Ex: [M.I] Prove that $x^n - y^n$ is divisable by $(x-y)$ for all integers $n \geq 0$, where $x \neq y$.

Let property $P(n)$ is defined on all integers $n \geq 0$, such that $(x-y) \mid x^n - y^n$ where $x \neq y$.

Step 1 (Basis): Show that $P(0)$ is true. i.e. Show $P(0)$ is true.

$$(x-y) \overset{?}{\mid} x^0 - y^0 \qquad x \neq y$$

$$(x-y) \mid 1 - 1$$

$$(x-y) \mid 0 \quad \checkmark$$

Step 2 (Inductive): Suppose that $P(K)$ is true pbac integer $K \geq 0$, such that

$$P(K) \longrightarrow (x-y) \mid (x^K - y^K), \; K \geq 0, \; x \neq y$$

{Inductive hypothesis}

Show that $P(K+1)$ is also true.

We have to show $\left[(x-y) \mid x^{K+1} - y^{K+1} \right]$

$\Rightarrow$ Using QRT

$$\left[\begin{array}{l} x^K - y^K = (x-y)q \\ x^{K+1} - y^{K+1} = (x-y)r \end{array} \right.$$

H.I

q, r is integer

[illegible] M.I [illegible]

Omar Oda

$$x^{k+1} - y^{k+1} = (\cdots\cdots) \cdots = (x-y)\,r$$

$$\hookrightarrow x^{k+1} - y^{k+1} = x x^k - y y^k$$

$$= \underbrace{x x^k} + \underbrace{x^k y - y y^k} - \underbrace{x^k y}$$

$$= [x x^k - x^k y] + [x^k y - y y^k]$$

$$= x^k[x-y] + y[\underbrace{x^k - y^k}]$$

Using H.I

$$= (x-y)\,x^k + y[(x-y)q]$$

$$= (x-y)\underbrace{[x^k + yq]}$$

$t \Rightarrow$ Integer

$$= (x-y)\,t$$

$\therefore$ ##

Q2: Using M.I prove that

$5 \mid 7^n - 2^n$ for all integer $n \geq 0$

$x = 7, \; y = 2$

$4 \mid 9^n - 5^n$ Digits

Q3: Prove that (M.I) $n^3 > 2n+1$, $n \geq 2$

Let property $P(n)$ is defined on all integers $n \geq 2$ such that

$$\boxed{n^3 > 2n+1 \leftarrow P(n)}$$

Step 1 (Basis): Show that $P(a)$ is true.
show that $P(2)$ is true

$2^3 \overset{?}{>} 2(2)+1 \Rightarrow 8 \overset{?}{>} 5$ ✓ true

Step 2 (Inductive): Suppose that $P(k)$ is true for some integer $k \geq 2$, such that

$k^3 > 2k+1$ ← $P(k)$ Inductive hypothesis

Show that $P(k+1)$ also true.

we have to show $\left[(k+1)^3 > \underbrace{2(k+1)+1} \right]$

Proof: $(k+1)^3 = k^3 + 3k^2 + 3k + 1 = (k+1)(k^2+2k+1)$

$> (2k+1) + 3k^2 + 3k + 1$ by substitute of M.I.H

$> 3k^2 + 5k + 2$ $\quad 2k+3$

$= 3k^2 + 3k + 2k + 2$

$= 3k^2 + 3k + 2(k+1) + 1 - 1$

$= (3k^2 + 3k - 1) + 2(k+1) + 1$

$> 2(k+1)+1$ ✓ for all $k \geq 2$

$3k^2 + 5k + 2$

$> 2k+3$ for all $k \geq 2$

✓ test

Prove (using M.I) that $n! > n^2$ for all $n \geq 4$

Let property $P(n)$ is defined on all integers $n \geq 4$

such that $\boxed{n! > n^2 \leftarrow P(n)}$

Step 1 (Basis): Show that $P(a)$ is true

Show $P(4)$ is true.

$$4! \overset{?}{>} 4^2 \Rightarrow 24 > 16 \checkmark$$

Step 2: (Inductive): Suppose that $P(k)$ is true for integer $k \geq 4$. Such that

$$k! > k^2 \longleftarrow P(k)$$

Inductive hypothesis

Show that $P(k+1)$ also true.

we have to show $(k+1)! > (k+1)^2$

Proof:

$$\begin{aligned}(k+1)! &= (k+1)\,k! \\ &> (k+1)\,k^2 \\ &> (k+1)(k+1) \\ &= (k+1)^2\end{aligned}$$

Since, $k^2 > (k+1)$ for all $k \geq 4$

$\therefore (k+1)! > (k+1)^2$ ⌘

Using M.I prove that $n^3 - n$ is divisable by 6. for all integer $n \geq 0$

Let property $P(n)$ is defined on all integers $n \geq 0$

Such that $\boxed{6 \mid n^3 - n \longleftarrow P(n)}$

OR (QRT) $n^3 - n = 6r$ for r is integer

Step1 (Basis) : Show that $P(0)$ is true. Show that $P(0)$ is true.

$6 \overset{?}{\mid} (0)^3 - (0) \Rightarrow 6 \overset{?}{\mid} 0$ ✓

Step 2 (Inductive): Suppose that $P(k)$ is true for an integer $k \geq 0$, such that

$6 \mid k^3 - k$ ← $P(k)$ Inductive hypothesis

QRT $\boxed{k^3 - k = 6q}$

Show that $P(k+1)$ is also true.

we have to show $\left[\, 6 \mid (k+1)^3 - (k+1) \,\right]$

$\Leftrightarrow$ QRT $(k+1)^3 - (k+1) = 6s$

$$(K+1)^3 - (K+1) = (K+1)\left[(K+1)^2 - 1\right]$$

$$= (K+1)\left[K^2 + 2K + \cancel{1} - \cancel{1}\right]$$

$$= K(K+1)(K+2)$$

$$= K(K^2 + 3K + 2)$$

$$= K^3 + 3K^2 + 2K$$

$$= \left[K^3 - K\right] + \left[3K^2 + 2K\right] + K$$

$$= 6q + \left[3K^2 + 3K\right]$$

$$= 6q + 3\left[K^2 + K\right]$$

$$= 6q + 3\left[K(K+1)\right]$$

$$= 6q + 3(2t)$$

$$= 6\underbrace{\left[q + t\right]}_{}$$

$$6r$$

QRT

$$\boxed{K^3 - K = 6q}$$

Substitute from H.I

even x odd = even

$2t$