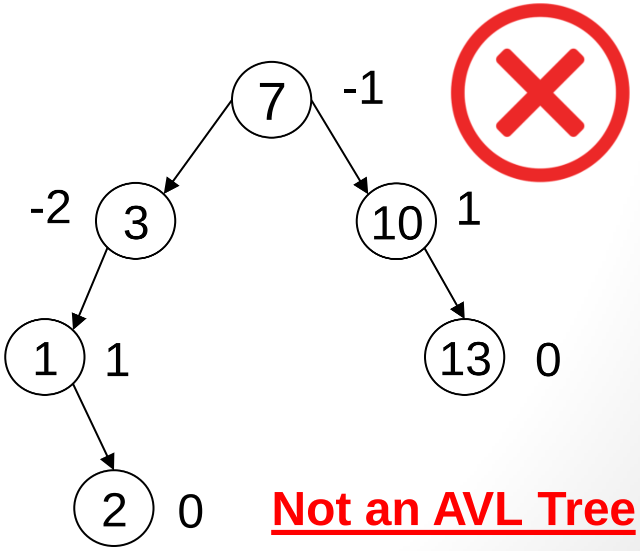
- Introduction
- What is an AVL Tree?
- AVL Tree Implementation.
- Why AVL Trees?
- Rotations.

What is an AVL Tree?

- An AVL (Adel’son, Vel’skii, & Lands) tree is a binary search tree with a height balance property:
    - For each node v, the heights of the subtrees of v differ by at most 1.
- A subtree of an AVL tree is also an AVL tree.
- An AVL node can have a balance factor of -1, 0, or +1.



AVL Tree



Not an AVL Tree

Why AVL Trees?

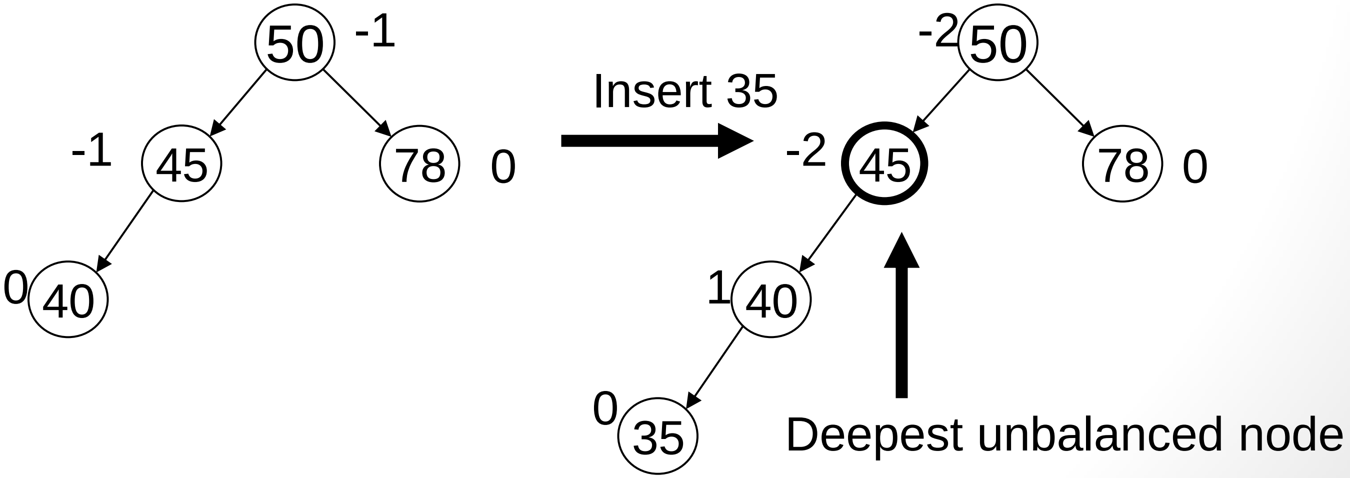
- Insertion or deletion in an ordinary Binary Search Tree can cause large imbalances.
- In the worst case searching an imbalanced Binary Search Tree is O(n).
- An AVL tree is rebalanced after each insertion or deletion.
  - The height-balance property ensures that the height of an AVL tree with n nodes is O(log n).
  - Searching, insertion, and deletion are all O(log n).

What is a Rotation?

- A rotation is a process of switching children and parents among two or three adjacent nodes to restore balance to a tree.

- **An insertion or deletion may cause an imbalance in an AVL tree.**

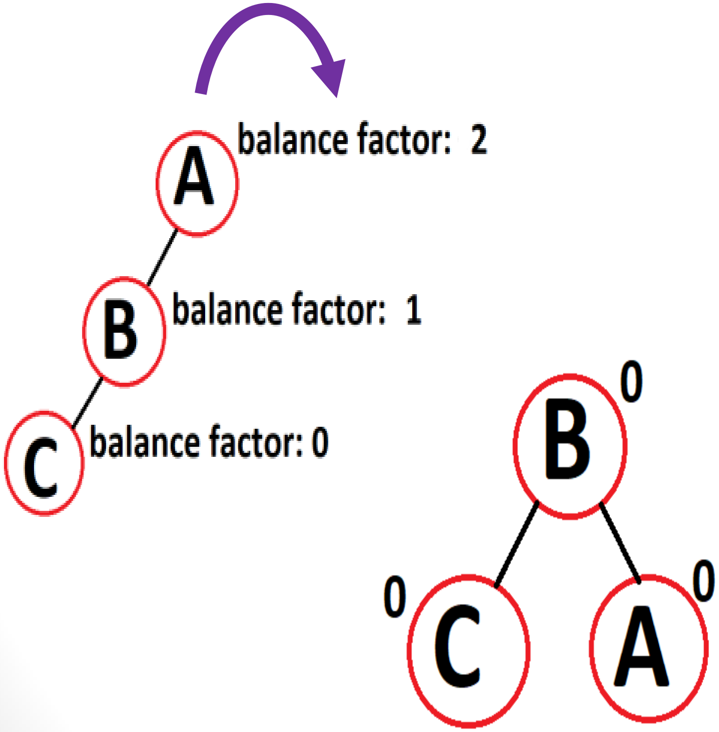
- The deepest node, which is an ancestor of a deleted or an inserted node, and whose balance factor has changed to **-2 or +2** requires rotation to rebalance the tree.

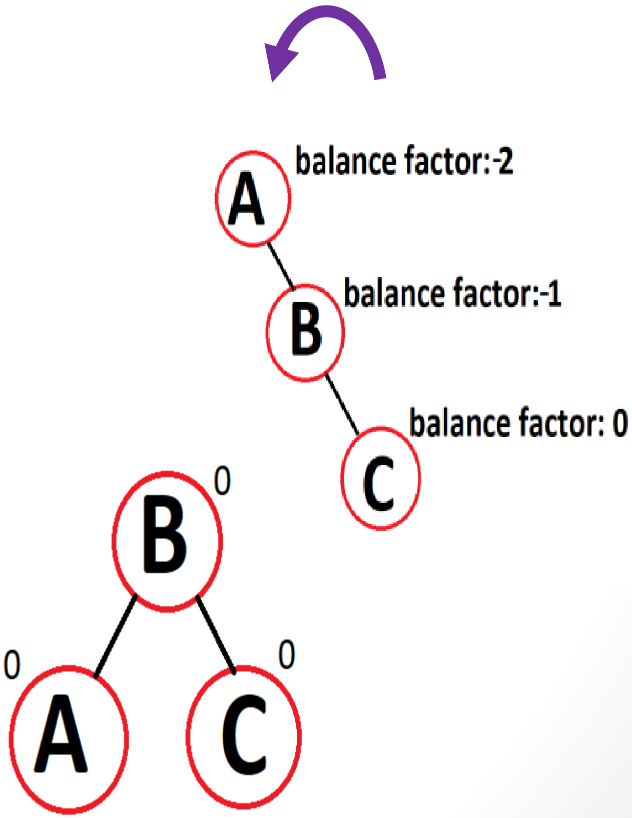


Single Rotation

- There are two kinds of single rotation:

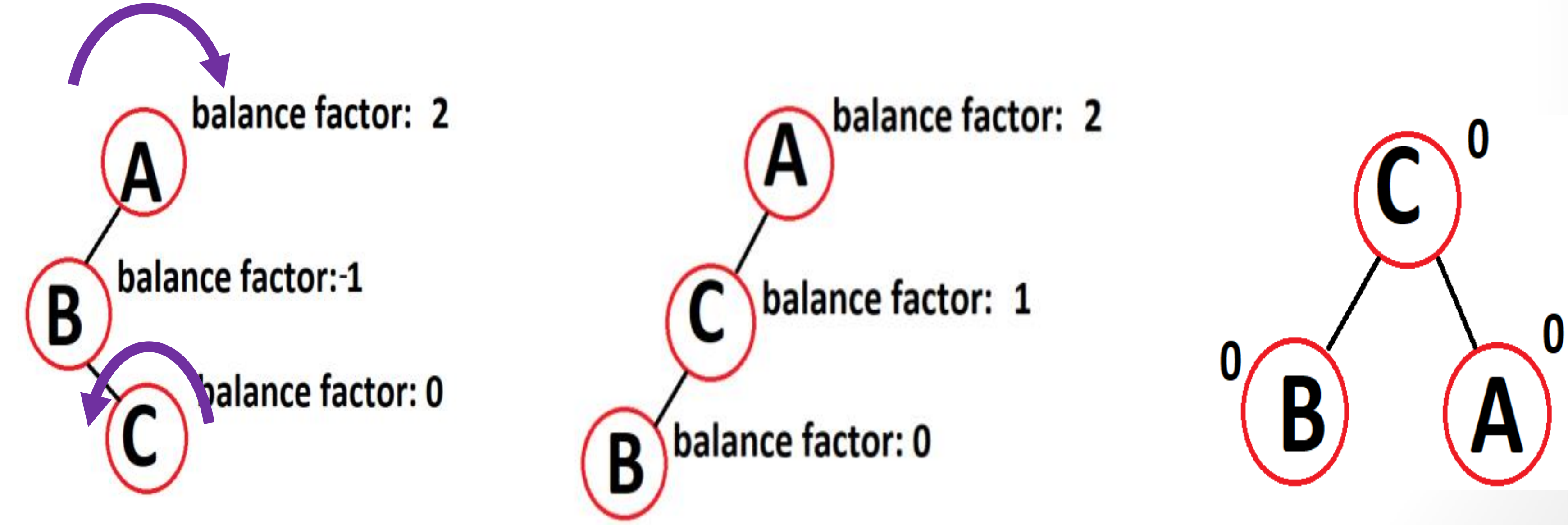
Right Rotation. **Left Rotation**.

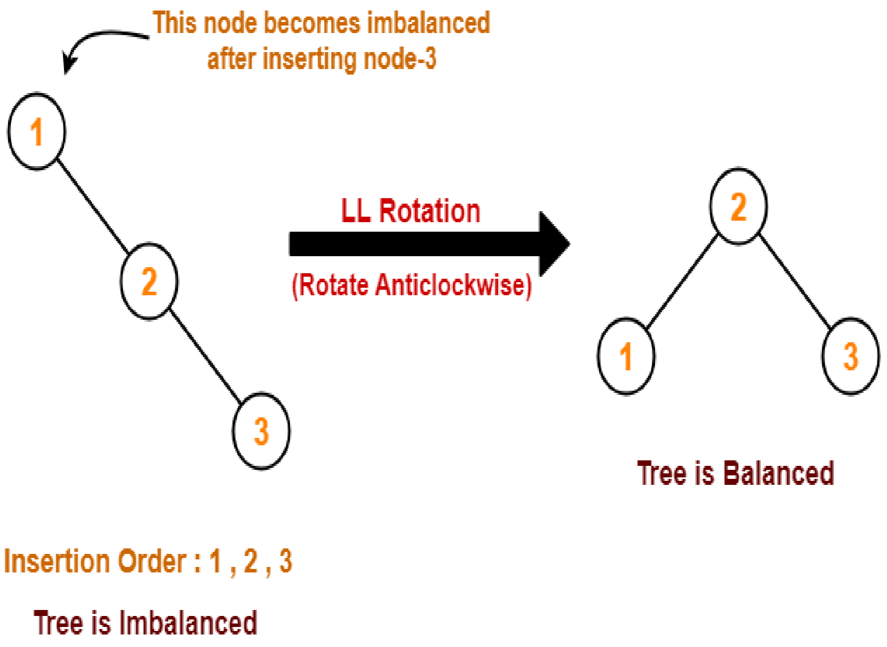




Double Rotation

- A double **right-left :**rotation is a **right rotation** followed by a **left rotation.**
- A double **left-right :**rotation is a **left rotation** followed by a **right rotation**.




This node becomes imbalanced after inserting node-3

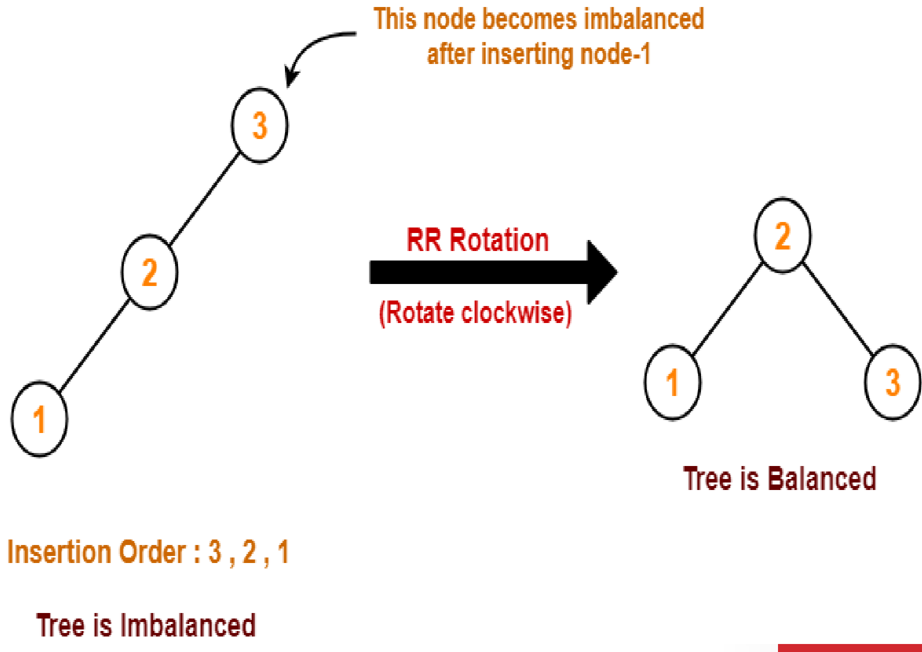
LL Rotation

(Rotate Anticlockwise)

Tree is Balanced

Insertion Order : 1 , 2 , 3

Tree is Imbalanced


This node becomes imbalanced after inserting node-1

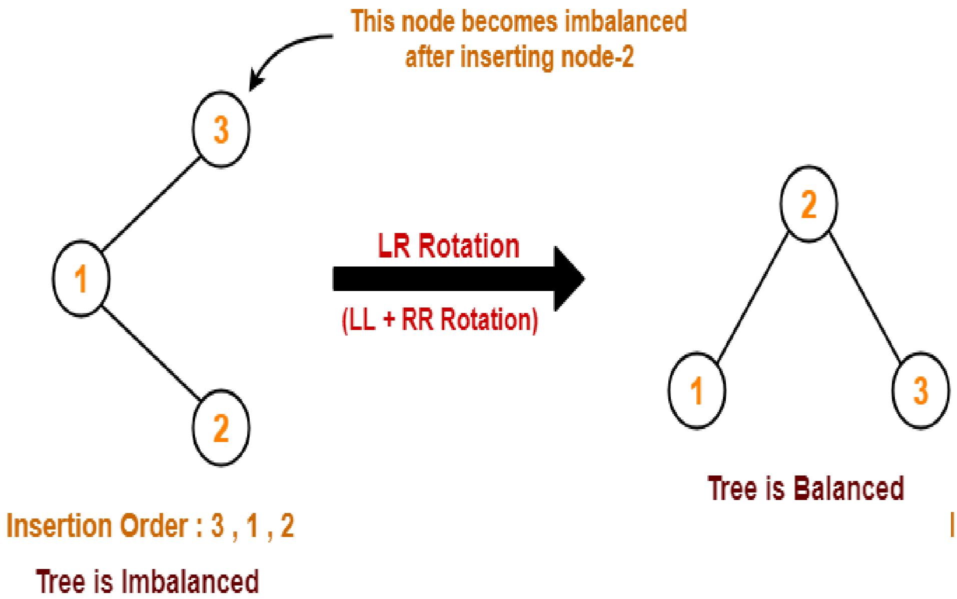
RR Rotation

(Rotate clockwise)

Tree is Balanced

Insertion Order : 3 , 2 , 1

Tree is Imbalanced


This node becomes imbalanced after inserting node-2

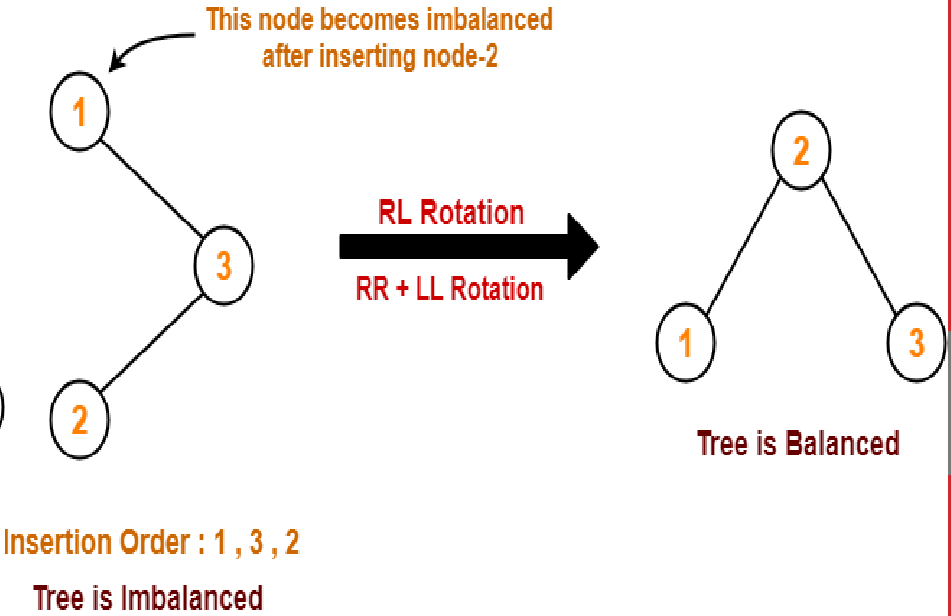
LR Rotation

(LL + RR Rotation)

Tree is Balanced

Insertion Order : 3 , 1 , 2

Tree is Imbalanced


This node becomes imbalanced after inserting node-2

RL Rotation

RR + LL Rotation

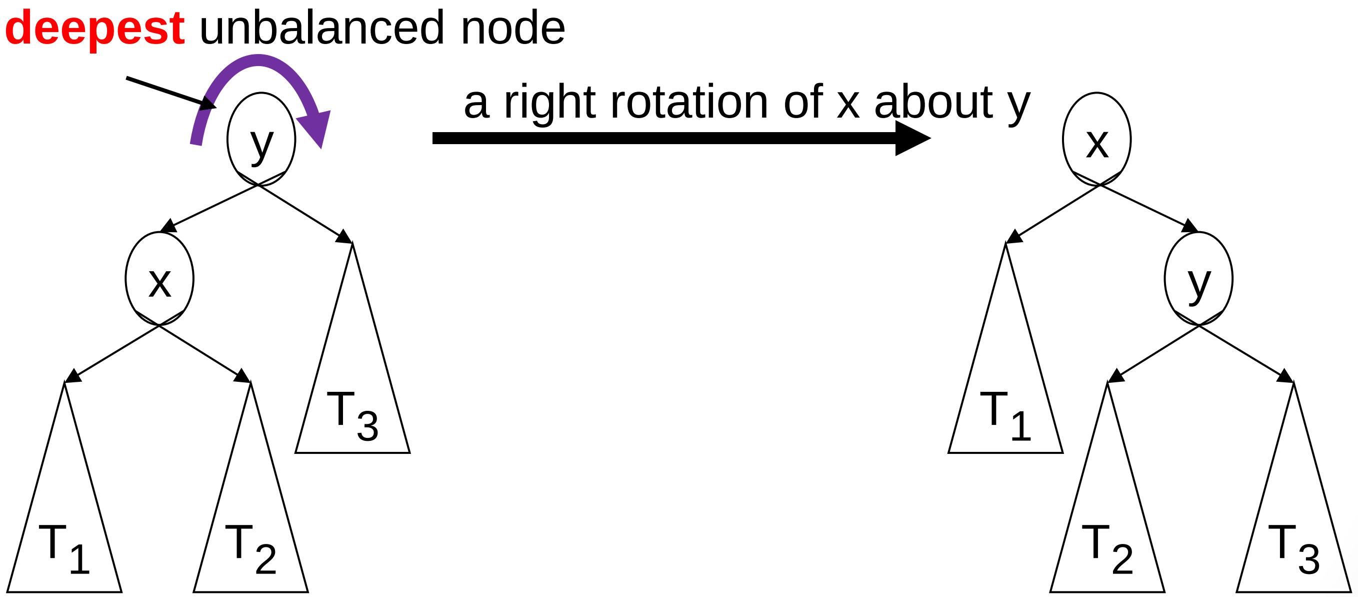
Tree is Balanced

Insertion Order : 1 , 3 , 2

Tree is Imbalanced

Single Right Rotation

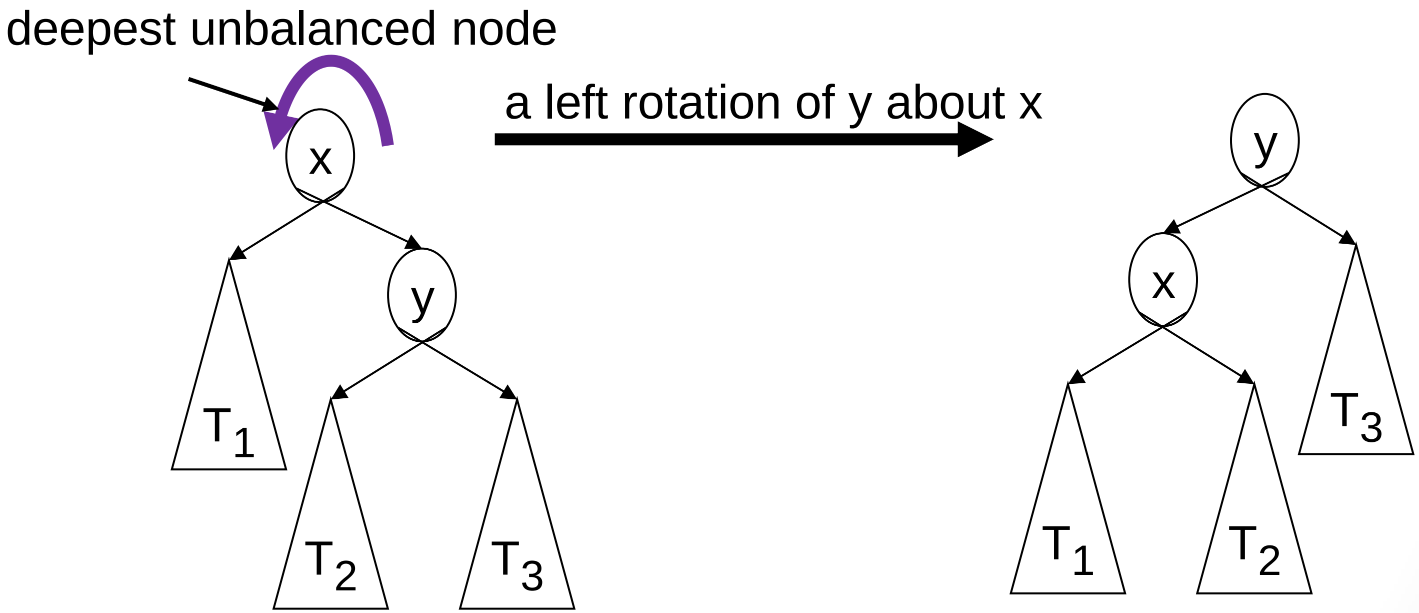
- Single right rotation:
  - The left child x of a node y becomes y's parent.
  - y becomes the right child of x.
  - The right child T_2 of x, **if any**, becomes the left child of y.



Note: The **pivot** of the rotation is the deepest unbalanced node

Single Left Rotation

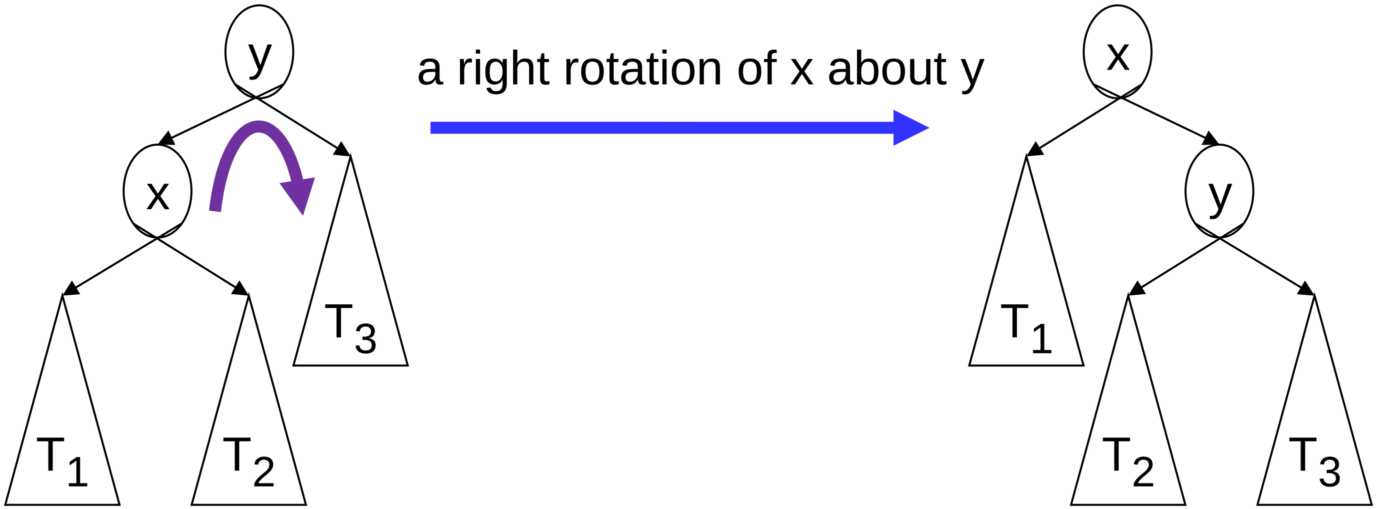
- Single left rotation:
  - The right child y of a node x becomes x's parent.
  - x becomes the left child of y.
  - The left child T_2 of y, **if any**, becomes the right child of x.



Note: The **pivot** of the rotation is the deepest unbalanced node

BST ordering property

- A rotation does not affect the ordering property of a BST.



BST ordering property requirement:

$T_1 < x < y$

$x < T_2 < y$

$x < y < T_3$

Similar

BST ordering property requirement:

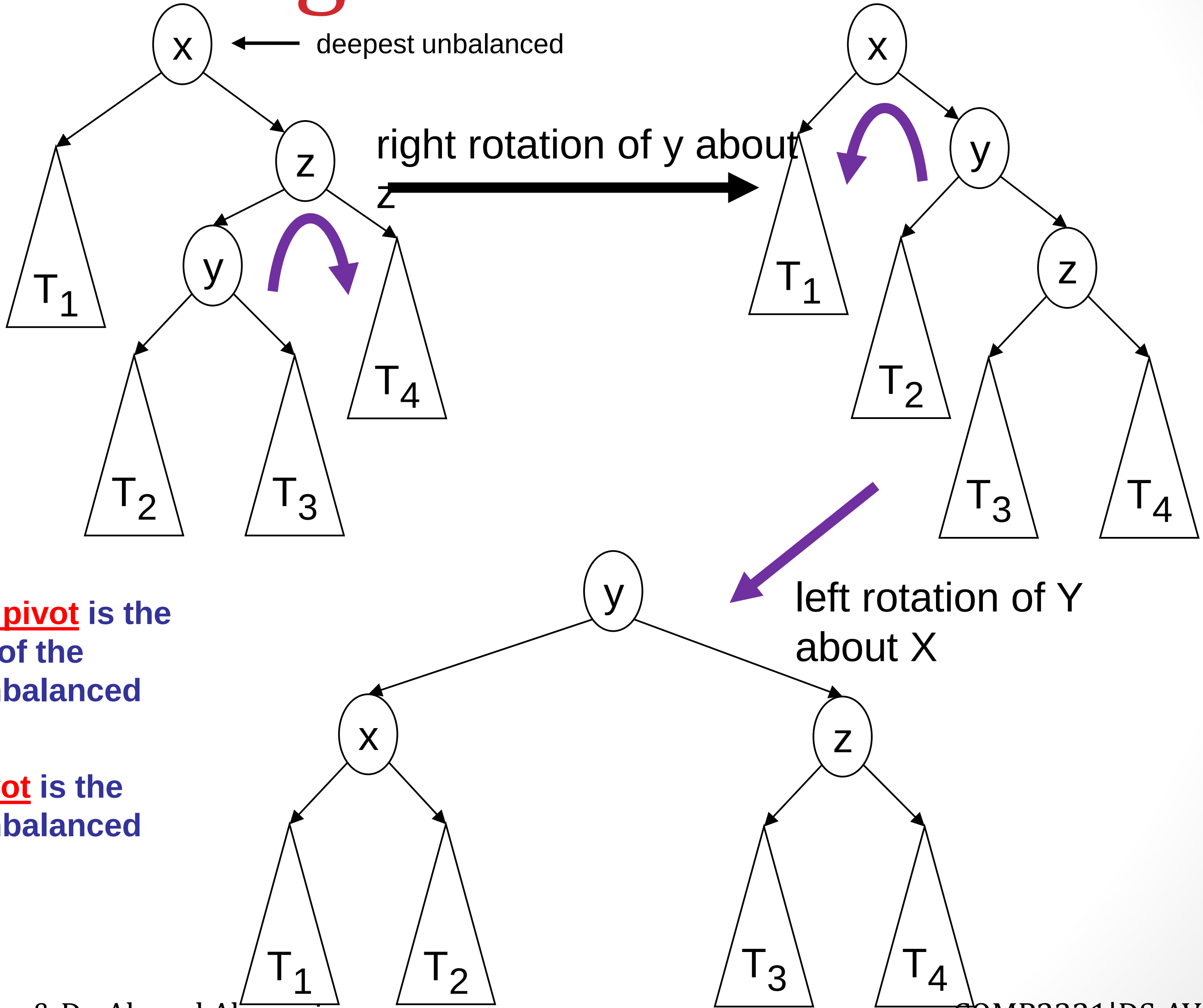
$T_1 < x < y$

$x < T_2 < y$

$x < y < T_3$

- Similarly for a left rotation.

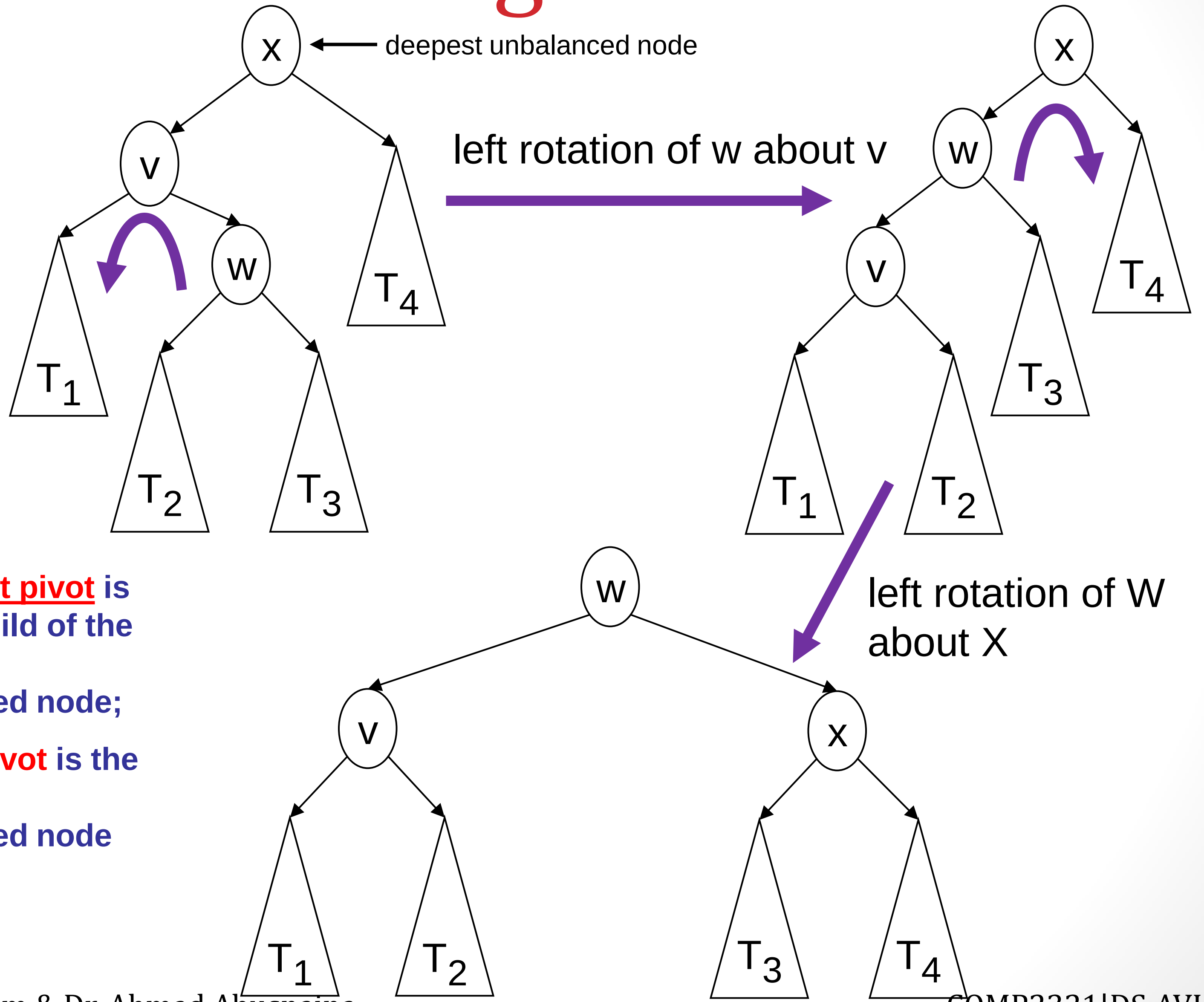
Double Right-Left Rotation



Note: First pivot is the right child of the deepest unbalanced node;

second pivot is the deepest unbalanced node

Double Left-Right Rotation



deepest unbalanced node

left rotation of w about v

left rotation of W about X

Note: <u>First pivot</u> is the left child of the deepest unbalanced node;

second pivot is the deepest unbalanced node

AVL Search Trees

- Inserting in an AVL tree
- Insertion implementation
- Deleting from an AVL tree

Insertion

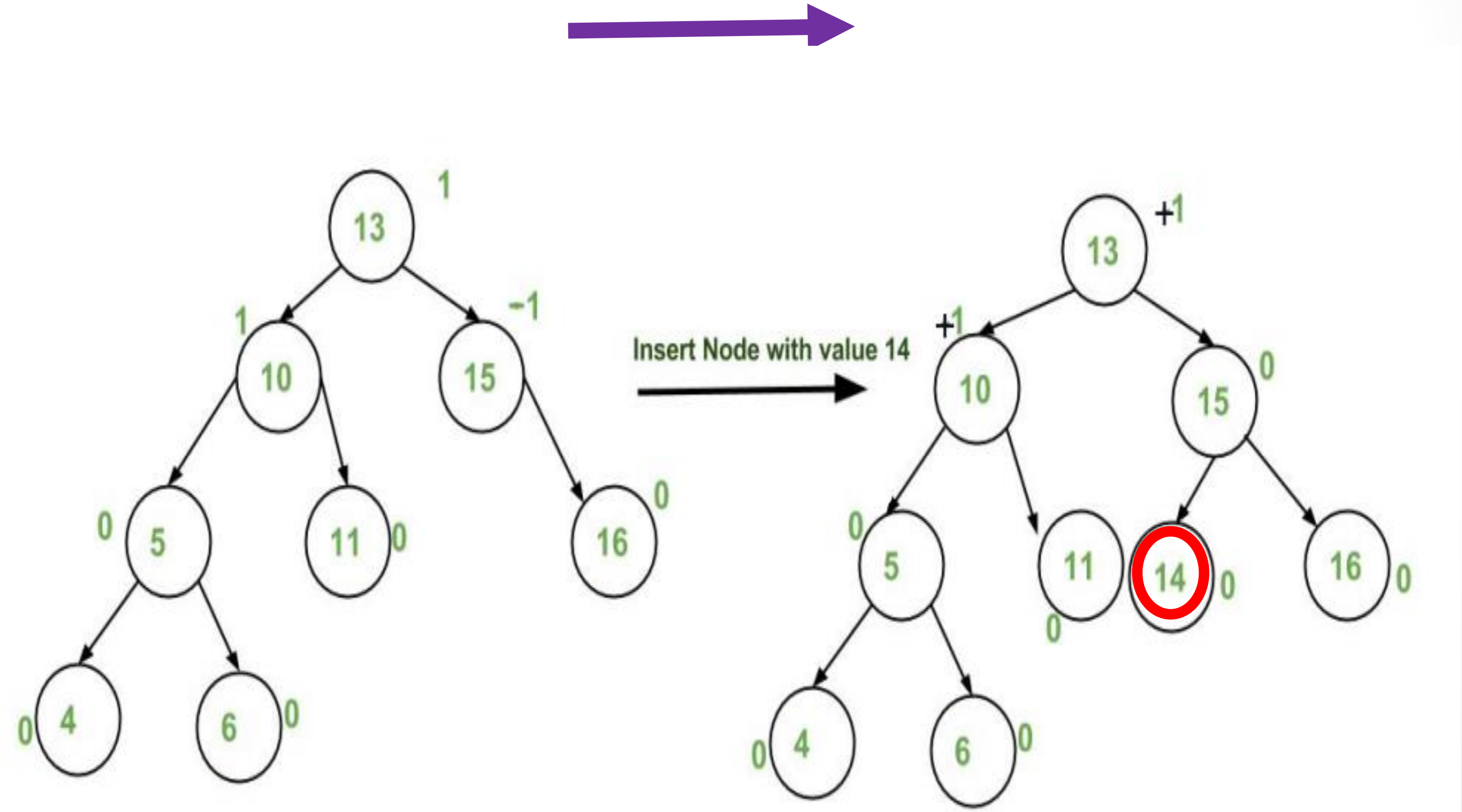
- Insert using a BST insertion algorithm.
- Rebalance the tree if an imbalance occurs.
- An imbalance occurs if a node's balance factor changes from -1 to -2 or from+1 to +2.
- Rebalancing is done at the deepest unbalanced ancestor of the inserted node.

- **There are three insertion cases:**
  1. Insertion that does not cause an imbalance.
  2. Same side (**left-left** or **right-right**) insertion that causes an imbalance.
     - Requires a single rotation to rebalance.
  3. Opposite side **(left-right or right-left)** insertion that causes an imbalance.
     - Requires a double rotation to rebalance.

Insertion: case 1

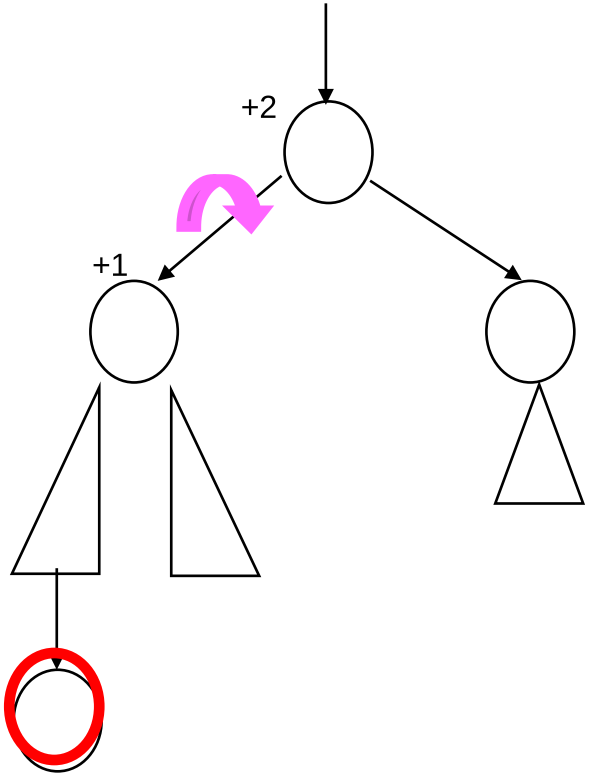
- Example: An insertion that does not cause an imbalance.

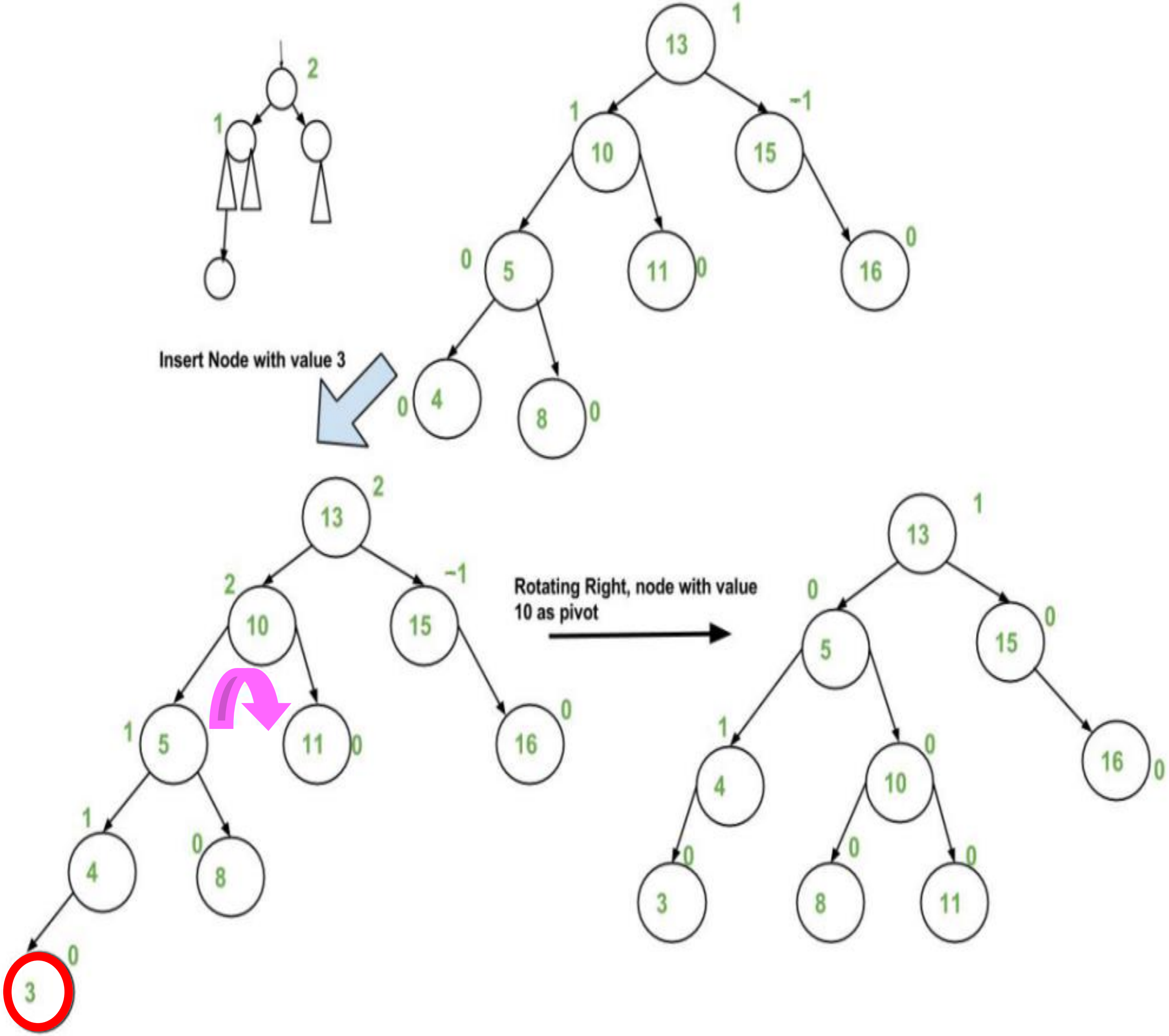
Insert 14



Insertion: case 2

- **Case 2a**: The lowest node (with a balance factor of -2) had a taller left-subtree and the insertion was on the left-subtree of its left child.
- Requires single right rotation to rebalance.

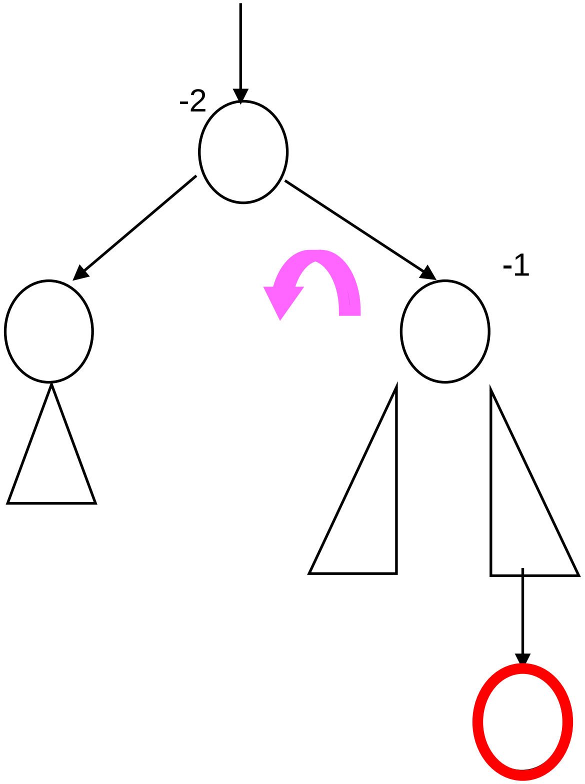



Insert Node with value 3

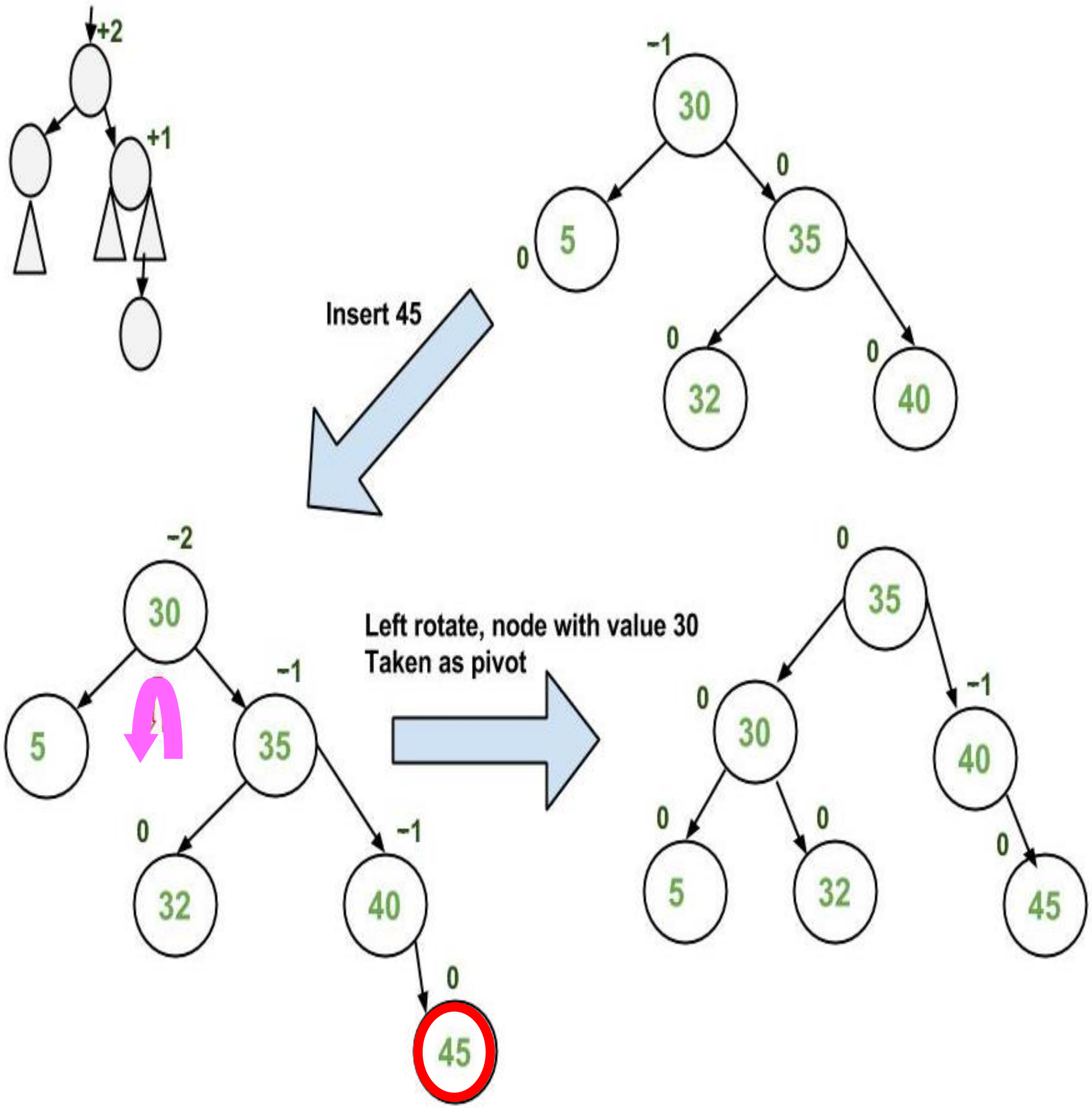
Rotating Right, node with value 10 as pivot

Insertion: case 2 (contd)

- Case 2b: The lowest node (with a balance factor of +2) had a taller right-subtree and the insertion was on the right-subtree of its right child.
- Requires single left rotation to rebalance.

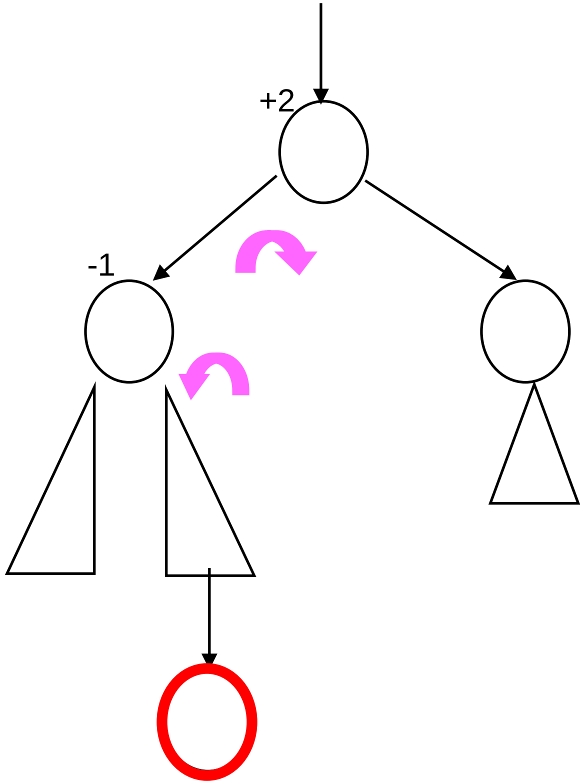


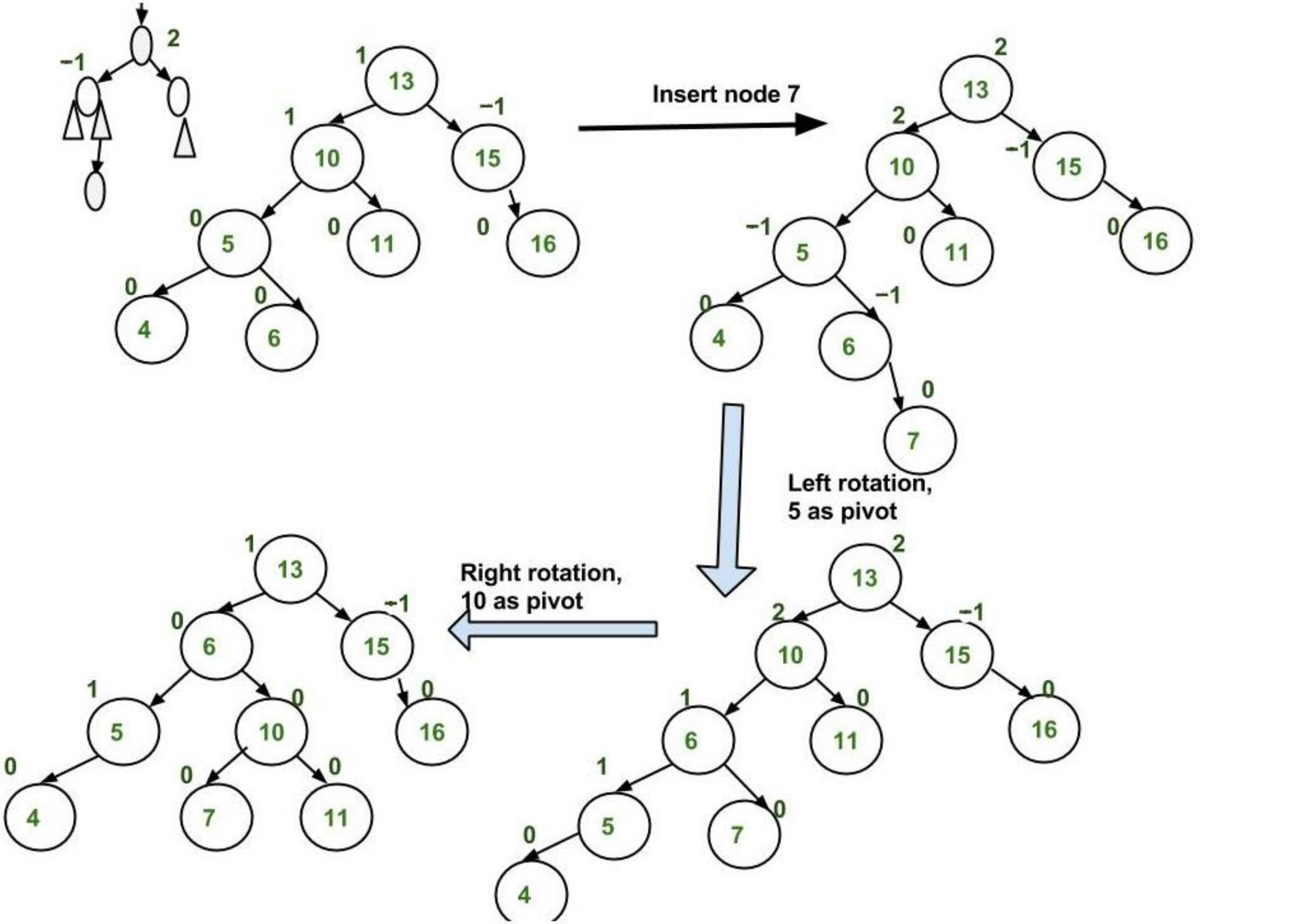
Example


+2
+1
Insert 45
-1
30
0
5
0
35
0
32
0
40
-2
30
5
-1
35
0
32
-1
40
0
45
Left rotate, node with value 30
Taken as pivot
0
35
0
30
-1
40
0
5
0
32
0
45

Insertion: case 3

- Case 3a: The lowest node (with a balance factor of -2) had a taller left-subtree and the insertion was on the right-subtree of its left child.
- Requires a double left-right rotation to rebalance.





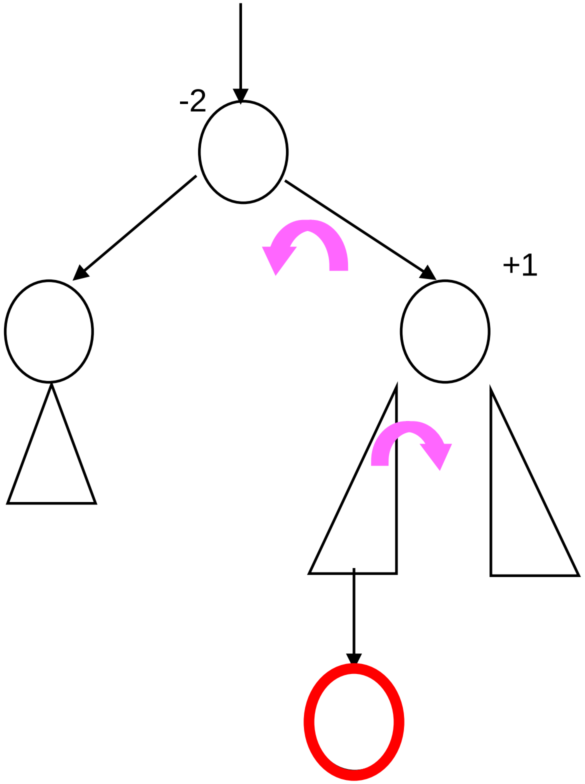
Insert node 7

Left rotation,
5 as pivot

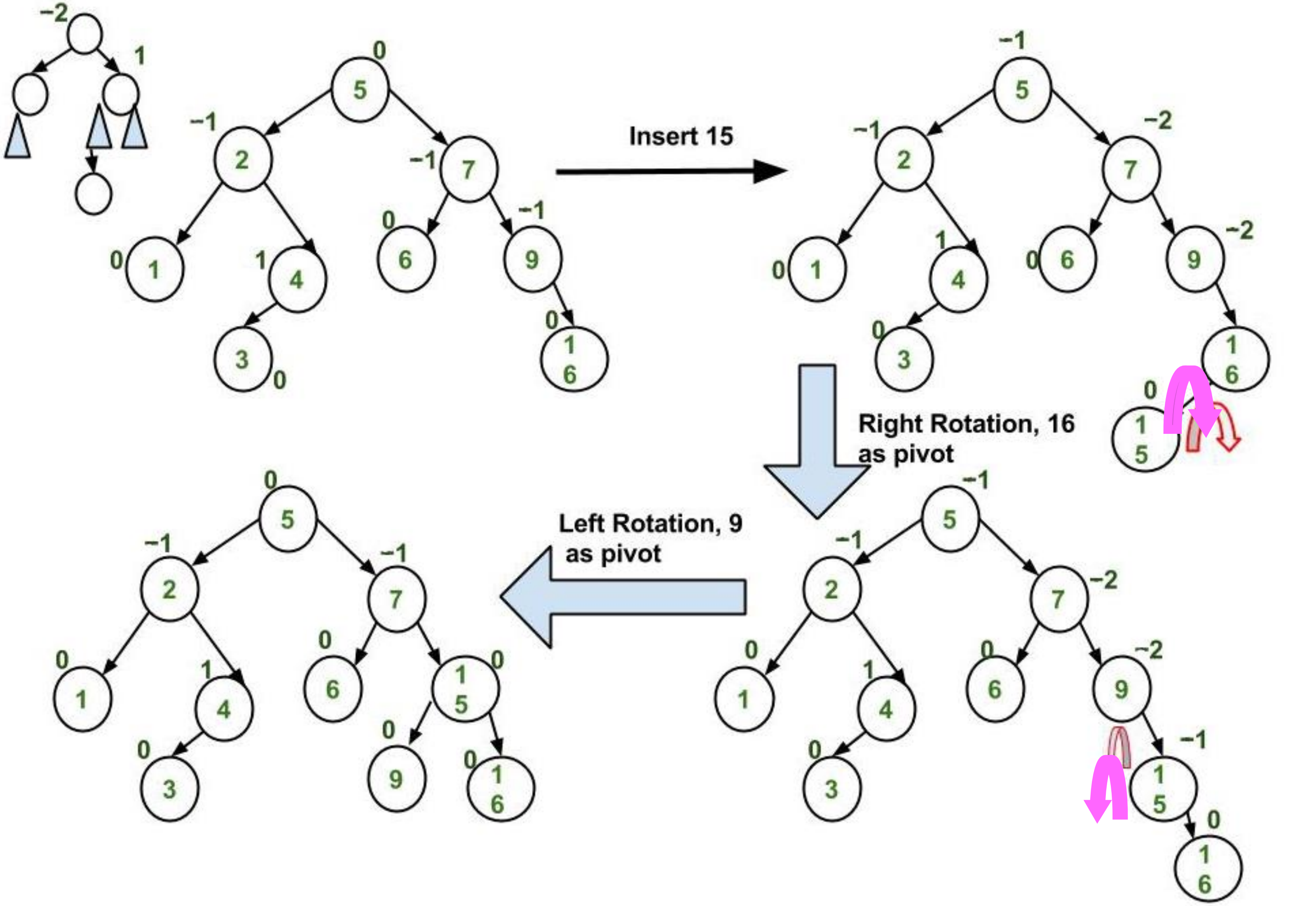
Right rotation,
10 as pivot

Insertion: case 3 (contd)

- Case 3b: The lowest node (with a balance factor of +2) had a taller right-subtree and the insertion was on the left-subtree of its right child.
- Requires a double right-left rotation to rebalance.



Example

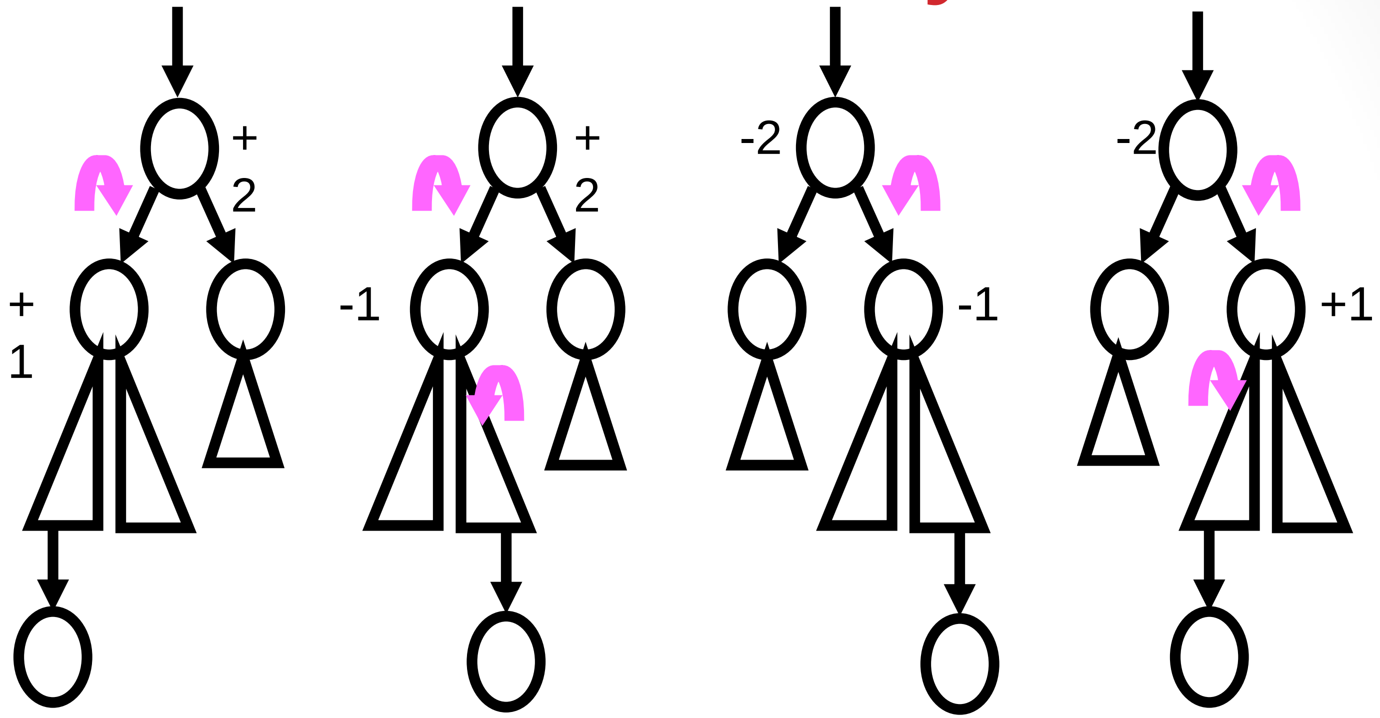


Insert 15

Right Rotation, 16 as pivot

Left Rotation, 9 as pivot

AVL Rotation Summary



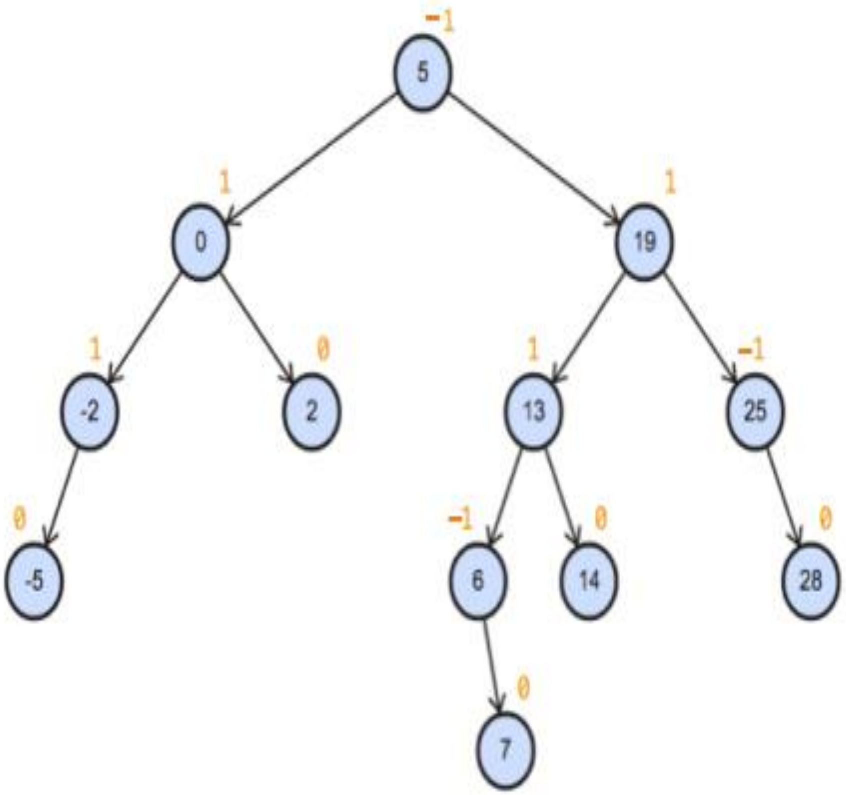
Single right rotation

Double left-right rotation

Single left rotation

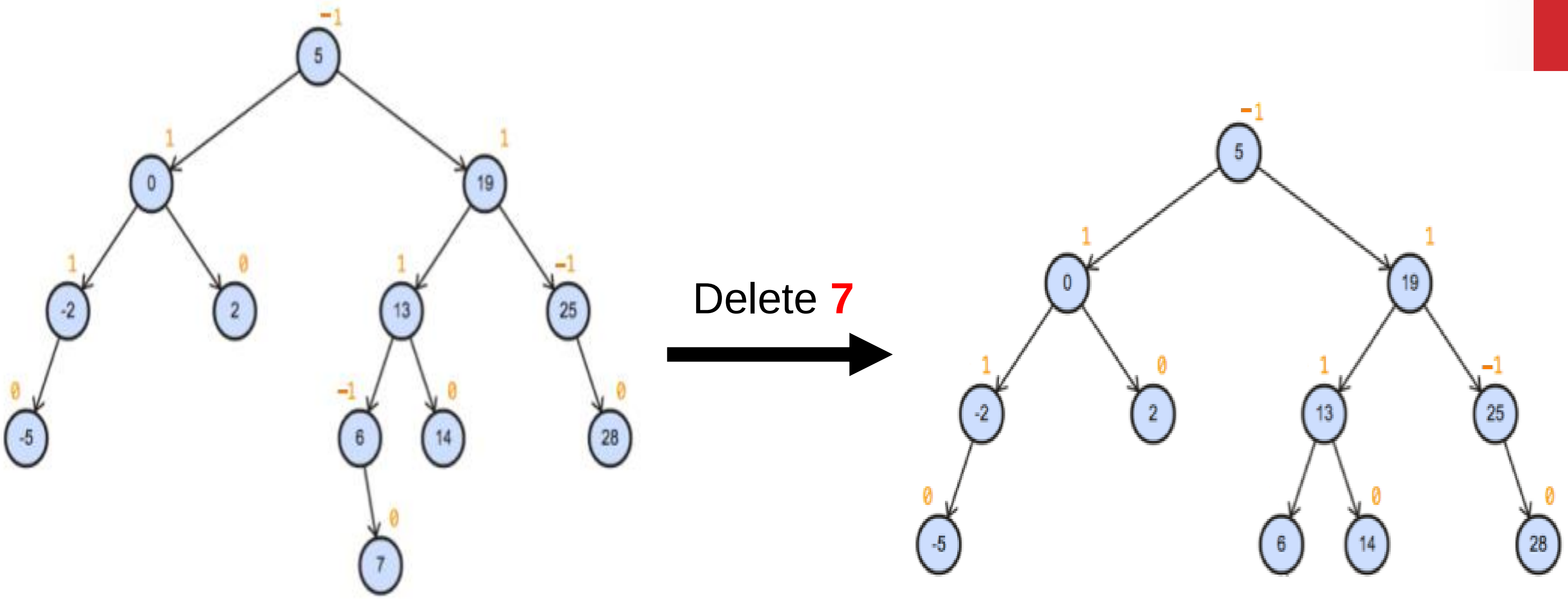
Double right-left rotation

Exercise: Insert into an initially empty AVL tree each of the following keys, in the order in which they appear in the sequence: **0, 25, 19, 5, -2, 28, 13, -5, 2, 6, 14, 7.**



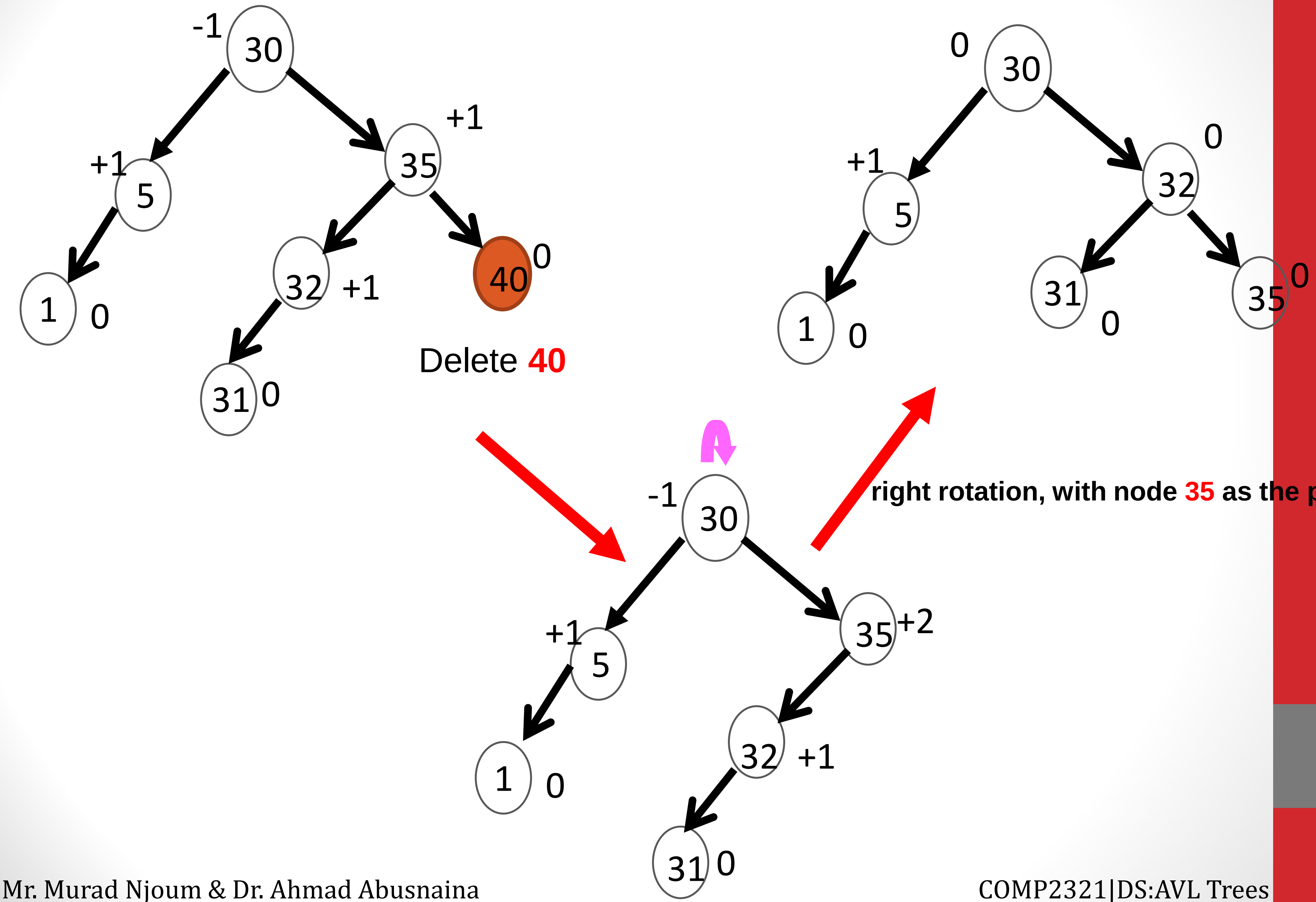
Deletion

- Delete by a BST deletion by copying algorithm.
- Rebalance the tree if an imbalance occurs.
- There are three deletion cases:
  1. **Deletion that does not cause an imbalance.**
  2. **Deletion that requires a single rotation to rebalance.**
  3. **Deletion that requires two or more rotations to rebalance.**
- Deletion case 1 example:



Delete 7

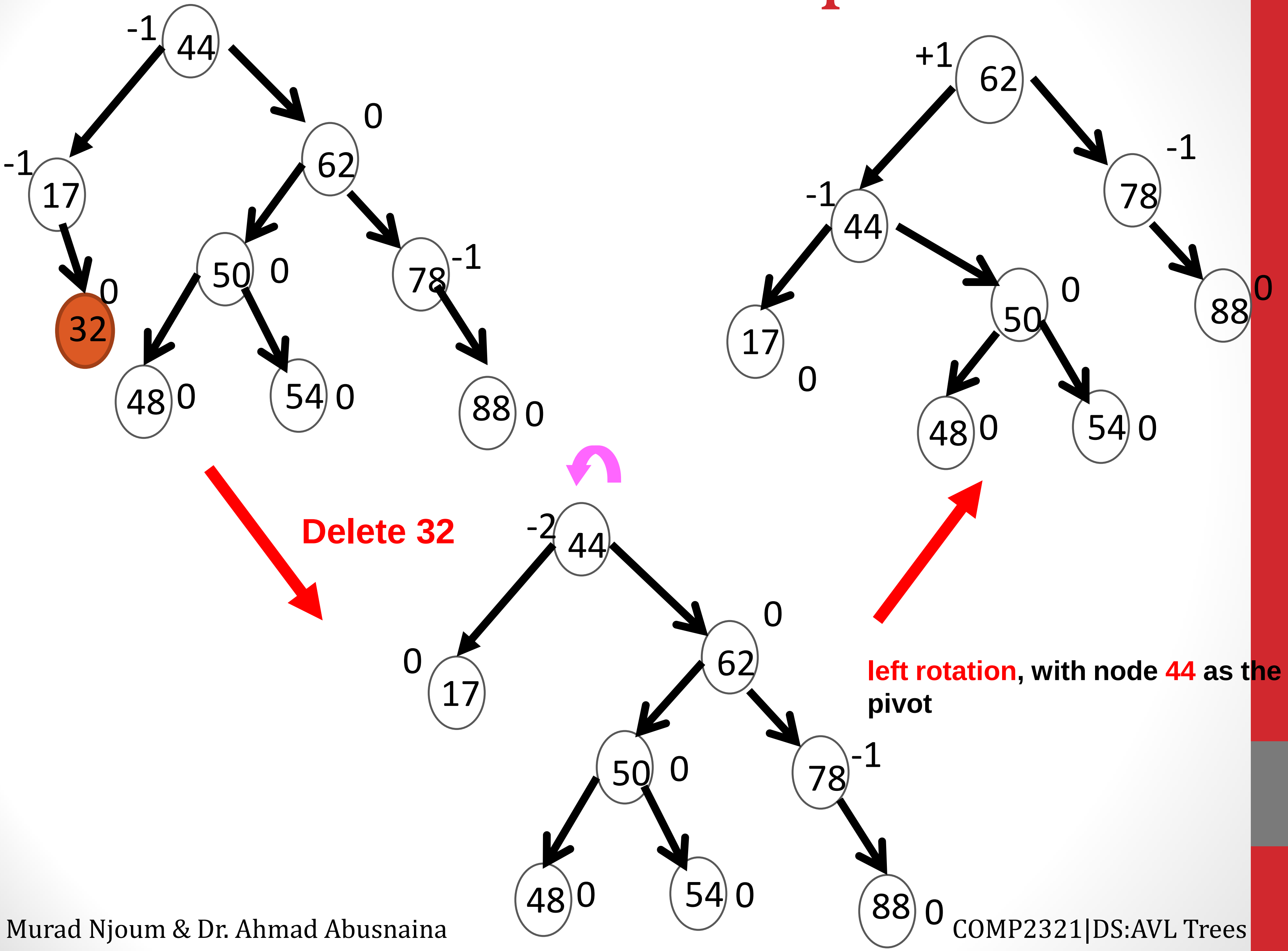
Deletion: case 2 examples



Delete **40**

right rotation, with node **35** as the pivot

Deletion: case 2 examples

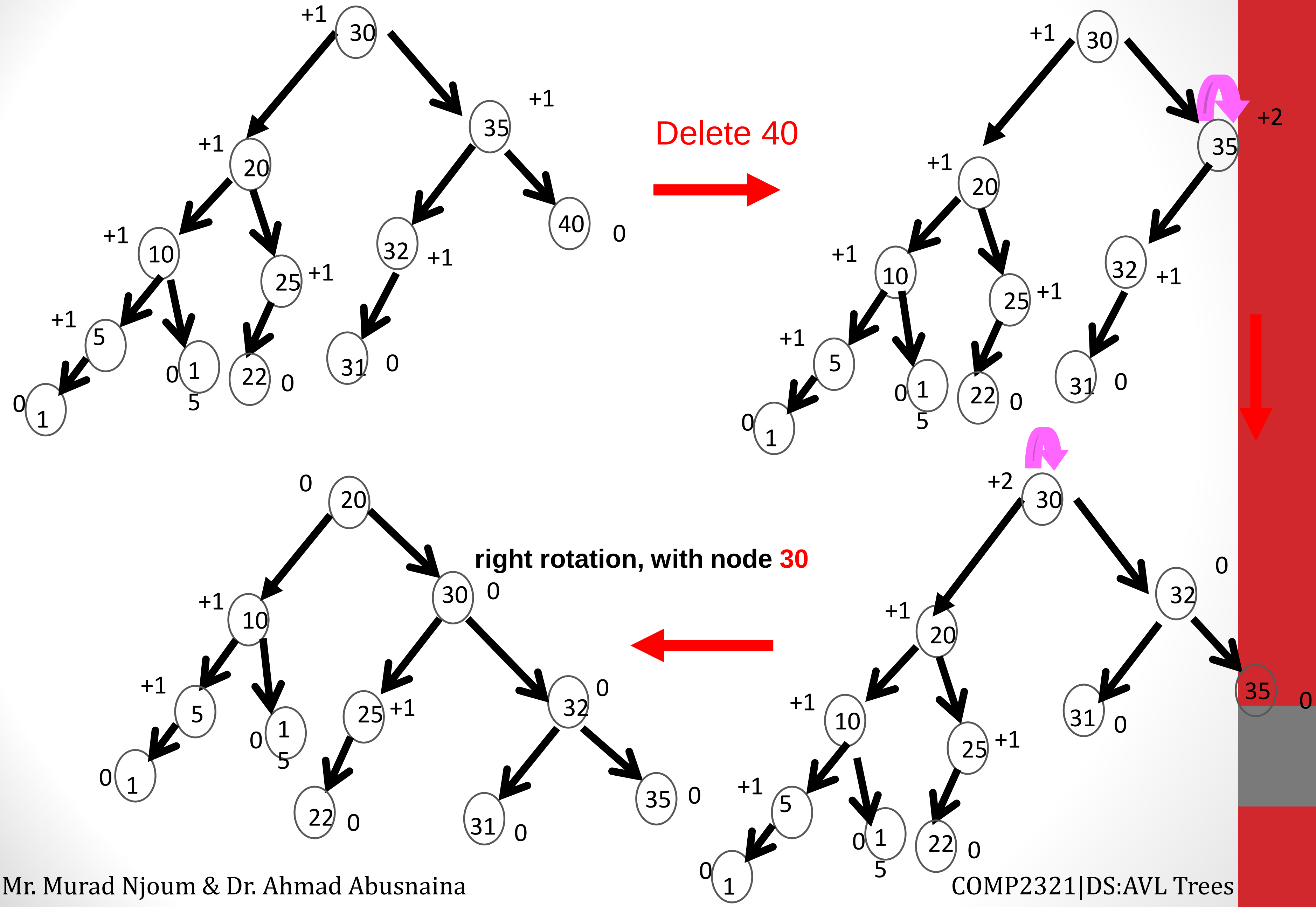


Delete 32

left rotation, with node 44 as the pivot

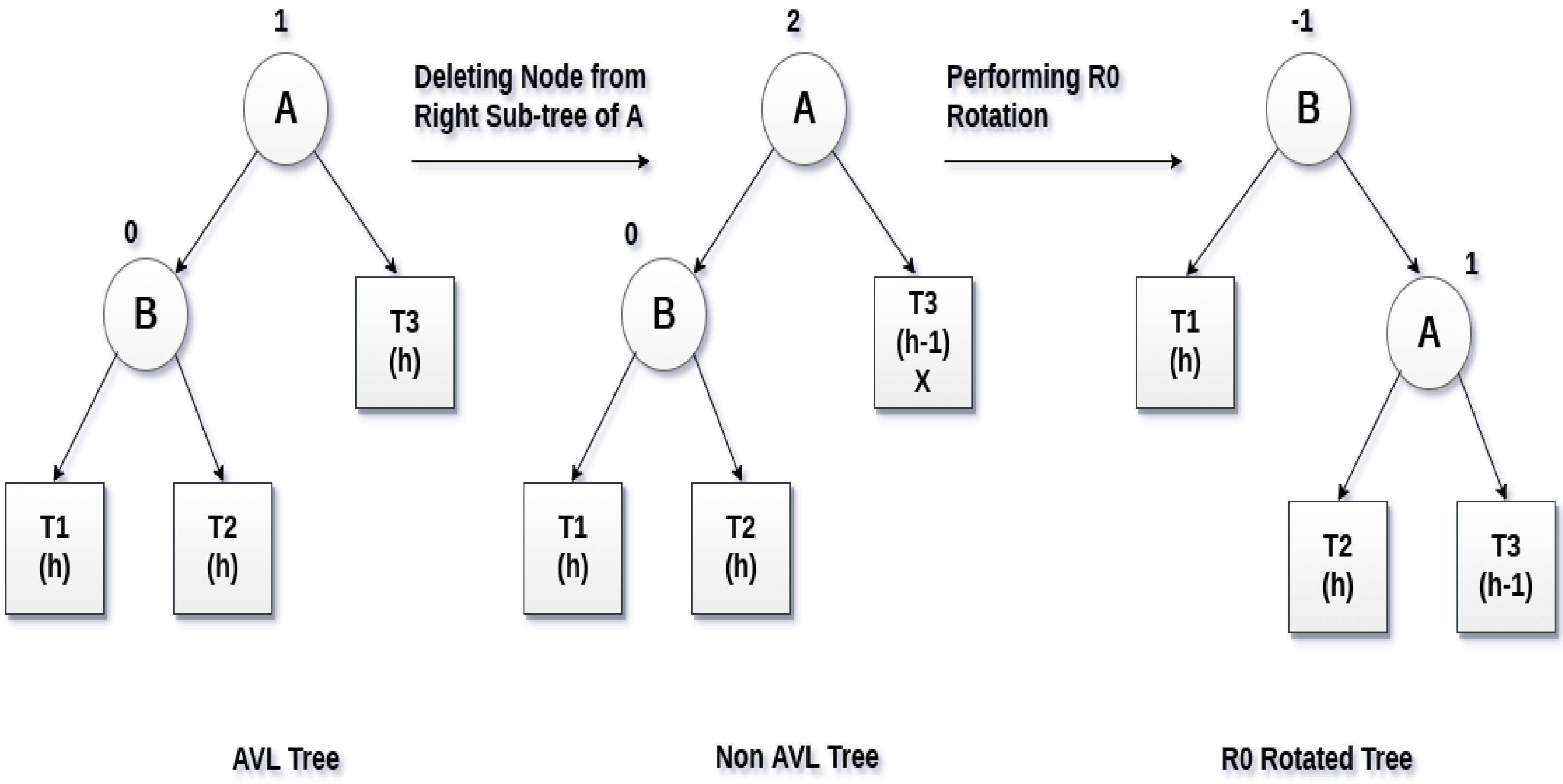
Deletion: case 3

right rotation, with node **35**

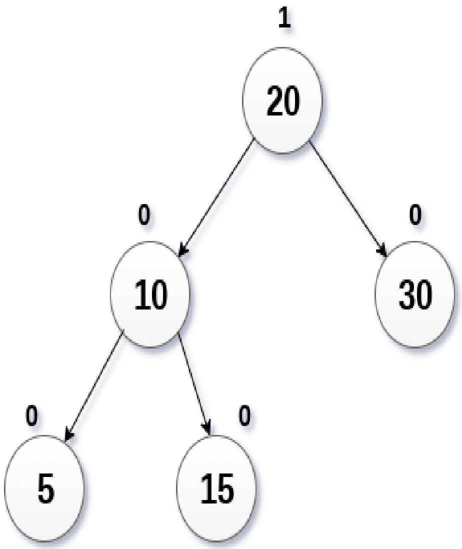


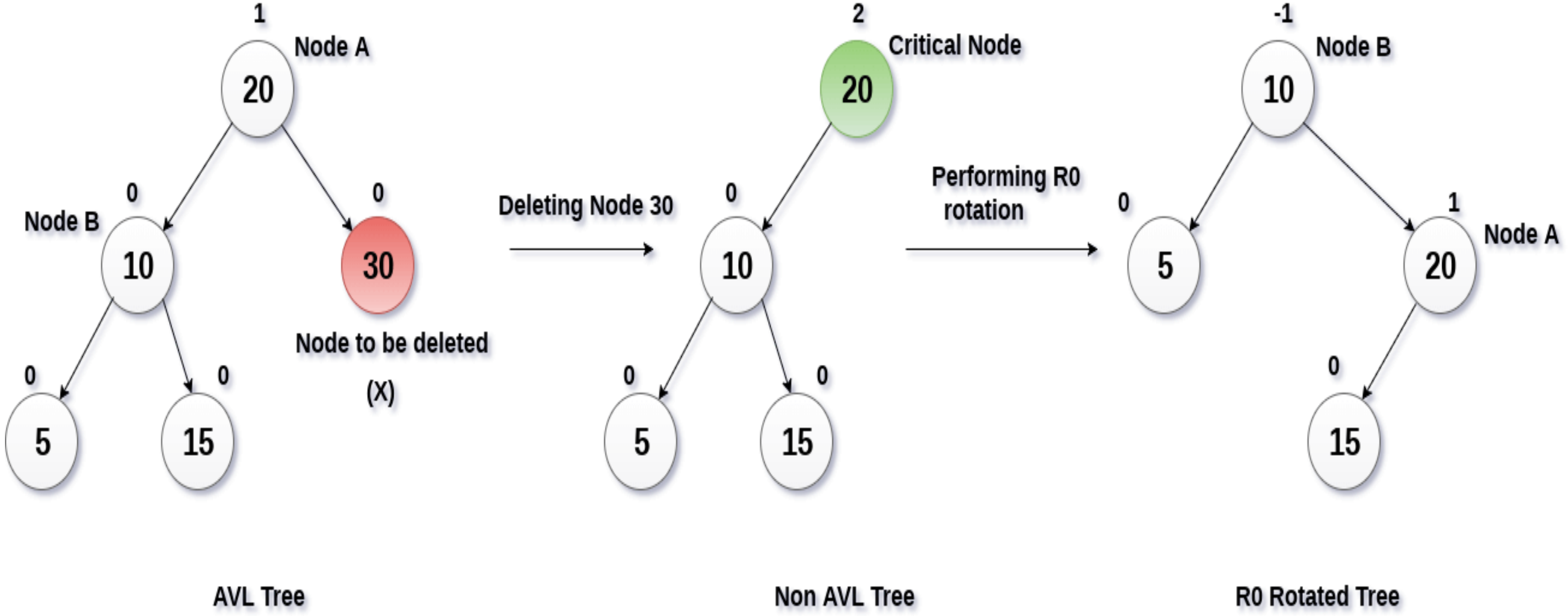
right rotation, with node **30**

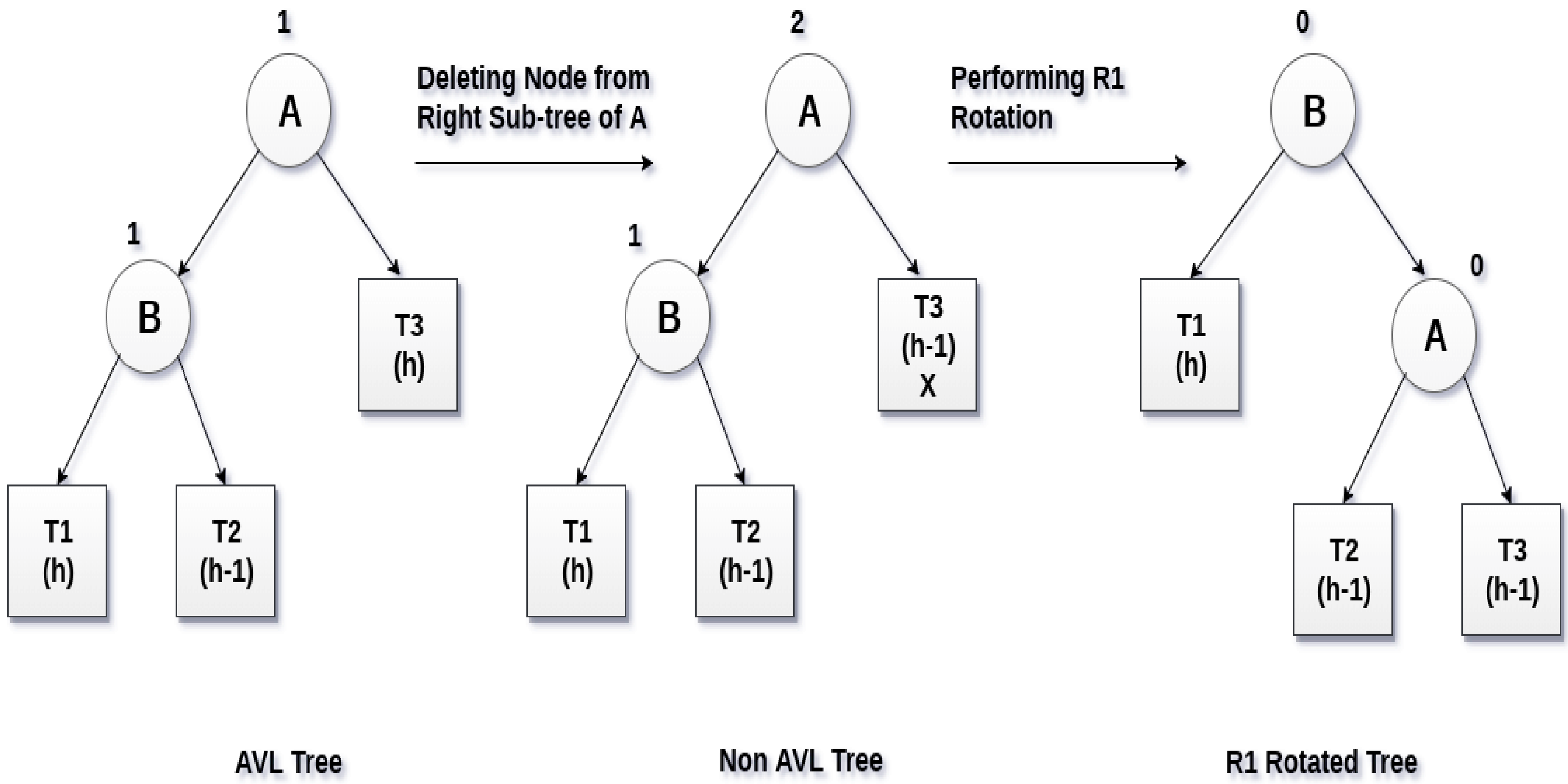
Deletion- In Depth- More Examples



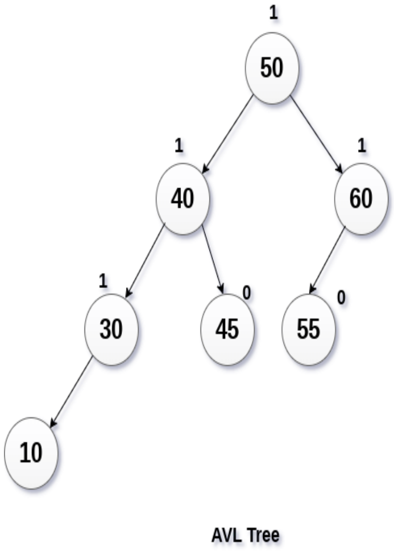
Example 1

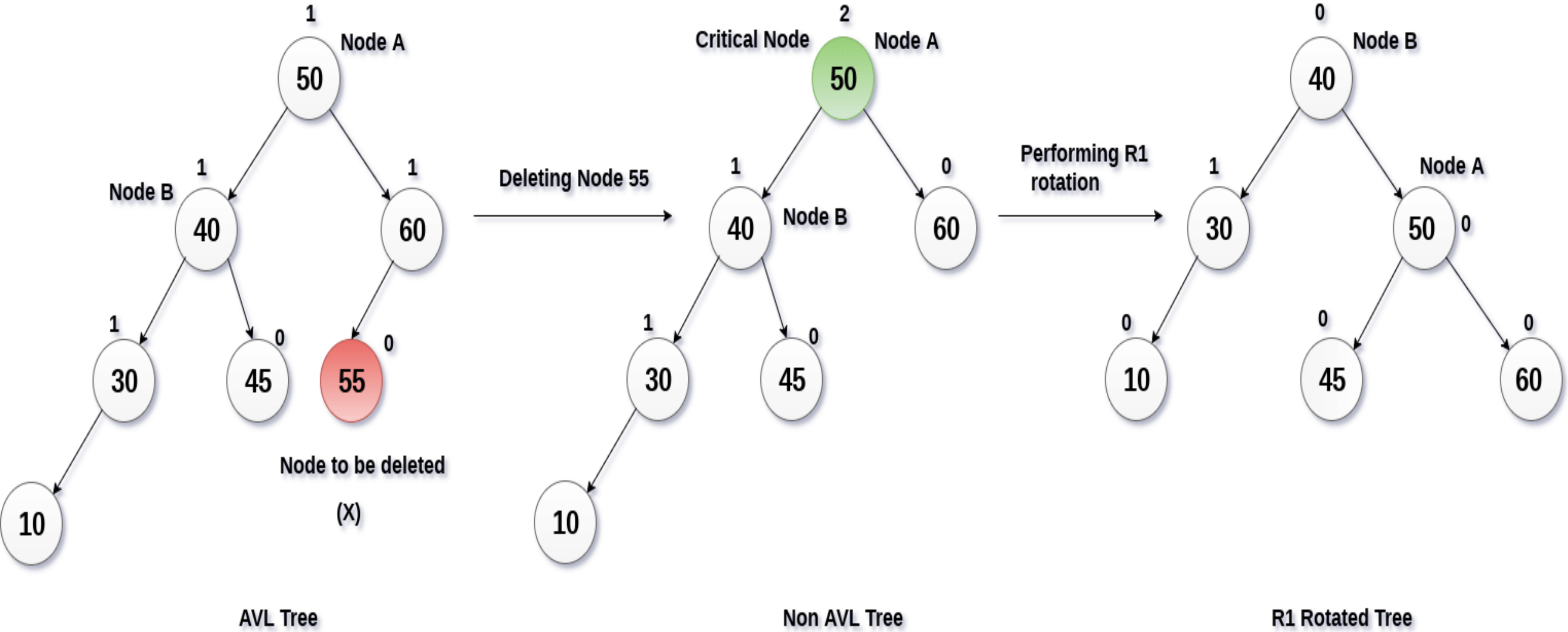

1
20
0
10
0
30
0
5
0
15

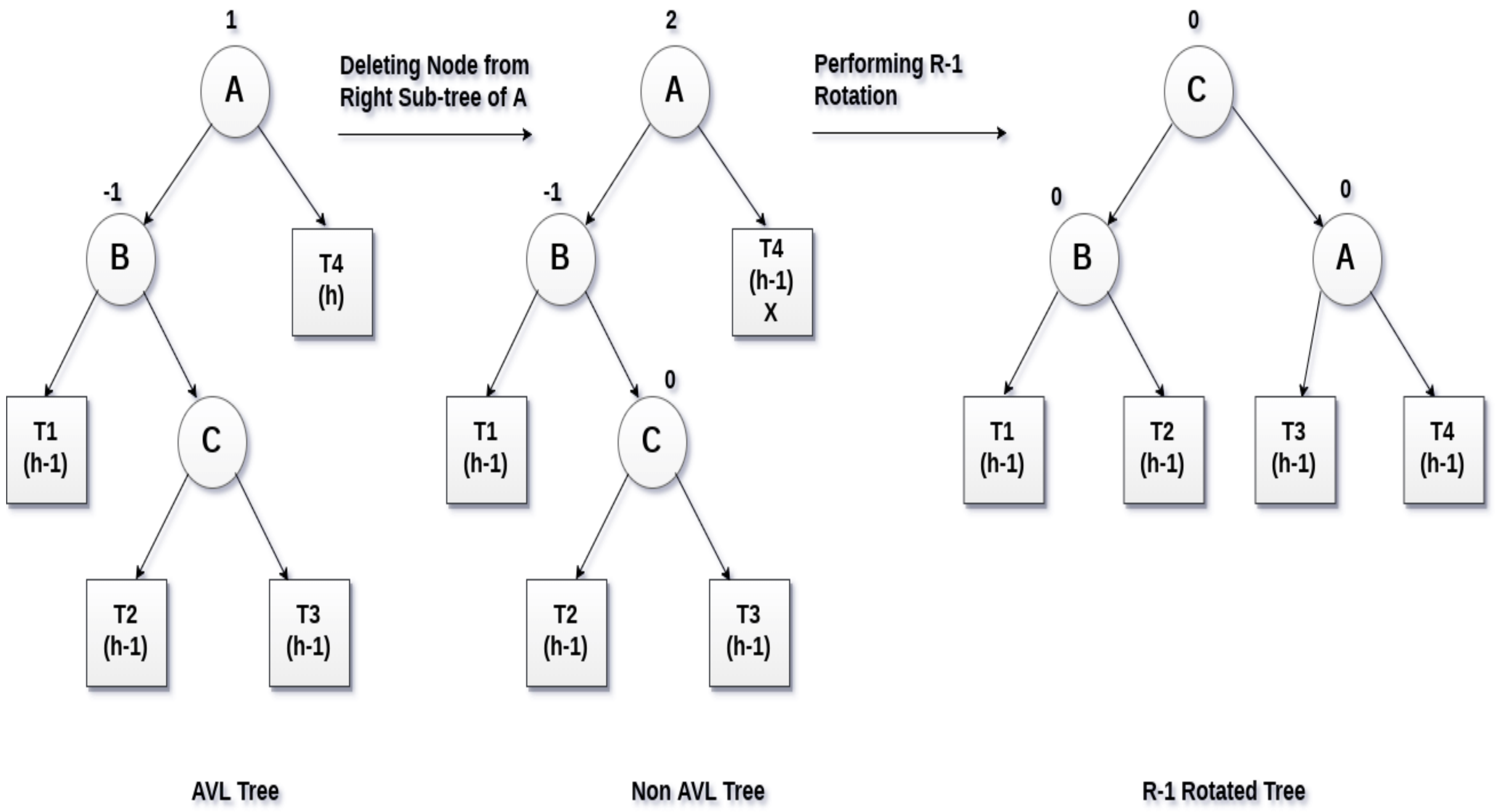

1
Node A
20
Node B
0
10
0
30
Node to be deleted
(X)
0
5
0
15
AVL Tree
Deleting Node 30
2
Critical Node
20
0
10
0
5
0
15
Non AVL Tree
Performing R0
rotation
-1
Node B
10
0
5
1
Node A
20
0
15
R0 Rotated Tree


1
A
Deleting Node from
Right Sub-tree of A
2
A
Performing R1
Rotation
0
B
1
B
T3
(h)
1
B
T3
(h-1)
X
T1
(h)
0
A
T1
(h)
T2
(h-1)
T1
(h)
T2
(h-1)
T2
(h-1)
T3
(h-1)
AVL Tree
Non AVL Tree
R1 Rotated Tree

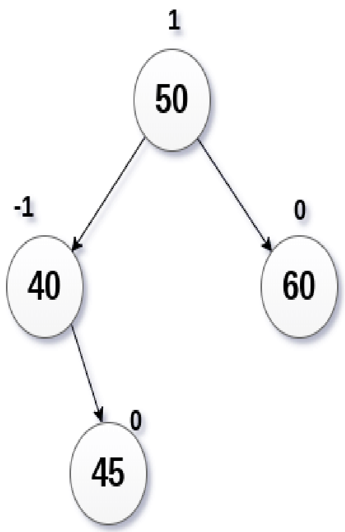
Example 2

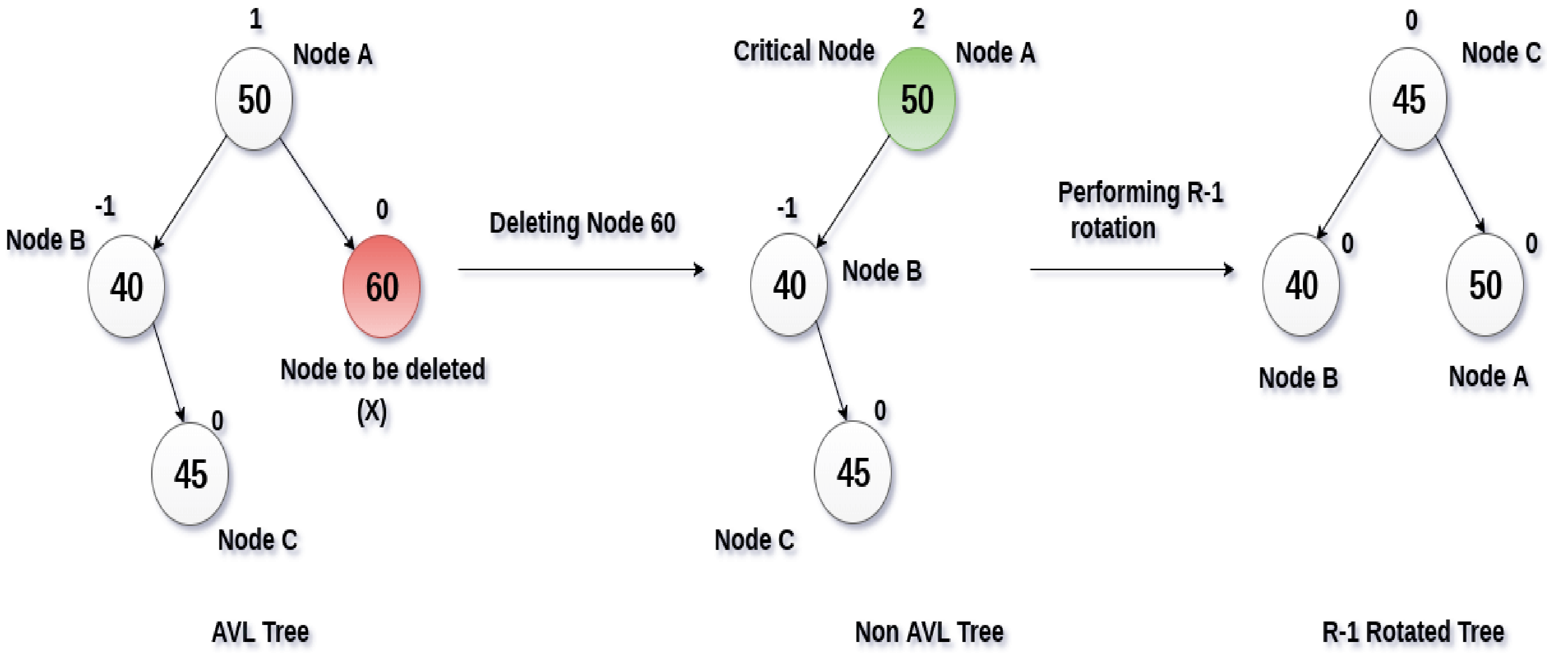

1
50
1
40
1
60
1
30
0
45
0
55
10
AVL Tree


1
Node A
50
Node B
1
40
1
60
1
30
0
45
0
55
10
Node to be deleted
(X)
AVL Tree
Deleting Node 55
Critical Node
2
Node A
50
1
40
Node B
0
60
1
30
0
45
10
Non AVL Tree
Performing R1
rotation
0
Node B
40
1
30
Node A
50
0
0
10
0
45
0
60
R1 Rotated Tree


1
A
Deleting Node from
Right Sub-tree of A
-1
B
T4
(h)
T1
(h-1)
C
T2
(h-1)
T3
(h-1)
AVL Tree
2
A
-1
B
T4
(h-1)
X
T1
(h-1)
0
C
T2
(h-1)
T3
(h-1)
Non AVL Tree
Performing R-1
Rotation
0
C
0
B
0
A
T1
(h-1)
T2
(h-1)
T3
(h-1)
T4
(h-1)
R-1 Rotated Tree

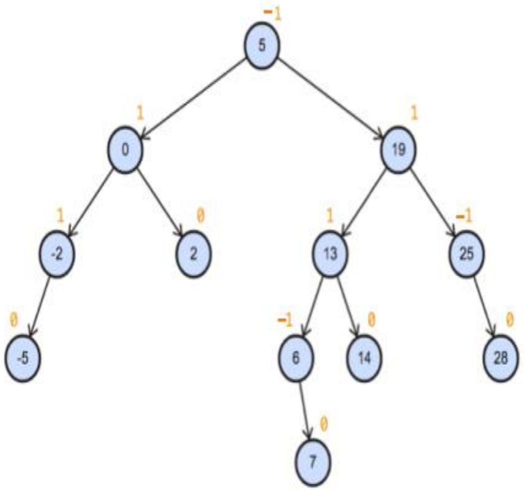
Example 3

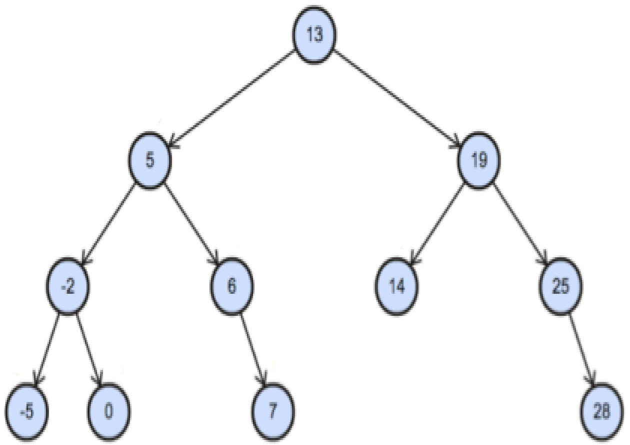

1
50
-1
0
40
60
0
45


1
Node A
50
-1
Node B
40
0
60
Node to be deleted
(X)
0
45
Node C
AVL Tree
Deleting Node 60
Critical Node
2
Node A
50
-1
40
Node B
0
45
Node C
Non AVL Tree
Performing R-1
rotation
0
Node C
45
40
0
50
0
Node B
Node A
R-1 Rotated Tree

Exercise (Previous Built AVL-Tree) :

A- Delete node 2





B- Delete root

C- Delete node 7, then 2 (Try it at home)

Exercise

- Rewrite the above codes for delete nodes from tree.

- Insert the following Number in AVL tree

  {20,50,30,15,3,45,17,25,12,11,7,19,14,2}

  Then Delete Number {45,20,15,25}

  Show your works after each step (Check Balance)

THANK YOU