

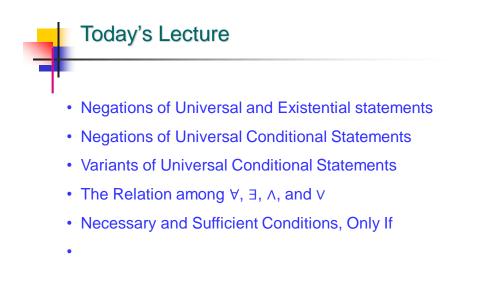
Chapter 3 The Logic of Quantified Statements (First Order (Predicate) Logic)

1



## 3.2: Predicates and Quantified Statements II

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The negation of a statement exactly expresses what it would mean for the statement to be false. Write negations for the following:

- **1**.  $\forall$  even integers x, x is positive.
- 2. I an integer n such that n has an integer square root.

Answers

- **1**.  $\exists$  an even integer x such that x is not positive.
- **2**.  $\forall$  integers *n*, *n* does not have an integer square root.

In general:

 $\neg(\forall x \text{ in } D, P(x)) \equiv \exists x \text{ in } D \text{ such that } \neg P(x)$  $\neg(\exists x \text{ in } D \text{ such that } P(x)) \equiv \forall x \text{ in } D, \neg P(x)$ 

<u>So</u>: The negation of a "for all" statement is a "there exists" statement, and the negation of a "there exists" statement is a "for all" statement.

4

4

Negation of Quantified Statements

The negation of the statement of the form  $\exists x \text{ in } D \text{ such that } Q(x)$ is logically equivalent to a statement of the form  $\forall x \text{ in } D, \neg Q(x)$ 

Symbolically:

 $\sim (\exists x \in D \ni Q(x)) \equiv \forall x \in D, \sim Q(x).$ 

Note: the negation of existential statement is logically equivalent to universal statement.

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5



Negate "Some integer *x* is positive and all integers *y* are negative."

**Solution:** Using all integers as the universe for *x* and *y*, the statement is  $\exists x . (x > 0) \land \forall y . (y < 0)$ . The **negation** is

by De Morgan's law  $\equiv \forall x . (x > 0) \lor \exists y . (y < 0)$  properties of negation  $\equiv \forall x . (x \le 0) \lor \exists y . (y \ge 0).$ 

Therefore, the **negation** is "Every integer *x* is non positive or there is an integer *y* that is nonnegative."

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### Negate "There is a student who came late to class and there is a student who is absent from class."

**Solution:** In symbols, if L(x) : "*x* came late to class" and A(x) : "*x* is absent from class," this statement can be written as  $\exists x \text{ st } L(x) \land \exists y \text{ st } A(y)$ .

Note that we must use a second variable *y*. By one of De Morgan's laws the negation can be written as

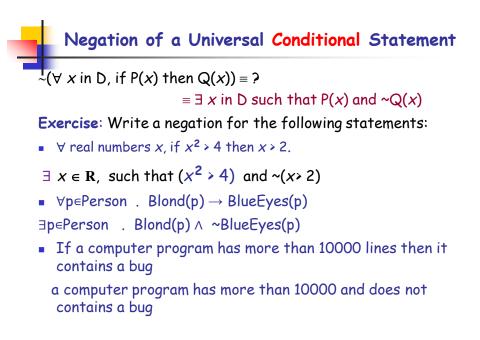
 $\sim (\exists x \text{ st } L(x)) \lor \sim (\exists y \text{ st } A(x)) \equiv \forall \mathbf{x}, \sim \mathbf{L}(\mathbf{x}) \lor \forall \mathbf{y}, \sim \mathbf{A}(\mathbf{x}).$ 

In English this is "No student came late to class or no student is absent from class."

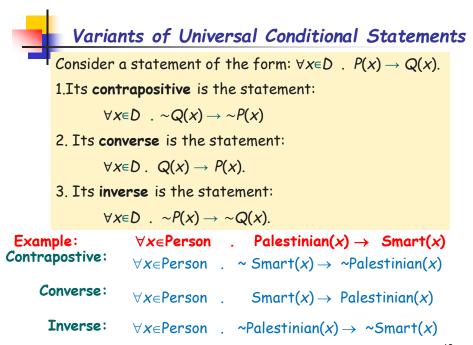
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 All Palestinians like Zatar Some Palestinians do not like Zatar Some Palestinians do not like Zatar All Palestinians Like Zatar All Palestinians do not like Zatar
♥ p∈ Prime . Odd(p) ∃p∈Prime . ~Odd(p)
Some computer hackers are over 40 All computer hackers are not over 40.
All computer programs are finite Some computer programs are not finite
No politicians are honest Some politicians are honest

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Variants of Universal Conditional Statements	
∀x ∈ R. More ∀x∈R . x >	Than(x,2) $\rightarrow$ MoreThan(x <sup>2</sup> ,4) 2 $\rightarrow$ x <sup>2</sup> > 4
Contrapostive:	$\forall x \in \mathbf{R}  .  x^2 \leq 4 \rightarrow x \leq 2$
Converse:	$\forall x \in \mathbb{R}$ . $x^2 > 4 \rightarrow x > 2$
Inverse:	$\forall x \in \mathbf{R}  x \leq 2  \rightarrow  x^2 \leq 4$

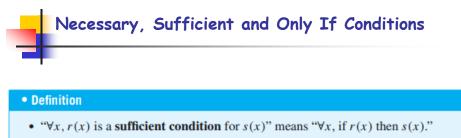
# The Relation among $\forall$ , $\exists$ , $\land$ , and $\lor$

If Q(x) is a predicate and the domain D of x is the set  $\{x1, x2, ..., xn\}$ , then the statements:

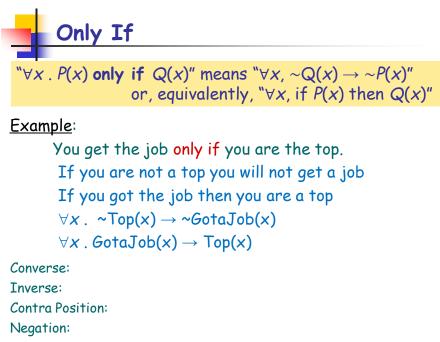
 $\forall x \in D, Q(x)$  and  $Q(x1) \land Q(x2) \land \cdots \land Q(xn)$ are logically equivalent.

Example: let Q(x) be "x·x=x" and D= $\{0,1\}$ . Then  $\forall x \in D, Q(x)$  can be rewritten as  $\forall$  binary digits x, x·x=x This is equivalent to (0·0=0) and (1·1=1), which can be rewritten in symbols as: Q(0) $\land$ Q(1).

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- " $\forall x, r(x)$  is a **necessary condition** for s(x)" means " $\forall x$ , if  $\sim r(x)$  then  $\sim s(x)$ " or, equivalently, " $\forall x$ , if s(x) then r(x)."
- " $\forall x, r(x)$  only if s(x)" means " $\forall x$ , if  $\sim s(x)$  then  $\sim r(x)$ " or, equivalently, " $\forall x$ , if r(x) then s(x)."



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## Necessary and Sufficient Conditions

" $\forall x . P(x)$  is a sufficient condition for Q(x)" means " $\forall x . P(x) \rightarrow Q(x)$ " " $\forall x . P(x)$  is a necessary condition for Q(x)" means " $\forall x, \sim P(x) \rightarrow \sim Q(x)$ " or, equivalently, " $\forall x, Q(x) \rightarrow P(x)$ "

#### Examples:

Squareness is a sufficient condition for rectangularity. If a shape is a square, then it is a rectangle.  $\forall x$ . Square(x)  $\rightarrow$  Rectangle(x)

To get a job, it is sufficient to be loyal. If one is loyal (s)he will get a job  $\forall x . \text{Loyal}(x) \rightarrow \text{GotaJob}(x)$ 

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15

## Necessary and Sufficient Conditions

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#### More examples:

Being smart is necessary to get a job.

If you are not smart you don't get a job

If you got a job then you are smart

 $\forall x : \text{~Smart}(x) \rightarrow \text{~GotaJob}(x)$ 

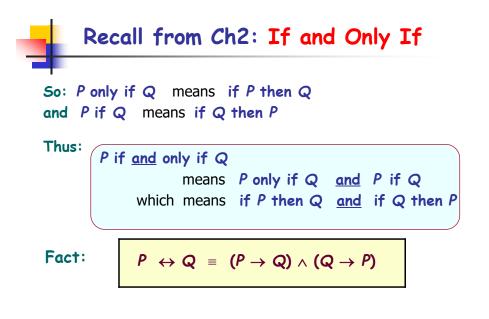
 $\forall x . GotaJob(x) \rightarrow Smart(x)$ 

#### Being above 40 years is necessary for being president of Palestine

 $\forall x . \ \text{~Above}(x, 40) \rightarrow \text{~CanBePresidentOfPalestine}(x)$ 

 $\forall$  x . CanBePresidentOfPalestine(x)  $\rightarrow$  Above(x, 40)

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Recall from Ch2: Only If	
P only if Q means if ~Q then ~P	
If Q didn't occur, then P didn't occur either	
Or, equivalently, if P then Q	
If P occurred then Q also had to occur.	