COMP 233 Discrete Mathematics

Chapter 3

The Logic of Quantified Statements (First Order (Predicate) Logic)

3.1 PREDICATES AND QUANTIFIED STATEMENTS I

What is First Order Logic?

Propositional Logic

P, Q	*Propositions*
$\neg P$	Negation
$P \wedge Q$	Conjunction
$P \vee Q$	Disjunction
$P \rightarrow Q$	Implication
$P \leftrightarrow Q$	Equivalence

We regard the world as *Propositions*

First Order Logic

P(x..y), Q(t,...s)	*Predicates*
$\neg P$	Negation
$P \wedge Q$	Conjunction
$P \vee Q$	Disjunction
$P \rightarrow Q$	Implication
$P \leftrightarrow Q$	Equivalence
$\forall$	Universal quantification
$\exists$	*Existential* quantification

We regard the world as *Quantified Predicates*

Predicates

A **predicate** is a sentence that contains a finite number of variables and becomes a statement when specific values are substituted for the variables.

Ex: $x^2 > 4$

- The **domain of a predicate variable** is the set of allowable values for the variable.
- **Ex:** Let $P(x)$ be the sentence "$x^2 > 4$" where the domain of x is understood to be the set of all real numbers. Then $P(x)$ is a predicate.
- **Question:** For what numbers x is $P(x)$ true?

  Ans: The set of all real numbers for which $x > 2$ or $x < -2$.

4

Predicate

- The word *predicate* refers to the part of a sentence that gives information about the subject.
- In the sentence "*Ali is a student at BZU,*"
  - the word *Ali* is the subject
  - the phrase *is a student at BZU* is the predicate.
- The predicate is the part of the sentence from which the subject has been removed.

Forming a Predicate

- predicates can be obtained by removing some or all of the nouns from a statement.
- For instance, let P stand for "is a student at BZU" and let Q stand for "is a student at."
- Then both P and Q are predicate symbols.
- The sentences
- P(x) = " x is a student at BZU"
- Q(x, y) = "x is a student at y"
- x and y are predicate variables.

Truth Set of a Predicate

The **truth set of a predicate** P(x) is the set of elements in the domain D of x for which P(x) is true.

We write

truth set of P(x) = $\{x \in D \mid P(x)\}$

How should we read this out loud?

"**the set of all** x in D **such that** P of x

Note: The vertical line denotes the words "such that" **for the set-bracket notation only**. In other contexts, the words "such that" are symbolized by "s.t." or "s. th."

Truth Set

- *Truth Set*.
  e.g., let $P(x)$ = "$x+1>x$".

- we could define the truth set as the set of integers.

8

Truth Set

- Find the truth set for the Q(*n*)
- Q(n) = "*n* is a factor of 8".

1. The domain of *n* is the set $\mathbf{Z}^+$ all +ve numbers

   The truth set is {1,2,4,8}

2. The domain of *n* is the set Z all integers numbers

   The truth set is {1,2,4,8,-1,-2,-4,-8}

Domains of numbers

Set Notation: $x \in A$ means that "x is an element of the set A," or "x is in A."

Important sets:

R, the set of all real numbers (on paper: $\mathbb{R}$)

Q, the set of all rational numbers (on paper: $\mathbb{Q}$)

Z, the set of all integers (on paper: $\mathbb{Z}$)

$\mathbf{R}^+$, the set of all positive real numbers

$\mathbf{Z}^{nonneg}$, the set of all nonnegative integers

Etc.

Arty of Predicates

$P(x_1, x_2\ , \ldots, x_n)$

Predicate Name

Arguments

Examples:

Unary Predicates: Person(Amjad), University(BZU)

Binary Predicates: StudyAt(Amjad, BZU)

Ternary Predicates StudyAt(Amjad,BZU, CS)

Quaternary Predicate: StudyAt(Amjad,BZU, CS, 2015)

n-ary Predicate: StudyAt(Amjad,BZU, CS, 2015, BA, ….)

Quantifier Expressions

- *Quantifiers* allow us to *quantify* (count) *how many* objects in the truth set satisfy a given predicate:

- "$\forall$" is the FOR ALL or *universal* quantifier.
- $\forall x\, P(x)$ means *for all* x in the truth set, P holds.

- "$\exists$" is the $\exists$XISTS or existential quantifier.
  $\exists x\, P(x)$ means there *exists* an x in the truth set (that is, one or more) such that $P(x)$ is true.

A **universal statement** is a statement saying that a certain property is true for all elements in a given set.

Ex: All students in this room are registered for COMP 233.

If a person in this room is a student, then that person is registered for COMP233.

The symbol ∀ stands for the words "for all"
– it is called the **universal quantifier**

- ∀ students *P* in this room, *P* is registered for COMP233.
- ∀ people *P* in this room, if *P* is a student, then *P* is registered for COMP233
- If *P* is a student in this room, then *P* is registered for COMP233.

The universal quantification is "implicit" in this statement.

13

Universal Quantifier ∀: Example

- Let *P*(*x*) be the *predicate* "*x* is full."
- Let the t.s. of *x* be parking spaces at BZU.
- The *universal quantification of P*(*x*),

  $\forall x\, P(x)$, is the *proposition:*
  - "All parking spaces at BZU are full." or
  - "Every parking space at BZU is full." or
  - "For each parking space at BZU, that space is full."

The Universal Quantifier ∀

- To prove that a statement of the form $\forall x\, P(x)$ is false, it suffices to find a **counterexample** (i.e., one value of x in the universe of discourse such that $P(x)$ is false)
  - e.g., $P(x)$ is the predicate "x>0"
  - Truth set = R
  - X=-2

15

Definition of Counterexample

Note: For a universal statement to be **false** means that there is **at least one** element of the set for which the property is false.

Definition: Given a universal statement of the form "∀ *x* in D, P(*x*)," a **counterexample** for the statement is a value of *x* for which P(*x*) is false.

Ex: True or false?

∀ COMP233 students *x*, *x* has studied COMP142.

Ans: The statement is false. __________ is a counterexample to the statement "∀ COMP 233 students *x*, *x* has studied calculus" because ____________ is a COMP 233 student and ___________ has not studied calculus.

16

Truth and Falsity of Universal Statements

- Let D ={ 1,2,3,4,5}, and consider the statement $\forall x \in D, x^2 \geq x$. Show that this statement is true.
- Check that "$x^2 \geq x$" is true for each individual x in D. $1^2 \geq 1$, $2^2 \geq 2$, $3^2 \geq 3$, $4^2 \geq 4$, $5^2 \geq 5$. Hence "$\forall x \in D, x^2 \geq x$" is true.

- B. Consider the statement $\forall x \in R, x^2 \geq x$.
- True or False
- Find a counterexample to show that this statement is false.
- Counterexample: Take x = 1/2.

Then x is in R (since 1/2 is a real number) and $(½)^2 = 1/4$ *not greater* 1/2 .

Hence "$\forall x \in R, x^2 \geq x$" is false

Given a property, an **existential statement** says that there is at least one element for which the property is true.

∃ stands for the words "there exists" or "there exists at least one" – it is called the **existential quantifier**

Ex: Rephrase the following statement in less formal language, and determine whether it is true or false.

∃ a COMP 233 student *x* such that *x* has studied calculus.

(existential statement)

Ans:

There is a COMP 233 student who has studied calculus.

Some COMP 233 student has studied calculus.

Some COMP 233 students have studied calculus.

At least one COMP 233 student has studied calculus.

Etc.

(Alternative formal version:

∃ *x* such that *x* is a COMP 233 student and *x* has studied calculus.)

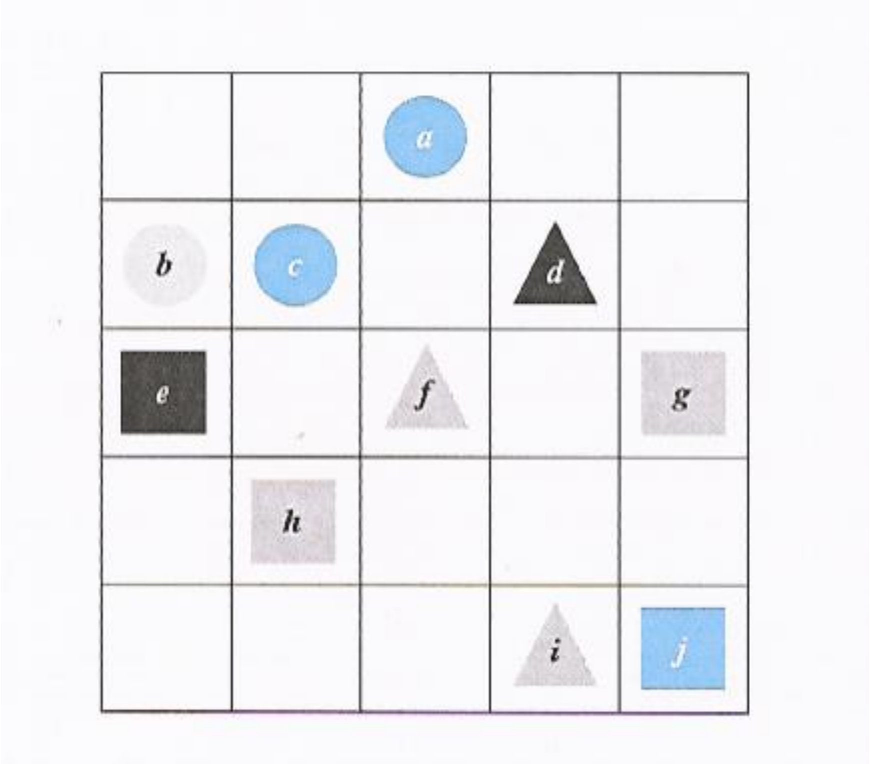
19

Existential Quantifier ∃ Example

- Let $P(x)$ be the *predicate* "x is full."
- Let the T.S. of x be <u>parking spaces at </u>BZU .
- The *universal quantification of* $P(x)$,

  $\exists x\, P(x)$, is the *proposition*:
  - "Some parking spaces at BZU are full." or
  - "There is a parking space at BZU that is full." or
  - "At least one parking space at BZU is full."

Tarski's World Example



True or False?

1. ∀ objects *x*, if *x* is a circle, then *x* is blue.

False, there is a circle, b, that is not blue.

2. ∃ an object *x* such that *x* is a square and *x* is to the left of c.

True, e is a square and e is to the left of c.

Exercises

Example 3.1.4 Truth and Falsity of Existential Statements

a. Consider the statement

$$\exists m \in \mathbf{Z}^+ \text{ such that } m^2 = m.$$

Show that this statement is true.

*In more formal versions of symbolic logic, the words *such that* are not written out (although they are understood) and a separate $\exists$ symbol is used for each variable: "$\exists m \in \mathbf{Z}(\exists n \in \mathbf{Z}(m + n = m \cdot n))$."

b. Let $E = \{5, 6, 7, 8\}$ and consider the statement

$$\exists m \in E \text{ such that } m^2 = m.$$

Show that this statement is false.

Solution

a. Observe that $1^2 = 1$. Thus "$m^2 = m$" is true for at least one integer m. Hence "$\exists m \in \mathbf{Z}$ such that $m^2 = m$" is true.

b. Note that $m^2 = m$ is not true for any integers m from 5 through 8:

$$5^2 = 25 \neq 5, \quad 6^2 = 36 \neq 6, \quad 7^2 = 49 \neq 7, \quad 8^2 = 64 \neq 8.$$

Thus "$\exists m \in E$ such that $m^2 = m$" is false. ■

Verbalizing Formal Statements

Write the following formal statements in an informal language:

$\forall x \in \mathbf{R} \cdot \ x^2 \geq 0$

The square of every real number is greater than or equal to zero

$\forall x \in \mathbf{R} \cdot \ x^2 \neq -1$

The square of any real number does not equal -1

$\exists m \in \mathbf{Z}^+ \cdot \ m^2 = m$

There is a positive integer that is equal to its square

$\forall x \in \mathbf{R} \cdot \ x > 2 \quad \rightarrow x^2 > 4$

If a real number is greater than 2 then its square is greater than 4

Different Writings

$\forall x \in Square$. Rectangle(x)
$\forall x$. If x is a square then x is a rectangle
$\forall$ *Squares x . x is a* a rectangle

Although the book uses this notation but it's not recommended as predicates are not clear.

$\forall p \in$Palestinian . Likes(p, Zatar)
$\forall p$. Palestinian(p) $\wedge$ Likes(p, Zatar)
All Palestinians like Zatar.

$\exists p \in$Person . Likes(p, Zatar)
$\exists p$. Person(p) $\wedge$ Likes(p, Zatar)
Some people like Zatar.

Formalize Statements

Write the following informal statements in a formal language:

All triangles have three sides

$\forall t \in Triangle \cdot \text{ThreeSided}(t)$

No dogs have wings

$\forall d \in \text{D}og \cdot \neg \text{HasWings}(d)$

Some programs are structured

$\exists p \in \text{Program} \cdot \text{structured}(p)$

If a real number is an integer, then it is a rational number

$\forall n \in \text{RealNumber} \cdot \text{Integer}(n) \rightarrow \text{Rational}(n)$

All bytes have eight bits

$\forall b \in \text{Byte} \cdot \text{EightBits}(b)$

No fire trucks are green

$\forall t \in \text{FireTruck} \cdot \neg \text{Green}(t)$

Quantifications might be Implicit

Formalize the following:

If a number is an integer, then it is a rational number.

$\forall n \in \text{RealNumber} \cdot \text{Integer}(n) \rightarrow \text{Rational}(n)$

If a person was born in Hebron then s/he is Khalili

$\forall x \in \text{Person} \cdot \text{BornInHebron}(x) \rightarrow \text{Khalili}(x)$

People who like Homos are smart

$\forall x \in \text{Person} \cdot \text{Like}(x, \mathit{Homos}) \rightarrow \text{Smart}(x)$

Universal Conditional Statements

Definition: Any statement of the following form is called a **universal conditional statement:**

$$\forall\ x \text{ in } D, \text{ if } P(x) \text{ then } Q(x).$$

For all — *a collection of objects* — *properties that x might satisfy*

Universal conditional statements are the most important form of statement in mathematics!

27

Example: A Universal Conditional Statement

$\forall$ real numbers x, if $x > 2$ then $x^2 > 4$.

Less formal versions:

1. All real numbers that are greater than 2 have squares that are greater than 4.
2. If a real number is greater than 2, then its square is greater than 4.

"Implicit" Quantification

More formal versions:

1. $\forall x \in \mathbf{R}$, if $x > 2$ then $x^2 > 4$.
2. $x > 2 \Rightarrow x^2 > 4$.

Notation: The symbol $\Rightarrow$ denotes a "universalized if-then."

So $x > 2 \Rightarrow x^2 > 4$ means $\forall x \in \mathbf{R}, x > 2 \rightarrow x^2 > 4$

28

3.2
PREDICATES AND QUANTIFIED STATEMENTS II

Negations of Quantified Statements

The negation of a statement exactly expresses what it would mean for the statement to be false. Write negations for the following:

1. $\forall$ even integers x, x is positive.
2. $\exists$ an integer n such that n has an integer square root.

Answers

1. $\exists$ an even integer x such that x is not positive.
2. $\forall$ integers n, n does not have an integer square root.

In general:

$\sim(\forall\ x \text{ in D, P}(x)) \equiv \exists\ x \text{ in D such that} \sim\text{P}(x)$
$\sim(\exists\ x \text{ in D such that P}(x)) \equiv \forall\ x \text{ in D,} \sim\text{P}(x)$

So: The negation of a "for all" statement is a "there exists" statement, and the negation of a "there exists" statement is a "for all" statement.

30

Negation of a Universal Conditional Statement

~(∀ x in D, if P(x) then Q(x)) ≡ ?

≡ ∃ x in D such that P(x) and ~Q(x)

Exercise: Write a negation for the following statement:

∀ real numbers x, if $x^2 > 4$ then $x > 2$.

∃Real number x, $x^2 > 4$ and x ≤ 2.

31

Negations of Quantified Statements

All Palestinians like Zatar

Some Palestinians do not like Zatar

Some Palestinians Like Zatar

All Palestinians do not like Zatar

$\forall\, p \in \text{Prime} \,.\, \text{Odd}(p)$

$\exists\, p \in \text{Prime} \,.\, \sim\text{Odd}(p)$

Some computer hackers are over 40

All computer hackers are not over 40

All computer programs are finite

Some computer programs are not finite

No politicians are honest

Some politicians are honest

Negations of Quantified Statements

$\forall x\ .\ P(x) \rightarrow Q(x)$

$\exists x\ .\ P(x) \wedge \sim Q(x)$

$\forall p \in \text{Person}\ .\ \text{Blond}(p) \rightarrow \text{BlueEyes}(p)$

$\exists p \in \text{Person}\ .\ \text{Blond}(p) \wedge \sim \text{BlueEyes}(p)$

If a computer program has more than 10000 lines then it contains a bug

a computer program has more than 10000 and does not contains a bug

Quantifier Equivalence Laws

- Definitions of quantifiers: If u.d.=a,b,c,…
  $\forall x\, P(x) \Leftrightarrow P(\mathrm{a}) \wedge P(\mathrm{b}) \wedge P(\mathrm{c}) \wedge \ldots$
  $\exists x\, P(x) \Leftrightarrow P(\mathrm{a}) \vee P(\mathrm{b}) \vee P(\mathrm{c}) \vee \ldots$

Variants of Universal Conditional Statements

Consider a statement of the form: $\forall x \in D \;.\; P(x) \rightarrow Q(x)$.

1. Its **contrapositive** is the statement:

   $\forall x \in D \;.\; \sim Q(x) \rightarrow \sim P(x)$

2. Its **converse** is the statement:

   $\forall x \in D \;.\; Q(x) \rightarrow P(x)$.

3. Its **inverse** is the statement:

   $\forall x \in D \;.\; \sim P(x) \rightarrow \sim Q(x)$.

$\forall x \in$ Person . Palestinian(x) $\rightarrow$ Smart(x)

Contrapostive: $\forall x \in$ Person . ~ Smart(x) $\rightarrow$ ~Palestinian(x)

Converse: $\forall x \in$ Person . Smart(x) $\rightarrow$ Palestinian(x)

Inverse: $\forall x \in$ Person . ~Palestinian(x) $\rightarrow$ ~Smart(x)

Variants of Universal Conditional Statements

$$\forall x \in \mathbf{R} \ .\ \text{MoreThan}(x,2) \rightarrow \text{MoreThan}(x^2,4)$$

$$\forall x \in \mathbf{R} \ .\ x > 2 \rightarrow x^2 > 4$$

Contrapostive: $\forall x \in \mathbf{R} \ .\ x^2 \leq 4 \rightarrow x \leq 2$

Converse: $\forall x \in \mathbf{R} \ .\ x^2 > 4 \rightarrow x > 2$

Inverse: $\forall x \in \mathbf{R} \ .\ x \leq 2 \rightarrow x^2 \leq 4$

Necessary and Sufficient Conditions

"$\forall x \,.\, r(x)$ is a **sufficient condition** for $s(x)$" means "$\forall x \,.\, r(x) \rightarrow s(x)$"

"$\forall x \,.\, r(x)$ is a **necessary condition** for $s(x)$" means "$\forall x, \sim r(x) \rightarrow \sim s(x)$"
or, equivalently, "$\forall x, s(x) \rightarrow r(x)$"

"$\forall x \,.\, r(x)$ **only if** $s(x)$" means "$\forall x, \sim s(x) \rightarrow \sim r(x)$"
or, equivalently, "$\forall x$, if $r(x)$ then $s(x)$"

Examples

Squareness is a sufficient condition for rectangularity.

If something is a square, then it is a rectangle.

$\forall x \,.\,$ Square(x) $\rightarrow$ Rectangular(x)

To get a job it is sufficient to be loyal.

If one is loyal (s)he will get a job

$\forall x \,.\,$ Loyal(x) $\rightarrow$ GotaJob(x)

Necessary and Sufficient Conditions

"$\forall x \,.\, r(x)$ is a **sufficient condition** for $s(x)$" means "$\forall x \,.\, r(x) \rightarrow s(x)$"

"$\forall x \,.\, r(x)$ is a **necessary condition** for $s(x)$" means "$\forall x, \sim r(x) \rightarrow \sim s(x)$"
or, equivalently, "$\forall x, s(x) \rightarrow r(x)$"

"$\forall x \,.\, r(x)$ **only if** $s(x)$" means "$\forall x, \sim s(x) \rightarrow \sim r(x)$"
or, equivalently, "$\forall x$, if $r(x)$ then $s(x)$"

Examples

Being smart is necessary to get a job.

If you are not smart you don't get a job

If you got a job then you are smart

$\forall x \,.\, \sim\text{Smart}(x) \rightarrow \sim\text{GotaJob}(x)$

$\forall x \,.\, \text{GotaJob}(x) \rightarrow \text{Smart}(x)$

Being above 40 years is necessary for being president of Palestine

$\forall x \,.\, \sim\text{Above}(x, 40) \rightarrow \sim\text{CanBePresidentOfPalestine}(x)$

$\forall x \,.\, \text{CanBePresidentOfPalestine}(x) \rightarrow \text{Above}(x, 40)$

Necessary and Sufficient Conditions

"$\forall x \,.\, r(x)$ is a **sufficient condition** for $s(x)$" means "$\forall x \,.\, r(x) \rightarrow s(x)$"

"$\forall x \,.\, r(x)$ is a **necessary condition** for $s(x)$" means "$\forall x, \sim r(x) \rightarrow \sim s(x)$"
or, equivalently, "$\forall x, s(x) \rightarrow r(x)$"

"$\forall x \,.\, r(x)$ **only if** $s(x)$" means "$\forall x, \sim s(x) \rightarrow \sim r(x)$"
or, equivalently, "$\forall x$, if $r(x)$ then $s(x)$"

Examples

You get the job only if you are the top.

If you are not a top you will not get a job

If you got the job then you are a top

$\forall x \,.\, \sim\text{Top}(x) \rightarrow \sim\text{GotaJob}(x)$

$\forall x \,.\, \text{GotaJob}(x) \rightarrow \text{Top}(x)$

3.3
STATEMENTS WITH MULTIPLE QUANTIFIERS

Multiple-Quantifier Statements

Statements with more than one quantifier

Examples: What do the following statements mean?

1. ∀ integers x, ∃ an integer y such that $y < x$.

This means: No matter what integer you might pick, there is an integer that is less than the one you picked.

For every integer, there exist another integer which it is less than it.

2. ∃ a positive integer x such that ∀ positive integers y, $x \leq y$.

This means: There is a positive integer that is less than or equal to every positive integer.

I.e., there is a smallest positive integer

41

Formalize these statements

There Is a Smallest Positive Integer

$\exists m \in Z^+ \ \forall n \in Z^+ . \text{LessOrEqual}(m,n)$

There Is No Smallest Positive Real Number

$\forall x \in R^+ \ \exists y \in R^+ \ . \ \text{Less}(y,x)$

Formalize these statements

The reciprocal of a real number a is a real number b such that ab = 1. The following two statements are true. Rewrite them formally using quantifiers and variables:

Every nonzero real number has a reciprocal.

$\forall u \in \text{NonZeroR}, \ \exists v \in \text{R} \ . \ uv = 1.$

There is a real number with no reciprocal.

$\exists c \in \mathbf{R} \ \forall d \in \mathbf{R},. \ cd \neq 1.$

The number 0 has no reciprocal.

A college cafeteria line has four stations: salads, main courses, desserts, and beverages. The salad station offers a choice of green salad or fruit salad; the main course station offers spaghetti or fish; the dessert station offers pie or cake; and the beverage station offers milk, soda, or coffee. Three students, Uta, Tim, and Yuen, go through the line and make the following choices:

Uta: green salad, spaghetti, pie, milk

Tim: fruit salad, fish, pie, cake, milk, coffee

Yuen: spaghetti, fish, pie, soda

These choices are illustrated in Figure 3.3.2.

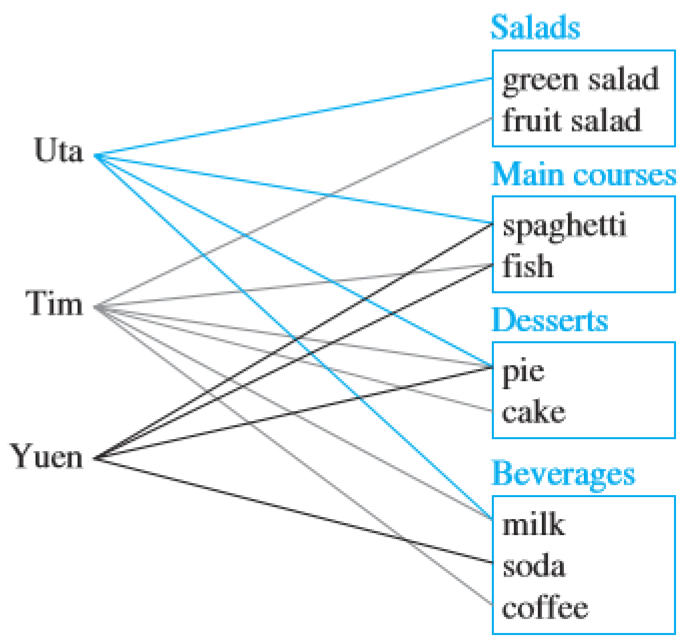
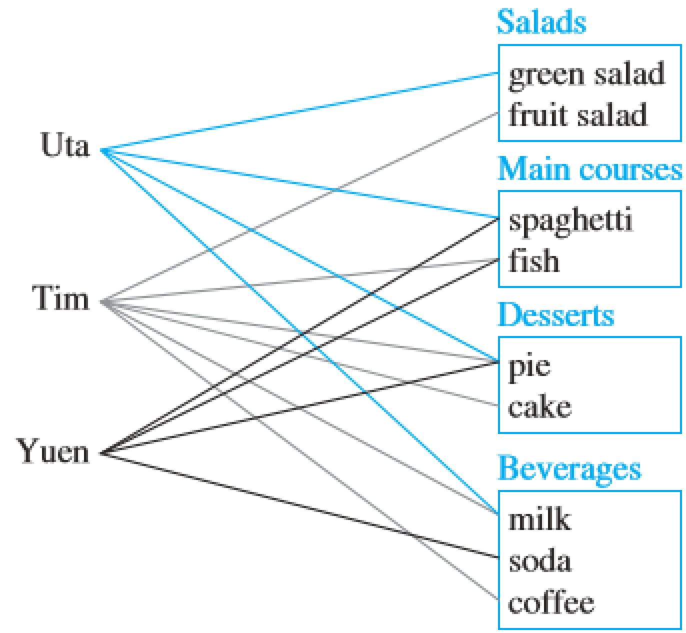


Figure 3.3.2



Write each of following statements informally and find its tr

a. $\exists$ an item I such that $\forall$ students S, S chose I.

b. $\exists$ a student S such that $\forall$ items I, S chose I.

c. $\exists$ a student S such that $\forall$ stations Z, $\exists$ an item I in Z such that S chose I.

d. $\forall$ students S and $\forall$ stations Z, $\exists$ an item I in Z such that S chose I.

Solution

a. There is an item that was chosen by every student. This is true; every student chose pie.

b. There is a student who chose every available item. This is false; no student chose all nine items.

c. There is a student who chose at least one item from every station. This is true; both Uta and Tim chose at least one item from every station.

d. Every student chose at least one item from every station. This is false; Yuen did not choose a salad. ■

Order of Quantifiers

Order matters		
	$\forall x \exists y . Loves(x,y)$ Everyone loves someone	$\exists x \forall y . Loves(x,y)$ Someone loves everyone
	$\forall y \exists x . Loves(x,y)$ Everyone loved by someone	$\forall y \exists x . Loves(y,x)$ Everyone loves someone
Order doesn't matters	$\forall x \forall y . Loves(x,y)$ $\forall x,y \;\; . Loves(x,y)$ Everyone loves everyone	$\exists x \exists y . Loves(x,y)$ $\exists x,y \;\; . Loves(x,y)$ someone loves someone

Order of Quantifiers Is Important!!

If $P(x,y)$= "x relies upon y," express the following in unambiguous English:

$\forall x \exists y \, P(x,y)$ = Everyone has *someone* to rely on.

$\exists y \forall x \, P(x,y)$ = There's someone whom *everyone* relies upon him

$\exists x \forall y \, P(x,y)$ = There's someone who relies upon *everyone* (including himself).

$\forall y \exists x \, P(x,y)$ = Everyone has *someone* who relies upon him.

$\forall x \forall y \, P(x,y)$ = *Everyone* relies upon *everyone*, (including themselves)!

Determine whether these two statements are true or false.

- $\forall x \in Z \;\; \exists y \in R^{*} (xy < 1)$

- $\exists y \in R^{*} \;\; \forall x \in Z (xy < 1)$

- Here, R^{*} denotes the set of all nonzero real numbers.

Negations of Multiply-Quantified Statements

~(∀ x in D, ∃ y in E such that P(x,y))

≡ ∃ x in D such that ∀ y in E, ~P(x,y)

~(∃ x in D such that ∀ y in E, P(x,y))

≡ ∀ x in D, ∃ y in E such that ~P(x,y)

49

Negations of Multiply-Quantified Statements

$\sim(\forall x \text{ in } D, \exists y \text{ in } E \text{ such that } P(x, y)) \equiv \exists x \text{ in } D \text{ such that } \forall y \text{ in } E, \sim P(x, y).$

$\sim(\exists x \text{ in } D \text{ such that } \forall y \text{ in } E, P(x, y)) \equiv \forall x \text{ in } D, \exists y \text{ in } E \text{ such that } \sim P(x, y).$

Examples:

$\sim (\forall x \; \exists y \; . \; Loves\,(x,y))$

$\exists x \; \forall y \; . \sim Love(x,y)$

$\sim(\exists x \; \forall y \; . \; Loves\,(x,y))$

$\forall x \; \exists y \; . \sim Love(x,y)$

Negations of Multiply-Quantified Statements

Not all people love someone.

~ (all people love someone)

$\sim(\forall x \exists y \,.\, \text{Love}(x,y))$

$\exists x \ \forall y \,.\, \sim\text{Love}(x,y))$

Some people do not love everyone

Not all people love everyone.

~ (All people love everyone)

$\sim \forall x \forall y \ \text{Like}(x, y)$

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