Ch. 7

Code

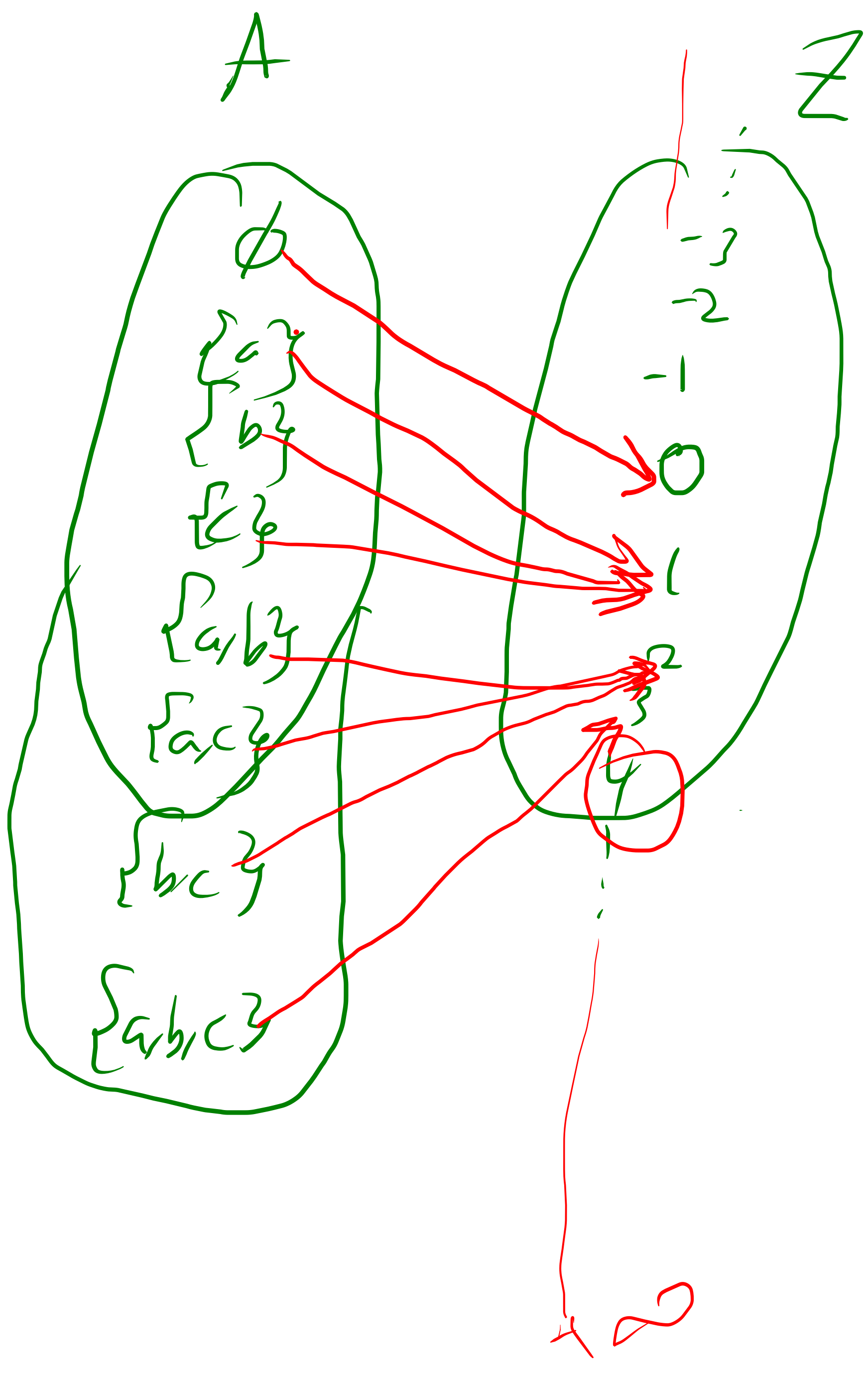
```
double fact(double x){
    if(x != 1)
    return (x+1)/(x-1);}
```

Show that if this function is bijective or not?

Question $F: P(\{a,b,c\}) \to Z$ for all A in Powerset

$F(A) = $ # of elements in A.

$$P(\{a,b,c\}) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$$



One-one? NOT $F(\{a\}) = F(\{b\}) = 1$

Onto? NOT

-1

$f(?) = \phi$

-1

$f(4) = \phi$

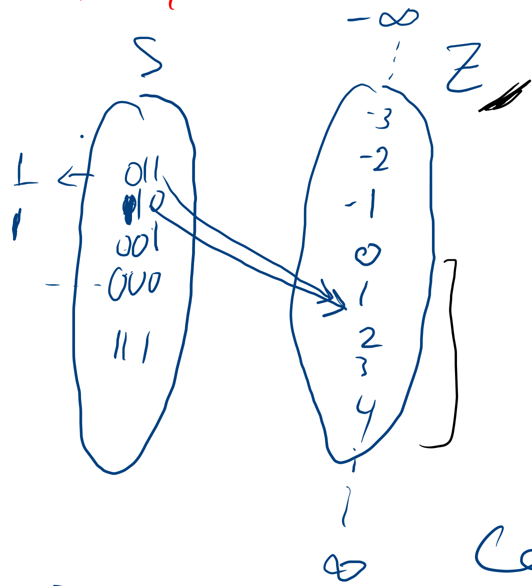
Question: S is set of all string of 0's and 1's and defined as follows

$$D : S \to \mathbb{Z}$$

$$D(s) = \#\text{ of } 1\text{'s} - \#\text{ of } 0\text{'s}$$

One-One ?

onto ?

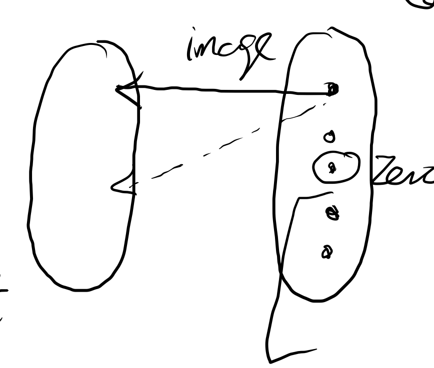


$\underbrace{\#\text{ of } 1\text{'s} - \#\text{ of } 0\text{'s}}$

Counter Example

$$\left.\begin{aligned} D(011) &= 1 \\ D(110) &= 1 \end{aligned}\right\} \text{ NOT One-to-One}$$

Codomain



Onto?

$D(1100000) = 2 - 5 = -3$

3 Cases $\quad \#1\text{'s} = \#0\text{'s} \Rightarrow \text{Zero} \in \mathbb{Z}$

$x > 0$ positive

$x < 0$ negative

Onto ✓

Ex: $S : Z^+ \to Z^+$

$S(8) = 8+4+2+1$
$= 15$

$S(n)$ = sum of positive divisors

Is this function One-to-One?

$S(1) = 1$

$S(2) = \{2,1\} \Rightarrow 3$

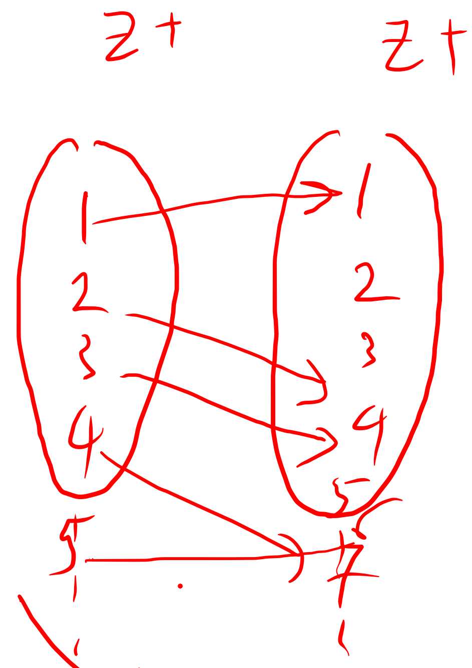
$S(3) = \{1,3\} = 4$

$S(4) = \{1,2,4\} = 7$

$S(5) = \{5,1\} = 6$

$S(6) = \{1,2,3,6\} = 12$

$S(11) = 12$



$S(6) = S(11) = 12$

NOT One-to-One

Is this function Onto?

NOT onto

$S^{-1}(2) = \phi$

$S^{-1}(5) = \phi$

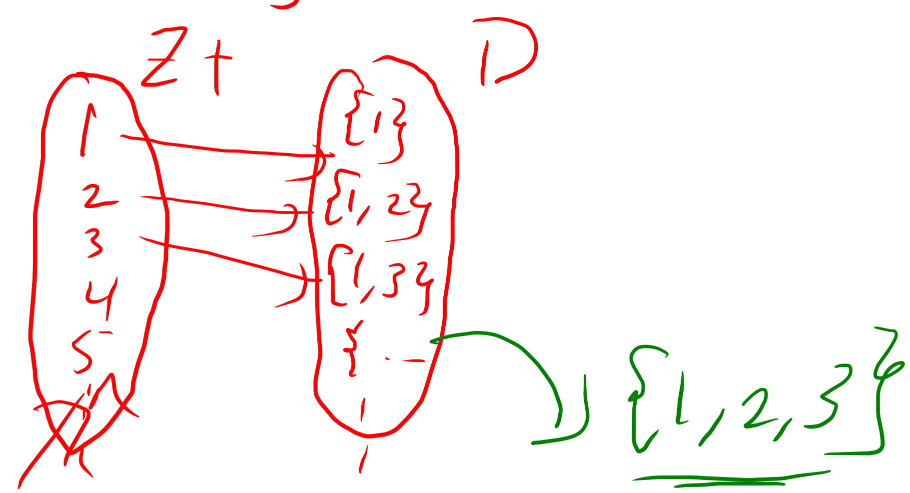
$T: Z^+ \to D$ D is finite subsets of positive integer.

$T(n) =$ set of all positive divisor of n

<u>One to One ?</u>

$T(1)=\{1\}$, $T(2)=\{1,2\}$, $T(3)=\{1,3\}$

$T(4)=\{1,2,4\}$, $T(5)=\{1,5\}$ $T(6)=\{1,2,3,6\}$



prove

if $T(n)=T(m)$, then $m=n$

$T(n)=\{1, \ldots\ldots, x\}$

$T(m)=\{1, \ldots\ldots, x\}$

Since $n=x$, $m=x$ $\therefore$ $\boxed{m=n}$

One to One

NOT Onto $T^{-1}(\{1,2,3\}) = \emptyset$

$J: Q \times Q \to R$

$J(r,s) = r + \sqrt{2}\, s$ for all $(r,s) \in Q \times Q$

Is this function bijective? (injective + surjective)

1-1 + onto

~ (injective: one-to-one):

$F: X \to Y$ $\forall x_1, x_2 \in X$, iff $F(x_1) = F(x_2)$

then $x_1 = x_2$.

$J_1 = J_2$, $J_1(r_1, s_1) = r_1 + \sqrt{2}\, s_1$

$J_2(r_2, s_2) = r_2 + \sqrt{2}\, s_2$

if $J_1 = J_2$, show that $[r_1 = r_2 \;\&\; s_1 = s_2]$ = one-to-one

$J_1 = J_2$

$\Leftrightarrow r_1 + \sqrt{2}\, s_1 = r_2 + \sqrt{2}\, s_2$

$r_1 - r_2 = \sqrt{2}\, s_2 - \sqrt{2}\, s_1$

$r_1 - r_2 = \sqrt{2}\,(s_1 - s_2)$

$\therefore \sqrt{2} = \dfrac{r_1 - r_2}{s_1 - s_2}$ form hypothesis

$(r_1, s_1) \in Q \times Q$

$(r_2, s_2) \in Q \times Q$

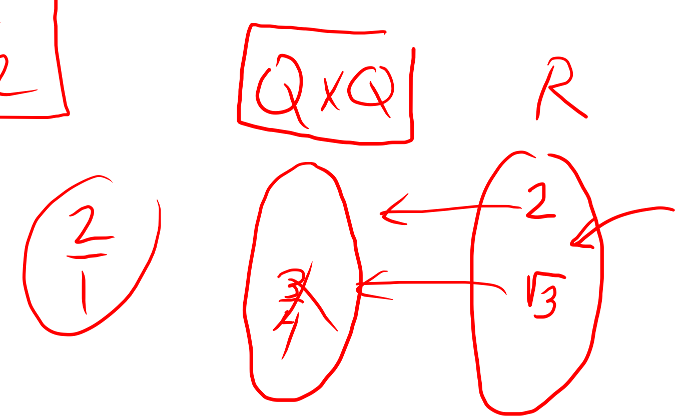
$\Rightarrow$ Rational # (Sub, +, *) closed.

If $s_1 \neq s_2$ Contradiction (Right side $\rightarrow$ ration)

left $\sqrt{2}$ × irrational

$\Rightarrow \therefore \boxed{s_1 = s_2} \Rightarrow \boxed{r_1 = r_2}$

$\therefore$ One-to-one



NOT onto! Counter Example

$\sqrt{3} \in R$ but $\sqrt{3} \notin Q \times Q$