Theorm: $1+2+3+\cdots\cdots+n = \frac{n(n+1)}{2}$ $\Rightarrow$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $P(n)$

for all integer $n \geq 1$

Using Mathmatical Induction prove that this theorm.

$[n \geq a]$

(1) * Let property $P(n)$ is equation

Show $1+2+3+\cdots\cdots+n = \frac{n(n+1)}{2} \leftarrow P(n)$

2 steps
1) basis
2) Inductive

1) Step 1 (basis): Show that $P(1)$ is true

Left side, Right side. $[L \overset{??}{=} R]$

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, $\sum_{i=1}^{1} i \overset{?}{=} \frac{1(1+1)}{2}$

$1 = 1$ ✓ true

Step 2 [inductive]: Show that for all $k \geq 1$

if $P(k)$ is true, then $P(k+1)$ is also true

Suppose that $P(k)$ is pbac for integer $k \geq 1$

Suppose that k is any integer with property $k \geq 1$

$$\sum_{i=1}^{k} i$$

$$\underbrace{1 + 2 + 3 + \cdots + k}_{} = \frac{k(k+1)}{2} \leftarrow \text{inductive hypothesis}$$

$\Rightarrow$ we want show that $P(k+1)$ is true.

$\therefore$ I want to show:

$$1 + 2 + 3 + \cdots + (k+1) = \frac{(k+1)(k+2)}{2}$$

Next term

$$\underbrace{1+2+3+\cdots+k}+(k+1)$$

$$\frac{k(k+1)}{2}$$

from inductive hypothesis

Left → Right

$$1+2+3+\cdots+(k+1)$$

$$\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2} \quad ??$$

$$= \frac{(k+1)(k+2)}{2} \quad \#$$

$$\Rightarrow \sum_{i=1}^{k+1} i = \sum_{i=1}^{k} i + (k+1)$$

$$\frac{k(k+1)}{2}$$ from Inductive hypothesis

$$= \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{k(k+1)+2(k+1)}{2} = \frac{k^2+k+2k+2}{2}$$

$$= \frac{k^2+3k+2}{2} = \frac{(k+1)(k+2)}{2}$$

#

$$\sum_{i=1}^{7} 2^i = \sum_{i=1}^{6} 2^i + 2^7$$

Evaluate:

1) $2+4+6+\cdots+500$

$$= 2[1+2+3+\cdots+250]$$

$$= 2\sum_{i=1}^{250} i = \cancel{2}\left[\frac{250 \times 251}{\cancel{2}}\right] = 62\,750$$

2) $5+6+7+\cdots+50$

$$\sum_{i=1}^{50} i - 10 = \frac{50(51)}{2} - 10 = 1265$$

3) $9+12+15+\cdots+150$

$$3[3+4+5+\cdots+50]$$

$$= \left(3\sum_{i=1}^{50} i\right) - 9 = 3\,\frac{(50)51}{2} - 9 = 3786$$

Theorm: $\sum_{i=0}^{n} r^i = \frac{r^{n+1} - 1}{r-1}$. r is real number $r \neq 1$

Using Mathmatical induction to prove this theorm.

1) Let Property $P(n)$ for all $n \geq 0$, all real number r, $r \neq 1$

$$\sum_{i=0}^{n} r^i = \frac{r^{n+1} - 1}{r-1} \quad r \neq 0 \quad \leftarrow P(n)$$

Step 1 (Basis): Show that $P(a)$ is true, $a = 0$, $P(0)$ is true for Both Sides. [Left and right]

Left: $\sum_{i=0}^{0} r^i = 1$, Right $\frac{r^{0+1} - 1}{r-1} = \frac{r-1}{r-1} = 1$

$\Rightarrow$ Left = right

$1 = 1$ ✓

Step2 [inductive]: Show that for all integer $k \geq 0$
if $P(k)$ is true, then $P(k+1)$ is also true.

Suppose that $P(k)$ is true for pbac, $k \geq 0$
k is integer number

Let k be any integer with $k \geq 0$ and suppose
that

$$\sum_{i=0}^{k} r^i = \frac{r^{k+1} - 1}{r - 1} \quad r \neq 1$$ ← Inductive hypothesis

We must show that $P(k+1)$ is true.

to show $$\sum_{i=0}^{k+1} r^i = \frac{r^{k+2}-1}{r-1} \longleftarrow P(k+1)$$

$$\sum_{i=0}^{k+1} r^i = \left(\sum_{i=0}^{k} r^i\right) + r^{k+1}$$

$$= \frac{r^{k+1}-1}{r-1} + r^{k+1} \Longrightarrow$$ by substitution of inductive hypothesis

$$= \frac{r^{k+1}-1}{r-1} + \frac{(r-1)\,r^{k+1}}{(r-1)} = \frac{\cancel{r^{k+1}} - 1 + r\,r^{k+1} \cancel{- r^{k+1}}}{r-1}$$

$$= \frac{r^{k+2}-1}{r-1}$$

$\longleftarrow P(k+1)$ is proved.

a. $1+3+3^2+\cdots+3^{n-2}$

✓ $\displaystyle\sum_{i=0}^{m-2} (3)^i = \frac{3^{m-1}-1}{2}$

$\left[r=3,\ r=4 \right.$
$\left. r=5 \right]$

b) $3^2+3^3+3^4+\cdots+3^m$

$3^2\left[1+3+3^2+\cdots+3^{m-2}\right]$

$= 3^2 \displaystyle\sum_{i=0}^{m-2} 3^i = 9\left[\frac{3^{m-1}-1}{2}\right]$