Numbering Systems

Computer Science Department

Outline

- Numbering Systems (NS)
  - Decimal, Binary, Octal & Hexadecimal Systems.
- Converting between NS.
- Binary Arithmetic (two’s complement)
- Data Representation
  - Integer, Float , and ASCII chars (7 bits+ parity bit)

A. Numbering Systems (NS)

1. Decimal System

- ❖Most People Use decimal representation to count.

- ❖In decimal there are **10** digits

  0,1,2,3,4,5,6,7,8,9

- ❖The base is **10**
- ❖We can Represent any value for these digits Ex: 754 ,

  123, 889, 345

Decimal System

Ex: 754

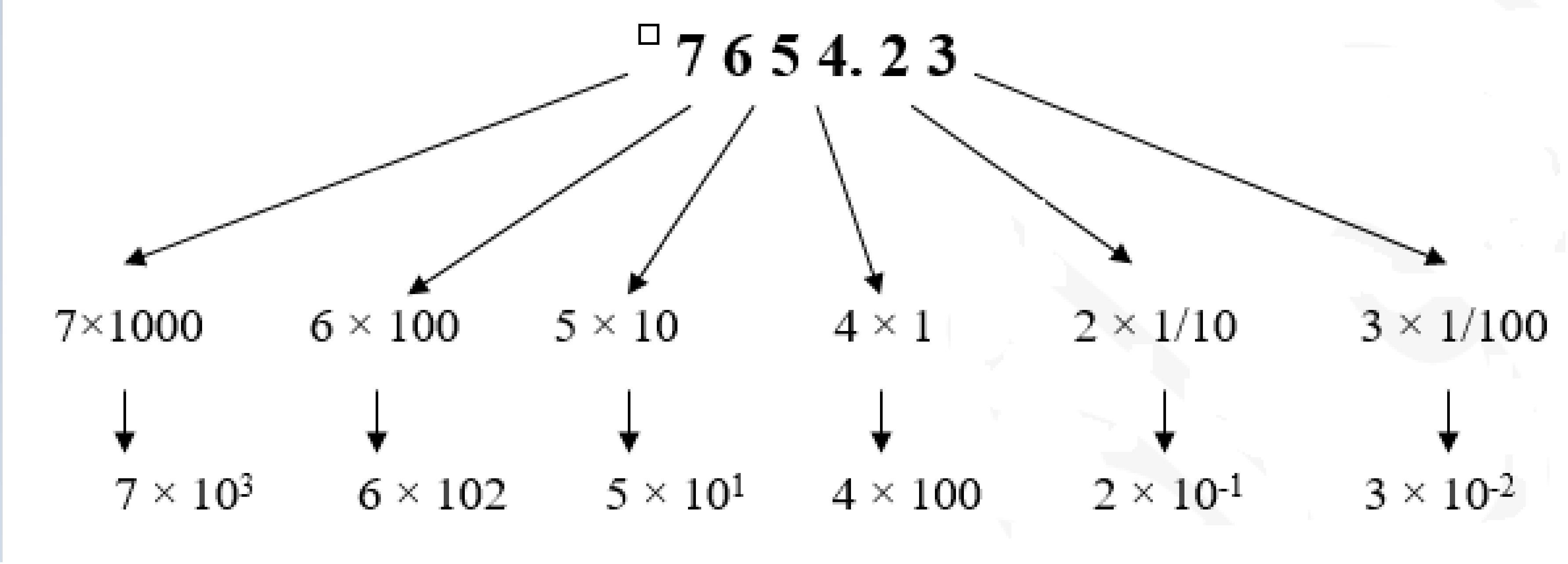
$$7.10^{2} + 5.10^{1} + 4.10^{0} = 700+50+4=754$$

base

Digit position

123 ???

Example: 7654.23



2. Binary System

- ❖Computer is not smart as a human .

- ❖Easy to make an electronic machine with two states: on and off , or 1 and 0.

- ❖In Binary there are **2** digits

0,1

The base is **2**

Binary System

- Each digit in binary number called **BIT.**

  1 0 1 0 , 4 digits, **How many bits ?**

  answer : 4 bits

- **4** bits form a **NIBBLE**.
- **8** bits form a **BYTE**.
- 1 0 1 0 0 0 1 1 **, How many Bits, Nibbles and Bytes?**

Answer :8 bits ,2 Nibbles and 1 byte

Binary System

❖Two bytes form a **WORD** and two words form a **DOUBLE WORD.**

EX:

0000 1111 1010 1010 : 16 bits , WORD

3. Octal System

- Uses 8 digits
  0,1,2,3,4,5,6,7

- The base is **8**

- **EX** $(123)_8$, $(156)_8$

4. Hexadecimal System

- Uses 16 digits
  0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

- The base is **16**

- **EX: 123h , 456h 0E120h**

Numbering Systems Conversion Table

Decimal Base-10	Binary Base-2	Octal Base-8	Hexadecimal Base-16
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

LAB:
Complete Previous Table to 32

16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17
24	11000	30	18
25	11001	31	19
26	11010	32	1A
27	11011	33	1B
28	11100	34	1C
29	11101	35	1D
30	11110	36	1E
31	11111	37	1F
32	100000	40	20

- **Suppose we need to develop new system with base 5,7 or 3?**

Base 5 : 0,1,2,3,4

Base 7 : 0,1,2,3,4,5,6

Base 3: 0,1,2

B. Converting between Numbering Systems

1. Binary –to– Decimal Conversion

The Process : ***Weighted Multiplication***

a) **Multiply** each bit of the *Binary Number* by it corresponding *bit-weighting factor* (i.e. Bit-0→2^0=1; Bit-1→2^1=2; Bit-2→2^2=4; etc).

b) **Sum up all** the products in step (a) to get the *Decimal Number*.

Example:

Convert the decimal number 0110_2 into its decimal equivalent.

0		1		1		0		
2^3		2^2		2^1		2^0		Bit-Weighting Factors
8		4		2		1		
0	+	4	+	2	+	0	=	6_{10}

$\therefore\ 0110_2 = 6_{10}$

Examples: Binary to Decimal

- 10110b

  $1*2^4+0*2^3+1*2^2+1*2^1+0*2^0=$

  $16+0+4+2= (22)_{10}$

1010b =?? , 0010b = ?? , 101b=??

Answer: **1010b=(10)** $_{10}$
0010b=(2) $_{10}$
101b=(5) $_{10}$

Example with fractions: $(1101.01)_2$ to Decimal

$(1101.01)_2 = 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 0 \times 2^{-1} + 1 \times 2^{-2}$

$= 1 \times 1 + 0 \times 2 + 1 \times 4 + 1 \times 8 + 0 \times 1/2 + 1 \times 1/4$

$= 1 + 0 + 4 + 8 + 0 + 0.25$

$= (13.25)_{10}$

2. Decimal –to– Binary Conversion

The Process : ***Successive Division***

a) Divide the *Decimal Number* by 2; the remainder is the LSB of *Binary Number* .

b) If the **quotation** is zero, the conversion is complete; else repeat step (a) using the quotation as the Decimal Number. The new remainder is the next most significant bit of the *Binary Number.*

Example:

Convert the decimal number 6_{10} into its binary equivalent.

$$2\overline{)6}^{\,3} \quad r = 0 \leftarrow \text{Least Significant Bit}$$

$$2\overline{)3}^{\,1} \quad r = 1$$

$$2\overline{)1}^{\,\mathbf{0}} \quad r = 1 \leftarrow \text{Most Significant Bit}$$

$\therefore \; 6_{10} = 110_2$

Dec → Binary Example 1

$(\mathbf{22})_{10}$

$(\mathbf{22})_{10} = (\ )_2$

Input	Result	Remainder
22/2	11	0
11/2	5	1
5/2	2	1
2/2	1	0
1/2	0	1

$(\mathbf{22})_{10} = (\mathbf{10110})_2$

Dec → Binary Example 2

Example:

Convert the decimal number 26_{10} into its binary equivalent.

Solution:

$$2\overline{)26}^{\,13} \quad r = 0 \leftarrow \text{LSB}$$

$$2\overline{)13}^{\,6} \quad r = 1$$

$$2\overline{)6}^{\,3} \quad r = 0$$

$$2\overline{)3}^{\,1} \quad r = 1$$

$$2\overline{)1}^{\,0} \quad r = 1 \leftarrow \text{MSB}$$

$\therefore\ 26_{10} = 11010_2$

Dec → Binary Example 3

Example:

Convert the decimal number 41_{10} into its binary equivalent.

Solution:

$$2\overline{)41} = 20 \quad r = 1 \leftarrow \text{LSB}$$

$$2\overline{)20} = 10 \quad r = 0$$

$$2\overline{)10} = 5 \quad r = 0$$

$$2\overline{)5} = 2 \quad r = 1$$

$$2\overline{)2} = 1 \quad r = 0$$

$$2\overline{)1} = 0 \quad r = 1 \leftarrow \text{MSB}$$

$\therefore \; 41_{10} = 101001_2$

Dec → Binary : More Examples (LAB)

a) $13_{10} = ?$

b) $22_{10} = ?$

c) $43_{10} = ?$

d) $158_{10} = ?$

Dec → Binary : More Examples (LAB)

a) 13_{10} = ? $1\,1\,0\,1_{\,2}$

b) 22_{10} = ? $1\,0\,1\,1\,0_{\,2}$

c) 43_{10} = ? $1\,0\,1\,0\,1\,1_{\,2}$

d) 158_{10} = ? $1\,0\,0\,1\,1\,1\,1\,0_{\,2}$

3. Binary –to– Octal Conversion

100101010b = $(\quad)_8$

100 101 010 = $(452)_8$

111000111b= $(\quad)_8$

111 000 111 = $(707)_8$

Binary to Octal (LAB)

- **100101011b =** $(453)_8$
- **101101011b =** $(\quad)_8$ **H.W**
- **100101001b =** $(\quad)_8$ **H.W**

4. Binary –to– Hexadecimal Conversion

10010101b = ( )$_h$

1001 0101 =(95h)

11100011b= (E3h)

5. Decimal –to– Hexadecimal Conversion

Let's convert the value $(39)_{10}$ to Hexadecimal

Input	Result	Remainder
39/16	2	7
2/16	0	2

$(39)_{10}$ = (27h)

(LAB Work)

Covert the following numbers to **decimal**

$(72)_{8}$, $(72)_{16}$, $(DE1)_{16}$

a. $(72)_{8} = (58)_{10}$
b. $(72)_{16} = (114)_{10}$
c. $(DE1)_{16} = (3553)_{10}$

Extra Exercises

Using pen and paper , solve the following questions :

a. $(AB)_{16} = (\quad)_2$

b. $(23)_4 = (\quad)_8$

c. $(35)_7 = (\quad)_8$

d. $(72E)_{16} = (\quad)_8$

Binary Addition

H.W

Solve Question 7 ,lab 1 ,page 9

Fractions & Conversions

Fractions

- Decimal to decimal (just for fun)

```
3.14 =>        4 x 10^-2 = 0.04
               1 x 10^-1 = 0.1
               3 x 10^0  = 3
                         -------
                           3.14
```

$$3.14 \Rightarrow \begin{array}{l} 4 \times 10^{-2} = 0.04 \\ 1 \times 10^{-1} = 0.1 \\ 3 \times 10^{0} = 3 \\ \hline \quad\quad\quad\quad 3.14 \end{array}$$

1. Fractions: Binary to Decimal Conversion

Example

10.1011 =>

$1 \times 2^{-4} = 0.0625$
$1 \times 2^{-3} = 0.125$
$0 \times 2^{-2} = 0.0$
$1 \times 2^{-1} = 0.5$
$0 \times 2^{0} = 0.0$
$1 \times 2^{1} = 2.0$

2.6875

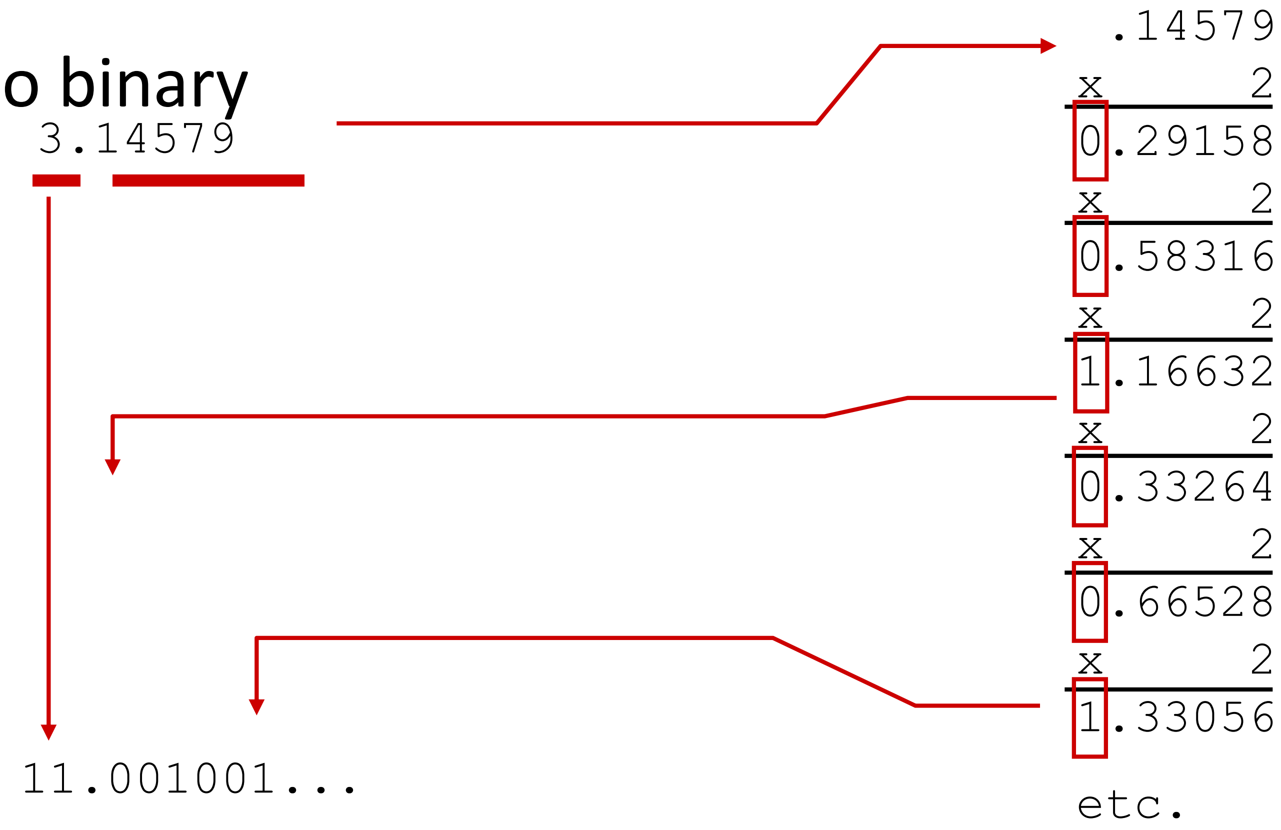
2. Fractions: Decimal to Binary Conversion

- Multiply the decimal number by 2 repeatedly.
- Use the integer part as the next digit each time, and then discard the integer
- When the fraction part is zero, we have an exact conversion
- Add trailing zeros to obtain the desired size
- **Example**: Convert ***0.625*** to binary

.625*2 = 1.25	.1
.25*2 = 0.50	.10
.5*2 = 1.00	.101

Fractions: 3.14579

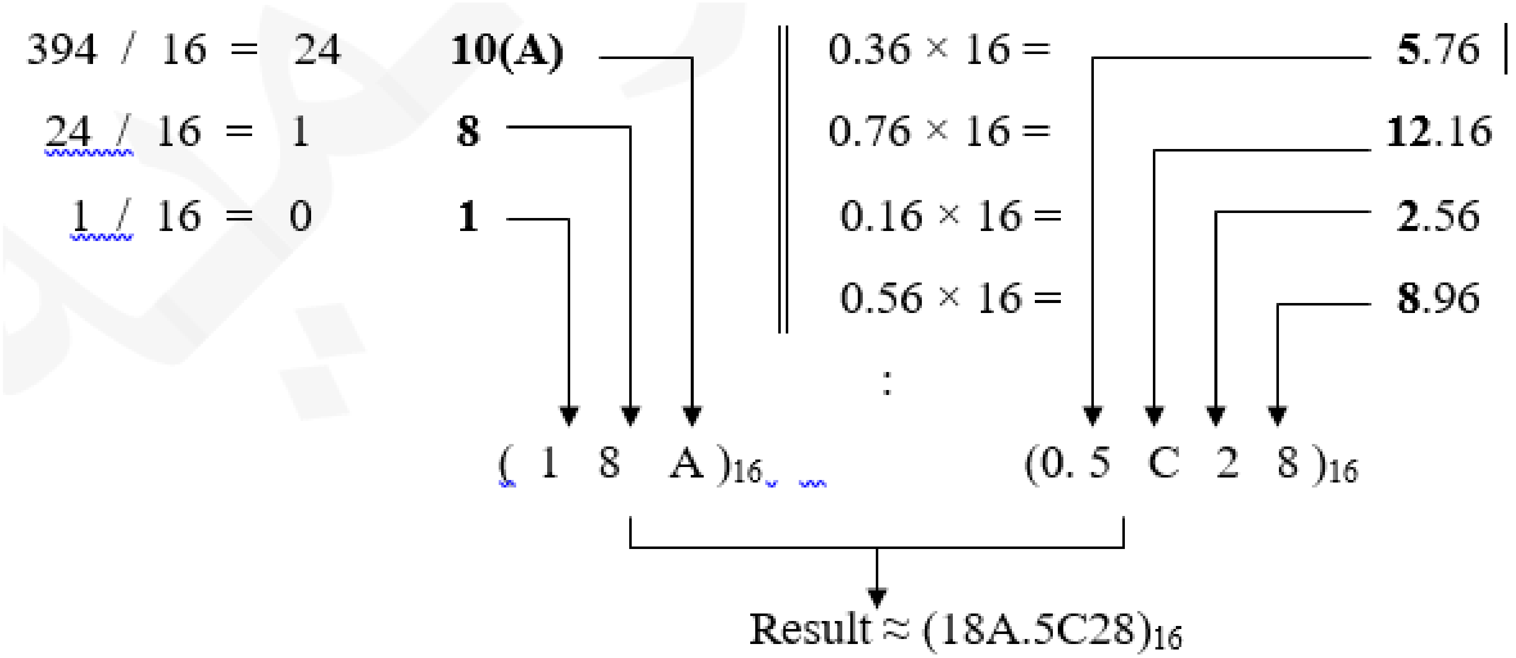
- Decimal to binary



Note: some decimal fractions don’t have an exact representation in binary.
0.375 does have! **(0.375 → 0.75 → 0.5 → 0)**
0.3 does not have an exact representation !

3. Fractions: Decimal to Hexadecimal Conversion

Example: Convert to Hex. :$(394.36)_{10}$



Exercise – (LAB)

Decimal	Binary	Octal	Hexa-decimal
29.8	11101.110011…	35.63…	1D.CC…
5.8125	101.1101	5.64	5.D
3.109375	11.000111	3.07	3.1C
12.5078125	1100.10000010	14.404	C.82