Prove: The square of any odd integer has the form 8m + 1 for some integer m.

*1) ~~Fo~~ Formal

$\forall_{n}$, n is odd, $\exists_m$ Such that

$n^2 = 8m + 1$

Proof: *(2) Suppose that n is odd that

[pbac]. [Show $n^2 = 8m+1$]

for m is any integer

$*$ $\Rightarrow$ n is odd $n = 2k+1$ for k is integer

$\Rightarrow$ $n^2 = (2k+1)^2$?? $8m+1$

$n^2 = (4k^2+4k+1)$

$= 4(k^2+k)+1$

$\frac{n^2-1}{4} = m$

let $m = k^2+k$ $\Rightarrow$ m is integer [$k*k + k$ ...]

$\therefore$ $\boxed{n^2 = 4m+1}$ α

$4(2m)+1$

$\Rightarrow 8m+1$ blind alley

$4 = \frac{8}{2}$

QRT $\quad n = dq + r, \quad 0 \leq r < d$

$\Rightarrow$ even, $2q$ // odd $2q+1$

$\Rightarrow$ $3q$, $3q+1$, $3q+2$ } Represent any integer

$q=0$
$q=1$ $\{0, 1, 2, 3, 4, 5, 6, 7, 8 \ldots\}$

q^2

$$\boxed{n = 4q + r} \quad 0 \leq r < 4$$

$r = 0, 1, 2, 3$

$2(2q)$ $\quad 2(2q+1)$

even?? odd?? $4q$, $4q+1$, $4q+2$, $4q+3$

$n^2 = 8m+1$

Case I $(\underline{\underline{4q+1}})$:

$$n^2 = (4q+1)^2 = 16q^2 + 8q + 1$$

$$= 8(\underline{2q^2 + q}) + 1$$

Let $m = 2q^2 + q \Longrightarrow$ integer

$$\boxed{\therefore n^2 = 8m+1}$$ # ✓

Case II $(\cancel{2}4q+3)$

$$n^2 = (4q+3)^2$$

$$= 16q^2 + 24q + 9$$

$$= 16q^2 + 24q + 8 + 1$$

$$= 8\underbrace{(2q^2 + 3q + 1)}_{m} + 1$$

Let $m =$

$\therefore \boxed{n^2 = 8m + 1}$

Prove that the sum of any four consecutive $\frac{39}{189}$ integer numbers has the form $4m+2$

$\forall$ Sum of 4 consecutive integer number has form $4m+2$ where m is integer

<u>Proof</u>: Suppose that n is [pobac], [show that sum $n, n+1, n+2, n+3$ has form $4m+2$]

__Proof:__ $n + (n+1) + (n+2) + (n+3)$

$$= \underline{\underline{4n+6}}$$

two __Cases:__

Case (n is odd): $n = 2K+1$ for K is integer

$$\Rightarrow 4(2K+1)+6 \quad \text{by Sub.}$$

$$= 8K+4+6 = 8K+10 \rightarrow 8+2$$

$$= 4(2K+2)+2$$

$$= 4m+2$$

. Case (n is even) : $n = 2k$ for some k
k is integer

$\Rightarrow$ $4n+6$

$$\Rightarrow 4n+6 = 4(2k)+6$$
$$= 8k+6$$
$$= 8k+4+2$$
$$= 4(2k+1)+2$$
$$= 4m+2$$

$\times\!\times$

Prove that for any integer n, n^2+5 is not divisable by 4.

$\forall n$, n is integer number. $4 \nmid n^2+5$

Suppose that n is [pbac]. Show $4 \nmid n^2+5$

Proof:

1) Case n is odd: $n = 2k+1$ for k is integer

$$(2k+1)^2+5 = 4k^2+4k+1+5$$

$$= 4\cancel{k}^2+4k+4+2$$

$$= 4(k^2+k+1)+2$$

$$= 4m+2$$

$r = 2$

Prove that for all integers n, if

$n \bmod 5 = 3$ then $n^2 \bmod 5 = 4$?

QRT $\quad n = dq + r \qquad r = 3$

$d = 5$

$\therefore n = 5q + 3 \qquad 0 \leq r < 5$

$\therefore n^2 = (5q+3)^2 = 25q^2 + 30q + 9 \rightarrow 5 + 4$

$= 5(5q^2 + 6q + 1) + 4$

$= 5m + 4$ ✓

Case n is even

$\cdot \ (2k)^2 + 5 = 4k^2 + 4 + 1$

$= 4(k^2 + 1) + 1$

$= 4\underset{\sim}{m} + 1$

Let $\boxed{m = k^2 + 1}$ $\Rightarrow$ m is integer

$\boxed{r = 1}$