Prove that the qouent of any two rational number is rational number.

Proof: 1) $\forall$ integer numbers. If a, and b are rational then a/b is rational

2) From the given a, b are rational

$\therefore$ $a = \frac{r}{s}$ $s \neq 0$, r, s are integer

$b = \frac{q}{t}$ $t \neq 0$, q, t " "

[Show] : a/b is rational

by substitution in $(a \& b)$

$$\frac{r/s}{q/t} = \frac{r}{s} \times \frac{t}{q} = \frac{rt}{sq}, \quad sq \neq 0$$

$$s \neq 0$$

Let $w = rt$ and $z = sq$, w, z are integer

Since $\frac{w}{z} \Rightarrow \therefore$ rational a, b, c integers

If $\underbrace{a \mid b \text{ and } b \mid c}$ $\longrightarrow$ $a \mid c$. <u>True</u>

$\Leftarrow)$ $a \mid b \Rightarrow b = aK_1$ — ① K_1: integer

$b \mid c \Rightarrow c = bK_2$ — ② K_2: integer

Substitute: 1 & 2

$$c = (aK_1)(K_2) = a\overbrace{(K_1K_2)}^{K} \quad \text{Assosiative}$$

$$c = aK \Rightarrow \frac{c}{a} = K \Rightarrow \boxed{a \mid c}$$

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If $a | b$ and $b | a \rightarrow a = b$ ✗

T or F?

• Ex: $a = 4$, $b = -4$, $a \neq b$

$\Leftrightarrow$ $b = ak_1$ $a = bk_2$ k_1, k_2 integer

by Sub.

$a = (ak_1)k_2$

$\cancel{a} = \cancel{a}(k_1 k_2)$

$1 = k_1 k_2$

$k_1 = 1$ $k_2 = 1$

$k_1 = -1$, $k_2 = -1$

$$n = \underbrace{p_1^{e_1} \cdot p_2^{e_2}} \cdot p_3^{e_3} \cdots\cdots p_k^{e_k}$$

Ex:

$$15 = 3 \times 5 = 3^1 \times 5^1$$

$$45 = 3 \times 3 \times 5 = 3^2 \times 5^1$$

$$27 = 3 \times 3 \times 3 = 3^3$$

$$54 = 2 \times 3 \times 3 \times 3 = 2^1 \times 3^3$$

$\left.\begin{matrix}\text{Even} \\ \text{odd}\end{matrix}\right\}$ C / Java

$n \left\{\begin{matrix}\text{is even} \\ \text{is odd}\end{matrix}\right\}$

~~q~~ divide by $\underline{2}$

$\therefore$ Using QRT

$n = dq + r$

$0 \le r < d$

$\Rightarrow \underline{\underline{d = 2}}$

$n = 2q + r$

$0 \le r < 2$

$r = 0, 1$

$\therefore n \left\{ \underset{\text{even}}{2q}, \underset{\text{odd}}{2q+1} \right\}$

EX: prove that any integer number can be written as the following sequence.

$$3q \,,\; 3q+1 \,,\; 3q+2$$

$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \ldots$

$\ldots \}$

$q=0$

$q=1$

$q=2$

using QRT

$n = dq + r \quad 0 \le r < d$

$n = 3q + r \quad 0 \le r < 3$

$r = 0, 1, 2$

$\therefore \; n = 3q \,,\; 3q+1 \,,\; 3q+2$

$\forall$ integer number if m and $m+1$ is consequative, then m is either odd or even and n is either odd or even.

$\cancel{A} = \cancel{m} + 1$

Show: Suppose that m is [pbac] element is either odd or even

[show that $m+1$] is either odd or even

Case(1): if m is even $\Rightarrow m = 2K$ $\quad K$: integer

Show $(m+1) = (2K+1) \Rightarrow$ odd

Case(2): if m is odd $m = 2S+1$ $\quad S$: integer

$m = (2s+1)$

$m+1 = (2s+1)+1 = 2s+2 = 2(s+1)$

let $t = s+1$ t is integer since

s integer , 1 intege

Integer + Integer = Int

$\therefore$

$m = 2t \Rightarrow$ definition of even.