Prove that

pbuc

For all $a, x \in B$ (Boolean set)

$a + x = 1$, $a \cdot x = 0$ then $x = \bar{a}$

$X = X \cdot 1$ because 1 is identity for .

$= X \cdot (a + \bar{a})$ Complement for +

$= X \cdot a + X \cdot \bar{a}$ distributive law for .

$= 0 + x \cdot \bar{a}$ hypothesis

$= a \cdot \bar{a} + x \cdot \bar{a}$ Complement law

$= \bar{a} \cdot a + \bar{a} \cdot x$ Commutative law

$= \bar{a} \cdot (a + x)$ distributive law

$= \bar{a} \cdot 1$ hypothesis

$= \bar{a}$ identity

Set

$1 \rightarrow U$

$A \cup A^c = U$

$\boxed{a + \bar{a} = 1}$

Prove that for all x, y, z in B where B is Boolean Set. If $x + y = x + z$ and $x \cdot y = x \cdot z$ then $y = z$.

$$
\begin{aligned}
y &= (y + x) \cdot y && \text{Absorption law} \\
&= y \cdot (y + x) && \text{Commutive law} \\
&= y \cdot (x + y) && = \\
&= y \cdot (x + z) && \text{hypothesis} \\
&= y \cdot x + y \cdot z && \text{distributive} \\
&= x \cdot y + y \cdot z && \text{Commutative} \\
&= x \cdot z + y \cdot z && \text{hypothesis} \\
&= z \cdot x + z \cdot y \\
&= z \cdot (x + y) \\
&= z \cdot (x + z) && \text{hypothesis} \\
&= z && \text{absorption law}
\end{aligned}
$$

For all sets A, B, C prove that

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

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we have to prove that

$$A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$$

and $$(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$$

Ex:

$A = \{x, y\}$

$B = \{1, 2\}$

$A \times B = \{(x,1), (x,2), (y,1), (y,2)\}$

Part I) prove: $A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$

Suppose that $(x,y) \in A \times (B \cup C)$, when x, y are pbac- element. Show that $(x,y) \in (A \times B) \cup (A \times C)$.

Since $(x,y) \in A \times (B \cup C)$ then $x \in A$ and $y \in (B \cup C)$

Case 1: $y \in B$. Since $x \in A$ and $y \in B$ by deffin of Cat. pr then $(x,y) \in (A \times B)$; Hence $(x,y) \in (A \times B) \cup (A \times C)$ by deffen of Union.

Case 2: $y \in C$ Since $x \in A$ and $y \in C$ by diff of Cat. p then $(x,y) \in (A \times C)$. Hence $(x,y) \in (A \times B) \cup (A \times C)$ by definition of Union.

Part-II) $(A\times B)\cup(A\times C) \subseteq A\times(B\cup C)$

Suppose that x, y are [illegible] element in set $(A\times B)\cup(A\times C)$. Show that $(x,y) \in A\times(B\cup C)$.

$\Rightarrow$ Since $(x,y) \in (A\times B)\cup(A\times C)$

$(x,y)\in(A\times B)$ or $(x,y)\in(A\times C)$

Case 1: $(x,y)\in(A\times B)$, $x\in A$ and $y\in B$ $\rightarrow$ Cart. product

Since $y\in B$, then $y\in(B\cup C)$. So $(x,y) \in A\times(B\cup C)$

Case 2: $(x,y)\in(A\times C)$. $x\in A$ and $y\in C$ $\Rightarrow$ Cart. p

Since $y\in C$, then $y\in(B\cup C)$ by def. of Union

So $(x,y)\in A\times(B\cup C)$

Case 1 + Case 2 (approved)

Hence $(x,y)\in A\times(B\cup C)$

3 Questions

Prove

$$(A - B) \cup (C - B) = (A \cup C) - B$$

(1) Algebra

(2) Element Argument

$= (A \cap B^c) \cup (C \cap B^c)$ by difference law

$= (B^c \cap A) \cup (B^c \cap C)$ Commutative law

$= B^c \cap (A \cup C)$ distributive law

$= (A \cup C) \cap B^c$ Commutative law

$= (A \cup C) - B$ difference law