Chapter 6

1) → prove by Element argument

2) → prove by Venn digram and Counter Example

3) → prove by algerba.

Element Argument

Ex: prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

[distributive].

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$\Rightarrow$ Two parts

show $\Rightarrow$ $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

and show $\Rightarrow$ $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

$\Rightarrow$ Starting Proof:

prove that: $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

Suppose that A, B, C are pbac sets.

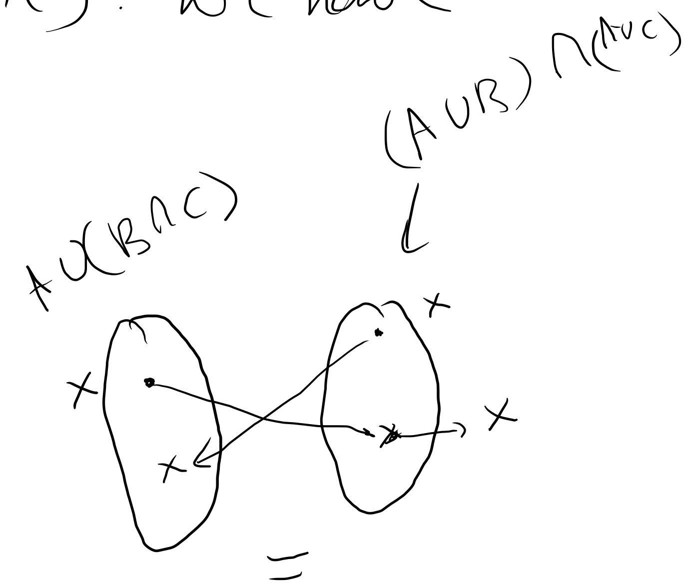
such that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

Suppose that x is pbac element of set $A \cup (B \cap C)$. We have

Final

to show that $x \in (A \cup B) \cap (A \cup C)$.

Since $x \in A \cup (B \cap C)$. (Hypothesis)



$\Rightarrow$ Cases:

Case 1: $x \in A$

Since $x \in A$, this mean $x \in A \cup B$ by definition of Union

and $x \in (A \cup C)$ by definition of Union

Hence, $x \in (A \cup B) \cap (A \cup C)$ by def. of $\cap$

intersection

Case 2: $x \in (B \cap C)$

Since $x \in (B \cap C)$ this mean

$x \in B$ and $x \in C$ by def. of $\cap$

Since $x \in B$ then $x \in (A \cup B)$ by def. on Union

Since $x \in C$ then $x \in (A \cup C)$ by def. of Union

Since $x \in A \cup B$ and $x \in (A \cup C)$

Hence $x \in (A \cup B) \cap (A \cup C)$ by def of intersection

Part II

Prove that $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

Suppose that x is pbac element

Case 1: $x \in A$ (hypothesis) $\Rightarrow$ Since $x \in A$, then $x \in A \cup (B \cap C)$ by defintion of Union.

Case 2: $x \notin A$

Since $x \notin A$, this mean $x \in B$ and $x \in C$

Since $x \in B$ and $x \in C \Rightarrow x \in B \cap C$ (by definition of $\cap$)

Hence, $x \in A \cup (B \cap C)$

Since Both Cases (1, 2), So $x \in A \cup (B \cap C)$

from Both Sides [Since we prove that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ and we prove $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$]

$\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$]

prove that $(A \cup B)^c = A^c \cap B^c$

$\Rightarrow$ Let A, B are two sets, phac sets

we have to show that $(A \cup B)^c \subseteq A^c \cap B^c$

and $A^c \cap B^c \subseteq (A \cup B)^c$

Part 1: We have to prove $(A \cup B)^c \subseteq A^c \cap B^c$

Proof: Suppose that x is phac element in set $(A \cup B)^c$. we have to prove that

$x \in A^c \cap B^c$.

Outline

$x \in (A \cup B)^c$

⋮

prove $x \in A^c \cap B^c$

Suppose $x \in (A \cup B)^c$,

$x \notin (A \cup B)$ Complement

$x \notin (A \text{ or } B)$

$\Leftrightarrow$ $x \notin A$ and $x \notin B$ by Demorgan law

$x \in A^c$ and $x \in B^c$ Complement

$x \in (A^c \cap B^c)$

$A^C \cap B^C \subseteq (A \cup B)^C$

Suppose that x is pbac element $x \in (A^C \cap B^C)$

Show that $x \in (A \cup B)^C$.

$A^C \cap B^C \Rightarrow x \notin A$ and $x \notin B$

$x \notin A \cup B$ by Demorgan law

$\therefore x \in (A \cup B)^C$ by Definition of Complement

Prove that for all sets A, B, C if

$A \subseteq B$, $B \subseteq C^c$, then $A \cap C = \emptyset$.

Suppose that x is a particular element such that

$x \in A$, then $x \in B$ by definition of subset.

Since $x \in B$, then $x \in C^c$

Since $x \in C^c$, then $x \notin C$ by definition of complement

$\Rightarrow$ Since $x \in A$ and $x \notin C$

$\therefore A \cap C = \emptyset$ ✓

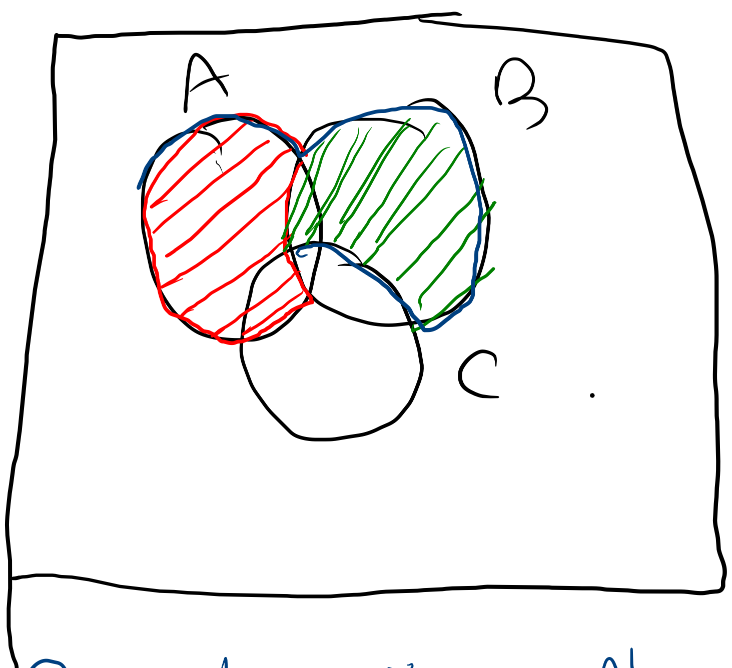
Proof by: Venn Diagrams:

prove that $(A-B) \cup (B-C) \neq A-C$

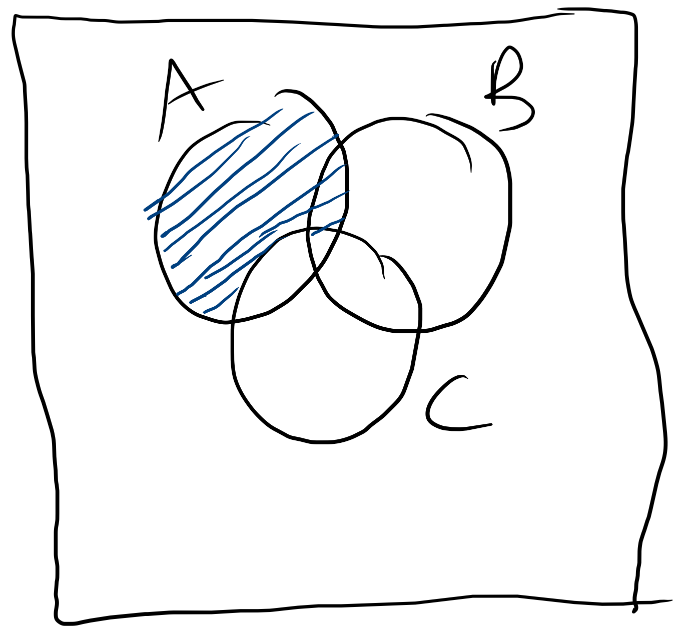
$A-B=\{1,4\}$

$B-C=\{2,3\}$

$A-C=\{1,2\}$



$\neq$



Counter Example

$A=\{1,2,4,5\}$

$B=\{2,3,5,6\}$

$C=\{4,5,6,7\}$

Algebra:

Prove that $A - (A \cap B) = A - B$

$A - (A \cap B) = A \cap (A \cap B)^c \longrightarrow$ By difference law

$= A \cap (A^c \cup B^c) \longrightarrow$ By demorgan law

$= (A \cap A^c) \cup (A \cap B^c) \longrightarrow$ By distributive law

$= \phi \cup (A \cap B^c) \longrightarrow$ By Complement law

$= (A \cap B^c) \longrightarrow$ By identity law of union.

$= A - B \longrightarrow$ By diffenence law

$(A-B)\cup(B-A)=(A\cup B)-(A\cap B)$

$=(A\cap B^c)\cup(B\cap A^c)$ By difference law.

$=[[(A\cap B^c)\cup B]\cap[(A\cap B^c)\cup A^c]]$ By distributive law

$=[B\cup(A\cap B^c)]\cap[A^c\cup(A\cap B^c)]$ By commutative law.

$=[(B\cup A)\cap(B\cup B^c)]\cap[(A^c\cup A)\cap(A^c\cup B^c)]$ By distributive law

$=[(B\cup A)\cap U]\cap[U\cap(A^c\cup B^c)]$ By d.f. Complem

$=(B\cup A)\cap(A^c\cup B^c)$ by identity law

$=(A\cup B)\cap(A^c\cup B^c)$ by Commutative law

$=(A\cup B)\cap(A\cap B)^c$ by Demorgan

$=(A\cup B)-(A\cap B)$ by difference law

$\#$

prove by Element argument // algebra

1) $(A-B) \cup (C-B) = (A \cup C) - B$

(2) algebra + Venn Diagram

$A \cap ((B \cup A^c) \cap B^c) = \emptyset$