

COMP2421 – DATA STRUCTURES AND ALGORITHMS

Heaps (Priority Queue)

Dr. Radi Jarrar

Department of Computer Science

Birzeit University



Heaps (Priority Queue)

- Queues are implemented under the FIFO principle.
- The first element to come in, the first element to be served.
- However, there has to be a scheme for cases in which there are some priorities.

Heaps (Priority Queue)

- Take an example a queuing system for printers.
- The first print job to come in is the first to be printed.
- Imagine you are printing a 2-paper report but unfortunately there are a few jobs that have been enqueued before. Each of these jobs of thousands of pages!
- This means you have to wait until all jobs have been finished.

Heaps (Priority Queue)

- There has to be a method so that short jobs to finish as fast as possible. These jobs have higher precedence over jobs that have already been running.
- However, some (not short) jobs are still important and they should have higher precedence in some cases! The Heaps data structures is a special kind of queue (called Priority Queue).
- Heaps use Binary Search Tree (BST) implementation. This gives $O(\log n)$ average running time for both operations.

MinHeap / MaxHeap

- **MinHeap:** a Binary Search Tree such that the data in each node is less than or equal to the data in the nodes' children.
- In MinHeap, the root is the minimum value in the tree.
- In the MaxHeap, the root is the maximum value in the tree.

Heaps (Priority Queue)

- There are two basic operations in Heaps:
 1. Insert (enqueue)
 2. DeleteMin (or max) (dequeue): finds, return, and delete the minimum element in the priority queue

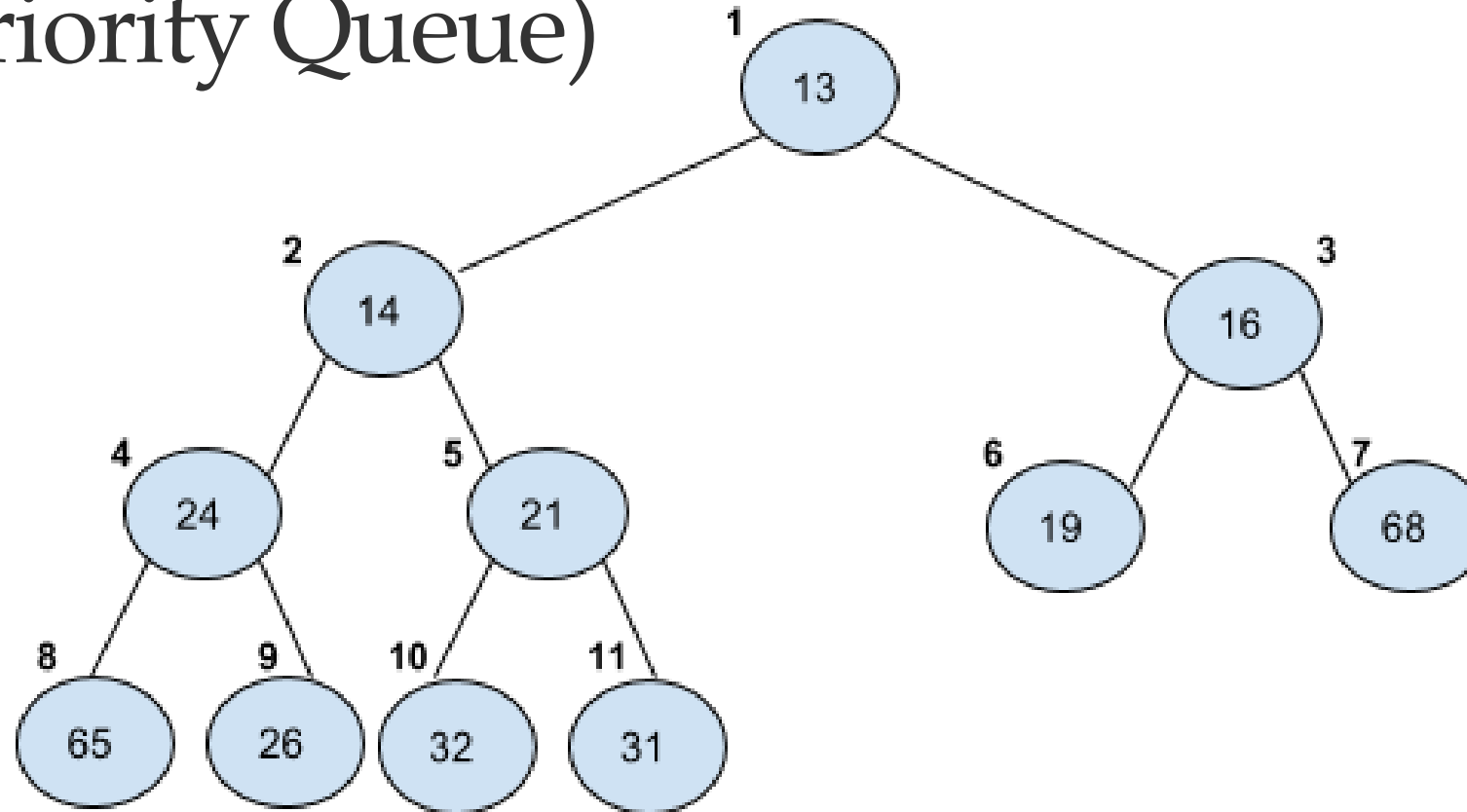
Heaps (Priority Queue)

- Heap Data Structure: it is an array that can be viewed as Binary Search Tree. It may also be called Binary Heaps.
- For any element at position i :

$$\text{Parent}(i) = i/2$$

$$\begin{aligned}\text{Child}(i) &= 2i \text{ left} \\ &= 2i + 1 \text{ right}\end{aligned}$$

Heaps (Priority Queue)



Array		13	14	16	24	21	19	68	65	26	32	31
Index	0	1	2	3	4	5	6	7	8	9	10	11

Notice that there is no need for pointers!

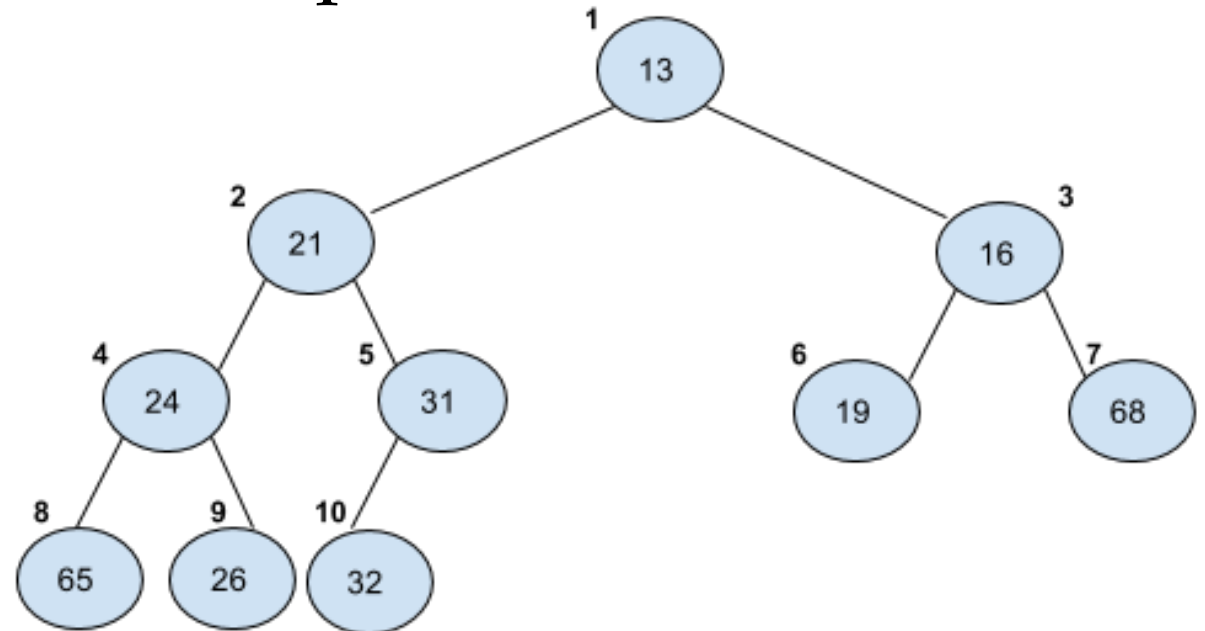
Heap Order Property

- Goal: find the min value quickly.
- It makes sense to have the minimum value at the root as we always delete min.
- Any node should be smaller than any of its descendants.
- For every node X , the key in the parent of X is smaller than or equal to the key in X .

Heaps - Example

- Insert(14) to the following MinHeap.

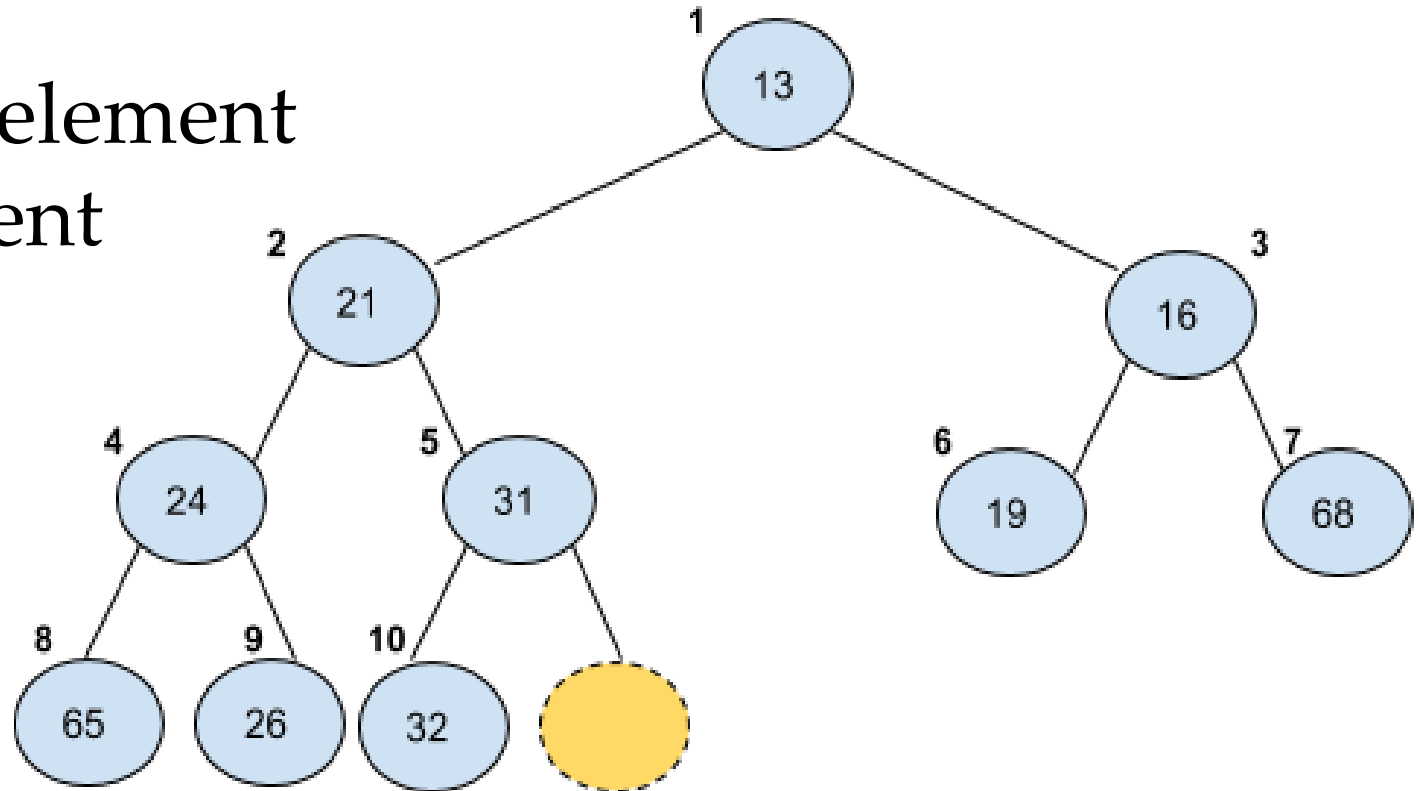
1. Find the next available spot in the tree to insert a new node



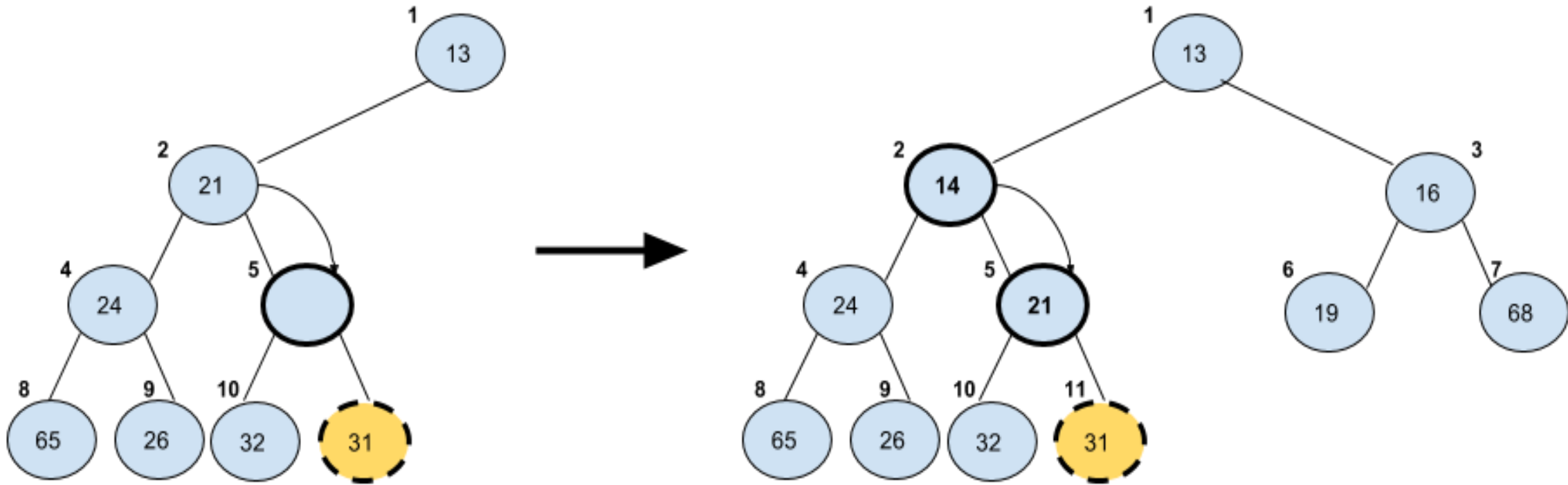
Heaps - Example

2. If X can be placed without violating the rule of heaps, then done.

3. Otherwise, slide the element that is in the hole's parent node into the hole.

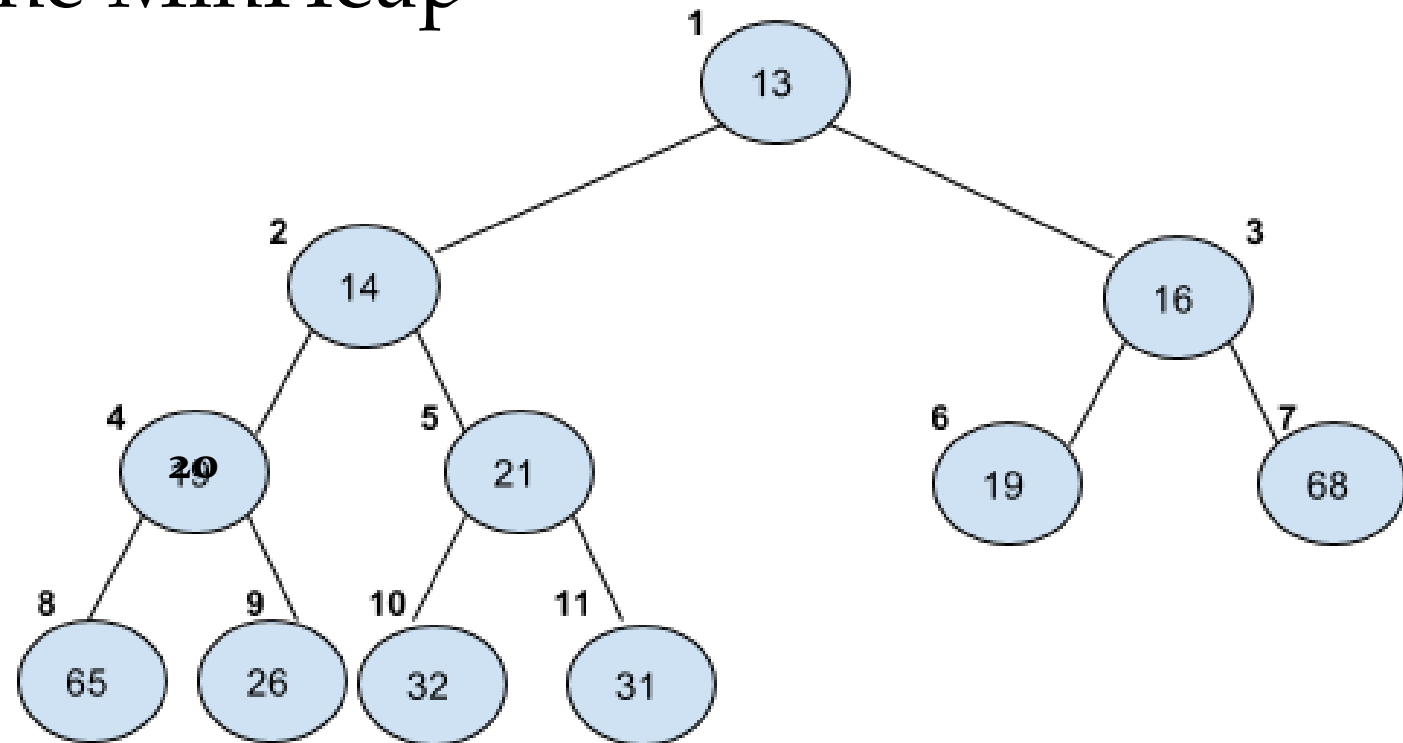


Heaps - Example



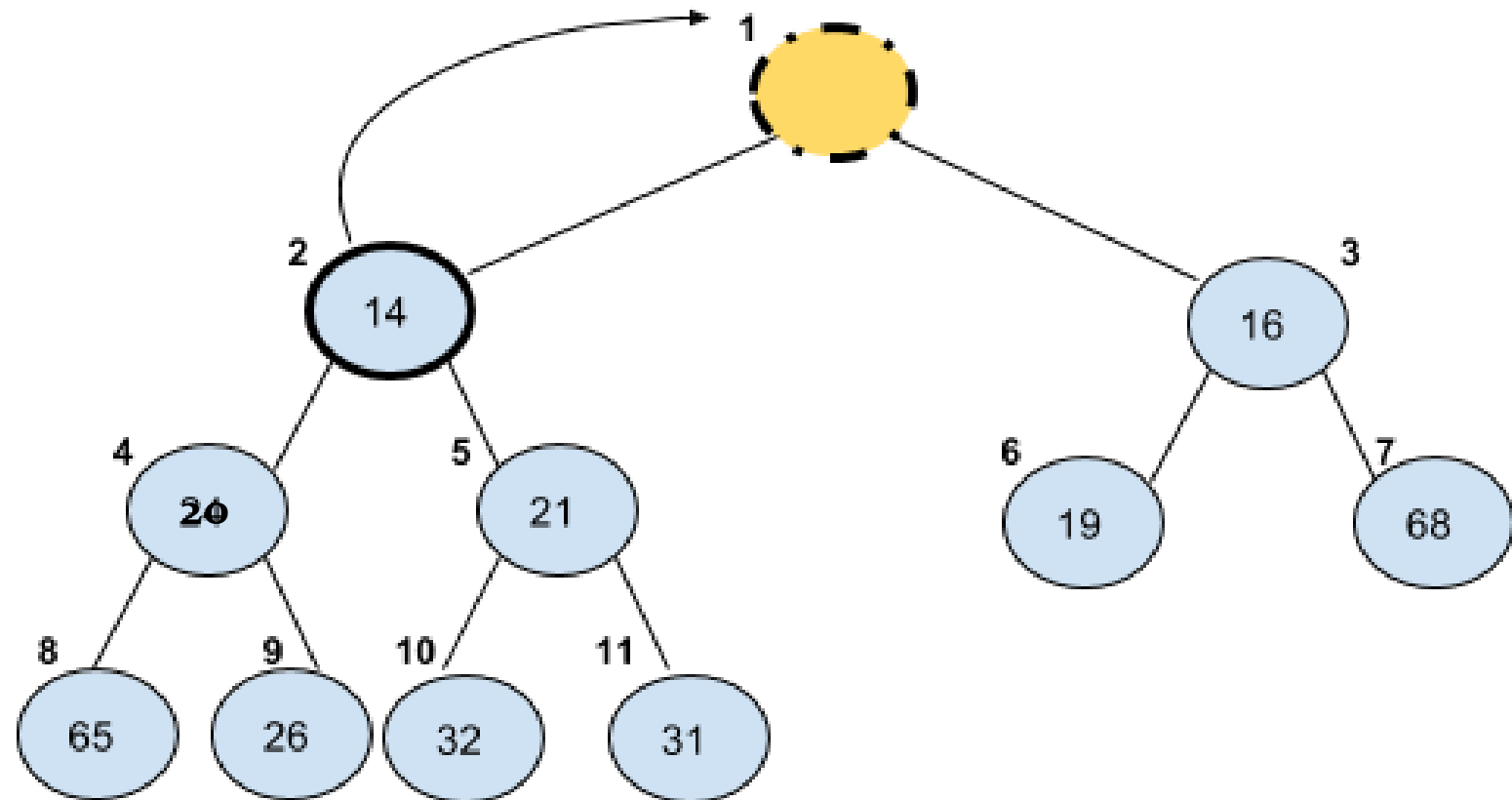
Heaps - Example

- Delete: We can delete either the min or max values. In our case, it is the min as we are operating on MinHeap.
- Example: Delete() from the MinHeap in the following heap



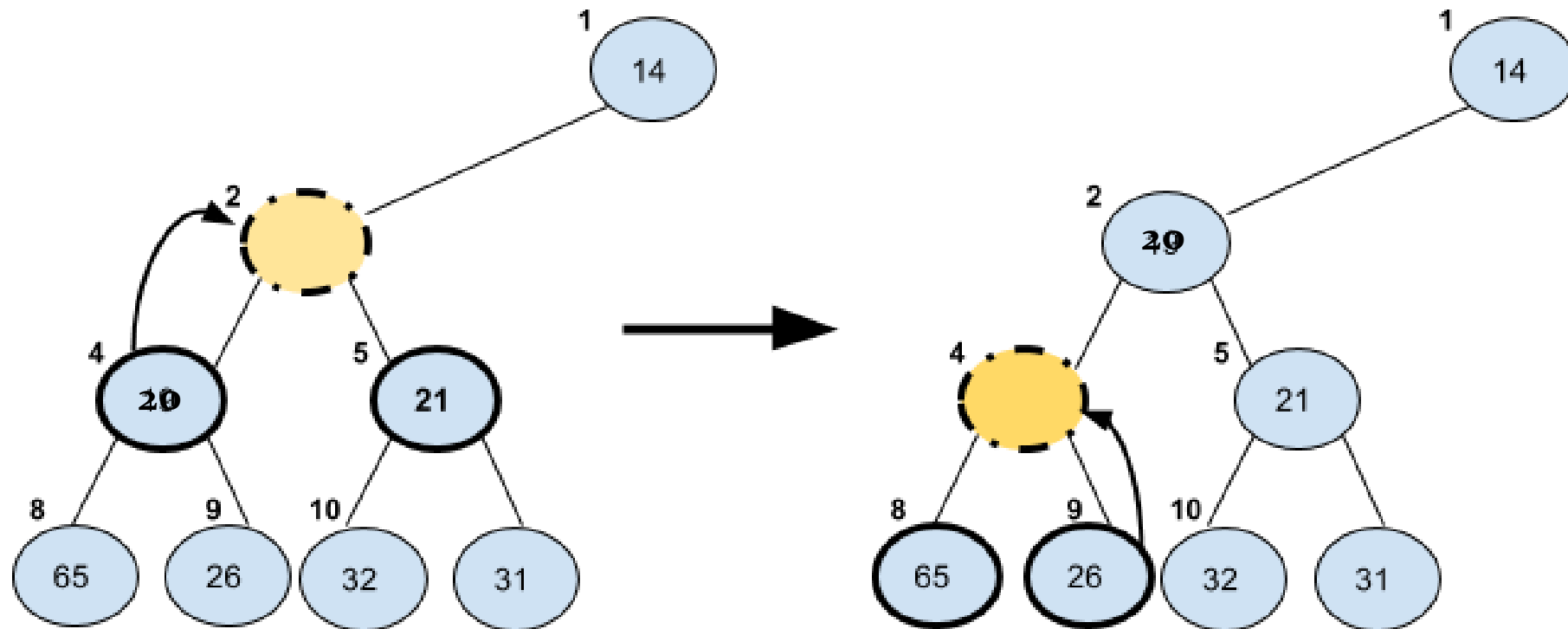
Heaps - Example

1. Find min and remove it. Keep a hole in its place (root).



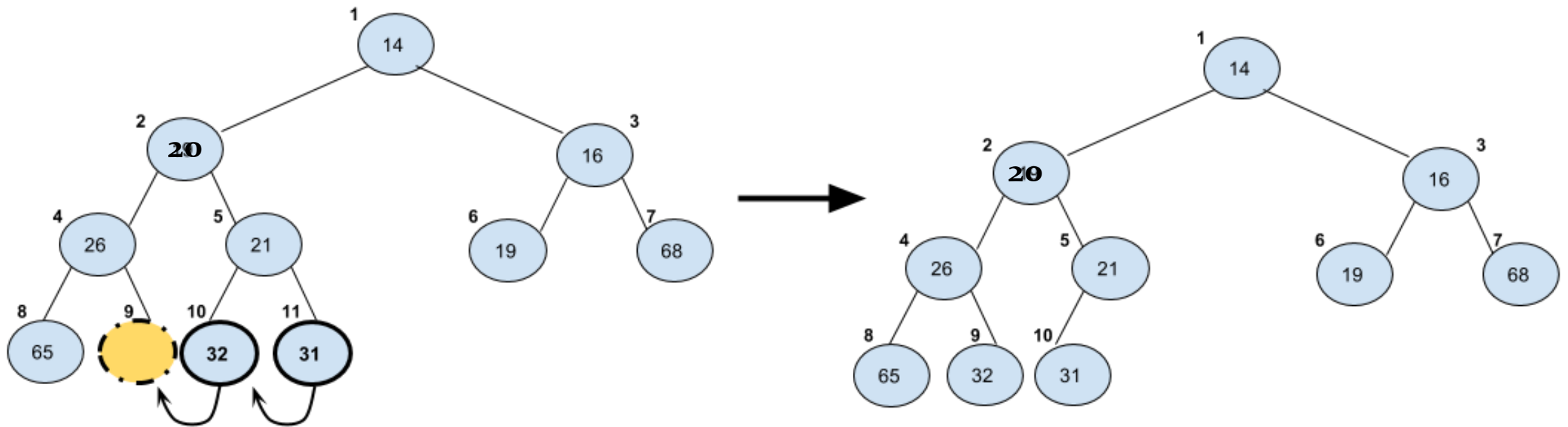
Heaps - Example

2. Push the smaller of the children up.



Heaps - Example

2. Slide the elements on the right to the left to fill the empty spot.



Applications

- Priority queue.
- Heap sort: sorting data using heaps.
- Selection of min/max from a set of data in $O(1)$ time.
- Graph algorithms: heaps are used in graph algorithms like implementing Dijkstra's algorithm.

Running time

	Insert	DeleteMin	FindMin
Binary Heap	$O(\log n)$	$O(\log n)$	$O(1)$