Mathmatical Induction Part II:

Ex: Prove using mathmatical induction that:

$$\prod_{i=2}^{n}\left(1-\frac{1}{i}\right)=\frac{1}{n} \quad \text{for all integer } n \geq 2.$$

Let Property $P(n)$ is defined for all integer $n \geq 2$, such that

$$\prod_{i=2}^{n}\left(1-\frac{1}{i}\right)=\frac{1}{n} \longleftarrow P(n)$$

Step1 (Basis): Show that $P(a)$ is true, i.e thats mean $P(2)$ is true for Both Sides

$$\prod_{i=2}^{2}\left(1-\frac{1}{i}\right)=\left(1-\frac{1}{2}\right)=\frac{1}{2}, \quad \frac{1}{2} \quad \therefore \frac{1}{2}=\frac{1}{2} \checkmark$$

Step 2: (Inductive): Suppose that $P(K)$ is true for some integer $K \geq 2$, that

$$\prod_{i=2}^{K}\left(1-\frac{1}{K}\right) = \frac{1}{K} \longleftarrow P(K)$$

inductive hypothesis

we want to show $P(K+1)$ is also true.

this mean: [Try to Prove: $\prod_{i=2}^{K+1}\left(1-\frac{1}{i+1}\right) = \frac{1}{K+1} \leftarrow P(K+1)$]

substitute inductive hypothesis

Proof: Left $\to$ Right

$$\prod_{i=2}^{K+1}\left(1-\frac{1}{i}\right) = \underbrace{\left(\prod_{i=2}^{K}\left(1-\frac{1}{i}\right)\right)}\left(1-\frac{1}{K+1}\right) = \frac{1}{K}\left(1-\frac{1}{K+1}\right)$$

$$= \frac{1}{K}\left(\frac{K+\cancel{1}-\cancel{1}}{K+1}\right) = \frac{1}{\cancel{K}}\left(\frac{\cancel{K}}{K+1}\right) = \frac{1}{K+1} \quad \#$$

Ex2: Using Mathmatical Induction Prove that

$2^{2n}-1$ is divisable by 3. $n \geq 0$

Let property $P(n)$ is defined for all integer $n \geq 0$

Such that $\boxed{3 \mid 2^{2n}-1, n \geq 0 \leftarrow P(n)}$

Step1 (Basis): Show that $P(0)$ is true. we want to show that $P(0)$ is true for Both side ($P(n)$)

QRT: $2^{2n}-1 = 3q + r$ (↓ Zero) $r=0$

$\therefore 2^{2n}-1 = 3q$

$P(0)$: $2^{2\cdot 0}-1 = 1-1 = 0$, $= 3q$ 0

$\boxed{0=0}$ ✓

$0 = 3q \Rightarrow \frac{0}{3} = q$ ✓

Step 2 (Inductive): Suppose that $P(K)$ is true
for integer $K \geq 0$, such that

$3 \mid 2^{2K} - 1$ ← $P(K)$

Inductive hypothesis

$K \Rightarrow \boxed{2^{2K} - 1 = 3r \quad \leftarrow P(K)}$

$= 3(2^{2K} + r)$

Show, this is true for $P(K+1)$ also.

$K+1 \Rightarrow 2^{2(K+1)} - 1 \stackrel{?}{=} 3q$

$2^{2(k+1)} - 1 = 2^{2k+2} - 1 = 2^{2k} \cdot 2^{2} - 1 = 4 \cdot 2^{2k} - 1$

$= 2^{2k}(3+1) - 1 = 3\,2^{2k} + \underbrace{2^{2k} - 1}$

$= 3\,2^{2k} + 3r$

$2^{2(k+1)} - 1 = 3q$

Ex 3: Using M.I Prove that

$$2n+1 < 2^n \quad n > 2$$

$\Rightarrow$ Let Property $P(n)$ is defined for all integer $n > 2$. Such that

$$\boxed{2n+1 < 2^n \quad n > 2 \quad \leftarrow P(n)}$$

Basis Step: Show that $P(a)$ is true. Show $P(3)$ is true for Property $P(n)$

$$2(3)+1 = 7 \overset{?}{<} 2^3 , \quad 7 < 8 \checkmark$$

<u>Step2: Inductive</u>: Suppose that $P(K)$ is true such that

phac integer $K > 2$

$\Rightarrow 2K+1 < 2^K \quad K > 2 \quad \leftarrow P(K)$

Inductive hypothesis

Show that $P(K+1)$ is true also.

We want to show $\left[\, 2(K+1)+1 < 2^{K+1} \quad K > 2 \,\right]$

Next step $\underline{K+1}$

$2(K+1)+1 = 2k+2+1 = 2K+3$

$= \boxed{(2K+1)}+2 < 2^K+2$

substitue by inductive hypothesis

$\cancel{2^{k+1}}$

$$< 2^k + 2$$

$$< 2^k + 2^k$$

$$< 2 \cdot 2^k$$

$$< 2^{k+1}$$

$$2(k+1)+1 < 2^{k+1}$$

for all $k \geq 2$

A

```c
int n;
double Sum=0, r;
Scanf("%d", &n);
if(r != 1)
{
    for(int i=0; i<=n; i++)
        Sum+=pow(r,i);
    printf("%f\n", Sum);
}
```

$$\sum_{i=0}^{n} r^i$$

$$\prod_{i=2}^{n}\left(1-\frac{1}{i}\right)=\frac{1}{n}$$

B

```c
int n;
double Sum=0;
Scanf("%d", &n);
Scanf("%lf", &r);
if(r != 1){
    Sum = (pow(r, n+1) -1) /(r-1);
    printf("%f", Sum");
}
```

$$\frac{r^{n+1}-1}{r-1}$$

Using Mathmatical Induction Prove that Both Codes Print Same Result ?

Is $2^{50}-1$ divides by 63?

a) True

b) False

c) Can't decided.

```c
int n;
Scanf("%d", &n);
if(n >= 0) {
    if ((int)(pow(2, 2*n)) - 1) % 3 == 0)
        printf("This is true");
    else
        printf("This is false");
}
```

$$3 \mid 2^{2n} - 1$$

Code always print Statement (This is true)

prove by M.I for all $n \geq 0$

$4 \mid 5^n - 1$ for $n \geq 0$

Idea

Using M.I.:

① $p(0)$ is true $5^0 - 1 = 1 - 1 = 0$ ✓

② $p(k+1)$

$4 \mid 5^{k+1} - 1$

$5^{k+1} - 1 = 4r$

$= 5\,5^k - 1 = (4+1)5^k - 1$

$= 4\,5^k + \boxed{5^k - 1}$

$4\,5^k + 4s = 4(\underbrace{5^k + s}_{w})$

$4 \quad w$

Define a sequence $a_1, a_2, a_3, \ldots$ as follows

$a_1 = 2$

$a_k = 5a_{k-1}$ for all integer $k \geq 2$

this sequence satisfy the property

$a_n = 2 \cdot 5^{n-1}$ for all $n \geq 1$

prove that using M.I. ?

$a_2 = 5a_1 = 5 \times 2 = 10$

$a_3 = 5a_2 = 5 \times 10 = 50$

$a_4 = 5a_3 = 5 \times 50 = 250$

L = R ✓

$a_n = 2 \cdot 5^{n-1}$ $n \geq 1$

$a_1 = 2 \cdot 5^{1-1} = 2 \cdot 5^0 = 2$

$a_2 = 2 \cdot 5^1 = 10$

$a_3 = 2 \cdot 5^2 = 50$

Inductive: Suppose that $P(k)$ is true for

P base integer $k \geq 1$

$a_k = 5a_{k-1}$.

$a_k = 2 \cdot 5^{k-1}$

next step $k+1$

$$\boxed{a_{k+1} = 2 \cdot 5^k}$$

$$a_{k+1} = 5a_k$$

$$= 5(2 \cdot 5^{k-1})$$

$$= 2 \cdot 5^k$$