

Prove: The square of any odd integer has the form $8m + 1$ for some integer m .

*1) Formal

$\forall n, n$ is odd, $\exists m$ such that

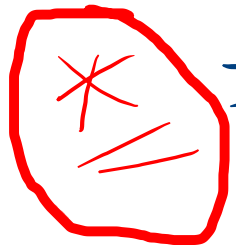
$$n^2 = 8m + 1$$

Proof: *2)

Suppose that n is odd that

[pbaac] - ^{Show} $n^2 = 8m + 1$

for m is any integer



n is odd $n = 2k+1$ for k is integer

$\Rightarrow n^2 = (2k+1)^2$ $??$ $8m+1$

$n^2 = (4k^2 + 4k + 1)$

$= 4(k^2 + k) + 1$

Let $m = k^2 + k \Rightarrow m$ is integer

$[k^2 + k]$

$\therefore n^2 = 4m + 1$

$\Rightarrow 4 \rightarrow (2m) + 1$
 $\Rightarrow 8m + 1$

blind alley

$n^2 - 1 = m$



$\sqrt{8/2}$

QRT $n = dq + r$, $0 \leq r < d$

✓ \Rightarrow even, $2q$ // odd $2q + 1$

✓ \Rightarrow $3q$, $3q + 1$, $3q + 2$ } Represent any integer

$q=0$
 $q=1$ { $0, 1, 2, 3, 4, 5, 6, 7, 8, \dots$ }

q^2

$n = 4q + r$

$0 \leq r < 4$

$r=0, 1, 2, 3$

Even?
Odd?

$4q, 4q + 1, 4q + 2, 4q + 3$

$$n^2 = 8m + 1$$

Case I (4q+1):

$$n^2 = (4q+1)^2 = 16q^2 + 8q + 1$$

$$= 8(2q^2 + q) + 1$$

Let $m = 2q^2 + q \implies$ integer

$$\boxed{\therefore n^2 = 8m + 1}$$

✓

Case II (~~4~~9 + 3)

$$n^2 = (49 + 3)^2 \quad 8 + 1$$

$$= 169^2 + 249 + 9$$

$$= 169^2 + 249 + 8 + 1$$

$$= 8(29^2 + 39 + 1) + 1$$

Let $m =$

m

\therefore

$$n^2 = 8m + 1$$

~~*~~

Prove that ^{the sum} of any four consecutive integer numbers has the form $4m+2$

\forall Sum of 4 consecutive integer number has form $4m+2$ where m is integer

Proof: Suppose that n is $[pba c]$ (show that sum $\cdot n, n+1, n+2, n+3$ has form $4m+2$)

Proof: $n + (n+1) + (n+2) + (n+3)$

$= \underline{\underline{4n+6}}$

Two Cases:



Case (n is odd): $n = 2k+1$ for k is integer

$\Rightarrow 4(2k+1) + 6$ by Sub.

$= 8k + 4 + 6 = 8k + 10 \rightarrow 8+2$
 $= 4(2k+2) + 2$
 $= 4m + 2$ ✘

Case (n is even):

$n = 2k$ for some k
 k is integer

$$\Rightarrow \underline{4n + 6}$$

$$\begin{aligned} \Rightarrow 4n + 6 &= 4(2k) + 6 \\ &= 8k + 6 \\ &= 8k + 4 + 2 \\ &= 4(2k + 1) + 2 \\ &= 4m + 2 \end{aligned}$$

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Prove that for any integer n , $n^2 + 5$ is not divisible by 4.

$\forall n$, n is integer number. $4 \nmid n^2 + 5$
Suppose that n is [pbac]. Show $4 \nmid n^2 + 5$

Proof:

1) Case n is odd:

$n = 2k + 1$ for k is integer

$$\begin{aligned}(2k+1)^2 + 5 &= 4k^2 + 4k + 1 + 5 \\ &= 4k^2 + 4k + 4 + 2 \\ &= 4(k^2 + k + 1) + 2 \\ &= 4m + 2 \quad \underline{\underline{r=2}}\end{aligned}$$

Prove that for all integers n , if

$$n \pmod{5} = 3 \quad \text{then} \quad n^2 \pmod{5} = 4?$$

QRT $n = dq + r$ $\underline{r=3}$
 $\underline{d=5}$

$$\therefore n = 5q + 3 \quad 0 \leq r < 5$$

$$\therefore n^2 = (5q + 3)^2 = 25q^2 + 30q + 9 \rightarrow 5+4$$

$$= 5(5q^2 + 6q + 1) + 4$$

$$= 5m + 4 \quad \checkmark$$

Case n is even

$$\begin{aligned} \cdot (2k)^2 + 9 &= 4k^2 + 4 + 1 \\ &= 4(k^2 + 1) + 1 \\ &= 4 \underbrace{m} + 1 \end{aligned}$$

Let $\boxed{m = k^2 + 1} \Rightarrow m$ is integer

$$\boxed{r = 1}$$