جامعة بيرزيت

Birzeit University

Computer Science Department – Engineering and Information Technology Faculty
Comp233, Discrete Mathematics, Dec 5, 2019
Winter 2019

Student Name: ____key________ Student *ID*: ____________

➢ **Instructor: Mr. Murad Njoum**

Question 1 (36%):

1) Out of 7 consonants and 4 vowels, how many words of 2 consonants and 3 vowels can be formed?

A. 5!
B. $\binom{7}{3}.\binom{4}{2}5^5$
C. $\binom{7}{2}.\binom{4}{3}.5!$
D. $\binom{7}{3}.\binom{4}{2}.5!$

2) In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one girl should be there?

A. 195
B. 209
C. 201
D. 212

3) In how many different ways can the letters of the word 'OPTICLL' be arranged so that the vowels always come together?

A. 4!3!
B. 5!3!
C. 6!2!
D. $\binom{7}{2}.\binom{7}{5}$

4) A question paper has two parts A and B, each containing 10 questions. If a student needs to choose 4 from part A and 8 from part B, in how many ways can he do that?

A. 8! 4!
B. $^{12}C_8 \times {}^{12}C_4$
C. $^{10}C_8 \times {}^{10}C_4$
D. None of them

5) In how many ways can 30 identical apples be divided among 11 boys?

A. $^{39}C_9$
B. 30!10!30!10!
C. $^{30}C_{10}$
D. $^{40}C_{11}$

6) A bag contains 3 yellow, 4 green and 3 blue balls. Three balls are drawn at random. What is the probability that none of the balls drawn is blue?

A. $\frac{1}{11}$
B. $\frac{10}{21}$
C. $\frac{9}{11}$
D. $\frac{7}{24}$

7) Two cards are drawn together from a pack of 52 cards. The probability that one is a club and one is a diamond?

A. $\frac{13}{51}$

B. $\frac{1}{52}$

C. $\frac{1}{26}$

D. $\frac{13}{102}$

8) Determine how many times "Hello World" will be printed when the algorithm segment is implemented and run if n=99.

```
for i := 1 to n
   for j := i to n
    print "Hello World"
   next j
 next i
```

A. 4950 B. 100X100 C.10,100 D. 5,050

9) For relation R on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, and whether it is transitive. $R = \{(1,1),(1,2),(3,1),(2,2),(3,3),(4,4)\}$

A. R is Reflexive
B. Symmetric but not transitive
C. Transitive neither symmetric nor reflexive
D. Equivalence

10) Determine whether the relation R on the set of all integers is reflexive, symmetric, transitive, or equivalence where $(x, y) \in R$ if and only if a $x \neq y$.

A) It's reflexive
B) It's symmetric
C) It's transitive
D) It's Equivalence

11) Determine whether the relation R on the set of all web pages is reflexive, symmetric, transitive, or equivalence where $(a, b) \in R$ if and only if everyone who has visited web page a has also visited web page b

A. Reflexive, Symmetric, and transitive
B. Reflexive, not Symmetric, and transitive
C. Reflexive, not Symmetric, and not transitive
D. Not Reflexive not Symmetric, and not transitive

12) What is your section number?
A. Section 1 B. Section C. Section 3 D. Section 4 E. Section 5

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A. 195
B. 209
C. 201
D. 212

3) In how many different ways can the letters of the word 'OPTICAL' be arranged so that the vowels always come together?

A. 4!3!
B. 5!3!
C. 6!2!
D. $\binom{7}{3}.\binom{7}{4}$

4) A question paper has two parts A and B, each containing 10 questions. If a student needs to choose 8 from part A and 4 from part B, in how many ways can he do that?

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A. $\frac{13}{51}$

B. $\frac{1}{52}$

C. $\frac{13}{102}$

D. $\frac{1}{26}$

8) Determine how many times "Hello World" will be printed when the algorithm segment is implemented and run if n=100.

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for i := 1 to n
   for j := i to n
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```

A. 4950 B. 100X100 C.10,100 D. 5,050

9) For relation R on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, and whether it is transitive. $R = \{(1, 1),(1, 2),(2, 1),(2, 2),(3, 3),(4, 4)\}$

A. R is Reflexive but not transitive
B. Symmetric but not transitive
C. Transitive neither symmetric nor reflexive
D. Equivalence

10) Determine whether the relation R on the set of all integers is reflexive, symmetric, transitive, or equivalence where $(x, y) \in R$ if and only if a $x \neq y$.

A) It's Equivalence
B) It's reflexive
C) It's symmetric
D) It's transitive

11) Determine whether the relation R on the set of all web pages is reflexive, symmetric, transitive, or equivalence where $(a, b) \in R$ if and only if everyone who has visited web page a has also visited web page b

A. Reflexive, Symmetric, and transitive
B. Reflexive, not Symmetric, and not transitive
C. Not Reflexive not Symmetric, and not transitive
D. Reflexive, not Symmetric, and transitive

12) What is your section number?
A. Section 1 B. Section C. Section 3 D. Section 4 E. Section 5

Question 2 (26 %):

Define ~ on Z by a ~ b if and only if $3a + b$ is a multiple of 4.

(a.) [12 %] Prove that ~ defines an equivalence relation.
(b.) [7 %] Find the equivalence class of 0.
(c.) [7 %] Find the equivalence class of 2.

Solution:
(a.)

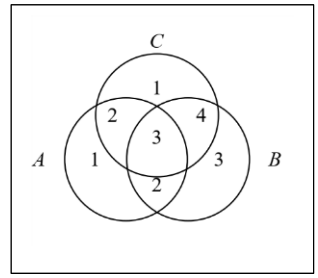
I. Reflexive: For any a ∈ Z, 3a + a = 4a is a multiple of 4, so a ~ a.
II. Symmetric: If a ~ b, then 3a + b = 4k for some integer k. Since (3a + b) + (3b + a) = 4(a + b), we see that 3b + a = 4(a + b) − 4k is a multiple of 4, so b ~ a.
III. Transitive: If a ~ b and b ~ c, then 3a + b = 4k for some integer k and 3b + c = 4l for some integer l. Since 4(k + l) = (3a + b) + (3b + c) = (3a + c) + 4b, we see that 3a + c = 4(k + l) − 4b is a multiple of 4 and hence, that a ~ c.

(b.)[0]= {x ∈ Z|x ~ 0} = {x|3x = 4k, for some integer k}. Now if 3x = 4k, k must be a multiple of 3. So 3x = 12l for some integer l ∈ Z and x = 4l. Therefore, 0 = 4Z = {0, 4, 8, 12, 16, ...}.

(c.)[2]= {x ∈ Z|x ~ 2} = {x|3x + 2 = 4k for some integer k} = {x|3x = 4k − 2 for some integer k}. Now if 3x = 4k − 2, then 3x = 3k + k − 2 and so k − 2 is a multiple of 3. Therefore, k = 3l + 2 for some integer l, 3x = 4(3l + 2) − 2 = 12l + 6 and x = 4l + 2. So 2 = 4Z + 2 = {2, 6, 10, 14, 18, ...}.

Question 3 (28%): **Part I (13 %) :**

In a group of 30 people, 8 speak English, 12 speak Spanish and 10 speak French. It is known that 5 speak English and Spanish, 5 Spanish and French, and 7 English and French. The number of people speaking all three languages is 3. How many do not speak any of these languages?



Solution:

Let E be the set of all English speakers, ➔ N(E)= 8
S the set of Spanish speakers ➔ N(S)=12
F the set of French speakers in our group➔ N(F)=10
$N(E\cap S) = 5$, $N(S\cap F) = 5$, $N(E\cap F) = 7$, $N(E\cap S\cap F) = 3$

$N(E\cup S\cup F) = N(E) + N(S) + N(F) - N(E\cap S) - N(S\cap F) - N(E\cap F) + N(E\cap S\cap F)$
$=8+12+10-5-5-7+3 = 16$
$N(E\cup S\cup F)^c = 30 - 16 = 14$

Part II (15 %)

A. (3%)A store sells 8 colors of balloons with at least 30 of each color. How many different combinations of 30 balloons can be chosen?

Total= $\binom{r+n-1}{r} = \binom{37}{30} = \frac{37!}{7!30!} = 10,295,472$

B. (4 %)If the store has only 12 red balloons but at least 30 of each other color of balloon, how many combinations of balloons can be chosen?

Total (**with picking at least 13 red**) = $\binom{17+8-1}{17} = \binom{24}{17} = \frac{24!}{7!17!} = 346,104$

Total (**at most 12-red balloon**) = 10,295,472−346,104 = 9,949,368

C. (4 %) If the store has only 8 blue balloons but at least 30 of each other color of balloon, how many combinations of balloons can be chosen?

Total (with at least 9 blue) = $\binom{21+8-1}{21} = \binom{28}{21} = 1,184,040$
Total (with at most 8 blue) = 10,295,472 − 1,184,040 = 9,111,432

D. (4 %) If the store has only 12 red balloons and only 8 blue balloons but at least 30 of each other color of balloon, how many combinations of balloons can be chosen?

Total (with at least 9B and 13R) = $\binom{8+8-1}{8} = \binom{15}{8} = 6435$

<u>Question 4 (15 %):</u> Given the following code in C language, what is the output?

```
#include <stdio.h>
int func(int n);
int main()
{ int a,b,c, i,j,k, n;

     n = 999, a = 0, b=0, c=1;
  for  (i= 1;i<=n; i++)
     {  if( func(i))
         a++;   }
for (j=1; j<=n;j++)
    {
      for (k=1; k<=j;k++)
         b+=a;
      c*=10*j; }

 printf("a=%d,b= %d, c=%d\n",a,b,c);
     return 0;}
```

```
int func(int x){
 int i=0,sum=0;
 while(x!=0)
     {
      sum+=x%10;
       x/=10;
      }
 if(sum ==10)
    return 1;
return 0;}
```

$X1 + X2 + X3 = 10$ ➔

Hence, a= $\binom{10+3-1}{10}$ $\binom{12}{10}$ $- 3 =$ *63 since 10 is not include in the equation above*

there are exactly as many doubles of integers (k, j) with 1 ≤ j ≤ k ≤ 999 as there are

2-combinations of integers from 1 through 999 with **repetition allowed.**

b=$a \sum_{i=1}^{999} i$=$\frac{i(i+1)}{2}$=63X $\frac{999\,(999+1)}{2}$=63X499,500=31,468,500

Output of c,

Output of C: $10\, n! = 10 \prod_{1}^{999} j$ =10*999!