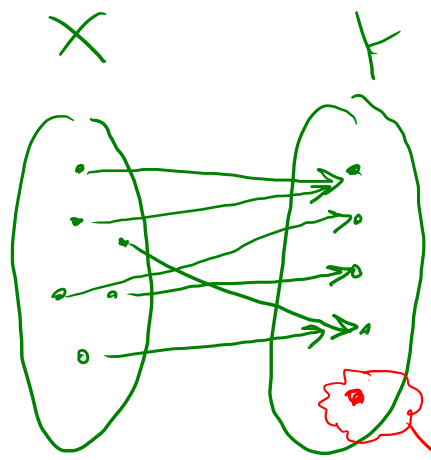
Onto: Every element in codomain has at least one image in domain. (Surjective)

Def: $F: X \to Y$ is onto $\iff \forall y \in Y, \exists x \in X$ such that $F(x) = y$

$F: X \to Y$ isn't onto $\iff \exists y \in Y$ such that $\forall x \in X, F(x) \neq y$



Function (✓) Test

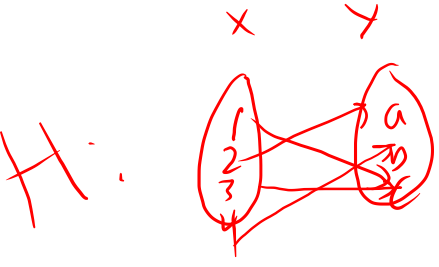
Onto (✓)

If it's Exist, then $\Rightarrow$ this function isn't onto.

Ex:

Let $X = \{1, 2, 3, 4\}$, $Y = \{a, b, c\}$

Define $H : X \to Y$ as follows

H: 

$H(1) = c$, $H(2) = a$, $H(3) = c$, $H(4) = b$

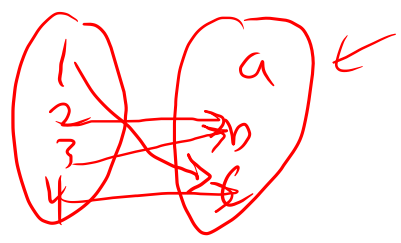
$K : X \to Y$ as follows

$K(1) = c$, $K(2) = b$, $K(3) = b$, $K(4) = c$

Is it onto

H? Test: function (Y), Yes it onto

K? Test: function (N), - {a}, NOT onto.

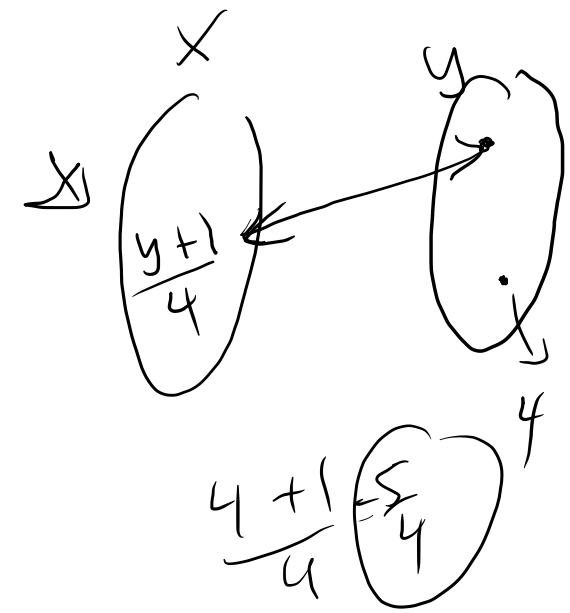
K: 

Ex: $f: \mathbb{R} \to \mathbb{R}$, $h: \mathbb{Z} \to \mathbb{Z}$ by rules.

$$f(x) = 4x - 1 \quad , \forall x \in \mathbb{R}$$

$$h(n) = 4n - 1 \quad , \forall n \in \mathbb{Z}$$

Is f onto? . Is h onto?



$f(x) = y \Rightarrow 4x - 1 = y$

$\therefore 4x = y + 1$

$\hookrightarrow x = \frac{y+1}{4} \in \mathbb{R}$ ✓ onto

$\Rightarrow h(n) = y \Rightarrow 4n - 1 = y \Rightarrow n = \frac{y+1}{4} \notin \mathbb{Z}$ not onto

law exponents

$$b^u * b^v = b^{u+v}$$

$$(b^u)^v = b^{vu}$$

$$\frac{b^u}{b^v} = (b)^{u-v}$$

$$(bc)^u = b^u c^u$$

log

$$\log(xy) = \log x + \log y$$

$$\log\left(\frac{x}{y}\right) = \log x - \log y$$

$$\log_b x^a = a \log_b x$$

$$\log_c x = \frac{\log_b x}{\log_b c}$$

~ one-to-one correspondence (bijective)

$F: X \to Y$ that is both one-to-one and onto.

Ex:

Let $A = \{a, b\}$ $\quad S = \{00, 01, 10, 11\}$

Define function $P(A) \to S$

domain → codomain

$P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

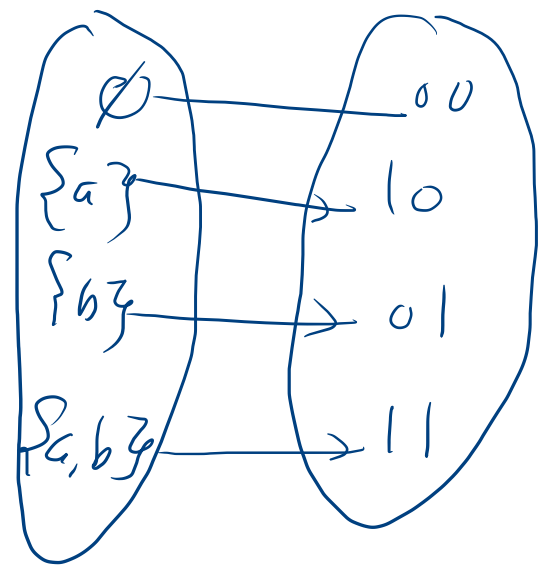
a is in A or not in A

b is in A or not in A

String

as following

	status of a	Status of b	
ϕ	not in	not in	00
$\{a\}$	in	not in	10
$\{b\}$	not in	in	01
$\{a, b\}$	in	in	11



Is this function one-one? ✓

Is this onto? ✓

bijective, 1-1 correspondance

Ex. Define $F: R \times R \to R \times R$, $(x, y) \in R \times R$

$F(x, y) = (x+y, x-y)$ Is this function one-to-one correspondance ? (bijective)

$\Rightarrow$ (one-to-one, onto) // prove or disprove

$F(x) = G(x)$ if $F(x_1) = F(x_2)$, then $x_1 = x_2$

if $F(x_1, y_1) = F(x_2, y_2)$, than $x_1 = x_2$, and $y_1 = y_2$

$\Leftrightarrow$ $F(x_1, y_1) = (x_1+y_1, x_1-y_1)$, $F(x_2, y_2) = (x_2+y_2, x_2-y_2)$

$\therefore$ $(x_1+y_1, x_1-y_1) = (x_2+y_2, x_2-y_2)$

$\therefore$ $x_1+y_1 = x_2+y_2$

$x_1-y_1 = x_2-y_2$ +

$2x_1 = 2x_2 \Rightarrow x_1 = x_2$

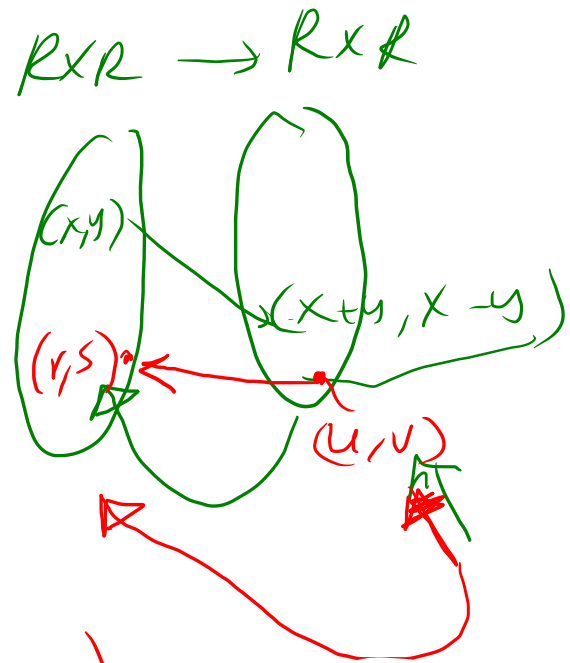
$x_1+y_1 = x_1+y_2$

$y_1 = y_2$

$\therefore$ one-to-one

Onto : $f(x) = y$.

$F(x,y) = (x+y, x-y)$



$F(r,s) = (u,v)$

$F(r,s) = (r+s, r-s)$

$u = r+s$, $v = r-s$

(Addition) $(5,-1) \leftarrow (4,6)$

$u = r+s$ +
$v = r+s$

$u+v = 2r \Rightarrow r = \frac{u+v}{2}$ ... (1)

$v = u+v - s \Rightarrow s = \frac{u+v}{2} - u$

$\frac{u+v-2u}{2} = \frac{v-u}{2}$

$\therefore F\left(\frac{u+v}{2}, \frac{v-u}{2}\right) = (u,v)$

Why $RXR \rightarrow RXR$?

Onto (✓)

Inverse Theorm: $F: X \to Y$ is one-to One

Correspondance $\Rightarrow$ that is (1-1 & onto)

then there is a function

$F^{-1}: Y \to X$ is defined as follows

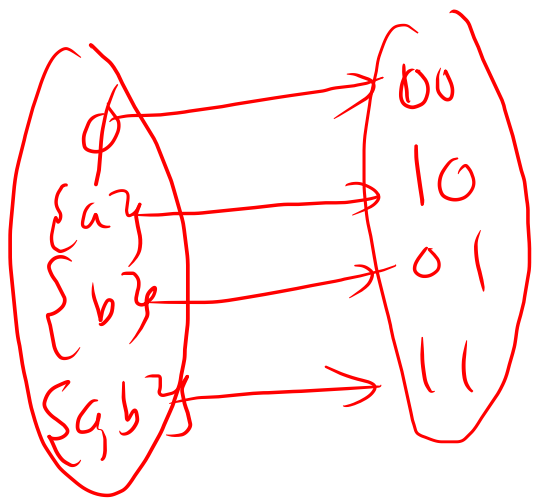
Given the element y in Y:

$F^{-1}(y) =$ the unique element x in X such that $F(x)$ equals y.

$$F^{-1}(y) = x \iff y = F(x)$$

h

Ex.



$h^{-1}(00) = \phi$

$h^{-1}(10) = \{a\}$

$h^{-1}(01) = \{b\}$

$h^{-1}(11) = \{a,b\}$

One-One, onto

$f : R \to R \quad f(x) = 4x - 1$

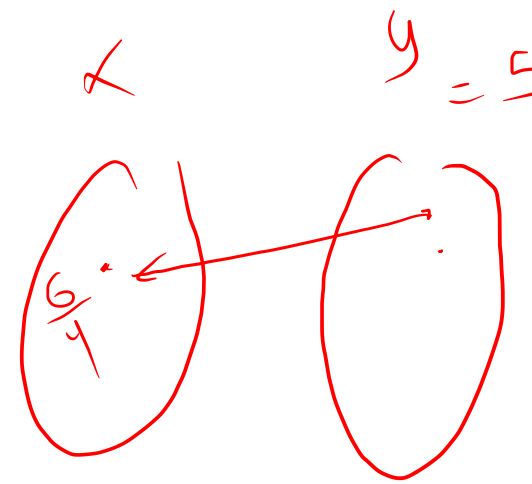
f^{-1} ?

$f(x) = y \Rightarrow 4x - 1 = y$

$x = \frac{y+1}{4}$

$f^{-1}(y) = x$ fun Def

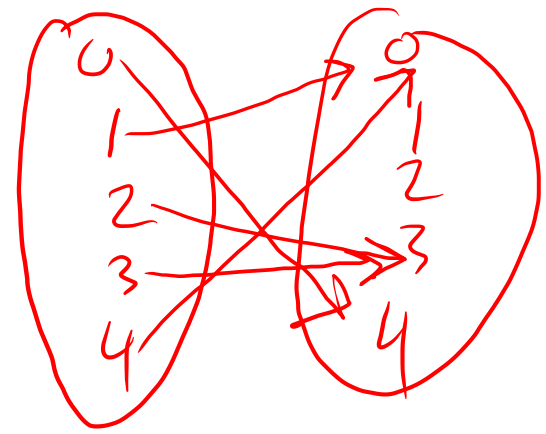
$$\boxed{f^{-1}(y) = \frac{y+1}{4}}$$



$D \rightarrow D$

$D = \{0, 1, 2, 3, 4\}$

$f(x) = (x^2 + 4) \% 5$



Not One-to One

NOT one

Ex:

```
int main(){
    int n=5;
    int A[n]={0,1,2,3,4};
    int B[n];
    fun(A,B,n);
    return 0;}

void fun(int A[],int B[], int n){
int i=0;
    for(i=0; i<n; i++)
        B[i]=(A[i]*A[i]+4)%5;
}
```

Is this function Bijective?

Ex:

```
double fact(double x) {
    if (x != 1)
    return (x+1)/(x-1); }
```

$f : R \to R$

$$f(x) = \frac{x+1}{x-1} \qquad x \neq 1$$

One-to-One? ✓

Onto? ✗