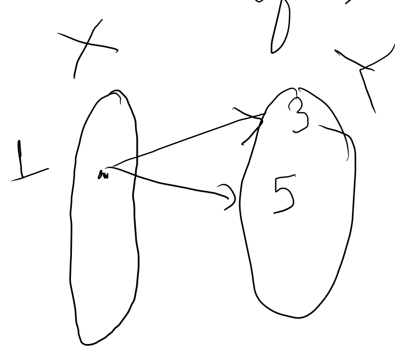
Chapter 7
functions

→ **well define function:**

Ex: $f: R \to R$

$f(x) \Rightarrow (x^2 + y^2) = 1$ Is this well define function?

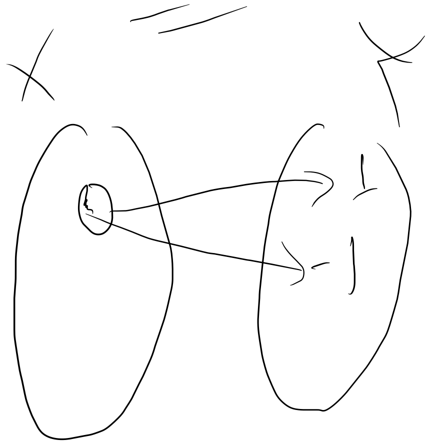


$x^2 + y^2 = 1$

Ex: let $x = 0 \Rightarrow y \Rightarrow 0^2 + y^2 = 1$

$y^2 = 1$

$\boxed{y = \mp 1}$



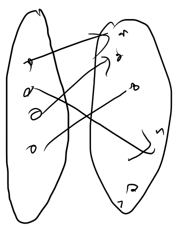
NOT well define.

Ex: $f : \mathbb{Q} \to \mathbb{Z}$, $f\left(\frac{m}{n}\right) = m$ for all integers m, n . $n \neq 0$

WDF ??

① Every $x \to y$

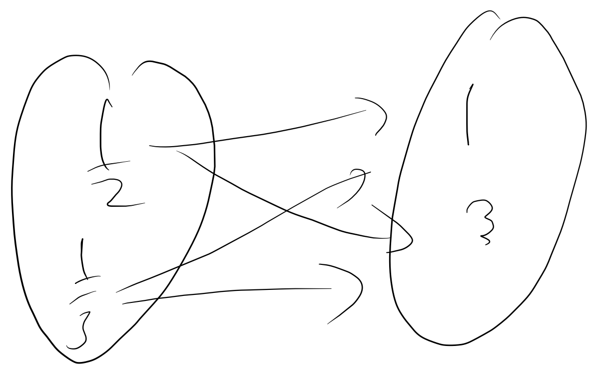
② –



Ex: $f\left(\frac{1}{2}\right) = 1$, $f\left(\frac{3}{6}\right) = 3$

$\frac{1}{2}$

counter



NWDF

$f\left(\frac{1}{3}\right) = 1$

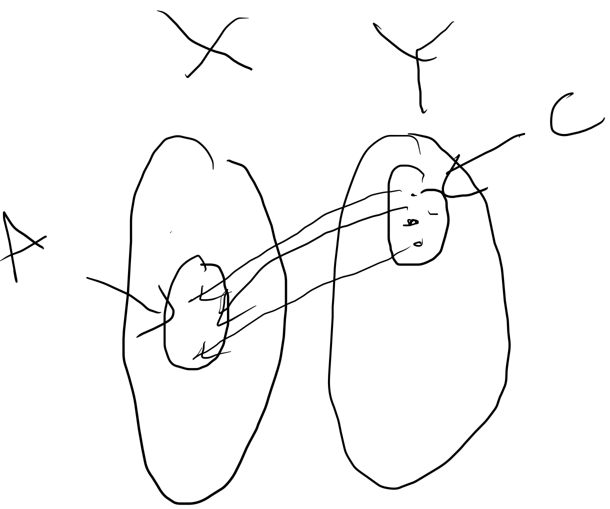
$f\left(\frac{3}{9}\right) = 3$

$\frac{3}{9} \to \frac{1}{3}$

Multiple choice

Σ?

Definition: If $f: X \to Y$ is a function



$A \subseteq X$ and $c \subseteq Y$, then

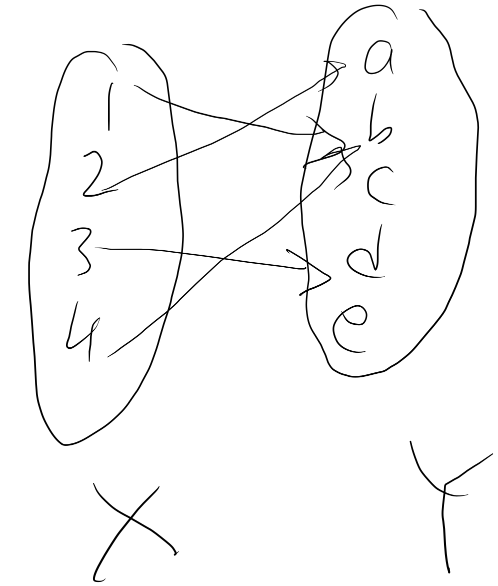
$$f(A) = \{ y \in Y \mid y = f(x) \text{ for some } x \text{ in } A \}$$

$$f^{-1}(C) = \{ x \in X \mid f(x) \in C \}$$

and $f(A)$ is called the image of A and $f^{-1}(C)$ is called the inverse of image C

Ex: $X = \{1, 2, 3, 4\}$, $Y = \{a, b, c, d, e\}$

$F : X \to Y$ by the following figure



$A = \{1, 4\}$, $C = \{a, b\}$

$D = \{c, e\}$

Find $F(A) = \{b\}$

$F(X) = \{a, b, d\}$

$F^{-1}(C) = \{1, 2, 4\}$

$F^{-1}(D) = \phi$

* **One-One function: (injective)**

(1-1 function): Every Element in domain has one image in codomain

Def: Let $F: X \to Y$ F is 1-1 (injective) iff $\forall_{x_1 x_2} \in X$

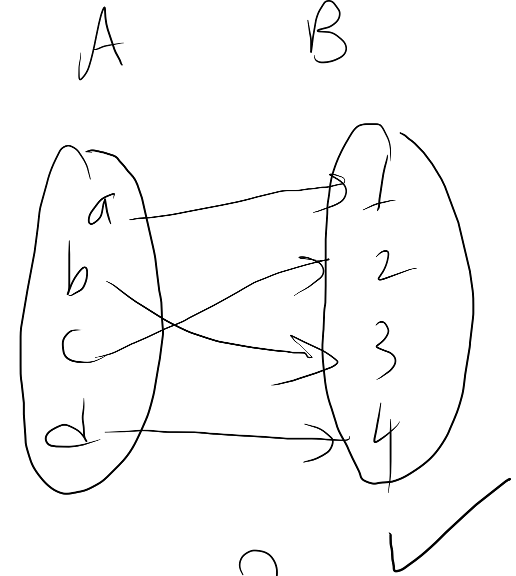
if $F(x_1) = F(x_2)$, then $x_1 = x_2$

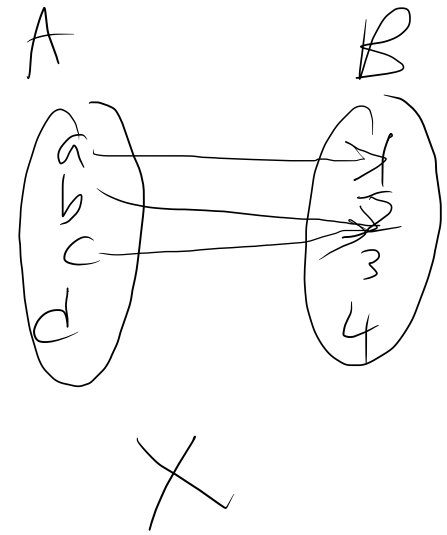
if $x_1 = x_2$ then $F(x_1) \neq F(x_2)$

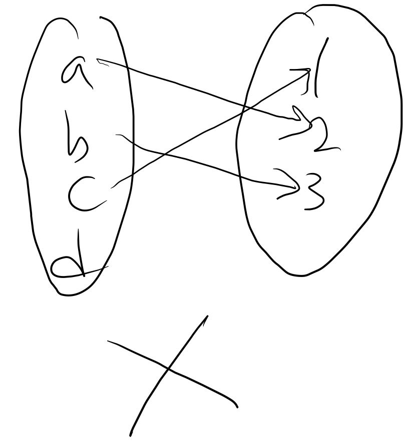
$F: X \to Y$ is 1-1 $\forall_{x_1 x_2} \in X$ iff $F(x_1) = F(x_2)$ then $x_1 = x_2$

Negation $F: X \to Y$ isn't 1-1 $\exists_{x_1 x_2} \in X$ $F(x_1) = F(x_2)$ and $x_1 \neq x_2$

Ex:







One-One?

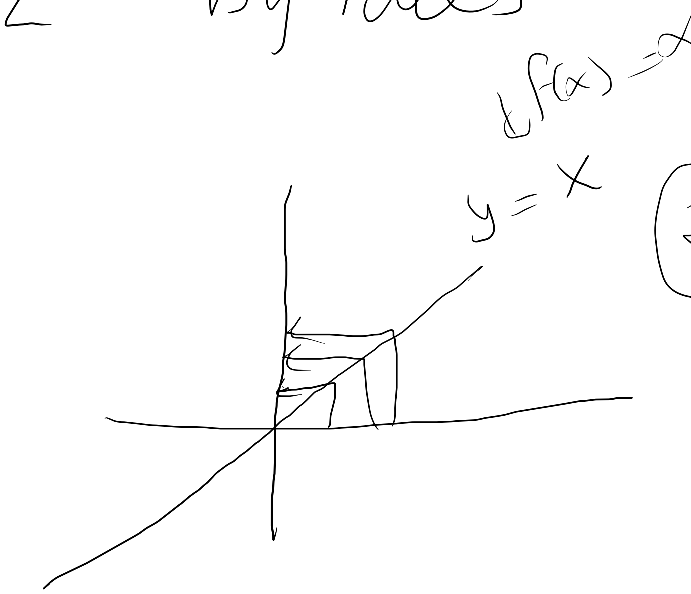
Ex: $f : \underset{Z}{R} \to \underset{Z}{R}$ and $g : Z \to Z$ by rules

$f(x) = 4x - 1$ all $x \in R$

$g(n) = n^2$ for all $n \in Z$

Is ~~F~~ one-one?

Is g one-One?



$R \to R$

$f(x) \Rightarrow$ One-One ✓

$\forall x_1, x_2 \in X$, $F(x_1) = F(x_2)$, then $x_1 = x_2$

x_1

$\therefore \quad f(x_1) = 4x_1 - 1 \quad$ and $\quad f(x_2) = 4x_2 - 1$

$f(x_1) = f(x_2) \xrightarrow{\text{It must}} x_1 = x_2$ One-One

$4x_1 - \cancel{1} = 4x_2 - \cancel{1} \rightarrow \frac{\cancel{4}x_1}{\cancel{4}} = \frac{\cancel{4}x_2}{\cancel{4}} \rightarrow \boxed{x_1 = x_2}$

double　　　　double　　　&

Ex:

```
int func(int num)
{
  return 4*(num-1); }
```

Yes?

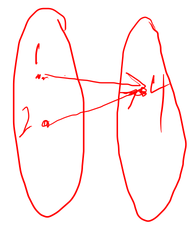
$func : \mathbb{Z} \longrightarrow \mathbb{Z}$

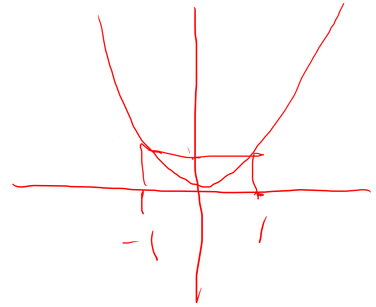
$f(n) = 4(n-1)$

Code

$g: \mathbb{Z} \to \mathbb{Z}$ on

$g(n) = n^2$





$\forall n_1, n_2 \in \mathbb{Z}$ if $g(n_1) = g(n_2)$, then $n_1 = n_2$

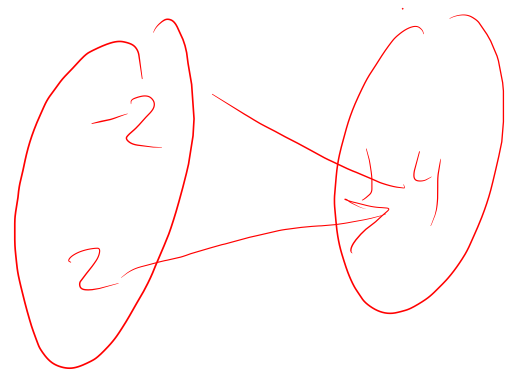
$\Leftarrow \quad g(n_1) = n_1^2 \quad , \quad g(n_2) = n_2^2$

$g(n_1) = g(n_2) \Rightarrow n_1^2 = n_2^2 \Rightarrow n_1 = \mp n_2$

$\boxed{\therefore n_1 \neq n_2}$

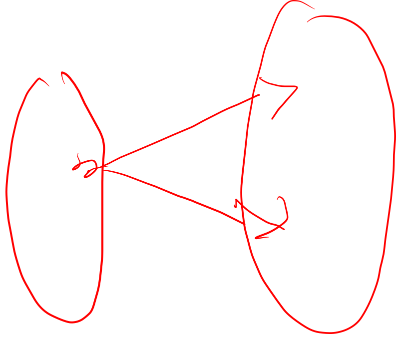
Example: $n_1 = -2$, $n_2 = 2$

$f(-2) = f(2) = 4$



But $n_1 \neq n_2$

$-2 \neq 2$



Define g : Mobile# $\rightarrow$ People

$g(x) =$ Person $\quad x \in$ Mobile#

Is this function One-One?

f : Fingerprint $\rightarrow$ People

$f(x) =$ Person $\quad x \in$ Fingerprint

Is this function One-One?

Application of One-One:

Hash function (Data Structure)

$\Rightarrow H(x) = x \bmod \#$

Ex: $H(x) = x \bmod 7$ — table size

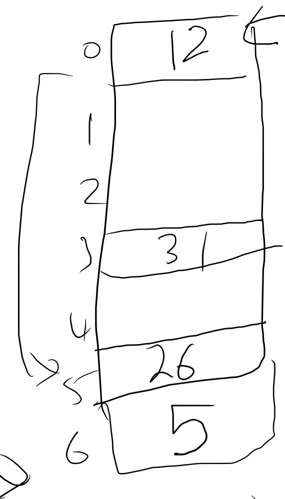
$4/7$

$H(26) = 26 \% 7 = 5$

$H(31) = 31 \% 7 = 3$

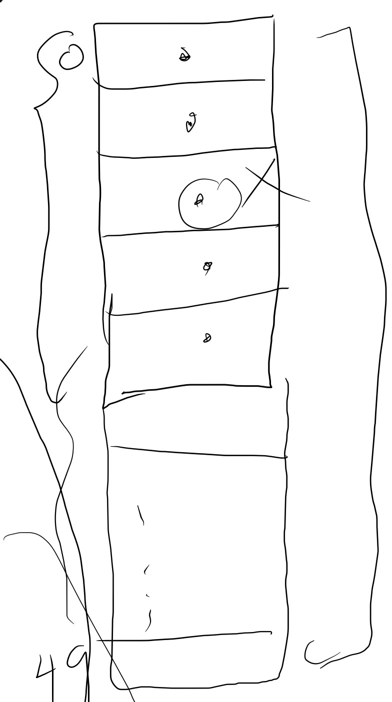
$H(5) = 5 \% 7 = 5$ — collision

$H(12) = 12 \% 7 = 5$

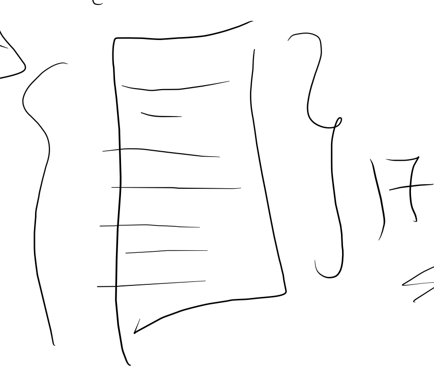


Array

int Array[100];



$7 \times 2 + 17$



Employee

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